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Abstract

This paper models the formation of R&D networks in an oligopolistic industry. In particular, it focuses on the coevolutionary process involving firms' technological capabilities, market structure and the network of interfirm technological agreements.

The main result of the paper is that the R&D network can work as a strong selection mechanism in the industry, creating ex post asymmetries among ex ante similar firms. This is due to a self-reinforcing, path-dependent process, in which events in the early stages of the industry affect firms' survival in the long run. In this framework, both market and technological externalities created by the formation of cooperative agreements play a crucial role. Although the R&D network creates profound differences at the beginning, which are reflected by an unequal distribution of links, it tends to eliminate them as it becomes denser and denser. The nature of the technological environment affects the speed of the transition and some of the characteristics of the industry in the long run.

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1. Introduction

Recently, interfirm technological agreements have played an important role in the innovative activity of high-tech industries (Hagedoorn, 2002). More and more innovation seems the result of joint R&D efforts and information sharing among firms, in a way that has lead some authors to talk about “the network (of collaborating firms) as the locus of innovation” (Powell et al., 1996). The shortening of the product life cycle, the increased competition and the complexity of the knowledge base required for innovation force firms to cooperate even in one of the fundamental source of competitive advantage. At the same time, from a policy point of view, technological cooperation has been considered (and consequently promoted) as a factor which positively affects industries’ and countries’ competitiveness in the US, Japan and Europe.

An impressive number of empirical studies in the fields of sociology, economics and business have thrown light on this phenomenon, although in a quite unsystematic way (see Hagedoorn et al., 2000, for a review). Similarly, from a theoretical point of view, a rich literature in the game theoretic industrial organization tradition has discussed the effects of R&D cooperation (Katz, 1986; d’Aspremont and Jacquemin 1988; Kamien et al., 1992), while a small but growing number of works have studied the dynamics and the effects of technological networks within an evolutionary framework (Gilbert et al., 2001; Ozman, 2003).

The present contribution falls within the theoretical literature, proposing a model that focuses on the dynamics of R&D network formation. The model is inspired by the recent papers by Goyal and Moraga (2001) and Goyal and Joshi (2003), which apply the tools of network games (Jackson and Wolisnki, 1996) to study the formation of R&D networks in a static framework. My model extends their analyses, considering explicitly the dynamic feedbacks between market competition and firms’ incentives to engage in collaboration.

The goal of the model is *not* to explain a list of stylized facts about interfirm technological agreements (by and large still to be produced), but rather to derive propositions involving the joint dynamics of R&D network and market structure in the context of a model embodying as *assumptions* some of the evidence on interfirm technological agreements. Both empirically and theoretically, the study of this *coevolution* seems a promising direction to pursue, basically missing in the current state of the literature.

The main result of the paper is that the R&D network can work as a strong selection mechanism in the industry, creating ex post asymmetries in ex ante similar firms. This is due to a self-reinforcing, path-dependent process, in which events in the early stages industry affect firms' survival in the long run. In this framework, both market and technological externalities created by the formation of cooperative agreements play a crucial role. Although it creates profound differences at the beginning, which are reflected by an unequal distribution of links across firms, the R&D network tends to eliminate them as it becomes denser and denser. The nature of the technological environment affects the speed of the transition and some of the characteristics of the industry in the long run.

The rest of the paper is organized as follows. Section 2 describes the model, whose analytical properties are the object of section 3. Section 4 presents results from numerical simulations. Finally, section 5 concludes.

2. *The model*

2.1 An informal description

Informally, the model can be summarized as follows. I consider the evolution of an industry where firms can introduce process innovations *only* through collaborations in an R&D activity, while remaining competitors in the market side¹. Firms produce a homogenous product, but they are generally different from the technological point of view: they have different levels of efficiency, which result in different levels of production costs, and different technological specializations, which allow complementarities to be exploited when firms collaborate.

I consider a discrete sequence of periods $t=0,1,2,\dots$. Each period can be divided in two sub-periods: the *networking* phase, where firms can modify the network structure according to a procedure described below, and a *market competition* phase, where firms, given the network structure, compete in the product market. Competition is à la Cournot, so that firms' different production costs are reflected in firms' different performances. Firms' efficiency level is the result of the history of R&D collaborations for each firm. R&D collaborative projects are modeled as pairwise relationships: for each pair of firms involved in a collaborative agreement, the cost of the project is assumed to be fixed, while its effect (a deterministic reduction in the production cost) depends upon the technological profiles of the two firms.

¹ I rule out the possibility of mergers (say, for antitrust reasons)

In the networking phase of each period, two firms are randomly drawn to change the current state of their pairwise relationship, *leaving the state of the remaining R&D network unaltered*. Two firms that are not collaborating can start a collaboration; two firms that are collaborating can decide to interrupt it. Capturing the bounded rationality of agents facing a complex evolution of network and technological capabilities, firms' decisions are based on the short run consequences on their profits. The resulting network for that given period determines firms' level of efficiency, firms' technological specializations and firms' performance, which will constitute the new initial conditions for the subsequent period.

2.2 Firms and market competition

I consider a market where n firms produce a homogenous product. However, firms are *heterogeneous* from the technological point of view. They are located in a bi-dimensional technological space, and they are identified by the vector $(\gamma_{it}, \alpha_{it})$. $\gamma \in [\gamma_0, 1)$ is a parameter measuring the productive efficiency of a firm. It determines unit cost of production according to:

$$c_{it} = c(1 - \gamma_{it}) \quad (1)$$

$\alpha_{it} \in (0, 1)$ characterizes the technological position of a firm, to be intended as its *technological specialization*. I assume that α does not affect *directly* the level of unit cost of production, but it is crucial in determining the value of collaborations.

I will sometime term $(\gamma_{it}, \alpha_{it})$ as *firm i 's technological capabilities*. Firms move over time in the technological space, and this is the effect of the network structure. Furthermore, I define $\gamma_t \in [\gamma_0, 1)^n$ as the n -dimensional vector of variable γ at time t for all the firms; similarly, $\alpha_t \in (0, 1)^n$ is the vector of all technological positions at time t .

Inverse demand is assumed to be linear:

$$p = A - Q \quad (2)$$

where Q is the total quantity produced by firms.

Firms are characterized by zero fixed costs of production. Given c_{it} , gross profits² are given by $\Pi_{it} = (p - c_{it})q_{it}$. Competition is *à la* Cournot, and it is assumed that firms play the (unique) Nash equilibrium in the one-stage game³. This means that the quantity produced by each firm at time t is:

$$q_{it}^* = \frac{a - n_t c_{it} + \sum_{j \neq i} c_{jt}}{n_t + 1} \quad (3)$$

where $n_t \leq n$ is the number of active firms (i.e. firms producing a strictly positive quantity) at t . I define N_t as the subset of such firms.

For sake of simplicity, firms that are inactive at time t are supposed to *exit* the market, never to reappear. This in particular implies that at the beginning of period $t+1$ all their existing links with other firms are severed, and since period $t+1$ onward they are no longer considered in the algorithm for network evolution. The discussion below on such an algorithm will make this point clearer. In equilibrium, gross profits are given by $\Pi_{it}^* = (q_{it}^*)^2$.

2.3 The effect of the R&D network

At each moment t , following the networking phase, the industry is characterized by an R&D network g_t . I define a binary variable $g_{ijt} \in \{0,1\}$: when $g_{ijt} = 1$, a collaborative link exists between firm i and j at time t . The network $g_t \in \{0,1\}^{\frac{n(n-1)}{2}}$ is then a collection of states for the pair-wise relationships among firms. I indicate with $g + g_{ij}$ the network obtained by replacing $g_{ij} = 0$ in a generic network g with $g_{ij} = 1$, and similarly with $g - g_{ij}$ I denote the network obtained by replacing $g_{ij} = 1$ with $g_{ij} = 0$. Furthermore I define $N_t(i) \equiv \{j \in N \setminus \{i\} : g_{ijt} = 1\}$, that is the set of firms that have a collaboration with firm i at time t .

Innovation is modelled as a deterministic reduction in the unit cost of production. A network structure corresponds to a list of collaborators for each firm. Suppose to take a generic firm i :

² Gross is referred to the cost of R&D. See below.

³ The assumed functional forms of demand and cost function, together with $A > c(1 - \gamma_0)$, assure the existence and uniqueness of equilibrium in the Cournot game (Wolfstetter,2000).

for i , collaboration with firm j at time t has a specific value v_{ijt} . The economic interpretation is as follows: whenever $g_{ijt} = 1$, firms i and j start a new R&D project together at time t , which allows them to reduce their unit cost of production to an extent that is function of v_{ijt} .

