Default, Credit Scoring, and Loan-to-Value: A Theoretical Analysis of Competitive and Non-Competitive Mortgage Markets

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Abstract

This study shows that when borrowers' default probability on the mortgage loan is unobservable to the lender, the latter can screen borrowers by their combined choice of loan-to-value (LTV) ratio and interest rate. It further demonstrates that when borrowers signal their default risk by acquiring a credit score, then a combined separating signaling and screening equilibrium is attained. If the signaling cost is sufficiently small, the combined signaling and screening equilibrium dominates the screening-only equilibrium under both competitive and non-competitive market frameworks. However, while, under the competitive setting, borrowers benefit from constituting a credit scoring signaling system, the prospective gain is shifted to lenders under imperfect competition. Finally, under both competitive and non-competitive combined signaling and screening equilibria, the study reveals that high and low risk borrowers, while acquiring distinct credit scores (and therefore paying different interest rates) might realize higher, lower, or identical LTV ratios. Hence, any empirical test of the relation between LTV ratio and default risk must incorporate the interrelation among the LTV ratio, credit score, and interest rate.

A crucial factor in the efficient operation of primary and secondary mortgage markets is the ability to correctly price default risk. Hence, extensive real estate economic research, both theoretical and empirical, is devoted to understanding the different aspects of mortgage default.[1] This paper builds on the asymmetric information literature to theoretically examine the mutual role of credit scoring, loan-to-value (LTV) ratio, and loan interest rate in separating borrowers by default risk in competitive and non-competitive mortgage markets.

The paper first shows that when borrowers' total financial resources are not perfectly observable to the lender, the latter can screen borrowers according to their default risk by their combined choice of LTV ratio and interest rate. This result stems from the fact that the wealthier the borrower is, the less costly it is for one to choose a low LTV ratio loan, ceteris paribus. In return, the borrower pays a lower interest rate.[2]

Moreover, when borrowers also signal their default risk by acquiring a credit score, then a combined separating signaling and screening equilibrium is attained. That is, borrowers are first partially separated into subsets according to their credit score,[3] and each subset is then screened by the choice of LTV ratio and interest rate to generate a combined fully separating signaling and screening equilibrium.

Ben-Shahar and Feldman (2003) develop a combined signaling and screening methodology, in which borrowers default risk is first partially separated by their credit record and then fully separated by their choice of a couplet of loan maturity and interest rate. They also show the conditions under which combined signaling and screening equilibrium (with credit record, loan maturity, and interest rate) Pareto dominates the screening equilibrium (with loan maturity and interest rate only) under a competitive market framework.

In contrast, credit scoring and the choice of LTV ratio and interest rate separate borrowers in the model presented here. Moreover, the analysis is extended to examine the effect of a non-competitive market environment on both equilibrium and welfare implications.

The paper shows that if the cost of the signaling mechanism (i.e., the credit scoring system) is sufficiently small, then the added welfare, gained by signaling, benefits borrowers (lenders) in a (non-) competitive market.[4]

It also follows from the analysis that in the combined signaling and screening equilibrium under both competitive and non-competitive mortgage markets, high and low risk borrowers, while acquiring distinct credit scores (which eventually affect their loan interest rate), might realize higher, lower, or identical LTV ratios. Hence, one implication of the model is that an empirical test of the relation between LTV ratio and default risk must incorporate the interrelation among the LTV ratio, credit score, and loan interest rate.[5]

Essentially, default in this framework is triggered by liquidity crunch; thus the degree of the default risk corresponds to the borrower's initial endowment. Lenders, however, cannot perfectly observe the borrower's endowment (which is only privately observed by the borrower) and, hence, cannot distinguish between borrowers' default risks. It is shown that, because low endowment is also associated with greater cost of both giving up on high LTV and establishing a good credit record, a correspondence emerges among credit scores, LTVs, and default risks. Moreover, it turns out that there exists an equilibrium under which borrowers' default risk is fully revealed by borrowers' attained credit score (signaling) and LTV (screening).

Stiglitz and Weiss (1981) were among the first to focus on the asymmetric information argument in credit markets to motivate rational credit rationing. They show that both the interest rate and the collateral on the loan may screen borrower's default risk. More recently, Brueckner (2000) and Harrison, Noordewier, and Yavas (2004) develop this intuition within the mortgage market framework.[6] Brueckner assumes that borrowers vary by default cost and that default follows a ruthless discretion. He derives a competitive separating screening equilibrium in which low (high) default cost—and thus riskier (safer)—borrowers self-select by choosing a high (low) LTV ratio for which they pay a greater (smaller) premium. Harrison et al. also examine how borrowers may be screened by their LTV ratio choice in a competitive environment. Specifically, they show that if default cost is sufficiently high (low), then a separating equilibrium is attained, where safer (riskier) borrowers self-select by choosing high LTV mortgage loans. Moreover, they show that there is a mid-level of default cost, which results in a pooling equilibrium. The required single crossing property for separation in this study is the borrower's probability of experiencing a reduction in income; borrowers with high probability are, of course, more likely to default. It should be noted, however, that our analysis differs from all of these studies, among other things, in considering the role of a credit scoring system and further focusing on non-competitive markets.

Studying mortgage origination under imperfect competition is of particular interest because of the frequently observed structure of mortgage markets worldwide. A growing body of literature presents empirical evidence of monopolistic power in the mortgage market of Canada (Heffernan, 1994), the United Kingdom (Devreux and Lanot, 2003; and Heffernan, 2006), the Netherlands (Swank, 1995), France (Gary-Bobo and Larribeau, 2004), as well as several other European Union countries (Neven and Roller, 1999). Furthermore, Hermalin and Jaffee (1996) claim that the two government-sponsored enterprises (GSEs)—Fannie Mae and Freddie Mac—effectively monopolize the secondary mortgage market in the United States, while Gan and Riddiough (2008, p. 6) add that the GSE sets ''the basis for the loan rates we see in the retail mortgage marketplace'' and ''as a result, the GSE in effect lends directly to consumers.'' According to Gan and Riddiough, the market dominance of the GSEs is achieved via regulatory barriers, government subsidies that are translated into lower cost of capital, and information advantage in credit evaluation. Hence, most retail mortgage originators simply ''passively apply the GSE's credit evaluation model to screen loan applicants and automatically sell qualified loans at guaranteed prices'' determined by the GSE. The authors conclude that ''despite a large number of retail mortgage lenders, vast segments of the U.S. residential mortgage market have strong monopolistic characteristics,'' (p. 4).[7] This growing evidence of the lack of competition in mortgage markets worldwide has led to an accumulating theoretical analysis of the non-competitive behavior of the players in the market and its implications; our study thus further extends this line of research (see, among others, Parlour and Rajan, 2001; Hyytinen, 2003; Niinimaki, 2004; Bond, Musto, and Yilmaz, 2006; and Gan and Riddiough, 2008).

The model is constructed in the next section. The screening equilibrium under competitive and non-competitive markets, respectively, is derived next, followed by the derivation of the combined signaling and screening equilibrium under competitive and non-competitive frameworks, respectively. Next, there is a comparison and examination of the welfare and distributional implications of the screening equilibrium and the combined signaling and screening equilibrium under both competitive and non-competitive frameworks. The paper closes with concluding remarks.

The Model

Consider a two-period world in which a risk-averse individual seeks to finance the purchase of an asset, the value of which equals A. The borrower is offered a mortgage loan in the first period, which the borrower is required to repay in full in the next period at a periodic interest rate cost of r. Denote the mortgage LTV ratio by L. Also, denote the borrower's initial endowment by w and suppose that in the second period the borrower's resources change to $w + x$, $x \in (\underline{x}, \overline{x})$, where x is a random variable with a differentiable density function $g(x)$ and some bounded variance and $\underline{x}$ $(\overline{x})$ is the minimum (maximum) realization of x.[8] The borrower's indirect utility function,[9] $\omega(\cdot)$, from the loan is then:

$$\omega(w, A, L, r) = u(w + LA) + \delta\left\{\int_{LA+LAr-w}^{\overline{x}} v(w + x - LA - LAr)g(x)dx + \int_{\underline{x}}^{LA+LAr-w} v(c)g(x)dx\right\}, \tag{1}$$

where $u(\cdot)$ and $v(\cdot)$ are the indirect utility functions in periods one and two, respectively, both assumed to exhibit positive first derivative and negative second derivative; w and $w + x$ are the individual's endowment in the first and second period, respectively; L is the LTV ratio; δ, $0 < \delta \le 1$, is a time preference factor; and c is a constant. Equation (1) asserts that for financing the purchase of an asset whose value equals A, the borrower first pays a sum equal to $(1 - L)A$ (i.e., obtains a net total of LA) in the first period. Then, in the second period, the borrower is required to repay $LA(1 + r)$. However, a repayment occurs only if the borrower's budget suffices, that is, if $w + x > LA(1 + r)$ [put differently, if x sustains $x > LA(1 + r) - w$]; otherwise, the borrower defaults.[10] In the latter case, the borrower is left with some fixed amount c.

