Real Exchange Rates Over the Past Two Centuries: How Important is the Harrod-Balassa-Samuelson Effect?

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Real Exchange Rates Over the Past Two Centuries: How Important is the Harrod-Balassa-Samuelson Effect?*

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Abstract

Using data since 1820 for the US, the UK and France, we test for the presence of real effects on the equilibrium real exchange rate (the Harrod-Balassa-Samuelson, HBS effect) in an explicitly nonlinear framework and allowing for shifts in real exchange rate volatility across nominal regimes. A statistically signi&cant HBS effect for sterling-dollar captures its long-run trend and explains a proportion of variation in changes in the real rate that is proportional to the time horizon of the change. There is signi&cant evidence of nonlinear reversion towards long-run equilibrium and downwards shifts in volatility during &xed nominal exchange rate regimes.

JEL classi&cation: F31, F41, C1.

Keywords: purchasing power parity; real exchange rate; nonlinear dynamics; Harrod-Balassa-Samuelson effect; productivity differentials.

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1 Introduction

In this paper, we investigate the influence of productivity differentials on the equilibrium level of the real exchange rate and the speed at which the real exchange rate converges towards that equilibrium. In doing so, we allow for movements in the equilibrium rate due to the influence of productivity differentials, as well as for nonlinearities in adjustment and the impact of nominal regimes on real exchange rate volatility, and we employ a long span of historical data for three countries, France, the United Kingdom and the United States, over a sample period that spans nearly two centuries.

Given that the real exchange rate is defined as the ratio of national prices expressed in a common currency, evidence of a long-run stable mean for the real exchange rate is a necessary condition for long-run purchasing power parity (PPP) to hold. The issue of whether or not the real exchange rate between major economies tends to revert towards a stable long-run equilibrium (i.e. whether the real exchange rate corresponds to a stationary stochastic process) has been a topic of considerable debate in the literature.[1,2] In short, even putting to one side certain econometric issues that have been raised concerning empirical research that has detected evidence of mean-reversion in real exchange rates,[3] these studies typically indicate a half life of shocks to the real exchange rate in the range of three to five years.[4] If we take as given that real shocks cannot account for the major part of the short-run volatility of real exchange rates (since it seems incredible that shocks to real factors such as tastes and technology could be so volatile) and that nominal shocks can only have strong effects over a time frame in which nominal wages and prices are sticky (which would presumably

[1] See Taylor and Taylor (2004) for a survey and critical discussion of this debate.

[2] This literature was largely spurred by the interest in testing for long-run relationships that followed the publication of Engle and Granger's seminal paper on cointegration and unit roots (Engle and Granger, 1987) and effectively tests for long-run *absolute* PPP (although see Flood and Taylor, 1996, and Coakley, Flood, Fuertes and Taylor, 2005, for tests of long-run *relative* PPP). Early unit-root studies of long run PPP (Taylor, 1988; Mark, 1990) could not reject the hypothesis of non-stationary real exchange rates using data for the recent float. However, Frankel (1986) and Lothian and Taylor (1997) showed that this may have been the result of the low power of univariate unit root tests. This led to a search for increased test power either through analysing panels of data for several real exchange rates (e.g. Abuaf and Jorion, 1990; Frankel and Rose, 1996; Lothian, 1997) or through analysing long spans of data (e.g. Frankel, 1986; Lothian and Taylor, 1997; Taylor, 2002).

[3] For example, Taylor and Sarno (1998) argue that widely used panel unit root tests are uninformative in this context, because rejection of the joint null hypothesis that each member of a set of real exchange rates is generated by a non-stationary process only implies that *at least one* series is generated by a stationary process, rather than that *all* of the series are generated by stationary processes.

[4] While much of this research has tended to test the hypothesis that long-run PPP does *not* hold (by formally testing a null hypothesis of non-cointegration of the nominal exchange rate and relative prices or of non-stationarity of the real exchange rate), a significant number of papers have tested the converse hypothesis— i.e. the hypothesis that long-run PPP *does* hold (by formally testing the null hypothesis of cointegration or of stationarity of the real exchange). See, e.g. Fisher and Park (1991), Culver and Papell (1999). In general, the results of this research have tended to favour long-run PPP. This strand of the literature has not, however, been without its share of debate over econometric methods (see e.g. Caner and Kilian, 2001).

give rise to a half life of adjustment much less than three to five years), then the apparent high degree of persistence of real exchange rates becomes problematic in the sense that there is no readily available economic rationale. Indeed, Rogoff (1996) has termed this finding of long half lives "the PPP puzzle".

Taylor, Peel and Sarno (2001) argue that the key both to detecting significant mean reversion in the real exchange rate and to solving Rogoff's PPP puzzle lies in allowing for nonlinearities in real exchange rate adjustment, so that the further the real exchange rate is from its long-run equilibrium, the stronger will be the forces driving it back towards equilibrium. The cause of this nonlinearity may be greater goods arbitrage as the misalignment grows (Parsley and Wei, 1996; Obstfeld and Taylor, 1997; Imbs, Mumtaz, Ravn and Rey, 2003; Sarno, Taylor and Chowdhury, 2004), or a growing degree of consensus concerning the appropriate or likely direction of movements in the nominal exchange rate among traders (Kilian and Taylor, 2003), or perhaps a greater likelihood of the occurrence and success of intervention by the authorities to correct a strongly misaligned exchange rate (Taylor, 1994, 2004, 2005; Sarno and Taylor, 2001; Reitz and Taylor, 2006).[5]

Parallel to the recent literature on nonlinearities in real exchange rate adjustment, researchers have also stressed the importance of real shocks to the underlying equilibrium real exchange rate (e.g. Engel, 1999, 2000; Engel and Kim, 1999). As discussed below, the idea that productivity shocks may affect the equilibrium real exchange rate—the so-called Harrod-Balassa-Samuelson (HBS) effect—has a fairly long history in economics (Harrod, 1933; Balassa, 1964; Samuelson, 1964). The empirical evidence on the Harrod-Balassa-Samuelson effect is surveyed in Froot and Rogoff (1995) and, more recently, in Taylor and Taylor (2004). In general, this research provides mixed results, with early studies such as Officer (1976b, 1982) finding little or no evidence of HBS effects and the preponderance of later studies finding at most very weak supporting evidence (e.g. Froot and Rogoff, 1991, 1995; Asea and Mendoza, 1994). Several very recent studies have, however, been more supportive (Chinn, 1999; Bergin, Glick and Taylor, 2004), and Bergin, Glick and Taylor (2004) suggest that the HBS effect may have been variable over time, perhaps due to variations in relative productivity differentials themselves, or other factors. A key point here is that if the equilibrium exchange rate is moving gradually over time, but statistical tests for real exchange rate stability assume that the equilibrium exchange rate is constant, then estimates of the speed of reversion towards the mean will be biased, and this bias may be at least partly responsible for Rogoff's PPP puzzle (Taylor and Taylor, 2004). Evidence suggestive of a bias arising from this source is provided by studies which have found that allowing for linear or nonlinear deterministic trends (which may be proxying for HBS effects) can

[5] Imbs, Mumtaz, Ravn and Rey (2005) argue that the PPP puzzle is largely due to aggregation bias resulting from using indices of prices to construct real exchange rates and heterogeneity in speeds of adjustment of relative prices at the disaggregated goods level. These authors note, however: "Nonlinear dynamics of *aggregate* real exchange rates may be fully compatible with—or at least observationally equivalent to—the argument about the importance of heterogeneity at the disaggregated level."

make a material difference in resolving the puzzles about whether and how fast the exchange rate moves to its PPP level (Taylor, 2002; Lothian and Taylor, 2000).

In this paper, we seek to contribute to this literature in several ways. In particular, we carry out an empirical analysis of real exchange rates and productivity differentials within a nonlinear framework, using a data set for the United States, the United Kingdom and France covering the period 1820-2001 (1820-1998 for investigations involving the franc). By proxying the level of productivity by real GDP per capita, this allows us to examine the HBS effect using a long-span of data over which productivity differentials would be expected to be important even between major economies.

The remainder of the paper is set out as follows. In the next section we discuss methods for modelling nonlinearity in real exchange rate adjustment, while in Section 3 we brie! y outline the theoretical rationale for the in! uence of productivity differentials on the long-run equilibrium real exchange rate. In Section 4 we discuss the evidence of shifting real exchange rate volatility across nominal exchange regimes and outline our empirical methods for allowing for these shifts. In the following section we describe our data set, and in Section 6 we present our empirical results. We provide some concluding comments and suggestions for future research in a &nal section.

