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STOCHASTIC EFFICIENCY MEASUREMENT: THE CURSE OF THEORETICAL CONSISTENCY

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The availability of efficiency estimation software – freely distributed via the internet and relatively easy to use – recently inflated the number of corresponding applications. The resulting efficiency estimates are used without a critical assessment with respect to the literature on theoretical consistency, flexibility and the choice of the appropriate functional form. The robustness of policy suggestions based on inferences from efficiency measures nevertheless crucially depends on theoretically well-founded estimates. This paper adresses stochastic efficiency measurement by critically reviewing the theoretical consistency of recently published technical efficiency estimates. The results confirm the need for a posteriori checking the regularity of the estimated frontier by the researcher and, if necessary, the a priori imposition of the theoretical requirements.

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I. Introduction

In the last 15 years applied production economics experienced a clear shift in its research focus towards the technical and allocative efficiency of decision making

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units. Parametric techniques as the stochastic production frontier model dominate the empirical literature of efficiency measurement (for a detailed review of different measurement techniques see e.g. Coelli et al. 1998 or Kumbhakar and Lovell 2000). The availability of estimation software – freely distributed via the internet and relatively easy to use – recently inflated the number of corresponding applications.[1] The results of the application of the econometric methods provided by these black box-tools are mostly not accompanied by a critical assessment with respect to the literature on theoretical consistency, flexibility and the choice of the appropriate functional form, running the risk of making improper policy recommendations.

This paper shows the importance of testing for the regularities of an estimated efficiency frontier based on flexible functional forms. The basic results of the discussion on theoretical consistency and functional flexibility are reviewed (Section II) and applied to the translog production function (Section III). Subsequently, stochastic efficiency measurement is discussed and some stochastic frontier applications are reviewed with respect to theoretical consistency (Section IV). It is argued that the economic properties of the estimation results have to be critically assessed, that the interpretation and calculation of efficiency have to be revised and that a basic change in the interpretation of the estimated functions is required.

II. Theoretical consistency, functional flexibility and domain of applicability

One of the essential objectives of empirical research is the investigation of the relationship between an endogenous (or dependent) variable y_j and a set i of exogenous (or independent) variables x_{ij} where subscript j denotes the j-th observation:

$$y_j = f(x_{ij}, ß_i) + \varepsilon_j . \tag{1}$$

In general the researcher has to make two basic assumptions with regard to the examination of this relationship. The first assumption specifies the functional form expressing the endogenous variable as a function of the exogenous variables. The second assumption specifies a probability distribution for the residual ε capturing the difference between the actual and the predicted values of the endogenous

[1] Since 1990 only with respect to agricultural economics more than 75 (about 5-10%) contributions have been made to *Agricultural Economics*, *American Journal of Agricultural Economics*, *European Review of Agricultural Economics*, *Review of Agricultural Economics* and *The Journal of Productivity Analysis* dealing with the estimation of stochastic efficiency frontiers.

variable. These two major assumptions about the underlying functional form and the probability distribution of the error term are usually considered as maintained hypotheses (see Fuss et al. 1978). Statistical procedures such as maximum likelihood estimation are used to estimate the relationship, i.e., the vector of the parameters β_i.

A. Lau's criteria

In general, economic theory provides no a priori guidance with respect to the functional relationships. However, Lau (1978 and 1986) has formulated some principle criteria for the ex ante selection of an algebraic form with respect to a particular economic relationship:[2] (i) *theoretical consistency*: the algebraic functional form chosen must be capable of possessing all of the theoretical properties required by the particular economic relationship for an appropriate choice of parameters. With respect to a production possibility set this would mean that the relationship in (1) is single valued, monotone increasing as well as quasi-concave implying that the input set is required to be convex.[3, 4] However, this indicates no particular functional form. (ii) *domain of applicability*: most commonly the domain of applicability refers to the set of values of the independent variables x_i over which the algebraic functional form satisfies all the requirements for theoretical consistency. Lau (1986) refers to this concept as the *extrapolative domain* since it is defined on the space of the independent variables with respect to a given value of the vector of parameters β_i. If, for given β_i, the algebraic functional form $f(x_i, \beta_i)$ is theoretically consistent over the whole of the applicable domain, it is said to be globally theoretically consistent or globally valid over the whole of the applicable domain. Fuss et al (1978) stress the *interpolative robustness* as the

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[2] The ex ante choice problem has to be distinguished from that of ex post choice which belongs to the realm of specification analysis and hypothesis testing.

[3] Monotone increasing implies that additional units of any input can never decrease the level of output, so all marginal productivities are non-negative. This is finally derived from the basic assumption of rational individual behaviour. Quasiconcavity is essentially equivalent to assuming that the law of the diminishing marginal rate of technical substitution holds. It implies that if x_i and x_k are capable of producing y, then their convex combination is also capable of producing y.

[4] In the following we only consider a production function relationship. However, the same arguments apply for a cost, profit, return or distance function each showing different exogenous variables. A general discussion would require relatively complex arguments without providing any further insights.

functional form should be well-behaved in the range of observations, consistent with maintained hypotheses and admit computational procedures to check those properties, as well as the *extrapolative robustness* as the functional form should be compatible with maintained hypotheses outside the range of observations to be able to forecast relations. (iii) *flexibility*: a flexible algebraic functional form is able to approximate arbitrary but theoretically consistent economic behaviour through an appropriate choice of the parameters.[5] The production function in (1) can be said to be *second-order flexible* if at any given set of non-negative (positive) inputs the parameters β can be chosen so that the derived input demand functions and the derived elasticities are capable of assuming arbitrary values at the given set of inputs subject only to theoretical consistency.[6] "Flexibility of a functional form is desirable because it allows the data the opportunity to provide information about the critical parameters." (Lau 1986, p. 1544). (iv) *computational facility*: this criteria implies the properties of 'linearity-in-parameters', 'explicit representability', 'uniformity' and 'parsimony'. For estimation purposes the functional form should therefore be linear-in-parameters, possible restrictions should be linear.[7] With respect to the ease of manipulation and calculation the functional form as well as any input demand functions derivable from it should be represented in explicit closed form and linear in parameters. Different functions in the same system should have the same 'uniform' algebraic form but differ in parameters. In order to achieve a desired degree of flexibility the functional form should be parsimonous with respect to the number of parameters. This to avoid methodological problems as multi-collinearity and a loss of degrees of freedom. (v) *factual conformity*: the functional form should be finally consistent with established empirical facts with respect to the economic problem to be modelled.[8]

[5] Alternatively flexibility can be defined as the ability to map different production structures at least approximately without determining the parameters by the functional form. The concept of flexibility was first introduced by Diewert (1973) and (1974). Lau (1986) and Chambers (1988) discuss local and global approximation characteristics with respect to different functional forms.

[6] This implies that the gradient as well as the Hessian matrix of the production function with respect to the inputs are capable of assuming arbitrary non-negative and negative semidefinite values respectively.

[7] If necessary a known transformation should be applied. Fuss et al. (1978) nevertheless stress that the tradeoff between the computational requirements of a functional form and the thoroughness of empirical analysis has to be weighted carefully.

[8] E.g., the well confirmed fact that the elasticities of substitution between all pairs of inputs are not all identical in the three or more-input case.