Therefore, such a value captures the opportunities for firm i to “learn” as a consequence of collaboration with firm j . In this framework, I refer to the process of learning as a process of knowledge “recombination”, an idea that dates back to Schumpeter and has been recently rediscovered also in formal models (Weitzman, 1998; Olsson, 2000). According to this interpretation, the creation of new knowledge relies on pre-existing knowledge (of the pair) as major inputs. In the model, firm i 's knowledge (i.e. its technological capabilities) is completely described by the vector $(\gamma_{it}, \alpha_{it})$. Being exposed to firm j 's knowledge in the collaboration, firm i recombines its knowledge and improves upon it to an extent that is increasing in firm j level of efficiency (which is taken as a proxy for learning opportunities) decreasing in firm i 's level of efficiency (capturing decreasing returns in learning) and depending on firms' relative technological positions according to a well specified function. Firm's technological positions are modified after collaboration, too⁴.

This representation of the learning process has the big advantage of parsimony, since the distribution of technological capabilities in the industry identifies both the outcome of market competition and the effects of technological collaboration.

More specifically, the value from collaboration is given by $v_{ijt} = f(d_t(i, j))\gamma_{jt-1}$. It is increasing in γ_{jt} , since the higher is the level of efficiency of your collaborator (the more it is “knowledgeable”), the more you can learn from it. It is also increasing in the value assumed by a function f , whose argument is given by the technological distance between firms, as defined by $d_t(i, j) = |\alpha_{it-1} - \alpha_{jt-1}|$. Some authors have argued that firms need to be technologically “not too distant, nor too near” for effective collaboration to take place (Nooteboom, 1999). This is because there are two opposing forces: if firms are distant, their different technological specializations can create opportunities for complementarities and synergies; but if they are too distant, they lack the “absorptive capacity” (Cohen and Levinthal, 1989) to learn from their collaborator and cognitive distance can harm effective communication. This conjecture has found empirical support (Mowery et al., 1998; Sampson, 2003) and it is reflected in the

⁴ A quite similar representation of knowledge, in the context of knowledge creation as knowledge recombination, can be found in Cowan et al. (2003).

particular functional form chosen for f , which is assumed to be a concave parabola (Cusmano,2002):

$$f(d_i(i, j)) = a_1 - \frac{a_2^2}{4a_3} + a_2 d_i(i, j) - a_3 d_i(i, j)^2 \quad (4)$$

$$a_1, a_2, a_3 > 0$$

$$f(d_i(i, j)) \geq 0 \forall d_i(i, j) \in [0,1]$$

The vector (a_1, a_2, a_3) identifies the technological characteristics of the industry. $\frac{a_2}{2a_3}$ is the optimal technological distance, as the result of the counterbalancing forces of absorptive capacity and search for complementarities⁵. a_1 is a measure of “technological opportunities”, being $a_1 = \max_d f(d)$.

Given the total value of collaboration $V_{it}(g_t) = \sum_{j \in N_t(i)} v_{ijt}$, γ_{it} is determined by

$$\gamma_{it} = 1 - e^{-\lambda L_{it}} \quad (5)$$

where

$$L_{it} = L_{it-1} + V_{it}(g_t) \quad \text{and} \quad \lambda > 0$$

Equation (5) captures the decreasing returns in the innovative process.

Finally, I assume that through collaboration firms modify their technological position. Formally:

$$\alpha_{it} = \rho \alpha_{it-1} + (1 - \rho) \sum_{j \in N_t(i)} \frac{\gamma_{jt-1}}{\Gamma_{it-1}} \alpha_{jt-1} \quad \text{if } N_t(i) \neq \{\emptyset\} \quad (6)$$

$$\alpha_{it} = \alpha_{it-1} \quad \text{otherwise}$$

$$\text{where } \Gamma_{it-1} = \sum_{j \in N_t(i)} \gamma_{jt-1}, \rho \in (0;1]$$

⁵ Parameters are assumed to be chosen in a way that the maximum point lies in the appropriate interval.

The final technological position of a firm at time t is a linear combination of its old technological position and a weighted average of technological positions of collaborating firms. A firm is weighted more if it has a high efficiency level (that implies more opportunities of learning). When $\rho < 1$, firms become technologically more “similar” to their collaborators. When $\rho = 1$ (so that technological positions are time-invariant), firms maintain their “identity” in the process of learning (when they recombine their knowledge).

2.4 The evolution of the network

Each period two firms among the ones still in the market are randomly chosen to possibly change their network state. Firms that are not currently collaborating can decide to form a collaborative link, firms that are already collaborating can sever the existing link. Each link has the same probability to be revised.

I assume that maintaining a collaborative link costs each firm a fixed amount $E > 0$ in each period. E has to be interpreted as the firm’s contribution to the joint R&D project. For a firm involved at time t in $|N_t(i)|$ collaborations, net profits are equal to $\Pi_{it} - |N_t(i)|E$.

The proposed algorithm can be reformulated as follows: each period, two firms are allowed to modify their portfolio of collaborations, starting a new collaboration between each other if it does not exist, or interrupting it if exists. The state of the remaining network is unaltered: all the other collaborations in which these firms are involved, and the collaborations of all the remaining firms are automatically confirmed. In other words, network at time $t-1$ and time t may differ only for the state of one link.

Suppose that at period t , the link ij (i.e. the potential or existing link involving firms i and j) is randomly chosen to be updated. Define $\Pi_i(g; \alpha, \gamma)$ as the profit for i resulting from market competition when the network is g , and the initial technological capabilities are given by (α, γ) . If $g_{ijt-1} = 1$, the link is severed if $\Pi_i(g_{t-1} - g_{ij}; \alpha_{t-1}, \gamma_{t-1}) > \Pi_i(g_{t-1}; \alpha_{t-1}, \gamma_{t-1}) - E$ or $\Pi_i(g_{t-1} - g_{ij}; \alpha_{t-1}, \gamma_{t-1}) \geq \Pi_j(g_{t-1}; \alpha_{t-1}, \gamma_{t-1}) - E$, while in the opposite case it is maintained. This simply means that a firm wants to sever an existing link if profits without the link and the saving on the R&D cost are higher than the profits with the link. If $g_{ijt} = 0$, the link is formed if $\Pi_i(g_{t-1} + g_{ij}; \alpha_{t-1}, \gamma_{t-1}) - E \geq \Pi_i(g_{t-1}; \alpha_{t-1}, \gamma_{t-1})$ and $\Pi_j(g_{t-1} + g_{ij}; \alpha_{t-1}, \gamma_{t-1}) - E \geq \Pi_j(g_{t-1}; \alpha_{t-1}, \gamma_{t-1})$. If a link does not exist, it is formed

when for both players the gain stemming from forming the link is higher than the R&D cost they have to sustain⁶.

In terms of behavioral assumptions, the proposed rule implies that agents are myopic, since they decide only on the basis of their current pay-off, but at the same time they have rational expectations *within a given period*, since during the networking phase at time t are able to predict *correctly* the marginal cost of their rivals at time t and the Nash equilibrium that will be played in the market phase.

The assumption of myopic behavior aims at the representing the bounded rationality of agents who face a highly complex and uncertain future evolution of the R&D network and of the technological capabilities of firms in the industry.

3. Analytical results

In this section I provide some analytical results. First, I consider the incentives to form collaborative links at the level of the *single pair* of firms. I will also show two numerical examples, for the set of parameters I will consider in the simulations. Then I will turn to the long run properties of the system. Although the stochastic process generated in the model is rather complex, a clear and intuitive result holds for the network state in the long run.

3.1 Firms' cooperative strategies

Let me introduce the following function:

$$F_i(\alpha_j, \gamma_j | (\alpha, \gamma)_{-j}) = \left(\frac{A - nc(1 - \gamma_i)(e^{-\lambda(\gamma_j f(d_{ij}))}) + c(1 - \gamma_j)e^{-\lambda f(d_{ij})\gamma_i} + \sum_{k \notin i, j} c_k}{n + 1} \right)^2 - \left(\frac{A - nc(1 - \gamma_i) + c(1 - \gamma_j) + \sum_{k \notin i, j} c_k}{n + 1} \right)^2$$

⁶ In order to avoid that with probability 1 no link is profitable at $t=0$, I assume that $E \leq E^*$, where

$$E^* = \left(\frac{a - (n-1)c(1 - \gamma_1^{\max}) + (n-2)c(1 - \gamma_0)}{n + 1} \right)^2 - \left(\frac{a - c(1 - \gamma_0)}{n + 1} \right)^2 \text{ and } \gamma_1^{\max} = 1 - e^{-\lambda(L_0 + a)}$$

Suppose to take a generic pair of firms i and j . Fix the technological capabilities of the other $(n-2)$ firms, and from $\gamma_k, k \in N/\{i, j\}$ derive the unit cost of such firms. Studying $F(\cdot)$ I can answer to the following question: how does the gross gain (i.e the variation in profits excluding R&D costs) for i of forming a link with firm j vary, as a function of j 's and i 's technological capabilities?⁷

In order to make computation easier, I write F as:

$$F_i(\alpha_j, \gamma_j | (\alpha, \gamma)_{-j}) = \left(\frac{c(1 - \gamma_j)(e^{-\lambda f(d_{ij})\gamma_i} - 1) - nc(1 - \gamma_i)(e^{-\lambda(\gamma_j f(d_{ij}))} - 1)}{n + 1} \right) (q_i(+ij) + q_i(-ij))$$

where $q_i(+ij)$ and $q_i(-ij)$ represent the quantities produced by firm i with and without the link with firm j respectively. The first factor represents a necessary condition for collaboration: the net effect of counterbalancing forces on firm i 's profits given by the reduction in its costs and in firm j costs must be positive, i.e. firms must increase the quantity they produce (and consequently their profits). Consistent with the existence of an interior solution, firms i and j are assumed to be close enough so that necessary condition is always satisfied.