Note that c is relatively small since it represents the residual wealth in the case that the borrower cannot repay the loan. It is thus assumed, without loss of generality, that $c = 0$.[11] Also, note that in order to rationalize the non-ruthless default on the part of the borrower (in the case where the asset value exceeds the loan value), it is further assumed that the transaction cost associated with selling the asset and repaying the loan is greater than the total cost of default.[12]

Then, by dividing the cash flow terms in Equation (1) by A, the arguments of the indirect utility function can be rewritten such that the purchased asset becomes the numéraire. That is, if $W = w/A$ and $X = x/A$, then the total indirect utility from the cash flow associated with a mortgaged asset—quantified in asset units—becomes:

$$\begin{aligned}\Omega(W, L, r) = U(W + L) \\ + \delta \left[\int_{L+Lr-W}^{\overline{X}} V(W + X - L - Lr) f(X) dX \right. \\ \left. + \int_{\underline{X}}^{L+Lr-W} V(0) f(X) dX \right],\end{aligned} \tag{2}$$

where $\Omega(\cdot)$, $U(\cdot)$, and $V(\cdot)$ is a transformation of $\omega(\cdot)$, $u(\cdot)$, and $v(\cdot)$, respectively, and $f(X)$ is a transformation of $g(x)$ when shifting from dollar units to the numéraire.

Now, denote the partial derivative of $U(W + L)$ and $V(W + X - L - Lr)$ by U' and V', respectively. Then, differentiating Equation (2) with respect to W produces, after reorganizing:

$$\frac{\partial \Omega}{\partial W} = U' + \delta \int_{L+Lr-W}^{\overline{X}} V' f(X) dX > 0. \tag{3}$$

The resulted inequality sign in Equation (3) implies that borrower utility increases, as expected, with initial endowment.

Likewise, differentiating $\Omega(\cdot)$ in Equation (2) with respect to r yields, after reorganizing:

$$\frac{\partial \Omega}{\partial r} = -\delta \int_{L+Lr-W}^{\overline{X}} V' L f(X) dX < 0, \tag{4}$$

and differentiating $\Omega(\cdot)$ in Equation (2) with respect to L yields, after reorganizing:

$$\frac{\partial \Omega}{\partial L} = U' - \delta \int_{L+Lr-W}^{\overline{X}} V'(1 + r)f(X)dX. \tag{5}$$

The middle expression in Equation (4) is always negative, which produces the inequality sign. That is, the borrower's utility is inversely affected by the loan interest rate. Furthermore, note that in Equation (5) for a sufficiently large value of U', the expression on the right-hand side is positive. Thus, consistent with conventional wisdom:

$$\frac{\partial \Omega}{\partial L} > 0. \tag{5a}$$

Equation (5) and the inequality in (5a) imply that the borrower generates particularly high utility from increasing the LTV ratio in the first period. Intuitively, on one hand, increasing the LTV ratio allows the borrower to purchase an asset that otherwise would have been unaffordable. On the other hand, increasing the LTV also raises the amount to be repaid. However, because of limited liability, repayment occurs only when the borrower's financial situation is relatively favorable. Consistent with conventional wisdom, it is assumed that the former effect overpowers the latter.

Now, denote the second partial derivative of $U(W + L)$ and $V(W + X - L - Lr)$ by U'' and V'', respectively. Then, twice differentiating $\Omega(\cdot)$ with respect to r and W produces:

$$\frac{\partial^2 \Omega}{\partial r \partial W} = -\delta\left\{V'(0)Lf(L + Lr - W) + \int_{L+Lr-W}^{\overline{X}} V''Lf(X)dX\right\}. \tag{6}$$

Note that the sign of $\partial^2\Omega/\partial r\partial W$ is ambiguous. That is, on one hand, due to risk aversion, $V'' < 0$. On the other hand, raising W increases the no-default probability and, thus, the probability that the required interest will eventually be repaid. It is therefore assumed that one effect cancels out the other,[13] that is:

$$\frac{\partial^2 \Omega}{\partial r \partial W} = 0. \tag{6a}$$

Similarly, twice differentiating $\Omega(\cdot)$ with respect to L and W generates:

$$\frac{\partial^2 \Omega}{\partial L \partial W} = U'' - \delta \Bigg\{ V'(0)(1 + r)f(L + Lr - W) + \int_{L+Lr-W}^{\overline{X}} V''(1 + r)f(X)dX \Bigg\}. \tag{7}$$

Again, the sign of the right-hand side of Equation (7) is indefinite. On one hand, due to risk aversion, the second derivative of the periodic indirect utility functions, U'' and V'', is negative. On the other hand, the probability of no-default rises with W and falls with L. According to conventional wisdom, however, the lower the initial endowment is, the greater are the benefits from increasing LTV. That is:

$$\frac{\partial^2 \Omega}{\partial L \partial W} < 0. \tag{7a}$$

Consider now a risk-neutral lender. The present value of the lender's profit function from the loan, $\pi(\cdot)$, is:

$$\pi(w, L, A, r, I) = -LA + \frac{1}{1 + r_f} \times \left[\int_{LA+LAr-w}^{\overline{x}} LA(1 + r)g(x)dx + \int_{\underline{x}}^{LA+LAr-w} Ig(x)dx \right], \tag{8}$$

where r_f, $r_f < r$, is the risk-free rate of return and I is the lender's income in the case that the borrower experiences default. Equation (8) states that the lender's profit consists of the first period cash outflow, $-LA$, and the second period income is $LA(1 + r)$ if no default occurs and I otherwise, all discounted by one plus the risk-free rate.[14]

In order to prevent arbitrage profits on the part of the lender, assume that $I < LA(1 + r_f)$. Hence, without loss of generality, let $I = 0$. Therefore, rewriting the lender's profit in Equation (8) in units of the numéraire (by dividing all cash flows by the cost of the purchased asset, A) produces:

$$\Pi(W, L, r) = -L + \frac{1}{1 + r_f} L(1 + r) \int_{L+Lr-W}^{\overline{X}} f(X)dX, \tag{9}$$

where $\Pi(\cdot)$ is a transformation of $\pi(\cdot)$ when shifting from dollars to the numèraire, $\Pi(\cdot) = \pi(\cdot)/A$, $X = x/A$, and $f(X)$ is a transformation of $g(x)$.

Now, differentiating $\Pi(\cdot)$ with respect to W yields:

$$\frac{\partial \Pi}{\partial W} = \frac{1}{1 + r_f} L(1 + r)f(L + Lr - W) > 0. \tag{10}$$

The right-hand side of Equation (10) is always positive, implying that the lender's profit rises with the borrower's endowment. Intuitively, a rise in the level of W reduces the default probability, thereby increasing the lender's expected revenue, ceteris paribus.

