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Welfare Analysis of the Number and Locations of Local Public Facilities

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Abstract

We develop a discrete or finite household model with congestable local public goods where the level of provision, the number of facilities and their locations are all endogenously determined in a purely normative context. We prove the existence of an equal-treatment identical-provision second best optimum, where all households are required to reach the same utility level, the provision of local public good is required to be the same at all facilities, and all facilities must serve the same number of consumers. Such an optimal public facility configuration may or may not be geographically centralized if there is only a single public facility site. Moreover, the optimal public facility configuration could be either concentrated (single site) or dispersed (multiple sites), depending crucially on the degree of congestability and the household valuation of the local public good as well as the commuting cost.

JEL Classification Numbers: D61, H41, R13

Keywords: Congestable Local Public Goods, Optimal Public Facility Configurations

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1. Introduction

In the literature on local public goods, there are many papers examining the effects of local public goods on the underlying spatial structures. These studies focus primarily on how the level public goods provision and the locations of public facility influence the locational choices of households and firms.[1] The presence of public goods serves as an agglomerative force, leading to the clustering of households and firms around a public facility. While this positive analysis is interesting, it is equally important to examine public facility configuration from a normative point of view. The number and locations of public facilities are crucial in determining the level of economic welfare in a community. However, due to technical barriers, this important task has not been explored completely. In this paper, we attempt such an endeavor by performing a general-equilibrium welfare analysis using a discrete (or finite) household model with congestable local public goods in which both the number of facilities and their locations are endogenously determined.[2]

In contrast with most previous studies in the area of public goods and spatial economics, where models with a continuum households are used, our paper employs a discrete household framework. This is because in continuum models, both the first and the second welfare theorems may fail even without public goods (see Berliant, Papageorgiou and Wang 1990). More importantly, due to a problem associated with sets of zero measure under the continuum setup, an optimum may not exist (see Berliant and Wang 1993). Thus, we prefer to use the discrete framework in order to circumvent these difficulties. Our model structure can be regarded as an extension of the discrete model of Berliant and Fujita (1992) to incorporate locally congestable public goods with the level of provision, the number and locations of the facility sites endogenously determined.

The second special feature of our paper is the consideration of locally congestable public goods. Most previous studies focus on the non-exclusive nature of local public goods (e.g., see Fujita 1986, Sakashita 1987,

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[1]See, for example, Arnott and Stiglitz (1979), Ellickson (1979), Scotchmer (1986), Wildasin (1987) and Thisse and Wildasin (1992).

[2]The discrete model is the original urban model of Alonso (1964). He proposes models with land prices that are additive across parcels, where no arbitrage is possible in equilibrium, as well as models with land prices that are not additive across parcels, such as pricing according to the value of the point in a parcel closest to the city center, where arbitrage is possible in equilibrium.

Thisse and Wildasin 1992, Peng 1996 and Hochman and Pines 1997), ignoring the possibility that public goods services may face local congestion as a function of the locationally dependent number of users. While Hochman (1982) considers local congestability, the public facility location in his paper is exogenously fixed. Local congestability is certainly observed in reality, as in the case of parks, public schools and libraries, where aggregate usage would affect the quality of services. Theoretically, it is also interesting to investigate how the presence of local congestability may affect the welfare properties of allocations in the spatial context.

The third feature of our paper is that we determine *both* the number and locations of public facilities and the level of public good provision endogenously. As in the literature, we restrict our focus to "identical provision" of the local public good with the same level of the local public good provided in each site and used by the same number of people. While Fujita (1986), Sakashita (1987) and Hochman and Pines (1997) endogenize the locations of public facilities, the number of public facilities is exogenously given as one in Fujita and as one or two in Sakashita as well as Hochman and Pines. Although Hochman and Pines make utility comparisons between the single and dual public facility cases, there is no endogenous mechanism pinning down the number of public facilities within the system. One may wonder whether the provision of a large central park/library or a few small community parks/libraries is more efficient in the spatial economy. Furthermore, the existing literature on public facilities in the spatial context assumes that the level of provision is exogenously fixed. In contrast, we endogenize the optimal allocation of resources between the public and the private goods but establish a sufficient condition such that the provision of the local public good is independent of the configuration of public facilities and the allocation of the private good.

Finally, in determining the optimal public facility configuration, our paper is the only one, to our knowledge, based on the concept of Pareto optimality. In particular, we construct a concept of "equal-treatment" identical-provision optimality that has all households achieving the same level of utility.[3] Fujita (1986) and Sakashita (1987) employ representative consumer utility maximization, which may be reinterpreted as an equally

[3]However, we will show at the end of Section 2 that with identical provision of local public goods, our concept of optimality generally forces a second best allocation since non-identical provision of local public goods may be Pareto dominant.

weighted social welfare maximum of households with concave utility functions. When land rents exit the system (as in Sakashita, for example), such command optimal outcomes need not be consistent with Pareto optimality. In comparing representative utility levels, Hochman and Pines (1997) consider an open model with an endogenous population of households. In their economy, the welfare of people outside the city is ignored and hence Pareto optimality is not well-defined. While our equal-treatment identical-provision optimality concept is well-defined in the general equilibrium sense, the feature of equal treatment allows us to compare our results with the contributions of other studies.

The main findings of this paper are as follows. First, under proper assumptions, we prove the existence of an equal-treatment identical-provision optimum and establish a sufficient condition for the level of public good provision to be independent of the public facility configuration and other endogenous variables. Second, given a single public facility, the equal-treatment optimal location of the public facility may not be at the geographic center and such an optimal location need not be unique. Third, with the number of public facilities endogenously determined, a sufficiently high degree of both congestability and commuting cost and a sufficiently low household valuation of the public good imply that the equal-treatment identical-provision optimal public facility configuration is always dispersed. Finally, we show the robustness of our main findings to the number of households and the possibility that an equal-treatment identical-provision optimum may be generally Pareto inferior to a non-identical provision feasible allocation.

In addition to constructing a well-defined and more complete normative analysis of the configuration of facilities providing congestable local public goods, our results contrast with those in the existing literature in several important dimensions. First, the possibility of a non-centralized public facility site and the possibility of multiple optimal public facility configurations contrast with all previous studies. Second, when there are two dispersed facility sites, the optimal locations need not be at the first and third quarter of the linear city, contrasting with Sakashita (1987). These discrepancies are mainly due to the difference between the finite versus continuum of households setups and the concept of equal-treatment identical-provision optimality versus command optimality; command optimality is commonly used in the balance of the literature. Third, our

conditions for generating dispersed public facility configurations contrast with those in Hochman and Pines (1997) where the commuting cost is required to be extremely high or low in order for a dispersed (two-location) configuration to emerge. In particular, in our framework the dispersed configuration becomes the unique outcome only when the commuting cost is sufficiently large.

In Section 2, we develop a simple spatial model of locally congestable public goods. Section 3 proves the existence of an equal-treatment identical-provision optimum. In section 4, we characterize optimal public facility configurations with two or more households, using a utility function allowing for the optimal level of public good provision to be independent of public facility configuration and private good allocation. Section 5 examines the case of non-identical provision of the local public good. Section 6 concludes the paper and provides avenues for future research.

2. The Economy

Consider a community accommodating N identical households that must locate in a location $i \in X = \{1, 2, 3, ... N\}$, where $N < \infty$. Each residential region has a unit length in a one-dimensional interval $L = [0, N]$, and is of homogeneous quality (except for the aspect of public goods service).[4] Each household is required to reside in a location. In the absence of vacant land or multiple occupancy, land allocation in our spatial economy becomes a simple assignment problem in which each lot is occupied by only one household.[5] Thus, household i consumes the interval $(i-1, i]$ and is conveniently labeled by the "front-door" location of its residence. The aggregate endowment of composite private good is $E > 0$.

Households are homogenous with respect to their taste (preferences). They enjoy consuming both composite private good and the local public good service provided by the community at certain assigned ***public***

[4]We conjecture that the definitions in Section 2 and the general results in Section 3 can be extended to the two dimensional setting, where consumers reside in squares rather than intervals.

[5]This is similar to the Koopmans and Beckmann (1957) linear assignment problem, meaning that it is infeasible to have more than one household residing in the same lot.

facility locations.[6] At a given location, a household has the nonnegative orthant of $\acute{u}^3$, $\acute{u}^3_+$, as his consumption set. There are three types of goods: (i) a composite consumption good, (ii) a local public good, whose facility service is valued by each household, and (iii) land. To simplify the analysis, we assume that the demand for land is perfectly inelastic - i.e., each household consumes a fixed, unit quantity of land. Thus, the consumption set reduces to $\acute{u}^2_+$.

The technology used by the community government to produce the single public good is constant returns to scale, generating one unit of public good for each unit of composite private good used as an input.[7] Thus, we let K denote both the composite consumption good used in production as well as the total supply of public good. This total supply is divided among the n facilities. The public good may be provided at one (***concentrated***) or more than one (***dispersed***) location. Denote the set of public facility locations as H δ L. When there is a single public facility site at η θ L, we have H = $\{\eta\}$. All households patronize this facility. When there are n sites, we have H = $\{\eta_1, \eta_2, ..., \eta_n\}$ where η_j θ L with a corresponding level of the local public good denoted by K_j, and where M_j is the number of households patronizing site j (j = 1, ..., n) (obviously, $\sum_{j=1}^{n} K_j = K$ and $\sum_{j=1}^{n} M_j = N$). Thus, by denoting the cardinality of a discrete set as $\mu(\cdot)$, we have $n = \mu(H)$ and the pair (n,H) summarizes the configuration of public facilities, i.e., both the number and locations of local public facility sites. Examples of such local public goods include schools, libraries, museums, parks, theater halls, public swimming pools, exercise fields or other recreational facilities.