I can show that the following propositions hold (the proofs can be found in the appendix):

Proposition 1 *Ceteris paribus, gains from the collaboration increase when firms' technological distance move towards the "optimal technological distance", and decrease otherwise.*

Proposition 2 *Ceteris paribus, the effect of an increase of γ_j on the gains from the collaboration is ambiguous. Possibly, an inverse U relation holds between γ_j and gains from collaboration.*

Proposition 3 *Ceteris paribus, the effect of an increase of γ_i on the gains from the collaboration is ambiguous. Possibly, an inverse U relation holds between γ_i and gains from collaboration.*

Proposition 4 *Ceteris paribus, gains from the collaboration decrease when the remaining firms' average efficiency increases.*

⁷ Notice that implicitly we are restricting our attention to the cases where the formation of the link does not lead to the exit of any firms.

The first proposition is indeed quite obvious. Proposition 2 is instead more interesting. The rationale for the possibly non-monotonic relationship is straightforward, however. High efficiency of a collaborator is good since your opportunities of learning increase and the extent it can learn from you is limited, but at the same time it is bad since efficiency is correlated with size. If a firm i 's potential collaborator is highly efficient, then it is "large". This makes i a "small" firm, in relative terms. Since I deal with process innovation, smaller firms have lower total gains per unit of cost reduction, and their incentive to collaborate and innovate, *ceteris paribus*, is smaller. This is the so-called "cost spreading" argument, which has been claimed to be one of the advantages in innovation by large firms, and it has found empirical support (Cohen and Klepper, 1996).

The nature of the opposing forces is symmetric in Proposition 3. If firm i is highly efficient, it assures great opportunities of learning to its potential collaborator, and the reduction in its unit cost is smaller in absolute value. At the same time firm i is "large": so that reduction in unit cost of production can be spread over a larger quantity.

Finally, the average efficiency of other firms (Proposition 4) comes into play through the usual channel: its effect on firm's size. Its increase decreases the gains from collaboration, since it makes the firm "smaller" in relative terms.

The results show the complex nature of the interaction between the technological and markets aspects concerning firms' incentives to collaborate. Furthermore they stress the feedbacks between firms' incentives and the evolution of the network. Network's evolution affects firms' incentive through market competition and opportunities for learning. In turn, the network changes according to firms' decision. Firms' strategies and the network *coevolve*, a point that has already been raised by business scholars (Koza, M. and Lewin, A., 1998).

Figure 1 and Figure 2 show the behavior of $F(\cdot)$ under the parameterization of the "standard" simulation discussed in the next session. In the first case (Figure 1), $\gamma_i = 0.35$ and

$$\sum_{k \neq i, j} \frac{\gamma_k}{n-2} = 0.35.$$

Firm i is sufficiently small so that the inverse U relationship between gains from collaboration and γ_j emerges. When γ_j is large enough (approximately 0.5), the negative effect on size prevails on the positive effect of technological opportunities. If instead $\gamma_i = 0.5$

(Figure 2), firm i 's size guarantees that an increase of γ_j monotonically increases the gains from collaborations.

Figure 1: Gains from collaboration-1

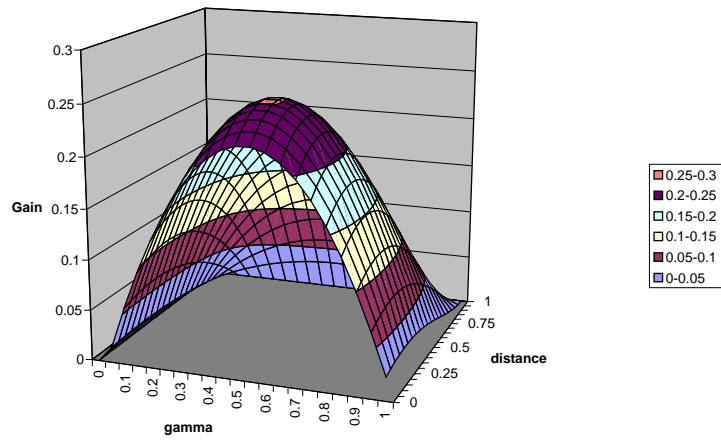
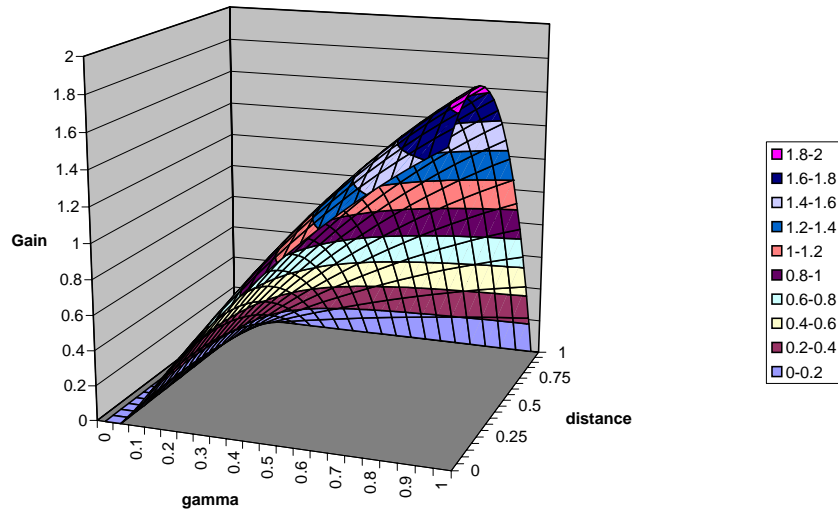


Figure 2- Gains from collaboration-2



3.2 The long run properties of the system

Although the stochastic process describing the evolution of the R&D occurs on a rather complicated state space, it is easy to derive clear results about the limit behavior of the network structure.

The industry at time t is completely characterized by the state $\{g_t, \gamma_t, \alpha_t\}$. Then, it is easy to verify that the underlying stochastic process satisfies the Markov property. Theorem 1, whose proof can be found in the appendix, concerns the long run properties of such a process.

Theorem 1 *As $t \rightarrow \infty$, each link is absent with probability 1. The absorbing states of the process are characterized by the empty network, and the set of these states is reached almost surely in the long run.*

The intuition behind this result is very simple and comes directly from the existence of marginal decreasing returns in the outcome of collaboration. Since innovative opportunities become smaller and smaller as firms continuously invest in R&D, while its cost is constant and strictly positive, it will come a time where forming or maintaining collaborative links is not convenient, irrespectively of other firms' technological positions. Loosely speaking, when (“almost”⁸) everything that could be discovered has been discovered, investing in R&D becomes unprofitable. Nevertheless, I am mainly interested in the transition phase of the system, per se and for the way it affects the final equilibrium is reached. This will be the subject of next section, where numerical simulations of the model are reported.

4. Simulation results

In this section I discuss some of the results emerging from a first series of numerical experiments performed on the model. Further analysis and a deeper inspection of the data are certainly required, but some interesting insights have already emerged.

In the “standard simulation”, I consider a situation where competition is rather tough at the beginning. Market size is $A=65$, 16 firms populate the industry at time 0, and their initial unit cost is about 47.56 ($c=50$, $L_{i0} = 5 \forall i \in N$). The initial network is empty. The “optimal”

⁸ Obviously, as the simulation will make clear, the precise quantification of “almost” is endogenous to the model.

technological distance is 0.25, and technological parameters are chosen in a way that the expected value of $f(d)$ is 0.5, ($a_1 = 0.56, a_2 = 0.5, a_3 = 1$), under the assumption of technological positions that are uniformly distributed along the interval (0,1). The R&D cost is rather “high”, $E=0.0230$, and corresponds to $0.975 * E^*$, where E^* is the largest R&D cost for which firms at optimal distance will form a link given their initial costs. $\rho = 1$, so that technological positions are time-invariant. I run the experiments for 1000 periods, by which a steady state is reached.

Figure 3 and 4 reports the results for the average of 40 replications.