Likewise, differentiating $\Pi(\cdot)$ with respect to r generates:

$$\frac{\partial \Pi}{\partial r} = \frac{L}{1 + r_f} \left[\int_{L+Lr-W}^{\overline{X}} f(X)dX - L(1 + r)f(L + Lr - W) \right], \tag{11}$$

and differentiating $\Pi(\cdot)$ with respect to L produces:

$$\frac{\partial \Pi}{\partial L} = -1 + \frac{1 + r}{1 + r_f} \times \left[\int_{L+Lr-W}^{\overline{X}} f(X)dX - L(1 + r)f(L + Lr - W) \right]. \tag{12}$$

The sign of the right-hand side of Equation (11) is ambiguous. Increasing the rate of return, r, raises the value repaid by the borrower and therefore increases profits. Concurrently, however, a greater r is associated with an increased probability of default, which decreases profits. It is assumed, according to market conventional wisdom, that the interest rate on the loan is in the range where the former effect overpowers the latter, implying that the rate of return on the loan positively affects profits.[15] That is:

$$\frac{\partial \Pi}{\partial r} > 0. \tag{11a}$$

The sign of the right-hand side of Equation (12) is also indefinite. On one hand, increasing L raises the total amount lent, thereby increasing the present value of the total net return if no-default occurs, $[(1 + r)/(1 + r_f) - 1]L$. On the other hand, increasing L raises the probability of the borrower's default, which diminishes lender's expected income. Thus, it is assumed that the level of L is in the range where increasing its value, ceteris paribus, produces an expected loss to the lender.[16] Hence:

$$\frac{\partial \Pi}{\partial L} < 0. \tag{12a}$$

Screening Equilibrium in a Competitive Market

Consider now a mortgage market with two types of borrowers, denoted by 1 and 2, who are differentiated by their initial endowment (in units of the numéraire), W. Let W_1 (W_2) be the initial endowment of borrower 1 (2), where $W_1 > W_2$.

Also, suppose that from the lenders perspective, both borrowers' second period wealth shock, X, arises ex ante from the same distribution.[17] Finally, suppose γ is the share of type 2 borrowers in the total borrowing population.

One can see from Equation (2) that borrower 2 is more likely to default than borrower 1, ceteris paribus. Suppose that each borrower's initial endowment is privately observed by the borrower and is unobservable to lenders.[18] Then, lenders cannot distinguish between the borrowers' default risk. Under competitive market conditions, however, borrowers can be screened by a self-selection process. That is, suppose (L_1, r_1) and (L_2, r_2) represent a combination of the LTV ratio and the interest rate on the loan offered by lenders and selected by borrower 1 and 2, respectively. Then:

Proposition 1: *There exists a competitive screening equilibrium in which the higher the borrower's default risk is, the greater are the LTV ratio and the interest rate on the loan. That is,* $L_2 > L_1$ *and* $r_2 > r_1$.

Proof: Given L_1, L_2, r_1, and r_2, and following Equations (2) and (9), a self-selection process occurs in a competitive market if the lender generates zero profit from the offered contracts:

$$\begin{aligned}\Pi(W_2, L_2, r_2) &= -L_2 + \frac{1}{1+r_f} L_2(1+r_2) \int_{L_2+L_2r_2-W_2}^{\overline{X}} f(x)dx \\ &= -L_1 + \frac{1}{1+r_f} L_1(1+r_1) \\ &\times \int_{L_1+L_1r_1-W_1}^{\overline{X}} f(x)dx = \Pi(W_1, L_1, r_1) = 0, \qquad (13)\end{aligned}$$

where borrower 1 chooses the combination of L_1 and r_1 over the alternative:

$$\begin{aligned}\Omega(W_1, L_1, r_1) &= U(W_1 + L_1) \\ &+ \delta\left[\int_{L_1+L_1r_1-W_1}^{\overline{X}} V(W_1 + X - L_1 - L_1r_1)f(X)dX\right. \\ &+ \left.\int_{\underline{X}}^{L_1+L_1r_1-W_1} V(0)f(X)dX\right] > U(W_1 + L_2) \\ &+ \delta\left[\int_{L_2+L_2r_2-W_1}^{\overline{X}} V(W_1 + X - L_2 - L_2r_2)f(X)dX\right. \\ &+ \left.\int_{\underline{X}}^{L_2+L_2r_2-W_1} V(0)f(X)dX\right] = \Omega(W_1, L_2, r_2), \qquad (14)\end{aligned}$$

and borrower 2 chooses the combination of L_2 and r_2 over the alternative:

$$\begin{aligned}\Omega(W_2, L_2, r_2) &= U(W_2 + L_2) \\ &+ \delta\left[\int_{L_2+L_2r_2-W_2}^{\overline{X}} V(W_2 + X - L_2 - L_2r_2)f(X)dX\right. \\ &+ \left.\int_{\underline{X}}^{L_2+L_2r_2-W_2} V(0)f(X)dX\right] \geq U(W_2 + L_1) \\ &+ \delta\left[\int_{L_1+L_1r_1-W_2}^{\overline{X}} V(W_2 + X - L_1 - L_1r_1)f(X)dX\right. \\ &+ \left.\int_{\underline{X}}^{L_1+L_1r_1-W_2} V(0)f(X)dX\right] = \Omega(W_2, L_1, r_1). \qquad (15)\end{aligned}$$

Given that $W_1 > W_2$ and following inequalities (10), (11a), and (12a), for Equation (13) to hold, it cannot be the case that both $r_1 \geq r_2$ and $L_1 \leq L_2$. Also, following inequalities (4) and (5a), for inequality (14) to hold, it may not be the case that both $r_1 \geq r_2$ and $L_1 \leq L_2$. Likewise, following inequalities (4) and (5a), then for inequality (15) to hold, it cannot be the case that both $r_2 > r_1$ and $L_2 \leq L_1$. Finally, it follows from inequalities (6a) and (7a) that for inequalities (14) and (15) to simultaneously hold, it cannot be the case that both $r_1 \geq r_2$ and $L_1 \geq L_2$. Hence, $r_2 > r_1$ and $L_2 > L_1$. □

Proposition 1 is consistent with the result obtained in Brueckner (2000) and in Harrison, Noordewier, and Yavas (2004) for the case where default costs are low. Intuitively, due to a lower initial endowment, it is more valuable for the riskier borrower to obtain a high LTV ratio than it is for the safer borrower, ceteris paribus. The high LTV ratio allows the borrower with the smaller initial endowment (and who thus experiences a greater default risk) to seek an asset that otherwise would have remained implausible. Put differently, it is less costly for the safer borrower who experiences a greater initial endowment to decrease the demanded LTV ratio on the loan. In return, however, the latter pays a lower interest rate.

Exhibit 1 in Appendix A demonstrates the existence of the equilibrium stated in Proposition 1. In this example, consider a four-borrower framework, where $W_1 > W_2 > W_3 > W_4$. The actual numbers in Exhibit 1 are of course imaginary and are for illustration only. They show, however, the concept under which the lower initial endowment is associated with greater LTV, higher interest rate, and greater default risk in equilibrium.

Screening Equilibrium in a Non-Competitive Market

Screening equilibrium can also prevail under imperfect competition market conditions.[19] For simplicity, consider a mortgage market with two lenders: *j* and *k*.

Proposition 2: *There exists a duopolistic Nash screening equilibrium in which the higher the borrower's default risk is, the greater are the LTV ratio and the interest rate on the loan. That is,* $L_2 > L_1$ *and* $r_2 > r_1$.

Proof: Let the lender's common knowledge strategy be to offer $(r_i, L_i) = (\hat{r}_i, \hat{L}_i)$, $i = \{1, 2\}$, if the other lender offers $(r_i, L_i) = (\hat{r}_i, \hat{L}_i)$, where $(\hat{r}_i, \hat{L}_i)$ is a combination of interest rate and LTV that maximizes the lender's profit, given the other lender's strategy and given the borrower types in the market [see Equations (13a)–(15a)]; otherwise, if the other lender offers $(r_i, L_i) = (\bar{r}_i, \bar{L}_i)$, where $(\bar{r}_i, \bar{L}_i) \succ_i (\hat{r}_i, \hat{L}_i)$ for some i and $\Pi(\bar{r}_i, \bar{L}_i, \cdot) \geq 0$, then offer $(r_i, L_i) = (\bar{r}_i, \bar{L}_i)$.[20] Let borrower i's strategy be to accept the best offer of (r_i, L_i), such that (r_i, L_i) is weakly preferred to $(\hat{r}_i, \hat{L}_i)$ by borrower i, where $(\hat{r}_i, \hat{L}_i)$ represents a locus generating utility level no less than $\hat{\Omega}_i$; otherwise, reject.

