POLITICAL PARTIES AND COALITION FORMATION

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Abstract

This chapter gives a flavour of recent theoretical work on coalition formation and political parties. I survey recent work on both pre-election coalition formation and post election coalition (or government) formation. A number of alternative rationales for the formation of parties are compared with the help of some illustrative examples.

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1 Introduction

Political parties have long been treated in both the theoretical political science and economics literature as unitary actors. Take for example the Downsian model of political competition (Downs (1957)), the multi dimensional spatial models (e.g Enelow and Hinich (1984) among others), and even the more recent models of redistributive politics (e.g Dixit and Londregan, Lindbeck and Weibull (1997)) . In many of these models important results about policy outcomes hinge on specific assumptions about party objectives.

This begs the question of what political parties actually are: what is the notion of party that theorists should be interested in? Are they agglomerations of policy positions of individual candidates (Osborne and Tourky, 2002) or are they informative brand names aimed at voters (Snyder and Ting, 2002)? Are they mechanisms to economise on the costs of standing for elections (Riviere, 1999, Osborne and Tourky, 2002) or simply credible commitment mechanisms (Levy, 2002, Morelli, 2002)?

One may go further and ask if we should really be looking at "parties" as coalitions of individuals or at coalitions of "parties" or groups of individuals –whether pre-election or post-election– since in many electoral systems it is such coalitions that affect actual policy outcomes.

If we care about choice in politics we should worry about the number of parties that a country has, their size and the platforms they take. Presumably systems that are dominated by one party– as in many African dictatorships– are less desirable than a multi-party system where parties have divergent platforms. On the other hand if party positions converge then outcomes may not be different between these two systems. Indeed, if these are important criteria for a well functioning democracy, we should also be interested in why different electoral systems generate different party structures. Duverger (1954) conjectured that majoritarian systems like the UK and USA would cause less parties to emerge than Proportional Representation systems used in some European countries (Duverger's Hypothesis). One of the most important informal "law" in political science, Duverger's (1963) Law states that Plurality Rule systems have a tendency towards a two party system. Recent research on political parties analyses exactly these types of questions.

This chapter attempts to overview of some of the theoretical work being done in this area. My aim is to present the most recent work which has not been covered elsewhere and so provide a flavour of the types of factors that economists and political scientists consider to be salient in party formation. Hence, this chapter is not meant to be an exhaustive survey, but rather to

focus on a few papers in detail.

The format followed is to categorise the models into those which consider parties to be coalitions of individual candidates who are motivated to form parties by electoral considerations (pre-electoral coalition formation) and those which focus on parties in the legislature: i.e. coalitions of groups of individual candidates who are interested in forming a government (post electoral coalition formation). The electoral motive is suppressed in these models. The benefits of forming coalitions here are economies of scale or the reduced uncertainty in outcomes, while the costs arise in the form of the political compromise over policy and the sharing of the private gains from office.

The chapter is organised as follows: Section 2 is concerned with Pre-electoral Coalitions, Section 3 with Post-election coalitions. Section 4 concludes.

2 Pre-electoral Coalitions

In order to understand the role of a party, we first need to understand what happens *without* parties: a world in which only independent candidates stand

for election. Such a world has been modelled recently by two sets of authors: Besley and Coate (1997) and Osborne and Slivinski (1996). These authors' were the first to endogenise the candidacy decision and the factors that influenced an individual citizen's decision to enter as a candidate or not, i.e. the trade off between the costs of candidacy and the benefits of getting the best policy for a particular candidate implemented. Candidates were assumed not to be able to credibly commit to anything but their own ideal policies. Thus policy platforms were inflexible in direct contrast to the Downsian model.

But it seems clear that forming coalitions in this set up would have dividends! Candidates could share the costs of candidacy if they made a "party" of like minded individuals, or they could improve their chances of electoral victory if they could join a coalition of individuals with different policy positions thus ensuring commitment to a *set* of policies rather than just their own best policies. Parties could also act as mechanisms to coordinate voters decisions. The first set of authors we survey considers exactly these variations on the citizen candidate model.

I will now focus on models of pre-electoral coalition formation. The common ingredients of most models involves (i) a policy space Q which can be uni or multi-dimensional Euclidean space, (ii) a private good X (iii) a

set of citizens partitioned (according to policy preferences)into N homogenous groups. It is assumed that citizens care only about policy, each citizen has an ideal policy and preferences are Euclidean (hence single peaked in uni-dimensional policy space). Thus, each group represents an ideal policy position. (iv) Finally some models have a cost of candidacy.

The models differ in the main motivations for parties to form, the predictions on the size and number of parties, the models and the equilibrium concepts employed. Among the motivations for pre-electoral coalitions to form, the first one we present is the cost-sharing motivation. Riviere (1999) and Osborne and Tourky(2002) are both models of parties as cost sharing organisations. Although the models are considerably different in detail, both assume uni-dimensional policy space. While Riviere (1999) assumes Plurality Rule, Osborne and Tourky (2002) present a more general model which is applicable to both Plurality Rule and Proportional Representation.

In Riviere (1999) the policy space is restricted to be three points on a line: $\mathcal{Q} \subset \mathcal{R} = \{-1, 0, 1\}$. Thus citizens are of three types ($N = 3$). The model is a modified version of the Besley-Coate (1997) Citizen candidate model. The median voter is not known at the party formation stage, otherwise only one party would ever form. There is a cost to entering as a candidate which is

assumed to be not "too large" so that entry is possible in the game.

As in Besley-Coate (1997) there are 4 stages: (i) the entry stage, (ii) the median voter is revealed (iii) voting tales place given the set of candidates (iv)policies are implemented.

Voting may be strategic, but weakly dominated strategies are eliminated iteratively. This equilibrium is called a refined voting equilibrium.

Two scenarios are compared: (1) Candidates are not allowed to share costs (2) Candidates can decide to form parties which are cost sharing organisations.

In case (2) a pre-entry stage is added where each citizen may simultaneously decide whether to become a party president or a member of a party (i.e. to nominate a president), or to stay independent.[1] Then the same four stages as in case (1) are played out.

Coalition proofness is demanded at the party formation stage in equilibrium. This ensures that parties will form – coordination failures are easy in Nash equilibrium: if all citizens expect others not to become members then parties are useless and no citizen will have an incentive to create one.

[1]The role of party presidents and members is asymmetric: only citizens of the same type can be in a party, only presidents can decide to enter as candidates, and costs are shared equally between all members, and citizens can be members of at most one party.

Moreover each type can have at most one party.

The equilibrium concept is essentially subgame perfectness with the additional refinements on equilibria at the entry stage (and pre-entry stage in Scenario (2)) –coalition proofness– and at the voting stage –iterated elimination of weakly dominated strategies. In scenario (1) the equilibrium concept is *Coalition-Proof Political Equilibrium* (CPPE). It consists of a vector s of entry decisions ($s \in \{0, 1\}$) and a function α which describes voting behaviour such that (i) s is a coalition proof Nash equilibrium (Bernheim, Peleg and Whinston, 1987) of the entry game given α(ii)for all non empty candidate sets C, for any state of nature I, $\alpha^I(C)$ is a voting equilibrium.

In scenario (2) the equilibrium concept is analogusly the subgame perfect equilibrium of the whole game now called a *Coalition proof Complete Political Equilibrium* (CPCPE).

