Noncooperative Oligopoly in Markets with a Continuum of Traders

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Abstract

In this paper, we study three prototypical models of noncooperative oligopoly in markets with a continuum of traders: the model of Cournot-Walras equilibrium of Codognato and Gabszewicz (1991), the model of Cournot-Nash equilibrium of Lloyd S. Shapley, and the model of Cournot-Walras equilibrium of Busetto et al. (2008). We argue that these models are all distinct and only the Shapley's model with a continuum of traders and atoms gives an endogenous explanation of the perfectly and imperfectly competitive behavior of agents in a one-stage setting. For this model, we prove a theorem of existence of a Cournot-Nash equilibrium.

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1 Introduction

In this paper, we reconsider the theory of noncooperative oligopoly in general equilibrium, focusing both on pure exchange economies, i.e., markets,

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with a continuum of traders (see Aumann (1964)), and with a continuum of traders and atoms (see Shitovitz (1973)). This theory has so far developed along two main lines of research. The first is the Cournot-Walras approach, initiated, in the context of economies with production, by Gabszewicz and Vial (1972), and, in the case of exchange economies, by Codognato and Gabszewicz (1991) (see also Codognato and Gabszewicz (1993), d'Aspremont et al. (1997), Gabszewicz and Michel (1997), Shitovitz (1997), Lahmandi-Ayed (2001), Bonnisseau and Florig (2003), among others). The second is the noncooperative market game approach, initiated by Shapley and Shubik (1977) (see also Dubey and Shubik (1977), Postlewaite and Schmeidler (1978), Mas-Colell (1982), Amir et al. (1990), Peck et al. (1992), Dubey and Shapley (1994), among others).

A relevant part of the work elaborated within these two lines of research has been concerned with the issue of the strategic foundation of the noncooperative oligopolistic interaction in general equilibrium. As stressed by Okuno et al. (1980), an appropriate model of oligopoly in general equilibrium should give a formal explanation of "[...] either perfectly or imperfectly competitive behavior may emerge endogenously [...], depending on the characteristics of the agent and his place in the economy" (see p. 22).

In this paper, we address this issue by concentrating on three prototypical models proposed in the literature: the model of Codognato and Gabszewicz (1991), based on a concept of Cournot-Walras equilibrium, which we call, following Gabszewicz and Michel (1997), homogeneous oligopoly equilibrium; the model of Cournot-Nash equilibrium originally proposed by Lloyd S. Shapley and further analyzed by Sahi and Yao (1989), and the model of Cournot-Walras equilibrium introduced by Busetto et al. (2008).

We first aim at establishing, in a systematic way, the relationships of the three concepts of equilibrium proposed in these models with the notion of Walras equilibrium; to this end, we consider, according to Aumann (1964), limit exchange economies, i.e., markets with an atomless continuum of traders and, according to Shitovitz (1973), mixed exchange economies, i.e., markets with a continuum of traders and atoms. Second, we investigate the relationships among those three concepts of equilibrium.

We reach the conclusion that the three notions of equilibrium are all distinct. In particular, we argue that the Shapley's model with an atomless continuum of traders and atoms is an autonomous description of the one-shot oligopolistic interaction in a general equilibrium framework, since even its closest Cournot-Walras variant, proposed by Busetto et al. (2008), may generate different equilibria. At the state of the art, it is the only model of noncooperative oligopoly which, according to Okuno et al. (1980), provides an endogenous explanation of the perfectly and imperfectly competitive behavior of agents in a one-stage setting. This motivates us to state and prove a theorem of existence for its Cournot-Nash equilibrium.

As regards the Cournot-Walras approach, instead, the model of Busetto et al. (2008) turns out to be a well-founded representation of the noncooperative oligopolistic interaction in general equilibrium, but only in a two-stage setting: this makes clear a fundamental characteristic of the Cournot-Walras equilibrium concept - namely its two-stage nature - which had remained implicit in all the previous models elaborated within this approach.

The paper is organized as follows. In Section 2, we build the mathematical model of a pure exchange economy where the space of traders is characterized as a complete measure space, with a purely atomic space of large traders and an atomless space of small traders. This model allows us to analyze, within a unifying structure, the different models proposed in the literature on noncooperative oligopoly in general equilibrium. Moreover, it provides the general analytical setup on the basis of which we prove our existence theorem. In Sections 3, 4, and 5, we introduce, respectively, the concept of homogeneous oligopoly equilibrium of Codognato and Gabszewicz (1991), the concept of Cournot-Nash equilibrium of the Shapley's model, and the concept of Cournot-Walras equilibrium of Busetto et al. (2008), and we analyze the relationships of each of them with the notion of Walras equilibrium. In Section 6, we compare the three different concepts of equilibrium. In Section 7, we state our existence theorem for a Cournot-Nash equilibrium of the mixed version of the Shapley's model. Appendix 1 contains the proofs of all the propositions and of a corollary. Appendix 2 contains the proof of existence.

2 The mathematical model

We shall consider a pure exchange economy with large traders, represented as atoms, and small traders, represented by an atomless sector. The set of traders is denoted by $T = T_0 \cup T_1$, where $T_0 = [0, 1]$ is the set of small traders and $T_1 = \{2, \dots, m+1\}$ is the set of large traders. If $a, b \in T_0$ satisfy $a \leq b$, let us write $[a, b) = \emptyset$ if $a = b$ and $[a, b) = \{t \in T_0 : a \leq t < b\}$ if $a < b$. Then, it is well known that the collection $\mathcal{S}_0 = \{[a, b) : a, b \in T_0 \text{ and } a \leq b\}$ is a semiring. Let $S_1 = \mathcal{P}(T_1)$ be the collection of all the subsets of T_1. It is also well known that this collection is an algebra. We denote by μ_0 and μ_1, respectively, the Lebesgue measure on $\mathcal{S}_0$ and the counting measure on $\mathcal{S}_1$. The following proposition gives a first characterization of the set of traders as a measure space.

Proposition 1. *The triplet $(T, \mathcal{S}, \mu)$ - where T is the set of traders, $\mathcal{S} = \{E \subset T : E = A \cup B,\ A \in \mathcal{S}_0,\ B \in \mathcal{S}_1\}$, and $\mu : \mathcal{S} \to [0, \infty]$ is a set function such that $\mu(E) = \mu_0(E \cap T_0) + \mu_1(E \cap T_1)$, for each $E \in \mathcal{S}$ - is a measure space.*

Let μ^* denote the Carathéodory extension of μ. Since $\mu^*(T) = 1+m < \infty$, the measure space $(T, \mathcal{S}, \mu)$ is finite. This implies that a subset E of T is μ-measurable if and only if $\mu^*(E) + \mu^*(E^c) = \mu^*(T)$. Denote by $\mathcal{T}$ the collection of all the μ-measurable subsets of T. The following proposition gives a characterization of the set of traders as a complete measure space.

Proposition 2. *The triplet $(T, \mathcal{T}, \mu^*)$ is a complete measure space and μ^* is the unique extension of μ on a measure on $\mathcal{T}$.*

Now, consider the triplet $(T_0, \mathcal{S}_0, \mu_0)$ which, as is well known, is a measure space. Denote by μ_0^* the Carathéodory extension of μ_0 and by $\mathcal{T}_0$ the collection of all the μ_0-measurable subsets of T_0. Then, it is also well known that the triplet $(T_0, \mathcal{T}_0, \mu_0^*)$ is a complete measure space and that μ_0^* is the unique extension of μ_0 to a measure on $\mathcal{T}_0$. Let $\mathcal{T}_{T_0} = \{E \cap T_0 : E \in \mathcal{T}\}$ be the restriction of $\mathcal{T}$ to T_0. The following proposition characterizes the restriction of the measure space $(T, \mathcal{T}, \mu^*)$ to T_0.

Proposition 3. *The triplet $(T_0, \mathcal{T}_{T_0}, \mu^*)$ is a measure space such that $\mathcal{T}_{T_0} = \mathcal{T}_0$ and $\mu^* = \mu_0^*$, where these measures are restricted to $\mathcal{T}_{T_0}$.*

Now, consider the triplet $(T_1, \mathcal{S}_1, \mu_1)$. It is easy to show that $\mu_1^* = \mu_1$, where μ_1^* denotes the Carathéodory extension of μ_1, and that $\mathcal{S}_1$ is the collection of all the μ_1-measurable subsets of T_1. This implies that the triplet $(T_1, \mathcal{S}_1, \mu_1)$ is a complete measure space. Let $\mathcal{T}_{T_1} = \{E \cap T_1 : E \in \mathcal{T}\}$ be the restriction of $\mathcal{T}$ to T_1. The following proposition characterizes the restriction of the measure space $(T, \mathcal{T}, \mu^*)$ to T_1.

Proposition 4. *The triplet* $(T_1, \mathcal{T}_{T_1}, \mu^*)$ *is a measure space such that* $\mathcal{T}_{T_1} = \mathcal{S}_1$ *and* $\mu^* = \mu_1$*, where these measures are restricted to* $\mathcal{T}_{T_1}$.

Let us now introduce the concept of an atom of a measure space (see Aliprantis and Border (1994), p. 303).

Definition 1. *Let* (X, Σ, μ) *be a measure space. A measurable set* A *is called an atom if* $\mu^*(A) > 0$ *and, for every subset* B *of* A*, either* $\mu^*(B) = 0$ *or* $\mu^*(A \setminus B) = 0$*. If* (X, Σ, μ) *has no atoms, then it is called an atomless measure space. If there exists a countable set* A *such that, for each* $a \in A$*, the singleton set* $\{a\}$ *is measurable with* $\mu^*(\{a\}) > 0$ *and* $\mu^*(X \setminus A) = 0$*, then the measure space* (X, Σ, μ) *is called purely atomic.*

From now on, to simplify the notation, μ^*, μ_0^*, μ_1^* will be denoted by μ, μ_0, μ_1, respectively. Then, the space of traders will be denoted by the complete measure space $(T, \mathcal{T}, \mu)$. By Propositions 3 and 4, it is straightforward to show that the measure space $(T_0, \mathcal{T}_{T_0}, \mu)$ is atomless and the measure space $(T_1, \mathcal{T}_{T_1}, \mu)$ is purely atomic. Moreover, it is clear that, for each $t \in T_1$, the singleton set $\{t\}$ is an atom of the measure space $(T, \mathcal{T}, \mu)$. A null set of traders is a set of Lebesgue measure 0. Null sets of traders are systematically ignored throughout the paper. Thus, a statement asserted for "all" traders, or "each" trader, or "each" trader in a certain set is to be understood to hold for all such traders except possibly for a null set of traders. The word "integrable" is to be understood in the sense of Lebesgue. Given any function $\mathbf{g}$ defined on T, we denote by ${}^0\mathbf{g}$ and ${}^1\mathbf{g}$ the restriction of $\mathbf{g}$ to T_0 and T_1, respectively. Analogously, given any correspondence $\mathbf{G}$ defined on T, we denote by ${}^0\mathbf{G}$ and ${}^1\mathbf{G}$ the restriction of $\mathbf{G}$ to T_0 and T_1, respectively. The following proposition reminds us that the integrability of $\mathbf{g}$ is equivalent to the integrability of ${}^0\mathbf{g}$ and ${}^1\mathbf{g}$.

Proposition 5. *A function* $\mathbf{g} : T \to R$ *is integrable if and only if* ${}^0\mathbf{g}$ *and* ${}^1\mathbf{g}$ *are integrable over* T_0 *and* T_1*, respectively.*

Moreover, it is well known that $\int_T \mathbf{g}(t)\, d\mu = \int_{T_0} {}^0\mathbf{g}(t)\, d\mu + \int_{T_1} {}^1\mathbf{g}(t)\, d\mu$, where $\int_{T_0} {}^0\mathbf{g}(t)\, d\mu = \int_{T_0} {}^0\mathbf{g}(t)\, d\mu_0$ and $\int_{T_1} {}^1\mathbf{g}(t)\, d\mu = \int_{T_1} {}^1\mathbf{g}(t)\, d\mu_1 = \sum_{t=2}^{m+1} \mathbf{g}(t)$, by Propositions 3 and 4.