2 Modelling Nonlinearity

As noted above, a number of authors have reported evidence of nonlinearity in real exchange rate adjustment. One particular statistical characterisation of nonlinear adjustment, which appears to work well for exchange rates, is the exponential smooth transition autoregressive (ESTAR) model (Granger and Teräsvirta, 1993; Teräsvirta, 1994, 1998; van Dijk, Teräsvirta and Franses, 2002).[6] In the ESTAR model, adjustment takes place in every period but the speed of adjustment towards the long-run mean varies with the extent of the deviation from the mean. An ESTAR model for a time series process $\{y_t\}$may

[6] For applications of the ESTAR model to exchange rates, see, e.g., Taylor and Peel (2000), Taylor et al. (2001) and Kilian and Taylor (2003).

be written:[7]

$$(y_t - \mu_0) = \sum_{j=1}^{p} \beta_j (y_{t-j} - \mu_0) + \left[\sum_{j=1}^{p} \beta_j^* (y_{t-j} - \mu_0)\right] \left[1 - \exp[-\theta(y_{t-d} - \mu_0)^2]\right] + \varepsilon_t \tag{1}$$

where $\varepsilon_t \sim N(0, \sigma_t^2)$, $\theta \in (0, +\infty)$ and μ denotes the mean or long-run equilibrium of the process. The exponential term $\left[1 - \exp[-\theta(y_{t-d} - \mu_0)^2]\right]$, a symmetrically inverse-bell-shaped function, is termed the transition function since it can be thought of as smoothly determining the transition of the autoregressive process between two extreme regimes, an inner regime and an outer regime. The inner regime corresponds to $y_{t-d} = \mu_0$, when the transition function vanishes and (1) becomes a linear AR(p) model:

$$(y_t - \mu_0) = \sum_{j=1}^{p} \beta_j (y_{t-j} - \mu_0) + \varepsilon_t. \tag{2}$$

The outer regime corresponds, for given θ, to $\lim_{|y_{t-d}-\mu_0| \to \infty} \left[1 - \exp[-\theta(y_{t-d} - \mu_0)^2]\right] = 1$, where (1) becomes a different AR(p) model:

$$(y_t - \mu_0) = \sum_{j=1}^{p} (\beta_j + \beta_j^*)(y_{t-j} - \mu_0) + \varepsilon_t \tag{3}$$

with a correspondingly different speed of mean reversion so long as $\beta_j^* \neq 0$ for at least one value of j.

In any particular application of the ESTAR model, of course, the parameters p and d must be chosen, and a number of selection procedures have been suggested in the literature (see Lundbergh, Teräsvirta and van Dijk, 2003 for a recent discussion of alternative methods of nonlinear model selection). In the present context, economic intuition suggests a presumption in favour of smaller values of the delay parameter d rather than larger values, in that it is hard to imagine why there should be very long lags before the real exchange rate begins to adjust in response to a shock, especially where one is using annual data. In the research reported below, we used the model procedure suggested by Granger

[7] It is more common to write a general ESTAR model in the form:

$$y_t = \beta_0 + \sum_{j=1}^{p} \beta_j y_{t-j} + \left[\beta_0^* + \sum_{j=1}^{p} \beta_j^* y_{t-j}\right] \left[1 - \exp[-\theta(y_{t-d} - c)^2]\right] + \varepsilon_t.$$

This, however, can be straightforwardly reparametarised as

$$y_t - \mu_0 = \sum_{j=1}^{p} \beta_j (y_{t-j} - \mu_0) + \left[\sum_{j=1}^{p} \beta_j^* (y_{t-j} - \mu_0^*)\right] \left[1 - \exp[-\theta(y_{t-d} - c)^2]\right] + \varepsilon_t\,,$$

where $\mu_0 = \beta_0/(1 - \sum_{j=1}^{p} \beta_j)$ and $\mu_0^* = -\beta_0^*/(\sum_{j=1}^{p} \beta_j^*)$. Now, unless $\mu_0 = \mu_0^*$ in this parameterisation, the process $\{y_t\}$ reverts towards a shifting mean, equal to μ_0 when $y_{t-d} = c$ (and the transition function vanishes); equal to $[\mu_0(1-\sum_{j=1}^{p} \beta_j) - \mu_0^* \sum_{j=1}^{p} \beta_j^*]/(1-\sum_{j=1}^{p} \beta_j^* - \sum_{j=1}^{p} \beta_j^*)$ when y_{t-d} is a long way away from c (and the transition function is equal to unity); and equal to some combination of these two values for intermediate deviations of y_{t-d} from c. Ruling this out by imposing $\mu_0 = \mu_0^*$, but allowing $c \neq \mu_0$, however, results in a model where the speed of reversion towards the mean of the process depends not upon the size of the deviation of y_{t-d} from the mean μ_0, but upon the size of the deviation from some other &xed point c, for which it is hard to attach an economic intuition. It therefore seems reasonable to assume further that $c = \mu_0$, resulting in the speci&cation of the ESTAR model as we have written it in (1) (following Taylor et al., 2001).

and Teräsvirta (1993) and Teräsvirta (1994). This involves &rst choosing the order of the autoregression, p, by an examination of the partial autocorrelation function of the series and then estimating an equation similar in form to (1) but with the second term on the right-hand side replaced with cross products of y_{t-j} and &rst, second and third powers in y_{t-d}, for various values of d. This can be interpreted as a third-order Taylor series expansion of (1). The resulting equation is nonlinear in some of the variables but is linear in the parameters, and so can be estimated by ordinary least squares, and a test of the exclusion restrictions on the power and cross-product terms in this estimated equation is then a test for linearity against a linear alternative. The value of d is then chosen as that which gives the largest value of this test statistic.In the Monte Carlo study of Teräsvirta (1994), this selection procedure was shown to work well in terms of choosing the correct value of the delay parameter.[8]

ESTAR models of the form (1) have been successfully applied to real exchange rates by, among others, Taylor et al. (2001) and Kilian and Taylor (2003), who effectively impose a constant value of the long-run equilibrium real exchange rate. In the analysis presented below, we extend this framework by introducing a potentially time-varying equilibrium value of the real exchange rate in order to allow for HBS effects. This can be analysed in the above framework by setting $\{y_t - \mu_0\} = \{q_t - \mu_t\}$ in (1), where q_t is the real exchange rate and μ_t is its time-varying equilibrium, so that the nonlinear ESTAR model employed in our investigation becomes:

$$(q_t - \mu_t) = \sum_{j=1}^{p} \beta_j (q_{t-j} - \mu_{t-j}) + \left[\sum_{j=1}^{p} \beta_j^* (q_{t-j} - \mu_{t-j})\right] \left[1 - \exp[-\theta(q_{t-d} - \mu_{t-d})^2]\right] + \varepsilon_t. \tag{4}$$

Our empirical speci&cation for the time-varying equilibrium real exchange rate μ_t is discussed in the next section.

Further, we also allow for shifts in variance in the error term $\{\varepsilon_t\}$, rather than assuming homoscedasticity as in previous studies of nonlinearity in real exchange rate movements.[9] As discussed above, this seems particularly appropriate since our data span a number of exchange rate regimes. The empirical speci&cation for the residual variance is discussed in Section 4.

3 Productivity Differentials and Long-Run Equilibrium Real Exchange Rates

According to the HBS framework (Harrod, 1933; Balassa, 1964; Samuelson, 1964), a country experiencing relatively high productivity growth will &nd that

[8] Note that this procedure can also be used to discriminate between an exponential form of the transition function, as in (1), and a logistic form, since third-order terms disappear in the Taylor series expansion of an exponential function and so should be insigni&cant in the auxiliary regression. For further details, see Granger and Teräsvirta (1993) , Teräsvirta (1994) or Lundbergh, Teräsvirta and van Dijk (2003).

[9] An exception to this is the recent study by Paya and Peel (2005).

its exchange rate tends to return to a level where its currency is overvalued on PPP considerations, and that the apparent degree of overvaluation on PPP grounds increases with the size of the differential in productivity between the home and foreign economies.

Suppose a country experiences productivity growth primarily in its traded goods sector, and that the law of one price (LOP) holds among traded goods in the long run. Productivity growth in the traded goods sector will lead to wage rises in that sector without the necessity for price rises, but workers in the nontraded goods sector will also demand comparable pay rises, and this will lead to a rise in the price of nontradables and hence a rise in the overall price index. Since the LOP holds among traded goods and, by assumption, the nominal exchange rate has remained constant, this means that the upward movement in the home price index will not be matched by a movement in the nominal exchange rate so that, if PPP initially held, the home currency must now appear overvalued on the basis of comparisons made using price indices expressed in a common currency at the prevailing nominal exchange rate. The crucial assumption is that productivity growth is higher in the traded goods sector.

We can analyse this issue more formally as follows. Consider an economy (Home) that has two sectors, one producing a composite tradable good and one producing a composite nontradable good. Consumer utility is a function of a consumption index that is itself a geometric weighted average of consumption in the tradeable and nontradable composite goods, so that the consumption-based price index will be a geometric weighted average of the Home prices of tradables and nontradables:[10]

$$P \equiv P_T^{\gamma} P_N^{1-\gamma}, \tag{5}$$

where P_N and P_T denote the price of nontradeables and tradeables, respectively, P is the consumer price index and γ $(0 < \gamma < 1)$ is a constant parameter. In the long run, labour is perfectly mobile between sectors so that workers receive the same long-run real wage in each sector, i.e. $W_T/P = W_N/P$, where W_T and W_N represent the nominal wage in the tradeable and nontradable sectors, respectively. Therefore, the nominal wage is also equalised across sectors in the long run: $W_T = W_N = W$, say. However, &rms in each sector pay a long-run nominal wage that is equal to the marginal revenue product of labour in that sector, i.e. $W_T = W = P_T A_T$ and $W_N = W = P_N A_N$, where A_N and A_T denote the marginal product of labour in the tradable and nontradable sectors respectively. Hence, we have:

$$P_N/P_T = A_T/A_N, \tag{6}$$

or, using (5):

$$P = P_T (A_T/A_N)^{(1-\gamma)}. \tag{7}$$

[10] See, e.g., Obstfeld and Rogoff (1996 pp. 226-8).