The main results are that without parties, the number of candidates who stand for election is decreasing in the cost of candidacy. When parties are allowed:(i)Active parties are Minimal Winning Coalitions: this conclusion is driven by the fact that members share the costs of candidacy, so each member must be "decisive" in the sense that without him the party would not form. (ii) Secondly the Duvergerian result that there is a tendency for only two

parties to form in equilibrium. The paper has a full characterisation of the CPCPE.

Intuitively this is a result of the fact that both extremist parties (all citizens are risk averse) may prefer not to run rather than not have a centrist candidate. Centrists therefore have a higher power in the collusion with an extremist party. Thus there are equilibria where only a centrist party and an extremist run. Compared to case (1) (no cost sharing) more candidates stand for election in case (2), as expected.

Thus parties in Riviere (1999) are viewed as coalitions of like minded individuals who share the fixed costs of campaigning. However self enforcing collusion between heterogenous parties can arise in equilibrium, in the sense that entry decisions of different parties are coalition proof.

Another set of authors that considers parties to be mainly cost sharing organisations is Osborne and Tourky (2002). Unlike Riviere (1999) however, they are more interested in modelling situations where a *group* of legislators makes policy decisions rather than a single party and candidates are not restricted to choosing their own ideal policy. Moreover the rule chosen for translating policy announcements by legislators into policy is that the median of the announcements is chosen. Thus parties in their model are a group

of legislators who vote for and support the same position. The reason for forming parties is costly participation and economies of scale to forming large parties.

The model consists of a set of players (finite) who decide whether or not to participate in decision making and of so which policy to champion. The policy space is unidimensional (it can be multidimensional as long as candidates can be ordered on a line according to their favourite positions) and the outcome is the median of the policies championed. The first assumption about payoffs is called *Costly Participation* and is defined as follows:

Costly Participation (C): If a participating player's switching to non participation does not change the policy, then her payoff increases.

The second assumption is called *Economies of Party Size(E)*and is defined as:

Economies of Party Size(E): If a participant's switching to a larger party does not change the median championed policy, then her payoff increases.

We discuss an example on committee voting from the paper in the Appendix. Example 1 shows how the model works. The main result is that Example 1 can be generalised and two party equilibria are characterised in the paper. Regarding equilibrium platforms, they show that the smaller is

the cost of participating, the closer will be the two parties positions to each other.

The authors weaken the economies of scale axiom to allow different types of games: e.g two stage games where players can be candidates and voters and in a variety of games satisfying these conditions they find the tendency towards a two party system, thus verifying Duverger's Law. Indeed, other outcome functions are explored in the paper that describe two stage games which satisfy similar axioms.

Notice that while Riviere (1999) has the entry decision subsequent to the party formation stage, Osborne and Tourky (2002) have a simultaneous entry and party formation decision. Moreover Riviere has exogenously given platforms, while this is endogenous in Osborne and Tourky. Even then some version of Duvergers Law seems to emerge in the two papers – in Riviere (1999) the motivating factor is that the policy compromise achieved by having two parties (one extreme and one centrist) rather than two extremes is favoured by the two extremists, while in Osborne and Tourky (2002) the motivating factor is cost savings and the fact that the outcome does not change if the median does not. If participation was costless it is a weakly dominating strategy to announce the true favourite position, however the two party

equilibrium is still a Nash equilibrium.[2]

Similar in spirit to the Osborne and Tourky (2002) model is one proposed by Gomberg, Marhuenda and Ortuno-Ortin (2001). However, the motivation is different – indeed, Gomberg et al assume that there are two parties to begin with. Their main contribution is a definition of political equilibrium and showing that it exists. They consider multi-dimensional policy spaces and introduce the notion of a party as a coalition of voter-members who support a given policy which depends on their primary electorate's composition. The overall social outcome is a weighted average of the parties positions, the weights being proportional to the share of votes received by each party. Equilibrium is defined as a situation where no coalition of voter members can

[2]A related paper by Gerber and Ortuno Ortin (1998) shows another two party emergence result. Here, the policy space is uni-dimensional and voting takes place on this space. Voting basically means that each player proposes a policy. The policy adopted is a compromise between the proposals made. There is a continuum of voters and each type has single peaked preferences. The outcome function is assumed to be continuous and superadditive. The main result is that a unique strong Nash equilibrium exists in the voting game which involves only two parties. The proposals are polarised, though the adopted policy is a compromise between these. The results do not generalise to the case of finite types since the continuity of the outcome function is necessary. Voting (participation) is costless.

deviate by changing their voting and obtain a preferred outcome. Existence of equilibrium is the main result.

In the appendix (Example 2) I show that in a simple uni-dimensional setting the platforms chosen by the two parties in the Gomberg et al solution are the same as the platforms chosen by the endoegenous parties in the Osborne and Tourky solution. Of course the main point in Gomberg et al is to generalise the solution to multi-dimensional spaces and prove existence.

Levy (2002A) models party formation differently: the main rationale for parties to form, in her view, is the ability of parties to solve the commitment problem of independent candidates. Parties are able to commit to a larger set of policies: in particular they are able to commit to any policy in the Pareto set of party members. The trade off that an individual citizen faces in joining a party is between the gain in terms of the probability of winning against the costs of compromising on policy. She defines the notion of *effectivness* of parties – parties are effective if the outcome is different when parties are allowed to form and when they are not. Her main result is that parties are not effective in a uni-dimensional policy space.

Levy's (2002A) model has N players: there is a representative agent in each group. There is a continuum of voters, and voters are assumed to vote

sincerely. The model has two phases of analysis: (i) The Platform game: Assume a partition, π of voters into coalitions, a typical coalition being denoted S. Each coalition chooses a policy in it's Pareto set in order to maximise it's chances for election. Not choosing a policy is akin to choosing not to run. The winning policy is chosen by plurality rule. For each policy chosen in an equilibrium of the platform game, expected utility is given using a continuous and concave utility function for each member of each party. An equilibrium of the platform game is a set of platforms $\{\delta_S\}_{S\subset\pi} \equiv \delta(\pi)$ such that δ_S is a policy in the Pareto set of its members given δ_{-S}. Proposition 1 shows that an equilibrium of this game exists. Now for each possible coalition structure choose an equilibrium of the corresponding platform game. Denote this pair as $(\pi, \delta(\pi))$. This yields an induced utility $U_i(\delta(\pi))$ for each player from this pair. An unstable partition is one that is not supported by any equilibrium of the platform game.

The stability concept is taken from Ray and Vohra (1997). Starting from a given partition, the only deviations allowed are to break parties into smaller ones. Deviations can be unilateral or multilateral but the deviators must be sub-coalitions of the existing coalitions. Also deviations take into account future deviations, both by members of their own coalitions and also members

of other coaltions. Credible threats are deviations to finer partitions which are stable themselves. Thus the definition is recursive. The technical details of the definition are presented in the Appendix.

Let k^* denote the dimensionality of a Euclidean space V that is spanned by the N ideal policies in society.

Her main result(Propositions 2 and 3, Levy (2002A)) is that parties are not effective when $k^* = 1$. The only stable outcome is the median voters ideal policy. When $k^* > 1$, parties are effective even if a Condorcet winner exists in the absence of parties. A sufficient condition for parties to be effective when the Condorcet Winner is a unique equilibrium in π^0, and preferences are Euclidean is that $k^* = N - 1$.