In the exchange economy, there are l different commodities. A commodity bundle is a point in R^l_+. An assignment (of commodity bundles to traders) is an integrable function $\mathbf{x}$: $T \to R^l_+$. There is a fixed initial assignment $\mathbf{w}$, satisfying the following assumption.

Assumption 1. $\mathbf{w}(t) > 0$, *for all* $t \in T$, $\int_{T_0} \mathbf{w}(t)\, d\mu \gg 0$.

An allocation is an assignment $\mathbf{x}$ for which $\int_T \mathbf{x}(t)\, d\mu = \int_T \mathbf{w}(t)\, d\mu$.

The preferences of each trader $t \in T$ are described by an utility function $u_t : R^l_+ \to R$, satisfying the following assumptions.

Assumption 2. $u_t : R^l_+ \to R$ *is continuous and strictly monotonic, for all* $t \in T$, *quasi-concave, for all* $t \in T_0$, *and concave, for all* $t \in T_1$.

Assumption 3. $u : T \times R^l_+ \to R$, *given by* $u(t, x) = u_t(x)$, *is measurable.*

A price vector is a vector $p \in R^l_+$. According to Aumann (1966), we define, for each $p \in R^l_+$, a correspondence $\mathbf{\Delta}_p : T \to \mathcal{P}(R^l)$ such that, for each $t \in T$, $\mathbf{\Delta}_p(t) = \{x \in R^l_+ : px \leq p\mathbf{w}(t)\}$, a correspondence $\mathbf{\Gamma}_p : T \to \mathcal{P}(R^l)$ such that, for each $t \in T$, $\mathbf{\Gamma}_p(t) = \{x \in R^l_+ : \text{ for all } y \in \mathbf{\Delta}_p(t), u_t(x) \geq u_t(y)\}$, and finally a correspondence $\mathbf{X}_p : T \to \mathcal{P}(R^l)$ such that, for each $t \in T$, $\mathbf{X}_p(t) = \mathbf{\Delta}_p(t) \cap \mathbf{\Gamma}_p(t)$.

A Walras equilibrium is a pair $(p^*, \mathbf{x}^*)$, consisting of a price vector p^* and an allocation $\mathbf{x}^*$, such that, for all $t \in T$, $\mathbf{x}^*(t) \in \mathbf{X}_{p^*}(t)$.

Finally, the following proposition shows that there is a measurable selector from the correspondence ${}^0\mathbf{X}_p$.

Proposition 6. *Under Assumptions 1, 2, and 3, for each* $p \in R^l_{++}$, *there exists an integrable function* ${}^0\mathbf{x}_p : T_0 \to R^l_+$ *such that, for each* $t \in T_0$, ${}^0\mathbf{x}_p(t) \in {}^0\mathbf{X}_p(t)$.

3 Homogeneous oligopoly equilibrium

We first consider the notion of Cournot-Walras equilibrium for exchange economies proposed by Codognato and Gabszewicz (1991). The concept of Cournot-Walras equilibrium was originally introduced by Gabszewicz and Vial (1972) in the framework of an economy with production. These authors were already aware that their notion of equilibrium raised some theoretical difficulties, as it depends on the rule chosen to normalize prices and profit maximization may not be a rational objective of firms. The reformulation of the Cournot-Walras equilibrium for exchange economies proposed by Codognato and Gabszewicz (1991) made it possible to overcome these problems, since it does not depend on price normalization and replaces profit maximization with utility maximization. This concept was generalized by Gabszewicz and Michel (1997) by means of a notion of oligopoly equilibrium for exchange economies (see also d'Aspremont et al. (1997), for another generalization of the concept). More precisely, the Cournot-Walras equilibrium introduced by Codognato and Gabszewicz (1991) corresponds to the case of homogeneous oligopoly equilibrium in the framework developed by Gabszewicz and Michel (1997). We adopt here their denomination.

In order to formulate the concept of homogeneous oligopoly equilibrium, we assume that the space of traders is as in Section 2. Moreover, we need to introduce the following variant of Assumption 2.

Assumption 2′. *$u_t : R^l_+ \to R$ is continuous, strictly monotonic, and strictly quasi-concave for all $t \in T$.*

Under Assumption 2′, for each $p \in R^l_{++}$, it is possible to define the small traders' Walrasian demands as a function ${}^0\mathbf{x}(\cdot, p) : T_0 \to R^l_+$, such that, for each $t \in T_0$, ${}^0\mathbf{x}(t, p) = {}^0\mathbf{\Delta}_p(t) \cap {}^0\mathbf{\Gamma}_p(t)$. It is also possible to show the following proposition.

Proposition 7. *Under Assumptions 1, 2′, and 3, the function ${}^0\mathbf{x}(\cdot, p)$ is integrable, for each $p \in R^l_{++}$.*

We also consider a particular specification of the exchange economy defined in Section 2, where the initial assignment of atoms is $\mathbf{w}(t) = (\mathbf{w}^1(t), 0, \ldots, 0)$, for all $t \in T_1$.

Consider now the atoms' strategies. A strategy correspondence is a correspondence $\mathbf{Y} : T_1 \to \mathcal{P}(R)$ such that, for each $t \in T_1$, $\mathbf{Y}(t) = \{y \in R : 0 \leq y \leq \mathbf{w}^1(t)\}$. A strategy selection is an integrable function $\mathbf{y} : T_1 \to R$ such that, for all $t \in T_1$, $\mathbf{y}(t) \in \mathbf{Y}(t)$. For each $t \in T_1$, $\mathbf{y}(t)$ represents the amount of commodity 1 that trader t offers in the market. We denote by $\mathbf{y} \setminus y(t)$ a strategy selection obtained by replacing $\mathbf{y}(t)$ in $\mathbf{y}$ with $y(t) \in \mathbf{Y}(t)$. Given a price vector $p \in R^l_{++}$ and a strategy selection $\mathbf{y}$, let ${}^1\mathbf{x}(\cdot, \mathbf{y}(\cdot), p) : T_1 \to R^l_+$ denote a function such that, for each $t \in T_1$, ${}^1\mathbf{x}^1(t, \mathbf{y}(t), p) = \mathbf{w}^1(t) - \mathbf{y}(t)$ and $({}^1\mathbf{x}^2(t, \mathbf{y}(t), p), \ldots, {}^1\mathbf{x}^l(t, \mathbf{y}(t), p))$ is, under Assumption 2′, the unique solution to the problem

$$\max_{x^2, \ldots, x^l} u_t(\mathbf{w}^1(t) - \mathbf{y}(t), x^2, \ldots, x^l) s.t. \sum_{j=2}^{l} p^j x^j = p^1 \mathbf{y}(t).$$

Let $\pi(\mathbf{y})$ denote the correspondence which associates, with each strategy selection $\mathbf{y}$, the set of the price vectors such that

$$\int_{T_0} {}^0\mathbf{x}^1(t,p)\,d\mu = \int_{T_0} {}^0\mathbf{w}^1(t)\,d\mu + \int_{T_1} \mathbf{y}(t)\,d\mu,$$

$$\int_{T_0} {}^0\mathbf{x}^j(t,p)\,d\mu + \int_{T_1} {}^1\mathbf{x}^j(t,\mathbf{y}(t),p)\,d\mu = \int_{T_0} {}^0\mathbf{w}^j(t)\,d\mu,$$

$j = 2,\ldots,l$. We assume that, for each $\mathbf{y}$, $\pi(\mathbf{y}) \neq \emptyset$ and $\pi(\mathbf{y}) \subset R^l_{++}$. A price selection $p(\mathbf{y})$ is a function which associates, with each $\mathbf{y}$, a price vector $p \in \pi(\mathbf{y})$.

Given a strategy selection $\mathbf{y}$, by the structure of the traders' measure space, Proposition 7, and the atoms' maximization problem, it is straightforward to show that the function $\mathbf{x}(t)$ such that $\mathbf{x}(t) = {}^0\mathbf{x}(t,p(\mathbf{y}))$, for all $t \in T_0$, and $\mathbf{x}(t) = {}^1\mathbf{x}(t,\mathbf{y}(t),p(\mathbf{y}))$, for all $t \in T_1$, is an allocation.

At this stage, we are able to define the concept of homogeneous oligopoly equilibrium.

Definition 2. *A pair $(\check{\mathbf{y}},\check{\mathbf{x}})$, consisting of a strategy selection $\check{\mathbf{y}}$ and an allocation $\check{\mathbf{x}}$ such that $\check{\mathbf{x}}(t) = {}^0\mathbf{x}(t,p(\check{\mathbf{y}}))$, for all $t \in T_0$, and $\check{\mathbf{x}}(t) = {}^1\mathbf{x}(t,\check{\mathbf{y}}(t),p(\check{\mathbf{y}}))$, for all $t \in T_1$, is a homogeneous oligopoly equilibrium, with respect to a price selection $p(\mathbf{y})$, if $u_t({}^1\mathbf{x}(t,\check{\mathbf{y}}(t),p(\check{\mathbf{y}}))) \geq u_t({}^1\mathbf{x},y(t),(t,p(\check{\mathbf{y}}\setminus y(t))))$, for all $t \in T_1$ and for all $y(t) \in \mathbf{Y}(t)$.*

Let us consider the relationship between the concepts of homogeneous oligopoly and Walras equilibrium. As is well known, within the Cournotian tradition, it has been established that the Cournot equilibrium approaches the competitive equilibrium as the number of oligopolists increases. Codognato and Gabszewicz (1993) confirmed this result. By considering a limit exchange economy à la Aumann, they were able to show that the set of the homogeneous oligopoly equilibrium allocations coincides with the set of the Walras equilibrium allocations.

On the other hand, they provided an example showing that this result no longer holds in an exchange economy à la Shitovitz, where strategic traders are represented as atoms. More precisely, by means of this example they proved that, within their Cournot-Walras structure, it was possible to avoid a counterintuitive result obtained by Shitovitz (1973) in the cooperative context. In his Theorem B, this author proved that the core allocations of a mixed exchange economy are Walrasian when the atoms have the same endowments and preferences (but not necessarily the same measure). In their example, Codognato and Gabszewicz (1993) considered an exchange economy with two identical atoms facing a atomless continuum of small traders and showed that, in this economy, there is a homogeneous oligopoly equilibrium allocation which is not Walrasian.

4 Cournot-Nash equilibrium

The model described in the previous section shares, with the whole Cournot-Walras approach, a fundamental problem, stressed, in particular, by Okuno et al. (1980). In fact, it does not explain why a certain agent behaves strategically rather than competitively.

Taking inspiration from the cooperative approach to oligopoly introduced by Shitovitz (1973), Okuno et al. (1980) proposed a foundation of agents' behavior based on the Cournot-Nash equilibria of a model of simultaneous, noncooperative exchange between large traders, represented as atoms, and small traders, represented by an atomless sector. Their model belongs to the line of research initiated by Shapley and Shubik (1977). Moreover, Okuno et al. (1980) were the first who aimed at showing that the unsatisfying result obtained by Shitovitz (1973) with his Theorem B could be avoided in the noncooperative context. In fact, they gave both an example and a proposition showing that, in their Cournot-Nash equilibrium model, the small traders always have a negligible influence on prices, while the large traders keep their strategic power even when their behavior turns out to be Walrasian in the cooperative framework considered by Shitovitz (1973). Nevertheless, the model they used incorporates very special hypotheses, since it considers only two commodities that no trader can simultaneously buy and sell.

In this section, we show that a more general result can be obtained by using the mixed version of a model of simultaneous, noncooperative exchange originally proposed by Lloyd S. Shapley and subsequently analyzed by Sahi and Yao (1989) in the case of exchange economies with a finite number of traders.

Let us assume that the space of traders is as in Section 2.