Equations (6) and (7) encapsulate the HBS condition that relatively higher productivity growth in the tradables sector will tend to generate a long-run rise in the relative price of nontradables and hence a rise in the overall price level. This translates into an appreciation of the real exchange rate through the law of one price, which is expected to hold among tradeable goods in the long run:

$$P_T^* = P_T S, \tag{8}$$

where an asterisk (here and below) denotes a variable in the trading economy (Foreign) or a Foreign coefficient and S is the exchange rate (the Foreign price of Home currency). If we assume that an equation similar to (5) (the definition of the consumer price index) holds for the Foreign economy, then an equation for the Foreign economy analagous to (7) can be derived by similar reasoning:

$$P^* \equiv P_T^*(A_T^*/A_N^*)^{(1-\gamma^*)}. \tag{9}$$

Equations (7), (8) and (9) then together imply the following expression for the long-run equilibrium real exchange rate, Q:

$$Q \equiv SP/P^* = (A_T/A_N)^{(1-\gamma)}/(A_T^*/A_N^*)^{(1-\gamma^*)}. \tag{10}$$

If the composition of consumption in terms of tradable and nontradable goods is similar in both countries (i.e. γ is close to γ^*), then (10) implies that Q will diverge from unity (the purchasing power parity level) according to whether productivity in the tradables sector relative to the non-tradables sector is greater in the Home or in the Foreign economy.

Suppose, however, that productivity in the nontradables sector in both the Home and Foreign economies is constant, then, taking logarithms of (10) we have:

$$q = \mu_0 + \mu_1 a_T - \mu_2 a_T^*, \tag{11}$$

where lower-case letters denote logarithms and the constant parameters μ_0, μ_1 and μ_2 are given by $\mu_0 = -(1-\gamma)a_N + (1-\gamma^*)a_N^*$, $\mu_1 = (1-\gamma) > 0$ and $\mu_2 = (1-\gamma^*) > 0$.

Equation (11) expresses the quintessence of the HBS effect: countries with relatively high levels of productivity will tend to have a less competitive equilibrium real exchange rate or, equivalently, rich countries will tend to have a higher exchange rate-adjusted price level on average.[11]

Ideally, one would like to have data on tradables sector productivity in order to investigate the HBS effect empirically. Over the long spans examined in this

[11] Note that the HBS effect can be mitigated by having a relatively high level of productivity in the nontradable goods sector. If however we assume $a_N \approx a_N^*$ and $\gamma \approx \gamma^*$, then $\mu_0 \approx 0$ in (11), so that variations in relative productivity in the tradeable goods sectors are entirely responsible for deviations from long-run PPP. In practice, estimates of μ_0 may vary from zero simply as a reflection of the arbitrary bases used in construction of the price indices.

paper, this is not available. If, however, productivity in the nontradables sector is assumed to be stagnant, then productivity in overall output will be directly proportional to tradables-sector productivity. If, in addition, we assume that the labour force is proportional to total population, then we can measure the productivity terms driving the HBS effect as the ratio of total national output i.e. real GDP to total population, as in the classic studies of Balassa (1964) and Officer (1976a,b). In our empirical analysis we maintain both of these assumptions to that $\{\mu_t\}$, the long-run equilibrium level of $\{q_t\}$ in the ESTAR model (4), is modelled as:[12]

$$\mu_t = \mu_0 + \mu_1 a_t - \mu_2 a_t^*, \tag{12}$$

where a_t^* and a_t are the logarithm of the ratio of real GDP to population in the Foreign and Home economies at time t, respectively.[13]

4 The Volatility of the Real Exchange Rate Across Nominal Regimes

As documented by Frankel and Rose (1995), there is an abundance of empirical evidence that convincingly argues that the volatility of real exchange rates tends to vary across nominal exchange rate regimes and, in particular, tends to be much higher during ! oating-rate regimes. Studies which have reached this conclusion from an analysis of postwar data include Mussa (1986, 1990), Eichengreen (1988), Baxter and Stockman (1989) and Flood and Rose (1995). The Baxter and Stockman (1989) and Flood and Rose (1995) studies are particularly interesting in that they demonstrate that, although both real and nominal exchange rates tend to be much more volatile during ! oating exchange rate regimes, the underlying macro fundamental variables display no such regime-speci&c shifts in volatility. In a more recent and wide-ranging analysis of the exchange rates of twenty countries over a period of a hundred years, Taylor (2002) &nds that the variance of the error term in simple autoregressive real exchange rate equations is almost perfectly correlated with the variance of the nominal exchange rate.

These studies suggest, therefore, that if one wishes to estimate a real exchange rate model spanning a number of nominal exchange rate regimes, it is

[12] In fact, as far the productivity of the nontradables sectors is concerned, we need only assume that there is no relative effect of nontradables sector productivity on the real exchange rate, not necessarily that nontradables sector productivity is constant. This follows because $\mu_0 = -(1-\gamma)a_N + (1-\gamma^*)a_N^*$ in (11). This term will be a non-zero constant if a_N and a_N^* are constant, but it will also be constant even if a_N and a_N^* are time-varying, so long as the terms $(1-\gamma)a_N$ and $(1-\gamma^*)a_N^*$ differ by a constant amount over time. This would follow where both nontradable-sector productivity growth and the share of nontradables in consumption were similar in the Home and Foreign economies.

[13] Although we have developed the HBS framework in terms of labour productivity rather than total factor productivity, similar results can be obtained relating to total factor productivity with a slightly more sophisticated model (Froot and Rogoff, 1991).

important to allow for shifts in volatility in the error term of the empirical model. In their long-span real exchange rate study, Lothian and Taylor (1996) explicitly acknowledge this issue and allow for shifts in volatility in a very general way by using heteroscedastic-robust estimation methods. In the present study, however, we speci&cally build in the possibility of shifts in volatility across nominal exchange rate regimes in designing our econometric model.[14]

We are particularly concerned that there may have been a downward shift in the volatility of real exchange rates during &xed nominal exchange rate regimes, such as the Bretton Woods and the interwar and classical gold standard periods. As demonstrated by Obstfeld, Shambaugh and Taylor (2004a, 2004b) and Reinhart and Rogoff (2004), however, it is important not simply to impose constraints according to official regime classi&cations but, rather, to use the data to determine *de facto* rather than *de jure* nominal exchange rate regimes. In particular, Obstfeld, Shambaugh and Taylor (2004a) test for *de facto* adherence to the classical Gold Standard for a number of countries, on the criterion of whether or not the end-of-month exchange rate against the pound sterling stays within $\pm 2\%$ bands over the course of a year. On the basis of this classi&cation, these authors &nd that the US dollar was *de facto* on the gold standard over the period January 1883 to June 1914, and the French franc over the period April 1872 to June 1914. Using a similar methodology, Obstfeld, Shambaugh and Taylor (2004b) &nd that the sterling-dollar rate was &xed *de facto* for the period April 1925 to August 1931 and the sterling-franc rate for the period August 1928 to August 1931. Under the Bretton Woods System, both exchange rates were pegged against the dollar from 1946 until the breakdown of the System around 1971, although sterling was devalued in September 1949 and again in November 1967. Hence, for our annual series, the sets of years during which the sterling-dollar and franc-sterling rates were *de facto* &xed according to Obstfeld, Shambaugh and Taylor (2004a, 2004b) are given by:[15]

$$Fix(US) = \{1883 - 1913, 1926 - 1930, 1946 - 1948, 1950 - 1966, 1968 - 1970\} \tag{13}$$

$$Fix(France) = \{1872 - 1913, 1928 - 1930, 1946 - 1948, 1950 - 1966, 1968 - 1970\} \tag{14}$$

Accordingly, if $\sigma^2_{i,t}$ is the residual variance at time t for country i ($i = US$ or $i = France$), we can allow $\sigma^2_{i,t}$ to vary across *de facto* &xed and ! oating nominal regimes fact by modelling it as:

$$\sigma^2_{i,t} = \sigma^2_{i,Float}[1 - I_t\{t \in Fix(i)\}] + \sigma^2_{i,Fix} I_t\{t \in Fix(i)\} \tag{15}$$

[14] Paya and Peel (2005) adopt an alternative method of allowing for heteroscedasticity in a nonlinear framework by employing a wild bootstrap procedure.

[15] We are grateful to Jay Shambaugh for helpful discussions and correspondence on this issue.

where $I_t\{.\}$ is an indicator variable, equal to unity when the statement in braces is correct. The parameters $\sigma^2_{i,Float}$ and $\sigma^2_{i,Fix}$ can then be estimated, along with those for the conditional mean, by maximum likelihood.

5 Data

For nominal exchange rates and aggregate prices, we used the series from Lothian and Taylor (1996) updated with data from the International Financial Statistics (IFS) CD-ROM data base.[16]

The real income data and population data used in this paper were constructed using a variety of sources. Data for UK real income for the period prior to 1864 were derived from Clark (2001). Data for UK real income for the periods 1864-69, 1870-1994 and 1995-2001 came from Feinstein (1972), Maddison (1995) and the IFS, respectively. Data for US real income came from Officer (2002) for 1791-1869, from Maddison (1995) for 1870-1994 and from the IFS thereafter. Data for French real income came from Toutain (1997) for 1815-1870, from Maddison (1995) for (1870-1994) and from the IFS thereafter. Data for UK population for 1791-1800 came from Populstat[17], for 1801-1980 from Mitchell (1988) and for the remaining period from the IFS. Data for US population came from Populstat for 1791- 1994 and from the IFS thereafter. Data for French population came from Mitchell (1998) for 1815-1869, from Maddison (1995) for 1870-1994 and from the IFS thereafter.