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[6]For notational simplicity, we follow the conventional wisdom assuming that each location contains only one resident. More realistically, one may imagine that each residential region x_i is occupied by a finite number of identical households. This generalization only requires minor modification of the definition of land partition without changing any of our main results.

[7]In practice, in most countries there are higher level authorities (city, state, region, federal). The higher tier may seek to influence or constrain the activities of local communities by direct methods (local authorities may be forbidden from enacting certain restrictive legislation or compelled to provide a minimum standard of public goods), or by taxation and subsidies (e.g., a matching grant). Also, it is possible that the cost of local public good provision may vary across locations, especially if it requires land as an input and if land is heterogeneous or the population distribution is not uniform. Nevertheless, these issues are far beyond the scope of the present paper.

Each household must incur a commuting cost to travel to the assigned public facility locations.[8] For simplicity, it is assumed that the amount of land required by the facility is negligible.[9] Moreover, we consider throughout the paper ***identical provision*** (henceforth called **IP**) of the local public good in the sense that when there are multiple public facility sites, the same level of local public good is provided at each site and the same number of households use each of them. Thus, when there are n sites, the identical provision of the public good implies that each site is equipped with a local public good of size $K_j = K/n$ and shared by a population of households of $M_j = N/n$ (for all j = 1, ..., n). While the feature of identical provision allows us to compare our results with those in these previous studies, we would like to alert the reader that in general, a non-identical provision of the public good may be Pareto dominant (see Section 5 for a detailed illustration).

Let Y_i d H be a subset of public facilities to which household i travels and denote y_i 0 Y_i as a travel destination of household i. In general, household i may be assigned to randomize, with probability $\theta(i,\eta_j)$ 0 [0,1] and $\sum_{\eta_j 0 H} \theta(i,\eta_j)$ ' 1, to travel to a particular destination η_j where $\theta(i,\eta_j) > 0$ for η_j 0 Y_i with $\mu(Y_i)$ # n and $\theta(i,\eta_j)$ = 0 for η_j ó Y_i.[10] Obviously, n is at most N under this setup and hence n 0 X. This randomization framework is designed to be consistent with the consideration of identical public good provision.[11]

The pecuniary cost of commuting, $T_i(y_i)$, varies with distance from the public facility location. Since any single point has zero measure, we do not have to differentiate between open and closed intervals and can utilize the ***inner-Hausdorff metric***, denoted d, to measure the distance between a single point and a half open

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[8]Hochman (1982) defined a concentrated local public good as a public good that is available to consumers only in specific locations to which households must travel to make use of the service. Fujita (1986) denoted this as a traveled-for public good. Thisse and Wildasin (1992) and Hochman and Pines (1997) employ the same concept in their definition of public goods.

[9]Fujita (1986), Sakashita (1987), Thisse and Wildasin (1992), Peng (1996) and Hochman and Pines (1997) make a similar assumption.

[10]Precisely, consider the case of $N = 3$ and $n = 2$ where the two facility sites are at $\eta_1 = 1$ and $\eta_2 = 2$. Thus, a possible travel pattern is: households 1 and 3 commute to $y_1 = \eta_1$ and $y_3 = \eta_2$, respectively, whereas household 2 randomizes by commuting to y_2 0 $Y_i = \{\eta_1, \eta_2\}$ with $\theta(2, \eta_1) = \theta(2, \eta_2) = 1/2$.

[11]An alternative is to follow Lucas (1990), assuming there is a continuum of household members in each household and the household head makes a joint decision for all members. Then, we can simply assume that half of the members of household i travel to one site and the remaining half to the other.

interval.[12] Specifically, the distance between household i's residential region in L and a public facility location y_i, denoted $d(i, y_i)$, is defined as: (i) $(y_i - i)$, if $y_i > i$ [rightward traveling pattern]; (ii) 0, if $i - 1 \# y_i \# i$ [non-traveling]; (iii) $(i - 1) - y_i$, if $y_i < i - 1$ [leftward traveling pattern], respectively. In this spatial economy, we assume that the commuting cost schedule (measured in terms of the composite good), T, satisfies:

(**A1**) (Commuting cost) $T_i = T(d(i,y_i))$ such that $T(0) = 0$, $T' > 0$ and $T'' \geq 0$.

For example, a linear commuting cost schedule can be expressed as:

$$T_i = t\,d(i,y_i),\ t > 0. \tag{1}$$

While we impose only (A1) in obtaining general theorems, the simple linear commuting cost schedule is used in Sections 4 and 5 below for a complete characterization of optimal public facility configurations. Denote the total commuting cost as $\mathcal{C}(n,H) = \sum_{i=1}^{N} \theta(i,y_i)\,T(d(i,y_i))$ where $Y_i \subseteq H \subseteq L$ for all i=1,...,N. For given $H = \{\eta_1, \ldots, \eta_n\}$, $n = \mu(H)$ and $K_j = K/n$, a set of random travel assignments $\Theta = \{\theta(i,\eta_j)\}_{\eta_j \in H;\, i=1,\ldots,N;\, j=1,\ldots,n}$ is called ***cost-minimizing*** if $y_i \in \operatorname{argmin}_{\eta_j \in H} d(i,\eta_j)$ for all $i \in X$ (i.e., every household is assigned to the closest public facility sites). For given n and $K_j = K/n$, a public facility configuration associated with a set of public facility sites H and a set of random travel assignments $\Theta = \{\theta(i,\eta_j)\}_{\eta_j \in H;\, i=1,\ldots,N;\, j=1,\ldots,n}$ is called ***aggregate-cost-minimizing*** if for all $H' \subseteq L$ and any other set of random travel assignments $\Theta' = \{\theta'(i,\eta_j)\}_{\eta_j \in H';\, i=1,\ldots,N;\, j=1,\ldots,n}$,

$$\mathcal{C}(n,H) \equiv \sum_{i=1}^{N} \theta(i,y_i)\,T(d(i,y_i)) \leq \sum_{i=1}^{N} \theta'(i,y_i')\,T(d(i,y_i'))$$

Let the quantity of composite good consumption and of public good consumption be given by z and G, respectively. Let α denote the ***degree of congestability*** of the public good. The amount of public service embodying congestion can therefore be specified as:[13]

[12] Importantly, the standard Euclidean metric is not applicable in this case. For further discussion, the reader is referred to Klein and Thompson (1984, p. 39) and Berliant and Wang (1993, pp. 130-131.)

[13] A local public good subject to the external effect of congestion is defined as a congestable local public good. For a discussion of congestable public goods, see Hochman (1982) and Starrett (1991).

(**A2**) (Consumption of public good) $G(K_j, M_j) = K_j(M_j)^{-\alpha}$, $\alpha \in [0,1]$.

Under identical provision, recall that the local public good allocated to each site is $K_j = K/n$ and each site is patronized by $M_j = N/n$ households. We can thus rewrite: $G = (K/n)/(N/n)^{\alpha}$. Notably, $\alpha = 0$ implies a pure public good, whereas $\alpha = 1$ implies a private good. For example, if all households are going to share only one concentrated local public good at a single facility of size K, then $G = K(N)^{-\alpha}$ which captures the case considered by Hochman (1982).

(**A3**) (Well-behaved preferences of households) $u(z, G)$ is twice continuously differentiable, $\partial u/\partial z > 0$, $\partial u/\partial G > 0$, and strictly concave.

An ***allocation*** is defined as a list of (i) nonnegative quantities of composite good $z_i \in \mathbb{R}_+$ consumed by each household i (i =1, ..., N), (ii) a configuration of public facilities described by a set of public facility sites $H \subset \{\eta_1, \eta_2, ..., \eta_n\} \subset L$ with the number of sites $n = \mu(H) \in X$, (iii) a nonnegative level of provision of the local public good $K_j \in \mathbb{R}_+$ at each location $\eta_j \in L$ (j =1, ..., n and $\sum_{j=1}^{n} K_j = K$), with the number of households patronizing each public facility site $M_j \in \mathbb{R}_+$ (j =1, ..., n and $\sum_{j=1}^{n} M_j = N$), (iv) a subset of location indexes $Y_i \subset H$ (i =1, ..., N) containing the public facility sites that household i patronizes, and (v) a randomization probability $\theta(i,\eta_j) \in [0,1]$ assigning the probability with which household i travels to a particular destination $\eta_j \in H$ with $\sum_{\eta_j \in H} \theta(i,\eta_j) = 1$, $\sum_{i=1}^{N} \theta(i,\eta_j) = M_j$, and $\theta(i,\eta_j) = 0$ for $\eta_j \notin Y_i$. In short, an allocation can be written as: $a = (\{z_i, Y_i\}_{i=1,...,N}, \{K_j, M_j\}_{j=1,...,n}, \{\theta(i,\eta_j)\}_{i=1,...,N;\, j=1,...n}, n, H)$.

Given $y_i \in Y_i \subset H$ (i = 1, ..., N), the ***material balance condition*** for the composite good is:

$$\sum_{i=1}^{N} [z_i + \sum_{y_j \in Y_i} \theta(i,y_j)\, T(d(i,y_i))] = E + K. \qquad \textbf{(2)}$$

We now define a ***feasible allocation*** as an allocation $a = (\{z_i, Y_i\}_{i=1,...,N}, \{K_j, M_j\}_{j=1,...,n}, \{\theta(i,\eta_j)\}_{i=1,...,N;\, j=1,...n}, n, H)$ satisfying the material balance condition (2).