Consider now the traders' strategies. Let $b \in R^{l^2}$ be a vector such that $b = (b_{11}, b_{12}, \dots, b_{ll-1}, b_{ll})$. A strategy correspondence is a correspondence $\mathbf{B} : T \to \mathcal{P}(R^{l^2})$ such that, for each $t \in T$, $\mathbf{B}(t) = \{b \in R^{l^2} : b_{ij} \geq 0,\, i, j =$

$1, \ldots, l$; $\sum_{j=1}^{l} b_{ij} \leq \mathbf{w}^i(t)$, $i = 1, \ldots, l\}$. A strategy selection is an integrable function $\mathbf{b} : T \to R^{l^2}$, such that, for all $t \in T$, $\mathbf{b}(t) \in \mathbf{B}(t)$. For each $t \in T$, $\mathbf{b}_{ij}(t)$, $i, j = 1, \ldots, l$, represents the amount of commodity i that trader t offers in exchange for commodity j. Given a strategy selection $\mathbf{b}$, we define the aggregate matrix $\bar{\mathbf{B}} = (\int_T \mathbf{b}_{ij}(t)\, d\mu)$. Moreover, we denote by $\mathbf{b} \setminus b(t)$ a strategy selection obtained by replacing $\mathbf{b}(t)$ in $\mathbf{b}$ with $b(t) \in \mathbf{B}(t)$.

Then, we introduce two further definitions (see Sahi and Yao (1989)).

Definition 3. *A nonnegative square matrix A is said to be irreducible if, for every pair i, j, with $i \neq j$, there is a positive integer $k = k(i, j)$ such that $a_{ij}^{(k)} > 0$, where $a_{ij}^{(k)}$ denotes the ij-th entry of the k-th power A^k of A.*

Definition 4. *Given a strategy selection* $\mathbf{b}$*, a price vector p is market clearing if*

$$p \in R_{++}^l, \sum_{i=1}^{l} p^i \bar{\mathbf{b}}_{ij} = p^j (\sum_{i=1}^{l} \bar{\mathbf{b}}_{ji}), \; j = 1, \ldots, l. \tag{1}$$

By Lemma 1 in Sahi and Yao (1989), there is a unique, up to a scalar multiple, price vector p satisfying (1) if and only if $\bar{\mathbf{B}}$ is irreducible. Denote by $p(\mathbf{b})$ the function which associates, with each strategy selection $\mathbf{b}$ such that $\bar{\mathbf{B}}$ is irreducible, the unique, up to a scalar multiple, market clearing price vector p.

Given a strategy selection $\mathbf{b}$ such that p is market clearing and unique, up to a scalar multiple, consider the assignment determined as follows:

$$\mathbf{x}^j(t, \mathbf{b}(t), p(\mathbf{b})) = \mathbf{w}^j(t) - \sum_{i=1}^{l} \mathbf{b}_{ji}(t) + \sum_{i=1}^{l} \mathbf{b}_{ij}(t) \frac{p^i(\mathbf{b})}{p^j(\mathbf{b})},$$

for all $t \in T$, $j = 1, \ldots, l$. It is easy to verify that this assignment is an allocation. Given a strategy selection $\mathbf{b}$, the traders' final holdings are defined as

$$\begin{aligned} \mathbf{x}^j(t) &= \mathbf{x}^j(t, \mathbf{b}(t), p(\mathbf{b})) \text{ if } p \text{ is market clearing and unique,} \\ \mathbf{x}^j(t) &= \mathbf{w}^j(t) \text{ otherwise,} \end{aligned}$$

for all $t \in T$, $j = 1, \ldots, l$.

This reformulation of the Shapley's model allows us to define the following concept of Cournot-Nash equilibrium for exchange economies with a continuum of traders (see Codognato and Ghosal (2000)).

Definition 5. *A strategy selection* $\hat{\mathbf{b}}$ *such that* $\bar{\hat{\mathbf{B}}}$ *is irreducible is a Cournot-Nash equilibrium if*

$$u_t(\mathbf{x}(t, \hat{\mathbf{b}}(t), p(\hat{\mathbf{b}}))) \geq u_t(\mathbf{x}(t, b(t), p(\hat{\mathbf{b}} \setminus b(t)))),$$

for all $t \in T$ *and for all* $b(t) \in \mathbf{B}(t)$.

Codognato and Ghosal (2000) already showed that, in limit exchange economies, the set of the Cournot-Nash equilibrium allocations of the Shapley's model and the set of the Walras equilibrium allocations coincide. It remains to be verified whether, according to Okuno et al. (1980), this equivalence no longer holds under the assumptions of Theorem B in Shitovitz (1973). We provide here a proposition and an example showing that, under those assumptions, the small traders always have a Walrasian price-taking behavior whereas the large traders have market power even in those circumstances where the core outcome is competitive.

Proposition 8. *For each strategy selection* $\mathbf{b}$ *such that* $\bar{\mathbf{B}}$ *is irreducible and for each* $t \in T_0$, *(i)* $p(\mathbf{b}) = p(\mathbf{b} \setminus b(t))$, *for all* $b(t) \in \mathbf{B}(t)$; *(ii)* $\mathbf{x}(t, b(t), p(\mathbf{b} \setminus b(t))) \in \mathbf{X}_{p(\mathbf{b})}(t)$, *for all* $b(t) \in$ *argmax*$\{u_t(\mathbf{x}(t, b(t), p(\mathbf{b} \setminus b(t)))) : b(t) \in \mathbf{B}(t)\}$.

More precisely, part (i) of Proposition 8 establishes that each small trader is unable to influence prices and part (ii) that all the best replies of each small trader attains a point in his Walras demand correspondence.

Example 1. *Consider the following specification of an exchange economy satisfying Assumptions 1, 2, and 3, where* $l = 2$, $T_1 = \{2, 3\}$, $T_0 = [0, 1]$, $\mathbf{w}(2) = \mathbf{w}(3)$, $u_2(x) = u_3(x)$, $\mathbf{w}(t) = (0, 1)$, $u_t(x) = (x^1)^{\alpha}(x^2)^{1-\alpha}$, $0 < \alpha < 1$, *for all* $t \in T_0$. *Then, if* $\hat{\mathbf{b}}$ *is a Cournot-Nash equilibrium, the pair* $(\hat{p}, \hat{\mathbf{x}})$ *such that* $\hat{p} = p(\hat{\mathbf{b}})$ *and* $\hat{\mathbf{x}}(t) = \mathbf{x}(t, \hat{\mathbf{b}}(t), \hat{p})$, *for all* $t \in T$, *is not a Walras equilibrium.*

Proof. Suppose that $\hat{\mathbf{b}}$ is a Cournot-Nash equilibrium and that the pair $(\hat{p}, \hat{\mathbf{x}})$ such that $\hat{p} = p(\hat{\mathbf{b}})$ and $\hat{\mathbf{x}}(t) = \mathbf{x}(t, \hat{\mathbf{b}}(t), \hat{p})$, for all $t \in T$, is a Walras equilibrium. Clearly, $\hat{\mathbf{b}}_{21}(t) = \alpha$, for all $t \in T_0$. Since, for each atom, at a Nash equilibrium, the marginal price (see Okuno et al. (1980)) must be equal to the marginal rate of substitution which, in turn, at a Walras equilibrium, must be equal to the relative price of commodity 1 in terms of commodity 2, we must have

$$\frac{dx_2}{dx_1} = -\hat{p}^2 \frac{\hat{\mathbf{b}}_{12}(t)}{\hat{\mathbf{b}}_{21}(t) + \alpha} = -\hat{p},$$

for each $t \in T_1$. But then, we must also have

$$\frac{\hat{\mathbf{b}}_{21}(2) + \alpha}{\hat{\mathbf{b}}_{12}(2)} = \frac{\hat{\mathbf{b}}_{21}(2) + \hat{\mathbf{b}}_{21}(3) + \alpha}{\hat{\mathbf{b}}_{12}(2) + \hat{\mathbf{b}}_{12}(3)} = \frac{\hat{\mathbf{b}}_{21}(3) + \alpha}{\hat{\mathbf{b}}_{12}(3)}. \tag{2}$$

The last equality in (2) holds if and only if $\hat{\mathbf{b}}_{21}(2) = k(\hat{\mathbf{b}}_{21}(3) + \alpha)$ and $\hat{\mathbf{b}}_{12}(2) = k\hat{\mathbf{b}}_{12}(3)$, with $k > 0$. But then, the first and the last members of (2) cannot be equal because

$$\frac{k(\hat{\mathbf{b}}_{21}(3) + \alpha) + \alpha}{k\hat{\mathbf{b}}_{12}(3)} \neq \frac{\hat{\mathbf{b}}_{21}(3) + \alpha}{\hat{\mathbf{b}}_{12}(3)}.$$

This implies that the pair $(\hat{p}, \hat{\mathbf{x}})$ such that $\hat{p} = p(\hat{\mathbf{b}})$ and $\hat{\mathbf{x}}(t) = \mathbf{x}(t, \hat{\mathbf{b}}(t), \hat{p})$, for all $t \in T$, cannot be a Walras equilibrium. ■

Notice that our example provides a result stronger than the proposition proved by Okuno et al. (1980) because it requires that atoms have not only the same endowments and preferences but also the same measure.

Proposition 8 and Example 1 clarify that the mixed version of the Shapley's model introduced in this section is a well-founded model of oligopoly in general equilibrium as it gives an endogenous explanation of strategic and competitive behavior. Therefore, it is immune from the criticism by Okuno et al. (1980).

5 Cournot-Walras equilibrium

In this section, we describe the concept of Cournot-Walras equilibrium proposed by Busetto et al. (2008), which makes it possible to overcome some problems inherent in the notion of equilibrium introduced by Codognato and Gabszewicz (1991). In particular, in the model proposed by these authors, oligopolists are characterized by a "twofold behavior," since they act à la Cournot in making their supply decisions and à la Walras in exchanging commodities. As we shall see below, in the model of Busetto et al. (2008), oligopolists always behave à la Cournot. This model can be viewed as a re-specification à la Cournot-Walras of the mixed version of the Shapley's model presented in Section 4. It can be formulated as follows.

Again the space of traders is as in Section 2.

As regards the atomless sector, like in Section 3, we can define, under Assumption 2′, the Walrasian demands as a function ${}^0\mathbf{x}(\cdot, p) : T_0 \to R^l_+$, such that for each $t \in T_0$, ${}^0\mathbf{x}(t, p) = {}^0\boldsymbol{\Delta}_p(t) \cap {}^0\boldsymbol{\Gamma}_p(t)$.

Consider now the atoms' strategies. Let $e \in R^{l^2}$ be a vector such that $e = (e_{11}, e_{12}, \ldots, e_{ll-1}, e_{ll})$. A strategy correspondence is a correspondence $\mathbf{E} : T_1 \to \mathcal{P}(R^{l^2})$ such that, for each $t \in T_1$, $\mathbf{E}(t) = \{e \in R^{l^2} : e_{ij} \geq 0,\, i, j = 1, \ldots, l; \sum_{j=1}^{l} e_{ij} \leq \mathbf{w}^i(t),\, i = 1, \ldots, l\}$. A strategy selection is an integrable function $\mathbf{e} : T_1 \to R^{l^2}$ such that, for all $t \in T_1$, $\mathbf{e}(t) \in \mathbf{E}(t)$. For each $t \in T_1$, $\mathbf{e}_{ij}(t)$, $i, j = 1, \ldots, l$, represents the amount of commodity i that trader t offers in exchange for commodity j. We denote by $\mathbf{e} \setminus e(t)$ a strategy selection obtained by replacing $\mathbf{e}(t)$ in $\mathbf{e}$ with $e(t) \in \mathbf{E}(t)$. Finally, we denote by $\pi(\mathbf{e})$ the correspondence which associates, with each $\mathbf{e}$, the set of the price vectors such that

$$\int_{T_0} {}^0\mathbf{x}^j(t, p)\, d\mu + \sum_{i=1}^{l} \int_{T_1} \mathbf{e}_{ij}(t)\, d\mu \frac{p^i}{p^j} = \int_{T_0} \mathbf{w}^j(t)\, d\mu + \sum_{i=1}^{l} \int_{T_1} \mathbf{e}_{ji}(t)\, d\mu, \quad (3)$$

$j = 1, \ldots, l$.

Assumption 4. *For each* $\mathbf{e}$, $\pi(\mathbf{e}) \neq \emptyset$ *and* $\pi(\mathbf{e}) \subset R^l_{++}$.