6 Empirical Results

6.1 Linear estimation results

As a preliminary examination of the data, we tested for the presence of unit roots in the processes generating the time series, under the maintained hypothesis of linearity, using standard linear unit root tests, the results of which are reported in Table 1.[18] In each case, consistent with the results of Lothian and Taylor

[16] For a full description of the earlier data and their sources see the appendix to Lothian and Taylor (1996). While we have extended our data set from that used in Lothian and Taylor (1996) to include an additonal ten years or so of data up to 2001, we have had to discard some observations at the beginning of the sample because the population data we use begins only in 1820. Nevertheless, the data set still spans over 180 years.

[17] http://www.library.uu.nl/wesp/populstat/populhome.html.

[18] In particular, following Perron (1988) and Lothian and Taylor (1996), we estimated equations of the form:

$$q_t = \kappa + \lambda(t - \frac{T}{2}) + \delta q_{t-1} + u_t$$

where T is the sample size and u_t is an error term. The following null hypotheses were then tested:

$$H_A : \delta = 1;\ H_B : (\kappa, \lambda, \delta) = (0, 0, 1);\ H_C : (\lambda, \delta) = (0, 1),$$

using either the standard t-statistics and F-statistics, τ_τ(although referred to the distributions calculated by Fuller, 1976 and Dickey and Fuller, 1981), or the the corresponding transforma-

(1996) (although using data sampled over a slightly different period), we are able to reject the unit root hypothesis at the &ve percent level or lower.

We then proceeded to estimate linear autoregressive models for each of the real exchange rates, with a lag length of one year, as suggested by examination of the partial autocorrelation function for each of the series. The results are reported in Table 2 and they are qualitatively similar to those reported by Lothian and Taylor (1996). Given the importance of data span in an analysis of low-frequency properties, it is perhaps not surprising, however, that the measured persistence of the two real exchange rates is slightly higher than that reported in our earlier work, where we used a slightly longer data set (1791-1990 for sterling dollar, as opposed to 1820-2001 in the present study, for example). Nevertheless, the point estimate of the autoregressive coefficient of 0.902 for sterling-dollar is close to the point estimate of 0.887 of Lothian and Taylor (1996), and implies a half-life of adjustment of 6.78 years. Again in line with Lothian and Taylor (1996), the results for the sterling-franc imply a faster speed of adjustment, with a point estimate of the autoregressive coefficient of 0.831 and a corresponding half-life estimate of 3.75 years.

In brief, therefore, the linear estimation results are noteworthy for two reasons, both of which serve to con&rm previous &ndings reported in the literature. First, it is possible to reject the unit root hypothesis at standard signi&cance levels using sufficiently long spans of data (Frankel, 1986; Lothian and Taylor, 1996, 1997). Second, although the unit root hypothesis can be rejected, the estimated half-lives of shocks to the real exchange rates involved are extremely slow— ranging from about 3.75 to 6.78 years. Given that the volatility of real exchange rates implies that they must be largely driven by nominal and &nancial shocks which one would expect to mean revert at a much faster rate, this evidence is con&rmatory of Rogoff's 'purchasing power parity puzzle' (Rogoff, 1996).

Note, however, that for sterling-franc there is signi&cant evidence of autoregressive conditional heteroskedasticity (ARCH) in the estimated residuals. Although we have used heteroscedasticity-robust estimated standard errors, this does suggest that it may be fruitful to try and model this heteroscedasticity di-

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tions of these statistics due to Phillips (1987) and Phillips and Perron (1988), $Z(\tau_\tau)$, $Z(\Phi_2)$ and $Z(\Phi_3)$.

Phillips and Perron (1988) and Schwert (1989) demonstrate that the Phillips-Perron non-parametric test statistics may be subject to distortion in the presence of moving-average components in the time series. Accordingly, as in Lothian and Taylor (1996), we therefore tested for the presence of moving-average components and could detect no statistically signi&cant such effects in either of the real exchange rate series.

If the unit root hypothesis cannot be rejected at this stage, then greater test power may be obtained by estimating the equation:

$$q_t = \kappa^* + \delta^* q_{t-1} + u_t^*$$

and testing the hypotheses:

$$H_D : \delta^* = 1; \; H_E : (\kappa^*, \delta^*) = (0, 1),$$

using the corresponding t-statistics and F-statistics, τ_μ and Φ_1 (again referred to the Dickey-Fuller distributions), or their Phillips-Perron transformations, $Z(\tau_\mu)$ and $Z(\Phi_1)$.

rectly. Alternatively, or in addition, the significant ARCH test statistic may simply be indicative of significant residual outliers, suggesting that the conditional mean is misspecified in the linear formulation.

6.2 Nonlinear estimation results

6.2.1 univariate estimation results

Bringing together the previous discussion on modelling nonlinearity, the Harrod-Balassa-Samuelson effect and regime-varying volatility, we can now summarise our empirical nonlinear model. We treat the UK as the Home economy and, for notational convenience, we introduce a country subscript on parameters and variables. Thus, $q_{France,t}$ is the real exchange rate between the UK and France and $q_{US,t}$ is the real exchange rate between the UK and the US. Further, treating the UK as the Home economy, Home productivity, denoted a_t in equation (12), becomes UK productivity at time t, denoted $a_{UK,t}$. The Foreign economy then becomes either France or the US, so that the Foreign productivity variable of equation (12), a_t^*, becomes either French or US productivity, denoted $a_{France,t}$ and $a_{US,t}$ respectively. The full empirical model may thus be written, for $i = US, France$:

$$\begin{aligned}(q_{i,t} - \mu_{i,t}) &= \textstyle\sum_{j=1}^{p} \beta_{i,j}(q_{i,t-j} - \mu_{i,t-j}) \\ &\quad + \left[\textstyle\sum_{j=1}^{p} \beta_{i,j}^*(q_{i,t-j} - \mu_{i,t-j})\right] \\ &\quad \times \left[1 - \exp[-\theta_i(q_{i,t-d} - \mu_{i,t-d})^2]\right] + \varepsilon_{i,t} \end{aligned} \tag{16}$$

$$\mu_{i,t} = \mu_{i,0} + \mu_{i,1}a_{UK,t} - \mu_{i,2}a_{i,t} \tag{17}$$

$$\varepsilon_{i,t} \sim N(0, \sigma_{i,t}^2) \tag{18}$$

$$\sigma_{i,t}^2 = \sigma_{i,Float}^2[1 - I_t\{t \in Fix(i)\}] + \sigma_{i,Fix}^2 I_t\{t \in Fix(i)\}. \tag{19}$$

As before, $I_t\{.\}$ is an indicator variable, equal to unity when the statement in braces is true and $Fix(US)$ and $Fix(France)$ are as defined in (13) and (14).

In practice, however, the final estimated models were simplified significantly due to the imposition of a number of insignificant restrictions. There was, moreover, no evidence of serial correlation beyond first-order on the basis of examination of the partial autocorrelation functions of the real exchange rates or from examination of the partial autocorrelation functions for the real exchange rate adjusted for relative productivity.[19] A &nal choice of &rst-order autoregression thus imposes the restrictions $\beta_{i,j} = 0$ and $\beta^*_{i,j} = 0$, for $j > 1$. The delay parameter, d, was chosen using the procedure suggested by Granger and Teräsvirta (1993) and Teräsvirta (1994), as outlined in Section 2 and, as anticipated, a delay of one year appeared to capture adequately the nonlinear dynamics of the ESTAR transition function ($d = 1$).[20] Further, the coefficient on foreign productivity, when estimated freely, was numerically close to and insigni&cantly different from being equal to that on domestic productivity, so that productivity was entered in relative terms ($\mu_{i,1} = \mu_{i,2}$). In addition, for both the US and France, the estimated value of $\mu_{i,0}$ was found to be insigni&cantly different from zero at the &ve percent level and was set to zero ($\mu_{i,0} = 0$). Finally, unrestricted estimates of $\beta_{i,1}$ and $\beta^*_{i,1}$ were numerically close to plus and minus unity, respectively, and the restrictions $\beta_{i,1} = 1$ and $\beta^*_{i,1} = -1$ could not be rejected at the &ve percent level and were imposed.