B. The concept of flexibility

It is important to have a more detailed look at the concept of flexibility. A functional form can be denoted as ‘flexible’ if its shape is only restricted by theoretical consistency. This implies the absence of unwanted a priori restrictions and is paraphrased by the metaphor of “providing an exhaustive characterization of all (economically) relevant aspects of a technology” (see Fuss et al. 1978). Each relevant aspect of the concept of second order flexibility is assigned to exactly one parameter: the level parameter, the gradient parameters associated with the respective first order variable, and the Hessian-parameters associated with the second order terms. As a functional form cannot be second-order flexible with fewer parameters, the number of free parameters provides a necessary condition for flexibility. With respect to a single-product technology with an n-dimensional input vector, a function exhaustively characterizing all of its relevant aspects should contain information about the quantity produced (one level effect), all marginal productivities (n gradient effects) as well as all substitution elasticities (n^2 substitution effects). As the latter are symmetric beside the main diagonal with n elements, only half of the off-diagonal elements are needed, i.e., $\frac{1}{2}n(n - 1)$. The number of effects an adequate single-output technology function should be capable of depicting independently of each other and without a priori restrictions amounts to a total of $\frac{1}{2}(n + 2)(n + 1)$. Hence a valid flexible functional form must contain at least $\frac{1}{2}(n + 2)(n + 1)$ independent parameters (see Hanoch 1970 and Feger 2000). Finally it has been shown that the function value as well as the first and second derivatives of a primal function can be approximated as well by the dual behavioural representation of the same technology (see Blackorby and Diewert 1979). With respect to the relation between the supposed true function and the corresponding flexible estimation function the following concurring hypotheses can then be formulated (see Morey 1986):

(i) *The estimated function is a local approximation of the true function*. This simply means that the approximation properties of flexible functional forms are only locally valid and therefore value, gradient and Hessian of true and estimated function are equal at a single point of approximation. As only a local interpretation of the estimated parameters is possible, the forecasting capabilities with respect to variable values relatively distant from the point of approximation are severly restricted.[9] In this case, at least the necessary condition of local concavity with

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[9] In the immediate neighbourhood of the approximation point each flexible functional form provides theoretically consistent parameters only if the true structure is theoretically consistent (see Morey 1986 and Chambers 1988).

respect to global concavity can be tested for every point of approximation (see Section IV).[10]

(ii) *The estimated function and the true structure are of the same functional form but show the desired properties only locally.* Most common flexible functions can either not be restricted to a well-behaved function without losing their flexibility (e.g., the translog function) or cannot be restricted to regularity at all. Points of interest in the true structure can be examined by testing the respective points in the estimated function. However, a positive answer to the question whether the estimated function and the true structure are still consistent with the properties of a well-behaved production function if the data does not equal the examined data set is highly uncertain. This uncertainty can only be illuminated by systematically testing all possible data sets.

(iii) *The estimated function and the true structure are of the same functional form and show the desired properties globally.* A flexible functional form which can be restricted to global regularity (e.g., the Symmetric Generalized McFadden Function, see Diewert and Wales 1987) without losing its flexibility allows for the inference from the estimation function to the true structure and hence allows for meaningful tests of significance as the model is theoretically well founded (see Morey 1986).[11] This approach of a flexible functional form promotes a concept of flexibility where the functional form has to fit the data to the greatest possible extent, subject only to the regularity conditions following from economic theory and independently depicting all economically relevant aspects. As Feger (2000) concludes: "The argument that any flexible functional form can approximate any other flexible functional form and any arbitrary data generation process does not suspend the researcher from the issue of reducing the specification error to the greatest possible extent in selecting the most appropriate functional form for the entire data." (see also Terrell 1995).

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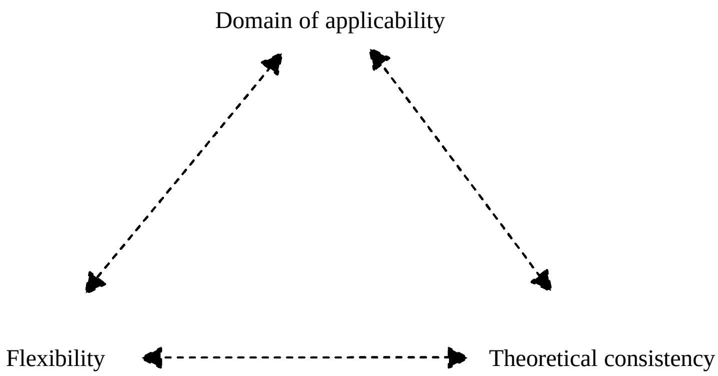
[10] Nevertheless, as initially Lau (1986, p. 418) pointed out, this does not always mean intrinsically concave. Morey (1986) raises the question about the location of the approximation point and stresses that there is no way to infer from the approximation function to the location of the approximation point. Commonly, the point of approximation is held to be located at some mean of variables over all observations. However, Feger (2000) stresses that this view emanates from erroneously interpreting the point of approximation and the point of expansion as synonyms.

[11] On the other side, a serious problem arises for the postulates of economic theory if a properly specified flexible function which is globally well-behaved is not supported by the data (see Feger 2000).

C. The magic triangle

As noted by Lau (1978), one should not expect to find an algebraic functional form satisfying all of his criteria (in general cited as Lau's incompatibility theorem). As one should not compromise on (at least) local theoretical consistency, computational facility or flexibility of the functional form, he suggests the domain of applicability as the only area left for compromises with respect to functional choice.[12]

Figure 1. The magic triangle of functional choice



As Figure 1 summarizes, for most functional forms there is a fundamental trade-off between flexibility and theoretical consistency as well as the domain of applicability. Production economists propose two solutions to this problem, depending on what kind of violation shows to be more severe (see Lau 1986 or Chambers 1988): the choice of functional forms which could be made globally theoretical consistent by corresponding parameter restrictions, here the range of flexibility has to be investigated; to opt for functional flexibility and check or impose theoretical consistency for the proximity of an approximation point (usually at the sample mean) only.

A globally theoretical consistent as well as flexible functional form can be considered as an adequate representation of the production possibility set. Locally theoretical consistent as well as flexible functional forms can be considered as an i-th order differential approximation of the true production possibilities. Hence, the translog function is considered as a second order differential approximation of the true production possibilities.

[12] Hence, even if a functional form is not globally theoretical consistent, it can be accomodated to be theoretically consistent within a sufficiently large subset of the space of independent variables.

III. The case of the translog production function

A prominent textbook example as well as the most often used functional form with respect to efficiency measurement is the Cobb-Douglas production function:

$$\ln y = a_0 + \sum_{i=1}^{n} a_i \ln x_i \,. \tag{2}$$

This function shows theoretical consistency globally if $a_i \geq 0$, but fails with respect to flexibility as there are only (n-1) free parameters. Similarily, the translog production function, probably the best investigated second order flexible functional form and certainly the one with the most applications, has to be noted:

$$f(x) = a_0 + \sum_{i=1}^{n} a_i \ln x_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} \ln x_i \ln x_j \,, \tag{3}$$

where symmetry of all Hessians by Young's theorem implies that $a_{ij} = a_{ji}$. It has $(n^2 + 3n + 2)/2$ distinct parameters and hence just as many as required to be flexible. By setting $A_{ij} = \Sigma_{i=1}^{n} \Sigma_{j=1}^{n} a_{ij}$ equal to a null matrix reveals that the translog function is a generalization of the Cobb Douglas functional form. The theoretical properties of the second order translog are well known (see, e.g., Lau 1986): it is easily restrictable for global homogeneity as well as homotheticity, correct curvature can be implemented only locally if local flexibility should be preserved, the maintaining of global monotonicity is impossible without losing second order flexibility.[13] Hence, the translog functional form is fraught with the problem that theoretical consistency can not be imposed globally. This is subsequently shown by discussing the theoretical requirements of monotonicity and curvature.

A. Monotonicity

As is well known with respect to a (single output) production function monotonicity requires positive marginal products with respect to all inputs:[14]

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[13] Feger (2000) claims that the translog entertains two advantages over all other specifications: first, it is extremely convenient to estimate, and second, it is likely to be a good specification for economic processes. Terrell (1996) applied a translog, generalized Leontief, and symmetric generalized McFadden cost function to the classical Berndt and Wood data. The results suggest that translog extensions to higher order could frequently outperform the Asymptotically Ideal Model (AIM) which is considered as today's state of the art.