Figure 3

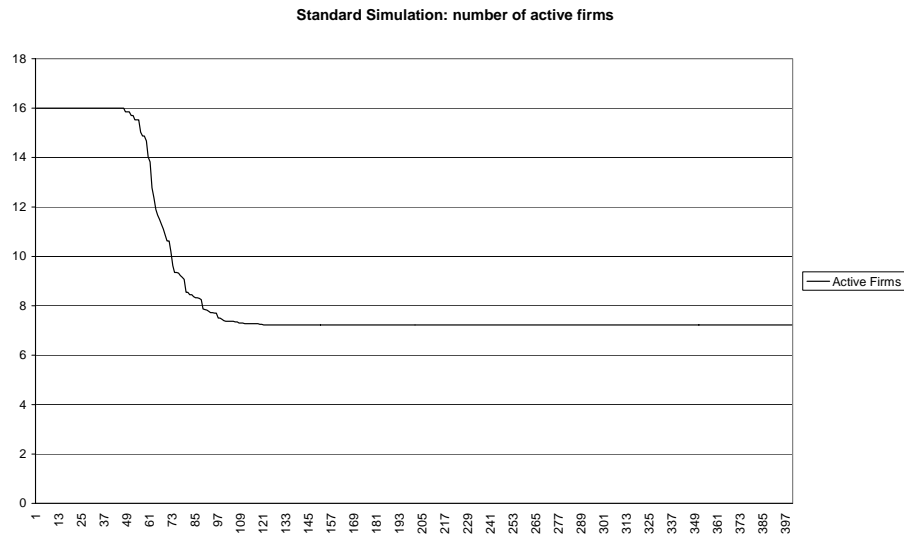


Figure 4

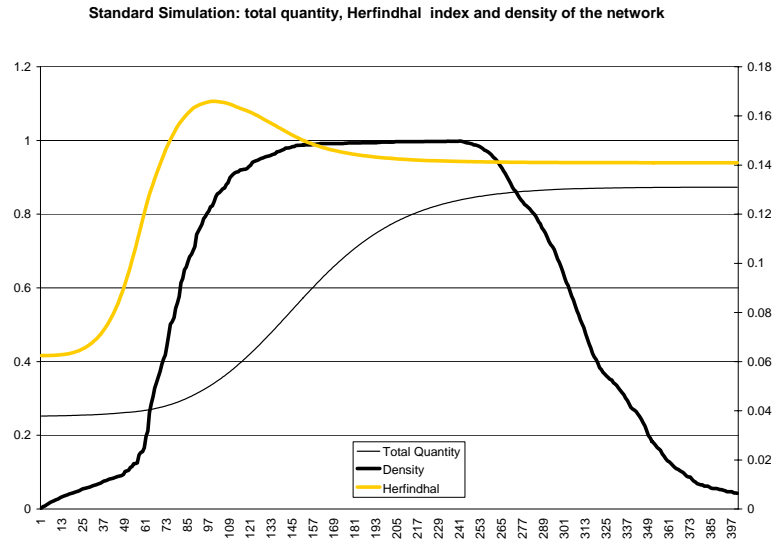


Figure 3 reports the number of active firms over time. The first immediate result is that the number of active firms has a sudden drop around period 45: a shakeout occurs. In the steady state, less than 8 firms on average are in the market, then slightly less than half of the initial number of firms. The shakeout (defined as a significant and rapid reduction in the number of firms active in the market) is indeed a typical feature of the evolution of industries in early stages, as represented by the theory of industry life cycles (Klepper, 1997). In the model, it is the process of network formation that creates the shakeout among firms that are symmetric at the beginning. In other words, the existence of a R&D network (i.e. the possibility for firms to form cost-reducing links) operates as a strong *selection mechanism*.

Figure 4 further elaborates on this point, and shows an interesting dynamics involving market structure and the network of collaborating firms.

The figure report the dynamics of three variables: total output produced in the market, normalized by market size ($\frac{Q}{A}$); market concentration, measured by the Herfindhal index; and network density, that is the fraction of existing links over the total number of possible links (considering only the firms still in the market). The scales of these variables are somewhat

different. For preserving readability and comparison of behavior over time, total quantity and density are to be read along the left axis, while the right axis is for the Herfindhal index.

The evolution of the industry can be described in the following terms. At the beginning the density of the network is growing relatively slowly. Since R&D costs are relatively high, market relatively small and the average level of efficiency in the industry low, firms need to find partners located almost at the optimal technological distance, and this process is assumed not be instantaneous. This creates differences in the relative competitiveness of firms, expressed by a sharp increase in the concentration index. In any case, given the low average level of efficiency in the market, the process of “knowledge recombination” is reflected by a limited growth rate for total output, which, given the assumptions, is only depending on the *average* efficiency of firms.

When the shakeout occurs, the time series for the network density has a break: as I will see better below, this is due to the fact that the firms exiting the market have typically no links, and then they were lowering the average. The process of links formation continues, however, until a *complete* network (density 1) emerges for around 100 periods. Concentration continues to grow, but then it starts declining when the density reaches a sufficiently high level: the network operates first as a mechanism creating different efficiency levels and then as a mechanism favoring the “catching-up” of relatively less efficient firms.

For around 100 periods, therefore, I can observe a sort of “steady state”, where almost equal size firms operate in a complete network.

The behavior of total output, reflecting the behavior of average efficiency, follows an S-shaped curve. The growth rate of total output is the highest during the formation of the network after the shakeout. In this period, the increasing density of the network, together with an increase in the average level of efficiency (creating more opportunities for recombination) and the fact that marginal decreasing returns are not limiting innovative opportunities yet, generate a high level of growth. Interestingly, the inflection point in the output series roughly corresponds to the time in which a complete network is formed. Then, the “steady state” in market structure and network dynamics is accompanied by a low growth of the average efficiency.

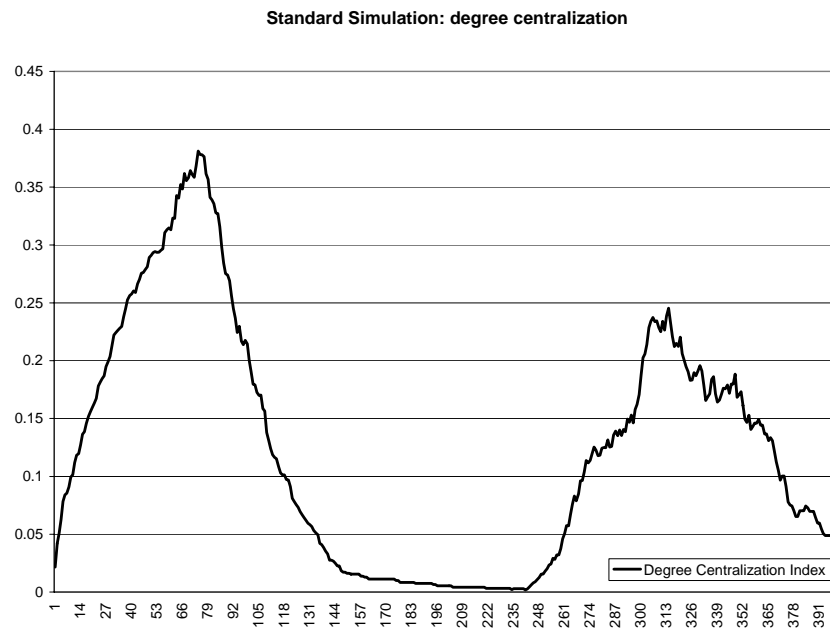
Since I model innovation as a process in which knowledge is both an input *and* an output, I can also interpret the results claiming that while in the early phases network formation mainly drives the creation of knowledge, in the late stage it is existence of a large pool of knowledge which preserves the incentive for firms to form new links (i.e. the cause-effect relation between network formation and knowledge creation is reversed while time elapses).

The final period occurs when technological opportunities have substantially been depleted. Total output and market shares stabilize, and simply some time is required for firms to sever their link. The final long run equilibrium is then reached when the empty network is finally obtained.

It is also interesting to look at the evolution of the network structure over time, especially for the phase immediately preceding and following the shakeout.

Figure 5 reports the behavior of the group degree centralization index over the simulation time.

Figure 5



This index takes a firm's degree (its number of links) as its centrality measure, and it basically summarizes how the links are distributed across firms. It takes value 0 when all the firms have the same number of links (as it happens in a regular network, like the complete network), and value 1 in a star, where there is one firm connected to all the others, and no other links exist (Wasserman and Faust, 1994).

The index shows a marked growth until the shakeout period: this implies, in substance, that in this phase links are more and more unequally distributed across firms. Then, the value of the

index falls down in a similar way, to reach the value of zero when the network becomes complete. Then it naturally grows again, when firms start removing their links, and comes back to zero, when the network is empty.

4.1 Discussion

Two main results deserve further explanations. The first major point is that, even firms are symmetric ex-ante, the opportunity of forming R&D links can generate profound asymmetries ex-post: in the long run, these are reflected in firms' survival.

Table 1 reports the statistics concerning the number of links for firms at period 40 (just before the shake-out) and their survival in the long run. The variable *link40* takes value 1 if the firm has at least one link at period 40; the variable *surviving* has value 1 if the firm survives the shakeout.