We next restrict our attention to the case of identical provision of the local public good. An ***identical-***

provision feasible allocation is a feasible allocation such that $K_j = K/n$ and $M_j = N/n$ for all j = 1, ... , n. An ***identical-provision optimum*** is an IP feasible allocation such that there is no other IP feasible allocation making no household worse-off and at least one better-off. Under assumption (A2) together with household randomization and identical provision, public good service becomes:

$$G(n,K) = K/(N^{\alpha} n^{1-\alpha}), \qquad \textbf{(3)}$$

which is constant across all households. The term $N^{\alpha}n^{1-\alpha}$ can be regarded as a "net congestion factor," increasing in the population and the number of public facility sites. Since all consumers are identical, the assignment of consumers to parcels is arbitrary and thus land allocation is trivial. Identical local public good provision enables us to write $K_j = K/n$ and $M_j = N/n$ for all j, so the IP optimality problem is simply given by:

$$\underset{\{z_i, y_i, \theta(i,y_i)\}_{i=1,\ldots,N}, K, n, H}{Max} \quad u(z_1, G(n,K)), \qquad \textbf{(4)}$$

s.t. (i) $u(z_i, G(n,K)) \geq v_i$, $i = 2,3,\ldots,N$;

(ii) *equation* (2); $z_i \geq 0$, $K \geq 0$, $n \geq 0$, $Y_i \in H$, $i = 1,2,\ldots,N$; $\sum_{i=1}^{N} \theta(i,\eta_j) = \frac{N}{n} \in \eta_j \subset H$.

This states the IP optimality problem as one of trying to maximize the well-being of household 1 subject to required utility levels for others in the economy (constraint (i)) and feasibility (equation (2)).

In the existing literature, an "optimal" configuration of public facilities is based on the concept of "command optimality" in which the community government determines the public facility configuration subject to (i) the competitive determination of the land rents (that often exit the system) and (ii) locational equilibrium (that equalizes households' utility). It is clear that such a concept is not well-defined in the Pareto sense - indeed, it is a mix of equilibrium and optimality concepts.[14] To be compatible with the literature, however, we modify the concept of IP optimality by considering ***equal treatment*** (**ET**) among all households. Specifically, an ***equal-***

[14]This problem has been pointed out by Berliant and ten Raa (1994, p. 638): "Most of the models [of public facility location] appear to use a mix of positive and normative concepts, equilibria and optima. At best, they can be interpreted as partial welfare analysis."

treatment identical-provision optimum (in short, an **ETIP** optimum) is an IP optimum such that all households reach the same utility, i.e., we have $u(z_i(x_i),G(n,K)) = v_o$ \$0 for all i = 1,..., N in problem (4).

Two comments are now in order. First, an ETIP optimum is Pareto optimal provided that the provision of public good at all facilities is the same in an equal utility Pareto optimum and the number of consumers served at each facility is the same at the Pareto optimum. That is, an ETIP optimum may fail to be first-best if it is welfare enhancing to alter the ETIP optimum to serve different numbers of consumers or provide different levels of local public goods at different sites. As discussed in Section 5 below, there may be an allocation with non-identical public good provision that Pareto dominates the allocations restricted to identical provision. Second, our definition of ETIP optimality basically determines the optimal distribution to consumers for given aggregate endowments of the composite good and land (E and L, respectively), as well as optimal public facility configurations in terms of the number and locations of sites. Thus, it does not involve an equilibrium concept, contrasting with the command optimality concept in previous studies.

3. Existence of an Equal-Treatment Identical Provision Optimum

In this section we establish theorems concerning the existence of an ETIP optimum and its general properties. An ETIP optimum can be obtained by solving the problem specified in (4) with $v_i = v_o$ for all i = 1,..., N and (4)(i) holding with equality.

To begin, we determine the optimal allocation between the public and the private composite good (see the Appendix for details of the calculations): $\sum_{i=1}^{N} \left(\frac{\partial u_i/\partial G}{\partial u_i/\partial z_i} \right) \frac{\partial G}{\partial K} = 1$, which is in essence the *Lindahl-Samuelson condition*. With regard to ETIP optimality, we define $\Lambda(K,n,z) \equiv \frac{1}{N} \frac{\partial u/\partial z}{(\partial u/\partial G)(\partial G/\partial K)}$ and show that the ETIP optimal provision of the local public good satisfies the following necessary condition (see the Appendix):

$$\Lambda(K,n,z) = 1 \qquad \textbf{(5)}$$

Denote $MRS_{Gz} \equiv (\partial u/\partial G)/(\partial u/\partial z)$ as the marginal rate of substitution between the public and the private composite goods. It may be more intuitive to use (3) to rewrite (5) as: $\Lambda(K,n,z) = \frac{N^{\alpha} n^{1-\alpha}}{N(MRS_{Gz})} = 1$. That is, the

sum of the marginal willingness-to-pay (i.e., $N(MRS_{Gz})$, under ETIP) is equal to the marginal cost of public good provision (i.e., the net congestion factor $N^{\alpha}n^{1-\alpha}$). Notably, the optimal composite good allocation, the optimal number of public facility sites and the optimal level of the public good are all jointly determined, making it difficult to characterize the ETIP optimum. We thus take the method adopted by Bergstrom and Cornes (1983), seeking a condition such that the optimal level of public good provision is not only uniquely determined but also *independent* of the public facilities configuration and the distribution of composite good. By examining (5), we impose:

Condition K: $\partial\Lambda/\partial n = \partial\Lambda/\partial z = 0,\ \partial\Lambda/\partial K > 0$.

Condition K is sufficient for (5) alone to pin down the optimal level of K, independent of n and z. An example satisfying Condition K will be provided shortly.

Lemma 1: *Under Assumptions (A2) and (A3) and Condition K, an ETIP optimum, if it exists, is associated with a uniquely determined level of public good that is independent of the public facility configuration.*

Proof: Since $\partial\Lambda/\partial n = \partial\Lambda/\partial z = 0$, we can now write $\Lambda = \Lambda_0(K)$ with $d\Lambda_0/dK > 0$. Thus, (5) can be inverted to determine the optimal provision $K = K_0$ where $K_0 \equiv \Lambda_0^{-1}(1)$ is independent of n and z. Q.E.D.

It is unnecessary to check the sufficiency of the Lindahl-Samuelson condition under ETIP because the independence of the local public good provision applies to all extreme points (not just global maxima).[15] To guarantee that the feasible set is non-empty, we consider:

Condition E: $K_0 < E$.

For illustrative purposes, an example is now in order:

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[15] The original form of the first-order condition should be written as $1/\Lambda(\cdot) = 1$. Those interested in the corresponding second-order condition with respect to the ETIP optimal allocation of the public good may totally differentiate the above expression and utilize the property of $\Lambda = \Lambda_0(K)$ obtained in Lemma 1 to derive such a condition: $-(d\Lambda_0/dK)/\Lambda_0^2 + -(\partial\Lambda/\partial K)/\Lambda_0^2 < 0$, which is ensured by Condition K.

Example: Consider $u(z,G) = \gamma z + \ln(G)$ where $\gamma > 0$. Then we have $\Lambda = \gamma K/N$ and hence (5) implies $K = K_0 / N /\gamma$, independent of any endogenous variables. It is clear that Condition E is met if $\gamma (E/N) > 1$, which is households' valuation of the average composite good endowment be greater than one.

Due to the endogenous number of public facilities, Bergstrom and Cornes' (1983) condition of a separable utility function that is quasi-linear in the composite good is insufficient in our model to guarantee an optimal level of aggregate public good provision independent of other endogenous variables. The imposition of Condition K simplifies the analysis greatly.

Under Assumptions (A1)-(A3) and Condition K and given the optimal number of facilities, we obtain another important necessary condition governing optimal travel assignments and public facility locations:

Lemma 2: *Under Assumptions (A1)-(A3) and Condition K, an ETIP optimum with an optimal number of public facilities given by n^*, if it exists, is associated with an aggregate-cost-minimizing travel assignment among all ETIP feasible travel assignments and public facility configurations with $n = n^*$. That is, let $(\{z_i, Y_i\}_{i=1,...,N}, \{K_j, M_j\}_{j=1,...,n^*}, \{\theta(i,\eta_j)\}_{i=1,...,N;\ j=1,...n^*}, n^*, H)$ be an ETIP optimum. Then $\int H$, $\mu(H) = n$, $\int \theta(i,y_i)$, $y_i \in Y_i$ $dH)$ $(i = 1,...,N)$,*

$$\sum_{i=1}^{N} \int_{y_i \in Y_i dH} \theta(i,y_i)\, T(d(i,y_i)) \le \sum_{i=1}^{N} \int_{y_i \in Y_i dH)} \theta(i,y_i)\, T(d(i,y_i)) \qquad \textbf{(6a)}$$

Moreover, the number of households patronizing a particular public facility site $j = 1, ..., n^*$ satisfies:

$$M_j = \sum_{k=1}^{n} \frac{1}{k} \mu(\{i \mid \eta_j \in Y_i dH, \mu(Y_i) = k\}) = \frac{N}{n}. \qquad \textbf{(6b)}$$

Proof: See the Appendix. Q.E.D.

Lemma 2 says that given the optimal number of public facilities, an optimal set of sites must be such that the aggregate travel cost is minimized. This property is crucial for determining households' travel patterns as well as the optimal configuration of public facilities.

In an ETIP optimum, public good service is uniform across households (as specified in (3)) and thus ET implies that the composite good consumption must be identical. Clearly, the total commuting cost $C(n,H)$ is non-increasing in n. The ETIP optimal composite good allocation to each household is then:

$$z = (E\&K\&C(n,H))/N. \quad \textbf{(7)}$$

Since C is a function of (n,H), G is a function of (n,K) and, from Lemma 1, $K = K_0$ (an exogenous constant), households' indirect utility must be a function of (n,H) alone. We denote this value by v(n,H).