[14] Barnett (2002, p. 199) notes: "In specifications of tastes and technology, econometricians often impose curvature globally, but monotonicity only locally or not at all. In fact monotonicity rarely is even mentioned in that literature. But without satisfaction of both curvature and

$$\frac{\partial y}{\partial x_i} > 0\ , \tag{4}$$

and thus non-negative elasticities. However, until most recent studies the issue of assuring monotonicity was neglected. Barnett et al. (1996), e.g., showed that the monotonicity requirement is by no means automatically satisfied for most functional forms, moreover violations are frequent and empirically meaningful. In the case of the translog production function the marginal product of input *i* is obtained by multiplying the logarithmic marginal product with the average product of input *i*. The monotonicity condition given in (4) holds for the translog specification if the following equation is positive:

$$\frac{\partial y}{\partial x_i} = \frac{y}{x_i} * \frac{\partial \ln y}{\partial \ln x_i} = \frac{y}{x_i} * \left(a_i + \sum_{j=1}^{n} a_{ij} \ln x_j \right) > 0 \tag{5}$$

Since both y and x_i are positive numbers, monotonicity depends on the sign of the term in parenthesis, i.e., the elasticity of y with respect to x_i. If it is assumed that markets are competitive and factors of production are paid their marginal products, the term in parenthesis equals input *i*'s share of total output, s_i.

By adhering to the law of diminishing marginal productivities, marginal products, apart from being positive should be decreasing in inputs implying the fulfillment of the following expression:

$$\frac{\partial^2 y}{\partial x_i^{\,2}} = \left[a_{ii} + \left(a_i - 1 + \sum_{j=1}^{n} a_{ij} \ln x_j \right) * \left(a_i + \sum_{j=1}^{n} a_{ij} \ln x_j \right) \right] * \left(y / x_i^{\,2} \right) < 0 \tag{6}$$

Again, this depends on the nature of the terms in parenthesis. These should be checked a posteriori by using the estimated parameters for each data point. Both restrictions (i.e., $\partial y/\partial x_i > 0$ and $\partial^2 y/\partial x_i^2 < 0$) should hold at least at the point of approximation.

B. Curvature

Whereas the first order and therefore non-flexible derivative of the translog, the Cobb Douglas production function, can easily be restricted to global quasi-concavity by imposing $a_i \geq 0$, this is not the case with the translog itself. The

monotonicity, the second-order conditions for optimizing behaviour fail, and duality theory fails."

necessary and sufficient condition for a specific curvature consists in the semi-definiteness of its bordered Hessian matrix as the Jacobian of the derivatives $\partial y/\partial x_i$ with respect to x_i: if $\nabla^2 Y(x)$ is negatively semi-definite, Y is quasi-concave, where ∇^2 denotes the matrix of second order partial derivatives with respect to (•). The Hessian matrix is negative semi-definite at every unconstrained local maximum[15], it yields with respect to the translog:

$$H = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} - \begin{pmatrix} s_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_n \end{pmatrix} + \begin{pmatrix} s_1 s_1 & \cdots & s_1 s_n \\ \vdots & \ddots & \vdots \\ s_n s_1 & \cdots & s_n s_n \end{pmatrix}, \tag{7}$$

where s_i denotes the elasticity of production:

$$s_i = \frac{\partial \ln y}{\partial \ln x_i} = a_i + \sum_{j=1}^{n} a_{ij} \ln x_j \ . \tag{8}$$

The conditions of quasi-concavity are related to the fact that this property implies a convex input requirement set (see in detail, e.g., Chambers 1988). Hence, a point on the isoquant is tested, i.e., the properties of the corresponding production function are evaluated subject to the condition that the amount of production remains constant. Given a point $\mathbf{x^0}$, necessary and sufficient for curvature correctness is that at this point $\mathbf{v'Hv} \leq 0$ and $\mathbf{v's} = 0$ where $\mathbf{v}$ denotes the direction of change.[16] Hence, contrary to the Cobb Douglas function quasi-concavity can not be checked for by simply considering the parameter estimates.

A matrix is negative semi-definite if the determinants of all of its principal submatrices are alternate in sign, starting with a negative one (i.e., $(-1)^k D_k \geq 0$ where D is the determinant of the leading principal minors and $k = 1, 2, \ldots, n$).[17] However, this criterion is only rationally applicable with respect to matrices up to the format 3 times 3 (see, e.g., Strang 1976), the most operational way of testing square numerical matrices for semi-definiteness is the eigen - or spectral decomposition:[18] Let **A** be a square matrix. If there is a vector $\mathbf{X} \in R^n \neq 0$ such that

[15] Hence, the underlying function is quasi-concave and an interior extreme point will be a global maximum. The Hessian matrix is positive semi-definite at every unconstrained local minimum.

[16] Which implies that the Hessian is negative semi-definite in the subspace orthogonal to $\mathbf{s} \neq 0$.

[17] Determinants of the value 0 are allowed to replace one or more of the positive or negative values. Any negative definite matrix also satisfies the definition of a negative semi-definite matrix.

[18] The eigen decomposition relates to the decomposition of a square matrix **A** into eigenvalues

$$\mathrm{AX} = \lambda \mathrm{X} \,, \tag{9}$$

for some scalar λ then λ is called the eigenvalue of **A** with the corresponding eigenvector **X**. Following this procedure the magnitude of the $m + n$ eigenvalues of the bordered Hessian have to be determined.[19] With respect to the translog production function curvature depends on the input bundle, as the corresponding bordered Hessian **BH** for the 3 input case shows:

$$\mathrm{BH} = \begin{pmatrix} 0 & f_1 & f_2 & f_3 \\ f_1 & f_{11} & f_{12} & f_{13} \\ f_2 & f_{21} & f_{22} & f_{23} \\ f_3 & f_{31} & f_{32} & f_{33} \end{pmatrix}, \tag{10}$$

where f_i is given in (5), f_{ii} is given in (6) and f_{ij} is

$$\frac{\partial^2 y}{\partial x_i \partial x_j} = \left[a_{ij} + \left(a_i + \sum_{j=1}^{n} a_{ij} \ln x_j \right) * \left(a_j + \sum_{i=1}^{n} a_{ij} \ln x_i \right) \right] * \left(y / x_i x_j \right) < 0 \,. \tag{11}$$

For some bundles quasi-concavity may be satisfied, but not for others. Hence, what can be expected is that the condition of negative-semidefiniteness of the bordered Hessian is met only locally or with respect to a range of bundles.

C. Graphical discussion

In order to provide a more comprehensive treatment of the properties of the translog function we discuss possible forms of isoquants (see Figure 2). We assume that inputs are normalised by their mean which we use as a reference point. The closed form of the graphs is due to the quadratic terms. Although, the graphs look very similar, the characteristics differ significantly. It becomes evident that simple inspection of the form of the isoquants is not sufficient to decide whether theoretical consistency holds or not.

The graphs in the lower left corner in panel C seem to be typical isoquants. However, the function is actually monotone decreasing and quasiconvex in that

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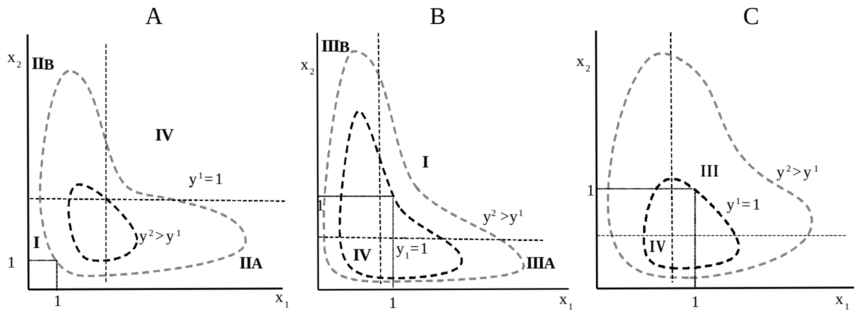
and eigenvectors and is based on the eigen decomposition theorem which says that such a decomposition is always possible as long as the matrix consisting of the eigenvectors of **A** is square.