A price selection $p(\mathbf{e})$ is a function which associates, with each $\mathbf{e}$, a price vector $p \in \pi(\mathbf{e})$ and is such that $p(\mathbf{e}') = p(\mathbf{e}'')$ if $\int_{T_1} \mathbf{e}'(t)\, d\mu = \int_{T_1} \mathbf{e}''(t)\, d\mu$. For each strategy selection $\mathbf{e}$, let ${}^1\mathbf{x}(\cdot, \mathbf{e}(\cdot), p(\mathbf{e})) : T_1 \to R^l_+$ denote a function such that

$${}^1\mathbf{x}^j(t, \mathbf{e}(t), p(\mathbf{e})) = \mathbf{w}^j(t) - \sum_{i=1}^{l} \mathbf{e}_{ji}(t) + \sum_{i=1}^{l} \mathbf{e}_{ij}(t) \frac{p^i(\mathbf{e})}{p^j(\mathbf{e})}, \quad (4)$$

for all $t \in T_1$, $j = 1, \ldots, l$. Given a strategy selection $\mathbf{e}$, taking into account the structure of the traders' measure space, Proposition 7, and Equation (3), it is straightforward to show that the function $\mathbf{x}(t)$ such that $\mathbf{x}(t) = {}^0\mathbf{x}(t, p(\mathbf{e}))$, for all $t \in T_0$, and $\mathbf{x}(t) = {}^1\mathbf{x}(t, \mathbf{e}(t), p(\mathbf{e}))$, for all $t \in T_1$, is an allocation.

At this stage, we are able to define the concept of Cournot-Walras equilibrium.

Definition 6. *A pair* $(\tilde{\mathbf{e}}, \tilde{\mathbf{x}})$, *consisting of a strategy selection* $\tilde{\mathbf{e}}$ *and an allocation* $\tilde{\mathbf{x}}$ *such that* $\tilde{\mathbf{x}}(t) = {}^0\mathbf{x}(t, p(\tilde{\mathbf{e}}))$, *for all* $t \in T_0$, *and* $\tilde{\mathbf{x}}(t) = {}^1\mathbf{x}(t, \tilde{\mathbf{e}}(t), p(\tilde{\mathbf{e}}))$, *for all* $t \in T_1$, *is a Cournot-Walras equilibrium, with respect to a price selection* $p(\mathbf{e})$, *if* $u_t({}^1\mathbf{x}(t, \tilde{\mathbf{e}}(t), p(\tilde{\mathbf{e}}))) \geq u_t({}^1\mathbf{x}(t, e(t), p(\tilde{\mathbf{e}} \setminus e(t))))$, *for all* $t \in T_1$ *and for all* $e(t) \in \mathbf{E}(t)$.

Let us investigate the relationship between the concepts of Cournot-Walras and Walras equilibrium. Here, we show, for the Cournot-Walras equilibrium, a result similar to those obtained, in limit exchange economies, by Codognato and Gabszewicz (1993) for the homogeneous oligopoly equilibrium, and by Codognato and Ghosal (2000) for the Cournot-Nash equilibrium. More precisely, we assume that the space of traders is denoted by the complete measure space $(T, \mathcal{T}, \mu)$, where the set of traders is denoted by $T = T_0 \cup T_1$, with $T_0 = [0, 1]$ and $T_1 = [2, 3]$, $\mathcal{T}$ is the σ-algebra of all measurable subsets of T, and μ is the Lebesgue measure on $\mathcal{T}$.

The following proposition establishes that, in this framework, the set of the Cournot-Walras equilibrium allocations coincides with the set of the Walras equilibrium allocations.

Proposition 9. *Under Assumptions 1,* $2'$*, 3, and 4, (i) if* $(\tilde{\mathbf{e}}, \tilde{\mathbf{x}})$ *is a Cournot-Walras equilibrium with respect to a price selection* $p(\mathbf{e})$, *there is a price vector* $\tilde{p}$ *such that* $(\tilde{p}, \tilde{\mathbf{x}})$ *is a Walras equilibrium; (ii) if* $(p^*, \mathbf{x}^*)$ *is a Walras equilibrium, there is a strategy selection* $\mathbf{e}^*$ *such that* $(\mathbf{e}^*, \mathbf{x}^*)$ *is a Cournot-Walras equilibrium with respect to a price selection* $p(\mathbf{e})$.

Proposition 9 has the following corollary assuring the existence of a Cournot-Walras equilibrium in limit exchange economies.

Corollary. *A Cournot-Walras equilibrium exists.*

The question whether the equivalence between the concepts of Cournot-Walras and Walras equilibrium still holds when the strategic traders are represented as atoms was dealt with by Busetto et al. (2008). They analyzed an exchange economy with two identical atoms facing an atomless continuum of traders and gave an example showing that, in this economy, there is a Cournot-Walras equilibrium allocation which is not Walrasian. We repropose here their example.

Example 2. *Consider the following specification of an exchange economy satisfying Assumptions 1,* $2'$*, 3, and 4, where* $l = 2$, $T_1 = \{2, 3\}$, $T_0 = [0, 1]$,

$\mathbf{w}(t) = (1, 0)$, $u_t(x) = lnx^1 + lnx^2$, *for all* $t \in T_1$, $\mathbf{w}(t) = (1, 0)$, $u_t(x) = lnx^1 + lnx^2$, *for all* $t \in [0, \frac{1}{2}]$, $\mathbf{w}(t) = (0, 1)$, $u_t(x) = lnx^1 + lnx^2$, *for all* $t \in [\frac{1}{2}, 1]$. *For this economy, there is a Cournot-Walras equilibrium allocation which does not correspond to any Walras equilibrium.*

Proof. The only symmetric Cournot-Walras equilibrium is the pair $(\tilde{\mathbf{e}}, \tilde{\mathbf{x}})$, where $\tilde{\mathbf{e}}_{12}(2) = \tilde{\mathbf{e}}_{12}(3) = \frac{1+\sqrt{13}}{12}$, $\tilde{\mathbf{x}}^1(2) = \tilde{\mathbf{x}}^1(3) = \frac{11+\sqrt{13}}{12}$, $\tilde{\mathbf{x}}^2(2) = \tilde{\mathbf{x}}^2(3) = \frac{1+\sqrt{13}}{20+8\sqrt{13}}$, $\tilde{\mathbf{x}}^1(t) = \frac{1}{2}$, $\tilde{\mathbf{x}}^2(t) = \frac{3}{10+4\sqrt{13}}$, for all $t \in [0, \frac{1}{2}]$, $\tilde{\mathbf{x}}^1(t) = \frac{5+2\sqrt{13}}{6}$, $\tilde{\mathbf{x}}^2(t) = \frac{1}{2}$, for all $t \in [\frac{1}{2}, 1]$. On the other hand, the only Walras equilibrium of the economy considered is the pair $(\mathbf{x}^*, p^*)$, where $\mathbf{x}^{*1}(2) = \mathbf{x}^{*1}(3) = \frac{1}{2}$, $\mathbf{x}^{*2}(2) = \mathbf{x}^{*2}(3) = \frac{1}{10}$, $\mathbf{x}^{*1}(t) = \frac{1}{2}$, $\mathbf{x}^{*2}(t) = \frac{1}{10}$, for all $t \in [0, \frac{1}{2}]$, $\mathbf{x}^{*1}(t) = \frac{5}{2}$, $\mathbf{x}^{*2}(t) = \frac{1}{2}$, for all $t \in [\frac{1}{2}, 1]$, $p^* = \frac{1}{5}$. ■

Therefore, the counterintuitive result established by Shitovitz (1973) with his Theorem B can be avoided also in the framework of Busetto et al. (2008). It remains to be analyzed, with reference to their Cournot-Walras model, the problem of the endogenous explanation of strategic and competitive behavior. We will go back to this issue in the next section, where we compare the different notions of equilibrium introduced above.

6 Homogeneous oligopoly, Cournot-Nash, and Cournot-Walras equilibrium

In this section, we propose to analyze, in a systematic way, the relationships among the three concepts of equilibrium presented above. We first show that the homogeneous oligopoly equilibrium concept proposed by Codognato and Gabszewicz (1991) and the Cournot-Walras equilibrium concept introduced by Busetto et al. (2008) differ. To this end, we provide the following example showing that there is a Cournot-Walras equilibrium allocation which does not correspond to any homogeneous oligopoly equilibrium.

Example 3. *Consider the following specification of an exchange economy satisfying Assumptions 1, 2′, 3, and 4, where* $l = 3$, $T_1 = \{2, 3\}$, $T_0 = [0, 1]$, $\mathbf{w}(t) = (1, 0, 0)$, $u_t(x) = 2x^1 + lnx^2 + lnx^3$, *for all* $t \in T_1$, $\mathbf{w}(t) = (1, 0, 0)$, $u_t(x) = lnx^1 + lnx^2 + lnx^3$, *for all* $t \in [0, \frac{1}{2}]$, $\mathbf{w}(t) = (0, 1, 1)$, $u_t(x) = x^1 + \frac{1}{2}lnx^2 + lnx^3$, *for all* $t \in [\frac{1}{2}, 1]$. *For this economy, there is a Cournot-Walras equilibrium allocation which does not correspond to any homogeneous* *oligopoly equilibrium.*

Proof. There is a unique Cournot-Walras equilibrium represented by the pair $(\tilde{\mathbf{e}}, \tilde{\mathbf{x}})$, where $\tilde{\mathbf{e}}_{12}(2) = \tilde{\mathbf{e}}_{12}(3) = \frac{1+\sqrt{241}}{48}$, $\tilde{\mathbf{e}}_{13}(2) = \tilde{\mathbf{e}}_{13}(3) = \frac{-1+\sqrt{97}}{24}$, $\tilde{\mathbf{x}}^1(2) = \tilde{\mathbf{x}}^1(3) = \frac{49-\sqrt{241}-2\sqrt{97}}{48}$, $\tilde{\mathbf{x}}^2(2) = \tilde{\mathbf{x}}^2(3) = \frac{1+\sqrt{241}}{44+4\sqrt{241}}$, $\tilde{\mathbf{x}}^3(2) = \tilde{\mathbf{x}}^3(3) = \frac{-1+\sqrt{97}}{28+4\sqrt{97}}$, $\tilde{\mathbf{x}}^1(t) = \frac{1}{3}$, $\tilde{\mathbf{x}}^2(t) = \frac{4}{11+\sqrt{241}}$, $\tilde{\mathbf{x}}^3(t) = \frac{2}{7+\sqrt{97}}$, for all $t \in [0, \frac{1}{2}]$, $\tilde{\mathbf{x}}^1(t) = \frac{9+\sqrt{97}+\sqrt{241}}{12}$, $\tilde{\mathbf{x}}^2(t) = \frac{6}{11+\sqrt{241}}$, $\tilde{\mathbf{x}}^3(t) = \frac{6}{7+\sqrt{97}}$, for all $t \in [\frac{1}{2}, 1]$. On the other hand, there is no interior symmetric homogeneous oligopoly equilibrium for the economy considered. ■

Codognato (1995) compared the mixed version of the model in Codognato and Gabszewicz (1991) with the mixed version of the Shapley's model. The point was the following: if the set of the equilibrium allocations of the model of homogeneous oligopoly equilibrium - where strategic and competitive behavior is assumed a priori - had coincided with the set of the equilibrium allocations of the Shapley's model - where strategic and competitive behavior is generated endogenously - then the notion of homogeneous oligopoly equilibrium could have been re-interpreted as the outcome of a game in which all agents behave strategically but those belonging to the atomless sector turn out to act competitively whereas the atoms turn out to have market power. Therefore, this equilibrium concept would have been immune from the criticism by Okuno et al. (1980).

Nonetheless, Codognato (1995) provided an example showing that the set of the homogeneous oligopoly equilibrium allocations does not coincide with the set of the Cournot-Nash equilibrium allocations. There could be two reasons for this result. The first is that the homogeneous oligopoly equilibrium, like the other Cournot-Walras equilibrium concepts, has an intrinsic two-stage nature, which cannot be reconciled with the one-stage Cournot-Nash equilibrium of the Shapley's model. The second is that, in the model of Codognato and Gabszewicz (1991), the oligopolists have the twofold behavior mentioned above, as they act à la Cournot in making their supply decisions and à la Walras in exchanging commodities whereas, in the mixed version of the Shapley's model, they always behave à la Cournot.

