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## An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy

#### 1. Introduction

In this paper, we develop a small, structural econometric model to be used in the quantitative evaluation of proposed rules for monetary policy. The quantitative evaluation of monetary policy rules has gained increased attention in recent years (see, e.g., McCallum, 1988, 1997; Taylor, 1993a; Bryant, Hooper, and Mann, 1993; Henderson and Mc-Kibbin, 1993; Feldstein and Stock, 1994; Leeper and Sims, 1994; Levin, 1996; and Fuhrer 1997a). Our approach differs from these studies in that we derive our econometric specification from an explicit model of intertemporal optimization on the part of both the suppliers and the purchasers of goods and services.<sup>1</sup>

Rigorously grounding our structural relations in optimizing individual behavior has two important advantages. The first is that we are able to respond to the well-known Lucas (1976) critique of econometric policy evaluation. Our analysis of hypothetical policy rules takes full account of the way that an understanding of the change in policy regime ought to affect the decision rules of private agents, and make them different than those that underlie the statistical correlations observed in past data.

We wish to thank Eduardo Loyo for superb research assistance, Ben Bernanke, Frank Diebold, Jeff Fuhrer, Bennett McCallum, and Chris Sims for helpful comments, and the NSF for research support.

<sup>1.</sup> Leeper and Sims (1994) made an ambitious effort of this kind, although, unlike us, they do not derive their price dynamics from producer optimization. Ireland (1997) is another recent study with an aim similar to ours, although our approaches differ considerably in their details.

Much of the recent work cited above does respond to the Lucas critique to at least some extent, by incorporating forward-looking specifications of at least some of the models' structural relations, and assuming rational (or model-consistent) expectations in analyzing alternative policies. But because no attempt is made to derive the complete structural model from internally consistent foundations in terms of individual optimization, doubts must remain as to whether the posited structural relations should genuinely be invariant to changes in the policy regime.

Demanding that one's structural relations be derived from individual optimization also has the advantage that evidence from other sources about the nature of the problems that individuals face can be used to corroborate the quantitative specifications that are used to explain the relations among aggregate time series. Ultimately, this is the only way in which the "observational equivalence" of a multitude of alternative possible structural interpretations of the comovements of aggregate series can be resolved. We make little attempt here at such validation of our proposed specification. But because our model parameters refer to concepts, such as the elasticity of firms' demand curves, or the average length of time that prices remain fixed, that have clear referents apart from the role of these parameters in our structural relations linking interest rates, inflation, and output in the economy as a whole, it becomes possible to consider the reasonableness of our specification on grounds other than simple statistical measures of goodness of fit.

The second advantage of an optimization-based approach is that the specification of individuals' decision problems that is used to explain the effects of monetary policy can also be used for purposes of welfare analysis. Of course, the analysis of the deadweight losses associated with alternative policies in terms of the individual preferences that account for the predicted responses to a policy change is by now the standard method of the public-finance literature. But this method has been little applied to problems of monetary policy, the main exception being analyses of the special issue of the costs of steady inflation (e.g., Lucas, 1993). Analyses of optimal monetary policy-or at least those that are based upon econometric models-consider instead the problem of minimizing one ad hoc loss function or another. Here we show, instead, how a utility-based measure of the deadweight loss associated with price-level instability can be derived, and how most of its parameters can be determined from our estimated structural equations. We can use this measure to address questions such as Summers's (1991) suggestion that a positive average rate of inflation is desirable in order to make it possible for nominal interest rates to be lowered as necessary for stabilization purposes. Not only does our econometric model allow us to discuss

quantitatively how much variability of nominal interest rates would be necessary for full stabilization, but our welfare measure in principle allows a direct comparison, in comparable units, of the deadweight losses associated with incomplete stabilization on the one hand and higher average inflation on the other.

Our analysis proceeds in five distinct steps. In the first step, we estimate a vector autoregression (VAR) model of the joint process of interest rates, inflation, and output. We use this VAR for two purposes. The first is to identify the actual monetary policy rule employed by the Fed. Following Taylor (1993b), we suppose that this rule is a reaction function that sets interest rates as a function of current and past values of output and inflation. The second purpose of this VAR is to estimate the way output, inflation, and interest rates respond to a stochastic disturbance to the monetary policy rule. Thus we learn how the economy responds to a monetary shock under the current monetary policy rule, using fairly standard "structural VAR" methodology (e.g., Cochrane, 1994; Leeper, Sims, and Zha, 1996).

In the second step, we postulate a simple theoretical model that can account for the estimated response of output and inflation to monetary policy shocks. In this model, we assume that there are impediments to the free adjustment of prices. In particular, we consider a variant of the Calvo (1983) model in which a firm's opportunity to change its price arrives stochastically and where, if this opportunity does not arise, the firm must keep its price constant. We choose the parameters of this extremely stylized model so that the model's predicted responses to monetary policy shocks match as closely as possible the estimated responses from the VAR.

In the third step, we combine the quantitative specification of the structural model (with the parameters obtained in step two) with the vector autoregression model to identify the shocks to the structural equations. The failure of the VAR to contain any stochastic singularities implies that the model's structural equations have residuals as well, which we can interpret in the context of the model as indicating stochastic variation in preferences and technology. We compute what these disturbances have been over our sample period, and use the VAR representation to determine the stochastic process followed by the real disturbances. One important advantage of our method of analyses is that, once we take into account these constructed disturbances, our model fits the data nearly as well as an unrestricted vector autoregression. This good fit provides an additional rationale for being interested in the model's implications concerning monetary policy.

In the fourth step, we use the quantitative model with parameters

estimated in step two and shock processes estimated in step three to simulate the consequences of hypothetical monetary policy rules. Using the estimated historical shock series, we can simulate alternative historical paths for the U.S. economy. In particular, we can compute the realizations of output, inflation, and interest rates under counterfactual rules. What is particularly attractive about this exercise is that, under the actual monetary rule estimated in step one, the simulated paths of output, inflation, and interest rates are identical to the actual paths. Thus, the simulations with counterfactual rules provide alternative historical paths for the U.S. economy.

In the fifth and final step, we use the parameters estimated in step two to compute the welfare consequences of different monetary rules. Moreover, we derive the rule that would have maximized the utility of our representative households given the shock processes obtained in step three.

## 2. The Effects of Monetary Policy Shocks under the Current Policy Regime

In this section, we describe our econometric characterization of the current monetary policy regime, and our estimates of the effects of monetary policy shocks under that regime. By monetary shocks we mean exogenous stochastic shifts in the feedback rule used by the Fed to set the Federal funds rate. Our interest in the effects of such shifts does not derive from a belief that they have played an important role in the generation of fluctuations in either output or inflation in the period with which we are concerned. Rather, we are interested in them because they can be econometrically identified without our having to commit ourselves to detailed assumptions about the true structural relations that determine output and inflation.

The monetary policy shocks and their effects cannot, of course, be identified without at least some weak *a priori* assumptions. In particular, we assume that recent U.S. monetary policy may be described by a feedback rule for the federal funds rate of the form

$$r_{t} = r^{*} + \sum_{k=1}^{n_{r}} \mu_{k}(r_{t-k} - r^{*}) + \sum_{k=0}^{n_{\pi}} \phi_{k}(\pi_{t-k} - \pi^{*}) + \sum_{k=0}^{n_{y}} \theta_{k} y_{t-k} + \epsilon_{t}, \qquad (2.1)$$

where  $r_t$  is the Federal funds rate in period t, and  $\pi_t$  is the rate of inflation between periods t - 1 and t,  $y_t$  is the percentage deviation of real GDP from trend, and  $r^*$  and  $\pi^*$  are long-run "target" values for the funds rate

and the rate of inflation respectively. The  $\epsilon_i$ 's represent exogenous monetary policy shocks, which are assumed to be serially uncorrelated. In assuming that monetary policy shocks may be identified with movements in the federal funds rate that cannot be predicted given the history of the funds rate, or by current and past values of other macro time series such as output and inflation, we follow a large part of the recent "structural VAR" literature on the identification of monetary policy shocks, beginning with Bernanke and Blinder (1992) and including Cochrane (1994) and Leeper, Sims, and Zha (1996). In our assumption of a feedback rule of the specific form (2.1) as a representation of current policy, we follow the monetary-policy "reaction function" literature, especially Taylor (1993b). Taylor (1993b) asserts that, at least under the chairmanship of Alan Greenspan (i.e., at least since late 1987), Fed policy has been well described by a rule of this kind.<sup>2</sup>

Identification of the monetary policy shocks  $\{\epsilon_i\}$  and estimation of the coefficients in (2.1) require a further identifying assumption about the correlation between  $\epsilon_i$  and the period-*t* endogenous variables. Our assumption is that a monetary policy shock at date *t* has no effect on either output or inflation during period *t*; the idea is that both pricing and purchasing decisions for period *t* are made prior to the realization of the shock, i.e., before the period-*t* funds rate is observed. (In the theoretical model proposed in the next section as an interpretation of our VAR results, this assumption is made explicit.) An alternative interpretation of our restriction is that pricing and purchasing decisions for period *t*, but on the basis of incomplete information—in particular, without information about current money-market conditions. The use of such *decision lags* as an identifying assumption is common in the structural VAR literature, beginning with Sims (1986). Under this assumption, equation (2.1) can be estimated using OLS.<sup>3</sup>

Note that this identifying assumption requires that we interpret any correlation between the period-*t* innovations in inflation or output and the period-*t* innovation in the funds rate as due to the way in which the Fed reacts to variations in inflation and output in setting the funds rate.

<sup>2.</sup> According to Taylor's much-discussed account of recent policy,  $n_r = 0$ ,  $n_\pi = 3$ ,  $n_y = 0$ ,  $\pi^* = 2\%/\text{year}$ ,  $r^* = 4\%/\text{year}$ , and the coefficients are given by  $\phi_k = 1.5/4$  for k = 0, 1, 2, 3, and  $\theta_0 = 0.5$ . Taylor presents these values as a rough rule of thumb rather than a precise quantitative specification. It is clear that actual policy involves a greater degree of interest-rate smoothing than the simple "Taylor rule" would predict; hence our allowance for lagged funds-rate terms in our generalization (2.1) of Taylor's rule.

<sup>3.</sup> Note that the constants  $r^*$  and  $\pi^*$  cannot be separately estimated from this equation alone. We are able to estimate them when we estimate our complete VAR model, by assuming that no equation of the three-variable VAR model contains a constant term, if the three state variables are all written in terms of deviations from their long-run values.

It is therefore necessary that we deny the existence of "information lags" in the monetary policy rule, if we are to avoid imposing any overidentifying restrictions at this stage in our characterization of the data. This is why the coefficients  $\phi_0$  and  $\theta_0$  are allowed to be nonzero in (2.1). We find, in fact, a significant positive coefficient  $\theta_0$ . This is because there is a significant positive correlation between the funds-rate innovation and the contemporaneous output innovation. Since we would expect a negative correlation, if any, if the current output realization had no effect on monetary policy (while a positive correlation is entirely plausible under the assumption that policy does respond to an output innovation within the quarter), the assumption of a decision lag for the Fed is unattractive, at least as regards the response to output innovations.<sup>4</sup>

We estimate (2.1) as part of a three-variable just-identified VAR model, where the variables included are the funds rate, the inflation rate, and detrended real GDP.<sup>5</sup> We estimate a complete system of this kind, so that we obtain not merely an estimated Fed reaction function, but also estimated impulse responses to monetary policy shocks under the policy regime characterized by that reaction function. The three variables included represent a minimal set for our purposes: they are the minimal set needed to allow us to estimate a monetary policy rule of the kind proposed by Taylor (1993b), and they allow us to model the effects of monetary policy on fluctuations in the three variables that central banks are most often supposed to concern themselves with as ultimate goal variables.

The sample period for our estimation of (2.1) runs from 1980:1 through 1995:2. We begin our sample in the first quarter of 1980, because it is widely recognized that a significant change in the U.S. monetary policy regime occurred around that time; thus at least one equation of our model, the monetary policy rule (2.1), cannot be expected to have remained invariant over a longer time period than the one that we use. Many, of course, would doubt that the monetary policy rule has remained unchanged since then. Conventional accounts of the succession of U.S. monetary policy regimes often identify important regime changes in late 1982 (the end of the Fed's experiment with targeting of nonborrowed reserves) and late 1987 (the transition from Volcker to Greenspan) as

4. The assumption made with regard to inflation innovations has little effect upon our results. Table 1 shows that the estimated coefficient  $\phi_0$  is in any event both small and statistically insignificant.

<sup>5.</sup> Our desire to model GDP leads us to use quarterly data. Our  $\{r_i\}$  series is the federal funds rate, annualized and averaged over the quarter. Our  $\{\pi_i\}$  series is the quarterly change in the log of the GDP deflator, also annualized. Finally, our  $\{y_i\}$  series is the log of real GDP, with a linear time trend removed.

well.<sup>6</sup> Our choice of the longest among several possible samples is determined by a desire to have long enough time series to allow estimation of an unrestricted VAR model. In fact, most VAR studies of the effects of monetary policy shocks make use of much longer data samples than ours. Our choice of a sample period represents a compromise between these two concerns.

Estimation of a VAR that includes (2.1) as one equation could be carried out in various ways. Perhaps the most obvious would be to estimate a recursive VAR with state vector  $[\pi_i, y_i, r_i]'$ , in which the causal ordering of the variables is the order in which they are listed. An alternative, that we follow here, is to estimate a recursive VAR with state vector

$$Z_t = [r_t, \ \pi_{t+1}, \ y_{t+1}]', \tag{2.2}$$

with the interest rate now first in the causal ordering. Specifically, we estimate a system of the form

$$T\overline{Z}_t = A\overline{Z}_{t-} + \overline{e}_t, \tag{2.3}$$

where the vector  $\overline{Z}_t$  is the transpose of  $[Z'_t, Z'_{t-1}, Z'_{t-2}]$ , *T* is a lower triangular matrix with ones on the diagonal and nonzero off-diagonal elements only in the first three rows, and *A* is a matrix whose first three rows contain estimated coefficients from the VAR. The first three rows of  $\overline{e}_t$ 's contain the VAR residuals, while the other elements are zero.<sup>7</sup>

This notation is unfamiliar in that some data for period t + 1 are included in the period-*t* state vector. The reason is that, according to our assumption about decision lags, these variables, which are only observed in period t + 1, are nonetheless determined on the basis of information that, according to our model, decision makers have in period *t*. We prefer this choice of notation because it allows us to describe the information used by decision makers in terms of the history of the vector { $Z_i$ }. Specifically, we can refer to the information set consisting of the history { $Z_{t-j}$ } for all  $j \ge 0$  as the *period-t information set*. Then the information used in choosing price changes that take effect in period t + 1and quantities to be purchased in period t + 1 may be taken to be exactly the period-*t* information set; similarly, the information used in

<sup>6.</sup> See, e.g., Strongin (1995) and Bernanke and Mihov (1995). Taylor (1993b) proposes his feedback rule only as an account of Fed policy since late 1987.

<sup>7.</sup> Three lags in (2.3) suffice to eliminate any significant serial correlation in the residuals, and the estimated coefficients on longer lags are also insignificant. Note that this form differs from the other, more familiar form in the numbers of lags of the various variables at the point of truncation.

setting the funds rate in period t + 1 is the period-*t* information set, except that the Fed's action involves a random disturbance  $\epsilon_{t+1}$  as well.

The first row of the estimated system (2.3) corresponds to the monetary-policy feedback rule (2.1), and the first element of the residual vector  $\bar{e}_i$  represents our identification of the monetary shock  $\epsilon_i$ . The coefficients of the estimated policy rule are displayed in Table 1. The estimated rule may be described as a generalized "Taylor rule," though the dynamics are more complex than in Taylor's simple specification. One way to measure the overall responsiveness of the funds rate to fluctuations in inflation and output is in terms of long-run multipliers, which indicate the eventual increase in the funds rate that would result in the case of a permanent change in the levels of inflation and output. These long-run multipliers are given by<sup>8</sup>

$$r - r^* = 2.13(\pi - \pi^*) + 0.47y. \tag{2.4}$$

Thus, as in Taylor's rule, an increase in GDP relative to trend raises the funds rate (and our estimated long-run multiplier is essentially the same as that indicated by Taylor), and an increase in inflation relative to its "target" level raises the funds rate by an even greater amount (so that short-term *real* interest rates rise). Our estimate of the sensitivity of the funds rate to inflation fluctuations is even stronger than Taylor's coefficient; this may well be due to our inclusion of the Volcker years in our sample. Differences in our dynamic specification include our finding of significant interest-rate smoothing (the coefficients  $\mu_k$  are all positive, and sum to 0.7), our finding that an increase in inflation does not begin to increase the funds rate until the following quarter, and our finding that the short-run multiplier for output is larger than the long-run multiplier (owing to the negative value for  $\theta_2$ ).