Unlike Riviere (1999) Levy assumes that there is a representative candidate in each group, given exogenously and that there is no cost sharing. But the dependence of the entry decision on the costs of campaigning and on risk aversion is similar in the two models.

In a companion paper, Levy (2002B), she verifies the robustness of the "party effectiveness" result under various stability concepts.

Morelli (2002) differs from the two authors above by looking at multi-district elections and Proportional Representation. He is concerned too with

whether "heterogenous" parties form in equilibrium, and whether parties are "effective", although these terms are interpreted differently from Levy (2002A). Parties are effective if they can be part of the government, without considering their effect on the actual policy implemented. The main question he is concerned with is the conditions under which the number of effective parties is larger under proportional representation than under Plurality voting (Duverger's (1954) hypothesis). Parties are *active* if they run for election in at least one district.

As in Levy (2002A) citizens are partitioned into N groups, each group composed of identical individuals (i.e. identical preferences). Preferences are single peaked and defined on a uni-dimensional policy space. There are three types of citizens (as in Riviere (1999)): so $\mathcal{Q} \subset \mathcal{R}x = \{-1, t_C, 1\}$, where t_C denotes the ideal policy of the centrist voters, $t_C \leq 0$. There are also three districts, with a measure 1/3 of total citizens and each with it's own distribution of voter preferences. Party leaders are given exogenously, and so are the set of potential candidates from each homogenous party. So with 3 districts, each district will have 3 potential candidates, with a total of 9 potential candidates given exogenously. The role of party leaders is to make coalitions of homogenous parties, called the *heterogenous* parties. The first

stage consists of a party formation stage. In the second stage each party decides whether to enter the election in each district. Then voting takes place. Voting can be sincere or "strategic" (since there is a continuum of voters this needs to be defined). Two electoral systems are considered – Plurality Rule (PV) and Proportional Representation (PR). They differ in the way that vote shares in each district translate into seats in legislature. There are three seats (one for each district). Under PV, a seat is allotted to a candidate if he wins (i.e. gets the maximum vote share) in a district. The party that gets a majority of seats will decide policy. If there is a tie then the centrist party gets to make the policy. Under PR, seats are allotted according to the Hare rule. Here we look first at the maximum vote share (summed across districts) for each party. This party (call it L) gets the first seat. Now we subtract $1/3$ from the total vote share for party L and compare with the original vote shares for the other two parties. The party than has the maximum vote share among these gets the second seat (call it C). Again, subtract $1/3$ from the total vote share for party C and compare with the remainder for part L, the remainder for party C and the total votes for party R. The party that gets the maximum vote share among these gets the third seat. The party that has a majority of seats makes the policy decision it has

chosen. If there is a tie again C gets to make policy.[3]

Exogenously given candidates get a utility from winning the election as well. There is a cost to entry, denoted c. Thus we may see candidates standing for election in a district even when they will not influence the policy given the voting equilibrium.

The strategic voting equilibrium needs to be described as it is unusual. Strategic voting is thought of as voting recommendations made by each heterogenous party to it's own group of voters across districts. Thus a voting recommendation by a heterogenous party composed of L and C would be a 3-tuple giving recommendations to it's voters (L and C voters) in district 1,2 and 3. An equilibrium with strategic voting is a Nash equilibrium in recommendations, i.e. given the voting recommendations of all other parties and assuming that all other voters follow the recommendations, a voter in party i should be weakly better off following the recommendation. Let π (as before) denote the partition into parties in equilibrium.

A voting equilibrium is *strong* if no *coalition* of parties $C \in 2^{\pi}$ can improve on the recommendations for voters in their coalition given the recommenda-

[3]This is an important difference from the way that PR is usually modelled – here policy is determined by the majority party in both Plurality and Proportional Representation and it is only the way in which winners are chosen that differs between the two.

tions of all other parties.

An equilibrium of the whole game calls for subgame perfection.

In this model politicans may want to run for election even if they do not matter in the determination of policy. The main question analysed is whether parties are "effective" i.e. does the party have at least one candidate who wins in the whole country?

Each group has an incentive to get it's own policy implemented. Suppose we focus on sincere voting and PV – if two districts had the same median voter then that group can always win two seats in PV, so no heterogenous parties form and at most two parties are effective, generically. The interesting case is when the median is different in all three districts. In this case, if all three parties stand and win one seat then the Centrist policy is the default option. Thus in the bargaining stage of the game both extreme parties offer the centrist voter his most preferred policy. This means that policy does not change in equilibrium whether heterogenous parties form or not. Indeed the equilibrium policy is always the centrist parties most preferred position and this is shown to be true as well for strategic voting (Proposition 1). Thus parties are *not* "effective" in the sense that policy cannot change due to party formation. Levy (2002A) showed this (see above) for a single district with

uni-dimensional policy space.

The main result is a full description of the configurations of voter preferences in the three districts under which PV has three active and effective parties (Corollary 2) and the configurations under which PR has at most two active and effective parties (Corollary 3). This allows us to come up with the conditions needed for Duverger's hypothesis to hold.

I compare the three models discussed above and their main results with the help of a simple example (taken from Levy (2002A) which is presented in the Appendix.

All the models discussed so far assume complete information on all sides at the voting stage. Synder and Ting (2002), in contrast, present a completely different view of parties. They view parties as "brand names" that inform voters credibly about the policy positions their members will take. In contrast to Levy (2002A) and Morelli (2002) who assume that candidates cannot credibly commit to any policy except their own most preferred point and therefore need parties to be able to offer other platforms, Snyder and Ting (2002) assume that candidates cannot commit to any platform at all and need parties to credibly signal their true policy preference. Parties act as screening devices a la Spence (1974). The main question is to explain

how the precise meanings associated with different parties (e.g. Republicans are fiscally conservative, Democrats are liberal etc) arise as equilibrium phenomena. The model is based on two assumptions: (i) party membership carries costs: this is similar to models discussed above, i.e. that joining a party means compromise. These costs are higher the futher away is the ideal policy of a candidate relative to the party position. (ii) Voters know little about preferences of candidates but much more about parties. This second assumption is in stark contrast to the Citizen Candidate models (Osborne and Slivinski (1996), Besley and Coate (1997)) and all the models considered before. Thus parties must be given exogenously in the model as is indeed the case.

There are three types of players in the model: parties, candidates and voters. There is a continuum of voters divided into a continuum of constituencies. Each constituency elects it's own (single) representative by plurality rule from among the candidates who stand. The winning candidates take office and implement policy. The policy space is uni-dimensional again $X = [-1, 1]$. Two situations are considered: one party only and two parties called L and R. Each party must choose it's platform in order to maximise the share of offices won by it's candidates. Candidates are driven by the

rewards of office and if elected, policy. Candidates ideal points are random draws from a uniform distribution on $[0, 1]$. However candidates cannot credibly communicate to voters except through their affiliation choice which is common knowledge. These choices are denoted by a – candidates may run unaffiliated ($a = 0$, independents) or join a party ($a = L, R$). The game consists of parties choosing platforms x_L and x_R. Then nature randomly draws an infinite sequence of candidates i.i.d from $[-1, 1]$. Candidates are offered affiliation with party L first and he may affiliate or not. Party R offers affiliation to the second candidate and so on. The remaining candidates may choose not to run or to run unaffiliated. Only one candidate is chosen to affiliate in each district. Then voting takes place and the winner is chosen by plurality rule. The median across districts is the point 0.