The relationship between the concepts of Cournot-Walras and Cournot-Nash equilibrium was analyzed by Busetto et al. (2008). They noticed that the allocation corresponding to a Cournot-Walras equilibrium in Example 2 also corresponded to a Cournot-Nash equilibrium as in Definition 5. Consequently, the Cournot-Nash equilibrium could be viewed as a situation in which all traders behave strategically but those belonging to the atomless sector have a negligible influence on prices. In effect, our Example 1 and Proposition 7 provide a more general proof of this fact. Thus, the strategic behavior of the atomless sector can be interpreted as competitive behavior.

On the other hand, at a Cournot-Walras equilibrium as in Definition 6, the atomless sector is supposed to behave competitively while the atoms have strategic power. This led Busetto et al. (2008) to conjecture that the set of the Cournot-Walras equilibrium allocations coincide with the set of the Cournot-Nash equilibrium allocations. They were able to show that this conjecture was false by means of the following example.

Example 4. *Consider the following specification of an exchange economy satisfying Assumptions 1, $2'$, 3, and 4, where $l = 2$, $T_1 = \{2, 3\}$, $T_0 = [0, 1]$, $\mathbf{w}(t) = (1, 0)$, $u_t(x) = lnx^1 + lnx^2$, for all $t \in T_1$, $\mathbf{w}(t) = (1, 0)$, $u_t(x) = lnx^1 + lnx^2$, for all $t \in [0, \frac{1}{2}]$, $\mathbf{w}(t) = (0, 1)$, $u_t(x) = x^1 + lnx^2$, for all $t \in [\frac{1}{2}, 1]$. For this economy, there is a Cournot-Walras equilibrium allocation which does not correspond to any Cournot-Nash equilibrium.*

Proof. The only symmetric Cournot-Walras equilibrium of the economy considered is the pair $(\tilde{\mathbf{e}}, \tilde{\mathbf{x}})$, where $\tilde{\mathbf{e}}_{12}(2) = \tilde{\mathbf{e}}_{12}(3) = \frac{-1+\sqrt{37}}{12}$, $\tilde{\mathbf{x}}^1(2) = \tilde{\mathbf{x}}^1(3) = \frac{11-\sqrt{37}}{12}$, $\tilde{\mathbf{x}}^2(2) = \tilde{\mathbf{x}}^2(3) = \frac{-1+\sqrt{37}}{14+4\sqrt{37}}$, $\tilde{\mathbf{x}}^1(t) = \frac{1}{2}$, $\tilde{\mathbf{x}}^2(t) = \frac{3}{7+2\sqrt{37}}$, for all $t \in [0, \frac{1}{2}]$, $\tilde{\mathbf{x}}^1(t) = \frac{1+2\sqrt{37}}{6}$, $\tilde{\mathbf{x}}^2(t) = \frac{6}{7+2\sqrt{37}}$, for all $t \in [\frac{1}{2}, 1]$. On the other hand, the only symmetric Cournot-Nash equilibrium is the strategy selection $\hat{\mathbf{b}}_{12}(2) = \hat{\mathbf{b}}_{12}(3) = \frac{1+\sqrt{13}}{12}$, $\hat{\mathbf{b}}_{12}(t) = \frac{1}{2}$, for all $t \in [0, \frac{1}{2}]$, $\hat{\mathbf{b}}_{21}(t) = \frac{5+2\sqrt{13}}{11+2\sqrt{13}}$ for all $t \in [\frac{1}{2}, 1]$, which generates the allocation $\hat{\mathbf{x}}^1(2) = \hat{\mathbf{x}}^1(3) = \frac{11+\sqrt{13}}{12}$, $\hat{\mathbf{x}}^2(2) = \hat{\mathbf{x}}^2(3) = \frac{1+\sqrt{13}}{22+4\sqrt{13}}$, $\hat{\mathbf{x}}^1(t) = \frac{1}{2}$, $\hat{\mathbf{x}}^2(t) = \frac{3}{11+2\sqrt{13}}$, for all $t \in [0, \frac{1}{2}]$, $\hat{\mathbf{x}}^1(t) = \frac{5+2\sqrt{13}}{6}$, $\hat{\mathbf{x}}^2(t) = \frac{6}{11+2\sqrt{13}}$, for all $t \in [\frac{1}{2}, 1]$, where $\hat{\mathbf{x}}(t) = \mathbf{x}(t, \hat{\mathbf{b}}, p(\hat{\mathbf{b}}))$, for all $t \in T$. ■

This confirms, in a more general framework, the nonequivalence result obtained by Codognato (1995). In this regard, it is worth noticing that, in both models compared in Example 4, large traders always behave à la Cournot. Therefore, this example removes one of the possible explanations of the nonequivalence proved by Codognato (1995), namely the twofold behavior of the oligopolists in the model of homogeneous oligopoly equilibrium. This suggested to Busetto et al. (2008) that the general cause of the nonequivalence between the concepts of Cournot-Walras and Cournot-Nash equilibrium had to be the two-stage implicit nature of the Cournot-Walras equilibrium concept, which cannot be reconciled with the one-stage Cournot-Nash equilibrium of the Shapley's model. For this reason, they introduced a reformulation of the Shapley's model as a two-stage game, where the atoms move in the first stage and the atomless sector moves in the second stage, and showed that the set of the Cournot-Walras equilibrium allocations coincides with a specific set of subgame perfect equilibrium allocations of this two-stage game. Therefore, they provided a strategic foundation of the Cournot-Walras approach in a two-stage setting.

7 An existence theorem

The analysis in the previous sections makes clear that a strategic foundation of the Cournot-Walras approach has been obtained only in a two-stage setting while, in a one-stage setting, the mixed version of the Shapley's model is immune from the criticism of Okuno et al. (1989), since it is able to endogenously explain the perfectly and imperfectly competitive behavior of agents. It provides an autonomous description of the one-shot oligopolistic interaction in a general equilibrium framework, since even its closest Cournot-Walras variant may generate different equilibria.

The fact that, within this model, traders' behavior has an endogenous foundation also permits us to overcome some technical problems which had so far made it difficult to prove the existence of equilibria in the models belonging to the Cournot-Walras line of research. In particular, in these models, equilibria may not exist, even in mixed strategies, since the Walras price correspondence may fail to be continuous. In the remainder of this paper, we prove a theorem of existence of a Cournot-Nash equilibrium for the mixed version of the Shapley's model. The construction of the mixed measure space of traders provided in Section 2 allows us to synthesize, in our proof, the techniques used by Sahi and Yao (1989) to show the existence of Cournot-Nash equilibria in finite exchange economies and those used to prove the existence of noncooperative equilibria in nonatomic games (see, Schmeidler (1973) and Khan (1985), among others). In proving our theorem, however, we had to deal with new technical problems in taking limits, that we have overcome by using the proof of the Fatou's lemma in several dimensions provided by Artstein (1979).

Moreover, in order to show our existence theorem, we need to introduce the following assumption on atoms' endowments and preferences.

Assumption 5. *There are at least two traders in T_1 for whom $\mathbf{w}(t) \gg 0$; u_t is continuously differentiable in R^l_{++}; $\{x \in R^l_+ : u_t(x) = u_t(\mathbf{w}(t))\} \subset R^l_{++}$.*

The theorem can be stated as follows.

Theorem. *Under Assumptions 1, 2, 3, and 5, there is a Cournot-Nash equilibrium $\hat{\mathbf{b}}$.*

8 Appendix 1. Proofs of the Propositions and the Corollary

Proof of Proposition 1. Since $\mathcal{S}_0$ is a semiring and $\mathcal{S}_1$ is an algebra, it easily follows that $\mathcal{S}$ is a semiring. Now, observe that

$$\mu(\emptyset) = \mu_0(\emptyset) + \mu_1(\emptyset) = 0.$$

Moreover, let $\{E_n\}$ be a disjoint sequence of $\mathcal{S}$ with $\cup_{n=1}^{\infty} E_n \in \mathcal{S}$. Then, we have

$$\begin{aligned}
&\mu(\cup_{n=1}^{\infty} E_n) = \mu_0((\cup_{n=1}^{\infty} E_n) \cap T_0) + \mu_1((\cup_{n=1}^{\infty} E_n) \cap T_1) \\
&= \mu_0(\cup_{n=1}^{\infty}(E_n \cap T_0)) + \mu_1(\cup_{n=1}^{\infty}(E_n \cap T_1)) \\
&= \sum_{n=1}^{\infty} \mu_0(E_n \cap T_0) + \sum_{n=1}^{\infty} \mu_1(E_n \cap T_1) \\
&= \sum_{n=1}^{\infty} (\mu_0(E_n \cap T_0) + \mu_1(E_n \cap T_1)) = \sum_{n=1}^{\infty} \mu(E_n).
\end{aligned}$$

Hence, μ is a measure on $\mathcal{S}$. ■

Proof of Proposition 2. It follows from the Carathéodory Extension Procedure Theorem (see Aliprantis and Border (1994), p. 289), since the measure space $(T, \mathcal{S}, \mu)$, being finite, is σ-finite. ■

Proof of Proposition 3. First, we shall show that, for every subset E of T_0, $\mu^*(E) = \mu_0^*(E)$. This can be done as follows

$$\begin{aligned}
&\mu^*(E) = \inf\{\sum_{n=1}^{\infty} \mu(E_n) : \{E_n\} \subset \mathcal{S},\, E \subseteq \cup_{n=1}^{\infty} E_n\} \\
&= \inf\{\sum_{n=1}^{\infty} \mu_0(E_n \cap T_0) + \sum_{n=1}^{\infty} \mu_1(E_n \cap T_1) : \{E_n\} \subset \mathcal{S},\, E \subseteq \cup_{n=1}^{\infty} E_n\} \\
&= \inf\{\sum_{n=1}^{\infty} \mu_0(E_n \cap T_0) : \{E_n\} \subset \mathcal{S},\, \{E_n \cap T_1\} = \emptyset,\, E \subset \cup_{n=1}^{\infty}(E_n \cap T_0)\} \\
&= \mu_0^*(E).
\end{aligned}$$

Now, observe that, since T_0 is μ-measurable, $\mathcal{T}_{T_0}$ is a collection of μ-measurable subsets of T_0. Let E be a subset of $\mathcal{T}_0$. By the σ-subadditivity of μ^*, we have

$$\begin{aligned}
&\mu^*(T) \leq \mu^*(E) + \mu^*(E^c) = \mu^*(E) + \mu^*((E^c \cap T_0) \cup T_1) \\
&\leq \mu^*(E) + \mu^*(E^c \cap T_0) + \mu^*(T_1).
\end{aligned}$$

On the other hand, since E is μ_0-measurable, we have

$$\begin{aligned}
&\mu^*(E) + \mu^*(E^c \cap T_0) + \mu^*(T_1) = \mu_0^*(E) + \mu_0^*(E^c \cap T_0) + \mu^*(T_1) \\
&= \mu_0^*(T_0) + \mu^*(T_1) = \mu^*(T_0) + \mu^*(T_1) = \mu^*(T).
\end{aligned}$$

This implies that $\mu^*(E) + \mu^*(E^c) = \mu^*(T)$, which, in turn, implies that E is μ-measurable, thereby showing that $\mathcal{T}_0 \subseteq \mathcal{T}_{T_0}$. Now, let E be a subset of $\mathcal{T}_{T_0}$. Since E is μ-measurable, we have

$$\mu^*(E) + \mu^*(E^c) = \mu^*(T).$$

By the σ-additivity of μ^* on $\mathcal{T}$, it follows that

$$\mu^*(E) + \mu^*(E^c \cap T_0) + \mu^*(T_1) = \mu^*(T_0) + \mu^*(T_1).$$

Therefore, we have $\mu_0^*(E) + \mu_0^*(E^c \cap T_0) = \mu_0^*(T_0)$ and this, in turn, implies that E is μ_0-measurable, thereby showing that $\mathcal{T}_{T_0} \subseteq \mathcal{T}_0$. Hence, $\mathcal{T}_{T_0} = \mathcal{T}_0$. ■

Proof of Proposition 4. First, by the same argument used in the proof of Proposition 3, it is possible to show that, for every subset E of T_1, $\mu^*(E) = \mu_1^*(E) = \mu_1(E)$. Now, observe that, since $\mathcal{S} \subseteq \mathcal{T}$, $\mathcal{S}_1 = \mathcal{S} \cap T_1 \subseteq \mathcal{T} \cap T_1 = \mathcal{T}_{T_1}$. On the other hand, $\mathcal{T}_{T_1} \subseteq \mathcal{P}(T_1) = \mathcal{S}_1$. Hence, $\mathcal{S}_1 = \mathcal{T}_{T_1}$. ■