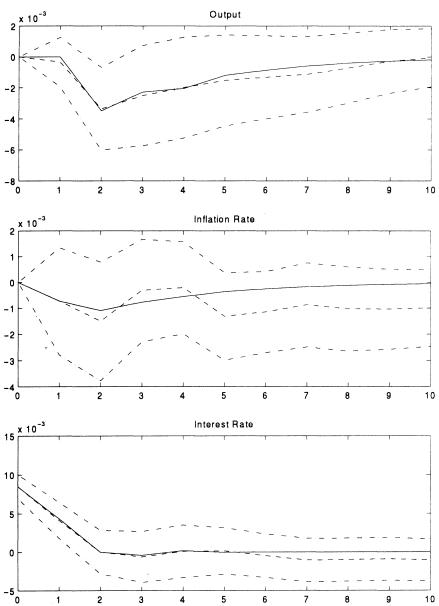
The complete estimated system also allows us to compute the response of output, inflation, and interest rates to a monetary policy shock. These impulse responses are plotted in the three panels of Figure 1. In each panel, the central dashed line indicates the point estimate of the impulse response function, while the two outer dot-dash lines indicate a confidence interval for each coefficient (plus and minus two times the standard error), based on analytic derivatives of the responses with respect to the parameters and on the variance-covariance matrix of the parameters.

<sup>8.</sup> Though these cannot be read off from the regression reported in Table 1 alone, we estimate long-run values  $\pi^* = 3.26\%$  and  $r^* = 6.25\%$ . We thus estimate a long-run average real funds rate of 3%. These values compare with Taylor's assumption of a 2% real rate and a 2% inflation target.

Independent variable	$R_t$	$\pi_{t+1}$	Y <sub>t+1</sub>
$\pi_{t+1}$			0.111 0.109
$R_t$		-0.085 0.133	-0.030 0.104
$R_{t-1}$	0.507	-0.078	-0.316
	0.129	0.141	0.110
<i>R</i> <sub><i>t</i>-2</sub>	0.023	0.194	0.382
	0.145	0.139	0.110
<i>R</i> <sub><i>t</i>-3</sub>	0.158	-0.096	-0.086
	0.109	0.106	0.083
$\pi_t$	-0.062	0.540	-0.099
	0.130	0.125	0.113
$\pi_{t-1}$	0.427	0.077	0.174
	0.152	0.156	0.121
$\pi_{t-2}$	0.300	0.379	-0.135
	0.157	0.156	0.128
Y <sub>t</sub>	0.624	0.260	1.335
	0.175	0.187	0.148
$Y_{t-1}$	-0.059	-0.297	-0.163
	0.306	0.293	0.230
$Y_{t-2}$	$\begin{array}{c} -0.418\\ 0.189\end{array}$	0.143 0.189	-0.258 0.148
R <sup>2</sup>	0.947	0.834	0.933
D.W.	2.07	2.13	1.83

#### Table 1 THE VECTOR AUTOREGRESSION SAMPLE: 1980:1–1995:2<sup>a</sup>

<sup>a</sup>Standard errors below estimates.



# Figure 1 ESTIMATED AND THEORETICAL RESPONSES TO A MONETARY POLICY SHOCK

Responses are plotted for a one-standard-deviation shock that raises the funds rate unexpectedly by about 0.8%. The estimated responses largely agree with conventional wisdom: interest rates are raised only temporarily (according our estimates, only for the first two quarters); output subsequently declines (not noticeably until two quarters later), but eventually returns to normal; and inflation also declines with a lag (with the greatest decline occurring two quarters later). These effects cannot be estimated with much precision (especially the effects on inflation). Nonetheless, they give us an idea of the features that our structural model should possess in order to be consistent with the data. In particular, a monetary tightening should temporarily lower both output and inflation; and these effects on output and inflation largely occur *after* short-term interest rates have returned to their normal level.

These results agree qualitatively with those that emerge from several recent VAR exercises, including Christiano, Eichenbaum, and Evans (1994) and, more relevantly (because he considers a three-variable VAR similar to ours), Cochrane (1994). In one of the many exercises reported in his paper, Cochrane (1994) estimates a VAR over the period 1959 to 1992 that includes quarterly observations of the federal funds rate as well as of the logarithms of output and of the price level.<sup>9</sup> When he includes a trend and computes the monetary policy shock by supposing that the systematic component of policy lets the federal funds rate at *t* react to output and the price level at *t*, he gets very similar impulse responses.

### 3. A Simple Model of Output and Inflation Determination

In this section we develop a simple equilibrium macroeconomic model, that we propose to use to interpret the fluctuations in the three time series of our VAR. The model is extremely rudimentary; we have reduced it to the essential elements necessary for a general-equilibrium account of the determination of output, inflation, and interest rates. The model presented here is intended more as an illustration of the method that we advocate than as a complete model of the U.S. economy. Nonetheless, we believe that it shows that it is possible to account for the

<sup>9.</sup> We get very similar impulse responses when we consider a longer sample that is similar to his. While the responses of output and inflation are more muted, we obtain qualitatively similar responses if, instead, we start our sample at the beginning of 1982. If the VAR includes only more recent observations, the effects of monetary shocks on output and inflation are much weaker and more poorly determined. This is probably due to the absence of significant monetary disturbances in the more recent period.

main features of these time series in terms of a model derived from optimizing behavior under rational expectations.

The model we use is an extension of the one used in Woodford (1996) to analyze the consequences of interest-rate feedback rules for monetary policy,<sup>10</sup> with additional decision lags (both for price and quantity variables) added in order to allow a better fit with the predictions of the VAR. The optimizing decision maker in our model is an infinite-lived representative household, which is both a consumer of all the goods produced in the economy and a producer of a single differentiated product. We index each household by *i* and let *i* vary continuously from zero to one. The objective of household *i*, looking forward from date t = 0, is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(C_{t'}^i \xi_t) - v(y_{t'}^i \xi_t)], \qquad (3.1)$$

where  $\beta$  is a discount factor,  $y_i^i$  is the output of the goods produced by household *i* at time *t*,  $\xi_i$  is a vector of random disturbances, and for each value of  $\xi_i$ , *u* is an increasing, concave function, and *v* is an increasing, convex function.<sup>11</sup> The argument  $C_i^i$  represents an index of the household's purchases of all of the continuum of differentiated goods produced in the economy, given by

$$C_{t}^{i} = \left(\int_{0}^{1} c_{t}^{i}(z)^{(\theta-1)/\theta} dz\right)^{\theta/(\theta-1)}$$
(3.2)

as in Dixit and Stiglitz (1977), where  $c_i(z)$  is the quantity purchased of good z, and the constant elasticity of substitution  $\theta$  is assumed to be greater than one.

For expositional purposes, it is easiest to think of these purchases by the household as purchases of nondurable consumption goods, and our model as one in which all output is nondurable and immediately consumed either by households or by the government. If capital goods are used in production, they are in fixed supply (and nondepreciating), and their allocation across firms (or households) cannot be changed. How-

- 10. Certain aspects of the structure of the model are discussed in more detail there. Note that here, unlike in the previous analysis, we consider only the case of a Ricardian fiscal policy, which allows us to avoid discussion of certain equilibrium conditions that play a key role in the earlier paper.
- 11. As is discussed further in Section 4, we can interpret v as a reduced-form representation of production costs in a model with firms and labor markets. Under this interpretation, v is convex both because there are diminishing returns to labor and because the marginal disutility of labor is increasing in labor.

ever, we seek to use the model to explain the dynamics of real GDP, and not just of consumption spending. Thus we actually interpret  $C_i^i$  as referring to the household's purchases of investment goods as well as consumption goods, and assume (at the price of an obvious oversimplification) that all such purchases are made to obtain immediate utility, which can be expressed (through a reduced-form or indirect utility function) as an increasing, concave function of total purchases at a given date. We ignore the effects of investment spending upon the evolution of productive capacity, in the hope that these effects are in any event not too significant at the frequencies with which we are most concerned in evaluating alternative monetary policies.<sup>12</sup> In making this simplifying assumption, of course, we follow a long tradition of macroeconometric modeling based upon more or less elaborate versions of the textbook IS– LM model.<sup>13</sup>

Perhaps the most surprising element of our preference specification, given that we are interested in monetary issues, is that we abstract from the liquidity services provided by money. This is no more than a simplification. Our model can be understood as a the limit of a model where real money balances provide utility but where, in the limit, these liquidity services are arbitrarily small. Alternatively, the model can be understood as one where utility is additively separable in real money balances, consumption, and goods supply, as in Woodford (1996). In this case, the model implies an additional first-order condition relating real balances to consumption and the interest rate. In the presence of an interest-rate rule, which is after all the focus of our analysis, this additional equilibrium condition simply determines the nominal level of money balances. Since this equilibrium condition plays no role in determining inflation, output, or interest rates, it can safely be ignored for our purposes.

When allocating a given amount of nominal spending at t,  $S_t^i$ , across all the different differentiated goods, the household maximizes (3.2) subject to the constraint that spending on all goods must not exceed  $S_t^i$ . This leads to the familiar Dixit–Stiglitz demand relations for relative quantities purchased as a function of relative prices. As usual, the total expenditure required to obtain a given quantity of the consumption aggregate (3.2) is given by  $S_t^i = P_t C_t^i$ , where  $P_t$  is the Dixit–Stiglitz price index defined by

$$P_{t} = \left(\int_{0}^{1} p_{t}(z)^{1-\theta} dz\right)^{l/(1-\theta)}.$$
(3.3)

12. This will be a subject of quantitative investigation in later work.

13. Similar simplifications are advocated, for example, in McCallum and Nelson (1997).

The intertemporal allocation of spending, in turn, amounts to choosing the path of the aggregate  $\{C_i\}$  defined in (3.2) to maximize (3.1) subject to a budget constraint that is written in terms of aggregate consumption expenditure.

We suppose that there are complete financial markets<sup>14</sup> and no obstacles to borrowing against future income, so that each household faces a single intertemporal budget constraint. Looking forward from date t, the intertemporal budget constraint of household i is of the form

$$E_t \sum_{T=t}^{\infty} \delta_{t,T} S_T^i \leq E_t \sum_{T=t}^{\infty} \delta_{t,T} [p_t(i)y_t^i - T_t] + A_{t,t}^i$$
(3.4)

where  $A_t^i$  denotes the nominal value of the household's financial assets at the beginning of period t,  $T_t$  denotes its net nominal (lump sum) tax obligation at date t, and  $\delta_{t,T}$  is the stochastic discount factor that defines the nominal present value at t of nominal income in any given state at date  $T \ge t$ . Because of the existence of complete financial markets, these stochastic discount factors are uniquely defined; a financial claim to a random nominal quantity  $X_t$  at date t has a nominal value of  $E_t(\delta_{t,T}X_t)$ . In particular, if  $R_t$  denotes the (gross) nominal interest rate on a riskless one-period bond purchased in period t, this interest rate must satisfy<sup>15</sup>

$$R_t = (E_t \delta_{t,t+1})^{-1}.$$
(3.5)

We assume that households must choose their index of purchases  $C_t$  at date t - 2.<sup>16</sup> We interpret this to mean that actions already taken prior to

- 14. It is important to note that by this we mean simply that a household is able to transfer purchasing power freely between dates and states of the world, through the exchange of state-contingent financial claims. We do not suppose that the state-contingent exchanges of real goods and services that occur in our model are contracted at some initial date, as in the Arrow-Debreu model. Nor do we suppose that households are able to arrange more complicated sorts of contracts that would allow them, in effect, to get around the constraints upon price-setting that we assume to exist in the spot markets for goods and services. The state-contingent financial claims that we imagine, however, do include the possibility of insuring oneself against the idiosyncratic income risk that households suffer because they change their prices at different dates.
- 15. Other financial claims can similarly be priced, but this is the only one that matters for our purposes, as we assume that the central bank conducts monetary policy by trading in the market for short-term nominal claims of this kind.
- 16. In fact, we simply require that C<sub>i</sub><sup>t</sup> be determined as of the beginning of period t 1, before the monetary policy shock in period t 1 is revealed, but at a time at which all period-t 1 goods transactions have been determined. In terms of our notational convention, this means that C<sub>i</sub><sup>t</sup> belongs to the date-t 2 information set. Note also that while we require that the index C<sub>i</sub><sup>t</sup> be determined at date t 2, the purchases of individual goods that are made in order to achieve this are determined only at date t 1—or by the beginning of period t—as these depend upon the period-t prices of the individual goods.

the realization of date-t - 1 monetary policy make it imperative that certain purchases be made during period t. Certain kinds of interestsensitive purchases do involve advance commitment of this kind; in particular, many kinds of investment projects take more than one quarter to complete, and abandonment of the project after initiation may be too costly to be contemplated expect in the case of quite extreme interim changes in market conditions.<sup>17</sup>

The household's optimal program of purchases then must satisfy

$$E_t u'(C_{t+2}^i;\xi_{t+2}) = E_t(\lambda_{t+2}^i P_{t+2})$$
(3.6)

at each date, where  $\lambda_t^i$  represents the household's marginal utility of nominal income at date t.<sup>18</sup> The marginal utilities of income at different dates and in different states in turn must satisfy

$$\lambda_t^i \delta_{t,T} = \beta^{T-t} \lambda_T^i \tag{3.7}$$

for any  $T \ge t$ . Conditions (3.6) and (3.7), and the requirement that (3.4) hold with equality, completely determine the household's optimal consumption plan, given its initial wealth, initial predetermined consumption level, and after-tax income expectations.

We furthermore assume that financial markets exist that allow households to insure one another against idiosyncratic income risk (which here results solely from differences in the time at which they change their prices). Assuming that all households have identical initial wealth, they will choose in equilibrium to completely pool their income risk, and we assume an equilibrium of this kind. As a result, in equilibrium the right-hand side of (3.4) has the same value for each household at any date, and households choose identical consumption plans and have identical marginal utilities of income. We can therefore drop the superscripts *i* in equations (3.6) and (3.7). Note also that it follows from (3.5) and (3.7) that the common marginal utility of income satisfies

$$\lambda_t = \beta E_t(R_t \lambda_{t+1}). \tag{3.8}$$

- 17. Our assumption thus amounts to a limiting case of a time-to-build model, in which the bulk of the expenditure connected with a project initiated at the beginning of period t 1 occurs during period t. Note that we could give an information-lag interpretation to this restriction upon household purchases: households choose their overall level of purchases in period t with knowledge of period-t 1 goods-market conditions, but before learning about period-t 1 money-market conditions, or about period-t conditions in either goods markets or money markets.
- 18. This quantity appears as the Lagrange multiplier associated with constraint (3.4). It is measured in units of period-t utility flow per dollar.

In our computation of the equilibrium responses to shocks in subsequent sections, we make use of a log-linear approximation to the equilibrium conditions of our model, expanding in terms of percentage deviations of various stationary state variables from their steady-state values (their constant values in the absence of all stochastic disturbances). In this log-linear approximation, (3.8) becomes

$$\hat{\lambda}_t = E_t(\hat{R}_t - \hat{\pi}_{t+1} + \hat{\lambda}_{t+1}),$$

where  $\hat{\lambda}_{\iota}$ ,  $\hat{R}_{\iota}$ , and  $\hat{\pi}_{\iota}$  denote percentage deviations in the stationary variables  $\lambda_{\iota}P_{\iota}$ ,  $R_{\iota}$ , and  $P_{\iota}/P_{\iota+1}$  respectively. (This last definition makes sense because we log-linearize all of our equations around a steady-state equilibrium in which the constant rate of inflation is zero.) We can furthermore solve this forward to obtain

$$\hat{\lambda}_{t} = \hat{r}_{t}^{l} \equiv \sum_{T=t}^{\infty} E_{t}(\hat{R}_{T} - \hat{\pi}_{T+1}), \qquad (3.9)$$

where  $\hat{r}_{t}^{l}$  defines percentage deviations in a long-run real rate of return. (In all of the equilibria that we analyze below, both interest rates and inflation follow stationary ARMA processes, as a result of which the infinite sum in (3.9) converges.)