By choosing it's platform a party implicitly chooses its mean and variance as it can anticipate the set of candidates who join it. These are the crucial determinants of electoral success. Voters like less uncertainty and platforms close to their own ideal points.

When only one party is allowed candidates may still choose to affiliate in order to reduce the uncertainty that voters have about their policy preferences. But (given the specifics of the model), it is shown that in such a case

the party can locate at 0 and win all districts. By locating at 0 the party will have the same mean as an unaffiliated candidate but will have a lower variance as extermist candidates will not join the party– it is not worthwhile for them. The party thus screens out candidates who are too far in the policy space. Thus equilibrium calls for consistency between the platform chosen and the ideal points of the candidates who join it.

When there are two parties, the equilibrium can be of two types: convergent or divergent depending on the value of the costs and benefits of winning for a candidate. When the relative benefits of holding office are low then the equilibrium is convergent: it involves both parties locating at the point 0 which will ensure that they each win half the districts. Unaffiliated candidates are sure to be defeated in every district. When the relative benfits of holding office are large however, they may get divergent equilibria that involves parties locating at a distance (the same distance) away from the mean and on opposite sides of 0.This location ensures (i) unaffiliated candidates lose all districts again because when the median is on the left side of the mean, the left party wins and vice versa.(ii) each party wins half the districts. The reason that o is no longer an equilibrium when the benefits of holding office are large is that more candidates will affiliate, with the result

that variance at 0 is very high. If one party locates at 0 (e.g. the R party) the other party (the L party) can do better by moving left of 0 in order to reduce it's variance and attract more voters – without losing out to unaffiliated candidates.

In this model parties are therefore viewed as entities that will choose platforms to attract certain types of candidates in order to maximise the chances of winning overall given the policy chosen by the other party. The factors that influence party membership are (i) the policy chosen by the other party (ii) the benefits for candidates to affiliate with a party (payoff from winning, and costs of standing, the costs of policy compromise) (iii) the degree to which uncertainty is decreased (and electoral success enhanced) by having a particular set of candidates relative to unaffiliated candidates.

3 Post-election Coalitions

The common thread linking all the authors mentioned so far is that an important factor in the party formation process is the anticipated voting behaviour that follows. However some authors suppress the role of voters and assume the legislative composition as given. The seminal work of Riker (1962) is the

first to study this kind of coalition behaviour. His predictions still form the basis for recent research on this topic: he focused on a narrow interpretation of the objectives of political parties, i.e. that they were interested in the spoils of office. Thus, the game is a zero sum game and this led to his insight called "the size principle". This is the notion that coalitions must be minimal winning coalitions, so as to maximise the gains from forming them. Traditionally the study of coalitions among political scientists has focused on post election coalitions between parties.[4] In the interest of brevity and non

[4]The literature has a long tradition in public choice. The themes that dominate have been: the move from uni-dimensional to multi-dimensional policy space and the resulting potential for chaos if decisions are made by plurality vote (e.g. McKelvey (1976)). Several co-operative game theoretic solutions were proposed to address this indeterminancy in the outcome of the legislative voting game. Thus eg the Core may fail to exist in multi-dimensional spaces. Some reviews of the extensive work on spatial theories of legislatures are Austen Smith (1983), Calvert(1986) and Shepsle(1986).The approach taken to solve this theoretical problem was to consider detailed versions of the legislative process: i.e. to introduce institutions as a way to introduce more structure in the problem and to make meaningful predictions. Shepsle (1979) e.g. proposed a *structure induced equilibrium* by adding a role for committees that specialised on making decisions on particular issues. In contrast to the much later model of Jackson and Moselle (2002)(discussed in detail above), Shepsle argues that the operation of the committee system will not allow issues

duplication I will focus on work that is more recent and refer the interested reader to the reviews mentioned below.

Jackson and Moselle (2002) model the benefits of party formation in the context of a legislative bargaining game. They use the legislative bargaining game of Baron and Ferejohn (1989) and extend it to the case where bargaining between legislators is taking place on two dimensions – a private good dimension and a public good dimension (whereas in the original legislative game bargaining was only over the distribution of a fixed amount of a private good. As in Levy (2002A) and Riviere (1999) the outcome of the game is compared when parties can form and when they cannot. The main question, in the context of this chapter, is to examine whether the equilibrium outcomes of the legislative game are different with and without parties – the question we saw in Levy(2002A) above: are parties *effective*? In terms of the extension of the simple legislative game to two dimensions the equilibria show that the two dimensions interact in interesting ways, even though legislators preferences are separable on the two dimensions. In short the motivation to form "parties" or coalitions may be to increase the power of a legislators' ideology or to garner extra benefits for their own constituencies

to be linked and it is not possible to make trades on policy issues.

in the budgetary process (the distributive dimension).

I will describe the model and then focus on one of the examples in the paper which illustrates the intuition to the main results.

The legislative game has n players (legislators), $n \geq 3$, is an odd number. A *decision* is a vector $(y, x_1, x_2, ..., x_n)$ consisting of an ideological decision y and a *distributive* decision which is about the division of a fixed pie of size X. The set of feasible public decisions is $[0, Y]$ and $Y \in [0, 1]$. The set of feasible distributive decisions consists of those that have $x_i \geq 0$, for all i and $\sum(x_i) = X$. Preferences of legislators are assumed to be separable in x_i and y. They are single peaked in y and strictly increasing in x_i for every y. We can order legislators in increasing order by their ideal points for the public good: $i \leq j$ iff $y_i \leq y_j$. They have a uniform discount rate $0 < \delta \leq 1$, the utility of reaching an agreement $(y, x_1, x_2, ..., x_n)$ at time t is $\delta^t u_i(y, x_i)$. The *legislative game* consists of a sessions $t = 1, 2, ...$ (can be infinite). At the beginnning of each session a legislator is chosen with probability p_i to make a proposal and the probabilties are the same in each session. Then legislators are called upon to vote "yes" or "no" on the proposal in a fixed order. The proposer can choose to propose on one dimension or both. The authors show that there is no loss of generality in restricting the rules of the game to be such

that both dimensions are proposed simultaneously (Proposition 1). A default utility is specified in case the game never ends. All actions are observable and the game is a perfect information game.

Equilibrium is a subgame perfect equilibrium of the game in stationery strategies (i.e. strategies that do not depend on history of past play). Since a legislator needs a majority to pass a proposal there are exactly $M = \frac{n-1!}{[(n-1)/2]!}$ sets of other legislators such that he can get a majority. A *simple equilibrium* is a stationery equilibrium where a legislator can randomise over a maximum of M proposals and each such proposal can be identified with a distinct coalition C of $(n-1)/2$ other legislators who vote yes on the proposal.

Since I are more interested in the pre game coalitions that form, I will not describe these results in detail but instead focus, as before, on an illustrative example taken from the paper. The main conclusions emerging from the legislative game are described in Section 6: (i) Simple equilibria always exist and in such equilibria (ii) both dimensions are considered together and a decision will be approved in the first session. (iii) Each legislator has a positive probability of being excluded from some decision that has a chance of being approved and (iv) generically there are at least two different ideological decisions that have a chance of being approved.

To understand the model better I present an example taken from Jackson and Moselle (2002) in the Appendix (example 4). A party is modelled as a binding agreement between the members to act as a single unit in the legislative game. The benefits to the legislators are measured relative to the payoffs in the legislative game. The set of utility levels possible for legislators in a coalition $\{i, j\}$ is described by the set of decisions that the coalition can make in the legislative game. The disagreement point is given by the expected utility of the legislator in the legislative game without parties.