Table 1

	<i>SURVIVING</i>		
<i>LINK40</i>	0	1	<u>Total</u>
0	303	3	306
1	48	286	334
<u>Total</u>	351	289	

The table clearly shows that firms exiting the market are firms without links. Furthermore, an inspection of the network structure in the initial phase shows that the network structure, at the shake-out, is typically given by a single component of connected firms, while remaining firms are disconnected. A first strong selection occurs between firms that *are* in network, and survive at the first shakeout, and firms that “are not able” to join the network “reasonably” soon⁹. The fact that firms without links eventually exit the market is not obviously surprising, since it is the natural consequence of the assumption that costs are reduced only through collaborations. The interesting point is the mechanism through which some firms are *excluded* by the R&D network.

Second, I need to explain also the evolution of the network structure, in particular the increase in centralization in the initial phase. The firms polarization in two groups of connected and

⁹ However, as the following example of a single run will illustrate, belonging to the main component is a necessary but not sufficient condition to survive.

disconnected firms is a candidate for a first basic explanation, but the evolution *within* the main component can also be an important determinant.

I will show that both the selection process and the evolution of the network structure are driven by a self-reinforcing, path-dependent process, in which events in the early stages of industry affect firms' centrality in the initial network with long term consequences in terms of survival (Arthur, 1990). Forming links at the very beginning (which in the model is due to random factors, and in the real world could correspond to different managerial practices, social contacts or other small "historical accident" affecting firms' networking propensity) propels a positive feedback mechanism that favors the centrality of such firms, and entraps excluded firms in their status. However, *among the surviving firm*, the negative feedbacks end up prevailing, and firms converge in market shares and efficiency levels.

The first mover advantage of firms forming links at the very beginning comes from the net effects of forces described in the previous section. Firms that are "lucky" and form links in the first periods become larger than the other firms. This increases their incentive to form new links, considering also that in this early phase decreasing returns are not substantial yet. At the same time, large firms are more efficient and competent (indeed, they are larger *because* they are more efficient) and they offer their collaborators more opportunities to learn. A complementarity exists between "large" and "small" firms: large firms are willing to cooperate because of the "cost spreading" argument and possibly because of the search for technological complementarities; small firms are willing to collaborate because of the high level of competences they can find. The final effect of this process is the tendency to reinforce the centrality of first movers firms, which results in the sharp increase of the centralization index. This process comes naturally to an end since the number of possible links to be formed is limited. This corresponds to the phase of industry maturity, when the network becomes complete.

At the same time, firms that are not able to form links in the initial phase are excluded by the subsequent process of the network formation: their incentive to start collaborations decreases because such firms are getting smaller and smaller, and they are a limited source of learning opportunities for their potential collaborators.

Overall, this suggests an industrial structure where one can identify three kinds of firms, identified by their position in the network in the initial phase: 1) isolated firms, which are never able to join the network, being trapped in a self-reinforcing mechanism of exclusion, and which end up exiting the market; 2) central actors, whose position is strongly path-dependent and that

can gain a (temporary) leadership in the market; 3) (temporarily) peripheral actors, that is firms that are active in the network in relatively laggard positions, but are destined to catch up with the leader, if able to survive the shakeout.

For a quantitative assessment, I have run two simple OLS regression on the data generated by the simulations. I considered the variation on the number of links between period 40 and period 10 as dependent variable (*newlink40*), and I regressed it on the number of times a firm has been called to change its network status from period 10 to period 40 (*newcalled40*) and on the number of links the firm have at period 10 (*link10*). In a sparse network, the first variable is clearly supposed to have a positive coefficient. Table 2 shows that, at the beginning of the life cycle, also the sign of the coefficient for the second variable is positive, and significant. We have the confirmation that the “Matthew effect”¹⁰ is at work here: firms that are more central at the very beginning are more likely to attract new collaborators in the following periods. This property is indeed quite often found in networks of alliances (see, for instance, Powell et al, 1996).

Table 2

<i>NEWLINK40</i>	<i>COEFF.</i>	<i>STD. ERR</i>	<i>T</i>	<i>P> T </i>
<i>newcalled</i>	0.2363637	0.0195672	12.08	0.000
<i>link10</i>	0.7089678	0.0574208	12.35	0.000
<i>constant</i>	0.3405201	0.0848473	-4.01	0.000
<i>Number of obs</i>	640			
<i>F(2,637)</i>	149.58			
<i>R-squared</i>	0.3196			

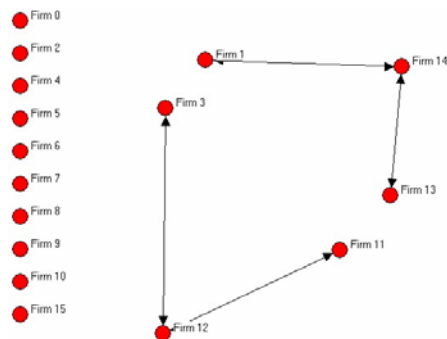
¹⁰ The term refers to the Gospel According to St Matthew: “For unto every one that hath shall be give, and shall have abundance: but from him that hath not shall be taken away even that which he hath”.

Concerning the selection process, the picture so far must be enriched including the role played by the externalities arising in the process of network formation. When two firms form a link, they always create a negative externality upon the remaining firms (“business stealing” effect). However, when two firms start to collaborate, this also creates a positive “technological externality”, but only for firms connected with these two firms. The new projects, indeed, increase the rate of growth of efficiency of the two partners, with a positive effect on the technological opportunities for their collaborator. The increasing network density is strongly penalizing for firms outside the active network at the beginning strongly penalized by the increase in the network density, since they find increasingly difficult to join the network.

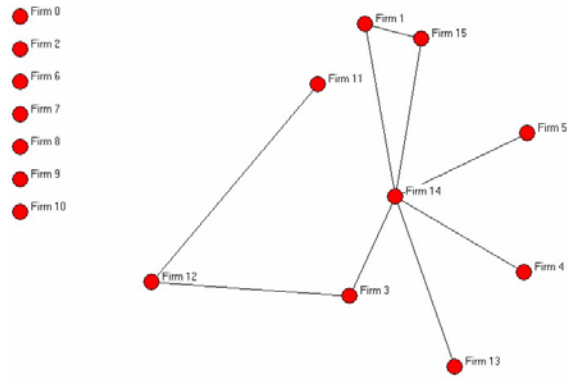
I consider now a single run as an illustrative example. In this history, the final number of surviving firms is 6. The shakeout occurs at period 67, when 7 firms (the ones without links) exit the market. However, the long run number of firms is reached only at period 91.

The following graphs show how network structure at period 20, 40 and 70, respectively. Table 2 summarizes the characteristics of firms at period 70, that is just after the shakeout (“Experience” is the total number of project performed by firms; “Surviving”, as before, is 1 if the firm is active in the market in steady state, 0 otherwise).

Single run: R&D network at t=20



Single run: R&D network at t=40



Single run: R&D network at t=70

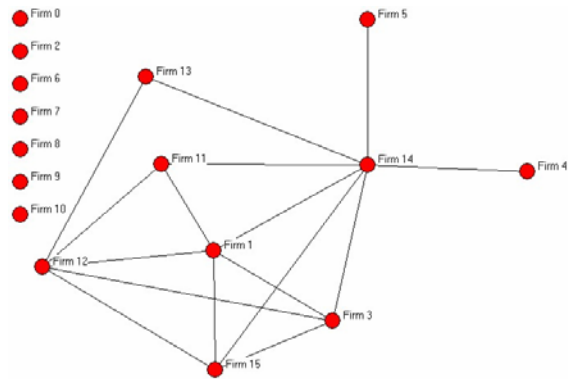


Table 2: Single run, standard simulation, market and network structure at $t=70$

Period 70	Firm 1	Firm 3	Firm 4	Firm 5	Firm 11	Firm 12	Firm 13	Firm 14	Firm 15
NumberOfLinks	5	4	1	1	3	5	2	7	4
Profit	6.788736	7.823927	0.259092	0.198898	1.07813	7.401257	0.823664	17.00422	5.964725
Quantity	2.627496	2.813526	0.531123	0.471061	1.071042	2.741579	0.932558	4.143093	2.461041
Alfa	0.284422	0.168737	0.259	0.896658	0.101301	0.013809	0.660202	0.477916	0.228852
Gamma	0.1084	0.112121	0.066473	0.065272	0.077271	0.110682	0.074502	0.138712	0.105071
Market Share	0.147674	0.15813	0.029851	0.026475	0.060196	0.154086	0.052413	0.232856	0.138319
Experience	146	160	34	33	72	181	60	280	134
Surviving	1	1	0	0	1	1	0	1	1

At period 20 the network is very sparse. Only four links are activated, but already two firms show prominence, firm 12 and firm 14, which are the center of the two star components. At period 40, the main component has a structure that resembles a star: firm 14 is very central, while firm 12 has remained stuck with its two initial collaborators. Finally, after the shakeout (period 70), the network is very dense, firm 14 has maintained its prominent position, with other firms catching up in the number of links. Notice how not all firms belonging to the R&D network will survive: not surprisingly, firms in a weak position (firm 4, 5 and 13) will finally exit the market.