Substituting (17) into (16), imposing these restrictions and rearranging, our &nal parsimonious empirical speci&cations were therefore of the form:[21]

$$\begin{aligned}[q_{i,t} - \mu_{i,1}(a_{UK,t} - a_{i,t})] \quad = \quad & [q_{i,t-1} - \mu_{i,1}(a_{UK,t-1} - a_{i,t-1})] \\ & \times \exp\left[-\theta_i[q_{i,t-1} - \mu_{i,1}(a_{UK,t-1} - a_{i,t-1})]^2\right] + \varepsilon_{i,t}\end{aligned} \tag{20}$$

$$\varepsilon_{i,t} \sim N(0, \sigma^2_{i,t}) \tag{21}$$

$$\sigma^2_{i,t} = \sigma^2_{i,Float}[1 - I_t\{t \in Fix(i)\}] + \sigma^2_{i,Fix} I_t\{t \in Fix(i)\}. \tag{22}$$

[19] As noted in Section 2, Granger and Teräsvirta (1993) suggest determining the order of the autoregression in STAR models by examination of the partial autocorrelation function (PACF). This is problematic in the present case, however, since we are jointly estimating the time-varying mean of the series to which we are simultaneously &tting an ESTAR model□ i.e.$(q_{i,t} - \mu_{i,t}) \equiv [q_{i,t} - (\mu_{i,0} + \mu_{i,1} a_{UK,t} - \mu_{i,2} a_{i,t})]$. While examination of the PACF for each of the real exchange rates and each of the productivity series did indeed suggest nothing greater than &rst-order serial correlation, it is well known that a linear combination of AR(1) processes may not necessarily be AR(1) (Granger and Morris, 1976). However, the PACF for the real exchange rate series adjusted for relative productivity, i.e. $[q_{i,t} - (a_{UK,t} - a_{i,t})]$, also appeared to be exhibit at most &rst-order serial correlation and, together with our intuitive preference for &rst-order model with annual data, it therefore seemed reasonable to proceed on this basis. We did, however, check for remaining serial correlation in the &nal estimated models (and found none).

[20] In addition, terms involving third-order powers of y_{t-d} were in every case insigni&cant in the estimated auxiliary regressions, implying that a logistic transition function could be rejected in favour of an exponential transition function.

[21] Note that the transition function in (20) is of the form exp[.] rather than the standard ESTAR transition function of the form $\{1 - \exp[.]\}$, as in (16). This is because, with a &rst-order autoregression ($\beta_{i,j} = 0$ and $\beta^*_{i,j} = 0$, for $j > 1$), the further restrictions $\beta_{i,1} = 1$ and $\beta^*_{i,1} = -1$ imply that deviations from long-run equilibrium follow a random walk in the close neighbourhood of equilibrium, when $\exp\left[-\theta_i[q_{i,t-1} - \mu_{i,1}(a_{UK,t-1} - a_{i,t-1})]^2\right] \approx \exp[0] = 1$, but become increasingly mean-reverting as the size of the deviation grows and $\exp\left[-\theta_i[q_{i,t-1} - \mu_{i,1}(a_{UK,t-1} - a_{i,t-1})]^2\right] \to 0$.

The univariate estimation results of this model, obtained by maximum likelihood estimation, are reported in Table 3. In both cases, a good &t is indicated, with the coefficient of determination in each case improving upon that obtained using a linear model (compare Table 2). Moreover, the residual diagnostics (calculated using the residuals standardized by the square root of the estimated variance function) are in each case satisfactory.[22] The major difference between the US and French results is that, for sterling-franc, the estimated coefficient $\widehat{\mu}_{France,1}$ was found to be insigni&cant at the &ve percent level and was set to zero.

These estimation results are noteworthy for a number of reasons. First, there is signi&cant evidence of nonlinear mean reversion, as shown by the fact that the estimated transition parameter $\widehat{\theta}_i$ is in both cases strongly signi&cantly different from zero. Note, however, that the ratio of this estimated coefficient to its standard error —the "t-ratio"— cannot be referred to the Student-t or normal distribution for purposes of inference, since under the null hypothesis $H_0 : \theta_i = 0$, $q_{i,t}$ follows a linear unit root process.[23] This introduces a singularity under the null hypothesis so that standard inference procedures cannot be used, analogously to the way in which standard inference procedures cannot be used in the usual Dickey-Fuller or augmented Dickey-Fuller tests for a linear unit root. Indeed, testing the null hypothesis $H_0 : \theta_i = 0$ is tantamount to a test of the null hypothesis against the alternative hypothesis of nonlinear mean reversion, rather than against the alternative of linear mean reversion.[24] Therefore, because the distribution of the estimator of θ_i is unknown under the null hypothesis, we calculated the empirical marginal signi&cance level of the ratio of the estimated coefficient to the estimated standard error by Monte Carlo methods under the null hypothesis that the true data generating process for the logarithm of both of the real exchange rate series was a random walk, with the parameters of the data generating process calibrated using the actual real exchange rate data over the sample period.[25] From these empirical marginal signi&cance levels (reported in square brackets below the coefficient estimates in Table 3), we see that the estimated transition parameter is signi&cantly different from zero with a marginal signi&cance level of virtually zero in each case. Since these tests may

[22] Note that these residual diagnostics should be treated only as indicative, since the standardized residuals are functions of estimated variance parameters.

[23] In addition, under the null hypothesis, $H_0 : \theta_i = 0$, the autoregressive parameters of the nonlinear part of the speci&cation are unidenti&ed; see Davies (1987), Hansen (1996).

[24] Our approach may thus be seen in some ways to be equivalent to unit root tests with the alternative of smooth transiton nonlinearity as developed by Kapetenios, Shin and Snell (2003). Eklund (2003) develops a joint test of nonstationarity and linearity &nds that the linear unit root hypothesis can be rejected in favour of nonlinear mean reversion for a number of real exchange rates, consistent with the approach in this paper and in Taylor, Peel and Sarno (2001).

[25] The empirical signi&cance levels were based on $5,000$ simulations of length 280, initialized at $q_1 = 0$, from which the &rst 100 data points were in each case discarded. At each replication a system of ESTAR equations identical in form to those reported in Table 3 was estimated. The percentage of replications for which a "t-ratio" for the estimated transition parameters greater in absolute value than that reported in Table 3 was obtained was then taken as the empirical marginal signi&cance level in each case.

be construed as nonlinear unit root tests, the results indicate strong evidence of nonlinear mean reversion for each of the real exchange rates examined over the sample period.

Second, the estimated coefficient for the relative productivity term, $\widehat{\mu}_{i,1}$ is strongly signi&cantly different from zero for the case of sterling-dollar (an asymptotic t-ratio of nearly eight) and is correctly signed according to the Harrod-Balassa-Samuelson effect: relatively higher US productivity generates a real appreciation of the equilibrium value of the dollar against the pound. For the case of sterling-franc, however, there is no signi&cant evidence of the HBS effect.[26]

6.2.2 joint estimation results

In order to gain efficiency in the estimation, we also estimated the US and French equations jointly by full information maximum likelihood (FIML), assuming a constant correlation coefficient between the French and US regression errors, so that the covariance matrix takes the form:

$$\begin{bmatrix} \varepsilon_{US,t} \\ \varepsilon_{France,t} \end{bmatrix} \sim N(O, \Sigma_t) \tag{23}$$

$$\Sigma_t = \begin{bmatrix} \sigma^2_{US,t} & \rho \cdot \sigma_{US,t} \cdot \sigma_{France,t} \\ \rho \cdot \sigma_{US,t} \cdot \sigma_{France,t} & \sigma^2_{France,t} \end{bmatrix} \tag{24}$$

where $\sigma^2_{i,t}$ ($i = US,\ France$) is as de&ned in (15) and ρ is the constant correlation coefficient. The joint estimation results are reported in Table 4.[27]

The FIML estimates of the residual variances are almost identical to those obtained using single-equation maximum likelihood, and the estimated correlation coefficient between the US and French residual series is strongly signi&cantly different from zero, with a point estimate of 0.169. Moreover, the HBS slope coefficient is again signi&cantly different from zero at the &ve percent level only for the US, for which there is a slight increase in the point estimate of this coefficient from 0.125 to 0.140. Perhaps the most striking aspect of the FIML estimation results, however, is the increase in the point estimates of the transition parameter, $\widehat{\theta}_i$, which increases from 2.594 to 3.023 for the US and from 3.064 to 3.218 for France. We again calculated the empirical distribution of the $t-$ratios for the estimated transition parameters, and they were each found to be highly signi&cantly different from zero.[28]

[26] These results are in line with the present authors conjecture in Lothian and Taylor (2000), based on an analysis of nonlinear trends in these real exchange rates.

[27] Since the franc ceased to exist after 1998, the joint estimation results are for the sample period 1820-1998.

[28] The empirical distributions of the $t-$ratios for θ_i were calculated similarly to the univariate case as described above (i.e. from Monte Carlo experiments in which the data generating process is a random walk), except that they were based on joint estimation of the French and US models.

Although we do not report any sophisticated residual diagnostics for the nonlinear FIML estimation results (since it is not clear what test diagnostic statistics would be applicable), for both France and the US, the &t and the &tted residuals were in fact almost identical to those of the univariate models reported in Table 3.

6.2.3 calculating the average speed of mean reversion

We proceeded to gain a measure of the mean-reverting properties of the estimated nonlinear models through calculation of their implied half-lives, using the models estimated by FIML.[29] Effectively, this involves comparing the impulse-response functions of the models with and without initial shocks. Thus, we examined the dynamic adjustment in response to shocks through impulse response functions which record the expected effect of a shock at time t on the system at time $t+j$. For a univariate linear model, the impulse response function is equivalent to a plot of the coefficients of the moving average representation (see e.g. Hamilton, 1994, p. 318). Estimating the impulse response function for a nonlinear model, however, raises special problems both of interpretation and of computation (Gallant, Rossi and Tauchen, 1993; Koop, Pesaran and Potter, 1996). In particular, with nonlinear models, the shape of the impulse-response function is not independent with respect to either the history of the system at the time the shock occurs, the size of the shock considered, or the distribution of future exogenous innovations. Exact estimates can only be produced — for a given shock size and initial condition — by multiple integration of the nonlinear function with respect to the distribution function each of the j future innovations, which is computationally impracticable for the long forecast horizons required in impulse response analysis.

