

# The Straight-Line Depreciation is Wanted, Dead or Alive

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## Abstract

We propose a simple axiomatic system that any depreciation method—complying with the core of the accounting of depreciation—must obey. We show that, while none of the prevalent depreciation methods (e.g., straight-line) ex ante conforms to these principles, the accredited proportional depreciation method not only maintains the axiomatic system, but also, for a plausible family of depreciation methods, is the unique method that complies with the axiomatic system. We further propose two consistency requirements of a depreciation method—partition consistency and dynamic consistency—and show that, in contrast to the commonly used methods, the proportional depreciation method is the only one to always sustain both. Our analysis may provide further resolution to the arguable evidence on the dominance of Funds From Operations over net income in measuring performance in the real estate industry.

Depreciation of tangible assets and amortization of intangible assets play a vital role in the financial statements of nearly all operating entities under different reporting regimes. Although these are non-cash items, they have a crucial effect on the entity's net income and, therefore, on its tax computations, investment policies, dividends, incentive schemes and compensations, loan covenants, regulatory requirements, financial ratios, etc.<sup>1</sup>

When it comes to the real estate industry (and to REITs in particular), depreciation is a major item.<sup>2</sup> Indeed, the Funds From Operations (FFO)—a financial measure that neutralizes the effect of depreciation and amortization by adding these items to the net income—is a commonly-used measure of performance. Yet, net income, which is highly affected by depreciation, is also valuable. For example, while strongly advocating for FFO over net income, Ben-Shahar and Tsang (2009) assert that at present there is no consensus, either among standard setters or academics, on which of these summary measures is more useful. Previous academic efforts present mixed evidence on the relative usefulness of net income and FFO [e.g., Fields et al. (1998), Gore and Stott (1998), Vincent (1999), Graham

and Knight (2000), and Higgins et al. (2006)]. All studies seem to show that both measures are, in general, value relevant. Moreover, Barth, Cram, and Nelson (2001) show that the depreciation item may predict future cash flows and maintains useful accrual information.

We show in this study that the arguable evidence regarding the limited usefulness of net income and, specifically, of the depreciation item in measuring performance may emerge from the inappropriate method by which depreciation is computed. By studying the REIT industry, Ben-Shahar and Tsang (2009) find that none of the accrual items excluded from FFO is significant in predicting future cash flows. They note that this finding is somewhat surprising for depreciation, the single largest reconciliation item, since accounting depreciation is supposed to satisfy the matching principle of revenue and expense and should not be of low quality. Indeed, we show that the matching principle (defined below) is likely to be violated by the prevailing depreciation methods including the commonly-used straight-line method.

Recently adopted accounting standards assert that “the method of amortization shall reflect the pattern in which the economic benefits of the intangible asset are consumed or otherwise used up. If that pattern cannot be reliably determined, a straight-line amortization method shall be used,” [FASB (2001a) item 12)].<sup>3</sup> Furthermore, the employed depreciation method should reflect the pattern in which “the asset’s economic benefits are consumed by the enterprise” [International Accounting Standards Board (IASB) (1982, paragraph 60)].<sup>4</sup> These more recently adopted standards thus manifest a practical revised approach toward depreciation and amortization, namely, that these methods should relate to the asset’s produced economic benefits. Hence, as a depreciation method whose charges do not generally correspond to the “pattern in which the asset’s economic benefits are consumed,” the straight-line method, while still most commonly employed, should no longer automatically apply but rather offer a default use only when the asset’s economic benefits cannot be “reliably determined.”<sup>5</sup>

Essentially, it is the matching principle that resides in the core of the approach for the accounting of depreciation, that is, the requirement for the correspondence between the asset’s generated cash flow pattern and the employed depreciation method. In the absence of this association, the need for impairments might emerge (FASB, 2001b)<sup>6</sup> even in cases where the fundamental economic parameters are unchanged (and as a result, a misleading signal regarding an allegedly poor asset performance might be conveyed to the public). To see this, consider, for example, a real estate asset whose original depreciable cost is equal to 1,000 and whose depreciation schedule is governed by a straight-line method over the 50 years of its expected useful life. Suppose further that the asset is projected to produce a total cash flow of 1,100 (for simplicity, suppose that the interest rate equals zero). It follows that at the end of the first year, the asset residual depreciable value is expected to be 980 (the cost of 1,000 minus a straight-line depreciation charge of 20). If, however, the first year’s cash flow were instead anticipated to equal 150 (as might be the case if, for example, a lease contract of a real estate asset is

structured that way) while the total cash flow were yet projected to remain at 1,100, the asset would be subject to an impairment of 30 at the beginning of the second period because its carrying value (980) would then exceed 950 (1,100 – 150). Hence, even though no change is expected in the fundamental economic parameters nor in the asset's performance in this simple example, impairment might occur when the depreciation method (straight-line in this case) does not ex ante conform to the matching principle; that is, it violates the requirement for the depreciation charges to reflect the use of the asset in the revenue-generating process.<sup>7</sup>

We suspect that the ongoing dominance of the straight-line method may, at least in part, be attributed to the absence of a straightforward translation of the matching principle into clear accounting guidance. This is where our analysis thus emerges: we propose an axiomatic system that translates the association between the depreciation method and the pattern in which the economic benefits of the real estate asset are consumed into a simple formal structure. This formal structure allows us to rigorously examine any possible depreciation method in light of the guidelines stated above.