The corresponding log-linear approximation to (3.6) is given by

$$-\bar{\sigma}E_t(\hat{C}_{t+2} - \bar{C}_{t+2}) = E_t\hat{r}_{t+2}^l, \qquad (3.10)$$

where the elasticity  $\tilde{\sigma}$  equals -u''C/u' (evaluated at the steady-state level of consumption),  $\hat{C}_t$  indicates the percentage deviation of  $C_t$  from its steady-state value, and  $\overline{C}_t$  is an exogenous disturbance (a certain function of the preference shock  $\xi_t$ ) indicating the level of consumption required at each point in time to maintain a certain constant marginal utility of consumption. Equation (3.10), together with the stipulation that  $\hat{C}_{t+2}$  be determined at date t, indicates how interest-sensitive purchases in period t + 2 depend upon interest-rate expectations at date t.

Total aggregate demand is assumed to be given by

$$Y_t = C_t + G_t, \tag{3.11}$$

where  $G_t$  represents exogenous variation in autonomous spending. While one natural interpretation of  $G_t$  is that it represents exogenous variation in government purchases, there are other possible interpretations as well. For instance,  $G_t$  could represent consumption purchases by liquidity-constrained consumers, who spend all their income (as in Campbell and Mankiw, 1989, except that we suppose that the real income of these consumers is an exogenous random process, rather than being a constant fraction of total income). Because of these various possible interpretations, we do not seek to identify  $G_t$  with the government purchases variable in the national accounts, any more than we wish to identify  $C_t$ with consumer expenditure. Rather, we focus only on the implications of our model for the evolution of  $Y_t$ . Note that we assume that  $G_t$  is determined by the beginning of period t (i.e., that it belongs to the date-t - 1information set), for consistency with the assumptions made in our identification of the monetary policy shocks from our VAR in Section 2.

Log-linearization of (3.11) yields

$$\hat{Y}_t = s_C \hat{C}_t + \tilde{G}_{t\prime}$$

where  $\hat{Y}_t$  denotes percentage deviations of  $Y_t$  from its steady-state value,  $\tilde{G}_t$  denotes the deviation of  $G_t$  from its steady-state value expressed as a percentage of the steady-state value of Y, and  $s_c$  is the steady-state value of the interest-sensitive share C/Y. Substituting this into (3.10) then yields the model's *IS equation* 

$$\hat{Y}_t = -\sigma^{-1} E_{t-2} \hat{r}_t^l + \hat{G}_{t'}$$
(3.12)

where  $\sigma \equiv s_C \tilde{\sigma}$ , and  $\hat{G}_t \equiv \tilde{G}_t + s_C E_{t-2} \overline{C}_t$  is a composite exogenous disturbance. The *aggregate demand block* of our model then consists of the monetary policy rule, the term-structure equation (3.9), and the IS equation (3.12).<sup>19</sup>

We now turn to our optimizing model of price setting and aggregate supply. The decision problem of price setters depends upon the demand that they face for their product. We suppose that autonomous expenditure  $G_i$  also represents an aggregate of purchases of individual goods, and we let this aggregate have the same form as (3.2), so that

$$G_t = \left(\int_0^1 g_t(z)^{(\theta-1)/\theta} dz\right)^{\theta/(\theta-1)}.$$

Moreover, we suppose that these individual purchases,  $g_t(z)$ , are chosen to maximize this aggregate for any given level of expenditure. As a

<sup>19.</sup> Note that this is the same basic structure as is used in the small model of Fuhrer and Moore (1995b). However, because our IS equation is derived from intertemporal optimization, there are some differences; for example, it is the past *expectation* of the *current* long-term real rate that affects current aggregate demand, in our specification, rather than the past value of the long-term real rate itself.

result, these purchases are made in the same proportions as those of consumers, so that the overall demand faced by an individual supplier satisfies

$$y_t^i = Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\theta}.$$
(3.13)

As is standard in models of monopolistic competition, we assume that an individual supplier regards itself as unable to affect the evolution of the variables  $Y_t$  and  $P_t$ , and so chooses its own price, taking the evolution of those variables as given.

The source of the real effects of monetary policy in our model is an assumption of decision lags in price setting. Following Calvo (1983), we assume that prices are changed at exogenous random intervals. Specifically, a fraction  $1 - \alpha$  of sellers get to choose a new price at the beginning any given period, whereas the others must continue using their old prices. Of those who get to choose a new price, a fraction  $\gamma$  start charging the new price during that period, whereas the remaining fraction  $1 - \gamma$  must wait until the next period to charge the new price, because (owing to a different organization of the markets for these goods) they must post their prices a quarter in advance. These assumed delays explain why no prices respond in the quarter of the monetary disturbance (as assumed in our identification of the policy shocks), and why the largest response of inflation to a monetary shock takes place only two quarters after the shock.<sup>20</sup>

Now let  $p_i^i$  denote the price chosen by a supplier that charges a new price beginning in period t, if the new price is chosen on the basis of period-t - i information (for i = 1,2). (The price chosen by all suppliers that choose on the basis of the same information set will be the same.) It then follows from (3.3) that the price index  $P_t$  will evolve according to

$$P_t = [\alpha P_{t-1}^{1-\theta} + (1-\alpha)\gamma(p_t^1)^{1-\theta} + (1-\alpha)(1-\gamma)(p_t^2)^{1-\theta}]^{1/(1-\theta)}.$$

Dividing both sides by  $P_t$  and log-linearizing, we obtain

$$\hat{\pi}_t = \gamma \hat{X}_t^1 + (1 - \gamma) \hat{X}_t^2, \tag{3.14}$$

where  $\hat{X}_{t}^{i} \equiv [(1-\alpha)/\alpha] \log (p_{t}^{i}/P_{t})$  for i = 1,2. Thus the overall rate of inflation depends upon the relative prices of the two types of suppliers with new prices in a given period.

20. Once again, our decision lags could represent information lags.

The optimal value of  $p_t^i$ , in turn, will be the price p that maximizes  $E_{t-i}\Phi_t(p)$ , where

$$\boldsymbol{\Phi}_{t}(\boldsymbol{p}) \equiv \sum_{j=0}^{\infty} \alpha^{j} \beta^{j} \left[ \lambda_{t+j} \left( 1 - \tau \right) \boldsymbol{p} \boldsymbol{Y}_{t+j} \left( \frac{\boldsymbol{p}}{\boldsymbol{P}_{t+j}} \right)^{-\theta} - \boldsymbol{v} \left( \boldsymbol{Y}_{t+j} \left( \frac{\boldsymbol{p}}{\boldsymbol{P}_{t+j}} \right)^{-\theta}; \boldsymbol{\xi}_{t+j} \right) \right],$$

and  $\tau$  is a constant tax rate that we assume the government levies upon sales of output. The first-order condition for this optimization problem is given by

$$E_{t-i}\Phi_t'(p_t^i) = 0. (3.15)$$

When these relations are log-linearized for i = 1,2, one finds that they imply that (up to a log-linear approximation) log  $p_t^2$  is equal to the conditional expectation at t - 2 of log  $p_t^1$ . It follows that

$$\hat{X}_{t}^{2} = E_{t-2}\hat{X}_{t}^{1} - \frac{1-\alpha}{\alpha}(\hat{\pi}_{t} - E_{t-2}\hat{\pi}_{t})$$

Equation (3.14) then implies that

$$\hat{\pi}_{t} = \frac{1}{1+\psi} \hat{X}_{t} + \frac{\psi}{1+\psi} E_{t-2} \hat{\pi}_{t}, \qquad (3.16)$$

where  $\psi \equiv (1 - \gamma)/\gamma \alpha$ , and where we now simply write  $\hat{X}_t$  for  $\hat{X}_t^1$ . Thus it suffices that we discuss the determination of  $p_t^1$ , and hence of  $\hat{X}_t$ .

Log-linearizing (3.15) for i = 1 around a zero-inflation steady state, we obtain

$$E_{t-1}\sum_{j=0}^{\infty} (\alpha\beta)^{j} \left[ (1+\omega\theta) \left( \frac{\alpha}{1-\alpha} \hat{X}_{t} - \sum_{s=1}^{j} \hat{\pi}_{t+s} \right) - (\omega+\sigma) (\hat{Y}_{t+j} - \hat{Y}_{t+j}^{s}) \right] = -\phi_{t-1}, \qquad (3.17)$$

where  $\omega$  equals  $v'' \hat{Y} / v'$  (evaluated at the steady-state level of output),

$$\begin{split} \phi_t &\equiv E_t [\hat{R}_{t+1} - \hat{\pi}_{t+2} - \sigma(\hat{Y}_{t+2} - \hat{G}_{t+2} - \hat{Y}_{t+1} + \hat{G}_{t+1})], \\ \hat{Y}_t^s &\equiv \frac{\omega}{\omega + \sigma} E_{t-1} \overline{Y}_t + \frac{\sigma}{\omega + \sigma} \hat{G}_t, \end{split}$$

and  $\overline{Y}_t$  is an exogenous disturbance (a certain function of the preference shock  $\xi_i$ ) that indicates the level of supply at each point in time required to maintain a certain constant marginal disutility of supply. Equation (3.17) says that, except for a term involving the real interest rates at *t*, the price charged by a firm that has an opportunity to change its price at t is proportional to a present discounted value of marginal costs. Marginal cost equals the price level plus a correction that depends on the level of aggregate activity, and must also be corrected for exogenous supply disturbances, represented by  $\hat{Y}_{t}^{s,21}$  A high level of the real interest rate at t relative to the expected growth rate of consumption from t to t + 1 reduces the price charged at t. Such a high real rate implies that the revenues generated from reductions in prices are actually more valuable than is implied by the current level of output. The reason for this is that the need to set consumption in advance implies that the current consumption is not a perfect indicator of the marginal utility of income. Rather, a high real interest rate relative to the rate of growth of consumption from t to t + 1implies that the marginal utility of income is relatively high at *t*, because people would, if they could, raise the current marginal utility of consumption by postponing some of their current consumption.

Quasidifferencing (3.17), along the lines of the derivation in Woodford (1996), leads to

$$\hat{X}_{t} = \beta E_{t-1} \hat{X}_{t+1} + \kappa (\hat{Y}_{t} - \hat{Y}_{t}^{s}) - \frac{\kappa}{\omega + \sigma} \phi_{t-1}, \qquad (3.18)$$

where

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)(\omega+\sigma)}{\alpha(1+\omega\theta)}$$

The *aggregate supply block* of our model then consists of equations (3.16) and (3.18).

Note that if we take conditional expectations of both (3.16) and (3.18) at date t - 2, we obtain simply

$$E_{t-2}\hat{\pi}_{t} = \kappa E_{t-2}(\hat{Y}_{t} - \hat{Y}_{t}^{s}) + \beta E_{t-2}\hat{\pi}_{t+1}.$$

21. We have defined  $\hat{Y}_{i}^{s}$  in terms of the supply disturbances affecting marginal cost at t that can be forecast on the basis of period-t - 1 information, because only these disturbances affect pricing decisions, given that all period-t prices are set on the basis of information at t - 1 or earlier.

This is identical, in conditional expectation, to the form of aggregate supply equation obtained from models such as those of Rotemberg (1982) and Calvo (1983), called by Roberts (1995) "the New Keynesian Phillips Curve." In this expectations-augmented Phillips curve, the exogenous disturbance  $Y_t^s$  plays the role of a "natural" rate of output for period *t*. Our aggregate supply relations here have the same implications as regards the relation that must exist between inflation and output fluctuations that can be forecast sufficiently far in advance; but they allow for a more flexible short-run relationship, due to the existence of additional decision lags beyond those present in the simplest discrete-time version of the Calvo model.

### 4. Estimation of Model Parameters

We now have a complete model of the determination of output, inflation, and interest rates, which consists of equations (2.1), (3.9), (3.12), (3.16) and (3.18). Apart from the coefficients of the monetary policy rule (2.1), the estimation of which we have discussed in the previous section, and the parameters specifying the stochastic processes for the real disturbances, which we consider in the following section, the model involves six parameters, the structural parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\sigma$ ,  $\theta$ , and  $\omega$ . Here we consider the estimation of these parameters so as to make the model's predictions regarding the effects of a monetary policy shock fit those estimated by the unrestricted VAR (and shown in Figure 1) as closely as possible.

Before proceeding further, it is perhaps worthwhile to comment briefly on our estimation strategy. We wish to estimate the model parameters so as to match theoretical with measured second moments of our three time series. The second moments of the data can be completely summarized by (1) the variances of the three orthogonal VAR innovations, and (2) the impulse response functions of the three variables to each of the three orthogonal innovations.<sup>22</sup> Obviously, we can match the observed variances of the three innovations by choosing appropriate variances for the exogenous disturbances in our model. We show in the next section that we can also completely match the estimated impulse responses of all three variables to the two innovations that are orthogonal to the identified monetary policy shock by appropriately specifying the stochastic processes of the "real" disturbances. This possibility is furthermore completely *independent* of the values we may have assigned

<sup>22.</sup> Note that the possibility of describing the data in this way does not depend upon having a structural interpretation of the orthogonal VAR innovations.

to the structural parameters, though it depends upon our use of OLS to estimate the parameters of the monetary policy rule. We do indeed obtain the monetary rule in this way; we use the estimates in the first column of Table 1 with the coefficients on output divided by 4 so that the estimates are identical to those we would have obtained if we had not annualized our interest rate and inflation series. Given this way of estimating the monetary policy rule, the *only* features of the data that provide any information about the structural parameters are the impulse responses to the monetary policy shock, shown in Figure 1.<sup>23</sup> It follows that it is appropriate to estimate these parameters so as to fit the estimated impulse response functions.

We now discuss the extent to which the structural parameters can be identified from the impulse responses. In fact, they are not all identified. Note first that  $\theta$  appears only in (3.18), where it matters only for the determination of  $\kappa$ . We may thus propose to estimate  $\kappa$  rather than  $\theta$ . With this change of variables, neither  $\alpha$  nor  $\gamma$  appears except in (3.16), where they matter only through their effect upon the ratio  $\psi \equiv (1 - \gamma)/\gamma \alpha$ . Thus the identified parameters are at most the five parameters  $\beta$ ,  $\kappa$ ,  $\sigma$ ,  $\omega$ , and  $\psi$ .

In fact, one can show that only four combinations of the structural parameters can be identified from the impulse responses to a monetary policy shock, given a particular feedback rule for the monetary policy. The parameters  $\beta$ ,  $\kappa$ , and  $\sigma$  are each identified, but only a single function of  $\omega$  and  $\psi$  is, rather than either of these being identified independently.<sup>24</sup> While the impulse responses do not fully identify  $\psi$ , they *can* be used to put a lower bound on it (given the possible range of variation in  $\omega$ ). Thus our introduction of a subset of producers who must determine price changes two quarters in advance (corresponding to  $\gamma < 1$ , so that  $\psi$  is strictly positive) does allow us to better fit the estimated impulse responses, even if the parameter  $\gamma$  cannot be identified.

The reason that, for a given  $\kappa$ ,  $\psi$  and  $\omega$  cannot be identified separately from the impulse response function is the following. The effect of a monetary shock at *t* on the expected evolution of prices, output, and

- 23. For clarity of presentation, these are displayed once again using annualized inflation and interest rates, even though the equations of the model involve quarterly interest and inflation rates.
- 24. The particular combination of parameters that matters for the impulse responses depends on the coefficients of the monetary policy rule. Thus both  $\omega$  and  $\psi$  matter for the model's predictions regarding the consequences of alternative monetary policy rules they even matter for the model's predictions regarding the effects of *monetary policy shocks* under alternative regimes. However, if we only observe the impulse responses under a *single* policy regime, we are unable to disentangle the two parameters without further information.

inflation starting from their position at t + 1 is independent of  $\psi$  and  $\omega$ . The reason is that for  $k \ge 1$  the expectation at t of  $\phi_{t+k}$  is zero (so that  $\omega$  does not affect the dynamic equations for periods beyond t + 1), while the expectation at t of  $\hat{\pi}_{t+k+1}$  is equal to the expectation at t of  $E_{t+k-1}\hat{\pi}_{t+k+1}$  (so that  $\psi$  does not affect the dynamic equations beyond t + 1). So the parameters  $\psi$  and  $\omega$  matter only for determining the response of inflation at t + 1 (since output at t + 1 is predetermined). But the response of just one variable at t + 1 cannot separately identify two parameters.