In the research reported in this paper, we calculated the impulse response functions, both conditional on average initial history and conditional on initial real exchange rate equilibrium, using the Monte Carlo integration method discussed by Gallant, Rossi and Tauchen (1993). The basic idea is to calculate a baseline forecast for a large number of periods ahead using the estimated model. We then calculate a second forecast but this time with a shock in the initial period. The difference between the baseline forecast path and the shocked forecast path then gives the impulse response function. In each case, the forecast path is calculated by simulating the model a large number of times and taking the average. The discrete number of years it takes for the effect of the shock on the level of the real exchange rate to dissipate by &fty percent is then taken as the estimated half life for that size of shock.[30]

We carried out two sets of simulations, one in which the real exchange rate is assumed to be at its long-run equilibrium prior to the shock, and one in which the real exchange rate response is calculated taking the average value of the real exchange rate over the Bretton Woods period as the initial value.[31]

[29] Using the models estimated by univariate maximum likelihood resulted in qualitatively identical results.

[30] This de&nition of the half-life may be problematic where the impulse response function is non-monotonic, since the effect of the shock on the level of the real exchange rate may drop below &fty percent of its initial value and then rise above it again. Fortunately, in the cases examined in this paper, this was not the case.

[31] All simulations were carried out using initial values of the variables corresponding to the post-Bretton Woods period — 1973-2001 for sterling-dollar, and 1973-1998 for sterling-franc. In our &rst estimation of the impulse response functions we condition on initial equilibrium by setting the initial lagged values of the real exchange rate equal to the estimated equilibrium level, given the lagged value of relative productivity and the estimated coefficient:

The estimated half-lives of the two real exchange rate models, calculated for six sizes of shock, conditional on average initial history over the post-Bretton woods sample periods period (1973 – 2001 for sterling-dollar, 1973 – 1998 for sterling-franc), or on initial equilibrium, are shown in Table 5.[32] They illustrate well the nonlinear nature of the estimated real exchange rate models, with larger shocks mean reverting much faster than smaller shocks and shocks conditional on average history mean reverting much faster than those conditional on initial equilibrium. In particular, for shocks of ten percent or less and conditional on average initial history, the half-life is in both cases two years, while larger shocks

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$\widehat{\mu}_{i,1}(a_{UK,t-1} - a_{i,t-1})$ for $i = US,\ France$, where $\widehat{\mu}_{i,1}$ denotes the values reported in Table 4, i.e. $\widehat{\mu}_{US,1} = 0.140$ and $\widehat{\mu}_{France,1} = 0$. We then used a total of $5,000$ replications to produce each next-step-ahead forecast in the sequence, conditional on the previous forecast, and took the average over the $5,000$ as the forecast value for that step. This is done for 20 steps ahead, with and without an additive shock at time t and the sequence representing the difference between the two paths is taken as the impulse response function. Since we use a large number of simulations, by the Law of Large Numbers this procedure should produce results virtually identical to that which would result from calculating the exact response functions analytically by multiple integration (Gallant, Rossi and Tauchen, 1993).

This procedure was then modified as follows in order to produce an estimate of the impulse-response function conditional on the average history of each of the real exchange rates. Starting at the first data point (for 1974), $q_{i,t-1}$ is set equal to $\{|q_{i,1973} - \widehat{\mu}_{i,1}(a_{UK,1973} - a_{i,1973})| + \widehat{\mu}_{i,1}(a_{UK,1973} - a_{i,1973})\}$. If $[q_{i,1973} - \widehat{\mu}_{i,1}(a_{UK,1973} - a_{i,1973})] > 0$, this is just $q_{i,1973}$ itself. If, however, $[q_{i,1973} - \widehat{\mu}_{i,1}(a_{UK,1973} - a_{i,1973})] < 0$, then $\{|q_{i,1973} - \widehat{\mu}_{i,1}(a_{UK,1973} - a_{i,1973})| + \widehat{\mu}_{i,1}(a_{UK,1973} - a_{i,1973})\}$ is the number which is an equal absolute distance above the estimated equilibrium value $\widehat{\mu}_{i,1}(a_{UK,1973} - a_{i,1973})$ as $q_{i,1973}$ is below it. This transformation is necessary because we consider only positive shocks and it is innocuous because of the symmetric nature of ESTAR adjustment below and above equilibrium. A 20-step forecast is then produced using 200 replications at each step, with and without a positive shock of size $log(1 + k/100)$ at time t, using the estimated ESTAR model, and realizations of the differences between the two forecasts are calculated and stored as before. We then move up one data point (hence setting $t - 1 = 1974$), and repeat this procedure. Once this has been done for every data point up to the end of the sample period, an average over all of the simulated sequences of differences in the paths of the real exchange rates with and without the shock at time t is taken as the estimated impulse response function conditional on the average history of the given exchange rate and for a given shock size.

[32] For linear time series models the size of shock used to trace out an impulse response function is not of particular interest since it serves only as a scale factor, but it is of crucial importance in the nonlinear case. In the present application we are particularly concerned with the effect of shocks to the level of the real exchange rate. Given a particular value of the log real exchange rate at time t, $q_{i,t}$ — whether this be the historical value or the estimated equilibrium level — a shock of k percent to the level of the real exchange rate involves augmenting $q_{i,t}$ additively by $log(1+k/100)$. (For small k, $log(1+k/100)$ is of course approximately equal to $k/100$. This approximation is not, however, good for the larger shocks considered in this paper.) This raises a problem, however, in the calculation of the half-lives, since although the natural measure might be the discrete number of years taken until the shock to the level of the real exchange rate has dissipated by a half — i.e. when the impulse response function falls below $log(1 + k/200)$ — this would make comparisons with previous research on linear time series models of real exchange rates difficult. Accordingly, although we define a k percent shock to the real rate as equivalent to adding $log(1 + k/100)$ to $q_{i,t}$, we calculate the half life as the discrete number of years taken for the impulse response function to fall below $0.5log(1+k/100)$, facilitating a comparison of our results with half lives estimated in previous studies. We considered six different sizes of percentage shock to the level of the real exchange rate, $k \in \{1, 5, 10, 20, 30, 40\}$. This allows us to compare and contrast the persistence of very large and very small shocks.

have a half life of one year or less. These results therefore accord broadly with those reported in Taylor et al. (2001), and shed some light on Rogoff's (1996) "PPP puzzle". Only for small shocks occurring when the real exchange rate is near its equilibrium do our nonlinear models consistently yield very long half lives in the range of three to five years or more, which Rogoff (1996) terms "glacial". Once nonlinearity is allowed for, even small shocks of one to five percent have a half life of two years or less, conditional on average history, and for larger shocks the speed of mean reversion is even faster.[33]

6.3 How important is the Harrod-Balassa-Samuelson Effect?

In Figure 1 we have plotted the sterling-dollar real exchange rate together with our measure of the Harrod-Balassa-Samuelson term, $HBS_t = \widehat{\mu}_{US,1}(a_{UK,t} - a_{US,t})$, where $\widehat{\mu}_{US,1}$is the fitted value of $\mu_{US,1}$ from Table 4. It is interesting how relative productivity captures the underlying trend depreciation of the real value of sterling against the dollar over this very long period. On the other hand, this raises the question of whether this common trend is purely a statistical artefact rather than an economic relationship. Our nonlinear estimation results do indicate that the Harrod-Balassa-Samuelson effect is strongly statistically significant in explaining movements in the equilibrium real exchange rate for sterling-dollar but not for sterling-franc over the one-hundred-and-eighty-year period under investigation. However, statistical significance is not quite the same thing as economic significance. In particular, if the Harrod-Balassa-Samuelson effect has been *economically* significant, then it should de better at explaining real exchange rate movements than complex time trends and we should also perhaps expect it to account for a substantial proportion of the variation in the real exchange rate over the sample period in question. Moreover, if reversion of the real exchaneg rate towards its fundamental equilibrium becomes stronger over longer time horizons, then the proportion of the variation in the real exchange rate explained by deviations from that equilibrium should be an increasing function of the time horizon. We investigated each of these issues.

6.3.1 trends, relative productivity and the real exchange rate

In Table 6, we report the results of some simple investigations of the importance of the HBS effect for sterling-dollar. In panel a) we report the results of regressing the real exchange rate onto the relative productivity term alone, $(a_{UK,t} - a_{US,t})$. The estimated slope coefficient is highly significant and the R^2 statistic reveals that the HBS effect appears to account for just over forty percent of variation in the real exchange rate over the last one-hundred-and-eighty years. This accords with Rogoff's (1996) intuition that real exchange rate variation is driven largely by nominal shocks (some sixty percent on our

[33] The 95% confidence bounds on the half lives were in every case less than one year in width.

measure) although a contribution of forty percent from the real side is clearly sizeable.