Theorem: (Existence of ETIP Optimum) *Under Assumptions (A1)-(A3) and Conditions K and E, there exists an equal-treatment identical-provision optimum.*

Proof: Denote the set of feasible allocations as A = $\{(\{z_i, Y_i\}_{i=1,...,N}, \{K_j, M_j\}_{j=1,...,n}, \{\theta(i,\eta_j)\}_{i=1,...,N;\ j=1,...n}, n, H) \mid \{z_i, Y_i\}_{i=1,...,N}$ and $\{K_j\}_{j=1,...,n}$ satisfy (2), $Y_i \subseteq H \ \forall i = 1,...,N$, $\sum_{k=1}^{n} \theta(i,\eta_j) \cdot M_j \le \eta_j \in H$, $\sum_{j=1}^{n} M_j = N$, $n = \mu(H) \in X$ and $H = \{\eta_1, ..., \eta_n\} \subseteq L\}$. Under (A1) and noting that $\mu(X) = N$ is finite, it is obvious that A is compact (closed and bounded). We next show that the set of IP feasible allocations is closed. Denote the set of IP feasible allocations as P = $\{ (\{z_i, Y_i\}_{i=1,...,N}, \{K_j, M_j\}_{j=1,...,n}, \{\theta(i,\eta_j)\}_{i=1,...,N;\ j=1,...n}, n, H) \in A \mid$ each site $\eta_j \in H$ has a local public good quantity $K_j = K/n$ and $\{Y_i\}_{i=1,...,N}$ and $\{\theta(i,\eta_j)\}_{i=1,...,N;\ j=1,...n}$ are consistent with $\sum_{k=1}^{n} \theta(i,\eta_j) \cdot M_j \le \sum_{k=1}^{n} \frac{1}{k} \mu(\{i \mid \eta_j \in Y_i, \mu(Y_i) \cdot k\}) \cdot N/n \ \forall \eta_j \in H$, i = 1, ... , N, j = 1, ... , n $\}$. The IP restrictions only involve randomization probabilities $\theta(i,\eta_j)$, which takes real values from a compact set [0,1], and quantities n, $y_i \in Y_i$ (i=1,...,N) and $\eta_j \in H$ (j=1,...,n), which all take integer values from 0 to N. Thus, the closedness property is trivial and P is compact as it is a closed subset of a compact set A. Further denote the set of ETIP feasible allocations as F = $\{ (\{z_i, Y_i\}_{i=1,...,N}, \{K_j, M_j\}_{j=1,...,n}, \{\theta(i,\eta_j)\}_{i=1,...,N;\ j=1,...n}, n, H) \in P \mid u(z_i, G(n,K)) = v_o \ \forall$ i = 1,..., N $\}$. We claim that F is closed. Under ETIP and Condition K, we can use Lemma 1 to obtain $K = K_0$ and Lemma 2 to pin down aggregate-cost-minimizing travel assignments. It is sufficient to focus on continuous quantities $Z = (z_1,...,z_N,\theta(1,\eta_1),...,\theta(N,\eta_1),\theta(1,\eta_2),...,\theta(N\&1,\eta_n),\theta(N,\eta_n))$. Consider now a sequence of ETIP allocations $a^R = (\{z_i^R, Y_i\}_{i=1,...,N}, \{K_0, M_j\}_{j=1,...,n}, \{\theta(i,\eta_j)\}_{i=1,...,N;\ j=1,...n}, n, H) \in F$ for all $R = 0, 1, 2, ...$ and it has a limit point $a = (\{z_i, Y_i\}_{i=1,...,N}, \{K_0, M_j\}_{j=1,...,n}, \{\theta(i,\eta_j)\}_{i=1,...,N;\ j=1,...n}, n, H) \in P$. By construction, $u(z_i^R, G(n,K_0)) =$

v_o $0 for all i = 1,..., N. Since u is continuous by (A3), we have $u(z_i),G(n,K_0)) = v_o$ and hence a 0 F, which proves the assertion. As F is a closed subset of a compact set P, F is compact.

Next, we show that F is nonempty. It is obvious that by constructing an IP public facility configuration (n,H) with $n = N$, $y_i = \eta_i$ 0 (i-1, i], $K_i = K_0/N$, $\theta(i,\eta_j) = 1$ for all η_i 0 (i-1, i], $\theta(i,\eta_j) = 0$ for all η_i ó (i-1, i], and M_i =1 (i=1,...,N), the corresponding total commuting cost is C(n,H)=0 and $G(n,K_0) = K_0/N$. Under Condition E, we have E-K_0 -C(n,H) > 0 and by choosing $z_i = (E\text{-}K_0)/N$, such allocations are ETIP feasible, implying F … .

Because n, y_i 0 Y_i (i=1,...,N) and η_j 0 H (j=1,...,n) are discrete and finite and u is continuous for each fixed ($\{Y_i\}_{i=1,...,N}$, n, H), the ETIP optimality problem has a continuous objective over a compact and nonempty domain F. By straightforward application of the Maximum Value Theorem implied by the Bolzano-Weierstrauss Theorem (Bartle (1976), pp. 154-155), an ETIP optimum exists. Q.E.D.

It is apparent that in establishing the theorem, all we need are the continuity of the households' utility function and the commuting cost function, together with Condition E and the assumption that given the monotone increasing properties of u in G and T in $d(i,y_i)$, G is non-increasing in n (this latter condition ensures that n is finite, bounded by N). The assumptions regarding differentiability, specific functional forms, or interior solutions imposed in (A1)-(A3) are not needed here, but are made for a complete characterization of the ETIP optimum (in section 4 below).

Remark 1: Recall that H = $\{\eta_1, \ldots, \eta_n\}$ and hence H and n are inter-related. For each n, define H((n) ' $argmin_H$ $C(n,H)$. While the total commuting cost $C(n,H((n))$ is non-increasing in n, the net congestion factor $N^{\alpha} n^{1\&\alpha}$ is strictly increasing in n (for $\alpha < 1$). Thus, under assumptions (A2) and (A3) as well as Condition K, equations (3) and (7) together with Lemma 1 ($K = K_0$) imply that per capita composite good consumption z is non-decreasing in n and G is strictly decreasing in n. An immediate consequence of these conflicting effects on $u(z,G)$ is that an ETIP optimum may be associated with an intermediate value of n 0 {2, ... , N-1}, i.e., an ETIP optimal public facility configuration need not be completely concentrated (n = 1) or completely dispersed ($n = N$).

Remark 2: There are in general a continuum of ETIP optimal public facility configurations, even when n is fixed. For example, consider the case of $n = 1$ and N as an odd number with a central region [**[**N/2**]**, **[**N/2**]**+1], where **[**D**]** is the Gauss operator of D 0 ú$_+$, defined as the largest integer that is less than or equal to D. We will show in Section 4 that under assumptions (A1)-(A3) and Conditions K and E, any η 0 [**[**N/2**]**, **[**N/2**]**+1] is ETIP optimal. When $n = 1$ and N is an even number, there is a unique ETIP optimum $\eta = N/2$, which is in between the front door of the site of the **[**N/2**]**-household and the back door of the site of the (**[**N/2**]**+1)-household.

We would like to remind the reader that the main purpose of this paper is to perform a *normative* analysis of local public facility configuration, which contrasts sharply with the existing literature based on a *positive* analysis of command optimality. Nevertheless, for comparison, we focus primarily on ETIP allocations, which are conceptually close to the allocations considered in previous studies.

4. Characterization of Optimal Public Facility Configurations

We characterize ETIP optimal public facility configurations explicitly given a linear commuting cost schedule specified as in (1). We focus on three issues: (i) whether an ETIP optimal configuration is uniquely determined, (ii) whether public facilities should be concentrated in one location or dispersed in more than one location, and (iii) whether a concentrated public facility should be geographically centralized.

4.1. Optimal Location of Public Facility: The Case of $n = 1$

It is important to note that the uniqueness/multiplicity properties of ETIP optimality depend crucially on the number of households. First we fix the number of facilities exogenously at 1.

Proposition 1: (ETIP optimal location of public facility with n =1) *Assume (A2), (A3), Conditions K and E, and the linear commuting cost schedule specified as in (1). Given n = 1, the set of* ETIP *optimal public facility configurations satisfies the following properties:*

(i) for even numbers N, it is unique and geographically centralized;

(ii) for odd numbers N > 1, it is a continuum consisting of any location in the central residential interval.

Proof: With $n = 1$, the random travel assignment is trivial in that $\mu(Y_i) = 1$ for all i 0 X. To prove part (i), consider an even number of households $N = 2$. As shown in Figure 1, for given unit commuting cost $t > 0$, we start with $\eta = 2$ (for the case of $\eta = 0$, the arguments are similar and hence omitted). In this case, the total commuting cost is $C = t$. Now, we move the public facility to $\eta = 1$. While the local public good service G is unchanged, the total composite good available for allocation to households increases from $E\text{-}K_0\text{-}t$ to $E\text{-}K_0$, resulting in a Pareto improvement. Similar arguments apply to any η 0 (1, 2). Therefore, the allocation $(z_i,K,n,H) = (E/N\text{-}K_0/2,K_0,1,\{1\})$ is ETIP optimal, where K_0 solves (5). The corresponding total commuting cost schedule with respect to the public facility location is depicted in bottom panel of Figure 1 from which one can see that $\eta = 1$ is associated with minimum total commuting costs, satisfying the requirement in (8). This analysis and the uniqueness result apply to the case with $n = 1$ and any even number N.