The complete set of parameters can thus be determined only if we introduce additional considerations. We propose to calibrate two of the unidentified parameters on the basis of independent evidence, allowing us to estimate the remaining parameters from the impulse responses. One parameter we calibrate is  $\alpha$ , which determines how frequently on average a producer changes his or her price. Given our assumed constraints upon price setting, our model implies that the mean time that a given price remains in effect is  $1/(1 - \alpha)$  quarters. We can thus choose a plausible value for this parameter based upon microeconomic evidence regarding the average length of time individual prices remain in effect.

Unfortunately, different studies of individual price adjustment report different estimates for this average length. At one extreme, Cecchetti (1986) reports that the newsstand prices of magazines stay constant for between 1.8 and 14 years. At the other, Dutta, Bergen, and Levy (1995) report that the price of orange juice at supermarkets stays constant for between 2 and 10 weeks. The findings of Carlton (1986), Blinder (1994), and Kashyap (1995) fall somewhere in between. Carlton (1986) shows that, depending on the product, prices are constant for between 4 and 13 months. Blinder (1994) reports that his interviewees kept prices constant for an average of 9 months.<sup>25</sup> Finally, Kashyap (1995) shows that products from the L. L. Bean catalogue keep their prices constant for between 11 and 30 months. This diversity of findings leads us to use Blinder's (1994) estimate both because it is relatively conservative and because it covers a broad range of industries. The result is that  $1/(1 - \alpha)$  equals 3 quarters, so that  $\alpha$  equals 0.66.

We can also calibrate  $\omega$  on the basis of data regarding labor costs. Note that this parameter gives the elasticity of the marginal disutility of producing output with respect to an increase in output. It is thus closely related to the elasticity of marginal cost with respect to output. To relate this parameter to measures of labor costs, we may further specify that

<sup>25.</sup> In his Table 4.1, he reports the fraction of his respondents for whom the frequency of price adjustment falls between various threshold levels. We had to make somewhat arbitrary assumptions to convert these frequency categories into mean lengths of constant prices.

output is produced via a production function Y = f(H), where H represents hours worked, and that the representative household has a disutility of working given by g(H), so that v(Y) is equal to  $g(f^{-1}(Y))$ . The equivalence with our earlier formulation is direct if the household uses its own hours in order to produce, as in the standard yeoman-farmer model. However, the same equilibrium conditions for output and prices are obtained if we assume that output is actually sold by firms that hire labor in a competitive labor market.<sup>26</sup> In this case, the model also determines the equilibrium wages in each segmented labor market.

This specification implies that v' = g'/f', as a result of which

$$\omega \equiv \frac{v'Y}{v'} = \left(\frac{g''H}{g'} - \frac{f''H}{f'}\right)\frac{f}{f'H}.$$
(4.1)

Wage-taking behavior by households implies that the wage in labor market *i* must satisfy  $w^i = g'(H^i)/u'(C)$ . Log-linearizing around the steady-state values of the state variables, and aggregating across labor markets, this yields

$$\frac{g''H}{g'}\frac{f}{f'H}=\epsilon_{wY}-\sigma,$$

where g''H/g' is the reciprocal of the Frisch elasticity of labor supply with respect to the real wage, and  $\epsilon_{wY}$  is the elasticity of the average real wage with respect to variation in Y, in the case of variations in output that are not associated with shifts in preferences or technology. Various instruments for such changes in economic activity are possible. Christiano, Eichenbaum, and Evans (1996) show, using a structural VAR to identify monetary policy shocks, that an increase in the federal funds rate that leads to a 0.4% reduction in output reduces real wages by about 0.1%, suggesting an elasticity of 0.25. Rotemberg and Woodford (1996) use a VAR to study the effects of oil price increases, and find that an oil shock that lowers output by about 0.25% lowers real wages by 0.1%, suggesting an elasticity of 0.4. Given this range of estimates, we set  $\epsilon_{wY} = 0.3.^{27}$ 

<sup>26.</sup> It is necessary, under this interpretation, to assume that the firms that sell goods the prices of which are chosen at different dates also hire from distinct labor markets, so that their wages need not move together despite competition in each of the segmented labor markets. This is not inconsistent with the assumption of competition, since there might be several firms at each "location" that share a local labor market and that all change their prices at the same time.

<sup>27.</sup> Note that this elasticity of the real wage with respect to variations in aggregate output agrees with that measured by Solon, Barsky, and Parker (1994), who do not instrument for technology or preference shocks.

Finally, assuming an isoelastic (Cobb–Douglas) production function, with an elasticity of output with respect to hours worked given by  $\eta$ , one finds that  $-f''f/(f')^2 = (1 - \eta)/\eta$ . Price-taking behavior on the part of firms implies furthermore that the wage in labor market *i* must satisfy  $w^i = f'(H^i)$ . It follows from this that the share of wages in the value of output in that sector should equal  $\eta/\mu^i$ , where  $\mu^i$  is the gross markup of price over marginal cost in sector *i* (Rotemberg and Woodford, 1997a). If markups are on average modest in size, then the value of  $\eta$  should not be much larger than the average labor share. We accordingly set  $\eta = 0.75$ , implying that  $-f''f/(f')^2 = 0.33$ . Substituting this, we find that  $\omega = 0.63 - \sigma$ . This restriction is imposed in our estimation of the values of these two parameters.

The parameter  $\beta$  is calibrated as well, not because it is not identifiable from the impulse responses, but because it is identifiable more directly from *first* moments of our data, and thus cannot plausibly be treated as a free parameter when trying to fit the second moments. Our model implies that  $\beta^{-1}$  should equal the gross real rate of return. Since this equals approximately 1.01 on average, we set  $\beta$  equal to 0.99.

We then choose values for our remaining three structural parameters,  $\kappa$ ,  $\sigma$ , and  $\phi$ , to ensure that the theoretical impulse responses are as close as possible to the empirical ones. We focus on the responses in the first four quarters after the shock, on the ground that we have the most confidence in the estimated impulse responses for the first year, and that it is only for these initial responses that we can reject the hypothesis that monetary policy shocks are irrelevant for our three variables. We thus choose the theoretical parameters that minimize the sum of squared differences between the theoretical and empirical impulse responses of output, inflation, and interest rates for quarters 0 through 4 following a monetary policy shock. In this optimization, we give equal weight to the three discrepancies that are depicted in Figure 1.

The parameters that minimize this criterion function are  $\kappa = 0.024$ ,  $\sigma = 0.16$ , and  $\psi = 0.88$ , which in turn imply  $\gamma = 0.63$ ,  $\omega = 0.47$ , and  $\theta = 7.88$ . With the possible exception of  $\omega$ , these parameter values are all fairly plausible.<sup>28</sup> The estimate of  $1/\sigma$  is substantially greater than the intertemporal elasticity of substitution typically found in the literature that analyzes nondurable consumption purchases. However, our estimate of  $1/\sigma$  indicates the elasticity of expected *output* growth with respect to the expected real return. Thus, we should expect a lower  $\sigma$  in our

<sup>28.</sup> The plausibility of these parameters runs counter to the suggestion of Chari, Kehoe, and McGrattan (1996) that models of this type are unable to reproduce the empirical persistence of output responses to monetary shocks without implausible parameters.

model since the purchases of both consumer durables and investment goods are likely to be more interest-sensitive.

We can use (4.1) together with our estimate of  $\omega$  to obtain an estimate of the Frisch elasticity of labor supply. Given that our assumptions imply that  $-f''f/(f')^2$  is equal to 0.33, the resulting Frisch elasticity is 9.5. This is certainly higher than the estimates obtained from microeconomic studies such as Pencavel (1986) and Card (1994). It is important to stress that this high labor supply elasticity is not necessary to match the empirical responses of the three series we have focused on. We could alternatively have imposed a Frisch elasticity of only 1.0, and still have obtained the same theoretical responses for our series, by maintaining the same  $\sigma$  and  $\kappa$ . The parameter  $\omega$  would then have to equal 1.66, while  $\gamma$  and  $\theta$  would be 0.71, and 7.61 respectively. Our reason for preferring the higher labor supply elasticity is that it rationalizes the weak observed response of real wages to a monetary disturbance. At the same time, we recognize the need for more research to reconcile the macroeconomic response of real wages with the microeconomic evidence concerning household labor supply.

Finally, the estimated elasticity  $\theta$  of the demand curve faced by a typical supplier is quite plausible. It implies a degree of market power that results in prices being set at a level 15% higher than marginal cost on average. The estimated elasticity of demand is thus neither so low as to imply implausibly large markups, nor so high as to make it implausible for firms to stagger their price adjustments.

In addition to displaying the empirical impulse responses to a monetary policy shock together with their confidence bands, Figure 1 also gives the theoretical responses corresponding to our estimated parameter values. (In each panel, the theoretical response is the solid line, while the estimated response is the middle dashed line.) As this figure indicates, the theoretical responses of output and interest rates match the estimated responses very closely. In particular, it is worth noting that our model accounts for both the magnitude and the degree of persistence of the effects on output of such a shock.

Our ability to match the output response may seem surprising given that the nominal interest rate has essentially returned to its steady-state value by the time output finally falls. However, the positive shock to interest rates lowers inflation and, as a result, raises the real interest rate for some time. This makes the output fall consistent with the IS equation (3.12). Because the increase in real interest rates is small relative to the fall in output, the value of  $\sigma$  that rationalizes these relative movements is also small.

The response of inflation is matched well for the first few quarters.

Subsequently, inflation reverts more quickly to its mean in the theoretical response, whereas the estimated response of inflation is much more protracted, if one can believe the point estimates. The problem is reminiscent of the criticism of the Taylor (1980) model of overlapping wage contracts by Fuhrer and Moore (1995a) as unable to explain inflation persistence. However, the confidence intervals indicate that the response of inflation is very poorly estimated in our sample, so that it is difficult to say that the data reject this aspect of the predictions of our model.

It is also worth remembering that the predictions of our model on this score (as on others) are themselves uncertain. In particular, they depend on the estimated coefficients of the monetary policy rule, which can hardly be estimated with great precision. Our method, which has estimated the monetary policy rule without any reference to the implications of the coefficients of this rule for the nature of the theoretical impulse responses of output and inflation to a monetary shock, makes it particularly unlikely that the theoretical impulse responses will match the unrestricted VAR estimates. A joint estimation strategy (in which the coefficients of the policy rule and the structural parameters are jointly chosen so as to match theoretical with estimated impulse responses) might well improve the fit of the impulse response of inflation.

### 5. Identification of Shock Processes

In this section, we construct time series for the three stochastic disturbances  $\epsilon_t$ ,  $\hat{G}_t$ , and  $\hat{Y}_t^s$ . We further show how the VAR can be used to infer the stochastic process that generates these variables. Finally, we show how to construct the response of our three endogenous variables to the shock processes for any given monetary policy rule. This allows us to construct counterfactual histories that, according to our model, would have taken place if the monetary authority had followed a different rule.

Note first that (2.3) can be premultiplied by  $T^{-1}$  to yield

$$\overline{Z}_t = B\overline{Z}_{t-1} + U\overline{e}_{t'}$$
(5.1)

where the matrix U consists of zeros except that its upper left 3-by-3 block consists of a lower triangular matrix with ones on the diagonal. Letting  $i_n$  denote the vector whose *n*th element is a one and whose other elements are zero, the historical time series for the monetary policy shock  $\epsilon_i$  can be derived from the relation

$$\boldsymbol{\epsilon}_{t} = (i_{1})'(\overline{Z}_{t} - B\overline{Z}_{t-1}). \tag{5.2}$$

We denote by  $\tilde{Z}_t$  the vector whose elements are the model's theoretical predictions concerning the elements of  $\overline{Z}_t$ , the vector of historical time series. (The need for separate notation will become clear when we introduce our simulation method; we will then distinguish between the *historical* law of motion of  $\overline{Z}_t$  and the *theoretical* law of motion of  $\tilde{Z}_t$ .) The structural equations of the previous section, (3.9), (3.12), (3.16), and (3.18), can be written in terms of the  $\tilde{Z}$ 's as

$$M'\tilde{Z}_{t} - N'\sum_{j=1}^{\infty} E_{t-1}\tilde{Z}_{t+j} = \hat{G}_{t+1},$$
(5.3)

$$P'E_{t-1}\tilde{Z}_t + R'E_t\tilde{Z}_{t+1} = \hat{Y}_{t+1}^s + \frac{\sigma}{\sigma + \omega}E_t(\hat{G}_{t+2} - \hat{G}_{t+1}).$$
(5.4)

Here (5.3) is obtained from (3.12) by substituting (3.9) to eliminate  $\hat{r}_{t}$  and recalling that, according to the model,  $\hat{Y}_{t+1}$  and  $\hat{\pi}_{t+1}$  are in the period-*t* information set. Similarly, (5.4) is obtained by substituting (3.16) into (3.18) to eliminate the  $\hat{X}$  variables. Note also that the time subscript is increased by one in both equations, so that the right-hand sides of both equations represent exogenous shocks in period *t*.

Assuming that the VAR correctly captures the stochastic process followed by the variables in  $Z_t$ , one can thus reconstruct the time series for  $\hat{G}_t$ and  $\hat{Y}_t^s$  by assuming that agents' expectations coincide with the VAR forecasts.<sup>29</sup> For instance, this implies that, under the policy regime that generates the historical data, agents' forecasts  $E_{t-1}\tilde{Z}_t$  are equal to  $B\overline{Z}_{t-1}$ . It then follows that we can reconstruct historical time series for  $\hat{G}_t$  and  $\hat{Y}_t^s$  using

$$s_{t} \equiv [\hat{G}_{t+1}, \hat{Y}_{t+1}^{s}]' = C\overline{Z}_{t-1} + D\overline{e}_{t}, \qquad (5.5)$$

$$C = \begin{bmatrix} M' - N'B(I - B)^{-1} \\ P' + R'B - \frac{\sigma}{\sigma + \omega} [N'B - M'(I - B)] \end{bmatrix} B, \qquad (5.6)$$

$$D = \begin{bmatrix} M' \\ R'B + \frac{\sigma}{\sigma + \omega} [M'(I - B) + N'B^{2}(I - B)^{-1}] \end{bmatrix} U.$$

These series are exogenous according to our model, so their realizations can be held fixed in simulations of counterfactual history under alternative policy regimes.

29. In inferring our shock series from the residuals of our structural equations, we extend the methodology of Parkin (1988).

Because our model involves forward-looking behavior, such simulations also require that we specify agents' beliefs regarding the stochastic processes generating the shock series. Given our proposed identification of the shock series, there is an obvious model-consistent specification of such beliefs: agents regard the vector of shocks  $s_i$ , from some period t =1 onward, as being generated by (5.5), together with the law of motion (5.1) for the stochastic process  $\overline{Z}_i$ , given a specification of the initial condition  $\overline{Z}_0$  and the distribution from which the white-noise innovations  $\overline{e}_i$  are drawn each period.

A complete simulation model would therefore consist of a specification of a monetary policy rule of the form (2.1), together with a specified distribution for the monetary policy shocks  $\epsilon_{i}$ ; the structural equations (5.3)–(5.4); and the law of motion for the real disturbances given by (5.1) and (5.5), together with the distribution of the shocks  $\overline{e}_{i}$ . Such a model would determine the evolution of  $\{\tilde{Z}_{i}, \overline{Z}_{i}, s_{i}\}$  given initial conditions  $\{\tilde{Z}_{0}, E_{0}\tilde{Z}_{1}, \overline{Z}_{0}\}$  and the white-noise shocks  $\{\epsilon_{i}, \overline{e}_{i}\}$ . The model could be used to simulate counterfactual history if we supply the historical shock series (computed above) for  $\{\epsilon_{i}, \overline{e}_{i}\}$ . In such a simulation, it is natural to specify initial conditions  $\tilde{Z}_{0} = \overline{Z}_{0}$  and  $E_{0}\tilde{Z}_{1} = B\overline{Z}_{0}$ , where  $\overline{Z}_{0}$ represents the historical data for the period immediately prior to the beginning of the simulation.<sup>30</sup> The simulation model can also be used to generate predictions about second moments of the elements of  $\tilde{Z}_{i}$ , in the stationary equilibrium associated with an arbitrary policy regime, by taking expectations over the distribution of possible realizations of the shocks.