The specified strategies are consistent with the following conditions for a separating equilibrium under the duopolistic market structure. Lender j maximizes profits under the constraint imposed by the other lender and the borrowers. That is, lender j:

$$\begin{aligned}
&\underset{L_1,L_2,r_1,r_2}{Max}\ \Pi_j(W_1, W_2, L_1, L_2, r_1, r_2, \gamma)\\
&\quad = \gamma\left[-L_2 + \frac{1}{1 + r_f} L_2(1 + r_2)\int_{L_2+L_2r_2-W_2}^{\overline{X}} f(x)dx\right]\\
&\quad + (1 - \gamma)\left[-L_1 + \frac{1}{1 + r_f} L_1(1 + r_1)\int_{L_1+L_1r_1-W_1}^{\overline{X}} f(x)dx\right],
\end{aligned} \tag{13a}$$

subject to lender k:

$$\begin{aligned}
&\Pi_k(W_1, W_2, L_1, L_2, r_1, r_2, \gamma)\\
&\quad = \gamma\left[-L_2 + \frac{1}{1 + r_f} L_2(1 + r_2)\int_{L_2+L_2r_2-w_2}^{\overline{X}} f(x)dx\right]\\
&\quad + (1 - \gamma)\left[-L_1 + \frac{1}{1 + r_f} L_1(1 + r_1)\int_{L_1+L_1r_1-W_1}^{\overline{X}} f(x)dx\right],
\end{aligned} \tag{13b}$$

and borrower 1 chooses the combination of L_1 and r_1 over the alternative:

$$\begin{aligned}
&\hat{\Omega}_1 \leq \Omega(W_1, L_1, r_1) = U(W_1 + L_1)\\
&\quad + \delta\left[\int_{L_1+L_1r_1-W_1}^{\overline{X}} V(W_1 + X - L_1 - L_1r_1)f(X)dX\right.\\
&\quad \left. + \int_{\underline{X}}^{L_1+L_1r_1-W_1} V(0)f(X)dX\right] > U(W_1 + L_2)\\
&\quad + \delta\left[\int_{L_2+L_2r_2-W_1}^{\overline{X}} V(W_1 + X - L_2 - L_2r_2)f(X)dX\right.\\
&\quad \left. + \int_{\underline{X}}^{L_2+L_2r_2-W_1} V(0)f(X)dX\right] = \Omega(W_1, L_2, r_2),
\end{aligned} \tag{14a}$$

and borrower 2 chooses the combination of L_2 and r_2 over the alternative:

$$\begin{aligned}
\hat{\Omega}_2 \leq \Omega(W_2, L_2, r_2) &= U(W_2 + L_2) \\
&+ \delta\left[\int_{L_2+L_2r_2-W_2}^{\overline{X}} V(W_2 + X - L_2 - L_2r_2)f(X)dX\right. \\
&+ \left.\int_{\underline{X}}^{L_2+L_2r_2-W_2} V(0)f(X)dX\right] \geq U(W_2 + L_1) \\
&+ \delta\left[\int_{L_1+L_1r_1-W_2}^{\overline{X}} V(W_2 + X - L_1 - L_1r_1)f(X)dX\right. \\
&+ \left.\int_{\underline{X}}^{L_1+L_1r_1-W_2} V(0)f(X)dX\right] = \Omega(W_2, L_1, r_1),
\end{aligned} \tag{15a}$$

where Π_j and Π_k are lender j's and k's profit, respectively, $\hat{\Omega}_i = \Omega(W_i, \hat{L}_i, \hat{r}_i)$, $i = \{1, 2\}$, and γ is the share of type 2 borrowers in the total borrowing population. Due to the complexity of this problem, a numerical solution is offered in order to demonstrate the existence of the equilibrium stated in Proposition 2. The demonstration is presented in Exhibit 2 in Appendix A for a four-borrower case, where $W_1 > W_2 > W_3 > W_4$. □

That is, under imperfect competition market conditions, the lenders maximize profits subject to the reaction function of other lenders in the market and subject to screening borrowers, while providing them a given level of utility. Proposition 2 shows that due to limited competition, the lenders can separate borrowers, such that $r_2 > r_1$ and $L_2 > L_1$, while extracting a greater share of the consumer surplus relative to the competition case (compare Exhibits 1 and 2 in Appendix A).

Combined Signaling and Screening Equilibrium in a Competitive Market

Now, suppose that prior to obtaining a mortgage loan, a borrower accumulates a credit history. Furthermore, when applying for a loan, the borrower is required to present the credit record to lenders and thereby attains a credit score.[21]

A good credit history, however, is attained at a cost: the higher the credit score, the more costly it is for the borrower to establish it. Consistent, for example, with Ben-Shahar and Feldman (2003), the cost of establishing a good credit record follows from the borrower's need to perform well in all pre-mortgage loans. The borrower is required, for example, to limit the accounts opened within the past twelve months, to maintain the balances on revolving credit sufficiently distant from the maximum limit, to maintain good public records (such as tax liens, judgments, or bankruptcies), to retain no recent credit card balances, to limit the amount of recent credit inquiries, etc.[22]

Denote the cost (in units of the numéraire) of establishing a credit history that generates a credit score by s, where a greater s is associated with a better credit score. Then, in the presence of a credit scoring system, Equation (2) becomes:

$$\begin{aligned}\Omega(W, L, r) = {} & U\left(W + L - \frac{s}{W}\right) \\ & + \delta\left[\int_{L+Lr-W}^{\overline{X}} V(W + X - L - Lr)f(X)dX\right. \\ & + \left.\int_{\underline{X}}^{L+Lr-W} V(0)f(X)dX,\right.\end{aligned} \tag{2a}$$

where the only difference between Equations (2) and (2a) is the subtraction of the cost of acquiring a credit score, s/W, in the latter equation as an argument of the first period indirect utility function. That is, the cost associated with maintaining a given credit record (and, thereby, attaining the corresponding credit score) declines with the level of one's initial wealth. Partially differentiating Equation (2a) with respect to s generates:

$$\frac{\partial\Omega}{\partial s} = -\frac{1}{W}U'\left(W + L - \frac{s}{W}\right) < 0, \tag{16}$$

where the inequality sign follows from the positive first derivative of the first period indirect utility function.

Further, twice differentiating Equation (2a) with respect to s and W yields:

$$\begin{aligned}\frac{\partial^2\Omega}{\partial s\partial W} = {} & \frac{1}{W^2}U'\left(W + L - \frac{s}{W}\right) \\ & - \frac{1}{W}U''\left(W + L - \frac{s}{W}\right)\left(1 + \frac{s}{W^2}\right) > 0.\end{aligned} \tag{17}$$

That is, the marginal cost (in utility terms) of increasing the credit score declines with the initial endowment.

Now, for simplicity, suppose that two feasible credit scores with costs s_h and s_l can be acquired by borrowers, where $s_h > s_l$. Thus, s_h (s_l) corresponds to a better (worse) credit score. Further, consider a case where there are four borrowers in the market, who are differentiated by their initial endowment, W. Let W_i be the privately observed initial endowment of borrower i, $i = \{1, 2, 3, 4\}$, where $W_1 > W_2 > W_3 > W_4$. It follows from Equation (2a) that the probability of default rises when shifting from borrower 1 to borrower 4, ceteris paribus.

Denote the LTV ratio, loan interest rate, and credit score selected by borrower i by L_i, r_i, $i = \{1, 2, 3, 4\}$, and s_m, $m = \{h, l\}$, respectively.

Proposition 3: *There exists a competitive combined signaling and screening equilibrium in which, within the entire borrowing population, safer (riskier) borrowers acquire the better (worse) credit score. Furthermore, within each subset of borrowers with identical credit scores, safer (riskier) borrowers attain a loan with a lower (higher) LTV ratio and a lower (higher) interest rate. That is,* $s_1 = s_2 = s_h$, $s_3 = s_4 = s_l$, $L_4 > L_3$, $L_2 > L_1$, $r_4 > r_3$, *and* $r_2 > r_1$.

Proof: See formulization of the equilibrium conditions of Proposition 3 in Appendix B. Also, due to the complexity of the problem, the existence of the equilibrium is demonstrated by a numerical solution in Exhibit 3 in Appendix A. □

Proposition 3 asserts that under a combined signaling and screening equilibrium, borrowers are first partially separated into subsets according to their signal (credit score). Then, depending on their credit score, they are offered a menu of mortgage loans varying by LTV ratio and interest rate, by which a self-selection process occurs and a fully revealing equilibrium is attained.