We next prove part (ii) by considering N odd. Consider Figure 2. We claim that any allocation with H = $\{\eta\}$ where η 0 [**[***N*/2**]**, **[***N*/2**]**+1] (i.e., the central region) is ETIP optimal. Suppose not. Consider the public facility site on the right of the central region with $\eta =$ **[***N*/2**]**+1 + ε, where $\varepsilon > 0$. Then moving the site to $\eta =$ **[***N*/2**]**+1 would result in an improved allocation as it reduces the total commuting cost by $2\varepsilon t$. Similar arguments apply for the case with the public facility site on the left of the central region and for any $\varepsilon > 0$. Next, we show that the relocation of the public facility site within the central region leaves the total commuting cost unchanged. Consider moving the site from $\eta =$ **[***N*/2**]**+1 to $\eta =$ **[***N*/2**]**+1 - ε, where ε 0 (0, 1). While the commuting cost of each household residing on the left of the central region is reduced by εt, that of each household on the right is raised by εt.[16] Since C is unchanged, these allocations are all ETIP optimal. Q.E.D.

The results in Proposition 1 contrast with findings in the framework with a continuum of households of Fujita (1986) and Sakashita (1987) in two important dimensions. First, in our model, an ETIP optimal location of the public facility need not be at the geographic center. Second, we find that multiple ETIP optimal configurations are possible in the sense that the optimal location of the public facility need not be unique and

[16]For illustrative purposes, we delineate the case of $N = 3$ and $\varepsilon = ½$ in Figure 2(a).

need not be geographically centralized. These discrepancies are mainly due to having a finite number instead of a continuum of households. It may be noted that as N goes to infinity (by expanding L to the entire extended real line), this multiplicity property is still robust. One may argue that if we fix the landscape of the entire residential region but take N to infinity, the optimal location of public facility may reduce to a single point at the geographical center. However, such an exercise is mathematically imprecise because the individual household lot size would approach zero (see Berliant and ten Raa 1991).

4.2. Optimal Number and Locations of Public Facilities: the Case of $N = 3$

We next turn to endogenizing the number of public facilities. When $N = 1$ or $N = 2$, the ETIP optimal facility location problem is trivial. When $N \geq 3$, it is possible that the ETIP optimal number of public facilities is no longer one. For illustrative purposes, it suffices to consider the case of $N = 3$. Given $\alpha < 1$ (i.e., G is not a pure private good), we claim:

Lemma 3: *Assume (A2), (A3), Conditions K and E, and the linear commuting cost schedule specified as in (1). Given N = 3, the ETIP optimal number of public facilities n can only be 1 or 2.*

Proof: This lemma can be easily proved by contradiction. Suppose $n = 3$ is ETIP optimal with one public facility of size $K_0/3$ in each residential interval (so no one commutes). Thus, $G(3,K_0) = K_0/3$. Consider an alternative ETIP allocation with $n = 2$, $H = \{1,2\}$ and $\theta(1,1) = \theta(3,2) = 1$ and $\theta(2,1) = \theta(2,2) = 1/2$ in which $C(2,\{1,2\}) = 0$. Now the public good service becomes $G(2,K_0) = (3/2)^{1-\alpha}(K_0/3)$, strictly greater than $K_0/3$ for any $\alpha < 1$. Thus, an allocation with $n = 3$ cannot be an ETIP optimum. Q.E.D.

Lemma 3 rules out the possibility of $n = 3$. Hence, there are three candidate ETIP optimal public facility configurations remaining to be examined with $N = 3$: (i) centrally concentrated (type-C), (ii) non-centrally concentrated (type-N) and (iii) dispersed (type-D). Notably, Proposition 1 implies that an ETIP optimum with $n = 1$ (concentrated) must have the public facility of size K_0 in the central residential interval. In this case, there are two possibilities: either $H = \{3/2\}$ (i.e., type-C) or $H = \{\eta\}$ with $\eta \in [1, 2] \setminus \{3/2\}$ (i.e., type-N). With regard to the dispersed case ($n = 2$), we claim:

Lemma 4: *Assume (A2), (A3), Conditions K and E, and the linear commuting cost schedule specified as in (1). Given N = 3, if the type-D public facility configuration is ETIP optimal, there are two locationally symmetric and identically provided public facilities of size K_0 /2 located at* η *= 1 and 2.*

Proof: To prove this lemma, we utilize Figure 2(b). In particular, we need to show that public facilities must be located at 1 and 2. Suppose not. Consider, say, an alternative ETIP feasible allocation with H = {1-ε, 2+ε}, θ(1,1-ε) = θ(3,2+ε) = 1, θ(2,1-ε) = θ(2,2+ε) = 1/2, and $z = (E - K_0 - \varepsilon t)/3$ where ε 0 (0,1). Then using the arguments in the proof of Proposition 1, we can obtain an improving reallocation with H = {1, 2}, θ(1,1) = θ(3,2) = 1, θ(2,1) = θ(2,2) = 1/2, and $z = (E - K_0)/3$), which contradicts the definition of ETIP optimality. By similar arguments, one can show that any other ETIP feasible allocations with n = 2 and H = {1+ε, 2-ε} is Pareto dominated by that with H = {1,2} as the latter is the only ETIP feasible allocation associated with zero total commuting cost. Q.E.D.

We are now left to examine each candidate configuration and to derive the corresponding conditions to support each case. In order to compute a household's indirect utility, we further assume that the utility function of the households takes the following form (satisfying Condition K):[17]

$$u(z,G) \text{ ' } \gamma z \% \ln(G) \qquad \textbf{(8)}$$

where $\gamma > 0$. From Lemma 1, we can solve the optimal provision of the local public good as $K_0 = N/\gamma$. Utilizing Lemmas 2-4, we can substitute the composite good allocation (7), and the public good service (3) under each type of facility configuration into the utility function (8) to obtain the household value for each type of public facility configuration as follows:

$$v^C \text{ ' } v^N \text{ ' } \gamma(E \& K_0)/3 \% \ln(K_0/3) \% (1 \& \alpha)\ln 3 \& \gamma t/3 \qquad \textbf{(9a)}$$

[17] Those interested in the conventional Cobb-Douglas utility functional form are referred to the Appendix. Since in that case Condition K is not satisfied, we instead assume that the level of the public good provision K is exogenously fixed. Under this assumption, the main conclusions are robust to functional form.

$$v^D = \gamma(E - K_0)/3 + \ln(K_0/3) + (1-\alpha)\ln 3 - (1-\alpha)\ln 2 \qquad \textbf{(9b)}$$

where the superscripts denote the types of public facility configurations and the corresponding aggregate commuting costs are: $C^C = C^N = t$ and $C^D = 0$.

To compare different types of public facility configurations, we need to check if they are Pareto rankable. This is easily done by comparing households' value for each case, as given by (9a) and (9b).

Proposition 2: (ETIP optimal location of public facility with $N = 3$) *Assume (A2), Condition E, the linear commuting cost schedule specified as in (1), and the utility function specified as in (8). When $N = 3$, an ETIP optimal public facility configuration satisfies the following properties:*

(i) *if* $t > 3(\ln 2)(1-\alpha)/\gamma$, *then an ETIP optimal public facility configuration is always dispersed;*

(ii) *if* $t < 3(\ln 2)(1-\alpha)/\gamma$, *then an ETIP optimal public facility configurations is always concentrated, either centralized or non-centralized.*

Proof: The proof follows immediately from comparing (9a) and (9b):

$$v^C = v^N \gtrless v^D \quad \Leftrightarrow \quad t \lessgtr 3(\ln 2)(1-\alpha)/\gamma \qquad \textbf{(10)}$$

provided that $\gamma E > 3$, which is ensured by Condition E. Q.E.D.

From (10), the determination of the optimal public facility configuration depends crucially on the commuting cost (t), the degree of non-exclusiveness ($1-\alpha$), and the household valuation of the public good ($1/\gamma$). Thus, within our congestable local public good framework with $\alpha < 1$, we can conclude from Proposition 2,

Proposition 3: *Assume (A2), Condition E, the linear commuting cost schedule specified as in (1), and the utility function specified as in (8). Given $N = 3$, an ETIP optimal public facility configuration is dispersed if the degree of congestability is large, the household valuation of the public good is low, and the unit commuting cost is sufficiently high; otherwise, an ETIP optimal public facility configuration is concentrated.*

Therefore, building a large central park (or library, or school) is not ETIP optimal in an economy with a high degree of public good congestability or a high commuting cost. A Pareto improvement results if the public good service is provided at dispersed facility sites with smaller size. This justifies the existence of local community parks, libraries and schools in reality.

Our results can be compared with those in previous studies. Sakashita (1987) considers the case of two pure public good facilities (although this number is exogenously given therein). He finds that in the absence of mutual accessibility between the two public facilities, the two public facility sites are at the first and third quarter of the linear city. In our model with N =3, when the (endogenously determined) ETIP optimal configurations are dispersed, the optimal public facility sites are at one-third and two-thirds of the total length of the linear city.

In another related paper, Hochman and Pines (1997) compare economic welfare with one or two public facilities given two distinct pure public goods in a model with a continuum of households. They conclude that two facilities (one for each pure public good) are optimal when the unit commuting cost is extremely large or extremely small. Our results suggest, however, that a dispersed configuration is optimal only when the unit commuting cost is sufficiently large. Moreover, it is worth noting that the optimality concept in their paper is completely different from our concept of ETIP optimum. Specifically, they consider an open model with an endogenous population of households, in which the welfare of those not within the model is ignored and hence Pareto optimality is not well-defined.

4.3. Optimal Number and Locations of Public Facilities: the General Case of $N > 3$ and $n \geq 1$

In Section 4.1, we considered the case $n = 1$ for any given N, whereas Section 4.2 examines the case of $N = 3$ for an endogenous determination of ETIP optimal n that need not be equal to one.[18] We now characterize the general case of $N > 3$ allowing for multiple public facility sites.