This method of simulation would have the property that, if the assumed monetary policy rule is the estimated historical one, and one feeds in the constructed historical shock series, the predicted series  $\{\bar{Z}_i\}$ coincides exactly with the historical data series  $\{\bar{Z}_i\}$ . Thus this method of identification of the shocks would allow a complete reconstruction of the historical data as the unfolding of a stationary rational expectations equilibrium. The intuition for this result is straightforward. Use of the monetary policy rule implied by the VAR ensures that we can perfectly reconstruct the behavior of interest rates as long as we are also able to match the behavior of output and inflation. Moreover, the proposed method for constructing  $\hat{G}_t$  and  $\hat{Y}_s^t$  ensures that, when current and past

<sup>30.</sup> This specification of initial conditions makes sense if we assume that the stationary equilibrium that results in the law of motion indicated by the VAR has been in effect up through period zero, and has been in effect long enough for all of the elements of Z<sub>0</sub> to have been determined under that regime.

values of output and inflation are equal to their actual values, the model is consistent with the VAR's predictions concerning future movements in these variables. Thus, as long as we start from an initial condition in which the model and the data agree, we are able to rationalize the evolution of all three series.

It is important to stress that this ability to reconstruct the historical series does not imply that our structural model is correct. Indeed, we would be able to reconstruct the historical series of output, inflation, and interest rates in the way described even for arbitrary values of the structural parameters. The model does, however, have testable predictions (which is what allows us to identify several of the structural parameters from moments of the data). Specifically, the identified monetary policy shock process  $\epsilon_i$  should be orthogonal to the real disturbances  $\hat{G}_i$  and  $\hat{Y}_i^s$ at all leads and lags. This restriction is not imposed in the above method of identification of the historical shock series. (In fact, the constructed historical shocks have the property that  $\bar{e}_{1t} = \epsilon_t$  in every period.<sup>31</sup>) One can avoid this problem in the case of the stochastic simulation model, by assuming that  $\overline{e}_{1t}$  and  $\epsilon_t$  are independent random variables, though drawn from identical distributions. But the resulting model of the datagenerating process is still subject to an internal inconsistency, in that if the model is true, it should not be possible to identify the four independent shocks  $\epsilon_{i}$ ,  $\overline{e}_{1i}$ ,  $\overline{e}_{2i}$ , and  $\overline{e}_{3i}$  from a three-variable VAR of the kind that we use.

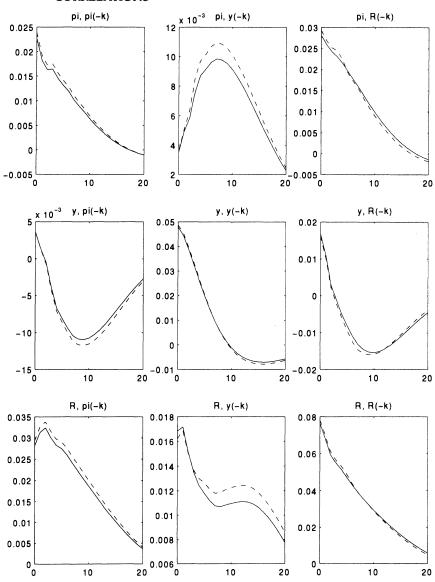
This inconsistency can be eliminated by modifying the above procedure for identification of the shocks. Specifically, we assume that the real disturbances  $s_t$  are generated by the laws of motion (5.1) and (5.5), but we assume that in these equations  $\overline{e}_{1t} = 0$  at all times, whereas  $\overline{e}_{2t}$ and  $\overline{e}_{3t}$  are again assumed to be white-noise random terms, independent of each other and of the monetary shocks, and drawn each period from distributions identical to those of the corresponding VAR residuals. This alternative stochastic simulation model has only three independent exogenous disturbances each period (two "real" shocks and the monetary policy shock). Under the assumption that such a model is correct, analysis of the VAR according to our method should (at least asymptotically) recover exactly the true stochastic processes generating the various shocks.

Correspondingly, for purposes of counterfactual historical simula-

<sup>31.</sup> This implies that constructed real and monetary shock series are not orthogonal, unless the matrices in (5.1) and (5.5) happen to imply that the shocks  $\overline{e}_{1i}$  have no effect upon  $s_i$ . This is in fact not the case for our estimated model, for otherwise the theoretical impulse responses to a monetary shock would perfectly match those estimated using the VAR.

tions, we do not construct the historical shock series by substituting the historical data  $\overline{Z}_{t-1}$  and VAR residuals  $\overline{e}_t$  into (5.5). Instead, we use the historical VAR residuals  $\overline{e}_{2t}$  and  $\overline{e}_{3t}$ , but set  $\overline{e}_{1t} = 0$  in all periods, and then simulate (5.1), starting from an initial  $\overline{Z}_0$  given by the historical data, to generate a modified sequence  $\{\overline{Z}_t\}$ . These modified  $\overline{Z}_{t-1}$  and  $\overline{e}_t$ are then substituted into (5.5) to construct the historical sequence of real disturbances. The method thus amounts to using not the residuals of the structural equations (5.3)-(5.4), but rather the component of those residuals that is orthogonal to the identified monetary policy shock and to all its lags. With this modification, the simulated paths using the estimated monetary policy rule no longer need to equal the actual paths. The extent to which the simulated data track the actual data then becomes a test of the accuracy of our structural model. As we show in the next section, our estimated model (with historical shock series constructed in the way just described) does quite a good job of accounting for the variations in real GDP, inflation, and the Federal funds rate since 1980, despite the specification error that is indicated by its failure to perfectly match the estimated impulse responses in Figure 1.

Another way of assessing the degree of correspondence between our model and the U.S. data is to compare the empirical auto- and crosscorrelation functions for our three series with the corresponding predictions of the stochastic simulation model, with the stochastic processes for the shocks specified as above. This comparison is shown in Figure 2, where in each of the nine panels, the solid line indicates the theoretical cross-correlation function, and the dashed line the cross-correlation function implied by the unrestricted VAR characterization of the U.S. data. It is clear that our model accounts for the second moments of the data to essentially the same degree as does the unrestricted VAR. Among other things, our model is able to perfectly reproduce the degree of persistence of inflation despite the criticism of a Taylor-style model of staggered price setting on this score by Fuhrer and Moore (1995a). It is also able to match the negative correlation of output with lagged nominal interest rates that King and Watson (1996) find cannot be explained by an optimizing model with Calvo-style staggered price setting similar to our own. Fuhrer (1997b) also draws attention to this correlation and suggests that a "backwards looking" IS curve is needed to explain it. We suspect that some of the difficulties faced by these previous authors in reconciling models of optimizing consumers and of staggered price setting with these features of the data may relate to the imposition of a priori restrictions upon the exogenous shock processes, for which there is no theoretical justification.



#### Figure 2 EMPIRICAL AND THEORETICAL AUTO- AND CROSS-CORRELATIONS

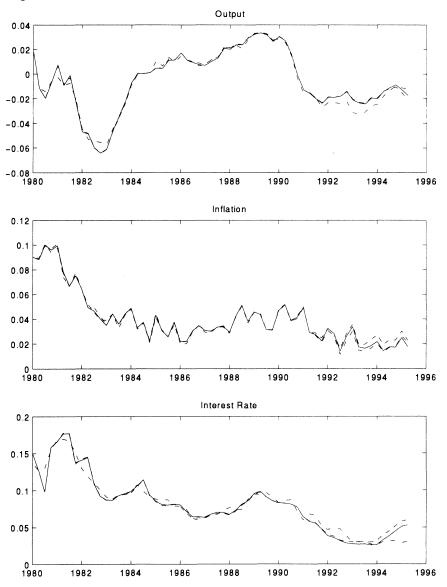
## 6. Simulation of Alternative Monetary Policy Rules

In this section we briefly illustrate how the simulation model built up in the previous two sections can be used to predict the consequences of alternative possible monetary policy rules of the form (2.1). We first display in Figure 3 the consequences of a feedback rule with the coefficients of the estimated historical policy rule. In each panel of this figure, the dashed line represents the actual data for the series in question, the solid line represents the simulation of our model assuming the historical sequence of monetary policy shocks as well as the historical series for the real shocks, and the dash-dot line represents a simulation in which the historical feedback rule for the funds rate is followed, but with the monetary policy shocks  $\epsilon_t$  set equal to zero.

One observes, first, that the first two plots track one another quite closely in each panel. Thus our model does quite well at accounting for the historical paths of output, inflation, and the funds rate, despite the fact that the theoretical and estimated impulse responses to a monetary shock do not perfectly coincide. The only very noticeable failure of our simulation model is in tracking the level of inflation from 1993 onward. The dash–dot line differs somewhat more from the solid line; this indicates the consequences, according to our model, of the random disturbances to monetary policy. Monetary policy shocks clearly have played some role; in particular, our simulations indicate that unexpectedly tight monetary policy in early 1982 deepened the 1982 recession, and that unexpectedly loose policy stimulated real activity in the period 1992–1993.

On the other hand, the simulations imply that relatively little of the variability in output or inflation in this period can be attributed to the monetary policy shocks. Table 2 shows this in a different way by reporting the predicted stationary variances of interest rates, output, and inflation under a variety of alternative policy rules. The first two rows of the table give these statistics for two regimes corresponding to the estimated feedback rule with and without the stochastic term. Comparison of the numbers in these two rows shows that, in the simulation of the historical policy regime, the monetary policy shocks are responsible, over the long run, for only 5.0% of the variance of deviations of real output from trend, and (perhaps more surprisingly) for only 1.3% of the variance of inflation.

But these results do *not* imply that monetary policy is unimportant. Nor do they necessarily absolve the Fed from any blame for the instability of output or inflation. What they mean is that it is the *systematic* part of recent monetary policy that has been of significance for recent eco-



#### Figure 3 ACTUAL AND SIMULATED PATHS

Policy	Var R	Var Y	Var π	Var (π-Eπ)	Var E(Y – Y <sup>s</sup> )	Loss from variability (L)
Historical: with shocks	7.64	4.79	2.28	0.66	12.14	3.43
without shocks	6.73	4.55	2.25	0.65	11.89	3.39
$\theta_{\pi} = 1.5, \ \theta_{y} = 0.5$	17.14	3.87	7.34	0.81	13.86	8.72
$\theta_{\pi}^{"}=1, \ \theta_{\mu}=5$	22.95	0.51	6.45	0.91	17.84	8.10
$\theta_{\pi} = 1, \ \theta_{y} = 5$ $\theta_{\pi} = 10, \ \theta_{y} = 0$	30.11	12.61	0.30	0.25	4.58	0.74
Minimum-L	732.89	18.77	0	0	0	0
Constrained-optimal	1.93	11.30	0.39	0.20	7.57	0.93

 
 Table 2
 VARIANCES OF OUTPUT, INFLATION, AND INTEREST RATES UNDER DIFFERENT MONETARY RULES

*Note:* For these computations, the interest rate and inflation are measured in annualized percentage points, and output is measured as a percentage deviation from trend.

nomic performance, not the stochastic variation in Fed policy (which, according to our estimates, has been minimal).

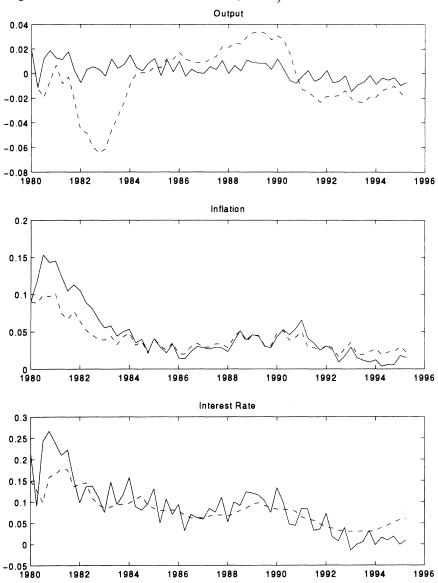
One can gain some understanding of the effect of alternative systematic monetary policies by comparing the predicted consequences of simple feedback rules of the kind discussed by Taylor (1993b),

 $r_t = \theta_{\pi} \pi_t + \theta_{y} y_t,$ 

under alternative values for the coefficients  $\theta_{\pi}$  and  $\theta_{y}$ . Rows 3–5 of Table 2 report predicted moments of the data for three possible choices of these coefficients. In row 3, we consider a "Taylor rule" with  $\theta_{\pi} = 1.5$ ,  $\theta_{y} = 0.5$ — values which are close to those used by Taylor to characterize current policy (his exact coefficients are in footnote 2).<sup>32</sup> According to our model, adherence to this rule would make a difference, since both inflation and interest rates would be significantly more variable.

Even sharper contrasts between policy rules are possible if we vary the coefficients of the "Taylor rule." Row 4 considers a policy in which instead  $\theta_{\pi} = 1$  and  $\theta_{y} = 5$ . The increased response to deviations of output from trend is predicted to reduce the variance of output fluctuations to about a tenth of its value under the historical policy regime. This stabilization of output, however, is accompanied by increased volatility of inflation and short-term nominal interest rates. (A counterfactual historical simulation assuming this policy rule is shown in Figure 4.) For purposes of contrast, row 5 of the table considers a "Taylor rule" in which  $\theta_{\pi} = 10$ 

32. The rule that we simulate here is not exactly Taylor's, since he assumes that the funds rate responds to the rate of inflation in the current and previous three quarters, while our rule assumes that it responds to the rate of inflation in the current quarter only.



### Figure 4 SIMULATED "TAYLOR RULE" ( $\theta_{\pi} = 1, \theta_{y} = 5$ )

and  $\theta_y = 0$ . The increased response to fluctuations in inflation is predicted to reduce the variance of inflation to about an eighth of its value under the historical policy. Inflation stabilization, however, is accompanied by increased volatility of both output and interest rates.

These comparisons show that according to our model, monetary policy matters a great deal for the behavior of both output and inflation, since either inflation or output can be stabilized to a much greater extent than it has been historically. This raises the obvious question of which policy rule results in more *desirable* patterns of fluctuations. We take this up in the next section.

### 7. The Welfare Loss from Price-Level Instability

We wish to consider the consequences of alternative monetary policy rules for the value achieved in equilibrium by the lifetime utility (3.1) of the representative household. The precise comparison that we propose to make is the following. Associated with any *stationary* rationalexpectations equilibrium of the kind discussed above (resulting from a time-invariant feedback rule for monetary policy) is an *unconditional* expected value of (3.1), averaging over all the possible histories of shocks that may have occurred prior to date zero. We propose to compare stationary equilibria in terms of the value of this unconditional expected utility. In this way, we take a "long-run" perspective in evaluating alternative policy rules; we do not consider the advantages that a particular rule may have that result from the nature of the particular fortuitous initial conditions that may exist at the time that one contemplates commitment to such a rule.

This objective is easily seen to be equivalent to maximization of the simpler objective function

$$W = E\left( u(C_t) - \int_0^1 v(y_t(z)) \, dz \right), \qquad (7.1)$$

where *E* refers to the unconditional expectation. This objective averages the disutility of working across households at a point in time, because, from our "long-run" perspective, any given household is equally likely to be in the situation of any one of the households (which differ, after all, only in the sequence of times at which they have been able to change the prices of the goods that they sell). By including the integral over *z* in (7.1), we do not need to interpret the expectation operator *E* as referring to an average over possible histories of opportunities for an individual seller to change its prices, but only an average over possible histories of the aggregate shocks (*i.e.*, the disturbances to preferences and technology).