Specifically, the subset of the safer borrowers (types 1 and 2) is separated from the subset of the riskier borrowers (types 3 and 4) by signaling: types 1 and 2 acquire a higher credit score than that acquired by types 3 and 4.[23] Moreover, within the same credit score subset, types are screened by their choice combination of LTV ratio and interest rate. As shown in the screening-only equilibrium presented earlier, in each subset, the borrower with the greater probability of default opts for the loan with the higher LTV ratio and the greater interest rate.

Hence, for example, borrowers 1 and 2, who both choose the better credit score at a cost of s_h (see the example in Exhibit 3, where $s_h = 0.019$) are separated by their distinct choice combination of LTV ratio and interest rate, where borrower 1 (2) selects a loan with a lower (higher) LTV ratio and a lower (higher) interest rate than that chosen by borrower 2 (1).

Furthermore, borrowers who acquire different signals (different credit scores) are separated into different subsets (classified into different risk categories). In equilibrium, however, it must hold that loans offered to borrowers in one subset will not motivate others to shift to that subset by selecting a different signal.

An empirical implication of the combined signaling and screening equilibrium is that the LTV is positively related to default risk only within each subset of credit scores but not over the entire population. In Exhibit 3, for example, one can see that borrowers 2 and 4 opt for a higher LTV than that chosen by borrowers 1 and 3, respectively. Yet, borrower 3's LTV is, at the same time, lower than that of borrowers 1 and 2, despite the former's greater default risk. This situation becomes possible due to the use of credit scores by which borrower 3 is categorized into the subset of the higher default risks. In this higher risk category, borrowers simply pay higher interest rates, ceteris paribus.[24]

Combined Signaling and Screening Equilibrium in a Non-Competitive Market

Combined signaling and screening equilibrium also attains under imperfect competition market conditions.

Proposition 4: *There exists duopolistic Nash combined signaling and screening equilibrium in which, within the entire borrowing population, safer (riskier) borrowers acquire the better (worse) credit score. Furthermore, within each subset of borrowers with identical credit scores, safer (riskier) borrowers attain a loan with a lower (higher) LTV ratio and a lower (higher) interest rate. That is,* $s_1 = s_2 = s_h$, $s_3 = s_4 = s_l$, $L_4 > L_3$, $L_2 > L_1$, $r_4 > r_3$, *and* $r_2 > r_1$.

Proof: The formulization of the necessary conditions of the equilibrium described in Proposition 4 is immediate following that of Propositions 2 and 3. Due to the complexity of the problem, the existence of the equilibrium is demonstrated by a numerical solution in Exhibit 4 in Appendix A. □

Proposition 3 argues that the combined signaling and screening equilibrium could be attained in a competitive mortgage market. Proposition 4 complements this argument by claiming that the combined signaling and screening equilibrium could also be attained under non-competitive conditions.

Welfare and Income Distribution Implications

It follows from Propositions 3 and 4:

Corollary: *Given a credit scoring system, both safer and riskier borrowers might achieve either high or low LTV ratios.*

Proof: Due to the complexity of the problem, the proof is shown by demonstration in Exhibits 3 and 4 in Appendix A. Note that under both competitive (Exhibit 3) and non-competitive (Exhibit 4) market frameworks, a combined signaling and screening equilibrium can be attained where, for example, $W_1 > W_2 > W_3$ and yet $L_2 > L_1 > L_3$. That is, a safer borrower attains either a higher or a lower LTV ratio.

Because of the partial separation generated by credit scores and the distribution into subsets that follows, borrowers who, *ex ante*, exhibit different levels of default risk, and who thus fall into a different subset of borrowers, might be offered either greater, lower, or identical LTV ratios in equilibrium. In that case, the difference in their ex ante default risk is only incorporated into the cost of their loan, that is, the requested interest rate.

The Corollary therefore implies that empirical research that examines the relationship between default risk and the attained interest rate must also incorporate the interrelation of the credit score.

Ben-Shahar and Feldman (2003) show the conditions under which the competitive combined signaling and screening equilibrium (with credit record, loan maturity, and interest rate) Pareto dominates the competitive screening equilibrium (with loan maturity and interest rate). Similarly, the model here shows that while the welfare of all lenders and some borrowers does not alter when shifting from screening equilibrium to combined signaling and screening equilibrium under competitive market conditions, the welfare of other borrowers may increase.

Furthermore, when shifting from a screening equilibrium to a combined signaling and screening equilibrium under imperfect competition market conditions, the produced gain reallocates in favor of lenders.

Proposition 5: *If signaling cost is sufficiently small, then in a competitive market, the combined signaling and screening equilibrium Pareto dominates the screening equilibrium. Moreover, while lenders are indifferent between the two equilibria, borrowers are better off under the combined signaling and screening equilibrium.*

Proposition 6: *If signaling cost is sufficiently small, then in a duopolistic market, the combined signaling and screening equilibrium Pareto dominates the screening equilibrium. Moreover, while borrowers are indifferent between the two equilibria, lenders are better off under the combined signaling and screening equilibrium.*

Proof: Due to the complexity of the problem, the proof is shown by demonstration in the exhibits in Appendix A. First, comparing Exhibits 1 and 3, where a numerical example is presented for the screening equilibrium and the combined signaling and screening equilibrium, respectively, one can immediately see that while lenders maintain zero profits under both competitive frameworks, borrowers experience a Pareto improvement shifting from the screening equilibrium (see Exhibit 1) to the combined signaling and screening equilibrium (see Exhibit 3). Particularly, focusing on the utility level, $\Omega(W_i, L_i, r_i)$, note that while borrower 3 and 4 maintain their utility level, borrowers 1 and 2 experience an increase in their utility. Similarly, comparing Exhibits 2 and 4, one can see that while all borrowers attain the same utility level when shifting from the screening equilibrium (see Exhibit 2) to the combined signaling and screening equilibrium (see Exhibit 4), lenders yet generate higher total profits under the latter (particularly, the profit generated from borrowers 1 and 2 rises from 0.05 to 0.12). □

The essential idea underlying Propositions 5 and 6 is that total welfare may increase when complementing the screening mechanism (the combination of LTV and interest rate menus) with a credit scoring system. If borrower types are first grouped into subsets according to their credit scores, then when progressing to the screening stage, the "good" (safer) borrowers in each post-signaling subset need only to separate themselves from the members of the subset as opposed to separating themselves from the entire population. According to Propositions 5 (6), the gain that derives under this altered structure will go to the borrower (lender), if the market is competitive (non-competitive).

Conclusion

The task of empirical and theoretical studies of mortgage default is, among other things, to improve the pricing of default risk, which is necessary for the efficient operation of both primary and secondary mortgage markets. In this context, the analysis of mortgage default is extended under asymmetric information. Most importantly, it is shown that under a system of credit scoring and a menu of LTV ratios and interest rates, a combined signaling and screening equilibrium is attained, where borrowers first signal their default risk by selecting a credit score, resulting in same-credit-score sub-groups of borrowers being formed. Then, lenders screen each sub-group by offering different combinations of LTV ratio and interest rate, thus producing a fully separating combined signaling and screening equilibrium.

The paper demonstrates the prevalence of the combined signaling and screening equilibrium under both competitive and non-competitive market frameworks. Moreover, the welfare and income distribution effect of supplementing a credit scoring system is examined under these two market settings. Ben-Shahar and Feldman (2003) show that borrowers are the beneficiary party from the establishment of a credit record signaling system (when screening by loan maturity and interest rate prevails) under a competitive market framework. This result is extended here by showing that lenders become the benefiting party from the establishment of a credit scoring signaling system (in addition to screening by a menu of LTV and interest rate) under an imperfect competition market setup.

While default in the model occurs due to liquidity crunch, it is possible to employ a similar setting, where default follows ruthless discretion. In that case, the value of the property (as opposed to borrower income) becomes the random variable, and default occurs if the latter falls below the loan balance. Future research may address this approach.

Furthermore, empirical research finds indecisive correlation between LTV ratio and default probability.[25] The model presented here suggests that for testing the relationship between LTV ratio and the propensity to default, one must incorporate the interrelation among LTV ratio, credit score, and interest rate. Specifically, it is predicted that within each same-credit-score sub-group, there is a positive correlation between LTV ratio and default probability. However, if categorization by credit scores is omitted from the analysis, one might falsely detect that no such relationship exists.