Proof of Proposition 5. See Kolmogorov and Fomin (1975) Theorem 4, p. 298, and Problem 6, p. 302. ∎

Proof of Proposition 6. First, observe that, for each $p \in R^l_{++}$, Assumption 2 implies that, for each $t \in T_0$, ${}^0\mathbf{X}_p(t) \neq \emptyset$. Moreover, from Aumann (1966), we know that the correspondence ${}^0\mathbf{X}_p$ is Borel measurable since the correspondences ${}^0\boldsymbol{\Delta}_p$ and ${}^0\boldsymbol{\Gamma}_p$ are Borel measurable and $\{(t,x) : x \in {}^0\mathbf{X}_p(t)\} = \{(t,x) : x \in {}^0\boldsymbol{\Delta}_p(t)\} \cap \{(t,x) : x \in {}^0\boldsymbol{\Gamma}_p(t)\}$. Finally, ${}^0\mathbf{X}_p$ is integrably bounded because $x^i \leq \frac{\sum_{j=1}^l p^j \mathbf{w}^j(t)}{p^i}$, $i = 1, \ldots, l$, for all $t \in T_0$ and for all x such that $x \in {}^0\mathbf{X}_p(t)$. But then, by Theorem 2 in Aumann (1965), there exists an integrable function ${}^0\mathbf{x}_p$ such that, for each $t \in T_0$, ${}^0\mathbf{x}_p(t) \in {}^0\mathbf{X}_p(t)$. ∎

Proof of Proposition 7. It is an immediate consequence of Proposition 6, since, for each $p \in R^l_{++}$, ${}^0\mathbf{x}(t,p) = {}^0\mathbf{X}_p(t)$, for all $t \in T_0$. ∎

Proof of Proposition 8. (i) It is an immediate consequence of Definition 3. (ii) It can be proved by the same argument used in the proof of part (i) of Theorem 2 in Codognato and Ghosal (2000). ∎

Proof of Proposition 9. (i) Let $(\tilde{\mathbf{e}}, \tilde{\mathbf{x}})$ be a Cournot-Walras equilibrium with respect to the price selection $p(\mathbf{e})$. First, it is straightforward to show that, for all $t \in T_1$, $\tilde{p}\tilde{\mathbf{x}}(t) = \tilde{p}\mathbf{w}(t)$, where $\tilde{p} = p(\tilde{\mathbf{e}})$. Let us now show that, for all $t \in T_1$, $\tilde{\mathbf{x}}(t) \in \boldsymbol{\Delta}_{\tilde{p}}(t) \cap \boldsymbol{\Gamma}_{\tilde{p}}(t)$. Suppose that this is not the case for a trader $t \in T_1$. Then, by Assumption 2′, there is a bundle $z \in \{x \in R^l_+ : \tilde{p}x = \tilde{p}\mathbf{w}(t)\}$ such that $u_t(z) > u_t(\tilde{\mathbf{x}}(t))$. By Lemma 5 in Codognato and Ghosal (2000), there exist $\lambda^j \geq 0$, $j = 1, \ldots, l$, $\sum_{j=1}^l \lambda^j = 1$, such that

$$z^j = \lambda^j \frac{\sum_{j=1}^l \tilde{p}^j \mathbf{w}^j(t)}{\tilde{p}^j},\ j = 1, \ldots, l.$$

Let $e_{ij}(t) = \mathbf{w}^i(t)\lambda^j$, $i, j = 1, \ldots, l$, for all $t \in T_1$. Substituting in Equation (4) and taking into account the fact that, by Equation (3), $p(\tilde{\mathbf{e}}) = p(\tilde{\mathbf{e}} \backslash e(t)) = \tilde{p}$, it is easy to verify that

$${}^1\mathbf{x}^j(t, e(t), p(\tilde{\mathbf{e}} \setminus e(t))) = \mathbf{w}^j(t) - \sum_{i=1}^l \mathbf{w}^j(t)\lambda^i + \sum_{i=1}^l \mathbf{w}^i(t)\lambda^j \frac{\tilde{p}^i}{\tilde{p}^j} = z^j,\ j = 1, \ldots, l.$$

But then, we have

$$u_t({}^1\mathbf{x}(t, e(t), p(\tilde{\mathbf{e}} \setminus e(t)))) = u_t(z) > u_t(\tilde{\mathbf{x}}(t)) = u_t({}^1\mathbf{x}(t, \tilde{\mathbf{e}}(t), p(\tilde{\mathbf{e}}))),$$

which contradicts the fact that the pair $(\tilde{\mathbf{e}}, \tilde{\mathbf{x}}))$ is a Cournot-Walras equilibrium. (ii) Let $(p^*, \mathbf{x}^*)$ be a Walras equilibrium. First, notice that, by Assumption $2'$, $p^* \in R^l_{++}$ and $p^*\mathbf{x}^*(t) = p^*\mathbf{w}(t)$, for all $t \in T$. But then, by Lemma 5 in Codognato and Ghosal (2000), for all $t \in T_1$, there exist $\lambda^{*j}(t) \geq 0$, $j = 1, \ldots, l$, $\sum_{j=1}^{l} \lambda^{*j}(t) = 1$, such that

$$\mathbf{x}^{*j}(t) = \lambda^{*j}(t)\frac{\sum_{j=1}^{l} p^{*j}\mathbf{w}^j(t)}{p^{*j}}, \ j = 1, \ldots, l.$$

Define now a function $\boldsymbol{\lambda}^* : T_1 \to R^l_+$ such that $\boldsymbol{\lambda}^{*j}(t) = \lambda^{*j}(t)$, $j = 1, \ldots, l$, for all $t \in T_1$ and a function $\mathbf{e}^* : T_1 :\to R^{l^2}_+$ such that $\mathbf{e}^*_{ij}(t) = \mathbf{w}^i(t)\boldsymbol{\lambda}^{*j}(t)$, $i, j = 1, \ldots, l$, for all $t \in T_1$. It is straightforward to show that the function $\mathbf{e}^*$ is integrable. Moreover, by using Equation (4), it is easy to verify that

$$\mathbf{x}^{*j}(t) = \mathbf{w}^j(t) - \sum_{i=1}^{l} \mathbf{e}^*_{ji}(t) + \sum_{i=1}^{l} \mathbf{e}^*_{ij}(t)\frac{p^{*i}}{p^{*j}},$$

$j = 1, \ldots, l$, for all $t \in T_1$. As $\mathbf{x}^*$ is an allocation, it follows that

$$\int_{T_0} \mathbf{x}^{*j}(t)\, d\mu + \int_{T_1} \mathbf{w}^j(t)\, d\mu - \sum_{i=1}^{l} \int_{T_1} \mathbf{e}^*_{ji}(t)\, d\mu + \sum_{i=1}^{l} \int_{T_1} \mathbf{e}^*_{ij}(t)\, d\mu \frac{p^{*i}}{p^{*j}} = \int_{T} \mathbf{w}^j(t)\, d\mu,$$

$j = 1, \ldots, l$. This, in turn, implies that

$$\int_{T_0} \mathbf{x}^{*j}(t)\, d\mu + \sum_{i=1}^{l} \int_{T_1} \mathbf{e}^*_{ij}(t)\, d\mu \frac{p^{*i}}{p^{*j}} = \int_{T_0} \mathbf{w}^j(t)\, d\mu + \sum_{i=1}^{l} \int_{T_1} \mathbf{e}^*_{ji}(t)\, d\mu,$$

$j = 1, \ldots, l$. But then, by Assumption 4, there is a price selection $p(\mathbf{e})$ such that $p^* = p(\mathbf{e}^*)$ and, consequently, $\mathbf{x}^*(t) = {}^0\mathbf{x}(t, p(\mathbf{e}^*))$, for all $t \in T_0$, and $\mathbf{x}^*(t) = {}^1\mathbf{x}(t, \mathbf{e}^*(t), p(\mathbf{e}^*))$, for all $t \in T_1$. It remains to show that no trader $t \in T_1$ has an advantageous deviation from $\mathbf{e}^*$. Suppose, on the contrary, that there exists a trader $t \in T_1$ and a strategy $e(t) \in \mathbf{E}(t)$ such that

$$u_t({}^1\mathbf{x}(t, , e(t), p(\mathbf{e}^* \setminus e(t)))) > u_t({}^1\mathbf{x}(t, \mathbf{e}^*(t), p(\mathbf{e}^*))).$$

By Equation (3), we have $p(\mathbf{e}^* \setminus e(t)) = p(\mathbf{e}^*) = p^*$. Moreover, it is easy to show that $p^{*1}\mathbf{x}(t, e(t), p(\mathbf{e}^* \setminus e(t)) = p^*\mathbf{w}(t)$. But then, the pair $(p^*, \mathbf{x}^*)$ is not a Walras equilibrium, which generates a contradiction. ■

Proof of the Corollary. From Aumann (1966), we know that, under Assumptions 1, $2'$, and 3, a Walras equilibrium exists. But then, by part (ii) of Proposition 9, this implies that a Cournot-Walras equilibrium exists. ■

9 Appendix 2. Proof of the Theorem

As in Sahi and Yao (1989), we shall first show the existence of a slightly perturbed Cournot-Nash equilibrium. Given $\epsilon > 0$, we define the aggregate bid matrix $\bar{\mathbf{B}}^\epsilon$ to be $\bar{\mathbf{B}}^\epsilon = (\int_T \mathbf{b}_{ij}(t)\, d\mu + \epsilon)$. Clearly, the matrix $\bar{\mathbf{B}}^\epsilon$ is irreducible. The interpretation is that an outside agency places fixed bids of ϵ for each pair of commodities (i, j). Given $\epsilon > 0$, we denote by $p^\epsilon(\mathbf{b})$ the function which associates to each strategy selection $\mathbf{b}$ the unique, up to a scalar multiple, price vector which satisfies

$$\sum_{i=1}^{l} p^i(\bar{\mathbf{b}}_{ij} + \epsilon) = p^j(\sum_{i=1}^{l}(\bar{\mathbf{b}}_{ji} + \epsilon),\ j = 1, \ldots, l. \tag{5}$$

Definition 7. *Given $\epsilon > 0$, a strategy selection $\hat{\mathbf{b}}^\epsilon$ is an ϵ-Cournot-Nash equilibrium if*

$$u_t(\mathbf{x}(t, \hat{\mathbf{b}}^\epsilon(t), p^\epsilon(\hat{\mathbf{b}}^\epsilon))) \geq u_t(t, b(t), p^\epsilon(\hat{\mathbf{b}}^\epsilon \setminus b(t)))),$$

for almost all $t \in T$ and for all $b(t) \in \mathbf{B}(t)$.

The following fixed point theorem, proven by Fan (1952) and Glicksberg (1952), is the basic tool we use to show our theorem.

Theorem (Fan-Glicksberg). *Let K be a nonempty, convex and compact subset of a locally convex space X. If ϕ is an upper semicontinuos mapping from K into K and if, for all $x \in X$, the set $\phi(x)$ is nonempty and convex, then there exists a point $\hat{x} \in K$ such that $\hat{x} \in \phi(\hat{x})$.*

Neglecting, as usual, the distinction between integrable functions and equivalence classes of such functions, we denote by $L_1(\mu, R^{l^2})$ the set of integrable functions taking values in R^{l^2} and by $L_1(\mu, \mathbf{B}(\cdot))$ the set of strategy selections (see Schmeidler (1973) and Khan (1985)). The locally convex space we shall working in is $L_1(\mu, R^{l^2})$ endowed with its weak topology. The following lemma provides us with the required properties of the set $L_1(\mu, \mathbf{B}(\cdot))$.