[19] Checking the definiteness of a $(2+x)(2+x)$ bordered Hessian ($x = 1, ..., n$) is not feasible as the determinant D_1 equals always zero.

Figure 2. Isoquants of a translog production function



(A) and (B) are theoretically consistent at the reference point, (C) is not. Roman numbers denote the properties of the graph y = 1 between the dashed lines. These numbers are not valid for the other isoquants

		Monotonicity	
		yes	no
Curvature	quasi-concave	**I**	**II**
	quasi-convex	**III**	**IV**

region, e.g., a correct shape is caused by the fact that both conditions for theoretical consistency are not satisfied. In fact, in panel C there is no region where the conditions hold. Panels A and B differ in so far as the function in A has a maximum whereas in B the function shows a minimum at the reference point. This differentiation has severe consequences for the region of consistent input values. In panel A the consistent values are located in the lower left corner. Moving along the graph would first lead to regions where the monotonicity requirement is violated (area II) and after that to the area in which the curvature condition is also not satisfied (area IV).[20] However, even where there is a region in which theoretical consistency is satisfied, the applicability of the estimation is rather limited, because an increase of factor input leads to a reduction of the valid region as a consequence of the monotonicity requirement. In fact, this range is limited to the maximum.

In panel B the theoretically consistent regions are located northeast of the reference point. Contrary to panel A, moving along the graph will lead to a region

[20] This kind of result is likely when the modes are smaller than the means of the variables.

in which the curvature condition is not satisfied anymore (III).[21] Moreover, the valid regions grow with an increase in inputs. Furthermore, no region exists where production starts to decline like is the case in panel A. Thus, panel B should be the preferred estimation result. Violation of theoretical consistency can be expected at relatively low levels of factor inputs.

As the translog function consists of quadratic terms it shows a parabolic form implying increasing as well as decreasing branches by definition causing inconsistencies regarding the monotonocity requirement ($\partial y/\partial x_i > 0$). Further violations of the curvature condition are caused by the logarithmic transformation of input variables. All functional forms showing these properties are finally subject to possible violations of their theoretical consistency. Unfortunately, all flexible functional forms commonly used in empirical economics belong to the same class as the translog function.

D. Theoretical consistency and flexibility

The preceeding discussion shows that there is a a trade-off between flexibility and theoretical consistency with respect to the translog as well as most flexible functional forms. Economists propose different solutions to this problem:

(i) Imposing globally theoretical consistency destroys the flexibility of the translog as well as other second-order flexible functional forms[22], as e.g., the generalized Leontief. However, theoretical consistency can be locally imposed on these forms by maintaining their functional flexibility. Ryan and Wales (2000) even argue that a sophisticated choice of the reference point could lead to satisfaction of consistency at most or even all data points in the sample.[23] Jorgenson and Fraumeni (1981) firstly propose the imposition of quasi-concavity through restricting **A** to be a negative semidefinite matrix. However, by imposing global consistency on the translog functional form Diewert and Wales (1987) note that the parameter matrix is restricted leading to seriously biased elasticity estimates.[24] Hence, the translog function would lose its flexibility.

[21] This kind of function will occur when the modes are larger than the means of the inputs.

[22] Second-order flexibility in this context refers to Diewert's (1974) definition where a function is flexible if the level of production (cost or profit) and all of its first and second derivatives coincide with those of an arbitrary function satisfying linear homogeneity at any point in an admissable range.

[23] In fact Ryan and Wales (1998, 1999, 2000) could confirm this for several functional forms in a consumer demand context as well as for the translog and generalized Leontief specification in a producer context. See also Feger (2000) and the recent example by Terrell (1996).

[24] Diewert and Wales (1987) illustrate that the Jorgenson-Fraumeni procedure for imposing

Any flexible functional form can be restricted to convexity or (quasi-)concavity with the above method – i.e., to local convexity or (quasi-)concavity. The Hessian of most flexible functional forms, e.g., the translog or the generalized Leontieff, are not structured in a way that the definiteness property is invariant towards changes in the exogenous variables (see Jorgenson and Fraumeni 1981). However, there are exceptions: e.g., the Hessian of the Quadratic does not contain exogenous variables at all, and thus a restriction by applying the Cholesky factorization suffices to impose regular curvature at all data points.[25]

(ii) Functional forms can be chosen which could be made globally theoretical consistent through corresponding parameter restrictions and by simultaneously maintaining flexibility. This is shown for the symmetric generalized McFadden cost function by Diewert and Wales (1987) following a technique initially proposed by Wiley et al. (1973). Like the generalized Leontief, the symmetric generalized McFadden is linearily homogenous in prices by construction, monotonicity can either be implemented locally only or, if restricted for globally, the global second-order flexibility is lost (see Feger 2000). However, if this functional form is restricted for correct curvature the curvature property applies globally.[26] Furthermore regular regions following Gallant and Golups' (1984) numerical approach to account for consistency by using, e.g., Bayesian techniques can be constructed with respect to flexible functional forms.[27]

IV. Implications for stochastic efficiency measurement

In recent years the research focus in production economics is the technical

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concavity will lead to estimated input substitution matrices which are "too negative semidefinite", i.e. the degree of substitutability will tend to be biased in an upward direction. If the elasticities are independent of the input vector by transformation (assuming $a_{ij} = 0$ for all i and j) the translog function looses its flexibility as it collapses to the Cobb Douglas form.

[25] It is woth noting that the Quadratic is disqualified for its incapability of being restricted with respect to linear homogeneity.

[26] Unfortunately, the second order flexibility property is in this case restricted to only one point.

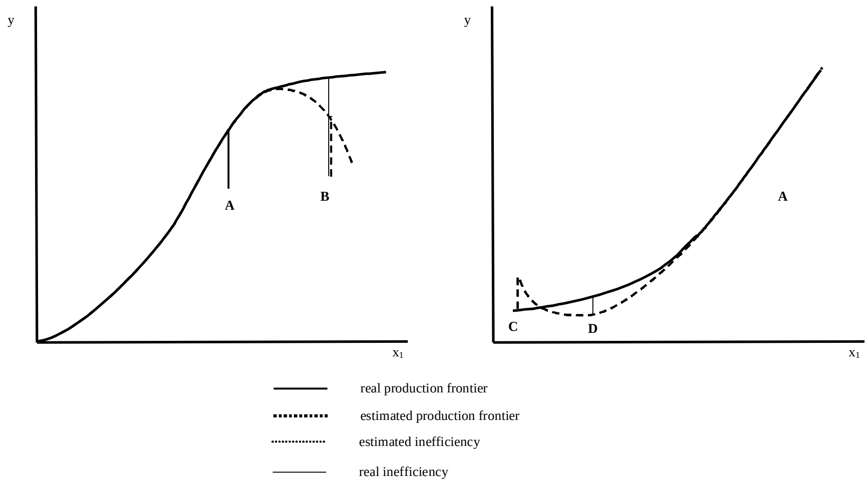
[27] To avoid the disturbing choice between inflexible and inconsistent specifications this approach imposes theoretical consistency only over the set of variable values where inferences will be drawn. Here the model parameters are restricted in a way that the resulting elasticities meet the requirements of economic theory for the whole range of variable constellations that are a priori likely to occur, i.e. a regular region is created. Alternatively one could apply a penalty function approach by including penalty terms linked to the individual derivatives in the respective likelihood function. We gratefully owe this last argument to an anonymous referee.

and allocative efficiency of netput bundles, instead of the structure and change of the production possibilities.[28] A typical representation of the production possibilities is given by the production frontier:

$$y = f(x) - \varepsilon, with\ 0 < \varepsilon < \infty\ . \tag{12}$$

This trend is accompanied by a shift in the interpretation insofar as the estimated results are not interpreted for the approximation point but for all input values. This is a necessary consequence of the shift of the research focus. While it is possible to investigate the structure of the production possibilities at any virtual production plan, efficiency considerations can only be performed for the individual observations. However, this in turn requires that the properties of the production function have to be investigated for every observable netput vector. The consequences of a violation of theoretical consistency for the relative efficiency evaluation will be discussed using Figures 3 and 4 by showing the effect on the random error term.