4.2 Comparative dynamics

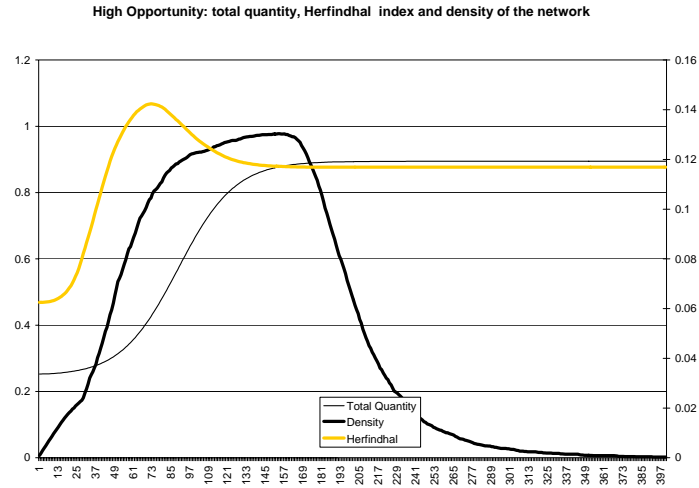
A natural question concerns possible exercises of comparative dynamics. In theory, several different parameterizations can be discussed. Here, I consider briefly two of them.

First, I increase technological opportunities. a_1, a_2, a_3 are chosen in a way that the expected value of $f(d)$ becomes 0.75 (instead of 0.5)¹¹. The opportunities for “knowledge” recombination within collaborative projects increase, making collaboration more attractive, *ceteris paribus*. Notice that high opportunity here does not mean that there is “more” to learn in the long run (unit cost is bounded from below, and it always (potentially) converges to 0), but simply that it is easier. The effect on market structure seems ambiguous, *a priori*. On one hand, more firms can engage in collaboration, especially at the beginning. On the other hand, the average efficiency growth rate is expected to be higher, and this is detrimental for the survival of firms that do not join immediately the network. As figures show, both effects are at work: with “high opportunities”, the equilibrium number of firms is higher (the long run level of concentration is lower), but the shakeout occurs typically earlier. Technological progress is faster, as expected.

¹¹ In particular, $a_1 = 0.84375$, $a_2 = 0.75$, $a_3 = 1.5$

Notice, finally, that the network does not reach density 1. This is easily explained by the fact that the faster depletion of innovative opportunities makes inconvenient the formation of links *before* a complete network is reached.

Figure 6



Finally, I have until now considered time invariant technological positions. Empirical evidence suggests indeed that interfirm technological agreements are important in explaining the movement of firms over time, and frequently (although not always), they tend to lead firms to become technologically more similar at the dyadic level (Mowery et al, 1998).

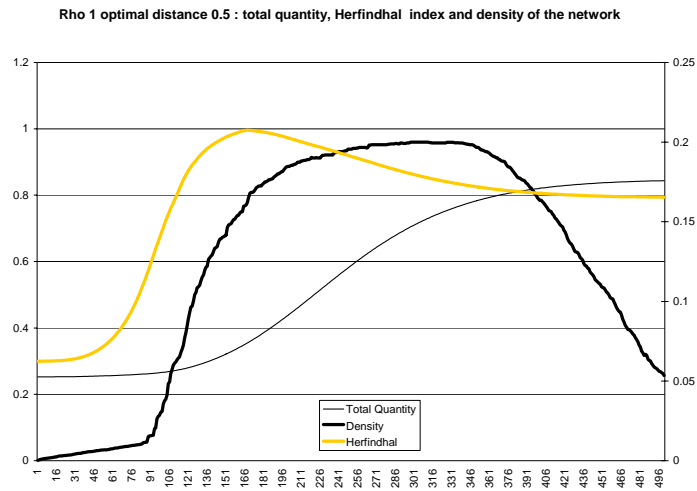
For a first study on the impact of variation of ρ on network evolution, I consider the case where technological heterogeneity matters in the outcome of collaboration, fixing the optimal distance at 0.5 (but keeping fixed the expected value of f)¹².

For this case, I run two sets of simulation, one with $\rho = 1$, the other with $\rho < 1$ ($\rho = 0.99$).

The results are reported in Figure 7 and 8.

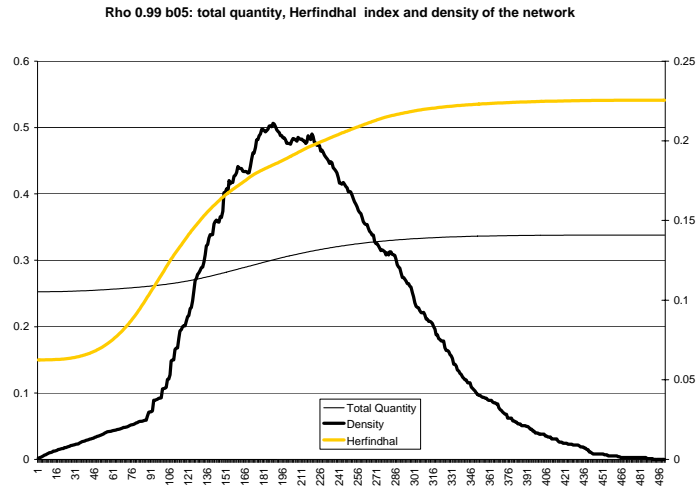
¹² $a_1 = 0.5517$, $a_2 = a_3 = 2.2069$

Figure 7



The first remarks concern the comparison between the first case and the standard simulation. Although the qualitative picture is rather similar, one can observe a slightly higher level of concentration in the long run. This is due to the relationship between the optimal distance and the initial distribution of technological positions. It is intuitive to see that, once the assumption of uniformly distributed firms is maintained, increasing the optimal technological distance over a certain threshold makes less likely to find a partner around the optimal level, especially for firms in the middle of the technological interval. Since at the beginning this is what really matters, more frictions are introduced in the search of a satisfying partner. Firms lucky enough to find right partners get a stronger advantage. Progress is less rapid, concentration is higher and the network less dense. This is clearly an example which shows that the hypothesis on the initial distribution of firms matters, especially for certain technological environments, because it affects the opportunity for cooperation in the industry. This aspect deserves further analysis in the future.

Figure 8



The changes when $\rho < 1$ are indeed quite radical. At the beginning the evolution is pretty much the same. This is not surprising, since in any case the process of technological convergence takes time. The real difference occurs after the shakeout. The process of network formation quite soon comes to an end. The reason for that is simple: the emergence of one single component inevitably lead to the overall convergence to a single technological position, which is detrimental for innovation. In the forty replications, the final value of the average technological distance lies in the interval $[0.002, 0.02]$. This implies that both technological progress and convergence in market shares stop.

This result shows the important role that entry, a factor not considered in the model, can actually play. In a “relatively” mature industry, in which the technological positions of incumbents have converged, new entrants have an important role to play. They can bring in a precious resource: different capabilities. This also can help the new firms to survive in the market, although less efficient, because of their role in the network. Extending the model to the role of new entrants is an interesting exercise that I plan to realize.

4.3 Extensions

In this section I check the robustness of the results with respect to two main assumptions of the models. First, I implement two other algorithms driving the formation of the R&D network;

second, I introduce, although in a very simple manner, an alternative way for cost reduction. Overall, the model exhibits robustness with respect to these changes.

Concerning the rules for links revision, it has been maintained the hypothesis of revision of *one* link per period. Given this restriction, two different algorithms have been considered. The first one can be defined as “socially oriented”, and it aims at capturing the idea that meetings are more likely between firms that have collaborators in common.

In practice, the algorithm works as follows:

- a) One firm is picked up randomly. Each firm has the same the probability to be chosen.
- b) With probability $\frac{|N_i(t)|}{n-1}$ the firm revises the state of one of its existing links; otherwise, the firms revise the state of one of its non-existing links.
- c) In the case of revision of an existing link, a firm $j \in N_i(t)$ is chosen with uniform probability.
- d) In the case of revision of a non-existing link, a given firm j is chosen by i to revise the state of the link with probability:

$$\frac{1 + |N_i(t) \cap N_j(t)|}{\sum_{k \in N_i(t)} 1 + |N_i(t) \cap N_k(t)|}$$

i.e. the probability of "meeting" is proportional to the number of collaborators that the two firms have in common.

The second algorithm will be labeled as “economically oriented”. It is meant to capture the active, “rational” firm’s search for optimal partners.

- a) One firm is picked up. Each firm has the same the probability to be chosen.
- b) For each $k \neq i$, net profits for i resulting from the meeting with k are computed. In particular, if the link ik does not exist, firm i correctly predicts the willingness of k to cooperate or not. I indicate with $\Pi_{it}(ik)$ such profits.
- c) The firm j that is actually chosen is given by $j \equiv \arg \max_{k \in N_i(t)} \Pi_{it}(ik)$

In case of ties, the one with the highest index is chosen.