We begin by proving the following Lemma:

[18] The case of $N = 2$ is trivial: the ETIP optimal configuration must be $n = 1$ and $\eta = 1$ (concentrated and centralized). It is therefore omitted from our discussion.

Lemma 5: *Assume (A2), Condition E, the linear commuting cost schedule specified as in (1), and the utility function specified as in (8). In an ETIP optimum, the following properties always hold:*

(i) for any even number N, there should be no more than N/2 public facilities;

(ii) for any odd number N that is a multiple of 3, there can be no more than 2N/3 public facilities;

(iii) for an odd number N that is not a multiple of 3, there may be as many as N -1 public facilities.

Proof: The proof of parts (i) and (ii) follows arguments similar to that of Lemma 3. When N is even, it is always Pareto dominant to have at least two adjacent households sharing one public facility at their back/front door location in which the public good service is higher than that with more than $N/2$ facilities. The proposed allocation has zero total commuting cost. When N is an odd number that is a multiple of 3, a configuration with three connected households sharing two public facilities as illustrated in Figure 2(b) always Pareto dominates an allocation with more than $2N/3$ facilities. Again, the proposed allocation has zero commuting cost.

For part (iii), it is sufficient to consider the case of $N = 5$. Since equal numbers of people must be served at all sites, there are five IP optimal public facility configuration candidates with n = 1, 2, 3, 4 and 5 and corresponding $H^*(1) \in [2, 3]$, $H^*(2) = \{1, 4\}$, $H^*(3) = \{1, 5/2, 4\}$, $H^*(4) = \{1, 2, 3, 4\}$ and $H^*(5) = \{\eta_j\}_{j=1,...,5}$ with $\eta_j \in [j-1, j]$ where $C(1, H^*(1)) = 4t$, $C(2, H^*(2)) = t$, $C(3, H^*(3)) = t/3$, $C(4, H^*(4)) = C(5, H^*(5)) = 0$. It is easily seen that if $K_0 < E < K_0 + t/3$ (a sufficient but not necessary condition), the ETIP optimal configuration of public facilities must be associated with $n = 4$ or 5 (since any allocation with n = 1, 2 or 3 is not IP feasible). These arguments together with Lemma 3 imply that for an odd number that is not a multiple of 3, it is possible to have an ETIP optimal configuration with $n = N - 1$ if the unit commuting cost is sufficiently large relative to the aggregate endowment. Q.E.D.

Notice that part (iii) of Lemma 5 is due in particular to the consideration of identical local public good provision that requires the number of consumers patronizing each site to be the same.[19] Although the completely dispersed ($n = N$) public facility configuration can never be optimal, it is possible to have the maximum dispersed

[19] We relegate the discussion of non-IP feasible allocations to Section 5 below.

($n = N$ - 1) public facility configuration as a candidate for ETIP optimality. In general, for any number $N > 3$, a maximum dispersed public facility configuration is ETIP optimal only if N is not a multiple of 2 or 3.

Before establishing the general proposition for $N > 3$ in Lemma 6 and Proposition 4 below, we would like to make two remarks. First, with regard to concentrated ($n = 1$) versus dispersed ($n > 1$) public facility configurations, one can easily see that Proposition 3 can be applied to the general case of $N > 3$. In particular, an ETIP optimal public facility configuration is always dispersed when there is a high degree of public good congestability, it is costly to commute and household valuation of the public good service is low. Otherwise, there are multiple ETIP optimal public facility configurations where the optimal configurations could be concentrated or dispersed. This characterization is robust with respect to the number of households $N > 3$.

Second, it may be interesting to examine the case of two optimal public facility sites ($n = 2$) to compare with Sakashita's (1987) result (assuming an appropriate set of parameters making $n = 2$ optimal, such as, $t > 3(\ln 2)(1\&\alpha)/\gamma$ under the utility function specification in (8) for $N = 3$). Recall that Sakashita's optimal public facility location at the first and third quarter of the landscape does not hold in our model with $N = 3$ (the ETIP optimal sites are at one third and two thirds). Yet Sakashita's result may occur in our model with $N = 4$ or any numbers that are multiples of 4. To obtain a general property, let us study the cases of $N = 5$, 6, 7, 9 and 10. When $N = 5$, the ETIP optimal configuration is $H = \{1, 4\}$. For $N = 6$, we have $H = \{\eta_1, \eta_2\}$ with $\eta_1 \in [1, 2]$ and $\eta_2 \in [4, 5]$; for $N = 7$, $H = \{2, 5\}$; for $N = 9$, $H = \{2, 7\}$; and for $N = 10$, $H = \{\eta_1, \eta_2\}$ with $\eta_1 \in [2, 3]$ and $\eta_2 \in [7, 8]$. For $N = 5$, 7 and 9, Sakashita's first and third-quarter configuration is not ETIP optimal. For the cases of $N = 6$ and $N = 10$, Sakashita's first and third-quarter configuration is only one of the ETIP optimal outcomes. Thus, his finding is not robust in general.

In general, we can characterize ETIP locations of public facilities for $N > 3$ given the linear commuting cost schedule as in (1) and the utility function specification as in (8). For illustrative purposes, we focus on a set of parameters such that an ETIP optimum satisfying cost-minimizing travel assignments is featured with $n^* = 2$. That is, the characterization of the ETIP public facility configuration delineated below is restricted to cases

with cost-minimizing travel assignments and an optimal number of sites equal to two.[20] We claim:

Lemma 6: *Assume (A2), Condition E, the linear commuting cost schedule specified as in (1), and the utility function specified as in (8). Given $N > 3$, the set of parameters resulting in a cost-minimizing ETIP optimal number of public facilities equal to two is nonempty.*

Proof: Recall that $H^{c}(n) = \mathrm{argmin}_H\ C(n,H)$. Substituting (3), (7) and $K = K_0$ into (8), we can obtain a condition in the case of $N > 3$ such that the utility values associated with $n = 1$ and $n = 2$ are equal: $C(1,H^{c}(1))\&C(2,H^{c}(2)) = N(\ln 2)(1-\alpha)/\gamma$. We then compute the lefthand side expression under various values of N. It is easily shown that for $N = 4$, $C(1,H^{c}(1))\&C(2,H^{c}(2)) = 2t$. Define $I_N \equiv \left\{2\% \sum_{J=5}^{N}\left(\left[\frac{J-3}{2}\right]-\left[\frac{J-5}{4}\right]\right)\right\}$. By induction, we can derive $C(1,H^{c}(1))\&C(2,H^{c}(2)) = tI_N$ for all $N \geq 5$. Let $t_c = N(\ln 2)(1-\alpha)/(\gamma I_N)$. One can then see that, for any t with $t > t_c$ approaching to zero from above, $n = 2$ is ETIP optimal among all allocations with cost-minimizing travel assignments. Q.E.D.

By Lemma 6, the set of parameters yielding a cost-minimizing ETIP optimal number of public facilities $n^* = 2$ is nonempty. Thus, we can show:

Proposition 4: *Assume (A2), Condition E, the linear commuting cost schedule specified as in (1), and the utility function specified as in (8). Given $N > 3$ and a set of parameters such that $\mathrm{argmin}_n\ C(n,H^{c}(n)) = 2$, ETIP optimal locations of public facilities under cost-minimizing travel assignments possess the following properties:*

(i) *when N is a multiple of 4, $H = \{N/4, 3N/4\}$;*

(ii) *when N is even but not a multiple of 4, $H = \{\eta_1, \eta_2\}$ with $\eta_1 \in [\,[N/4], [N/4]+1\,]$ and $\eta_2 \in [\,N-[N/4]-1, N-[N/4]\,]$;*

(iii) *when N is an odd number, $H = \{\,[(N+1)/4]\,,\ N-[(N+1)/4]\,\}$.*

Proof: Case (i) is trivial. For case (ii), divide the entire region evenly into two where each half has a public facility site. By symmetry, we focus on the left half, $L_1 \equiv [0, N/2]$. Since N is not a multiple of 4, L_1 must

[20] We note that it is very difficult to find general location-patterns of public facilities for ETIP optima associated with non-cost-minimizing travel assignments.

contain an odd number of households. Since there is only one public facility in L_1, we can apply Proposition 1 (ii) to conclude that the ETIP optimal site must be in the central residential interval in L_1, which is [**[***N*/4**]**, **[***N*/4**]** +1]. By symmetry, the site in the right half is obtained.

Next, consider case (iii). Since it is always suboptimal to have both sites in the central region, we begin by removing the household residing in the central region (denoted i_m) and denoting L_2 / [0, **[***N*/2**]**] which must contain one of the two sites. There are now two subcases to investigate. First, if L_2 contains an even number of households (i.e., **[***N*/2**]** is even), by Proposition 1 (i), the ETIP optimal public facility location must be at the geographic center of L_2. Then we add back the middle household i_m. By randomization, only half of that household involves commuting to the public facility site in L_2, we claim that adding back i_m would not change the optimal public facility location. Suppose not, say, the facility site shifts toward i_m (it is trivial that a shift away from i_m can never be optimal). In this case, the reduction of the commuting cost incurred by i_m is half of the increased commuting cost by household 1, thus leading to a higher total commuting cost and contradicting the optimality condition. Therefore, the optimal location is at $\eta = $ **[***N*/2**]**/2. Second, if L_2 contains an odd number of households (i.e., **[***N*/2**]** is odd), from case (ii) above, the optimal location is the central interval of L_2, [**[***N*/4**]**, **[***N*/4**]**+1]. Of course, when we add back i_m, only the upper bound of this interval is optimal, i.e., $\eta = $ **[***N*/4**]**+1. Notice that by the same arguments as above, the optimal allocation would not shift over to the adjacent interval toward i_m. Combining the two subcases, we obtain the ETIP optimal location in L_2 as $\eta = $ **[**(*N*+1)/4**]** and that the analogous result holds for the right half as well.[21] Q.E.D.