Figure 3. Violation of monotonicity

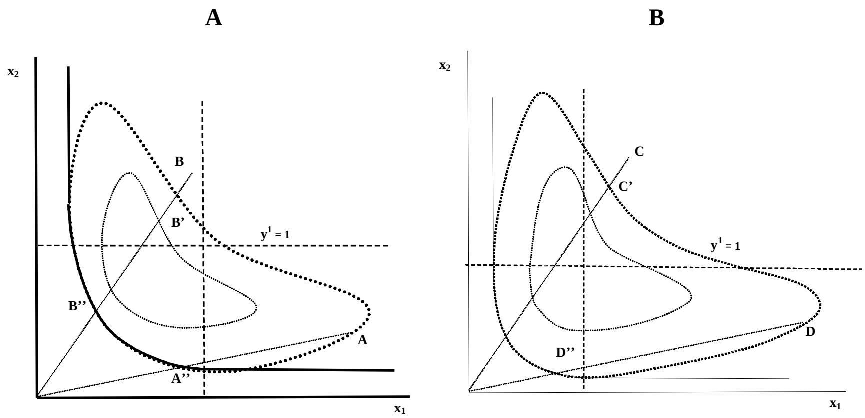


As becomes clear the estimated relative inefficiency equals the relative inefficiency for the production unit A with respect to the real production function.

[28] Typical issues investigated concern separability, homotheticity as well as the impact of technological change (see, e.g., Chambers 1988). In general, the results were interpreted for the approximation point only.

As the estimated function violates the monotonicity critera for parts of the function the estimated relative inefficiency of production unit B understates the real inefficiency for this observation. The same holds for production unit C which actually lies on the real production frontier, whereas the estimated relative inefficiency for production unit D again understates the real inefficiency. Figure 4 shows the implications as a result of irregular curvature of the estimated efficiency frontier:

Figure 4. Violation of quasi-concavity



As illustrated by Figure 2A, area I shows theoretical consistency. The dotted line describes an isoquant of the estimated production function. The relative inefficiency of the input combination at production unit B measured against the estimated frontier (at B') understates the real inefficiency which is obtained by measuring the input combination against the real production frontier at point B''. Observation A lies on the estimated isoquant and is therefore measured as full efficient (point A). Nevertheless this production unit produces relatively inefficient with respect to the real production frontier (see point A''). The same holds for production unit D (real inefficiency has to be measured at point D''). Finally relative inefficiency of observation C detected at the estimated frontier (C') corresponds to real inefficiency for this production unit as the estimated frontier is theoretical consistent.

The graphical discussion clearly shows the implications for efficiency measurement: theoretical inconsistent frontiers over- or understate real relative inefficiency and hence lead to severe misperceptions and finally inadequate as well as counterproductive policy measures with respect to the individual production unit in question. However, a few applications exist considering the need for theoretical consistent frontier estimation: e.g., Khumbhakar (1989), Pierani and Rizzi (2001), Christopoulos et al. (2001), Craig et al. (2003), and Sauer and Frohberg (2006) estimated a symmetric generalized McFadden cost frontier by imposing concavity and checking for monotonicity (these studies use a non-radial approach, except for Craig et al. who uses a shadow cost frontier). Here global curvature correctness is assured by maintaining functional flexibility. O'Donnell (2002) applies Bayesian methodology to impose regularity constraints on a system of equations derived from a translog shadow cost frontier. However, the vast majority of existing efficiency studies uses the error components approach by applying an inflexible Cobb-Douglas production function or a flexible translog production function without checking or imposing monotonicity as well as quasi-concavity requirements.

A. Examples: Testing for local consistency of technical efficiency estimates

Although the majority of applications with respect to stochastic efficiency estimation uses the Cobb-Douglas functional form we subsequently focus on applications using the translog production function to derive efficiency judgements. This, as we outlined earlier, because of the relative superiority of flexible functional forms: in our opinion the Cobb-Douglas functional form should not be used for stochastic efficiency estimations any longer.

Theoretical consistency of the estimated function should be ideally tested and proven for all points of observation, which requires for the translog specification beside the parameters of estimation also the output and input data on every observation. Most contributions fail to satisfactorily document the applied data set at least with respect to the sample means. However, the following exemplary analysis uses a number of translog production function applications published in recent years focusing on agriculture related issues. Here monotonicity – via the gradient of the function with respect to each input by investigating the first derivatives – as well as quasi-concavity – via the bordered Hessian matrix with respect to the input bundle by investigating the eigenvalues – are checked for the individual local approximation point at the sample mean. Table 1 shows the results of the exemplary regularity tests (see Table A1 in Appendix for the numerical details of the regularity tests performed).

Kumbhakar and Hjalmarrson (1993) investigated the efficiency of 608 Swedish farms engaged in milk production for the period 1968 to 1975 considering labor, material, land and capital as inputs. All first derivatives with respect to inputs

Table 1. Examples of local irregularity of translog production function models

Study: Authors (year) Country	Data set: No. obs., years (Model) Output	Inputs	Monotonicity (for every Input)	Diminishing marginal productivity (for every Input)	Quasi-concav. (of input bundle)	Local regular. (monotone & quasi-concave)
I) Kumbhakar and	608, 1968-1975	Labor	x	x		
Hjalmarrson (1993)	Dairy Output	Material	x	x		
Sweden		Land	x	0	0	0
		Capital	x	x		
II) Kumbhakar and	4890, 1976-1988	Fodder	0	0		
Heshmati (1995)	Diary Output	Material	0	0		
Sweden		Labor	x	x		
		Capital	0	0		
		Grass	x	x	0	0
		Land	x	x		
		Pasture	0	x		
		Age	0	x		
III) Battese and	330, 1986-1991	Land	x	x		
Broca (1997)	(Model 1)	Labour	0	0		
Pakistan	Wheat Output	Fertiliser	x	x	0	0
		Seed	x	0		
IV) Brümmer (2001)	185, 1995 & 1996	Labour	x	x		
Slovenia	*(Models 95 & 96)*	Land	0	0	0	0
	Total Farm Output	Intermediates	0	0		
		Capital	x	0		
V) Ajibefun, Battese	67, 1995	Land	x	0		
and Daramola (2002)	Total Crop Output	Labour	x	x		
Nigeria		CapitalHired	x	x	0	0
		Labour	x	x		
VI) Alvarez and	196, 1993-1998	Labour	0	0		
Arias (2004)	Milk Output	Cows	x	x		
Spain		Feedstuff	x	0	0	0
		Land	0	0		
		Roughage	x	0		
VII) Kwon and	1026, 1993-1997	Land	x	x		
Lee (2004)	*(Models 93 – 97)*	Labour	x	x		
Korea	Rice Output	Capital	x	x		
		Fertiliser	0	0	0	0
		Pesticides	x	x		
		Others	x	x		

Note: Evaluated at the sample means due to lack of data on each observation. x – fulfilled, 0 – not fulfilled. A study by Brümmer and Loy (2000) that is not reported showed similar inconsistencies.