In particular, we require that the depreciation charges computed ex ante by any depreciation method sustain the following basic principles:

1. The periodic depreciation charge is non-negative (because, unlike a valuation method, depreciation is a method for allocating an asset's cost that is always non-negative; moreover, note that a negative depreciation charge is potentially a source of generating unlimited income and thereby may, among other things, become a simple tool for earnings management);
2. The periodic depreciation charge is no greater than same period's expected cash flow (in both nominal and present value terms);
3. The periodic depreciation charge maintains the essential aspect of the matching principle, i.e., for all period  $i$  and  $j$  of the asset's useful life, if period  $i$ 's cash flow (that is associated with the asset) is no smaller than period  $j$ 's cash flow, then  $i$ 's depreciation charge and net income after deduction of depreciation must be no smaller than those of  $j$ .

Several key results follow from these three basic requirements. First, it turns out that principle (2) further implies the core of the impairment principle, namely, that a profitable asset at the time of purchase (that is, an asset whose associated present value is no less than its cost) should never be subject to recognition of an impairment loss during its useful life as long as there is no change in the fundamental economic variables.<sup>8</sup> Secondly, it follows that, under plausible circumstances, each of the commonly prevailing depreciation methods (straight-line, double-declining, and sum-of-the-years-digit methods) violates one or more of principles (1)–(3).<sup>9</sup>

Moreover, principles (1)–(3), in fact, appear, under different names, in the literature of fair division and cost sharing (e.g., Moulin, 2002). Particularly, the

association between the periodic cash flow and the periodic depreciation charge is consistent with the intuition underlying a fair division: according to its contribution to the consumption of the benefits, each period should “participate” in sharing (be charged for) the cost of the asset. Following the fair division literature and particularly Moulin (1988), we thus propose and examine the proportional depreciation method, which directly depends on the cash flow profile of the asset. We show not only that the proportional method always conforms to principles (1)–(3), but also that the proportional method becomes the unique depreciation method that conforms to these principles when we supplement the axiomatic system with the requirement that any periodic depreciation charge merely depends on the asset’s total cost, the periodic cash flow, and the asset’s total associated cash flow (while maintaining independence of all other aspects of the cash flow profile such as the cash flow distribution over time).

Finally, following the fair division literature (e.g., Hart and Mas-Colell, 1989; Aumann and Hart, 1992; and Moulin, 2002), we propose two requirements of consistency of depreciation methods and examine the prevailing methods in their light. First, the Partition Consistency property requires that the depreciation charges are, in effect, robust to any sub-period division of the asset’s useful life. That is, that the attained periodic depreciation charges maintain, independently whether they are computed on either, for example, annual, semi-annual, quarterly, or even unequal time division into any sub-period structure (e.g., three and nine months).<sup>10</sup> The second consistency property—Dynamic Consistency—requires that, *ceteris paribus*, subsequent to the determination of the depreciation charges, a re-computation of the depreciation at any time period along the asset’s useful life should generate a depreciation scheme (for the remaining time periods) that is identical to the one originally determined at the time of purchase. In practice, this property guarantees that if there is no change in the economic fundamentals during the asset’s useful life, then the original computation of the depreciation charges will sustain. We show that, in contrast to the commonly-used methods, the proportional depreciation method is the only one that always sustains both consistency properties.

We should emphasize that in the presented analysis we neither consider the information value of the potential depreciation methods nor do we search for more informative methods (e.g., Feltham and Ohlson, 1996). Also, we examine neither incentive schemes nor goal congruence issues associated with depreciation (e.g., Dutta and Reichelstein, 2005). Finally, we ignore any tax implications associated with depreciation.<sup>11</sup>

In Section 2, we develop the axiomatic system and examine its implications for the commonly-used depreciation methods. We consider the proportional depreciation method in Section 3. In Section 4, we summarize and discuss empirical implications.

### The Axiomatic System and its Implications

Consider a real estate asset whose depreciable cost is  $A$ . The asset is expected to generate a stream of cash flow (after deduction of maintenance costs and before interest)  $Y = (y_1, \dots, y_T)$ , where the cash flow attributed to period  $i$  ( $i = 1, \dots, T$ ),  $y_i$ , is generated by the operating income of that period, and  $T$  denotes the number of periods during which the asset will be in service (that is,  $T$  represents the useful life of the real estate asset). We assume that there is certainty with respect to the cash flow,<sup>12</sup> that  $Y$  is independent of the cash flow of other assets, and that the cash flow sustains:

$$Y_i \geq 0 \text{ for all } i = 1, \dots, T. \quad (1)$$

We further assume that:

$$\sum_{i=1}^T \frac{y_i}{(1+r)^i} \geq A, \quad (2)$$

where  $r$  is the cost of capital associated with the particular asset. Inequality (2) thus implies that the asset represents an ex ante economically efficient investment. Finally, we assume that the salvage value of the asset is, without loss of generality, equal to zero.