Let $u_i^{NB}(\{i, j\})$ denote the utility that i gets from party $\{i, j\}$ using the Nash bargaining solution to split the gains from party formation. The following definition applies to the three player game discussed in the paper.

Definition: A party $\{i, j\}$ *is stable if* $u_i^{NB}(\{i, j\}) \geq u_i^{NB}(\{i, k\})$ *and* $u_j^{NB}(\{i, j\}) \geq u_j^{NB}(\{j, k\})$

In our example therefore the stable parties are $\{1, 2\}$ and $\{2, 3\}$. A number of illustrative examples in the paper show that with different parameter values there can be very different stable parties emerging. Some interesting comparitive statics are derived.

Thus Jackson and Moselle (2002) conclude that parties are effective in the sense that legislative outcomes are significantly altered when parties can

form but that the changes depend on parameters like preference intensities.

Jackson and Moselle (2002) focus on the differences in outcomes when parties can form as opposed to when they cannot when there are two dimensions in the legislative game. They do not have predictions of the types of parties that form, apart from some examples. Can we say anything about the types of coalitions that form? When legislators care about power as well as policy outcomes, this question is addressed in the uni-dimensional case by Bandhopadhyaya and Oak (2002). They consider a model of legislative coalition formation under a Proportional Representation (PR) system.

While the role of voters was suppressed in the literature reviewed above, Austen Smith and Banks (1988) and Baron (1993) are models that combine voting in elections with coalition formation at the legislative stage. Both focus on proportional representation. Austen Smith (1986) focuses on multi district plurality rule elections but does not model the legislative game explicitly. In addition to deriving predictions about the equilibrium coalitions that emerge, these authors also derive the electoral platforms chosen by the parties endogenously. In this sense, these models are more general than the models of post election coalitions I reviewed above.

The idea in Austen Smith and Banks (1988) is that voters will antici-

pate the coalitions that form at the legislative stage and vote strategically to achieve their best policy. Although there are 3 parties competing on a uni-dimensional policy space, there is also a distributional dimension to the legislative game in that parties can also bargain over the spoils of office. Parties are interested in minimising the difference between their electoral platforms and the policy ultimately implemented as well as the spoils of office.

The timing is: first the three parties announce electoral platforms, then an election takes place and vote shares are determined. Finally policy is decided by bargaibning between teh different parties. The weights in the bargaining game are determined by the share of votes of each party: this happens endogenously through the legislative game where the party with the highest vote share gets to propose a policy and distribution of the private good first. If the proposal is accepted by a majority it is implemented, otherwise the party with the next highest vote share gets to make a proposal and so on. The subgame perfect equilibrium of this is the solution to the legislative game for given vote shares. Finally policy is implemented. A party needs at least s votes to get into the legislature where $s \in [3, \frac{1}{3}n]$ is odd and n denotes the number of voters. Vote shares of the parties determine their

weights in the implemented policy. If a party has an absolute majority in votes then it can implement it's own announced policy and corner all the private benefits. Assume that each party gets at least s votes.

Coalition formation in the legislative stage consists of each party making a proposal consisting of a coalition, a policy and a distribution of private benefits. The members of the coalition can accept or reject the proposal. This proposal is passed if a majority of the coalition members decide to accept it. Otherwise in the next time period the party with the second highest vote share attempts to make a winning coalition and so on. If no government has formed after the third party makes its attempt then a caretaker government is formed which makes its decision equitably. Equilibrium in this game is a subgame perfect equilibrium of the whole game. The legislative game thus differs in the detailed description from that of Jackson and Moselle (2002), in particular that proposals are made simultaneously in the latter model. However the inefficiency of the legislative bargaining process is also a feature of the Austen Smith-Banks model. This inefficiency is driven by the risk aversion of party members and the uncertainty in the equilibrium outcome. The legislative game is solved using a non-cooperative bargaining approach – an idea employed subsequently by Jackson and Moselle (2002). An important

insight from the coalition formation or legislative stage is that coalitions that form are minimal winning coalitions (almost by definition) but not necessarily of minimum size (that is not a choice variable as party sizes are determined by proportional representation). Also winning coalitions are not necessarily connected. I.e. there may be equilibria where the two extreme parties form a coalition – it is the parties with the highest and lowest weights that form a coalition. This is because the party who proposes would prefer to do so when it has to compromise less. The electoral equilibrium is fully characterised. Unlike plurality rule the legislative influence of a party is not monotonic in vote shares. The expected policy in equilibrium is the median voters position, though realised policy may be different. Finally not all voters vote sincerely in equilibrium.

Some conclusions are similar to and anticipate the results of Jackson and Moselle (2002): legislators with extreme preferences may end up forming a coalition (unconnected coalitions), the median voter is an anchor for the different policy positions in equilibrium, and the inefficiency of the outcomes. Baron (1993) has a very similar model to Austen Smith and Banks (1988) except that parties aim to maximise the aggregate welfare of their supporters and voting is sincere. The main result is that unlike Plurality voting systems

there may not be a convergence of policy positions.

4 Conclusions

So, what are the main insights that emerge from this survey of party formation? The first important insight is that although most models of politics are uni-dimensional, it is not an innocuous assumption. Indeed the question of whether parties as an analytical construct are important to study may depend crucially on this assumption. Both Levy (2002A) and Jackson and Moselle (2002) show, in different settings that parties are effective when the issue space is multi-dimensional, and not necessarily when the issue space is uni-dimensional.

What are the motivations to forming parties? As aptly summarised by Strom (1990) the literature has focused on three main motivations: office seeking, policy seeking and vote seeking. Most of the models have similar motivating factors: economies of party size (Osborne and Tourky (2002), cost sharing (Riviere, (1999)), greater ability to commit to and obtain good policy positions (Levy (2002A), Morelli (2001), Jackson and Moselle (2002), Austen Smith and Banks (1988)). The costs to forming parties are the costs

of compromise in many of the models.

A number of the papers surveyed confirm the minimal winning coalition idea of Riker (1962), in particular when there is a distributive dimension to party formation (e.g. Riviere (1999), Austen-Smith and Banks (1988), Bandopadhyaya and Oak (2002)) . However, many instances are shown where parties are not connected (e.g. Jackson and Moselle (2002), Austen-Smith and Banks (1988)).

Many of the papers focus on showing the conditions under which Duverger's (1954) Law holds. Thus e.g. Riviere (1999) shows the conditions under which two parties emerge endogenously in single district Plurality Rule elections while Morelli (2001) shows the configurations of voter preferences under which Duverger's (1954) hypothesis (that there are typically more parties under Proportional Representation than under Plurality Rule) is true.

What determines the number of parties? Among the authors for whom this is an endogenous variable: Levy(2002A) seems to suggest that it is the original partition of groups of citizens with identical preferences (although this is not a prediction of the model): her example 2 shows that when there are 3 groups, there will be two parties in equilibrium. In Morelli (2001) it is

the electoral institution and the original preference distribution that predicts the number of parties that form, while in Austen Smith and Banks (1988) and Jackson and Moselle (2002) it is the legislative bargaining game and the gains from trade and ultimatley the preference intensities of legislators that determine the number of legislative coalitions that will form. On the other hand some papers simply show the conditions under which a two party result emerges (e.g Morelli (2001), Osborne and Tourky (2002)). Of course, while Osborne and Tourky (2002) have sufficient conditions that guarantee obtain duvergerian outcomes for every distribution of preferences and for every electoral system, Morelli (2001) shows that preferences and electoral systems determine everything.