In panel b) of Table 5 we have reported the results of regressing relative productivity onto a cubic trend.[34] In Lothian and Taylor (2000), we found that a cubic trend was significant when added to a real exchange rate autoregression for sterling-dollar, and we conjectured that this term was in fact proxying for HBS effects. The fact that the cubic trend is able to explain some 97 percent of the variation in the relative productivity term appears to confirm this conjecture. In Lothian and Taylor (2000) we also pointed out, however, that a cubic trend in the HBS effect was to be expected on economic grounds also, given the increasing dominance of the UK over the US as an industrial power in the earlier part of the sample period, and the rise and subsequent dominance of the US over the UK in the later part of the sample period.

In panel c) of Table 5, we report the results of regressing the HBS-adjusted real exchange rate i.e.$[q_{US,t} - \widehat{\mu}_{US,1}(a_{UK,t} - a_{US,t})]$ onto its own lagged value and the cubic trend terms. The cubic trend terms are found to be individually and jointly insignificantly different from zero, consistent with the results and conjectures of Lothian and Taylor (1997). In addition, note that, also consistent with the analysis and conjectures of Lothian and Taylor (2000), the estimated half life of adjustment drops dramatically in the HBS-adjusted autoregression, (from the estimate of 6.78 years reported for the unadjusted sterling-dollar real exchange rate in panel a) of Table 2) to 3.19 years. Although these results are clearly only indicative, especially given the importance we have demonstrated of allowing for nonlinear adjustment in real exchange rates, they are nevertheless striking.

6.3.2 explaining real exchange rate variation due to HBS effects at different time horizons

While the finding that HBS effects accounted for about forty percent of real exchange rate variation for sterling-dollar over the whole sample period so that some sixty percent of the variation is due to nominal factors it seems likely that the contribution of real factors to real exchange rate movements will vary over different time horizons. In particular, it seems reasonable to expect nominal variablity to dominate mostly at shorter horizons, with real effects becoming more important at longer horizons.[35] In order to investigate this possibility, we estimated long-horizon regressions of the form

$$(q_{US,t+k} - q_{US,t}) = \alpha + \gamma_k[(a_{UK,t+k} - a_{US,t+k}) - (a_{UK,t} - a_{US,t})] + \nu_t \quad (25)$$

where α and γ_k are regression parameters, ν_t is the regression residual (which will in general be serially correlated for $k > 1$, since overlapping forecast errors

[34] The term cubic trend, is understood here to denote a function of time including terms in t and t^2 as well as t^3.

[35] Indeed, this seems to be the import of Rogoff's (1996) analysis of real exchange rate movements.

will contain some common information). By regressing the change in the real exchange rate from period t to period $t+k$ onto the change in relative productivity over the same period, this regression will capture the amount of variation in the $k-$year change in the real exchange rate that can be explained by the $k-$year change in the HBS effect.[36] Thus, if nominal rather than real effects dominate real exchange rate movements over short horizons, then we should expect a low R^2 for regressions with low values of k and increasing values of the R^2 as k increases.

The results of estimating the long-horizon regression for values of k from one to ten years are given in Table 7.[37] They are in accordance with our intuition. At the shortest horizon of one year, the change in productivity accounts for less than one percent of the variation in the annual change in the real exchange rate and the estimated value of γ_k has a p-value (marginal level of statistical signi&cance) of 0.47. It is not until the time horizon reaches &ve years that the estimated slope parameter becomes signi&cantly different from zero at the &ve percent level, with around four percent of the &ve-year real exchange rate change explained by the HBS effect. The signi&cance of the HBS effect reaches its peak at seven years, when nearly nine percent of the seven-year real exchange rate change is explained, after which it declines. By the tenth year, however, relative productivity is still signi&cant□ albeit at only the ten percent level□ in explaining the ten-year real exchange rate change, with around four percent explained.

7 Conclusion

A reading of the empirical literature on real exchange rates and purchasing power parity suggests a number of in! uences worthy of investigation. The &rst is the effect of real variables on the equilibrium levels of real exchange rates over the long run, and in particular the in! uence of relative productivity differentials□ the Harrod-Balassa-Samuelson effect. A second issue concerns the possibility of

[36] This is a variant on the standard long-horizon regression of the the $k-$period change of a variable onto its deviation from equilibrium at time t. In the present context, the standard long-horizon regression would take the form

$$(q_{US,t+k} - q_{US,t}) = \alpha + \gamma_k[q_{US,t} - \widehat{\mu}_{US,1}(a_{UK,t} - a_{US,t})] + \nu_t$$

A regression of this kind would be uninformative for our purposes, however, since the concept of real exchange rate equilibrium involves an element of pure PPP as well the HBS effect, and we wish to isolate the in! uence of the latter alone.

Long-horizon regressions have long been used in the &nance literature (see e.g. Campbell, Lo and MacKinlay, 1997). For applications to exchange rates, see Mark (1995), Chen and Mark (1996) and Kilian and Taylor (2003).

[37] It is well known that asymptotic critical values for the t-test statistics for the slope coef&cients in long-horizon regressions are severely biased in small samples. In order to mitigate these size distortions, empirical marginal signi&cance levels can be calculated based on the bootstrap approximation of the &nite sample distribution of the test statistic under the null hypothesis of no exchange rate predictability. The marginal signi&cance levels reported in Table 7 were computed using the bootstrap algorithm for long-horizon regressions described in detail in Kilian and Taylor (2003).

nonlinear adjustment of real exchange rates to their long-run equilibria. A third relates to differences in real exchange rate volatility across nominal exchange rate regimes.

We have investigated all three sets of in! uences in the research reported in this paper. To do so, we have estimated exponential smooth transition autoregressive (ESTAR) models for real sterling-dollar and real sterling-franc exchange rates in which we include relative real per capita income as a proxy for relative productivity and in which we allow for possible shifts in the variance of the errors. The data set that we use spans nearly two centuries and thereby allows not only enhanced test power but also provides an environment in which the various factors that in principle can affect real exchange-rate behaviour have sufficient scope to operate.

While we &nd evidence of signi&cant nonlinearities in adjustment for both exchange rates, we &nd signi&cant evidence of HBS effects for sterling-dollar but not for sterling-franc. There is also evidence of shifting real exchange rate volatility for both exchange rates, with higher volatility recorded during ! oating nominal exchange rate regimes.

We then go on to analyse the impulse-response functions for shocks of varying magnitudes to the two real exchange rates. In both instances, these show greatly increased speeds of adjustment vis-à-vis those estimated with linear autoregressive models for all but the very smallest shocks. Conditional on average initial history, the estimated half lives for large shocks of twenty per cent or more are only one year; for small shocks in the range of one to &ve percent they range from one to two years depending upon the exact magnitude of the shocks.

While the HBS effect is able to explain some forty percent of the variation in the level of the sterling-dollar real exchange rate over the whole sample period, we found that the in! uence of real effects on the real exchange rate varies according to the time horizon considered. In particular, long-horizon regressions of the $k-$year change in the real exchange rate onto the $k-$year change in relative productivity revelealed that at the shortest horizon of one year, HBS effects account for only a tiny proportion of the change in the real exchange rate. The proportion explained increases with the length of the time horizon, however, until it peaks at the seven-year horizon, when HBS effects explain around nine percent of the seven-year change in the real exchange rate.

This research might be fruitfully extended in a number of directions. First, investigation of the Harrod-Balassa-Samuelson effect in a nonlinear framework could be carried out for other countries, especially those that have experienced high rates of growth relative to the base country.[38] Second, the analysis could be repeated, focusing on the recent ! oating-rate period, and perhaps employing nonlinear panel estimation methods for a group of countries. Third, the framework used in this paper could be extended to a multivariate nonlinear system involving nominal exchange rates and relative prices as well as productivity differentials, in order to examine the relative speed of adjustment of nominal exchange rates and relative prices to deviations from the equilibrium real exchange

[38] See e.g. Chinn (1999) and Chinn and Dooley (1999).

rate.[39]

[39] See, e.g. Cheung, Lai and Bergman (2004).

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Table 1: Linear Unit Root Tests for Real Exchange Rates
a) Sterling-*Dollar 1820-2001*

τ_μ	τ_τ	Φ_1	Φ_2	Φ_3
-3.19	-3.44	4.89	4.06	6.07

$Z(\tau_\mu)$	$Z(\tau_\tau)$	$Z(\Phi_1)$	$Z(\Phi_2)$	$Z(\Phi_3)$
-3.23	-3.69	5.23	4.62	6.91

b) Sterling-*Franc 1820-1998*

τ_μ	τ_τ	Φ_1	Φ_2	Φ_3
-3.72	-3.73	6.96	4.92	7.32

$Z(\tau_\mu)$	$Z(\tau_\tau)$	$Z(\Phi_1)$	$Z(\Phi_2)$	$Z(\Phi_3)$
-3.86	-3.85	7.48	5.21	7.76

Note: The null hypotheses for each of the test statistics are given in footnote 16 in the text and de&ned in Perron (1988). A Newey-West window of width 4 was used for the non-parametric corrections (Newey and West, 1987), although experiments with different band-widths led to little difference in the results. The asymptotic critical values for the statistics at various test sizes are as follows (Fuller, 1976; Dickey and Fuller, 1981):