For the asset whose depreciable cost equals  $A$ , denote the depreciation charge that is allocated to period  $i$  by  $d_i$ ,  $i = 1, \dots, T$ , and let the vector  $(d_1, \dots, d_T)$  denote a depreciation method, where the vector  $(d_1, \dots, d_T)$  is the depreciation charges from an ex ante perspective and where  $\sum_i d_i = A$ .

Now, let  $\langle A, y_1/(1+r), \dots, y_T/(1+r)^T \rangle$  be a depreciation problem. We define a depreciation problem as one composed of two components: an asset with an original cost equal to  $A$  and a vector consisting of the present value of the cash flow generated by the asset over the  $T$  periods of its expected useful life. We then propose an axiomatic framework by which we attempt to capture a set of basic requirements to which any depreciation method that solves the depreciation problem must obey, while conforming to fundamental accounting principles.

**Axiom 1:** (*Non-Negativity*)

$$d_i \geq 0 \text{ for all } i = 1, \dots, T.$$

**Axiom 2:** (*Upper-Bound*)

$$\frac{y_i}{(1+r)^i} - d_i \geq 0 \text{ for all } i = 1, \dots, T.$$

**Axiom 3:** (*Matching Principle*)

$$\text{For all } i, j = 1, \dots, T, \text{ if } \frac{y_i}{(1+r)^i} \leq \frac{y_j}{(1+r)^j} \text{ then } d_i \leq d_j$$

$$\text{and } \frac{y_i}{(1+r)^i} - d_i \leq \frac{y_j}{(1+r)^j} - d_j.$$

Axiom 1 simply states that, under the assumption of non-negative periodic cash flow associated with a given asset, the periodic depreciation charge must also be non-negative (as noted earlier, depreciation is not a valuation method but a method for allocating an asset's cost that is, of course, always non-negative. Moreover, negative depreciation charges are undesired from the earnings management perspective). Axiom 2 corresponds to a boundary condition of the matching principle: the recognition of expenses by the association of costs and revenues on a cause and effect basis. Axiom 2 thus guarantees that no period ever experiences depreciation that is greater than the present value of the corresponding produced cash flow. Axiom 3 accounts for the main aspect of the matching principle: greater cash flow (in present value terms) must be associated with no smaller depreciation; and, at the same time, the attained order of cash flow (in present value terms) after depreciation cannot be reversed due to the employed depreciation method. Hence, a clear and direct correspondence is established between the income and the expense involved in generating it. In Axioms 2 and 3, the depreciation charges are considered in nominal terms — otherwise, the asset cannot be entirely depreciated (and, of course, at the time of purchase, the present value and the nominal value of the cost are equal). The cash flow appears in present value terms as the analysis is conducted from an *ex ante* perspective.<sup>13</sup> Hereafter, we refer to the set of Axioms 1-3 as the axiomatic system.<sup>14</sup>

We define an impairment-free depreciation method as:

**Definition 1:** *Given a depreciation problem, we say that a depreciation method is impairment-free if it ex ante maintains the impairment principle (in the absence of changes in the asset performance or interest rate it does not require a recognition of impairment loss), i.e., if for all  $i = 1, \dots, T$  and for all  $l = 1, \dots, T - 1$ , we have  $A - \sum_{i=1}^l d_i \leq \sum_{i=l+1}^T y_i / (1+r)^{i-l}$ .*

According to recent developments in generally accepted accounting principles (GAAP), when formally reporting on its financials, the firm is required to test that the recoverable amount of each of its depreciable assets is greater than their carrying cost (FASB, 1995, 2001b; and IASB, 2004). Accordingly, the impairment-free depreciation method requires that the net book value of the asset (after accumulated depreciation) at any time period of the asset's useful life should never be greater than the present value of its future cash flow. In other words, given a profitably producing asset and no unexpected changes in the economic fundamentals, the depreciation method should not lead to the recognition of an impairment loss.

Following the axiomatic system and given Definition 1, a preliminary result immediately emerges.

**Proposition 1:** *Any depreciation method that conforms to Axiom 2 is impairment-free. That is, if for all  $i = 1, \dots, T$ ,  $y_i / (1 + r)^i - d_i \geq 0$ , then for all  $l = 1, \dots, T$   $A - \sum_{i=1}^l d_i \leq \sum_{i=l+1}^T y_i / (1 + r)^{i-l}$ .*

**Proof:** See the Appendix.

The axiomatic system thus further implies that an asset whose depreciation maintains Axiom 2 will not be subject to impairment due to “depreciation assumptions that are not adjusted appropriately,” (FASB, 2001b).<sup>15</sup>

We further claim:

**Proposition 2:** *For each of the prevailing depreciation methods (i.e., straight-line, sum-of-the-years-digit, and double-declining) there exist depreciation problems for which the axiomatic system is violated.*

**Proof:** See the Appendix.

That is, the commonly employed depreciation methods (which all merely depend on the asset's useful life and are independent of the cash flow profile) might not conform to the requirements stated under the axiomatic system (which is sensitive to the cash flow profile).<sup>16</sup> It turns out that the simplification of the prevailing methods is obtained at a cost: assuming that (or acting as if) there is a simplistic pattern of cash flow might inherently produce, under plausible circumstances, the violation of the matching principle that further carries to economically unjustified recognition of impairment loss.