In terms of predictions about which policy will ultimately be implemented – the asymmetric importance of the median is a common theme both in unidimensional and multi-dimensional models. Finally equilibrium concepts in most papers surveyed here were non cooperative, most called for subgame perfection. An exception is Levy (2002B) which explicitly uses some cooperative game theoretic equilibrium concepts as well. Jackson and Moselle (2002) use a combination approach– they employ the nash bargaining solution as well as non cooperative bargaining in their model.

I will conclude by observing that an important question is whether, given the importance of the median in most models, parties exaggerate the role of the majority. In practice parties coexist with other coalitions – like interest groups or coalitions of voters– so that outcomes are far from those predicted in this survey. Indeed coalitions may form between parties and their "clients" in ways that are detrimental to the majority. Apart from Morelli (2001) most authors do not focus on the welfare aspects of party formation and this remains an open (albeit normative) question in this newly emerging literature.

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Appendix

Example 1 (Osborne and Tourky (2002)):

Each member of a committee of n individuals has to choose whether to champion a policy (assumed to be in an interval X) and if so which one. The payoff function depends on the actual policy chosen which is the median of the announced positions. If a person participates she incurs a cost $c_i(k(x))$ where k is the number of people announcing the same policy $x \in X$. The cost is positive and decreasing in k. Each player has the strategy set $X \bigcup \theta$ where θ stands for no participation. This payoff function (see Example 2.1 in Osborne and Tourky (2002)), satisfies the two axioms C and E above. So what equilibria might we expect? What drives the two party equilibrium result? The equilibria are shown with the help of some figures (see Figure 4 in the paper). Suppose there is an odd number of participants: then the equilibria must be either (i) Only one independent candidate announces a policy which is the median of the true favourite positions of participants (ii) there are exactly two parties of equal size and one independent in the middle (median of the true favourite policies) position. If there is an odd number of candidates then either (i) there are exactly two independent candidates on either side of the true median (ii) two equally sized parties with $k \geq 2$ on

either side of the true median (iii) A single party on one side of the median with $k \geq 3$ members and one independent and a party on the other side with $k-1$ members. (iv) Two independents, one on either side of the median, flanked by two parties.

The intuition for the result is the following: the trade off between joining a party and choosing to be an independent is the savings in costs vs the costs of compromise. However if a legislator's favourite position is to the left of the median, then joining a party who is also left of the median in a way that does not change the median position is a strategy that makes the individual no worse off in terms of policy outcome but makes him strictly better off in terms of cost savings. Thus left leaning legislators are better off joining a left leaning party and similarly right leaning legislators are better off joining a right leaning party.

Thus, for example let us consider why it is an equilibrium for an even number of participants to agglomerate at exactly two positions. Suppose the number of players is 6 and the policy space is $X = [0, 1]$. Preferences are single peaked and given by $u_i(x) = -(x - x_i)^2$, where x_i represents the ideal point of player i. They can be ordered on the policy space from 0 to 1 depending on their ideal point. Since the number of participants is even the

median is halfway between the two middle positions. For simplicity assume that we have two people at 0 one at 1/4, two at 3/4 and one person at 1. Then the mean of the two medians is 1/2. Assume that the function $c(k)$ (cost as a function of the number of members of the party) is given by C/k. The outcome function takes the announced positions and maps them to the unique median if the number of announcements is odd and takes the mean of the two median positions if the number of announcements is even.

Then if $C \leq \frac{9}{16}$ there is a two party equilibrium with the left party position x given by 1/4 and the right party position y given by 3/4. To see this: notice that members of the left party whose favourite point is to the left of 1/4 do better by participating than by not (gain $= -[(1/2) - 0]^2 + [(3/4) - 0]^2 = \frac{5}{16}$ while the cost per capita is $C/3 = \frac{3}{16}$. The members of the left party at 1/4 just break even and are indifferent between participating and not. By symmetry this is true for the right party as well. Hence this is an equilibrium and so is any other pair (x', y') that gives the same medians and satisfies $x' < 1/4$ and $y' > 3/4$. This is because if $x' > 1/4, y' < 3/4$(to maintain the same medians) we have that the gain for members located at 1/4 is strictly less than $\frac{3}{16}$, so that all such members will choose not to participate.

No player can profitably deviate from such a two party location (x, y).

If any member leaves he will change the outcome to the other parties position and we checked that he prefers to participate given the other player's positions. Members do not want to move further to the left given this configuration as they do not change the outcome but they have higher costs.

There can also be two party equilibria in this example with each party composed of two members each, if costs are given by $C = \frac{75}{576}$.. Eg let $x = 1/3$ and $y = 3/4$, so that the mean is $\frac{13}{24}$. At this level of cost, the candidates with favourite position at $3/4$ will be indifferent between announcing $3/4$ and not participating. So lets assume that one of them participates and the other does not. Candidates at the position $1/4$ do not participate as the costs are too high relative to their gain. Candidates at 0 will participate in the left party and the candidate at 1 will participate in the right party.

Example 2 (Osborne and Tourky (2002), Gomberg et al (2001)):

Let us analyse a simpler version of the Gomberg et al model with a unidimensional policy space and a finite set of agents. We may take the same example (Example 1 above). In this model however the number of parties is *fixed* at two. Let us assume that the actual policy implemented will be the weighted mean of the two party positions, with weights given by the share of votes. I want to show that $x = 1/4, y = 3/4$ with members of the left party

being given by all the citizens who have ideal points to the left of 1/4 and members of the right party given by all citizens who have ideal points on the right of 3/4 is an equilibrium. Once these platforms are announced no coalition of agents can do better by switching to voting for the other party, as this will shift the implemented policy away from their preferred point. So this is an equilibrium in the sense of Gomberg et al. Of course the main point is to generalise to the multi dimensional case and prove existence. The idea in Osborne and Tourky (2002), on the other hand, is to show how a two party equilibrium arises in a uni-dimensional space (or a multi dimensional one that can be ordered by best points).

Levy (2002A), Definition of stable partition:

Denote the finest partition structure, the single member coalition partition, by π^0 and since no deviations are possible for this, $(\pi^0, \delta(\pi^0))$ is defined as stable for all $\delta(\pi^0)$).

Let $R(\pi)$ denote the set of coalition structures that are refinements of π. A coalition $\tilde{\pi}$ is *induced* from π if $\tilde{\pi}$ is formed by breaking a coalition π into two. Suppose that for some π, all stable coalitions with their respective equilibria are defined for all $\pi' \in R(\pi)$.

Definition: $(\pi^1, \delta(\pi^1))$ *is sequentially blocked by* $(\pi', \delta(\pi'))$ *if* $\exists$ *a sequence*

$\{(\pi^1, \delta(\pi^1)), (\pi^2, \delta(\pi^2)), ..., (\pi^m, \delta(\pi^m))\}$, *such that:*

1. $(\pi^1, \delta(\pi^1)) = (\pi, \delta(\pi))$, $(\pi^m, \delta(\pi^m)) = (\pi', \delta(\pi'))$, *and for every* $j = 2, ..., m$ *there is a deviator* S^j *that induces* π^j *from* π^{j-1}.