5. On the Non-Identical Provision of the Local Public Good

Up to this point, we have restricted our attention to the case of identical local public good provision. In this Section, we study the consequences of relaxing the IP assumption. In particular, we illustrate that an ET

[21] The reader may find that as *N* approaches infinity, the ETIP optimal facility sites become **[**(*N*+1)/4**]**/*N* 6 1/4 and (*N* - **[**(*N*+1)/4**]**)/*N* 6 3/4, reducing to the first-quarter and third-quarter rule found in Sakashita (1987). However, the ETIP optimal allocation of land parcels in the limit becomes zero for each household. Thus, as argued by Berliant and ten Raa (1991), we cannot simply take a limit for our finite model to converge to Sakashita's continuum model.

feasible allocation with non-identical provision of a local public good can Pareto dominate any IP feasible allocation. That is, by imposing,

Condition V: $\max\left\{\left(\frac{3\alpha}{\gamma}\right)\left[4\ln 2 - 5\ln\left(\frac{5}{3}\right)\right], \frac{15(1-\alpha)}{2\gamma}\ln\left(\frac{3}{2}\right)\right\} < t < \frac{15(1-\alpha)}{\gamma}\ln\left(\frac{4}{3}\right)$,

we claim:

Proposition 5: *Assume (A2), Conditions E and V, the linear commuting cost schedule specified as in (1), and the utility function specified as in (8). There exists a non-identical provision feasible allocation with equal treatment strictly Pareto dominating the ETIP optimum.*

Proof: It is sufficient to verify this by examining the special case of $N = 5$ and by showing that an ETIP optimal allocation features $n^* = 3$ and is dominated by a non-IP feasible allocation, as ET can always be achieved by reallocation of the composite good under Condition K.

First, we show that the ETIP optimum in the case of $N = 5$ features an optimal number of facility locations $n^* = 3$ under proper conditions. Denote the highest utility values among all ETIP feasible allocations for given n as v_n. Defining $\bar{v} \equiv \gamma(E - K_0)/5 + \ln K_0 - \alpha\ln 5$, straightforward manipulations yield:

$$v_1 = \bar{v} - (4/5)\gamma t \qquad \textbf{(11a)}$$

$$v_2 = \bar{v} - (1-\alpha)\ln 2 - (1/5)\gamma t \qquad \textbf{(11b)}$$

$$v_3 = \bar{v} - (1-\alpha)\ln 3 - (1/15)\gamma t \qquad \textbf{(11c)}$$

$$v_4 = \bar{v} - (1-\alpha)\ln 4 \qquad \textbf{(11d)}$$

$$v_5 = \bar{v} - (1-\alpha)\ln 5 \qquad \textbf{(11e)}$$

It is obvious from (11d) and (11e) that $v_4 > v_5$. Consider,

$$\frac{15(1\&\alpha)}{2\gamma}\ln\left(\frac{3}{2}\right) < t < \frac{15(1\&\alpha)}{\gamma}\ln\left(\frac{4}{3}\right) \qquad \textbf{(12)}$$

From (11b), (11c) and (11d) and given $t > 0$ and $\alpha < 1$, the first inequality of (12) implies that $v_3 > v_2$ whereas the second inequality of (12) implies $v_3 > v_4$. Under the first inequality of (12), it must also be true that $t > [5(1\&\alpha)/(3\gamma)]\ln 2$ and hence we can show from (11a) and (11b) that $v_2 > v_1$. That is, condition (12) is sufficient to ensure the indirect utility ordering: $v_3 > \max\{v_1, v_2, v_4, v_5\}$ and, therefore, the ETIP optimum features $n^* = 3$. More specifically, this ETIP optimum is associated with a set of public facility sites H = {1, 5/2, 4}, a provision of the local public good at each site of $K_j = K_0/3$ with $K_0 = 5/\gamma$ and j = 1, 2, 3, a travel assignment $\theta(1, 1) = \theta(5, 4) = \theta(3, 5/2) = 1$, $\theta(2, 1) = \theta(4, 4) = 2/3$ and $\theta(2, 5/2) = \theta(4, 5/2) = 1/3$, and an allocation of the composite good $z = [E \& K_0 \& t/3]/5$.

Next, we consider a non-IP, ET feasible allocation with H = {1, 5/2, 4}, the public good provision of $K_j = K_0/3$, a (cost-minimizing) travel assignment $\theta(1, 1) = \theta(2, 1) = \theta(3, 5/2) = \theta(4, 4) = \theta(5, 4) = 1$, and an allocation of the composite good $z_3 = \left[E \& K_0 \& 4(\alpha/\gamma)\ln 2\right]/5$ and $z_i = \left[E \& K_0 \% (\alpha/\gamma)\ln 2\right]/5$ (i = 1,2,4,5). Obviously, this latter alternative allocation results in $C(3, H(3)) = 0$, lower than that under the above specified ETIP optimum in which $C(3, H^*(3)) = t/3$. Further assume that the unit commuting cost is sufficiently high, satisfying: $t > [3\alpha/\gamma][4\ln 2 \& 5\ln(5/3)]$. Then this alternative non-IP, ET feasible allocation is Pareto superior to the ETIP optimum and generates a net per capita utility gain of $\{\gamma t/3 \ \& \ \alpha[4\ln 2 \& 5\ln(5/3)]\}/5 > 0$.

Under Condition V, the non-IP, ET feasible allocation proposed above strictly Pareto dominates the ETIP optimum and thus dominates any ETIP feasible allocations. Q.E.D.

6. Concluding Remarks

We have examined a model with a finite number of households and with a congestable local public good where the number and locations of facility sites and the level of public good provision are endogenously determined. We prove the existence of an ETIP optimum and show that there may be a continuum of ETIP optimal public facility configurations and an ETIP optimal public facility location need not be central

geographically. An ETIP optimal public facility configuration must be dispersed with multiple facility sites if the degree of congestability and the unit commuting cost are high and the household valuation of the local public good is low. Otherwise, the ETIP optimal public facility configuration is concentrated.

Along these lines, there are at least three possible avenues of interest for future research. First, one may explore the theoretical front, examining the nonemptiness of an ETIP core (thus refining the set of ETIP optima) and the validity of the first and second welfare theorems based on Lindahl or competitive spatial equilibrium. Second, one may relax the assumption of identical households to allow for, say, two types of households, with heterogeneous tastes as in Wang (1990). One may study whether an optimal public facility is geographically biased toward the rich or a consumer who likes the public good. Finally, it is possible to consider more than one public good. When all public goods are necessities, it is interesting to examine whether a concentrated public facility configuration may still emerge where different public goods are provided at the same site.

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Figure 1: Public Facility Configuration and Total Commuting Costs with $N = 2$

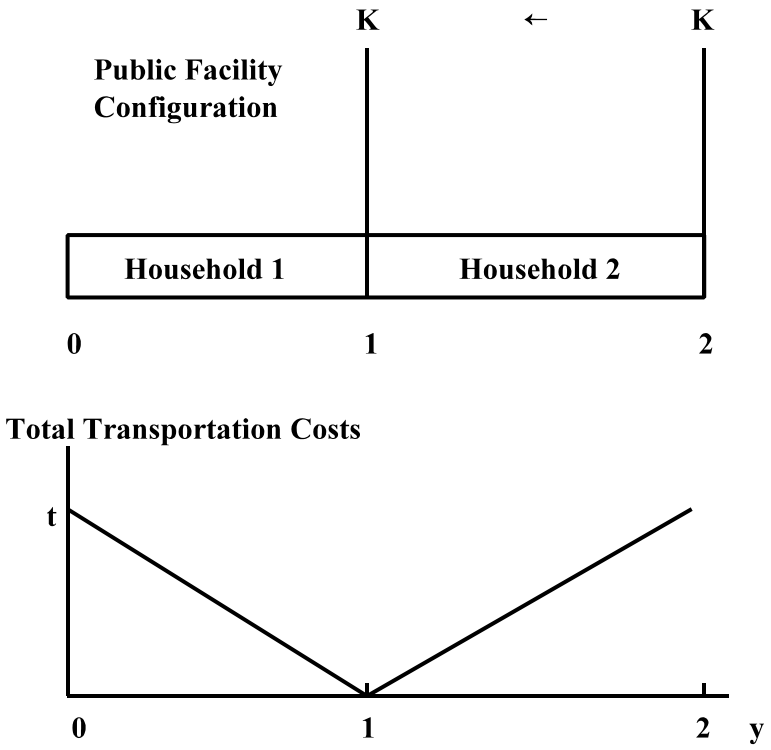
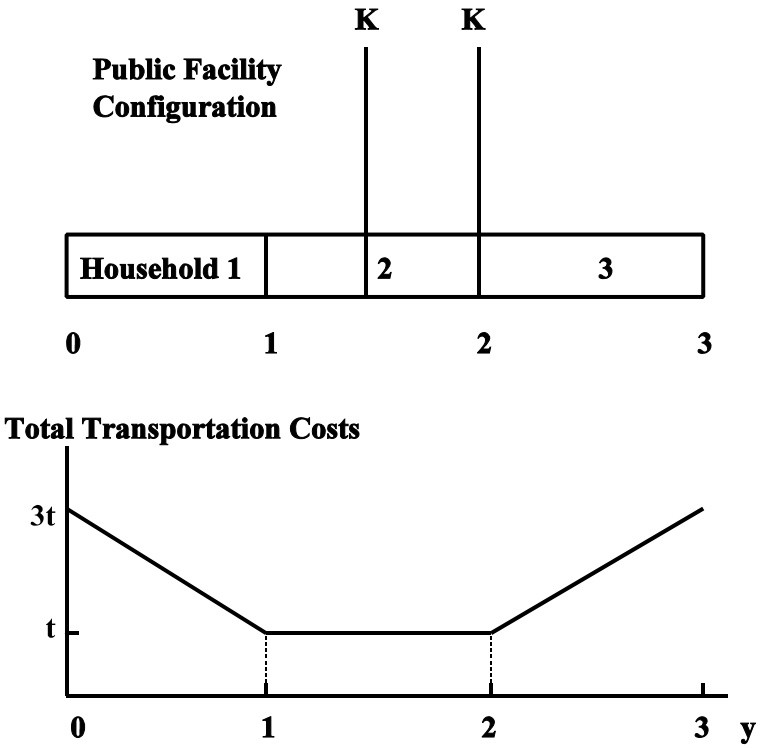
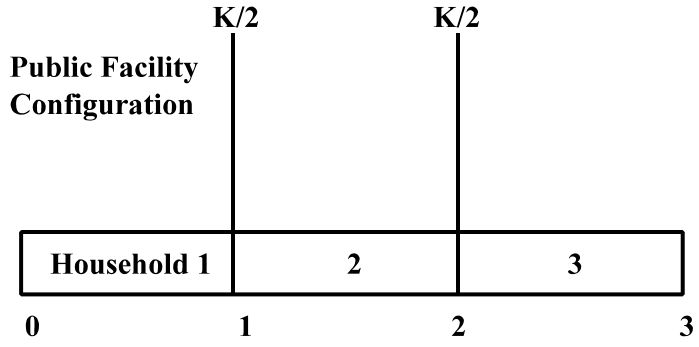