Appendix A

Exhibit 1: Demonstrating the existence of the equilibrium stated in Proposition 1, where $U(\cdot) = \ln(\cdot)$, $V(\cdot) = \ln(\cdot)$, and $f(\cdot)$ is a uniform density function on the domain [0, 3]. That is, $X \sim U(0, 3)$. In addition, $r_f = 0.02$ and $\delta = 0.9$. $\Omega(W_i, L_i, r_i)$ represents borrower i's indirect utility from own mortgage loan and $\Omega(W_i, L_j, r_j)$, $\Omega(W_i, L_k, r_k)$, and $\Omega(W_i, L_m, r_m)$ represent i's indirect utility from the mortgage selected by borrowers j, k, and m, respectively, where $j, k, m \neq i$ and appear in an increasing order.

	Borrower 1	Borrower 2	Borrower 3	Borrower 4
W_i	0.75	0.65	0.55	0.45
L_i	0.790	0.806	0.829	0.876
r_i	0.047	0.111	0.196	0.349
$\Omega(W_i, L_i, r_i)$	0.9144	0.8317	0.7440	0.6507
$\Omega(W_i, L_j, r_j)$	0.9141	0.8316	0.7434	0.6490
$\Omega(W_i, L_k, r_k)$	0.9133	0.8313	0.7439	0.6500
$\Omega(W_i, L_m, r_m)$	0.9106	0.8292	0.7429	0.6507
$\Pi(W_i, L_i, r_i)$	0	0	0	0
$\int_{L_i+L_ir_i-W_i}^{\bar{X}} f(X)dX$	0.974	0.918	0.853	0.756

Exhibit 2: Demonstrating the existence of the equilibrium stated in Proposition 2, where $U(\cdot) = \ln(\cdot)$, $V(\cdot) = \ln(\cdot)$, and $f(\cdot)$ is a uniform density function on the domain [0, 3]. In addition, $r_f = 0.02$, $\delta = 0.9$, and each borrower type contains the same number of borrowers. $\Omega(W_i, L_i, r_i)$ represents borrower i's indirect utility from own mortgage loan and $\Omega(W_i, L_j, r_j)$, $\Omega(W_i, L_k, r_k)$, and $\Omega(W_i, L_m, r_m)$ represent i's indirect utility from the mortgage selected by borrowers j, k, and m, respectively, where $j, k, m \neq i$ and appear in an increasing order.

	Borrower 1	Borrower 2	Borrower 3	Borrower 4
W_i	0.75	0.65	0.55	0.45
L_i	0.729	0.743	0.763	0.795
r_i	0.113	0.177	0.260	0.385
$\Omega(W_i, L_i, r_i)$	0.8764	0.7911	0.7005	0.6036
$\Omega(W_i, L_j, r_j)$	0.8763	0.7908	0.6994	0.6012
$\Omega(W_i, L_k, r_k)$	0.8758	0.7910	0.7001	0.6025
$\Omega(W_i, L_m, r_m)$	0.8736	0.7893	0.6996	0.6036
$\Pi(W_i, L_i, r_i)$	0.05	0.05	0.05	0.05
$\int_{L_i+L_ir_i-W_i}^{\bar{X}} f(X)dX$	0.980	0.925	0.863	0.783

Exhibit 3: Demonstrating the existence of the equilibrium stated in Proposition 3, where $U(\cdot) = \ln(\cdot)$, $V(\cdot) = \ln(\cdot)$, and $f(\cdot)$ is a uniform density function on the domain [0, 3]. In addition, $r_f = 0.02$ and $\delta = 0.9$. $\Omega(W_i, L_i, r_i)$ represents borrower i's indirect utility from own mortgage loan and $\Omega(W_i, L_j, r_j)$, $\Omega(W_i, L_k, r_k)$, and $\Omega(W_i, L_m, r_m)$ represents i's indirect utility from the mortgage selected by borrowers j, k, and m, respectively, where $j, k, m \neq i$ and appear in an increasing order.

	Borrower 1	Borrower 2	Borrower 3	Borrower 4
W_i	0.75	0.65	0.55	0.45
L_i	0.842	0.865	0.829	0.876
r_i	0.076	0.154	0.196	0.349
s_i	0.019	0.019	0	0
$\Omega(W_i, L_i, r_i)$	0.9193	0.8347	0.7440	0.6507
$\Omega(W_i, L_j, r_j)$	0.9190	0.8346	0.7432	0.6432
$\Omega(W_i, L_k, r_k)$	0.9133	0.8313	0.7439	0.6447
$\Omega(W_i, L_m, r_m)$	0.9106	0.8292	0.7429	0.6507
$\Pi(W_i, L_i, r_i)$	0	0	0	0
$\int_{L_i+L_ir_i-W_i}^{\bar{X}} f(X)dX$	0.948	0.884	0.853	0.756

Exhibit 4: Demonstrating the existence of the equilibrium stated in Proposition 4, where $U(\cdot) = \ln(\cdot)$, $V(\cdot) = \ln(\cdot)$, and $f(\cdot)$ is a uniform density function on the domain [0,3]. In addition, $r_f = 0.02$, $\delta = 0.9$, and each borrower type contains the same number of borrowers. $\Omega(W_i, L_i, r_i)$ represents borrower i's indirect utility from own mortgage loan and $\Omega(W_i, L_j, r_j)$, $\Omega(W_i, L_k, r_k)$, and $\Omega(W_i, L_m, r_m)$

represents i's indirect utility from the mortgage selected by borrowers j, k, and m, respectively, where j, k, $m \neq i$ and appear in an increasing order.

	Borrower 1	Borrower 2	Borrower 3	Borrower 4
W_i	0.75	0.65	0.55	0.45
L_i	0.816	0.909	0.763	0.795
r_i	0.301	0.534	0.260	0.385
s_i	0.019	0.019	0	0
$\Omega(W_i, L_i, r_i)$	0.8764	0.7911	0.7005	0.6036
$\Omega(W_i, L_j, r_j)$	0.8763	0.7900	0.6956	0.5928
$\Omega(W_i, L_k, r_k)$	0.8758	0.7910	0.6993	0.5994
$\Omega(W_i, L_m, r_m)$	0.8736	0.7893	0.6996	0.6036
$\Pi(W_i, L_i, r_i)$	0.12	0.12	0.05	0.05
$\int_{L_i+L_i r_i-W_i}^{\bar{X}} f(X)dX$	0.895	0.752	0.863	0.783

Appendix B

Equilibrium conditions of Proposition 3. Following Equations (2a) and (9), a combined signaling and screening separation by credit score, LTV ratio, and interest rate in a competitive market is attained in equilibrium when the following is maintained:

The lender generates zero profits from all offered contracts:

$$\Pi(W_1, L_1, r_1) = \Pi(W_2, L_2, r_2) = \Pi(W_3, L_3, r_3) = \Pi(W_4, L_4, r_4) = 0, \tag{A1}$$

where borrower 1 chooses the combination of s_h, L_1, and r_1 over the three alternatives:

$$\Omega(W_1, L_1, r_1, s_h) = U\left(W_1 + L_1 - \frac{s_h}{W_1}\right)$$

$$+ \delta\left[\int_{L_1+L_1r_1-W_1}^{\overline{X}} V(W_1 + X - L_1 - L_1r_1)f(X)dX\right.$$

$$\left. + \int_{\underline{X}}^{L_1+L_1r_1-W_1} V(0)f(X)dX\right] > U\left(W_1 + L_2 - \frac{s_h}{W_1}\right)$$

$$+ \delta\left[\int_{L_2+L_2r_2-W_1}^{\overline{X}} V(W_1 + X - L_2 - L_2r_2)f(X)dX\right.$$

$$\left. + \int_{\underline{X}}^{L_2+L_2r_2-W_1} V(0)f(X)dX\right] = \Omega(W_1, L_2, r_2, s_h), \tag{A2a}$$

$$\Omega(W_1, L_1, r_1, s_h) = U\left(W_1 + L_1 - \frac{s_h}{W_1}\right)$$

$$+ \delta\left[\int_{L_1+L_1r_1-W_1}^{\overline{X}} V(W_1 + X - L_1 - L_1r_1)f(X)dX\right.$$