showed positive signs at the sample mean and therefore fulfilled the monotonicity criterion (see Table 1). However, the second derivative with respect to land revealed to be non-negative and therefore indicates non-observance of the law of diminishing productivity. Hence checking the eigenvalues of the corresponding bordered Hessian matrix, the latter turned out not to be negative semi-definite and the estimated production frontier does not fulfill the curvature criterion of quasi-concavity. Kumbhakar and Heshmati (1995) estimated technical efficiency for a panel of Swedish Dairy Farms by a multi-step approach. They used fodder, material, labor, capital, grass fodder, cultivated land, pasture land as well as the age of the farmers as input variables. Evaluated at the sample mean only 3 of 8 inputs fulfilled the monotonicity requirement. The estimated function was not quasi-concave. Battese and Broca (1997) estimated technical efficiencies of 109 wheat farmers in Pakistan over the period 1986 to 1991 using land, labor, fertilizer and seed as inputs. Only model 2 fulfilled the monotonicity requirements for all four inputs. Both models evaluated at the sample means failed to adhere to quasi-concavity. Brümmer (2001) attempted to analyse the technical efficiency of 185 private farms in Slovenia for the years 1995 and 1996. For both years the estimated function showed to be non-monotone in the inputs land and intermediates. The estimated translog frontiers do not fulfill the curvature requirement of quasi-concavity. Ajibefun, Battese and Daramola (2002) aimed to investigate factors influencing the technical efficiency of 67 crop farms in the Nigerian state of Oyo for the year 1995. The authors used land, labor, capital as well as hired labour to estimate a translog production frontier. However, the estimated function showed to be monotone in all inputs but not quasi-concave for the input bundle. Alvarez and Arias (2004) tried to find evidence on the relationship between technical efficiency and the size of 196 dairy farms in Spain for the period 1993 to 1998. For the inputs labour and land the estimated frontier showed to be non-monotone at the sample means. The estimated production frontier's curvature is not correct. Finally Kwon and Lee (2004) estimated stochastic production frontiers for the years 1993 to 1997 with respect to Korean rice farmers. All efficiency frontiers showed to be non-monotone for the input fertilizer and do not fulfill the curvature requirement of quasi-concavity. To sum up: 100% of arbitrarily selected translog production frontiers fail to fulfill (at least) local regularity at the sample means.

Hence, as the investigated frontiers are flexible but not regular (at least at the sample mean) derived efficiency scores are not theoretically consistent and therefore are not an appropriate basis for the formulation of policy measures focusing on the relative performance of the investigated decision making units.

B. Choosing a point of approximation: Imposing curvature and verifying consistency

In the case of a translog frontier model, quasi-concavity can be imposed at a reference point (usually at the sample mean) following Jorgenson and Fraumeni (1981) who firstly proposed the (a priori) restriction of the Hessian to be a negative semidefinite matrix. This is briefly exemplified for a two input translog function as the deterministic kernel of a stochastic cost efficiency model:

(i) Let us suppose the cost function version of a translog functional form employing two inputs (x_1, x_2) to produce one output y at the cost c:

$$c(x) = a_0 + \sum_{i=1}^{2} a_i \ln w_i + \frac{1}{2}\sum_{i=1}^{2} \sum_{j=1}^{2} a_{ij} \ln w_i \ln w_j + b_y \ln y + b_{yy} \ln y^2 \tag{13}$$
$$+ \frac{1}{2}\sum_{i=1}^{2} \gamma_{iy} \ln w_i \ln y .$$

Hence, one would obtain the Hessian **H**

$$\text{H} = \begin{pmatrix} \frac{\partial^2 c}{\partial w_1 \partial w_1} & \frac{\partial^2 c}{\partial w_1 \partial w_2} \\ \frac{\partial^2 c}{\partial w_2 \partial w_1} & \frac{\partial^2 c}{\partial w_2 \partial w_2} \end{pmatrix}, \tag{14}$$

which is subsequently replaced by the negative product of a lower triangular matrix **D** times its transpose **D'**

$$-(\text{DD}') = -\left[\begin{pmatrix} d_{11} & 0 \\ d_{21} & d_{22} \end{pmatrix} * \begin{pmatrix} d_{11} & d_{12} \\ 0 & d_{22} \end{pmatrix}\right] = \begin{pmatrix} -d_{11}d_{11} & -d_{12}d_{11} \\ -d_{21}d_{11} & -d_{21}d_{12} - d_{22}d_{22} \end{pmatrix}. \tag{15}$$

Imposing curvature at a reference point (usually the sample mean) is attained by setting

$$a_{ij} = -(\text{DD}')_{ij} + a_i \delta_{ij} + a_i a_j \tag{16}$$

where $i, j = 1, \ldots, \mathbf{n}$, $\delta_{ij} = 1$ if $i = j$ and 0 otherwise and $(\mathbf{DD'})_{ij}$ as the ij-th element of **DD'** with **D** a lower triangular matrix.[29] If our point of approximation is the sample

[29] Alternatively one can use Lau's (1978) technique by applying the Cholesky factorization **A** = **-LBL**' where **L** is a unit lower triangular matrix and **B** as a diagonal matrix.

mean all data have to be divided by their mean transferring the approximation point to an (n + 1)-dimensional vector of ones. At this point the elements of **H** do not depend on the specific input price bundle.

In a cross-sectional context such a stochastic cost frontier can be written as

$$\mathrm{E}_i \geq c\left(y_i, w_i; \beta\right) * \exp\left(v_i\right), \tag{17}$$

where [$c(y_i, w_i; ß)*exp(v_i)$] is the stochastic cost frontier consisting of two parts: a deterministic part $c(y_i, w_i; ß)$ common to all producers and a producer-specific random part $exp(v_i)$ capturing the effects of random shocks on each producer. The deterministic kernel is represented by (13) and for our single-output case would be simply reformulated by inserting the specific elements according to (16)

$$c(\mathrm{y}, \mathrm{w_i}; ß) = a_0 + a_1 \ln w_1 + a_2 \ln w_2 + \frac{1}{2}\left(-d_{11}d_{11} + a_1 - a_1 a_1\right)\ln w_1^2 \tag{18}$$

$$+\frac{1}{2}\left(-d_{12}d_{11} - a_1 a_2\right)\ln w_1 \ln w_2 + \frac{1}{2}\left(-d_{12}d_{12} - d_{22}d_{22} + a_2 - a_2 a_2\right)\ln w_2^2$$

$$+b_y \ln y + b_{yy} \ln y^2 + \frac{1}{2}\gamma_{1y} \ln w_1 \ln y + \frac{1}{2}\gamma_{2y} \ln w_2 \ln y$$

However, the elements of **D** are nonlinear functions of the decomposed matrix, and consequently the resulting estimation function becomes nonlinear in parameters. Hence, linear estimation algorithms are ruled out even if the original function is linear in parameters. By choosing an arbitrary example of two inputs (and constraining for symmetry and homogeneity) one could obtain the following parameter estimates (see Table 2):[30]

[30] Here the usual estimation statistics is not shown as it is irrelevant for explanatory purposes. A non-linear least square regression was applied in LIMDEP 8.0 by using 130 cross-sectional observations with respect to the arbitrary chosen case of maize production in Ethiopia by applying the inputs land and seed.

Table 2. Arbitrary example of a two-input translog cost frontier kernel

Parameter	Coefficient
a_0	0.00017(e)
a_1	0.63343(e)
a_2	0.50609(e)
d_{11}	-0.14659(e)
d_{22}	0.37403(e)
d_{12}	-0.30761(e)
h_{11}	-0.02149(c)
h_{22}	-0.23453(c)
h_{12}	-0.04509(c)

Notes: Point of approximation is sample mean; (e): by non-linear estimation; (c): by matrix calculation.