Lemma 1. *The set $L_1(\mu, \mathbf{B}(\cdot))$ is nonempty, convex and weakly compact.*

Proof. For each $i = 1, \ldots, l$, let $\lambda_{ij} \geq 0$, $\sum_{j=1}^{l} \lambda_{ij} = 1$. Since $\mathbf{w}$ is an assignment, the function $\mathbf{b} : T \to R_+^{l^2}$ such that, for each $t \in T$, $\mathbf{b}_{ij}(t) = \lambda_{ij}\mathbf{w}^i(t)$, $i, j = 1, \ldots, l$ belongs to $L_1(\mu, \mathbf{B}(\cdot))$. The fact that $L_1(\mu, R^{l^2})$ is a vector space and the fact that, for each $t \in T$, $\mathbf{B}(t)$ is convex imply that $L_1(\mu, \mathbf{B}(\cdot))$ is convex. Finally, the weak compactness of $L_1(\mu, \mathbf{B}(\cdot))$ may be proved following Khan (1985). First, notice that $\sup_{\mathbf{b} \in L_1(\mu, \mathbf{B}(\cdot))} \int_T |\mathbf{b}_{ij}|\, d\mu < \infty$, $i, j = 1, \ldots, l$. Let $\epsilon > 0$. For each $j = 1, \ldots, l$, there exists a $\delta_j > 0$ (depending upon ϵ) such that $|\int_E \mathbf{w}^j(t)\, d\mu| \leq \epsilon$, for all measurable sets E with $\mu(E) \leq \delta_j$ (see Problem 18.6 in Aliprantis and Burkinshaw (1990b), p. 127). This implies that, if $\mu(E) \leq \delta = \min\{\delta_1, \ldots, \delta_l\}$, then, for all $\mathbf{b} \in L_1(\mu, \mathbf{B}(\cdot))$, $\int_E |\mathbf{b}_{ij}(t)|\, d\mu \leq \epsilon$, $i, j = 1, \ldots, l$. This, by the Dunford-Pettis theorem (see Diestel (1984), p. 93), in turn implies that $L_1(\mu, \mathbf{B}(\cdot))$ has a weakly compact closure. Now, let $\{\mathbf{b}^n\}$ be a Cauchy sequence of $L_1(\mu, \mathbf{B}(\cdot))$. Since $L_1(\mu, R^{l^2})$ is complete, $\{\mathbf{b}^n\}$ converges in the mean to an integrable function $\mathbf{b}$. But then, there exists a subsequence $\{\mathbf{b}^{k_n}\}$ of $\{\mathbf{b}^n\}$ such that $\mathbf{b}^{k_n} \to \mathbf{b}$ a.e. (see Theorem 21.5 in Aliprantis and Burkinshaw (1990a), p. 159). The compactness of $\mathbf{B}(t)$, for each $t \in T$, implies that $\mathbf{b} \in L_1(\mu, \mathbf{B}(\cdot))$. Hence $L_1(\mu, \mathbf{B}(\cdot))$ is norm closed and, since it is also convex, it is weakly closed (see Corollary 4 in Diestel (1984), p. 12). ■

Given $\epsilon > 0$, let $\alpha : L_1(\mu, \mathbf{B}(\cdot)) \to L_1(\mu, \mathbf{B}(\cdot))$ be a mapping such that $\alpha(\mathbf{b}) = \{\mathbf{b} \in L_1(\mu, \mathbf{B}(\cdot)) : \mathbf{b}(t) \in \alpha_t(\mathbf{b})$, for almost all $t \in T\}$ where, for each $t \in T$, the mapping $\alpha_t : L_1(\mu, \mathbf{B}(\cdot)) \to \mathbf{B}(t)$ is such that $\alpha_t(\mathbf{b}) = \operatorname{argmax}\{u_t(\mathbf{x}(t, b(t), p^\epsilon(\mathbf{b} \setminus b(t)))) : b(t) \in \mathbf{B}(t)\}$. The following lemma provides us with the required properties of α.

Lemma 2. *Given $\epsilon > 0$, the mapping $\alpha : L_1(\mu, \mathbf{B}(\cdot)) \to L_1(\mu, \mathbf{B}(\cdot))$ is an upper semicontinuous mapping such that, for all $\mathbf{b} \in L_1(\mu, \mathbf{B}(\cdot))$, the set $\alpha(\mathbf{b})$ is nonempty and convex.*

Proof. Let $\epsilon > 0$ be given. Consider a trader $t \in T_1$. By Lemma 4 in Sahi and Yao (1989), we know that α_t is an upper semicontinuos mapping such that, for all $\mathbf{b} \in L_1(\mu, \mathbf{B}(\cdot))$, $\alpha_t(\mathbf{b})$ is nonempty, compact and convex. Now, consider a trader $t \in T_0$. Given $\mathbf{b} \in L_1(\mu, \mathbf{B}(\cdot))$, Proposition 8 implies that $u_t(\mathbf{x}(t, b(t), p^\epsilon(\mathbf{b} \setminus b(t)))) = u_t(\mathbf{x}(t, b(t), p^\epsilon(\mathbf{b})))$, for all $b \in \mathbf{B}(t)$. Therefore, for all $\mathbf{b} \in L_1(\mu, \mathbf{B}(\cdot))$, α_t is nonempty and compact, by the continuity of the function $u_t(\mathbf{x}(t, b(t), p^\epsilon(\mathbf{b})))$ over the compact set $\mathbf{B}(t)$, and convex, by Assumption 2. The upper semicontinuity of α_t is a straightforward consequence of the Maximum Theorem (see Berge (1997), p. 116). Now, given a strategy selection $\mathbf{b} \in L_1(\mu, \mathbf{B}(\cdot))$, by Proposition 6, there exists an integrable function $\mathbf{x}_{p^\epsilon(\mathbf{b})} : T_0 \to R^l_+$ such that, for each $t \in T_0$, $\mathbf{x}_{p^\epsilon(\mathbf{b})}(t) \in \mathbf{X}_{p^\epsilon(\mathbf{b})}(t)$. By Lemma 5 in Codognato and Ghosal (2000), for each $t \in T_0$, there exist

$\lambda^j(t) \geq 0$, $\sum_{j=1}^{l} \lambda^j(t) = 1$, such that

$$\mathbf{x}^j_{p^\epsilon(\mathbf{b})}(t) = \lambda^j(t) \frac{\sum_{j=1}^{l} p^{\epsilon j}(\mathbf{b})^0\mathbf{w}^j(t)}{p^{\epsilon j}(\mathbf{b})}, \; j = 1, \ldots, l.$$

Define a function $\boldsymbol{\lambda} : T_0 \to R^l_+$, such that, $\boldsymbol{\lambda}(t) = \lambda(t)$, for each $t \in T_0$. Since $\mathbf{x}_{p^\epsilon(\mathbf{b})}$ and ${}^0\mathbf{w}$ are integrable functions with respect to μ_0 and $\sum_{j=1}^{l} p^{\epsilon j}(\mathbf{b})^0\mathbf{w}^j(t) \gg 0$, for all $t \in T_0$, $\boldsymbol{\lambda}$ is a integrable function with respect to μ_0. Now, define a function ${}^0\mathbf{b}^* : T_0 \to R^{l^2}_+$ such that ${}^0\mathbf{b}^*_{ij}(t) = {}^0\mathbf{w}^i(t)\boldsymbol{\lambda}^j(t)$, $i, j = 1, \ldots, l$, for all $t \in T_0$. The function ${}^0\mathbf{b}^*$ is integrable with respect to μ_0 and hence, by Proposition 3, with respect to μ. Moreover, by Theorem 2 in Codognato and Ghosal (2000), $\mathbf{b}^*(t) \in \alpha_t(\mathbf{b})$, for each $t \in T_0$. Let ${}^1\mathbf{b}^* : T_1 \to R^{l^2}_+$ be a function such that ${}^1\mathbf{b}^*(t) \in \alpha_t(\mathbf{b})$, for each $t \in T_1$. The function ${}^1\mathbf{b}^*$ is integrable with respect to μ_1 and hence, by Proposition 4, with respect to μ. But then, by Proposition 5, $\alpha(\mathbf{b})$ is nonempty. The convexity of $\alpha(\mathbf{b})$ is a straightforward consequence of the convexity of $\alpha_t(\mathbf{b})$, for all $t \in T$. Finally, the upper semicontinuity of α may be proved following Khan (1985). Since $L_1(\mu, \mathbf{B}(\cdot))$ is weakly compact, we can show the upper semicontinuity of α by showing that its graph is closed in $L_1(\mu\mathbf{B}(\cdot)) \times L_1(\mu\mathbf{B}(\cdot))$ (see the Corollary in Berge (1997), p. 112). Let $\{\mathbf{b}^\nu, \mathbf{b}^{*\nu}\}$ be a net converging to $(\mathbf{b}, \mathbf{b}^*)$ where $\mathbf{b}^{*\nu} \in \alpha(\mathbf{b}^\nu)$. The set $\{\mathbf{b}^\nu, \mathbf{b}^{*\nu}\} \cup (\mathbf{b}, \mathbf{b}^*)$, being a subset of $L_1(\mu, \mathbf{B}(\cdot) \times L_1(\mu, \mathbf{B}(\cdot))$, is relatively weakly compact (see Theorem 2.11 in Aliprantis and Border (1994), p. 30). By the Eberlein-Smulian Theorem (see Aliprantis and Border (1994), p. 200), the set $\{\mathbf{b}^\nu, \mathbf{b}^{*\nu}\} \cup (\mathbf{b}, \mathbf{b}^*)$ is also relatively weakly sequentially compact. This, in turn, implies that there exists a sequence $\{\mathbf{b}^n, \mathbf{b}^{*n}\}$, extracted from the net $\{\mathbf{b}^\nu, \mathbf{b}^{*\nu}\}$, which converges weakly to $(\mathbf{b}, \mathbf{b}^*)$ (see Problem 17L in Kelley and Namioka (1963), p. 165). Now, for each $t \in T$, denote by $L_s\{\mathbf{b}^{*n}(t)\}$ the set of limit points of the sequence $\{\mathbf{b}^{*n}(t)\}$ and by $\mathrm{co}L_s\{\mathbf{b}^{*n}(t)\}$ the set of convex combinations of these limit points. For each $t \in T$, the fact that α_t is compact-valued and upper semicontinuous and the fact that $\mathbf{B}(t)$ is compact imply that $L_s\{\mathbf{b}^{*n}(t)\} \subseteq \alpha_t(\mathbf{b})$ and this, together with the fact that $\alpha_t(\mathbf{b})$ is convex, in turn, implies that $\mathrm{co}L_s\{\mathbf{b}^{*n}(t)\} \subseteq \alpha_t(\mathbf{b})$. Since the sequence $\{\mathbf{b}^{*n}\}$ converges weakly to $\mathbf{b}^*$ and is uniformly integrable (see Hildenbrand (1974), p. 52), by Proposition C in Artstein (1979), we have $\mathbf{b}^*(t) \in \mathrm{co}L_s\{\mathbf{b}^{*n}(t)\}$ and so we are done. ■

Now, we can prove the existence of an ϵ-Cournot-Nash equilibrium.

Lemma 3. *For each $\epsilon > 0$, there is an ϵ-Cournot-Nash equilibrium $\hat{\mathbf{b}}^{\epsilon}$.*

Proof. It is a straightforward consequence of Lemmas 1 and 2 and the Fan-Glicksberg Theorem. ■

As in Sahi and Yao (1989), we introduce the concept of δ-positivity.

Definition 8. *For $\delta > 0$, the function $\mathbf{B}^{\delta} : T \to R^{l^2}$ is a δ-positive strategy function if $\mathbf{B}^{\delta}(t) = \mathbf{B}(t) \cap \{b \in R^{l^2} : \sum_{i \notin J} \sum_{j \in J}(b_{ij} + b_{ji}) \geq \delta,\ \text{for each } J \subseteq \{1, \ldots, l\}\}$, for each $t \in T_1$ with $\mathbf{w}(t) \gg 0$; $\mathbf{B}^{\delta}(t) = \mathbf{B}(t)$, for the remaining traders $t \in T$.*

An ϵ-Cournot-Nash equilibrium $\hat{\mathbf{b}}^{\epsilon}$ is called δ-positive if, for almost all $t \in T$, $\hat{\mathbf{b}}^{\epsilon}(t) \in \mathbf{B}^{\delta}(t)$. For each $t \in T_1$, let $\delta^*(t) = \frac{1}{m}\min\{\mathbf{w}^1(t), \ldots, \mathbf{w}^l(t)\}$ and $\delta^* = \min\{\delta^*(t) : \delta^*(t) > 0,\ t \in T_1\}$. Given $\epsilon > 0$, let $\alpha^{\delta^*} : L_1(\mu, \mathbf{B}(\cdot) \to L_1(\mu, \mathbf{B}(\cdot))$ be a mapping such that $\alpha^{\delta^*}(\mathbf{b}) = \{\mathbf{b} \in L_1(\mu, \mathbf{B}(\cdot) : \mathbf{b}(t) \in \alpha_t^{\delta^*}(\mathbf{b}),\ \text{for almost all } t \in T\}$ where, for each $t \in T$, $\alpha_t^{\delta^*}(\mathbf{b}) = \alpha_t(\mathbf{b}) \cap \mathbf{B}^{\delta^*}(t)$. The following lemma is a strengthening of Lemma 4.