### 334 · ROTEMBERG & WOODFORD

We furthermore simplify our analysis by taking a second-order Taylor series approximation to our objective (7.1). (More details of these calculations can be found in Rotemberg and Woodford, 1997b.) This has the advantage of allowing us to derive an approximate loss function that can be evaluated using *only* the log-linear approximate characterization of the equilibrium, obtained by solving the equations derived in Section 3. Another advantage is that we obtain a loss function that can be written as a weighted sum of contributions from the variances of various endogenous series, which allows direct comparison of our conclusions with the *ad hoc* loss functions typically assumed in the literature.

At the cost of not being able to evaluate the effect of monetary policy on the long-run level of output, we suppose that changes in monetary policy are accompanied by changes in the constant income-tax rate  $\tau$  so that this tax is optimal in each case. This ensures that, roughly speaking, the *average* level of output is optimal and independent of monetary policy. Our idea here is to separate the issue of the welfare losses associated with *fluctuations* in output from those due to a suboptimal *average* level of output, due (for example) to the presence of monopolistic competition or distorting taxes, and to make monetary policy responsible solely for the minimization of the former losses, assuming that other policy instruments will be used to ensure the desired average level of output. This assignment of tasks to policy instruments makes sense if, as a practical matter, the tax code can affect the long-run level of output but cannot be adjusted rapidly enough to be used to ensure an optimal response to stochastic disturbances.

In Rotemberg and Woodford (1997b), we show that a second-order approximation to W takes the form

$$W = -\frac{1}{2} u_c Y \left[ (\sigma + \omega) \operatorname{var} \left[ E_{t-2} (\hat{Y}_t - \hat{Y}_t^s) \right] + (\theta^{-1} + \omega) E \operatorname{var}_z \log y_t(z) \right] (7.2)$$

plus terms that are of third or higher order in the amplitude of the shocks, and terms that are unaffected by monetary policy. (Such terms are similarly neglected in the expressions that follow.) Thus our measure of deadweight loss depends upon the variability of aggregate output around the "natural rate," but also upon the dispersion of output levels across producers of different goods. The second term, in turn, depends solely upon the degree of price dispersion, since the demand curve (3.13) implies that

$$\operatorname{var}_{z} \log y_{t}(z) = \theta^{2} \operatorname{var}_{z} \log p_{t}(z).$$
(7.3)

Finally, our-price setting equations imply that

$$E\left[\operatorname{var}_{z} \log p_{t}(z)\right] = \frac{\alpha}{(1-\alpha)^{2}} \left[ \operatorname{var}\left(E_{t-2}\hat{\pi}_{t}\right) + (1+\psi) \\ \operatorname{var}\left(\hat{\pi}_{t} - E_{t-2}\hat{\pi}_{t}\right) + (E\hat{\pi}_{t})^{2} \right].$$
(7.4)

Thus the degree of price dispersion that exists on average increases with average inflation, with the variability of the rate of inflation forecasted two quarters in advance, and with the variability of unexpected inflation. Substituting (7.3) and (7.4) into (7.2), one obtains

$$W = -\frac{1}{2}u_{c}Y\theta(1+\theta\omega)\alpha(1-\alpha)^{2} \left[\frac{(1-\alpha)\kappa}{(1-\alpha\beta)\theta} \operatorname{var} E_{t-2}(\hat{Y}_{t}-\hat{Y}_{t}^{s}) + \operatorname{var} (E_{t-2}\hat{\pi}_{t}) + (1+\psi)\operatorname{var} (\hat{\pi}_{t}-E_{t-2}\hat{\pi}_{t}) + (E\hat{\pi}_{t})^{2}\right].$$
(7.5)

This is the welfare criterion in terms of which we shall compare alternative policies. Note that only two structural parameters, namely  $\psi$  and  $(1-\alpha)\kappa/[(1-\alpha\beta)\theta]$ , matter for the *relative* ranking of alternative policies (as opposed to a computation of the absolute size of the deadweight loss).

One can furthermore show that the factor in square brackets in (7.5) equals  $L + \pi^{*2}$ , where  $\pi^*$  is the average inflation rate and

$$L = \frac{(1 - \alpha)}{\kappa (1 - \alpha \beta) \theta} \text{var } E_{t-2}(\hat{X}_t - \beta \hat{X}_{t+1}) + \text{var } E_{t-2} \hat{X}_t$$
$$+ \frac{1}{1 + \psi} \text{var } (\hat{X}_t - E_{t-2} \hat{X}_t).$$
(7.6)

*L* is the welfare loss from output and inflation variability, denominated in the units of the variance of inflation. This loss obviously reaches an absolute minimum when  $\hat{X}_t$  is a constant, so that inflation is constant. On the other hand, the remaining loss term in (7.5) is minimized when average inflation is zero. Thus the welfare measure *W* achieves its theoretical maximum value of zero when inflation is constant and equal to zero, which is to say, when prices are constant over time.<sup>33</sup>

33. Our finding that price stability is optimal in our model is closely related to King and Wolman's (1996) argument that, in their closely related model, price stability leads output to behave as it would if prices were flexible. Note that this conclusion depends upon a number of special features of the model developed here, in particular, upon the assumption that the existence of nominal price rigidities is the only distortion that prevents equilibria from necessarily being optimal.

We momentarily ignore the choice of the average inflation rate, and show that there is a policy that ensures that  $\hat{X}_t$  is constant so that *L* is zero.<sup>34</sup> The only effects of a nonzero  $\pi^*$  on our equilibrium are to increase the interest rate in each period by  $\pi^*$  while  $\hat{X}$  is increased by a constant as well. Substituting constants for both  $\hat{X}_t$  and  $\hat{\pi}_t$  into the structural equations (3.9), (3.12), and (3.18), we see that an equilibrium with steady inflation involves

$$\hat{Y}_t = E_{t-2}\hat{Y}_t^s + \hat{G}_t - E_{t-2}\hat{G}_t$$
(7.7)

and

$$E_{t-1}\hat{K}_{t} = \omega[(\hat{G}_{t} - \hat{Y}_{t}^{s}) - E_{t-2}(\hat{G}_{t} - \hat{Y}_{t}^{s})] + \sigma E_{t-1}[(\hat{G}_{t} - \hat{Y}_{t}^{s}) - (\hat{G}_{t+1} + \hat{Y}_{t+1}^{s})], \qquad (7.8)$$

where we have neglected the constants. It is then easily verified that all of the structural equations are satisfied if these two are, and  $\hat{X}_i$  and  $\hat{\pi}_i$  are constant. This establishes the possibility, in principle, of complete inflation stabilization.

Note that (7.8) only determines  $E_{t-1}\hat{R}_t$ . This is the only restriction upon the path of short-term nominal interest rates implied by price stability. To avoid adding unnecessary noise to interest rates, the central bank should also ensure that the actual value of  $R_t$  (as opposed to only its expectation at t - 1) is given by the right-hand side of (7.8). This has the additional advantage that it economizes on the information requirements of the central bank, since it makes interest rates at t depend only on the periodt - 1 information set.

Using our estimated processes for the real shocks, we now consider the fluctuations in output and interest rates that would obtain under such a first-best policy. Such an equilibrium would have required output to vary much more than it did under historical policy; Table 2 indicates that this minimum-*L* policy would have led to a variance of output nearly four times as large as the variance implied by the historical policy. This is mainly due to the highly volatile character of our inferred series for the "supply" disturbances  $\hat{Y}_t^s$ . According to our model, the reason output movements have been so much smaller under the actual policy is

<sup>34.</sup> We do not analyze in this paper the form of the interest-rate feedback rule that achieves this stationary equilibrium. It is unlikely that only one feedback rule supports this optimum; alternative rules may support the same stationary equilibrium but differ in the behavior specified for the central bank off the equilibrium path. However, it is worth noting that in general the rule must be more complicated than any member of the family (2.1). The implementation issue is taken up in the context of a simplified version of our model in Bernanke and Woodford (1997).

that the actual policy consistently "leans against the wind," so that the interest rate is increased whenever output rises. As has been pointed out by numerous authors [see, *e.g.*, Rotemberg (1983), Ireland (1997), and Aiyagari and Braun (1996)], such countercyclical policy is not appropriate in response to supply shocks of the sort represented by  $\hat{Y}_t^s$ . With sticky prices an increase in  $\hat{Y}_t^s$ , which reduces marginal costs, tends to lower prices and thus raise output. However, if interest rates are raised in response to the output increase, prices fall by less than marginal cost, so this fall in prices is not sufficient for output to increase by the amount that  $\hat{Y}_t^s$  increases. A policy of price stability requires that the monetary authority accommodate the increase in output required by the increase in  $\hat{Y}_t^s$ . As a result, output becomes more variable if  $\hat{Y}_t^s$  is variable.

The path of interest rates that would have achieved complete inflation stabilization involves very large swings in interest rates and is remarkably choppy. In particular, as Table 2 indicates, the variance of the funds rate along this path is 733 (a standard deviation of 27 percentage points), while the variance of the funds rate under the historical policy is only 7.6 (a standard deviation only a tenth that large). One consequence of this is that such a policy is not consistent with a low average interest rate (and inflation rate) unless the nominal interest rate can be negative. Thus, as suggested by Summers (1991), the zero nominal-interest-rate floor poses an impediment to stabilization policy with a low average level of inflation.<sup>35</sup> Given our parameters, complete stabilization of inflation would require an average inflation rate near 50% per year to keep the federal funds rate positive in all quarters over our sample period. But that would imply other sorts of welfare losses, both those indicated by  $(E\hat{\pi}_t)^2$  in (7.5) that result from the increased dispersion of prices across suppliers, as well as the more conventional "shoe-leather costs" (from which our model abstracts).

Thus our analysis leads to the conclusion that completely *stable* inflation is inconsistent with a low *average* inflation rate. This occurs in our model because we find there to be fairly large fluctuations in  $\hat{Y}_{t}^{s}$ . Insulating prices from the effects of these supply shocks requires very large swings in the interest rate if, as seems plausible and as is implied by our parameters, relatively large movements in interest rates are needed to change the prices that firms choose to set. Because of the costs of having

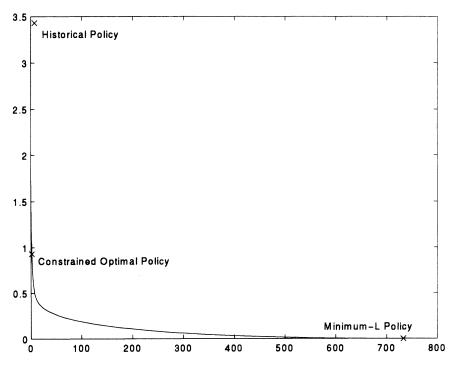
<sup>35.</sup> The discussion of the model in Section 3 has ignored this floor, because that model abstracts altogether from the fact that money balances are held. If, however, we introduce liquidity services from non-interest-earning money into (3.1), we obtain an additional equilibrium condition representing the demand for money. This equilibrium condition will be inconsistent with an equilibrium nominal interest rate that is negative in any period. Note that the introduction of a demand for money need have no effect other than imposing  $R_t \ge 1$  for all t upon the system of equations derived in Section 3.

to maintain a high average rate of inflation, it is likely to be desirable to accept some degree of inflation variability for the sake of reducing the size of the swings in nominal interest rates required in order to stabilize inflation. It is thus of interest to consider the costs, in terms of a higher value of L, that must be accepted in order to reduce the variability of the Federal funds rate. We thus consider the nature of the equilibrium that achieves the minimum possible value of (7.6) subject to a constraint of the form

$$\operatorname{var} \hat{R}_t \le v_{\mathsf{R}}.\tag{7.9}$$

Figure 5 shows the trade-off between the constraint parameter  $v_R$  and the minimum attainable level of the welfare loss *L* from inflation variability, which, as noted above, is expressed in units of the variance of inflation. This figure, which has the variance of the federal funds rate on the horizontal axis, shows that the minimum attainable loss *L* is a convex function of  $v_R$ . In particular, the deadweight loss from inflation variation

## Figure 5 THE TRADEOFF BETWEEN INTEREST-RATE VOLATILITY AND WELFARE LOSS L



hardly rises as the variance of nominal interest rates is reduced from its optimal value of 733 to a value less than one-seventh that size (corresponding to a standard deviation on the order of 10 percentage points). Further reductions in the volatility of interest rates have larger effects on welfare, but even reducing the variance of the funds rate to something near its recent level requires an increase in *L* that is only a fraction of the deadweight loss associated with current policy, according to our estimates.

The question then becomes which point in Figure 5 is optimal once one recognizes that points with more volatile interest rates require higher average inflation rates. Here we pursue a crude approach to this problem by imposing the constraint that the average federal funds rate must be no smaller than k times the standard deviation of the funds rate, for some k > 0.36 Since the average (or steady-state) funds rate is given by  $r^* = \rho + \pi^*$ , where  $\rho$  is the steady-state real rate of return (determined by the rate of time preference), this constraint can be written in the form

$$\rho + \pi^* \ge k\sigma(\hat{R}). \tag{7.10}$$

This constraint indicates how a higher degree of variability of the funds rate requires a higher target rate of inflation  $\pi^*$ . We can then ask which, among the log-linear approximate equilibria characterized earlier that satisfy (7.10) in addition to the other equilibrium conditions, achieves the highest value of (7.5).

Under our log-linear approximate characterization of equilibrium, the value of L achieved in any stationary equilibrium is independent of the target inflation rate  $\pi^*$  around which inflation fluctuates. (Our approximate equilibrium conditions are derived by log-linearizing around a steady state with zero inflation, but continue to represent a valid approximation as long as  $\pi^*$  is small enough.) Thus we can consider, on the one hand, the lowest value of L consistent with a given value of  $\sigma(\hat{R})$ , given the structural equations used to derive Figure 5, and on the other hand, the lowest value of  $\pi^{*2}$  consistent with the value of  $\sigma(\hat{R})$ , given (7.10). The sum of these two terms, expressed as functions of  $\sigma(\hat{R})$ , is minimized by the unique value of  $\sigma(\hat{R})$  for which

$$\{-2\lambda\sigma(\hat{R})\} + \{2k[k\sigma(\hat{R}) - \rho]\} = 0,$$

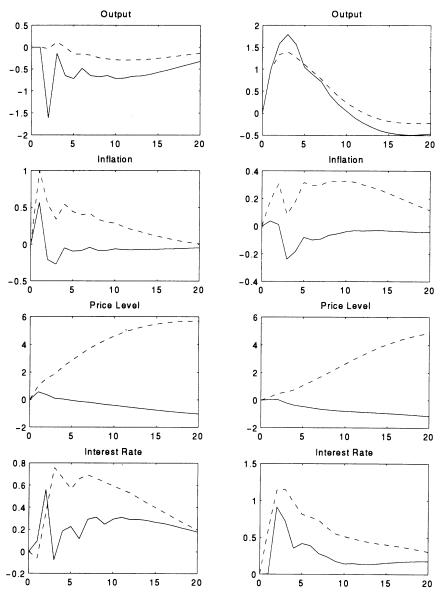
<sup>36.</sup> In the event that the exogenous shocks have bounded supports, this is a *sufficient* condition for nonnegativity of the funds rate at all times, and *necessary* within the class of equilibria in which the state variables all are *linear* functions of the shocks. This makes a natural case to consider, given our use of linearization methods here to characterize equilibria. In general, however, the optimal equilibrium subject to the constraint that the funds rate must always be nonnegative is unlikely to belong to the class of linear solutions.

where the two terms in curly braces represent, respectively, the derivatives of *L* and  $\pi^{*2}$  with respect to  $\sigma(\hat{R})$ , and where  $-\lambda$  (with  $\lambda > 0$ ) is the slope of the locus graphed in Figure 5. We assume  $\rho = 3\%$ /year, as indicated by the long-run values  $r^*$  and  $\pi^*$  resulting from historical policy (according to our VAR) and k = 2.26, which is the largest value such that the historical equilibrium (according to our VAR) would satisfy (7.10). Then this condition is satisfied at the point in Figure 5 where  $\lambda =$ 0.22 and var  $\hat{R} = 1.93$ , which requires a target inflation rate of at least  $\pi^*$ = 0.14%/year. This is a positive rate of inflation, as conjectured by Summers, but a trivially small one. Furthermore, this calculation neglects other costs of inflation, such as the costs of economizing on money balances that are emphasized in much of the literature. Taking account of such costs would only make the optimal average inflation rate even lower, as would the choice of a lower value for k.