(ii) If we now proceed to (a posteriori) check the theoretical consistency of our estimated deterministic kernel we have to show that the first derivatives of (18) are positive (monotonicity), the own-price second derivatives are negative and finally the Hessian is negative semi-definite (concavity). Table 3 shows the relevant derivatives of (18) as well as the eigenvalues (or alternatively the principal minors[31]) of the Hessian at the point of approximation, here the Hessian does not depend on the input price bundle (a vector of ones):

Table 3. First and second derivatives at the point of approximation (sample mean)

Derivative	Value	Eigenvalue	Value
$\frac{\partial c}{\partial w_1} = f_1$	0.63343	$e_1 = \frac{1}{2}\left[(h_{11}+h_{22})+\sqrt{\left[4h_{12}h_{12}+(h_{11}-h_{22})^2\right]}\right]$	-0.01234
$\frac{\partial c}{\partial w_2} = f_2$	0.50610	$e_2 = \frac{1}{2}\left[(h_{11}+h_{22})-\sqrt{\left[4h_{12}h_{12}+(h_{11}-h_{22})^2\right]}\right]$	-0.24368
$\frac{\partial^2 c}{\partial w_1 \partial w_1} = h_{11}$	-0.02149	$\det(H_1) = (h_{11})$	-0.02149
$\frac{\partial^2 c}{\partial w_2 \partial w_2} = h_{22}$	-0.23453	$\det(H_2) = (h_{11}h_{22} - h_{12}h_{12})$	0.00301

[31] These have to alternate in sign, beginning with a negative one, those of $\det(H_1)$.

As becomes evident the estimated cost frontier kernel is monotone in its input prices, has negative second own-price derivatives and is finally concave at the point of approximation. It can be expected that input price bundles in the neighbourhood of the approximation also provide the desired output. The transformation even moves the observations towards the approximation point and thus increases the likelihood of getting theoretically consistent results at least for a range of observations (see Ryan and Wales 2000). Imposing curvature globally is attained by setting $a_{ij} = -(\mathbf{DD'})_{ij}$, however, this would destroy the flexibility of the translog cost function.

V. The need for consistent and flexible efficiency measurement

The preceeding discussion aims at highlighting the compelling need for a critical assessment of efficiency estimates with respect to the current evidence on theoretical consistency, flexibility as well as the choice of the appropriate functional form. The application of a flexible functional form as the translog specification by the majority of technical efficiency studies is adequate with respect to economic theory.[32] However, most applications do not adequately test for whether the estimated function has the required regularities of monotonicity and quasi-concavity, and hence run the risk of making improper policy recommendations. The test for theoretical consistency for an arbitrary selected sample of translog production frontiers published in agricultural economic journals in the recent 10 years reveals the significance of this problem for efficiency measurement.

The researcher has to check a posteriori for the regularity of the estimated frontier, which means checking these requirements for each and every data point with respect to the translog specification. If these requirements do not hold, they have to be imposed a priori to estimation as briefly outlined in the text. While imposing global regularity leads to a significant loss of functional flexibility, local imposition requires a differentiated interpretation: if theoretical consistency holds for a range of observations, this ‘consistency area’ of the estimated frontier should be determined and clearly stated to the reader. Estimated relative efficiency scores hence only hold for observations which are part of this range. Alternatively flexible functional forms – as, e.g., the symmetric generalized McFadden – could be used which can be accomodated to global theoretical consistency over the whole range

[32] Unless there is strong a priori information on the true functional form, flexibility should be maintained as much as possible (see, e.g., Lau 1986).

of observations. Furthermore one should always check for a possibility of using dual concepts such as the profit or cost function with respect to the efficiency measurement problem in question.[33] Policy measures based on such efficiency estimates are not subject to possible inadequacy and a waste of scarce resources. Here exemplary applications already exist in the literature.

Appendix

Table A1. Numerical details of regularity tests performed

Study	Monotonicity ($\partial y/ \partial x_i > 0$)	Diminishing marginal productivity ($\partial^2 y/ \partial x_i^2 < 0$)	Quasi–concavity Eigenvalues of bordered Hessian matrix ($E_i = 0$)
I)	Input 1: 0.07571	Input 1: -0.00002	E1: -0.58005
	Input 2: 1.76208	Input 2: -0.00487	E2: **0.00079**
	Input 3: 0.60774	Input 3: **0.06243**	E3: -181.13829
	Input 4: 0.26717	Input 4: -0.00033	E4: **0.63627,** E5: **181.13849**
II)	Input 1: **-1.44259**	Input 1: **3.24172E-05**	E1: **2116.84741**
	Input 2: **-0.44539**	Input 2: **2.36834E-05**	E2: **46.42065**
	Input 3: 0.189542	Input 3: -1.33923E-06	E3: **0.04901**
	Input 4: **-0.59149**	Input 4: **1.04829E-05**	E4: -1.55354E-06
	Input 5: 8.56558	Input 5: -0.00516	E5: -0.07129
	Input 6: 1586.66	Input 6: -33.4089	E6: -0.00564
	Input 7: **-1408.62**	Input 7: -0.86203	E7: -2137.260
	Input 8: **-146.971**	Input 8: -26.3370	E8: -18.40785, E9: -68.18484
III)	Input 1: 1115.82115	Input 1: -47.18914	E1: **1298.53011**
	Input 2: **-1.17838**	Input 2: **0.00133**	E2: -1321.70761
	Input 3: 5.23465	Input 3: -0.01544	E3: **0.01271**
	Input 4: 26.37129	Input 4: **0.00042**	E4: -0.02751, E5: -23.99859
IV)	Input 1: 1474.20723	Input 1: -198.88438	E1: -2.10927
Model 1995	Input 2: **-0.05921**	Input 2: **3.34786E-06**	E2: -240882.7599
	Input 3: **-172.24372**	Input 3: **20.03483**	E3: **1.93102E-06**
	Input 4: 5.12042	Input 4: **0.00445**	E4: **240710.0172,** E5: **0.00681**

[33] As Lau (1986, p. 1558) notes: "With regard to specific applications, one can say that as far as the empirical analysis of production is concerned, the surest way to obtain a theoretically consistent representation of the technology is to make use of one of the dual concepts such as the profit function, the cost function or the revenue function."

Table A1. (Continued) Numerical details of regularity tests performed

Study	Monotonicity ($\partial y / \partial x_i > 0$)	Diminishing marginal productivity ($\partial^2 y / \partial x_i^2 < 0$)	Quasi–concavity Eigenvalues of bordered Hessian matrix ($E_i = 0$)
V)	Input 1: 545.51798	Input 1: **325.59682**	E1: -473.82527
	Input 2: 63.39966	Input 2: -0.07723	E2: **756.14889**
	Input 3: 210.64866	Input 3: -2.32279	E3: -0.61524
	Input 4: 1.22185	Input 4: -0.00026	E4: **41.48851,** E5: -0.00035
VI)	Input 1: **-13848.63785**	Input 1: **3208.26404**	E1: -13276.23262
	Input 2: 269.10386	Input 2: -11.85909	E2: **16174.03199**
	Input 3: 2.70035	Input 3: **1.22526E-05**	E3: -116.13557
	Input 4: **-4609.10832**	Input 4: **474.94612**	E4: -3.9745E-05
	Input 5: 20.27928	Input 5: **0.00236**	E5: **889.68296,** E6: **0.00672**
VII)	Input 1: 2483.90355	Input 1: -1973.7690	E1: **1685.90046**
Model 1993	Input 2: 1.56905	Input 2: -0.01193	E2: -3659.58336
	Input 3: 6.03447	Input 3: -0.00561	E3: -18709.41058
	Input 4: **-0.82598**	Input 4: **0.00551**	E4: **18709.53378**
	Input 5: 5.89932	Input 5: -0.00916	E5: **0.00538**
	Input 6: 9.51835	Input 6: -0.08145	E6: -0.02303, E7: -0.32609

Notes: bold not consistent with economic theory. Regularity tests also failed for study IV, model 1996 and study VII, models 1994-97.