Figure 2: Public Facility Configuration and Total Commuting Costs with $N = 3$

(a) The Case of Concentrated Configuration



(b) The Case of Dispersed Configuration



Appendix

A. Derivation of ETIP Optima:

To solve problem (4) with $v_i = v_o$ for all i = 1,..., N and (4)(i) holding with equality, we note that random travel assignments have non-zero values only for sites $\eta_j \in Y_i$ (i = 1,...,N; j = 1,...,n) and thus we can focus exclusively on $\Theta(Y_i) \equiv \{\theta(i,y_i)\}_{y_i \in Y_i}$. We can set up the Lagrangian function as:

$$\Phi(\{z_i,Y_i,\Theta(Y_i)\}_{i=1,\ldots,N},K,n,H) = u(z_1,G(n,K)) + \sum_{i=2}^{N} \lambda_i^C\{u(z_i,G(n,K))\} + \lambda^Z\{E - K - \sum_{i=1}^{N}(z_i + T(d(x_i,y_i)))\} \quad \text{(A-1)}$$

where $G(n,K) = K/(N^{\alpha}\, n^{1-\alpha})$ and the multipliers λ^C and λ^Z are, in sequential order, associated with the two constraints given in (4).

The first-order conditions are:

$$\partial u_1/\partial z_1 - \lambda^Z = 0, \quad \text{(A-2a)}$$

$$\lambda_i^C(\partial u_i/\partial z_i) - \lambda^Z = 0, \quad i = 2,\ldots,N \quad \text{(A-2b)}$$

$$(\partial u_1/\partial G)(\partial G/\partial K) + \sum_{i=2}^{N} \lambda_i^C(\partial u_i/\partial G)(\partial G/\partial K) = \lambda^Z, \quad \text{(A-2c)}$$

$$\Phi(\{z_i,Y_i,\Theta(Y_i)\},K,n,H) \geq \Phi(\{z_i,Y_i',\Theta'(Y_i')\},K,n',H'), \quad \forall\ Y' \subset H' \subset L,\ H' \neq H,\ n' = \mu(H') \leq X,\ \Theta' \neq \Theta, \quad \text{(A-2d)}$$

For any n, (A-2a) and (A-2b) together imply:

$$(\partial u_1/\partial z_1)/(\partial u_i/\partial z_i) = \lambda_i^C. \quad \text{(A-3)}$$

Substituting (A-2a) and (A-3) into (A-2c) to eliminate λ^Z and λ_i^C, we obtain:

$$(\partial u_1/\partial G)(\partial G/\partial K) + \sum_{i=2}^{N} \frac{\partial u_1/\partial z_1}{\partial u_i/\partial z_i}(\partial u_i/\partial G)(\partial G/\partial K) = \partial u_1/\partial z_1, \quad \text{(A-4a)}$$

or, by rearranging terms,

$$\sum_{i=1}^{N} \frac{1}{\partial u_i/\partial z_i}(\partial u_i/\partial G)(\partial G/\partial K) = 1. \tag{A-4b}$$

Under ETIP, all individuals have the same public good service G and receive $u(z_i, G) = v_0$. Thus, z_i are the same for all i and (A-4b) becomes:

$$N\left(\frac{\partial u/\partial G}{\partial u/\partial z}\right)\frac{\partial G}{\partial K} = 1. \tag{A-5}$$

Define $\Lambda(K,n,z) \equiv \frac{1}{N}\frac{\partial u/\partial z}{(\partial u/\partial G)(\partial G/\partial K)}$. Then, equation (5) follows immediately from (A-5).

Next, from (A-2a), $\lambda^Z > 0$, which together with (A-1) implies $\psi(\{z_i,Y_i,\Theta(Y_i)\},K,n,H)$ is strictly decreasing in the aggregate commuting cost. Thus, letting $(\{z_i, Y_i, \Theta(Y_i)\}_{i=1,...,N}, \{K_j, M_j\}_{j=1,...,n}, n, H)$ be an ETIP optimum, then $\forall H^*$, $\mu(H^*) = n$, $\forall \theta^*(i,y_i)$, $y_i \in Y_i^*$ dH^* $(i = 1,...,N)$, the optimal choice given by (A-2d) satisfies:

$$\sum_{i=1}^{N}\int_{y_i\in Y_i} \theta(i,y_i)\,T(d(i,y_i)) \geq \sum_{i=1}^{N}\int_{y_i^*\in Y_i^*} \theta^*(i,y_i^*)\,T(d(i,y_i^*)) \tag{A-6}$$

This yields the necessary condition (6a) for the optimization problem (4).

Note that under Condition E, (A-3) and (A-6) together with the second constraint of (4) jointly determine the optimal locations of public facilities and the indirect utility level v(n,H) for a given n with $K = K_0$. By comparing $v(n,H)$ for each n ∈ X, the optimal number of public facilities n is therefore pinned down.

B. An Alternative Example: the Case of Cobb-Douglas Utility Function

Assume instead that the utility function of the households takes the Cobb-Douglas form, as in the conventional literature where K is fixed exogenously:

$$u[z,G(n)] = z^{\gamma}[G(n)]^{1-\gamma}, \tag{B-1}$$

where $\gamma \in (0, 1)$ with $1-\gamma$ capturing the household valuation of the public good. The exogeneity of K is required since Condition K is no longer met under the Cobb-Douglas utility function specification. Consider $N = 3$. Utilizing Lemmas 2-4, we can combine (3), (7) and (B-1) to yield:

$$v^C = v^N = \frac{K^{1-\gamma}[(E-K)-t]^{\gamma}}{3^{\alpha(1-\gamma)+\gamma}} \tag{B-2a}$$

$$v^D = \frac{K^{1-\gamma}(E-K)^{\gamma}}{2^{(1-\alpha)(1-\gamma)}3^{\alpha(1-\gamma)+\gamma}}. \qquad \text{(B-2b)}$$

where $C^C = C^N = t$ and $C^D = 0$. Define:

$$B(\alpha,\gamma) = 1-(\frac{1}{2})^{(1-\alpha)(\frac{1-\gamma}{\gamma})} \qquad \in (0,1) \qquad \text{(B-3)}$$

which depends positively on the degree of non-exclusiveness (1-α) and each household's valuation of the public good (1-γ). We now prove:

Proposition 2N: (ETIP optimal location of public facility with N=3) *Assume (A2), Condition E, the linear commuting cost schedule specified as in (1), and the utility function specified as in (B-1). When N = 3, an ETIP optimal public facility configuration satisfies the following properties:*

(i) *if $B(\alpha,\gamma)(E-K) < t$, then an ETIP optimal public facility configuration is always dispersed;*

(ii) *if $B(\alpha,\gamma)(E-K) > t$, then an ETIP optimal public facility configuration is always concentrated, either centralized or non-centralized.*

When $B(\alpha,\gamma)(E-K) = t$, an ETIP optimal public facility configuration may be dispersed or concentrated.

Proof: Comparing (B-2a) and (B-2b), one gets:

$$v^C = v^N \gtrless v^D \quad \Leftrightarrow \quad B(\alpha,\gamma)(E-K) \gtrless t, \qquad \text{(B-4)}$$

which completes the proof. Q.E.D.

From (B-3), $B(\alpha,\gamma)$ depends positively on the degree of non-exclusiveness and the household valuation of the public good.

We next remind the reader that the characterization of the general case of $N > 3$ with multiple public facility sites (i.e., the properties in Lemma 5 and Proposition 4) remain valid under the Cobb-Douglas utility function specification, so long as K is exogenously fixed.

With regard to non-IP local public good provision, we can also show that Proposition 5 still holds for Cobb-Douglas utility if the level of public good provision is fixed exogenously. The modification of the argument in the text is straightforward.

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