	10%	5%	2.5%	1%
τ_μ, $Z(\tau_\mu)$	-2.57	-2.86	-3.12	-3.43
τ_τ, $Z(\tau_\tau)$	-3.12	-3.41	-3.66	-3.96
Φ_1, $Z(\Phi_1)$	3.78	4.59	5.38	6.43
Φ_2, $Z(\Phi_2)$	4.03	4.68	5.31	6.09
Φ_3, $Z(\Phi_3)$	5.34	6.25	7.16	8.27

Table 2: Estimated Linear Autoregressions

a) *Sterling-Dollar 1820-2001*

$$\underset{}{\widehat{q}_{US,t}} = \underset{(-1.401)}{-.007} + \underset{(28.188)}{0.902}\; q_{US,t-1}$$

$$\begin{aligned} R^2 &= 0.82;\;\; SER = 6.45\% \\ AR(1) &= [0.08];\;\; ARCH(1) = [0.25];\;\; HL = 6.78. \end{aligned}$$

b) *Sterling-Franc 1820-1998*

$$\widehat{q}_{France,t} = \underset{(-1.286)}{-.009} + \underset{(12.043)}{.831}\; q_{France,t-1}$$

$$\begin{aligned} R^2 &= .65;\;\; SER = 7.5\%; \\ AR(1) &= [0.85];\;\; ARCH(1) = [0.00];\;\; HL = 3.75. \end{aligned}$$

Note: Figures in parentheses below estimated coefficients are asymptotic $t-$ratios, claculated using heteroscedastic-consistent estimated standard errors (White, 1980); &gures in square brackets are marginal signi&cance levels. R^2 is the coefficient of determination, SER is the standard error of the regression, $AR(1)$ is a lagrange multiplier statistic for &rst-order serial correlation of the residuals, $ARCH(1)$ is a lagrange multiplier statistic for &rst-order autoregressive heteroscedasticity in the residuals, and HL is the implied estimated half-life of real exchange rate shocks.

Table 3: Estimated Nonlinear Models: Single-Equation Maximum Likelihood

a) *Sterling-Dollar 1820-2001*

$\widehat{\mu}_{US,1}$	$\widehat{\theta}_{US}$	$\widehat{\sigma}^2_{US,Float}$	$\widehat{\sigma}^2_{US,Fix}$
0.125	2.594	0.005	0.002
(2.246)	(2.577)	(8.125)	(6.797)
	[0.009]		

$$
\begin{aligned}
R^2 &= .83; \\
AR(1) &= [0.12]; \quad ARCH(1) = [0.46]; \\
NL-ESTAR &= [0.55]; \quad NL-LSTAR = [0.61].
\end{aligned}
$$

b) *Sterling-Franc 1820-1998*

$\widehat{\mu}_{France,1}$	$\widehat{\theta}_{France}$	$\widehat{\sigma}^2_{France,Float}$	$\widehat{\sigma}^2_{France,Fix}$
0.00	3.064	0.007	0.003
$(-)$	(9.575)	(12.009)	(20.793)
	[0.001]		

$$
\begin{aligned}
R^2 &= .67; \\
AR(1) &= [0.74]; \quad ARCH(1) = [0.83]; \quad HBS(\mu_{France,1} = 0) = [0.15]; \\
NL-ESTAR &= [0.67]; \quad NL-LSTAR = [0.77].
\end{aligned}
$$

Note: Figures in parentheses below estimated coefficients denote the ratio of the estimated coefficient to the estimated standard error (the asymptotic '$t-$ratio'); figures in square brackets are marginal significance levels. The marginal significance levels for the null hypotheses $H_0 : \theta_i = 0$ were calculated by Monte Carlo methods, as described in the text. R^2 is the coefficient of determination, SER is the standard error of the regression, $AR(1)$ is a lagrange multiplier statistic for first-order serial correlation of the residuals and $ARCH(1)$ is a lagrange multiplier statistic for first-order autoregressive heteroskedasticity in the residuals. $HBS(\mu_{France,1} = 0)$ is a Wald test statistic for the parameter on relative productivity to be zero in the sterling-franc equation. $NL-ESTAR$ and $NL-LSTAR$ are lagrange multiplier statistics for the hypothesis of no remaining nonlinearity of the ESTAR and LSTAR (logistic smooth transition autoregressive) varieties, respectively.

Table 4: Estimated Nonlinear Models: Joint Estimation by Full-Information Maximum Likelihood

$\widehat{\mu}_{US,1}$	$\widehat{\theta}_{US}$	$\widehat{\sigma}^2_{US,Float}$	$\widehat{\sigma}^2_{US,Fix}$
0.140	3.023	0.005	0.002
(2.999)	(3.246)	(8.463)	(6.656)
	[0.009]		

$R^2 = .83$

$\widehat{\mu}_{France,1}$	$\widehat{\theta}_{France}$	$\widehat{\sigma}^2_{France,Float}$	$\widehat{\sigma}^2_{France,Fix}$
0.00	3.218	0.007	0.003
(−)	(11.121)	(12.474)	(20.283)
	[0.001]		

$R^2 = .70$

$\widehat{\rho}$
0.169
(3.460)

Note: Estimation period is 1820-1998. Estimation method is full information maximum likelihood. Figures in parentheses below estimated coefficients denote the ratio of the estimated coefficient to the estimated standard error (the asymptotic "t−ratio"); figures in square brackets are marginal significance levels. The marginal significance levels for the null hypotheses $H_0 : \theta_i = 0$ were calculated by Monte Carlo methods, as described in the text.

Table 5: Estimated Half-Lives for the Nonlinear Models

a) *Conditional on average initial history*

Shock (%):	40	30	20	10	5	1
Sterling-Dollar	1	1	1	2	2	2
Sterling-Franc	1	1	1	2	2	2

b) *Conditional on initial exchange rate equilibrium*

Shock (%):	40	30	20	10	5	1
Sterling-Dollar	1	1	2	4	6	9
Sterling-Franc	1	1	2	3	4	6

Note: Half lives of real exchange rate shocks were calculated by Monte Carlo methods based on the model estimates reported in Table 4, as described in the text.

Table 6: The Harrod-Balassa-Samuelson Effect and the Sterling-Dollar Exchange Rate

a) *Regression of real exchange rate onto relative productivity*

$$\underset{}{\widehat{q}_t} = \underset{(1.171)}{0.032} + \underset{(7.945)}{0.243} \; (a_{UK,t} - a_{US,t})$$

$$R^2 = 0.43; \quad SER = 12.85\%.$$

b) *Regression of relative productivity onto cubic trend*

$$(a_{UK,t} - a_{US,t}) = \underset{(-4.890)}{-0.120} + \underset{(6.239)}{8.107 \times 10^{-3}} \; t - \underset{(-10.432)}{1.769 \times 10^{-4}} \; t^2 + \underset{(10.071)}{5.997 \times 10^{-7}} \; t^3$$

$$R^2 = 0.97; \quad SER = 9.34\%.$$

c) *Autoregression of HBS-adjusted real exchange rate with a cubic trend*

$$\begin{aligned} [\widehat{q}_t - \widehat{\mu}_{US,1}(a_{UK,t} - a_{US,t})] = {} & \underset{(1.002)}{.020} + \underset{(18.279)}{0.805} \; [q_{t-1} - \widehat{\mu}_{US,1}(a_{UK,t-1} - a_{US,t-1})] \\ & + \underset{(0.149)}{1.346 \times 10^{-4}} \; t - \underset{(-0.977)}{1.150 \times 10^{-5}} \; t^2 + \underset{(1.408)}{6.188 \times 10^{-8}} \; t^3 \end{aligned}$$

$$R^2 = 0.78; \quad SER = 6.35\%; \quad W(No\ Trends) = [0.08]; \quad HL = 3.19.$$

Note: Figures in parentheses below estimated coefficients are are asymptotic $t-$ratios, calculated using heteroscedastic-consistent estimated standard errors (White, 1980); &gures in square brackets are marginal signi&cance levels. HBS_t is the Harrod-Balassa-Samuelson effect: $HBS_t = \widehat{\mu}_{US,1}(a_{UK,t} - a_{US,t})$, where $\widehat{\mu}_{US,1}$ is the estimated value of $\mu_{US,1}$ in Table 4. R^2 is the coefficient of determination, SER is the standard error of the regression, $W(No\ Trends)$ is a Wald test for the joint signi&cance of the three trend parameters, and HL is the implied estimated half life of real exchange rate shocks.

Table 7: The Short and Long-Horizon Contribution of HBS Effects to Sterling-Dollar Real Exchange Rate Variation

k	p − value of $\widehat{\gamma}_k$	$\mathbf{R}^2_k$,
1	0.478	0.002
2	0.793	0.000
3	0.369	0.006
4	0.158	0.021
5	0.031	0.044
6	0.003	0.066
7	0.000	0.087
8	0.001	0.070
9	0.013	0.050
10	0.093	0.036

Note: The Table shows the coefficient of determination, R^2_k, and the marginal signi&cance level ($p-$value) for $\widehat{\gamma}_k$ in the long-horizon regression

$$(q_{US,t+k} - q_{US,t}) = \alpha + \gamma_k[(a_{UK,t+k} - a_{US,t+k}) - (a_{UK,t} - a_{US,t})] + \nu_t$$

for values of k from 1 to 10. The marginal signi&cance levels were calculated using the bootstrap algorithm described in Kilian and Taylor (2003).

Figure 1: Real Sterling-Dollar and the HBS Effect