$$\left. + \int_{\underline{X}}^{L_1+L_1r_1-W_1} V(0)f(X)dX\right] > U\left(W_1 + L_3 - \frac{s_l}{W_1}\right)$$

$$+ \delta\left[\int_{L_3+L_3r_3-W_1}^{\overline{X}} V(W_1 + X - L_3 - L_3r_3)f(X)dX\right.$$

$$\left. + \int_{\underline{X}}^{L_3+L_3r_3-W_1} V(0)f(X)dX\right] = \Omega(W_1, L_3, r_3, s_l), \tag{A2b}$$

$$\Omega(W_1, L_1, r_1, s_h) = U\left(W_1 + L_1 - \frac{s_h}{W_1}\right)$$

$$+ \delta\left[\int_{L_1+L_1r_1-W_1}^{\overline{X}} V(W_1 + X - L_1 - L_1r_1)f(X)dX\right.$$

$$\left. + \int_{\underline{X}}^{L_1+L_1r_1-W_1} V(0)f(X)dX\right] > U\left(W_1 + L_4 - \frac{s_l}{W_1}\right)$$

$$+ \delta\left[\int_{L_4+L_4r_4-W_1}^{\overline{X}} V(W_1 + X - L_4 - L_4r_4)f(X)dX\right.$$

$$\left. + \int_{\underline{X}}^{L_4+L_4r_4-W_1} V(0)f(X)dX\right] = \Omega(W_1, L_4, r_4, s_l), \tag{A2c}$$

borrower 2 chooses the combination of s_h, L_2, and r_2 over the three alternative combinations:

$$
\begin{aligned}
\Omega(W_2, L_2, r_2, s_h) &= U\left(W_2 + L_2 - \frac{s_h}{W_2}\right)\\
&\quad + \delta\left[\int_{L_2+L_2r_2-W_2}^{\overline{X}} V(W_2 + X - L_2 - L_2r_2)f(X)dX\right.\\
&\quad \left. + \int_{\underline{X}}^{L_2+L_2r_2-W_2} V(0)f(X)dX\right] > U\left(W_2 + L_1 - \frac{s_h}{W_2}\right)\\
&\quad + \delta\left[\int_{L_1+L_1r_1-W_2}^{\overline{X}} V(W_2 + X - L_1 - L_1r_1)f(X)dX\right.\\
&\quad \left. + \int_{\underline{X}}^{L_1+L_1r_1-W_2} V(0)f(X)dX\right] = \Omega(W_2, L_1, r_1, s_h), \qquad \text{(A3a)}
\end{aligned}
$$

$$
\begin{aligned}
\Omega(W_2, L_2, r_2, s_h) &= U\left(W_2 + L_2 - \frac{s_h}{W_2}\right)\\
&\quad + \delta\left[\int_{L_2+L_2r_2-W_2}^{\overline{X}} V(W_2 + X - L_2 - L_2r_2)f(X)dX\right.\\
&\quad \left. + \int_{\underline{X}}^{L_2+L_2r_2-W_2} V(0)f(X)dX\right] > U\left(W_2 + L_3 - \frac{s_l}{W_2}\right)\\
&\quad + \delta\left[\int_{L_3+L_3r_3-W_2}^{\overline{X}} V(W_2 + X - L_3 - L_3r_3)f(X)dX\right.\\
&\quad \left. + \int_{\underline{X}}^{L_3+L_3r_3-W_2} V(0)f(X)dX\right] = \Omega(W_2, L_3, r_3, s_l), \qquad \text{(A3b)}
\end{aligned}
$$

$$
\begin{aligned}
\Omega(W_2, L_2, r_2, s_h) &= U\left(W_2 + L_2 - \frac{s_h}{W_2}\right)\\
&\quad + \delta\left[\int_{L_2+L_2r_2-W_2}^{\overline{X}} V(W_2 + X - L_2 - L_2r_2)f(X)dX\right.\\
&\quad \left. + \int_{\underline{X}}^{L_2+L_2r_2-W_2} V(0)f(X)dX\right] > U\left(W_2 + L_4 - \frac{s_l}{W_2}\right)\\
&\quad + \delta\left[\int_{L_4+L_4r_4-W_2}^{\overline{X}} V(W_2 + X - L_4 - L_4r_4)f(X)dX\right.\\
&\quad \left. + \int_{\underline{X}}^{L_4+L_4r_4-W_2} V(0)f(X)dX\right] = \Omega(W_2, L_4, r_4, s_l), \qquad \text{(A3c)}
\end{aligned}
$$

borrower 3 chooses the combination of s_l, L_3, and r_3 over the three alternative combinations:

$$\begin{aligned}
\Omega(W_3, L_3, r_3, s_l) &= U\left(W_3 + L_3 - \frac{s_l}{W_3}\right) \\
&\quad + \delta\left[\int_{L_3+L_3r_3-W_3}^{\overline{X}} V(W_3 + X - L_3 - L_3r_3)f(X)dX\right. \\
&\quad \left. + \int_{\underline{X}}^{L_3+L_3r_3-W_3} V(0)f(X)dX\right] > U\left(W_3 + L_1 - \frac{s_h}{W_3}\right) \\
&\quad + \delta\left[\int_{L_1+L_1r_1-W_3}^{\overline{X}} V(W_3 + X - L_1 - L_1r_1)f(X)dX\right. \\
&\quad \left. + \int_{\underline{X}}^{L_1+L_1r_1-W_3} V(0)f(X)dX\right] = \Omega(W_3, L_1, r_1, s_h),
\end{aligned} \tag{A4a}$$

$$\begin{aligned}
\Omega(W_3, L_3, r_3, s_l) &= U\left(W_3 + L_3 - \frac{s_l}{W_3}\right) \\
&\quad + \delta\left[\int_{L_3+L_3r_3-W_3}^{\overline{X}} V(W_3 + X - L_3 - L_3r_3)f(X)dX\right. \\
&\quad \left. + \int_{\underline{X}}^{L_3+L_3r_3-W_3} V(0)f(X)dX\right] > U\left(W_3 + L_2 - \frac{s_h}{W_3}\right) \\
&\quad + \delta\left[\int_{L_2+L_2r_2-W_3}^{\overline{X}} V(W_3 + X - L_2 - L_2r_2)f(X)dX\right. \\
&\quad \left. + \int_{\underline{X}}^{L_2+L_2r_2-W_3} V(0)f(X)dX\right] = \Omega(W_3, L_2, r_2, s_h),
\end{aligned} \tag{A4b}$$

$$\begin{aligned}
\Omega(W_3, L_3, r_3, s_l) &= U\left(W_3 + L_3 - \frac{s_l}{W_3}\right) \\
&\quad + \delta\left[\int_{L_3+L_3r_3-W_3}^{\overline{X}} V(W_3 + X - L_3 - L_3r_3)f(X)dX\right. \\
&\quad \left. + \int_{\underline{X}}^{L_3+L_3r_3-W_3} V(0)f(X)dX\right] > U\left(W_3 + L_4 - \frac{s_l}{W_3}\right) \\
&\quad \delta\left[\int_{L_4+L_4r_4-W_3}^{\overline{X}} V(W_3 + X - L_4 - L_4r_4)f(X)dX\right. \\
&\quad \left. + \int_{\underline{X}}^{L_4+L_4r_4-W_3} V(0)f(X)dX\right] = \Omega(W_3, L_4, r_4, s_l),
\end{aligned} \tag{A4c}$$

and borrower 4 chooses the combination of s_l, L_4, and r_4 over the three alternative combinations:

$$\Omega(W_4, L_4, r_4, s_l) = U\left(W_4 + L_4 - \frac{s_l}{W_4}\right)$$
$$+ \delta\left[\int_{L_4+L_4r_4-W_4}^{\overline{X}} V(W_4 + X - L_4 - L_4r_4)f(X)dX\right.$$
$$\left.+ \int_{\underline{X}}^{L_4+L_4r_4-W_4} V(0)f(X)dX\right] > U\left(W_4 + L_1 - \frac{s_h}{W_4}\right)$$
$$+ \delta\left[\int_{L_1+L_1r_1-W_4}^{\overline{X}} V(W_4 + X - L_1 - L_1r_1)f(X)dX\right.$$
$$\left.+ \int_{\underline{X}}^{L_1+L_1r_1-W_4} V(0)f(X)dX\right] = \Omega(W_4, L_1, r_1, s_h), \tag{A5a}$$