2. $(\pi', \delta(\pi'))$ *is stable.*

3. $(\pi^j, \delta(\pi^j))$ *is not stable for any* $\delta(\pi^j)$ *and* $1 < j < m$.

4. $U_i(\delta(\pi')) > U_i(\delta(\pi^{j-1}))$ *for all* $j = 2, ..., m$ *and* $i \in S$.

Then $(\pi, \delta(\pi))$ is stable if there is no $(\pi', \delta(\pi'))$ for $\pi' \in R(\pi)$ that sequentially blocks $(\pi, \delta(\pi))$.

Example 3 : Comparison of Riviere (1999), Levy (2002A), Morelli (2001):

There are 3 districts each of which has $N = 3$. We can assume that each group is composed of n_i citizens. The citizens in each group i have the preferences: $u_i(x) = -(i_x - x)^2$ where i_x denotes the ideal point of voter $i \in \{a, b, c\}$. Let $a_x = 0, b_x = 1, c_x = 2$. The policy space is restricted to be the interval $[0, 2]$. No group has an absolute majority in district 1 (so the median is b), while a and b have absolute majorities in districts 2 and 3 respectively. We consider Plurality Rule and sincere voting.There is a cost to candidacy c. Riviere and Levy (2002A) focus on single district solutions.

Thus in district 1, Levy predicts that no (heterogenous) parties form: if all three types run, the equilibrium will have b winning the election. If a, b form a party against c they can offer anything in $(0, 1]$ and win, while if b, c form a party against a, then they can offer anything in $[1, 2)$ and win. What about ac against b? This coalition can win only if they offer the median position 1 otherwise b can enter and win on this platform. To see which coalitions are stable: the trivial one is stable by definition and the only other coalitions that are stable will have policy platform offered being equal to 1 otherwise b can deviate by itself and do better. The grand coalition is stable if the platform is 1 again: hence there are multiple stable partitions but the policy outcome is the same. This is Levy's main result: for policy outcomes to be different from the benchmark case of no parties, we need more than one dimension in the policy space . Riviere (1999) focuses on how many members there would be within the homogenous groups. Morelli (2001) and Levy (2002A) treat this as exogenously given and the cost sharing motivation for forming parties is not considered. Riviere (1999) requires some uncertainty over the median voter– without this only the median candidate would ever enter. So let us focus on district 1 and assume that there are n citizens in each group (i.e. whose preferences are common knowledge to begin with). In stage 1 citizens

in the three groups announce their membership: they can choose to announce themselves as president or members of a party or stay independent (parties are restricted to be homogeneous)[5]. Let N' denote the total number of citizens in the district. The ideology of the other $N'-3n$ citizens is uncertain and determines the state of the world. The probability that the median voter is a, b or c type is $3/10, 4/10, 3/10$ respectively. Once citizens make their announcements parties are formed and in the second stage candidates announce whether they run or not. Candidates can be independents or party presidents only. The median type is then realised and voting takes place. Finally policies are implemented by the winner. The expected gain from standing for election for a candidate depends on the other candidates who stand for election (as this determines the probability of winning) as well as the cost and the number of members willing to share the cost. Would candidates from a, b, c stand independently? The answer depends on costs of candidacy – candidates are trading off the cost against the policy gain of winning. If costs are small enough we can have all three candidates standing. Now let us consider the costs to be high enough so that no independent candidate will consider standing. When parties are allowed to form, the fact that equilibria

[5]This is like the membership game of Levy 2002(B)

at the party formation stage are coalition proof means that coalitions will be *minimal winning coalitions*, i.e only as many members will join the party as are needed to make the party president of party i stand for election given the rest of the candidates[6]. The size of the minimal coalition for a party of type i is determined by the gain to a member of i from a candidate of type i running for election given the rest of the candidate set. Denote by $M_i(C)$ the size of the minimal winning coalition for party i given the candidate set C, i.e. assuming that all candidates $C/\{i\}$ stand for election. Consider a typical member of party a. His preferences over the set of candidates running for election are the following: $(a) \succ_a (a,b) \succ_a (a,b,c) \succ_a (b,c) \succ_a (c)$. A candidate of type c has symmetric preferences to these but a candidate of type b is slightly different: $(b) \succ_b (a,b) \succ_b (a,b,c) \succ_b (a,c) \succ_b (a)$. He is indifferent between (a,b) and (b,c) and between (a) and (c).

For the following calculations I assume that the ideal policy of a is -1, that of b is 0 and that of c is 1, in order to have the same numbers as Riviere (1999) has. Also I assume, as in the paper, that default utility when no-one stands$= -4$. These preferences imply that $M_a(a) \leq M_a(a,b) \leq M_a(a,c) \leq$

[6]I.e. that makes the president indifferent between standing and not standing. Note that the entry decision is taken only after the size of parties is determined, thus it can be made contingent on the size.

$M_a(a,b,c)$. Indeed, we can explicitly calculate these: given cost of candidacy $=\delta > 4$, M_a is the number that equates the per capita cost of candidacy in the party to the expected gain from standing for election given that no other candidate is standing: in this case $\frac{\delta}{M_a(a)} = 4$ since the gain from standing is 4. Thus $M_a(a) = \frac{\delta}{4}$. Similarly $\frac{\delta}{M_a(a,b)} = \frac{3}{10}$, so $M_a(a,b) = \frac{10\delta}{3}$, and in the same way $M_a(a,c) = \frac{\delta}{2}$, $M_a(a,b,c) = \frac{10\delta}{3}$. The numbers for party C are symmetric. For party b we have:$M_b(b) = \frac{\delta}{4}$,$M_b(b,c) = M_b(a,b) = \frac{10\delta}{7}$ and $M_b(a,b,c) = \frac{10\delta}{4}$.

Consider what happens when all three parties have candidates: can this be an equilibrium (CPCPE)? No, because all three parties can do better by deviating collectively to a situation where the centrist party b wins. To illustrate the caculation, consider party a. If all three run, this party gets $\frac{3}{10}(-4) + \frac{4}{10}(-1)$ whereas if only b runs, a gets -1.

Can it be an equilibrium for a centrist party b to be the only one running? This situation is less straightforward and more interesting as parties are using their size strategically to support this equilibrium. The answer is yes: it is clear that no coalition involving the b party can succeed as these citizens are getting their best possible policy. However there is a prisoner's dillemma between the a types and the c types:in the absence of party size as a strategic

variable, it is a dominant strategy for them to enter given that the candidate set is (b): this prisoners dillemma exists in the absence of any commitment mechanism. But note that in this model, the size of the party is a credible commitment device[7]. Given that the size of the b party is such that it can support a candidate set (b) but nothing bigger, this means that the extremist parties (say a) expect to reach the situation (a, c) or (a) if they enter the race. The size of party c is such that they can support (a, c) so both extremist parties expect to reach (a, c) if they enter, which is worse for them than b standing alone.

It can also be an equilibrium for one extremist party to stand against the centrist party. Consider the situation where a, b stand for election. Why does the party c not deviate and stand as well (since the outcome (a, b, c) is preferred by them)? Again because the centrist party can use its size strategically to convince the c party that if it were to enter the outcome would be (a, c) rather than (a, b, c) and the c party prefers the situation (a, b) to the situation (a, c) by risk aversion. The main result is this: the set of rational expectations that supports the two party equilibrium is much bigger than the set that supports the one (centrist) party equilibrium, in the

[7]The entry decision takes place after the party formation stage

sense that the latter requires more restrictions on the size of parties.