In contrast, by using the greater information regarding the expected future cash flow available to the firm today (especially in cases of already leased real estate)—as it is demanded by GAAP—we now explore an alternative depreciation method that not only conforms to the axiomatic system, but is also highly applicative and maintains essential properties of consistency.

### The Proportional Depreciation Method

Consider the depreciation method that for every depreciation problem the proportion of the periodic depreciation charge to the total cost of the asset (net of salvage value) is equal to the proportion of that period's operating income (or production units) to the total operating income (or total production units) in present value terms. This method is sometimes referred to as the units of production method.<sup>17</sup> Namely, each period depreciation charge sustains:

$$d_i = \frac{\frac{y_i}{(1+r)^i}}{\sum_i \frac{y_i}{(1+r)^i}} \times A \text{ for all } i = 1, \dots, T. \quad (3)$$

We refer to the depreciation method in (3) as the proportional depreciation method.<sup>18</sup>

We should emphasize that while we assume, to simplify the analysis, that there is certainty with respect to the cash flow and, thus, that the ex ante and ex post earnings generated by the asset are the same, the application of the proportional method is more general. That is, at the end of each accounting period when the periodic income is revealed and the depreciation charge is recorded, the managers may re-assess their forecast of the remaining future earnings based on the best available information and, accordingly, re-compute the periodic depreciation using the proportional method. If ex post periodic earnings and the re-assessed future earnings turn out to be different from the projected ex ante, then future depreciation charges that are re-computed by the proportional method will reflect those changes. (This dynamic consistency property of the proportional method is further discussed below—see Definition 3 and Propositions 6 and 7.)<sup>19</sup>

It immediately follows:

**Proposition 3:** *The proportional depreciation method sustains the axiomatic system.*

**Proof:** See the Appendix.

Consider now the family of depreciation methods in which the periodic depreciation charge  $d_i$ ,  $i = 1, \dots, T$ , maintains:

$$d_i = f_i \left[ \sum_{i=1}^T \frac{y_i}{(1+r)^i}, \frac{y_i}{(1+r)^i}, A \right] \text{ for all } i = 1, \dots, T, \quad (4)$$



where,  $f(\cdot)$  denotes a function. Note that Equation (4) requires that the depreciation charge at any single time period merely involves the present value of the entire sum of cash flow generated along the asset's useful life (but not the distribution of the cash flow among the time periods), the present value of the corresponding periodical cash flow, and the original cost of the asset. Essentially, the family of depreciation methods that comply with (4) does not require any prior information regarding the specific future cash stream. For each time period, given the asset's total economic value at the time of purchase and the period's cash flow, the depreciation charge for that period is immediately determined by the function. We thus refer to the family of depreciation methods that conform to (4) as the partial information methods.<sup>20</sup>

Following Moulin (1988), we now claim:

**Proposition 4:** *For  $T \geq 3$  and for all depreciation problems, the proportional depreciation method is the unique partial information method that satisfies the axiomatic system.*

**Proof:** See the Appendix.

Hence, within the set of partial information depreciation methods, the proportional method is the only one that maintains the axiomatic system. For every other depreciation method in this family, there exists a depreciation problem such that the depreciation charges, computed by that depreciation method for that problem, violate the fundamental requirements formulated in the axiomatic system.

In addition to the requirements conveyed by the axiomatic system, we now propose two essential consistency properties of depreciation methods: partition consistency and dynamic consistency. We then examine the depreciation methods in light of these properties.

### Partition Consistency

Consider a real estate asset whose expected useful life is 200 quarters. Suppose that the adopted depreciation method (whatever it may be) produces an initial depreciation scheme such that the depreciation charge in the first quarter equals 100 and at each subsequent quarter it equals to that of the previous quarter plus 10 (that is  $d_1 = 100$  and  $d_i = d_{i-1} + 10$  for  $i = 2, \dots, 200$ ). Partition consistency then requires that, ceteris paribus, if the asset were to be depreciated, say, annually (rather than quarterly), then employing once again the same depreciation method should result in equivalent depreciation charges; that is, 460 for the first year (i.e.,  $100 + 110 + 120 + 130$ ), 620 for the second year (i.e.,  $140 + 150 + 160 + 170$ ) and so on. The partition consistency property thus requires that the depreciation charges are, in effect, robust to any partition of the entire depreciation period. The annual depreciation charge should maintain, independently of whether the depreciation is computed on, for example, an annual, semi-annual, or quarterly basis. Partition consistency is particularly important in environments where both interim and annual (aggregating the interim periods) financial reporting prevails.