$$\Omega(W_4, L_4, r_4, s_l) = U\left(W_4 + L_4 - \frac{s_l}{W_4}\right)$$
$$+ \delta\left[\int_{L_4+L_4r_4-W_4}^{\overline{X}} V(W_4 + X - L_4 - L_4r_4)f(X)dX\right.$$
$$\left.+ \int_{\underline{X}}^{L_4+L_4r_4-W_4} V(0)f(X)dX\right] > U\left(W_4 + L_2 - \frac{s_h}{W_4}\right)$$
$$+ \delta\left[\int_{L_2+L_2r_2-W_4}^{\overline{X}} V(W_4 + X - L_2 - L_2r_2)f(X)dX\right.$$
$$\left.+ \int_{\underline{X}}^{L_2+L_2r_2-W_4} V(0)f(X)dX\right] = \Omega(W_4, L_2, r_2, s_h), \tag{A5b}$$

$$\Omega(W_4, L_4, r_4, s_l) = U\left(W_4 + L_4 - \frac{s_l}{W_4}\right)$$
$$+ \delta\left[\int_{L_4+L_4r_4-W_4}^{\overline{X}} V(W_4 + X - L_4 - L_4r_4)f(X)dX\right.$$
$$\left.+ \int_{\underline{X}}^{L_4+L_4r_4-W_4} V(0)f(X)dX\right] > U\left(W_4 + L_3 - \frac{s_l}{W_4}\right)$$
$$+ \delta\left[\int_{L_3+L_3r_3-W_4}^{\overline{X}} V(W_4 + X - L_3 - L_3r_3)f(X)dX\right.$$
$$\left.+ \int_{\underline{X}}^{L_3+L_3r_3-W_4} V(0)f(X)dX\right] = \Omega(W_4, L_3, r_3, s_l), \tag{A5c}$$

Endnotes

1 See, for example, the empirical studies of Crawford and Rosenblatt (1995) and Epley, Kartono, and Haney (1996) and the theoretical works of Kau, Keenan, and Kim (1993), and Kau and Keenan (1999).

2 As described later in this section, this result is also consistent with Brueckner (2000) and Harrison, Noordewier, and Yavas (2004). Also, note that the entire analysis of the article is devoted to mortgage loans that have a certain level of default risk. The loans in focus are, therefore, assumed to exhibit a LTV ratio beyond a minimum threshold level.

3 In recent years, the FICO® score has become the widely spread standard representing borrowers' credit records examined by lenders in processing loan applications. See Straka (2000) for more information on credit scorings and Collins, Harvey, and Nigro (2002) on their limitations.

4 Wang, Young, and Zhou (2002) argue that if screening of default risk is sufficiently costly, it may also be optimal for the strategic lender to randomly reject workout requests, where the likelihood of this strategy rises when borrowers cannot observe the lenders' screening policies.

5 Reinforcement of this prediction can be found in Campbell and Dietrich (1983), who find that the relationship between LTV ratio and the likelihood to default is negative for the 80%–90% LTV range, and positive beyond the 90% level. As pointed out by the authors, this result might be due to adverse selection. Also, in examining multifamily mortgage loans, Archer, Elmer, Harrison, and Ling (2002) find that the LTV ratio does not significantly contribute to the explanatory power of default. Both studies, however, do not incorporate the interrelated effect among credit score, LTV, and interest rate.

6 Other papers that examine mortgage default under asymmetric information include, for example, Posey and Yavas (2001), Ben-Shahar (2006), and Ben-Shahar, Benchetrit, and Sulganik (forthcoming).

7 For more on the lack of competition in the U.S. mortgage market, see Beyer, Dziobek, and Garrett (1999), Van Order (2000), Harvey and Nigro (2003), and Downing, Jaffee, and Wallace (2005).

8 It is assumed that $\underline{x}$ ($\bar{x}$) is sufficiently small (large) to yield default (loan repayment) as will be seen next.

9 As opposed to focusing on the indirect utility function, one could alternatively choose to model the problem focusing on the user costs function. The latter would of course require some adjustments.

10 That is, default is motivated by liquidity crunch as opposed to ruthless maximization of the value of the default option. For empirical evidence that undermines the pure ruthless discretion assumption in this context, see, for example, Deng, Quigley, and Van Order (1996), Yang, Buist, and Megbolugbe (1998), and Deng and Quigley (2002). Also, note that the framework presented is consistent with Stiglitz and Weiss (1981), who also consider initial wealth as the underlying force that produces different levels of risk on the part of the borrowers.

11 If, instead, c is maintained to be small and positive, the results will sustain, however, the equations that follow will be somewhat less tractable.

[12] That is, if the borrower's total transaction costs associated with default are denoted by T, then it is necessary to assume that $-T < A - LA(1 + r) - B$, where B is the total transaction costs accompanying the sale of the asset and the repayment of the loan.

[13] In fact, it is sufficient to assume in (6a) that $\partial^2\Omega/\partial r\partial W \leq 0$. That is, the negative marginal utility that follows an increase in the interest rate becomes less meaningful as the initial endowment becomes greater.

[14] One might argue that the lender's income in case of default, I, might be a function of the borrower's wealth, W. In this case, the results may sustain, however, additional conditions will be required. Also, note that discounting using the risk-free rate is not a necessary assumption for the results that follow.

[15] This assumption most likely holds for the prime mortgage market. As, for example, in Ben-Shahar and Feldman (2003), the effect presented by Jaffee and Russell (1976), where the lender's profit may fall with the loan rate due to adverse selection, is ignored.

[16] Obviously, for some relatively low levels of L, the lender is better off increasing the LTV as this extends the volume of the business. It is assumed here, however, that the levels of L are in the range in which a marginal increase in the LTV is no longer favorable, as its unfavorable effect on the probability of default dominates. See further justification for this assumption in the empirical evidence found, for example, in Deng, Quigley, and Van Order (1996) and Lekkas, Quigley, and Van Order (1993). For other theoretical justification, see, for example, Stiglitz and Weiss (1981).

[17] This, of course, does not entail that the ex post shocks are also identical.

[18] Indeed, borrowers are required to disclose their income and other financial resources when applying for a mortgage loan. Yet, moral hazard motivates borrowers to misrepresent their true financial status. Recognizing the effect of moral hazard, lenders thus foresee the limitation of relying on borrowers' reported financial statements.

[19] For other studies in which non-competitive equilibrium under asymmetric information is derived, see, for example, Besanko and Thakor (1987), Villas-Boas and Schmidt-Mohr (1999), Armstrong and Vickers (2001), and Rochet and Stole (2002).

[20] Note that the assumptions underlying the described Nash equilibrium are different than those under Bertrand equilibrium: while under the latter there exists a price competition, where each lender conjectures that the other lender maintains its price fixed (here the price is determined by the combination of LTV and interest rate), it is assumed here that each lender knows that the other lender will respond to a price change.

[21] FICO credit scores are affected, among other things, by delinquencies, too many accounts opened within the last twelve months, short credit history, balances on revolving credit near the maximum limits, public records (such as tax liens, judgments, or bankruptcies), no recent credit card balances, too many recent credit inquiries, too few revolving accounts, and too many revolving accounts.

[22] Also, it follows from the rational expectations assumption that the credit score attained ex post is correctly anticipated ex ante (during the pre-mortgage years in which the borrower manages debt).

[23] In the attained separating equilibrium, wealthier (i.e., safer) borrowers acquire a better credit score than that acquired by poorer (i.e., riskier) borrowers. However, note that the cost of the credit score is inversely related to the initial endowment—that is, in order to attain a credit score s, one needs to ''invest'' an amount equal to s/W [see Equation (2a)]. Hence, wealthier borrowers do not necessarily ''invest'' more in acquiring a better

credit score. In the cases where the investment in the credit score does reduce the wealth difference among borrowers with different initial endowments, it is yet assumed that the amount invested in the credit score (s/W) is sufficiently small and does not change borrowers' wealth ranking overall.

[24] As shown next, this empirical prediction holds for non-competitive markets as well.

[25] See, for example, Campbell and Dietrich (1983), Lekkas, Quigley, and Van Order (1993), Capozza, Kazarian, and Thomson (1997), and Archer, Elmer, Harrison, and Ling (2002).

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