References

Ajibefun, Igbekele A., George E. Battese and Adebiyi G. Daramola (2002), "Determinants of technical efficiency in smallholder food crop farming: Application of stochastic frontier production function", *Quarterly Journal of International Agriculture* **41**: 225 – 240.

Alvarez, Antonio and Carlos Arias (2004), "Technical efficiency and farm size: A conditional analysis", *Agricultural Economics* **30**: 241 – 250.

Barnett, William A. (2002), "Tastes and technology: Curvature is not sufficient for regularity", *Journal of Econometrics* **108**: 199 – 202.

Barnett, William A., Milka Kirova, and , Meenakshi Pasupathy (1996), "Estimating policy invariant deep parameters in the financial sector, when risk and growth matter", *Journal of Money, Credit, and Banking* **27**: 1402 – 1430.

Battese, George E., and Sumiter S. Broca (1997), " Functional forms of stochastic frontier production functions and models for technical inefficiency effects: A comparative study for wheat farmers in Pakistan", *Journal of Productivity Analysis* **8**: 395 – 414.

Blackorby, Charles, and Erwin W. E. Diewert (1979), "Expenditure functions, local duality, and second order approximations", *Econometrica: Journal of the Econometric Society* **47**: 579 – 601.

Brümmer, Bernhard, and Jens-Peter Loy (2000), "The technical efficiency impact of farm credit programmes: A case study of Northern Germany", *Journal of Agricultural Economics* **51**: 405 – 418.

Brümmer, Bernhard (2001), "Estimating confidence intervals for technical efficiency: The case of private farms in Slovenia", *European Review of Agricultural Economics* **28**: 285 – 306.

Chambers, Robert (1988), *Applied Production Analysis: A Dual Approach*, Cambridge, MA, Cambridge University Press.

Christopoulos, Dimitris, John Loizides, and Efthymios G. Tsionas (2001), "Efficiency in European railways: Not as inefficient as one might think", *Journal of Applied Economics* **4**: 63 - 88.

Coelli, Tim, S. Prasado, and George E. Battese, (1998), *An Introduction to Efficiency and Productivity Analysis*, Boston., Dordrecht and London: Kluwer Academic.

Craig, Steven, J. Airola, and Tipu Manzur (2003), "The effect of institutional form on airport governance efficiency", Department of Economics, University of Houston.

Diewert, Erwin W. and Terence J. Wales (1987), "Flexible functional forms and global curvature conditions", *Econometrica* **55**: 43 - 68.

Diewert, Erwin W. (1973), "Functional forms for profit and transformation functions", *Journal of Economic Theory* **6**: 284 – 316.

Diewert, Erwin W. (1974), "Functional forms for revenue and factor requirements", *International Economic Review* **15**: 119 – 130.

Feger, Fritz (2000), *A Behavioural Model of the German Compound Feed Industry: Functional Form, Flexibility, and Regularity*, dissertation, Göttingen (http://webdoc.sub.gwdg.de/diss/2000/feger/).

Fuss, Melvyn, and Daniel McFadden (1978), *Production Economics: A Dual Approach to Theory and Applications, Vol. 1: The Theory of Production, Vol. 2: Applications of the Theory of Production,* Amsterdam, North-Holland.

Gallant, A. Ronald, and Gene H. Golub (1984), "Imposing curvature restrictions on flexible functional forms", *Journal of Econometrics* **26**: 295 – 321.

Hanoch, Giora (1970), "Generation of new production functions through duality", Discussion Paper 118, Harvard Institute of Economic Research, Cambridge, MA.

Jorgenson, Dale W., and Berta M. Fraumeni, (1981), "Relative prices and technical change", in E. R. Berndt, ed., *Modeling and Measuring Natural Resource Substitution*, Cambridge, M.I.T. Press.

Kumbhakar, Subal (1989), "Estimation of technical efficiency using flexible functional form and panel data", *Journal of Business & Economic Statistics* **7**: 253 – 258.

Kumbhakar, Subal C. and Lennart Hjalmarsson (1993), "Technical efficiency and technical progress in Swedish dairy farms", in H. O. Fried, C.A.K. Lovell and S.S. Schmidt, eds., *The Measurement of Productive Efficiency – Techniques and Applications*, New York, Oxford University Press.

Kumbhakar, Subal C., and Almas Heshmati (1995), "Efficiency measurement in Swedish dairy farms: An application of rotating panel data, 1976 – 88", *American Journal of Agricultural Economics* **77**: 660 – 674.

Kumbhakar, Subal C., and Knox C. A. Lovell (2000), *Stochastic Frontier Analysis*, Cambridge, MA, Cambridge University Press.

Kwon, Oh. S., and Hyunok Lee (2004), "Productivity improvement in Korean rice farming: Parametric and non-parametric analysis", *The Australian Journal of Agricultural and Resource Economics* **48**: 323 – 346.

Lau, Lawrence J. (1978), "Testing and imposing monotonicity, convexity and quasi-convexity constraints", in: Fuss, Melvyn; and D. McFadden, eds., (1978), *Production Economics: A Dual Approach to Theory and Applications*, Amsterdam, North-Holland.

Lau, Lawrence J. (1986), "Functional forms in econometric model building", in Z. Griliches and M. D. Intriligator, eds., *Handbook of Econometrics* vol. 3, New York, North-Holland Elsevier.

Morey, Edward R. (1986), "An introduction to checking, testing, and imposing curvature properties: The true fundtion and the estimated function", *Canadian Journal of Economics* **19**: 207 – 235.

O'Donnell, Chris J. (2002), "Parametric estimation of technical and allocative efficiency in U.S. Agriculture", in: Ball, E. and G. W. Norton, eds., *Agricultural Productivity: Measurement and Sources of Growth*, Boston, Kluwer.

Pierani, Pierpaolo, and Pier L. Rizzi (2001), "Technology and efficiency in a panel of Italian dairy farms: A SGM restricted cost function approach", *Agricultural Economics* **29**: 195-209.

Ryan, David L., and Terence J. Wales (1998), "A simple method for imposing local curvature in some flexible consumer demand systems", *Journal of Business and Economic Statistics* **16**: 331 – 338.

Ryan, David L., and Terence J. Wales (1999), "Flexible and semiflexible consumer demands with quadratic Engel curves", *The Review of Economics and Statistics* **81**: 277 – 287.

Ryan, David L., and Terence J. Wales (2000), "Imposing local concavity in the translog and generalized Leontief cost functions", *Economic Letters* **67**: 253 – 260.

Sauer, Johannes, and Klaus Frohberg (2006), "Allocative efficiency of rural water supply - A globally flexible SGM cost frontier", *Journal of Productivity Analysis* (forthcoming).

Strang, Gerald (1976), *Linear Algebra and its Applications*, New York, Academic Press.

Terrell, Dek (1995), "Flexibility and regularity properties of the asymptotically ideal production model", *Econometric Reviews* **14:** 1 – 17.

Terrell, Dek (1996), "Incorporating monotonicity and concavity conditions in flexible functional forms", *Journal of Applied Econometrics* **11**: 179 – 194.

Tsionas, Efthymios G. and George C. Bitros (2004), "A consistent approach to cost efficiency measurement", *Oxford Bulletin of Economics and Statistics* **66:** 49 – 69.

Wales, Terence J. (1977), "On the flexibility of flexible functional forms", *Journal of Econometrics* **5**: 183 – 193.

Wiley, David E., William H. Schmidt, and William J. Bramble (1973), "Studies of a class of covariance structure models", *Journal of the American Statistical Association* **68**: 317 – 323.