Formally, let  $\langle A, y_1/(1+r), \dots, y_T/(1+r)^T \rangle$  once again be a depreciation problem and let  $D$  be a depreciation method yielding  $(d_1, \dots, d_T)$ . Let  $P = \{P_1, \dots, P_k\}$ ,  $k \leq T$ , be a partition of  $\{1, \dots, T\}$  such that for  $i$  and  $j$ ,  $j > i$ ,  $i, j \in P_m$  if and only if  $i, i+1, \dots, j-1, j \in P_m$ . To every partition  $P$ , associate a vector  $(y_1^*, \dots, y_k^*)$ , where  $y_j^* = \sum_{i \in P_j} y_i / (1+r)^i$  and consider the depreciation problem  $\langle A, y_1^*, \dots, y_k^* \rangle$ . Let  $(d_1^*, \dots, d_k^*)$  be its solution according to  $D$ . Consider now the derived series of depreciation problems  $\langle d_m^*, y_s/(1+r)^s, \dots, y_\nu/(1+r)^\nu \rangle$ , where  $s, \dots, \nu \in P_m$ ,  $\sum_{l=s}^\nu y_l / (1+r)^l = y_m^*$ , and  $m = 1, \dots, k$ .

**Definition 2:** A depreciation method maintains the partition consistency property if for every depreciation problem, every feasible partition, and every  $m$ ,  $s$ , and  $i$ , the depreciation method sustains:

$$d_i \left( d_m^*, \frac{y_s}{(1+r)^s}, \dots, \frac{y_\nu}{(1+r)^\nu} \right) = d_{s+i-1} \left( A, \frac{y_1}{1+r}, \dots, \frac{y_T}{(1+r)^T} \right).$$

We then argue:

**Proposition 5:** The proportional depreciation method satisfies the partition consistency property.

**Proof:** See the Appendix.

While the depreciation charges that are derived by the proportional method are robust to any time partition of the asset's useful life, note that the straight-line method maintains the partition consistency property only in cases where each element in the partition is of equal length (otherwise, the straight-line method is of course undefined). Finally, it is straightforward to see that the sum-of-the-years-digit and the double-declining methods do not retain the partition consistency property.

### Dynamic Consistency

Consider a real estate asset whose depreciable cost is 775 and whose useful life is five periods. Suppose that the adopted depreciation method (whatever it may be) produces an initial depreciation scheme of 400, 200, 100, 50, and 25 (sum up to 775) for the five periods, respectively. Dynamic consistency then requires that, ceteris paribus, if the same asset were acquired after the first period at a cost of 375 and had a useful life of only the remaining four periods, then employing once again the same depreciation method should result in identical depreciation charges, i.e., 200, 100, 50, and 25, for the remaining four periods, respectively. That is, dynamic consistency requires that, from the time of purchase, re-computing the depreciation charges for the remaining periods using the same depreciation method at any time period along the asset's useful life produces the original depreciation

allocation for those remaining periods. The dynamic consistency property thus maintains an important implication for financial analysis. In essence, it guarantees that if there are no changes in the economic fundamentals, the depreciation charges that are computed for the asset’s residual useful life do not vary at any time along the asset’s useful life.

Formally:

**Definition 3:** *A depreciation method maintains the dynamic consistency property if for all  $i = 1, \dots, T - l$  and  $l = 1, \dots, T - 1$ , the depreciation method sustains:*

$$\begin{aligned}
 d_i & \left[ A - \sum_{j=1}^l d_j, \frac{y_{l+1}}{(1+r)^{l+1}}, \dots, \frac{y_T}{(1+r)^T} \right] \\
 & = d_{l+i} \left[ A, \frac{y_1}{(1+r)^1}, \dots, \frac{y_T}{(1+r)^T} \right].
 \end{aligned}$$

According to Definition 3, the dynamic consistency property thus requires consistency with respect to cash flow that is discounted back to the date of purchase. We then claim:

**Proposition 6:** *The proportional, straight-line, and sum-of-the-years-digit depreciation methods satisfy the dynamic consistency property.*

**Proposition 7:** *The double-declining depreciation method does not maintain the dynamic consistency property.*

**Proofs:** See the Appendix.

The proportional depreciation method thus receives further support by satisfying the dynamic consistency property.

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## Conclusion

There is mixed evidence in REIT studies regarding the relative usefulness of net income and FFO as performance measures (e.g., Fields, Rangan, and Thiagarajan, 1998; Gore and Stott, 1998; Vincent, 1999; Graham and Knight, 2000; and Ben-Shahar and Tsang, 2009). In the analysis presented here, we show that the limited usefulness of net income (and, particularly, of the depreciation item) in measuring performance might be due to the inappropriate methods by which depreciation is computed, i.e., the use of methods that do not reflect the pattern in which the asset’s economic benefits are consumed by the enterprise. Accordingly, current accounting standards practically attempt to force firms to maintain the matching and the impairment principles by declaring that the methods of depreciation and amortization should be associated with the asset’s produced economic benefits,

[e.g., FASB (2001a item 12) and IASB (1982, paragraph 60)]. Despite these guidelines, the straight-line (the most prevalent method), the sum-of-the-years-digit, and the double-declining depreciation methods apparently have yet retained their popularity in the market.

We devise a simple axiomatic system by which we capture the essence of the above accounting standards and show that none of the prevailing depreciation methods sustains the spirit the required guidelines. Moreover, we examine the accredited proportional depreciation method and show not only that it conforms to the revised standards, but also that, within a plausible family of depreciation methods, it is the unique method that complies with the axiomatic system. Finally, we introduce two time-period consistency properties of depreciation and amortization—Partition Consistency and Dynamic Consistency—and show that, in contrast to the commonly-used methods, the proportional depreciation always sustains both consistency properties.