Lemma 4. *For each $\epsilon > 0$, there is a δ^*-positive ϵ-Cournot-Nash equilibrium $\hat{\mathbf{b}}^{\epsilon}$.*

Proof. Let $\epsilon > 0$ be given. By Lemma 6 in Sahi and Yao (1989), we know that, for each $\mathbf{b} \in L_1(\mu, \mathbf{B}(\cdot))$, $\alpha_t^{\delta^*}(\mathbf{b})$ is nonempty, for each $t \in T_1$ with $\mathbf{w}(t) \gg 0$. But then, by the same argument of Lemma 4, $\alpha^{\delta^*}(\mathbf{b})$ is nonempty. The convexity of $\alpha^{\delta^*}(\mathbf{b})$ is a straightforward consequence of the convexity of $\alpha_t(\mathbf{b})$ and $\mathbf{B}^{\delta^*}(t)$, for all $t \in T$. The upper semicontinuity of α^{δ^*} can be proved using the same argument as that of Lemma 4 since, for all $t \in T$, $\alpha_t^{\delta^*}$ is upper semicontinuos, by the upper semicontinuity of α_t and the nonemptyness and compactness of $\mathbf{B}(t)$ (see Theorem 2' in Berge (1997), p. 114). This completes the proof since all the assumptions of the Fan-Glicksberg Theorem are satisfied. ■

Let $\epsilon_n = \frac{1}{n}$, $n = 1, 2, \ldots$. By Lemma 4, for each $n = 1, 2, \ldots$, there is a δ^*-positive ϵ-Cournot-Nash equilibrium $\hat{\mathbf{b}}^{\epsilon_n}$. The fact that the sequence $\{\int_{T_0} {}^0\hat{\mathbf{b}}^{\epsilon_n}(t)\, d\mu_0\}$ belongs to the compact set $W = \{b_{ij} \in R^{l^2} : 0 \leq b_{ij} \leq \int_{T_0} \mathbf{w}^i(t)\, d\mu_0,\ i, j = 1, \ldots, l\}$, the sequence $\{{}^1\hat{\mathbf{b}}^{\epsilon_n}\}$ belongs to the compact set $\prod_{t \in T_1} \mathbf{B}^{\delta^*}(t)$ and the sequence $\hat{p}^{\epsilon_n}$, where $\hat{p}^{\epsilon_n} = p(\hat{\mathbf{b}}^{\epsilon_n})$, for each $n = 1, 2, \ldots$, belongs, by Lemma 9 in Sahi and Yao (1989), to a compact set P, implies that the sequence $\{\int_{T_0} {}^0\hat{\mathbf{b}}^{\epsilon_n}(t)\, d\mu_0, {}^1\hat{\mathbf{b}}^{\epsilon_n}, \hat{p}^{\epsilon_n}\}$ belongs to the compact set

$W \times \prod_{t\in T_1} \mathbf{B}^{\delta^*}(t) \times P$. This, in turn, implies that it has a subsequence (which we denote in the same way to save in notation) which converges to an element of the set $W \times \prod_{t\in T_1} \mathbf{B}^{\delta^*}(t) \times P$ (see Problem D in Kelley (1955), p. 238). Since the sequence $\{{}^0\hat{\mathbf{b}}^{\epsilon_n}\}$ satisfies the assumptions of Theorem A in Artstein (1979), there is a function ${}^0\hat{\mathbf{b}}$ such that ${}^0\hat{\mathbf{b}}(t)$ is a limit point of ${}^0\hat{\mathbf{b}}^{\epsilon_n}(t)$ for almost all $t \in T_0$ and such that the sequence $\{\int_{T_0} {}^0\hat{\mathbf{b}}^{\epsilon_n}(t)\, d\mu_0\}$ converges to $\int_{T_0} {}^0\hat{\mathbf{b}}(t)\, d\mu_0$. Moreover, ${}^0\hat{\mathbf{b}}(t) \in \mathbf{B}^{\delta^*}(t)$, for almost all $t \in T_0$, because ${}^0\hat{\mathbf{b}}(t)$ is the limit of a subsequence of $\{{}^0\hat{\mathbf{b}}^{\epsilon_n}(t)\}$, for almost all $t \in T_0$. Since the sequence $\{{}^1\hat{\mathbf{b}}^{\epsilon_n}\}$ converges to a point ${}^1\hat{\mathbf{b}} \in \prod_{t\in T_1} \mathbf{B}^{\delta^*}(t)$, the sequence $\{\int_{T_1} {}^1\hat{\mathbf{b}}^{\epsilon_n}(t)\, d\mu_1\}$ converges to $\int_{T_1} {}^1\hat{\mathbf{b}}(t)\, d\mu_1$. But then, by Proposition 5, the sequence $\{\int_T \hat{\mathbf{b}}^{\epsilon_n}(t)\, d\mu\}$ must converge to $\int_T \hat{\mathbf{b}}(t)\, d\mu$. Since the sequence $\{\hat{p}^{\epsilon_n}\}$ converges to a price vector $\hat{p} \in P$, by the continuity of (5), $\hat{p}$ and $\int_T \hat{\mathbf{b}}(t)\, d\mu$ must satisfy (1). Moreover, since, by Lemma 9 in Sahi and Yao (1989), $\hat{p} \gg 0$, Lemma 1 in Sahi and Yao (1989) implies that $\hat{\hat{\mathbf{B}}}$ is completely reducible. But then, since $\hat{\mathbf{b}} \in L_1(\mu, \mathbf{B}^{\delta^*}(\cdot))$, by Remark 3 in Sahi and Yao (1989), $\hat{\hat{\mathbf{B}}}$ must be irreducible. In order to conlcude that $\hat{\mathbf{b}}$ is a δ^*-positive ϵ-Cournot-Nash equilibrium, we have to show that $u_t(\mathbf{x}(t, \hat{\mathbf{b}}(t), \hat{p})) \geq u_t(\mathbf{x}(t, b(t), p(\hat{\mathbf{b}} \setminus b(t))))$, for almost all $t \in T$ and for all $b(t) \in \mathbf{B}(t)$. Let $\hat{\hat{\mathbf{B}}} \setminus b(t)$ denote the aggregate matrix corresponding to the strategy selection $\hat{\mathbf{b}} \setminus b(t)$ and let $\hat{\hat{\mathbf{B}}}^{\epsilon_n} \setminus b(t)$ denote the aggregate matrix corresponding to the strategy selection $\hat{\mathbf{b}}^{\epsilon_n} \setminus b(t)$, for each $n = 1, 2, \ldots$. As in Sahi and Yao (1989), we proceed by considering the following possible cases.

Case 1. $t \in T_1$ and $b(t) \in \mathbf{B}(t)$ is such that $\hat{\hat{\mathbf{B}}} \setminus b(t)$ is completely reducible. Clearly, $\hat{\hat{\mathbf{B}}}^{\epsilon_n} \setminus b(t)$ is irreducible, for each $n = 1, 2, \ldots$, and so is $\hat{\hat{\mathbf{B}}} \setminus b(t)$, by Remark 3 in Sahi and Yao (1989). Since the sequence $\{\int_T \hat{\mathbf{b}}^{\epsilon_n} \setminus b(t)(t)\, d\mu\}$ converges, by the same argument given above, to $\int_T \hat{\mathbf{b}} \setminus b(t)(t)\, d\mu$ and since, by Lemma 2 in Sahi and Yao (1989), prices are cofactors, the sequence $\{p^{\epsilon_n}(\hat{\mathbf{b}}^{\epsilon_n} \setminus b(t))\}$ converges to $p(\hat{\mathbf{b}} \setminus b(t))$. Consequently, the sequence $\{\mathbf{x}(t, b(t), p^{\epsilon_n}(\hat{\mathbf{b}}^{\epsilon_n} \setminus b(t)))\}$ converges to $\mathbf{x}(t, b(t), p(\hat{\mathbf{b}} \setminus b(t)))$. The fact that the sequence $\{\mathbf{x}(t, \hat{\mathbf{b}}^{\epsilon_n}(t), \hat{p}^{\epsilon_n})\}$ converges to $\mathbf{x}(t, \hat{\mathbf{b}}(t), \hat{p})$ and the fact that $u_t(\mathbf{x}(t, \hat{\mathbf{b}}^{\epsilon_n}(t), \hat{p}^{\epsilon_n})) \geq u_t(\mathbf{x}(t, b(t), p^{\epsilon_n}(\hat{\mathbf{b}} \setminus b(t))))$, for each $n = 1, 2, \ldots$, allow us to conclude, by Assumption 2, that $u_t(\mathbf{x}(t, \hat{\mathbf{b}}(t), \hat{p})) \geq u_t(\mathbf{x}(t, b(t), p(\hat{\mathbf{b}} \setminus b(t))))$.

Case 2. $t \in T_1$ and $b(t) \in \mathbf{B}(t)$ is such that $\hat{\hat{\mathbf{B}}} \setminus b(t)$ in not completely re-

ducible. The fact that the sequence $\{\mathbf{x}(t, \hat{\mathbf{b}}^{\epsilon_n}(t), \hat{p}^{\epsilon_n})\}$ converges to $\mathbf{x}(t, \hat{\mathbf{b}}(t), \hat{p})$ and the fact that $u_t(\mathbf{x}(t, \hat{\mathbf{b}}^{\epsilon_n}(t), \hat{p}^{\epsilon_n}) \geq u_t(\mathbf{w}(t))$, for each $n = 1, 2, \ldots$, imply, by Assumption 2, that $u_t(\mathbf{x}(t, \hat{\mathbf{b}}(t), \hat{p})) \geq u_t(\mathbf{w}(t)) = u_t(\mathbf{x}(t, b(t), p(\hat{\mathbf{b}} \setminus b(t))))$.
Case 3. $t \in T_0$ and $b(t) \in \mathbf{B}(t)$. Clearly, the matrix $\hat{\hat{\mathbf{B}}} \setminus b(t)$ is irreducible and, by Proposition 7, $p^{\epsilon_n}(\hat{\mathbf{b}}^{\epsilon_n} \setminus b(t)) = p^{\epsilon_n}(\hat{\mathbf{b}}^{\epsilon_n})$, for each $n = 1, 2, \ldots$, and $p(\hat{\mathbf{b}} \setminus b(t)) = p(\hat{\mathbf{b}})$. Since $\hat{\mathbf{b}}(t)$ is a limit point of the sequence $\{\hat{\mathbf{b}}^{\epsilon_n}(t)\}$, it is a limit of a subsequence (which we denote in the same way to save in notation) of this sequence. But then, the fact that the sequence $\{\mathbf{x}(t, b(t), \hat{p}^{\epsilon_n})\}$ converges to $\mathbf{x}(t, b(t), \hat{p})$, the fact that the sequence $\{\mathbf{x}(t, \hat{\mathbf{b}}^{\epsilon_n}(t), \hat{p}^{\epsilon_n})\}$ converges to $\mathbf{x}(t, \hat{\mathbf{b}}(t), \hat{p})$ and the fact that $u_t(\mathbf{x}(t, \hat{\mathbf{b}}^{\epsilon_n}(t), \hat{p}^{\epsilon_n}) \geq u_t(\mathbf{x}(t, b(t), \hat{p}^{\epsilon_n}))$, for each $n = 1, 2, \ldots$, imply that $u_t(\mathbf{x}(t, \hat{\mathbf{b}}(t), \hat{p})) \geq u_t(\mathbf{x}(t, b(t), \hat{p}))$. ■

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