Figure 9 and 10 reports the Herfindhal index, the total output and network density for the same parameterization of the “Standard” case, when the algorithms of network formation are respectively the “socially” oriented algorithm and the “economically” oriented-one.

Figure 9

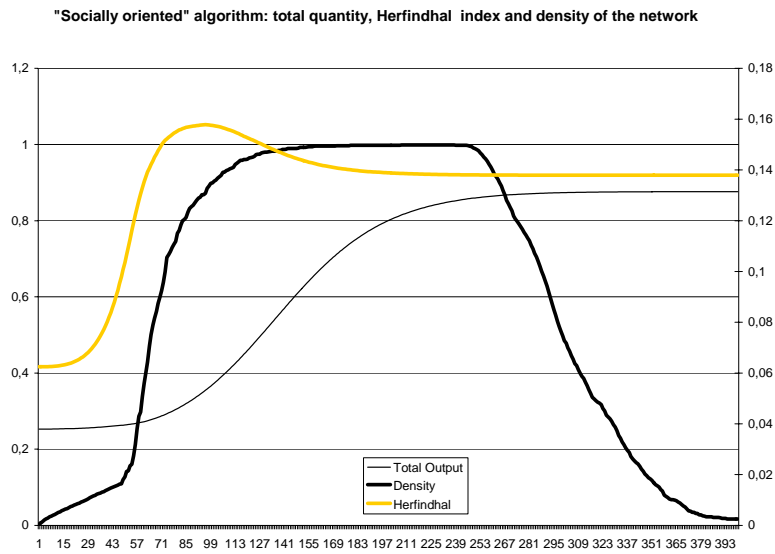
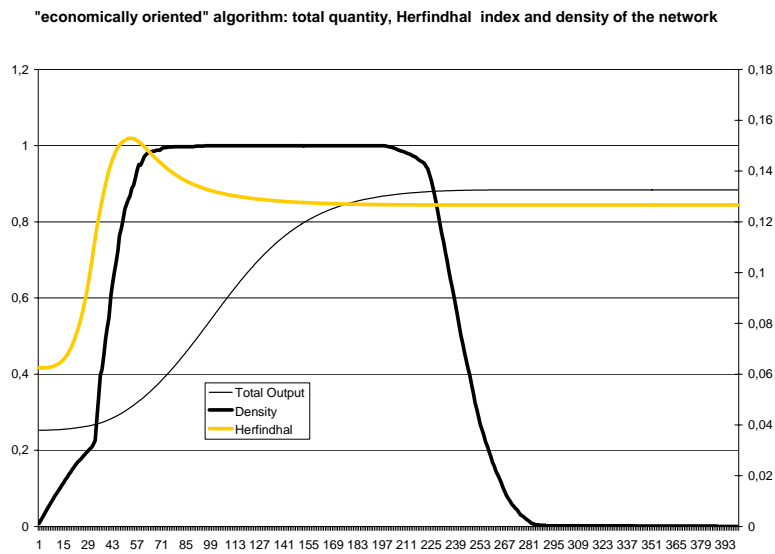


Figure 10



The effects of the “socially” oriented algorithm are negligible. The results are easy to interpret. First, what is crucial in the model are the first links formed, when the self reinforcing mechanism is at the work. Since at the beginning the network is sparse, the probability of meeting is basically uniform, and the differences are necessarily of minor importance. When the network has reached a sufficiently high density (i.e. in the periods just preceding the shake out), firms active in the network become significantly more likely to meet. But these firms are also the more likely to start cooperation, since they are larger and more competent. The effect, then, is simply to make the convergence towards the complete network slightly more rapid, and consequently the shake-out slightly more rapid, without an impact on the qualitative behavior of the series.

The “economically oriented” algorithm has instead a more significant effect. This is similar to an increase in technological opportunities: the shakeout occurs earlier, but involves fewer firms. This algorithm substantially reduces the frictions in the network formation, and then it leads to a stronger role of the first mover advantage. Larger firms at the very beginning have more incentives to form new agreements, so they can look around for complementarities among the “small firms”; small firms can look for the largest firm. In this way, more links are formed: the shakeout is anticipated (because the exclusion process starts in advance) but involves fewer firms. In any case, the selection process is quite strong.

In terms of alternative way of cost reduction, a very simple formulation has been considered. I relax the assumptions that costs can be reduced only through collaboration. Each period, each firm is assumed to start an “in-house” R&D project. More generally, other factors (for instance, learning by doing) can lead to this reduction in costs. The strongest assumption is that this process of cost reduction does not require any investment by the firm. Introducing explicitly an R&D cost (say a fixed cost similar to the costs required for cooperative R&D) in the framework of a simultaneous game would create a problem of multiple equilibria, when firms are close enough in efficiency level (i.e. size). Even if one assumed some rule to pick up one equilibrium, this would be too complex to implement.

In economic terms, this assumption can be justified by claiming that collaborative projects are typically started for larger, more costly (and with higher benefits) projects than in-house R&D. The assumption of no cost approximates a situation where each firm can cover the costs of internal R&D in any case, and the costs can be consequently not modelled. Furthermore, in the present context, I introduce in-house R&D to check the robustness of the results, and not to

fully model the choice between in-house and cooperative R&D. Here, one major point is to check the robustness of the selection result due to the network formation. With positive costs (and indivisibility), very small firms would not invest in R&D alone either (for the cost spreading argument). Then the “no cost” situation can be interpreted an upper bound for outcome of the selection process: selection cannot be stronger than the case of “costless” R&D.

Following the notation of the paper, I label v_{it} the value of such an in-house project. I consider two possible formulations:

$$v_{it} = \beta f(0) \gamma_{it}$$

$$v_{it} = \beta f(0) k$$

In the first case, I deal with a cumulative process: more competent firms have more valuable in-house projects; in the second case, instead, the value is independent from firm's level of efficiency. This second case is clearly more favorable to 'laggard' firms, and it is introduced mostly as a benchmark case.

Figure 11

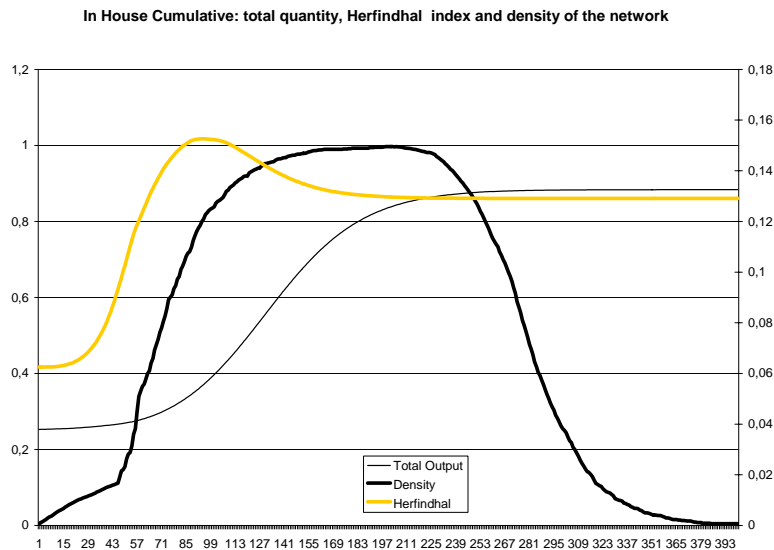
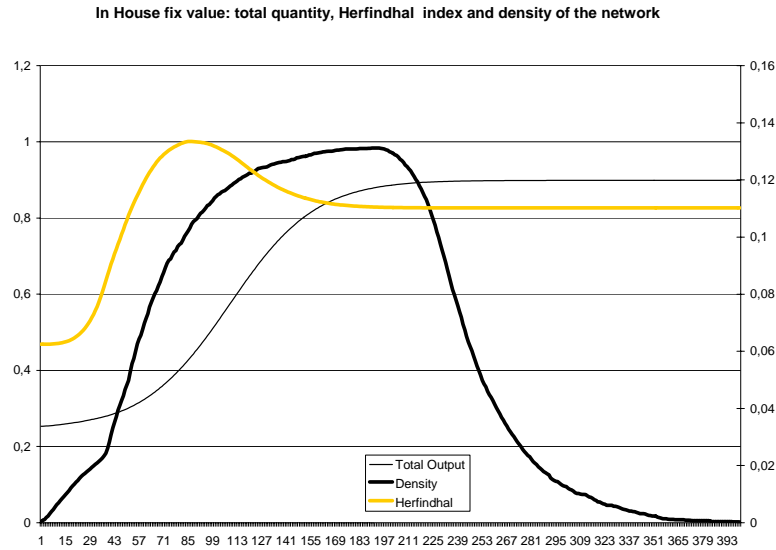


Figure 12



The effect of this modification (Figure 11 and 12) goes indeed in the predicted direction: selection is less strong. In the "cumulative" version (Figure 11, with $\beta = 0.4$), results are very similar to the "standard simulation". Indeed, in this formulation, in house R&D and cooperative R&D are complementary, in the sense that starting cooperative projects increases the value of in-house R&D, and the presence of in-house R&D increases the incentive of cooperative R&D through its effect on size. Then the two effects, strengthening and weakening the selection process, substantially cancel out.