It should be noted that while the proportional method adopts the greater availability of information regarding a firm's future cash flow that follows from the developments in financial reporting (e.g., IASB, 2004; and FASB, 2001b), it, essentially, requires only the information regarding the ongoing periodical cash flow generated by the asset, the total present value of the future cash flow, and the asset's cost at the time of purchase.

Several research questions emerge from our study. Given that all the prevailing depreciation methods potentially violate the requirement to refrain from non-economic-based impairments, are there any guidelines that the players in the market may use to distinguish between economic- and non-economic-based impairments? That is, once impairment is announced, are there any observed indications by which investors can conclude whether the impairment is due to unexpected real economic development or simply a result of the employed depreciation method? And, moreover, empirically, is there any depreciation method that produces more non-economic-based impairments than others? And if so, what are the particular characteristics of the industries where these impairments are announced?

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## Appendix

### Proof of Proposition 1

Given that  $\sum_i d_i = A$ , we have for all  $l = 1, \dots, T$ :

$$A - \sum_{i=1}^l d_i = \sum_{i=l+1}^T d_i. \quad (A1)$$

However, given that the depreciation methods conform to Axiom 2, we can further develop the right-hand side of (A1) and argue that for all  $l = 0, \dots, T - 1$ :

$$\sum_{i=l+1}^T d_i \leq \sum_{i=l+1}^T \frac{y_i}{(1+r)^i} \tag{A2}$$

and then:

$$\begin{aligned} \sum_{i=l+1}^T \frac{y_i}{(1+r)^i} &\leq (1+r)^l \sum_{i=l+1}^T \frac{y_i}{(1+r)^i} \\ &= \sum_{i=l+1}^T \frac{y_i}{(1+r)^{i-l}} \end{aligned} \tag{A3}$$

From (A1)-(A3) the result immediately follows. ■

Proof of Proposition 2

It is straightforward to see that the straight-line depreciation method (where  $d_i = A/T$  for all  $i$ ) may violate Axiom 2; the sum-of-the-years-digit method [where  $d_i = (T - i + 1)A/\sum_{i=1}^T i$  for all  $i$ ] may violate Axioms 2 and 3; and the double-declining method [where  $d_i = 2(A/T)(1 - 2/T)^{i-1}$  for all  $i = 1, \dots, s$ , where  $s$  is such that  $(1 - 2/T)^{s-1} > 1/2$  and  $(1 - 2/T)^s \leq 1/2$ ] may violate Axioms 2 and 3. ■

Proof of Proposition 3

Note that  $d_i$  in Equation (3) satisfies Axiom 1 because the right-hand side of (3) is always non-negative; also,  $d_i$  in (3) satisfies Axiom 2 because of the combination of Equations (2) and (3); Finally, one can immediately see that  $d_i$  in (3) also satisfies Axiom 3. ■

Proof of Proposition 4

It is obvious that  $d_i$  in Equation (3) sustains the condition in Equation (4). We now show uniqueness by applying Moulin (1988). Particularly, following Equation (4), we have:

$$\sum_{i \in S} d_i = f_S \left[ A, \sum_{i \in S} \frac{y_i}{(1+r)^i}, \sum_{i \in T \setminus S} \frac{y_i}{(1+r)^i} \right]$$

for any coalition  $\phi \neq S \subset \{1, \dots, T\}$ , (A4)

where  $f_S$  corresponds to  $S$ . Now fix  $S = \{1, 2\}$  and apply (A4):

$$d_1 + d_2 = f_{1,2} \left[ A, \frac{y_1}{(1+r)} + \frac{y_2}{(1+r)^2}, \sum_{i \geq 3} \frac{y_i}{(1+r)^i} \right]. \tag{A5}$$

Denote  $\sum_{i=1}^T y_i / (1+r)^i$  by  $\bar{y}$ . Then:

$$\begin{aligned} & f_1 \left[ A, \frac{y_1}{(1+r)}, \bar{y} \right] + f_2 \left[ A, \frac{y_2}{(1+r)^2}, \bar{y} \right] \\ &= f_{1,2} \left[ A, \frac{y_1}{(1+r)} + \frac{y_2}{(1+r)^2}, \bar{y} - \frac{y_1}{(1+r)} - \frac{y_2}{(1+r)^2} \right]. \tag{A6} \end{aligned}$$

That is, for any  $\bar{y}$  and  $A$ , the left-hand side of (A6) depends only on  $y_1 / (1+r) + y_2 / (1+r)^2$  (as long as the latter does not coincide with  $\bar{y}$ ). This is the functional equation of Jensen; and its solution, given the requirements in Axioms 1 and 2 for all  $A$ ,  $Y$ , and  $i$ , dictates (3). ■