Let us consider what happens when a party is interested in maximising the number of districts it can win. How many parties can form in such a case and what platforms do they offer (Morelli (2001))? a has an absolute majority in district 2 and c in district 3. If no heterogenous parties form, in the voting subgame we will have each group winning one seat only and the policy implemented is the median of the three, i.e. 1. If instead we have a, b together vs c then a, b will win the first two and c the third district, and vice versa if we have c, b vs a. In each of these cases the policy implemented must still be 1 because otherwise in the bargaining subgame the other extreme party will offer 1 to b and this will be accepted. Competition in bargaining leads to the median policy (for the country) being offered in every equilibrium. There is no (strict) incentive for heterogenous parties to form. Thus all three groups are effective in the sense of Morelli but not effective in the sense of Levy.

What happens with PR? With the distribution of preferences above, i.e. the median is different in every district, suppose we have the following additional restrictions: party a has a (sincere) share of votes which is the maximum across the three districts but is strictly less than $1/2$ the total votes. It also has a sincere share of votes that would allow it to get another seat

under the Hare quota if all three parties run in every district. Thus if all three parties run the policy would be 0. The other extreme party c will however decide not to run in this situation if the net gains from running are less than the policy gain of letting the b party win. Thus there will be at most two effective parties under PR in this scenario while there were three parties under PV. This situation is the opposite of what is predicted by Duverger's Hypothesis. In order to see situations where the hypothesis does hold, consider the preferences where no party has a majority in any district but across districts using the Hare quota gives each party one seat if they all run. In this case all parties are effective under PR. Under PV however it would pay for a hetrogenous party to form so that they can win two seats and implement 0. So there would be only two effective parties in equilibrium under PV.

Example 4(Jackson and Moselle (2002) Example 1):

There are 3 legislators. $Y = X = 1$, $\delta = 1$. Each legislator has an equal chance of being recognised (called upon to make a proposal). Their peaks are $\hat{y}_1 = 0$,$\hat{y}_2 = 1/2$,$\hat{y}_3 = 1$. Thus the median is $1/2$. The preferences of legislators are given by the quasi-linear utility functions: $u_i = -b_i|y - \hat{y}_i| + x_i$. Possible majority coalitions for any one legislator comprise of one other leg-

islator. Legislators have well defined ex ante expected utilities for a given strategy profile and given the stationarity these are also the continuation utilities denoted v_i, if the current proposal is not accepted. A proposal from i to j denoted y_{ij}, x_{ij} is a proposal that *includes* legislators i and j i.e. promises them a utility at least as high as the continuation utilities and *excludes* legislator k $(k \neq, i, j)$ if his utility from that proposal is strictly less than his continuation utility. Let $b_1 = 1, b_2 = 3$ and $b_3 = 6$. Thus the Marginal Rate of Substitution of public for private goods is given by b_i for each legislator i. To find the equilibrium, we need to find the probabilities a_{ij} that legislator i will make a proposal to j, as well as the proposal vectors y_{ij}, x_{ij}. The ex-ante utility of each legislator (say legislator 1)is given by $v_i = 1/3[a_{12}(u_1(y_{12}, x_{12})) + a_{13}(u_1(y_{13}, x_{13}))] + 1/3[a_{21}(u_1(y_{21}, x_{21})) + a_{23}(u_1(y_{23}, x_{23}))] + 1/3[a_{31}(u_1(y_{31}, x_{31})) + a_{32}(u_1(y_{32}, x_{32}))]$. (Note that $\sum_i(a_{ij}) = 1$.)

In a simple stationery equilibrium, the utilities from the proposals that are accepted must equal the continuation utilities, proposals made by a legislator in equilibrium must give him a utility at least as high as his continuation utility and proposals that exclude a legislator must give a utility strictly less than his continuation utility. The solution to this is: $a_{12} = a_{23} =$

$a_{31} = 1$. The corresponding decisions are: $y_{12} = \frac{1}{2} - \frac{1}{6}$, $x_{12} = (1,0,0)$,; $y_{23} = \frac{1}{2} + \frac{1}{6}$, $x_{23} = (0,1,0)$, and $y_{31} = 1, x_{31} = (1,0,0)$. Let us check the conditions for equilibrium: in the solution we msut have $u_1(y_{12}, x_{12}) \geq u_1(y_{31}, x_{31}) = v_1 > u_1(y_{23}, x_{23})$. Note that $u_1(y_{12}, x_{12}) = \frac{5}{6}, u_1(y_{31}, x_{31}) = 0$, and $u_1(y_{23}, x_{23}) = -\frac{5}{6}$. And $v_1 = \frac{1}{3}(\frac{5}{6}) + \frac{1}{3}(0) + \frac{1}{3}(-\frac{5}{6})$, thus confirming the condition above. Similarly for legislators 2 and 3.[8]Thus equilibrium utility values of the legislators are $v_1 = 0, v_2 = -\frac{1}{2}, v_3 = -2$. Note that this is not an efficient allocation, e.g. the decision $y = \frac{3}{4}, x = (\frac{1}{2}, \frac{1}{2}, 0)$ would give strictly higher utilities to each of the three players of $u_1 = \frac{1}{4}, u_2 = -\frac{1}{4}$ and $u_3 = -\frac{3}{4}$. This is because (1) legislators are risk averse and (2)the fact that for sufficiently high discount factors there is always a positive probability that a legislator will be excluded from some proposal – see Proposition 4. Thus legislators can gain from binding agreements that guarantee a certain utility.

A party is modelled as a binding agreement between the members to act as a single unit in the legislative game. The benefits to the legislators are measured relative to the payoffs in the legislative game. The set of utility levels possible for legislators in a coalition $\{i, j\}$ is described by the

[8]This solution is generalised in the paper to different parameter values for the b_i's.

set of decisions that the coalition can make in the legislative game. The highest utility that legislator 2 can make e.g if a coalition forms between 2 and 3, is given by equilibrium utility for 2 when 2 proposes to 3. Since in equilibrium this is accepted by 3 in the legislative game, it must give 3 at least his disagreement payoff. Thus this is the best that 2 can do while 3 gets at least his disagreement utility. In cases when there is no equilibrium proposal from a legislator to the other person in his coalition, a direct computation must be done. Thus, in the same example: since in equilibrium 3 does not make a proposal to 2 we must compute his highest utility directly: keeping 2 at his disagreement utility, $v_2 = -\frac{1}{2}$ the best that 3 can do is the decision $y = 1, x = (0, 1, 0)$ and this gives the utility $u_3 = 0$. Thus we get a linear utility possibility frontier by varying y between $\frac{2}{3}$ and 1. The disagreement utilities are given by the continuation utilities in the legislative game: $v_2, v_3 = -\frac{1}{2}, -2$. The Nash barganining solution is applied to this problem and this gives the utilities associated with the two players forming a party. For further details of finding the utility possibility frontier when both y and x change in the optimal decisions, see the paper. Finally, calculating a utilities in this way we get that legislator 1 prefers the party $\{1, 2\}$ to the party $\{1, 3\}$ and 2 is indifferent between parties $\{1, 2\}$ and $\{2, 3\}$

and 3 is indifferent between the parties $\{1, 3\}$ and $\{2, 3\}$.