Hence, because there exist only small gains in terms of reduction of L from raising the variability of interest rates beyond what is consistent with zero average inflation, the trade-off indicated in Figure 5 is favorable towards keeping inflation low. If there are additional, independent reasons for the Fed to prefer not to have a highly variable funds rate (as discussed, for example, by Goodfriend, 1991), then these would justify choice of a point even further to the left in Figure 5, and hence of an even lower target rate of inflation.

It is of particular interest to compare the constrained-optimal policy that minimizes L while keeping the average inflation rate at 0.14% with the actual policy of the Fed. This constrained-optimal policy has the immediate advantages that its average level of inflation and its variance of interest rates are considerably smaller than under the historical policy. Moreover, as the figure and Table 2 indicate, the loss from variability (L) under this policy is only about a fourth as large as under the estimated historical policy. The variance of output doubles relative to the historical policy, but according to the model this is desirable as well, since output is kept closer to the "natural rate."

To illustrate how the constrained-optimal policy would differ from actual policy, Figure 6 plots the impulse responses of output, inflation, and interest rates to the two real shocks, under the constrained-optimal policy and under our estimate of historical policy. The two shocks are orthogonalized as follows. One period-*t* shock (the one considered in the second column) is defined as the innovation in period-*t* + 1 autonomous spending—*i.e.*, the part of  $G_{t+1}$  that could not be forecast on the basis of output, inflation, and interest rates through period *t*. The other period-*t* shock (the one considered in the first column) is defined as that component of the innovation in the "natural rate" of output  $\hat{Y}_t^s$  that is orthogonal



### Figure 6 IMPULSE RESPONSES UNDER ACTUAL AND CONSTRAINED-OPTIMAL POLICY

to the innovation in autonomous spending. The specific shock represented in the first column is a *supply shock* that increases equilibrium inflation  $\hat{\pi}_{t+1}$  (under the historical policy rule) by 1% more than one would have expected given the level of autonomous spending; the shock represented in the second column is an *autonomous spending shock* that raises real spending  $\hat{Y}_{t+1}$  by 1%.

In each panel of the figure, the dashed line indicates the impulse response to the shock under the historical policy, while the solid line indicates the response that would occur under the constrained-optimal policy. Under historical policy, both types of shocks are inflationary, leading to increases in inflation that persist for many quarters, and so to a large eventual cumulative increase in the price level. Under the constrained-optimal policy, prices also rise slightly on impact. But what is striking about the constrained-optimal policy is that it ultimately leads prices to fall in response to these inflationary shocks. Thus inflationary shocks are accompanied by expected deflation in subsequent quarters. The result is that the constrained-optimal policy not only stabilizes *inflation* to a greater extent than under current policy, it also stabilizes the *price level* to a considerable extent.

Another striking difference between historical and constrained-optimal policy has to do with the effect of supply shocks on real activity. Under historical policy, real output is largely insulated from the effects of supply shocks, which instead result in persistent fluctuations in inflation. Under the constrained-optimal policy, instead, an adverse supply shock results in relatively large increase in interest rates and a sharp transitory contraction of real activity. It is for this reason that (as Table 2 indicates) the constrained-optimal policy involves much greater output variability than has occurred under the historical policy.

By contrast, output movements in response to the autonomous spending shock are quite similar under historical and constrained-optimal policy. Historical policy has allowed output to respond to these shocks, but according to our model it is desirable for this to occur. The nominal interest rate rises less under the constrained-optimal policy, and returns more quickly to normal. However, real interest rates are not significantly lower during the transition back to the steady state, because this policy induces deflation as discussed above.

Table 2 also provides a nice contrast between this constrained-optimal policy and the "Taylor rule" whose coefficients are  $\theta_{\pi} = 10$  and  $\theta_{y} = 0$ . These two policies induce about the same variance of inflation and output while also having similar losses from variability *L*. However, the simple "Taylor rule" achieves this by having interest rates react aggressively to inflation, and this leads interest rates to be very volatile. Our

constrained-optimal rule, by contrast, allows interest rates to be less variable by tailoring the dynamic response to shocks more appropriately.

### 8. Conclusions

This paper has provided a method for computing optimal monetary policy in the context of an optimizing model that fits the U.S. data nearly as well as an unrestricted vector autoregression. The two basic ingredients of this method are a vector autoregression of the variables of interest and an optimizing model that predicts the evolution of these variables. As long as the model can match the estimated impulse responses of the variables to a monetary shock, the method can be applied easily, because it is straightforward to fit the response of the model to the other shocks. Thus, the method can accommodate much richer vector autoregressions than the one we have considered, as well as more elaborate models. In this paper we have worked with a minimal model, both to show how this method can be applied and to show that even very simple optimizing models can fit the data rather well. Even so, it would be desirable to have a model that deals explicitly with investment and the resulting capital accumulation as well as with labor-market variables.

In addition to providing a method of analysis, we have also been able to reach interesting conclusions regarding optimal monetary policy. In particular, the complete stabilization of inflation appears to require fairly large swings in interest rates, which, given a zero interest-rate floor, require a high average interest rate and thus a high inflation rate. Thus, the existence of this floor limits the degree to which it is desirable to stabilize inflation. On the other hand, it appears to be possible, at least in principle, both to lower the average inflation rate and to stabilize inflation more than has been done historically in the U.S. While this requires that inflationary shocks still be allowed to increase inflation transitorily, such shocks must be followed by deflation shortly thereafter. The result is that neither surges in autonomous spending nor adverse supply shocks lead to long-run increases in prices.

Our specific conclusions as to the desirable responses of output, inflation, and interest rates to stochastic disturbances may well be sensitive to the particular optimizing model we have considered and, specifically, to the absence of other types of stochastic disturbances, such as timevarying labor-market distortions and changes over time in firms' desired markups of price over marginal cost. These are issues that only further investigation of other, more elaborate optimizing models can settle. Our main hope with this paper is precisely to shift the debate over optimal monetary policy so that it will involve different optimizing models, all of which fit the data reasonably well, instead of involving equations that fit well by construction but that have only a tenuous connection to explicit behavioral hypotheses at the microeconomic level.

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### Comment

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### 1. Introduction

This paper espouses several admirable goals. First, it aims to build a simple rational-expectations macroeconometric model from an optimizing framework. At least two advantages attach to this approach. The first is that to the extent that the model is built upon stable micro parameters, it has some hope of addressing the Lucas critique (1976). Of course, the model is not certain to be invulnerable to the critique by virtue of its

micro foundations. But it has a hope of being robust to the critique. Second, the welfare function for policy analysis arises naturally from the agents' objective function. Many pervious studies have used *ad hoc* objective functions, such as weighted combinations of the variances of output around potential and inflation around its target. It is not obvious that such metrics appropriately reflect the loss of welfare associated with employment and price fluctuations.

A second goal of the paper is to estimate the model, seriously confronting the structure with the important dynamic properties of output, inflation, and nominal interest rates as summarized in a vector autoregression. This impulse is essential if one's ultimate goal is to use the model to deliver advice to monetary policymakers. It is too easy to write an elegant theoretical model, and too difficult to write a model that also replicates key dynamic elements found in the data. Policymakers will rightly be leery of the former, and at least somewhat more comfortable with the latter.

The paper then uses the model to perform counterfactual policy exercises (how would a different policy have altered the outcome?) and to compute an "optimal" monetary policy. Both of these model exercises are of inherent interest to practical policymakers and to researchers.

In my discussion of the paper, I want to focus on a few fundamental issues.

- 1. Is the Rotemberg–Woodford model still subject to the Lucas critique?
- 2. How efficiently are the behavioral parameters estimated, and how generalizable is their estimation technique?
- 3. How does monetary policy work in the model?
- 4. How important are the innovations (the disturbance terms) to the empirical success of the model?

To anticipate, I will suggest that the model is still subject to the Lucas critique in two important ways, that the behavioral parameters could be estimated with a more efficient use of information, that monetary policy works in a nonstandard way in the model, and that the dynamics in the disturbance terms are crucial to the empirical performance of the model.

### 2. The Lucas Critique

The paper claims that its modeling strategy adequately addresses the Lucas critique. I think it has only partially done so. The Lucas critique is, after all, a theoretical assertion that ultimately rests on empirical testing. Much of the critique is deflected when we include rational expectations in our models, so that agents alter their view of the future when policymakers and other alter the systematic component of their behavior.

Some more of the critique *may* be deflected—although we cannot know this *a priori*—when we use optimizing foundations for our models. The logic is, of course, that an individuals' taste parameters may be assumed to be fairly stable in response to the behavior of policymakers (or others), whereas the reduced-form consumption rule that arises from the interaction of taste parameters, budget constraints, and expectations of future resources should not be assumed to be stable. When a model represents aggregate consumer choice with a single parameter for the intertemporal elasticity of substitution, a single parameter reflecting the time rate of preference, and so on, we must admit the possibility that these aggregated concepts may not be time-invariant. The effects of the changing demographic composition of the population and the introduction of new products and technologies may render aggregate "micro" parameters unstable. In any event, we can and must test whether such models are in fact stable across time and regimes.

Such testing would be easy to do in this paper. The authors have already taken a stand on when a break in monetary regime occurred (1979). They can easily allow for a separate reaction function before 1980 and test for the stability of the other structural parameters before and after 1980. Such a test would go a long way towards empirically establishing the robustness of the model specification to the Lucas critique.

I will return to a second Lucas-critique concern in a moment. The thrust of the argument is that the disturbance terms identified in the paper are at least as likely to be subject to the Lucas critique as are the behavioral parameters.

### 3. Estimating the Structural Parameters

The behavioral parameters are chosen so as to match the structural model's and the VAR's response to a federal-funds-rate shock. However, as Rotemberg and Woodford document in their framework, the funds-rate shock accounts for no more than 5% of the variation in the endoge-nous variables. While this estimator should deliver consistent estimates of the behavioral parameters, it is certainly inefficient with respect to the full set of information in the vector autocovariance function used as a metric by Rotemberg and Woodford later in the paper. The funds-rate impulse responses contain some of the second-moment information in the data, weighted in a particular way. The autocovariance function has *all* the second-moment information in the VAR. An alternative to the impulse-response estimator is to fit the model to the autocovariance

function by maximizing the model's likelihood. This method is both computationally feasible and econometrically efficient.<sup>1</sup>

Of more concern is the extent to which Rotemberg and Woodford's estimation strategy can be generalized. It is well known that it is not possible to map reduced-form responses to (orthogonalized) VAR innovations into structural model responses to model innovations. The mapping is one-to-one in the case studied in this paper: the VAR-derived reaction function is common to both the VAR and the structural model. But as Rotemberg and Woodford show in Section 5 of their paper, the correspondence is nontrivial for the other disturbances in their model. In general, such a mapping will not be possible, and thus the estimator cannot be used for most cases.

### 4. Monetary Policy in the Model

Monetary policy behaves differently in this model than in conventional descriptions. The reason is that there are essentially no lags in this model other than those in the reaction function. The one exception is consumption, which is assumed to be predetermined two periods ahead. I don't know what economic behavior motivates this assumption, although it certainly helps the model to fit the delayed response of output to a federal-funds-rate shock in the VAR. Without this assumption, if monetary policy did not respond with a lag, then the model would have literally no dynamics (other than the dynamics in the disturbance terms). Everything would jump immediately to its equilibrium (even with sticky prices).

This sounds very different from standard depictions of monetary policy. Many have cited Friedman's "long and variable lags" of the effects of policy. The Fed needs to look ahead because it is steering a huge, inertial tanker that responds very gradually to its actions. That sense is totally absent in Rotemberg and Woodford's model. In fact, in their model, the Fed is driving an extremely responsive Lamborghini, but for no particular reason it moves the steering wheel very gradually, as if it were at the wheel of a 1972 Ford Pinto. This logical disconnect between the Fed's inertial behavior and the economy it attempts to influence allows the structural model to exhibit some persistent dynamics, but one has to wonder why the Fed would act in that way if the economy really behaved as depicted here.

<sup>1.</sup> Estimating the structural parameters in this way took less than one minute on a Sun workstation.

### 5. The Role of the Disturbance Terms

Rotemberg and Woodford identify disturbance terms that embody complex dynamics, which, when added to the behavioral equations in the model, capture almost all of the covariance information in the VAR. What does this exercise tell us? In short, I think it tells us that they should wave the white flag. It seems fair to ask economists to surrender when they are forced to conclude that the dynamics present in the data can only be explained by things whose dynamics arise for reasons that we can't explain.

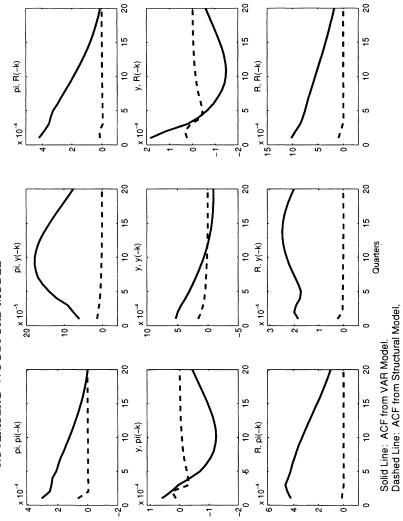
However, the extent to which the disturbance terms account for the dynamics is not as clear as it could be in the paper. Therefore, I propose a model diagnostic to determine the contribution made by the disturbances: I exclude the disturbances from the model, and compute the resulting "error-free" autocovariance function.<sup>2</sup> As Figure 1 of this comment shows, the resulting autocovariance functions don't look at all like those in Figure 2 of the paper. For Rotemberg and Woodford's model, this diagnostic tells us that the disturbance terms are responsible for explaining virtually all of the dynamics evident in the VAR.<sup>3</sup>

Put in a more positive light, Rotemberg and Woodford's paper raises a fundamental question about where to draw the line between a model's ability to generate dynamics based on its behavioral equations and based on its error processes. Microeconomic shocks to taste and technology might well be serially correlated, so one should not by default assume that they are i.i.d. The question is how much the model should rely on its disturbances for its dynamics.

Finally, it is not clear why the authors use VAR-based expectations to solve for the disturbance terms. This procedure implies a mixed-bag model with structural parameters derived from the impulse response assuming rational expectations, and disturbances that are computed taking those parameters as given and using VAR expectations. Below, I sketch a method that retrieves the disturbance terms jointly with the structural parameters, assuming rational expectations throughout.

- 2. To estimate the autocovariance function, I compute the disturbance terms implied by Rotemberg and Woodford's model and parameters, using methods documented in Fuhrer and Moore (1995). I then "whiten" the disturbances, regressing each on its own lags and the lags of other disturbances, and calculate the residual covariance matrix from these residuals. This residual covariance matrix, together with the structural parameters, uniquely determines the autocovariance function implied by the model. I checked the sensitivity of the autocovariance function to the estimation of the residual covariance matrix; the qualitative results presented here in Figure 1 are completely unaffected by modest changes in the estimated covariance matrix.
- 3. An autocovariance function computed using alternative parameters estimated via FIML produce a nearly identical plot.

# Figure 1 COMPARISON OF AUTOCOVARIANCE FUNCTIONS: VAR VS. ROTEMBERG-WOODFORD MODEL



Solid line: ACF from VAR model. Dashed line: ACF from structural model.

### 5.1 A DISTURBANCE-TERM LUCAS CRITIQUE

Are the disturbance processes obtained by Rotemberg and Woodford likely to be stable across time and across policy regimes? It is difficult to see why they would be. The disturbances have no theoretical basis, and are not based on optimizing, rational agent behavior. As a result, the Lucas critique applies in full force to the disturbances—which is particularly bad news for a model that relies so heavily on its disturbances to produce dynamics.