*Proof of Proposition 5*

The proportional depreciation method sustains the condition:

$$\begin{aligned} & d_i \left( d_m^*, \frac{y_s}{(1+r)^s}, \dots, \frac{y_v}{(1+r)^v} \right) \\ &= d_i \left( \sum_{n=s}^v \frac{y_n}{(1+r)^n} \times \frac{A}{\sum_{i=1}^T \frac{y_i}{(1+r)^i}}, \frac{y_1}{1+r}, \dots, \frac{y_T}{(1+r)^T} \right), \tag{A7} \end{aligned}$$

where the right-hand side of (A7) can be developed into:

$$\begin{aligned}
 d_i & \left( \sum_{n=s}^v \frac{y_n}{(1+r)^n} \times \frac{A}{\sum_{i=1}^T \frac{y_i}{(1+r)^i}}, \frac{y_1}{1+r}, \dots, \frac{y_T}{(1+r)^T} \right) \\
 & = \frac{y_{s+i-1}}{(1+r)^{s+i-1}} \times \left( \sum_{n=s}^v \frac{y_n}{(1+r)^n} \times \frac{A}{\sum_{i=1}^T \frac{y_i}{(1+r)^i}} \right),
 \end{aligned}
 \tag{A8}$$

and the right-hand side of (A8) can be reduced into:

$$\begin{aligned}
 & \frac{y_{s+i-1}}{(1+r)^{s+i-1}} \times \left( \sum_{n=s}^v \frac{y_n}{(1+r)^n} \times \frac{A}{\sum_{i=1}^T \frac{y_i}{(1+r)^i}} \right) \\
 & = \frac{y_{s+i-1}}{(1+r)^{s+i-1}} \times \frac{A}{\sum_{i=1}^T \frac{y_i}{(1+r)^i}}.
 \end{aligned}
 \tag{A9}$$

However, the right-hand side of (A9) can be re-written:

$$\begin{aligned}
 & \frac{y_{s+i-1}}{(1+r)^{s+i-1}} \times \frac{A}{\sum_{i=1}^T \frac{y_i}{(1+r)^i}} \\
 & = d_{s+i-1} \left( A, \frac{y_1}{(1+r)^1}, \dots, \frac{y_T}{(1+r)^T} \right).
 \end{aligned}
 \tag{A10} \blacksquare$$

Proof of Proposition 6

The proportional depreciation method sustains dynamic consistency because every  $d_i$  of this method satisfies:

$$\begin{aligned}
 d_{i+1} & \left[ A - A \frac{\sum_{j=1}^i \frac{y_j}{(1+r)^j}}{\sum_{j=1}^T \frac{y_j}{(1+r)^j}}, \frac{y_{i+1}}{(1+r)^{i+1}}, \dots, \frac{y_T}{(1+r)^T} \right] \\
 & = A \frac{\frac{y_i}{(1+r)^i}}{\sum_{j=1}^T \frac{y_j}{(1+r)^j}} \left[ 1 - \frac{\sum_{j=1}^{i-1} \frac{y_j}{(1+r)^j}}{\sum_{j=1}^T \frac{y_j}{(1+r)^j}} \right]. \tag{A11}
 \end{aligned}$$

However the right-hand side of (A11) is equal to  $A[y_i/(1+r)^i][\sum_{j=1}^T y_j/(1+r)^j]$ , which, in turn, equals:

$$d_i \left[ A, \frac{y_1}{(1+r)^1}, \dots, \frac{y_T}{(1+r)^T} \right].$$

The straight-line depreciation method sustains dynamic consistency because every  $d_i$  of this method satisfies:

$$d_i = \left( A - \frac{i-1}{T} A \right) \frac{1}{T-i-1}. \tag{A12}$$

However, the right-hand side of (A12) can be reduced to  $A/T$ .

Finally, the sum-of-the-years-digit depreciation method sustains dynamic consistency because every  $d_i$  of this method satisfies:

$$d_i = \left( A - A \frac{\sum_{t=i-1}^T t}{\sum_{t=1}^T t} \right) \frac{i-1}{\sum_{t=1}^{T-i-1} t} \tag{A13}$$

However, the right-hand side of (A13) can be reduced to  $A(i-1)/\sum_{t=1}^T t$ . ■



### Proof of Proposition 7

Computing the depreciation charges under the double-declining method for the first two periods at  $t = 0$ , we get  $2A/T$  for the first period and, thus,  $(2A/T)(1 - 2/T)$  for the second period. Yet if we then re-compute the depreciation charge for the second period at  $t = 1$ , we get  $(2/T - 1)(A - 2A/T)$ . Note, however, that  $(2A/T)(1 - 2/T) \neq (2/T - 1)(A - 2A/T)$  for all  $A \neq 0$  and  $T$ . ■