In the second case, the effect of decreased concentration is stronger. In this case (Figure 12), $\beta = 0.2$ and $k=0.5$, which means that in-house RD is equivalent to a collaboration with a firm having the same technological position and efficiency 0.1. However, the main point here is that the results of the model respond "smoothly" to a limited ability of firm to progress autonomously in cost reduction: the logic in the arguments put forth in the previous sub-section is still valid.

5. Conclusion

In this paper I have presented a model of dynamic R&D network formation, in which the focus is explicitly on the joint dynamics of market structure, firms' technological capabilities and

network evolution. First, through an analytical experiment, I have spelled out more clearly the effects of market competition and technological opportunities on the firms' incentive to collaborate. Then, I have showed results from numerical simulations. They clearly show the general importance of R&D networks as powerful selection mechanism, leading firms that are not able to join the network or that occupy weak positions to exit the market. At the same time, in the long run the network levels out the differences among the surviving firms, through a process of "densification" of the network that leads to the emergence of a complete or almost complete network in the phase of industry maturity. Also, I have also shown how the rate of technical progress, in the form of "high opportunity" or availability of partners, can affect the industry structure in the long run; I have pointed out the detrimental effect on innovation generated by a slow process of technological converge among firms; I have shown that the model is robust to the proposed modifications of the network formation algorithm and to a simple introduction of in-house R&D.

Two final remarks. From a theoretical point of view, it is interesting to compare my results with the analyses by Goyal and Joshi (2003) and Goyal and Moraga (2001). When they restrict their attention to symmetric networks, they generally show the stability of the complete network. However, in the simplified framework with three firms, Goyal and Moraga show also the stability of some forms of asymmetric networks, in which possibly one of the firms can be forced out of the market when excluded by the collaboration. In general, they maintain a role for asymmetric networks in having profound effects on market structure, a claim that is consistent with the empirical evidence on the firms' motivation to engage in collaboration (Hagedoorn, 1993). In a sense, my model reconciles these two results, assigning symmetric and asymmetric networks different roles in different phases of the industry.

From the empirical point of view, systematic analyses of the role and effect of R&D network on industry evolution are still missing. However, the model seems consistent with the appreciative argument on the emergence of "knowledge-based networked oligopolies" (Delapierre and Mytelka, 1998). In sectors like pharmaceuticals and ICT, a denser and denser network is emerging, involving the big players at the global level. Furthermore, this web of alliances constitutes a significant barrier to entry (in the model, a barrier to survival), when rapid technical progress and strong competition make impossible a stand-alone strategy.

Overall, the first results of the model seem promising. Much work, however, has to be done in data analysis; several experiments out of the basic model, as well as some extensions, seem also interesting to pursue. All this is left for future version of this paper.

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Appendix

Proof of Proposition 1

Simple derivations show that:

$$\frac{\partial F}{\partial d(i, j)} = \lambda(a_2 - a_3 d(i, j))c \left[\frac{n\gamma_j(1 - \gamma_i)(e^{-\lambda(\gamma_j f(d_{ij}))} - 1) - \gamma_i(1 - \gamma_j)(1 - e^{-\lambda f(d_{ij})\gamma_i})}{n + 1} \right]$$

As long as firms are close enough, the second factor is positive¹³. The sign of the derivative is then determined by $a_2 - a_3 d(i, j)$, which is positive if firms' distance is lower than the optimal one, and negative otherwise.

Proof of Proposition 2

Deriving one obtains:

$$\begin{aligned} \frac{\partial F}{\partial \gamma_j} = & c \left[\frac{n\lambda f(d(i, j))(1 - \gamma_i)e^{-\lambda\gamma_j f(d(i, j))} + 1 - e^{-\lambda\gamma_i f(d(i, j))}}{n + 1} \right] [q_i(+ij) + q_i(-ij)] \\ & + c \left[\frac{(1 - \gamma_j)(e^{-\lambda f(d_{ij})\gamma_i} - 1) - n(1 - \gamma_i)(e^{-\lambda(\gamma_j f(d_{ij}))} - 1)}{n + 1} \right] \left[\frac{dq_i(+ij)}{d\gamma_j} + \frac{dq_i(-ij)}{d\gamma_j} \right] \end{aligned}$$

The quantities in the first two square brackets are positive, so it is the first addend. The sign of the second addend depends on

$$\frac{dq_i(+ij)}{d\gamma_j} + \frac{dq_i(-ij)}{d\gamma_j} = \frac{c \left[\lambda f(d(i, j))n(1 - \gamma_i)e^{-\lambda\gamma_j f(d(i, j))} - e^{-\lambda\gamma_i f(d(i, j))} - 1 \right]}{n + 1}$$

which is negative for λ sufficiently small.

From the study of the second derivative, it can be shown that $\frac{\partial^2 F}{\partial \gamma_j^2} < 0$ for λ sufficiently

small. Then the point (if any) where the derivative becomes 0 must be a maximum point. If gains from the collaboration are positive, there are consequently three possible cases: the

¹³ Notice however that the condition of positivity here is stricter than the necessary condition of positive gains from collaboration.

increase in γ_j 1) has always a positive effect; 2) has always a negative effect; 3) has a positive effect initially, and then has a negative effect.

Proof of Proposition 3

Deriving one obtains:

$$\frac{\partial F}{\partial \gamma_i} = c \left[\frac{-\lambda f(d(i, j))(1 - \gamma_j)e^{-\lambda \gamma_i f(d(i, j))} + n(e^{-\lambda \gamma_j f(d(i, j))} - 1)}{n + 1} \right] [q_i(+ij) + q_i(-ij)]$$

$$+ c \left[\frac{(1 - \gamma_j)(e^{-\lambda f(d_{ij})\gamma_i} - 1) - n(1 - \gamma_i)(e^{-\lambda(\gamma_j f(d_{ij}))} - 1)}{n + 1} \right] \left[\frac{dq_i(+ij)}{d\gamma_i} + \frac{dq_i(-ij)}{d\gamma_i} \right]$$

The first addend is negative, while, if the necessary condition for positive gain holds, the sign of the second addend depends on $\left[\frac{dq_i(+ij)}{d\gamma_i} + \frac{dq_i(-ij)}{d\gamma_i} \right]$.

It shows that:

$$\left[\frac{dq_i(+ij)}{d\gamma_i} + \frac{dq_i(-ij)}{d\gamma_i} \right] = c \left[\frac{n(1 + e^{-\lambda \gamma_j f(d_{ij}))} - (1 - \gamma_j)\lambda f(d(i, j))e^{-\lambda \gamma_i f(d(i, j))}}{n + 1} \right]$$

The first quantity in square brackets is larger than 1, while the second is smaller than 1 for $\lambda f(d(i, j))$ small. Their difference is then positive.

The overall effect is ambiguous. Studying the second derivative, one gets $\frac{\partial^2 F}{\partial \gamma_i^2} < 0$ for

λ sufficiently small. Then the point (if any) where the derivative becomes 0 must be a maximum point. There are consequently three possible cases: the increase in γ_i 1) has always a positive effect; 2) has always a negative effect; 3) has a positive effect initially, and then a negative effect.

Proof of Proposition 4

The proposition comes directly from:

$$\frac{\partial F}{\partial \sum_{k \neq i, j} c_k} = \frac{2}{n + 1} \left[\frac{(1 - \gamma_j)(e^{-\lambda f(d_{ij})\gamma_i} - 1) - n(1 - \gamma_i)(e^{-\lambda(\gamma_j f(d_{ij}))} - 1)}{n + 1} \right]$$

Proof of theorem 1

I consider the situation where a stable oligopolistic structure has emerged, in the sense that the number of firms will remain constant in the future (the market structure at time t will be maintained in all the subsequent periods if $\frac{A - n_t c_{it}}{n_t + 1} > 0 \forall i \in N_t$). I have to prove

that $\lim_{t \rightarrow \infty} \Pr(g_{ijt} = 1) = 0 \forall ij \in N_t^2$. If $\lim_{t \rightarrow \infty} \Pr(g_{ijt} = 1) \neq 0$ I would have $\lim_{t \rightarrow \infty} \gamma_i = \lim_{t \rightarrow \infty} \gamma_j = 1$.

By continuity of $F(o)$ (where this is the gain function defined in section 4.1), this implies $\lim_{t \rightarrow \infty} F_i(\alpha_j, \gamma_j) = \lim_{t \rightarrow \infty} F_j(\alpha_i, \gamma_i) = 0$. But then, since $E > 0$, the link will asymptotically become unprofitable. Given that each link is updated with a positive probability, it will be severed with probability 1 as $t \rightarrow \infty$, and then I have the initial claim.