# 6. An Internally Consistent Estimate of Complex Error Processes

Rotemberg and Woodford use a mixture of rational and VAR-based expectations to derive the behavioral parameters and the error processes. One can derive both jointly, under the assumption of rational expectations. A general linear rational-expectations model may be written

$$\sum_{i=-k}^{l} H_i E_{t-j} Z_{t+i} = \epsilon_t,$$

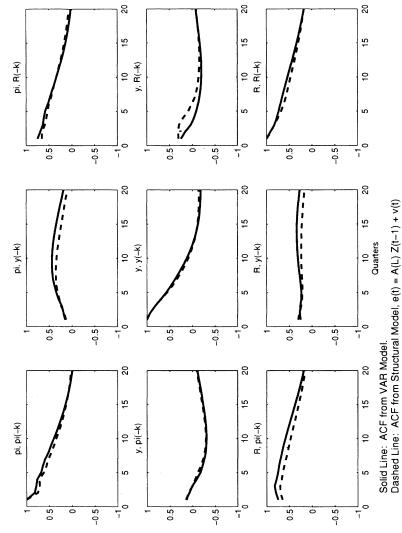
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where the  $H_i$  are square coefficient matrices, the  $Z_{t+i}$  are endogenous variables, and the  $\epsilon_t$  are innovations. This definition is sufficiently general to include different expectations, viewpoint dates, and complex error structures. In particular, the disturbances to the behavioral equations may follow any vector ARMA process, or they may depend on lagged Z's as in Rotemberg and Woodford's model, by a suitable expansion of the state space. A simple example of an error structure that fits into the framework is

$$E_{t}(R_{t+2} - \pi_{t+3} + \sigma Y_{t+2} - Y_{t+1}) + u_{t} = 0, u_{t} - a\pi_{t-1} - bu_{t-1} + c\epsilon_{t-1} = \epsilon_{t},$$

where the first equation is the authors' IS curve, and the disturbance term,  $u_{tr}$  has an ARMA (1,1) structure, and also depends on lagged inflation.

Rotemberg and Woodford solve for the model disturbances as a particular linear combination of lagged observables and VAR innovations [see their equation (5.5)]. However, note that by doing so, the two "real" disturbances are no longer orthogonal in general, since the Z's are obviously not orthogonal. It is possible to write this specification into the Figure 2 COMPARISON OF AUTOCORRELATION FUNCTIONS: VAR VS. ROTEMBERG-WOODFORD MODEL



Solid line: ACF from VAR model. Dashed line: ACF from structural model, e(t) = A(L) Z(t-1) + v(t).

model from the start, and to estimate the structural parameters and the error processes jointly, assuming rational expectations throughout.<sup>4</sup>

I perform this exercise, estimating the structural parameters of Rotemberg and Woodford's model jointly with an error process that has the simple structure

 $u_t = A(L)Z_{t-1} + \epsilon_t.$ 

I impose orthogonality of the funds-rate error with respect to the two "real" errors in this estimation. The parameters  $\kappa$ ,  $\psi$ ,  $\sigma$ , and A(L) are estimated jointly, via maximum likelihood. While the resulting structural parameter estimates differ from Rotemberg and Woodford's estimates (due to the different expectational assumption), Figure 2 of this comment shows that this technique also provides a near-perfect match between the model's and the VAR's vector autocorrelation function.<sup>5</sup>

### 7. Summary

Because the paper aspires to use its optimization-based econometric framework for monetary analysis, one must ultimately provide an answer to the central question: Could I comfortably use Rotemberg and Woodford's model for monetary policy analysis and advice? All in all, the answer to this question for me must be no, because:

- 1. The model doesn't really empirically address the Lucas critique for the structural parameters, and thus may be subject to instability across different policy regimes.
- 2. The estimation method used to obtain the behavioral parameters is inefficient and not generalizable.
- 3. The link between inertial monetary policy and the jump behavior of the rest of the model seems peculiar. This suggests that, other than the reaction function, the model does not jibe well with the data.
- 4. As evidence of this proposition, the model relies almost exclusively on its error structure to replicate the dynamics in the VAR. It is difficult to imagine providing advice to an FOMC member of the following kind: "monetary policy should ease because the disturbances
- 4. No computational constraint prevents us from modeling the error processes jointly with the structural parameters. King and Watson (1995) and Anderson and Moore (1985) utilize techniques that handle these cases perfectly well.
- 5. My estimate of the intertemporal elasticity of substitution  $(1/\sigma)$  is quite a bit smaller than their estimate,  $\kappa$  is considerably larger, and  $\psi$  is consistent with a large range of values for  $\alpha$ 's and  $\gamma$ 's in the price specification. The innovations—the  $\epsilon_t$ 's—are (of course) whitened at these estimates.

have all been large and positive of late, and we expect them to persist at those levels for the next four to eight quarters."

5. The error processes cannot be assumed stable with respect to regime shifts, and are thus subject to the Lucas critique.

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### Comment

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In my own work on monetary policy rules—e.g., McCallum (1988, 1997)—I have favored a research strategy centering around the rules' *robustness*, in the following sense: Because there is a great deal of professional disagreement as to the proper specification of a structural macroeconomic model, it seems likely to be more fruitful to strive to design a policy rule that works reasonably well in a variety of plausible quantitative models, rather than to derive a rule that is optimal in any one particular model. Obviously, Rotemberg and Woodford have chosen the latter, more conventional strategy over the one that I have promoted.<sup>1</sup> A supporter of the robustness approach does not need to disapprove of their study, however, especially if he/she views the paper's model-building contribution as more significant than the policy-rule exercises. Furthermore, the robustness approach can operate at the level of the research community rather than the individual; Rotemberg and Woodford's model could in principle be one of those in which I would want a policy rule to perform reasonably well.

In fact, there is much to be said in favor of attempts such as theirs to build small quantitative macro models in which the agents are depicted as solving dynamic optimization problems, but with some type of nomi-

<sup>1.</sup> The robustness approach is also recommended by Blanchard and Fischer (1989, p. 582) and implicitly by Bryant, Hooper, and Mann (1993).

nal price and/or wage stickiness built in so as to create a significant link between monetary policy actions and real output responses.<sup>2</sup> This type of study potentially combines the theoretical discipline of real businesscycle analysis with the empirical discipline of econometric modeling. As the authors clearly explain, such an approach offers in principle the prospect of yielding a quantitative model that is structural and therefore potentially immune to the Lucas critique. This is, in fact, just the type of analysis that Lucas's work was pointing to in the 1970s before the macro profession got somewhat sidetracked by the notion that cyclical fluctuations might be due almost entirely to technology shocks. Today there is a lot of promising work of this type going on; Rotemberg and Woodford mention a few studies, and there are several others discussed and analyzed by Nelson (1997a, 1997b). It should be mentioned, I think, that this line of work was pioneered in a yet to be published paper by King (1990).

Thus I find the general outline of Rotemberg and Woodford's paper to be praiseworthy and their execution to be extremely skillful and sophisticated. But there are a few features of their model that seem unattractive, and I find their policy-rule analysis, as well as their model-testing exercises, to be unconvincing. Accordingly, the rest of this comment will be devoted to criticisms, even though there is much to praise in the paper.

One of my main objections is to Rotemberg and Woodford's assumption that the central bank knows the value of the current quarter's real output rate when setting the interest rate—its policy instrument—for that quarter. That this assumption is seriously counterfactual is illustrated in a recent paper by Ingenito and Trehan (1996), in which they update the San Francisco Fed's statistical procedure for estimating the current guarter's real GDP.<sup>3</sup> In this procedure, the estimate is based on observations on monthly data (such as industrial production, employment levels, etc.) during the quarter, as well as on lagged values of the real GDP itself. A total of 34 such monthly series are considered, with the final estimating equation using two of them. What this study indicates is that the estimates of a current quarter's real GDP made about two-thirds of the way through the quarter have a root-mean-square (RMS) error of about 1.5% (annualized). That figure suggests a 95%confidence interval that is about 6 percentage points in width. And even at the *end* of the quarter the RMS error is almost as large. So a 95% confidence internal for the quarter's real GDP growth rate could include negative values and also values above 4% (annualized)—i.e., both recession and boom values! Consequently, to assume that monetary policy-

2. The reference here is to links at cyclical frequencies, not to departures from superneutrality that require substantial effects on the stock of capital.

3. An earlier version is discussed by Trehan (1992). On the topic, also see Meltzer (1987).

makers can respond to actual GDP and inflation rates seems highly unrealistic.<sup>4</sup> Figure 14 of Cochrane's (1994) paper shows that this matters somewhat for the shape of impulse response functions such as Rotemberg and Woodford's Figure 1; a much more serious issue will be discussed shortly.

I am also uncomfortable with the assumed model restriction that monetary policy actions have no effect on consumption demand and output until two quarters later. A one-quarter delay would seem defensible, if questionable, but not two quarters. The 1980 imposition and removal of credit controls was admittedly not a monetary policy action of the usual sort, but it was a demand-side action taken via the central bank—and its effects clearly showed response lags of less than two months.

For their specification of price-setting behavior, Rotemberg and Woodford begin with the Calvo–Rotemberg model, which has recently become the closest thing there is to a standard specification—see Roberts (1995). But then they make two modifications. First, they assume that prices newly set in period *t* are based only on information from periods t - 1 and earlier. Then, second, they assume that for a substantial fraction of the sellers there is an additional quarter's delay before the price change goes into effect. These modifications, like the assumption regarding the consumption lag, help to make the model's responses to monetary policy shocks more similar to those that are found empirically, but they do so in a rather arbitrary manner. Invoking such lags seems at least somewhat inconsistent with the objective of "rigorously grounding our structural relations in optimizing individual behavior" (Section 1).

In response to such objections to their model, Rotemberg and Woodford might reply that it performs quite well empirically, as evidenced by their Figures 2 and 3. In the first of these, the autocorrelation and crosscorrelation functions for output, interest, and inflation are shown to match very closely those of an unrestricted VAR, whereas in the latter the simulated time paths are shown to match very closely the actual historical paths (when the historical policy rule is used). It should be pointed out, however, that in presenting these matches the authors do not indicate what fraction of the variables' behavior is explained by the motivated portion of the model's relationships, as opposed to that portion that comes from its serially correlated disturbances. My preconference guess was that much of the model's "explanatory" power comes from the predictable part of the disturbances, so that the autocorrelation and cross-correlation plots would look much less impressive if this por-

<sup>4.</sup> Although it must be said that the interval is narrower for inflation.

tion due to the disturbances were eliminated, thereby making the plots more nearly comparable to those in Fuhrer and Moore (1995).<sup>5</sup> The results in Fuhrer's comment in this volume are supportive of that guess.

The foregoing argument should not be interpreted as a claim that only white-noise residuals should be permitted in a model. There are good reasons to believe that (e.g.) preference and technology shocks are likely to be quite persistent, closer to random walks than to white noise. And the lagged output terms in Fuhrer and Moore's IS function plays a rather similar role to the autocorrelated disturbances in Rotemberg and Woodford's model. But Fuhrer and Moore's lagged output terms are clearly visible; the reader can see what they are and then consider whether they can be sensibly rationalized. So my argument could be viewed as a suggestion that Rotemberg and Woodford report information concerning the autocorrelation properties of their disturbances so that readers can make more informed judgments as to whether their model's empirical performance is or is not impressive.

Let us now consider Rotemberg and Woodford's simulations with alternative policy rules. Here I think that they are quite correct in their finding that "relatively little of the variability in output or inflation in this period can be attributed to the monetary policy shocks" (Section 6) and also to emphasize that "these results do *not* imply that monetary policy is unimportant" (Section 6). Instead, it is the systematic part of the Fed's policy behavior that has been of major quantitative importance, at least for the time period studied and probably for the entire postwar era.

There are, however, some aspects of the simulation results with alternative policy rules that are in my judgment highly questionable. In particular, the results reported in Rotemberg and Woodford's Table 2 indicate that by increasing the strength of the policy responses to current departures of output and inflation from their target values, i.e., by increasing  $\theta_y$  and  $\theta_{\pi}$ , the variances of those departures can be sharply reduced relative to historical policy and relative to the Taylor-rule settings with  $\theta_y = 0.5$  and  $\theta_{\pi} = 1.5$ . But, as was emphasized above, actual central banks have to respond to lagged values of  $y_t$  and  $\pi_t$ , or to expectations of current or future values that are based on lagged observations of various (endogenous) variables. Accordingly, the possibility that explosive oscillations will be caused by excessively strong feedback—often termed instrument instability—becomes prominent. Suppose, for example, that the analyst were to adopt the policy-rule values in row 4 or row 5 of Table 2 ( $\theta_y = 5$ ,  $\theta_{\pi}$ 

<sup>5.</sup> In the Fuhrer-Moore (1995) model, all disturbances are white noise, so the correlation functions reflect only the motivated portion of its relationships. Their model does include, however, lagged output and real interest-rate terms whose motivation is somewhat dubious.

= 1 or  $\theta_y = 0$ ,  $\theta_{\pi} = 10$ ) but used the previous quarter's observations on  $y_t$  and  $\pi_t$  in the rule rather than the current quarter's. Then there would be a real danger of instrument instability.<sup>6</sup> More generally, I believe that the advice for central banks to respond much more strongly to signals indicating that target variables are differing from their target values<sup>7</sup> is truly dangerous. And for basically the same reason—that the assumed policy-rule specification is nonoperational—I am dubious about some of the other policy results reported by Rotemberg and Woodford, even though they are nicely illustrative of the potential for models built in the praise-worthy manner that their introduction promotes.

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- 6. In the four-variable VAR model used in McCallum (1997), both of these policy-rule settings result in highly explosive oscillations. I would not claim that this VAR model is an attractive vehicle for policy simulations, but I would urge Rotemberg and Woodford to consider simulations in their model using a policy rule that is realistic with respect to available information.
- 7. Similar advice has been offered in other recent papers, such as Ball (1997).

### Discussion

Olivier Blanchard asked for clarification regarding the results of the counterfactual monetary policy experiments. In particular, he wanted to know whether he should take seriously the finding that, by using a different policy rule, the Fed could have avoided the recession of the early 1980s while still bringing down the rate of inflation. Woodford responded that a crucial assumption underlying the exercise was that the alternative policy rule was credible. He agreed that this was not an innocuous assumption. Matthew Shapiro added that another possible reason for the result was that the nominal inertia in the model applies to prices rather than to inflation; with "sticky" inflation (which is suggested by some empirical studies), the recessionary costs of a disinflation might well be greater.

Torsten Persson reiterated discussant Jeffrey Fuhrer's concern that the model's dynamics may come more from the exogenous shock processes than from its internal propagation mechanisms. He drew the analogy to the critique of time-to-build models made by Cogley and Nason, that "you get out what you put in." Robert Hall followed up on the propagation issue, suggesting that in neglecting the labor market the authors were missing an important source of persistence. For example, after people lose their jobs during a recession it takes time to reestablish worker–job matches. John Haltiwanger concurred, citing the 1981–1982 recession as a case in which most of the decline in employment came from job destruction. In his own work, he has found much of the reaction to a monetary policy shock estimated in simple VARs to coincide with fluctuations in the rate of job destruction.

Robert King questioned the reasonableness of the model's assumption that consumption is set two periods in advance. Because consumption is the interest-sensitive component of demand in the model, he wondered how this assumption affected the determination of the real interest rate, and whether in the short run the real interest rate is "disconnected" from the driving forces of the model.

Christopher Carroll questioned the validity of welfare analyses in a model that is so highly aggregated. He noted, for example, that the magnitude of aggregate consumption fluctuations may greatly understate the variability in consumption experienced by some families, because most of the variation in employment is due to lost jobs rather than a reduction in hours. Thus a given variation in aggregate employment, for example, may have larger overall effects on welfare than is calculated in the paper.

Finally, the authors responded to some issues raised by the dis-

cussants. Rotemberg remarked that it would be useful to investigate Bennett McCallum's concern that lagged reactions by the Fed might induce instrument instability. He thought that instability was less likely to be an issue in the structural model, as opposed to in the reduced-form VAR, since forward-looking agents would anticipate large interest-rate changes by the Fed if prices were adjusted in the current period. Woodford disagreed with a suggestion by Fuhrer that the paper's estimation method is not internally consistent because the theoretical impulse responses are calculated assuming no structure to the other disturbances. He said that, in fact, all of the impulse responses were computed under the assumption of independence of the policy and nonpolicy disturbances. He also defended the absence of explicit lag terms in the model, on the grounds that it was useful to keep the model as simple as possible subject to the requirement that it is able to fit the impulse responses from the VAR. Finally, Woodford took issue with a criticism of the model made by McCallum, that it does not explicitly include money balances. Woodford noted that a money demand equation could easily be appended; he also suggested that more formal modeling of money demand was not worthwhile in this context, as the distortions due to the use of money in transactions are small when the nominal interest rate is low, as is the case in the simulations.