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### Endnotes

- <sup>1</sup> The analysis presented here refers to both the amortization of intangible assets and the depreciation of tangible assets. We will thus interchangeably use these terms.
- <sup>2</sup> Note that according to Financial Accounting Standards Board (1987, par. 34), even land that is generally not subject to depreciation might be depreciated: In cases where the fertility of the land diminishes quickly (such as in a landfill, gravel pit, or farm land), depreciation of the cost of land may be appropriate. Also, note that depreciation charges maintain their significant effect in the case of equity evaluations that are based on financial ratios.
- <sup>3</sup> The most common method of depreciation and amortization for financial reporting purposes is the straight-line method that simply equally allocates the cost of the asset (net of salvage value) over the asset useful life [on the prevalence of the different depreciation methods see, for example, McFarland (1990)].
- <sup>4</sup> Recall that financial statements in the United States are prepared according to Generally Accepted Accounting Principles (GAAP), especially, according to the Financial Accounting Standards Board (FASB) statements, whereas financial reporting in Europe is generally in accordance with the standards set by the International Accounting Standards Board (IASB).
- <sup>5</sup> The latter may particularly apply to income-generating properties that, unlike machinery, often allow for relatively reliable estimations of future income streams due to the long-lived lease contracts that are associated with them.
- <sup>6</sup> According to U.S. accounting standards, an impairment loss is recognized only if the carrying amount of a long-lived asset is not recoverable from its undiscounted cash flows. In that case, an impairment loss is measured as the difference between the carrying amount and the fair value of the asset (FASB, 2001b).
- <sup>7</sup> It should be emphasized that the numerical examples in the article are chosen so as to clarify the presented ideas. The specific figures in each numerical example may, of course, change in order to more closely replicate real world scenarios.
- <sup>8</sup> FASB sheds some light on this impairment issue: In FASB (2001b), the Board notes that it also considers but does not adopt an alternative approach to impairments that will require different measures for different impairments. At one end, an asset might be Impaired because depreciation assumptions are not adjusted appropriately. At the other extreme, an asset might be impaired because of a major change in its use. The Board, however, does not develop a workable distinction between the two situations that would support the use of different measures. For more on the accounting of impairments, see FASB (2001b) and IASB (2004).

- <sup>9</sup> Recall that the sum-of-the-years-digit and the double-declining are accelerated depreciation methods, in which period  $i$ 's depreciation charge is equal to  $A(T - i + 1)/\sum_{i=1}^T i$  for all  $i$  in the former and to  $2(A/T)(1 - 2/T)^{i-1}$  for all  $i = 1, \dots, s$ , where  $s$  is such that  $(1 - 2/T)^{s-1} > 0.5$  and  $(1 - 2/T)^s \leq 0.5$  in the latter, and where  $A$  is the asset's total cost to be depreciated and  $T$  is the number of periods of the asset's useful life.
- <sup>10</sup> Partition consistency is particularly significant in those environments where interim financial reporting prevails.
- <sup>11</sup> For recent studies that estimate depreciation for real estate assets, see, for example, Fisher, Smith, Stern, and Webb (2005) and Harding, Rosenthal, and Sirmans (2008). For the analysis of depreciation within a game theoretic framework, see, for example, Aparicio and Sanchez-Soriano (2008) and Ben-Shahar and Sulganik (2009).
- <sup>12</sup> Alternatively,  $Y$  can be viewed as a vector whose components represent the periodic expected cash flow.
- <sup>13</sup> Importantly, our analysis may of course further include the financial statement perspective according to which all figures (including cash flow) are considered in nominal terms—simply, plug  $r = 0$  into Axioms 2 and 3.
- <sup>14</sup> As was previously implied, Axioms 1–3 appear, under different names, in the literature of fair division and cost sharing (Moulin, 2002). It is unsurprising, then, that solution concepts borrowed from that literature adhere to the axiomatic system and serve as feasible depreciation methods.
- <sup>15</sup> Also, note that an asset whose original recoverable amount is greater than its original carrying cost (that is, its recoverable amount is greater than its carrying cost at  $t = 0$ ) cannot be subject to impairment at any  $t > 1$ , if there is no unfavorable change in  $Y$  and/or  $r$  and/or the required net capital investment. This may be considered as a rational constraint on the accounting method.
- <sup>16</sup> There are, of course, certain depreciation problems for which these commonly used methods do conform to Axioms 1–3.
- <sup>17</sup> The proportional method is conceptually similar to the units-of-production method; however, it considers the cash flow as opposed to the production stream (e.g., FASB, 1985). Note further that the proportional method, by corresponding to the periodical cash flow, leaves little room for earnings management.
- <sup>18</sup> While the proportional depreciation method could, in general, be applied to any potential asset, it is particularly applicable to real estate, where the future operating income could be more reliably anticipated. Also, note that the “legitimacy” of the proportional depreciation method is already apparent from U.S. accounting standards (FASB, 1985), where it is stated that capitalized costs are amortized based on current and future revenue for each product (with an annual minimum equal to the straight-line amortization over the remaining estimated economic life of the asset). This “mixed” method, however, may violate Axiom 2. Finally, the proportional depreciation method may also be formulated in nominal terms as opposed to present value ones.
- <sup>19</sup> We thank the anonymous referee for raising the question regarding the effect of a potential discrepancy between forecasted and ex post earnings (which follows unanticipated physical, functional, or economic obsolescence to the property) on the use of the proportional depreciation method. Also, on the relative quality of earnings forecasts in the real estate industry and specifically in REITs, see Downs and Guner (2006). Also, see Nourse (1994).

<sup>20</sup> In the context of Moulin (1988), this family of methods is referred to as the decentralizability set. Also, note that the straight-line depreciation method further belongs to the partial information family; however, as shown earlier, it may violate Axiom 2.

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