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A Productivity Model of City Crowdedness

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Abstract: Population density varies widely across U.S. cities. A simple, static general equilibrium model suggests that moderate-sized differences in cities' total factor productivity can account for such variation. Nevertheless, the productivity required to sustain above-average population densities considerably exceeds estimates of the increase in productivity caused by such high density. In contrast, increasing returns to scale may be able to sustain multiple equilibria at below-average population densities.

Keywords: Population density, productivity, urban agglomeration

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1 Introduction

Crowdedness varies hugely across U.S. cities. Among metropolitan areas with a population of at least 100,000 in 2000, the most crowded (New York City) had a population density forty-nine times that of the least crowded (Dothan, Alabama). Among municipalities with a population of at least 100,000 in 2000, the ratio between the most and least crowded (New York City and Chesapeake, VA) was forty-five.

Economic theory can account for such variation by assuming that more crowded areas are characterized by higher productivity. But are the required productivity differences plausible? Conversely, the very existence of cities is often attributed to increases in productivity that are *caused* by dense concentrations of economic activity (Marshall, 1890; Jacobs, 1969). How do the increases in productivity required to sustain higher density compare with estimates of the higher productivity engendered by such higher density?

To answer these questions, the present paper lays out and calibrates a simple, static general equilibrium model of city crowdedness. Homogenous individuals choose to live and work in one of two local economies. They derive utility from consumption of a traded good and housing. Firms in each economy produce the traded good and housing using land, capital and labor. Total factor productivity is assumed to vary exogenously between the two economies. In equilibrium, each economy must offer individuals the same level of utility and provide capital with the same rate of return. The resulting model is similar to those in Henderson (1974, 1987, 1988), Haurin (1980), Upton (1981), and Haughwout and Inman (2001).

A baseline calibration of the model suggests that accounting for the observed variation in crowdedness requires productivity differences among cities on the order of 40 percent. Such productivity differences are far less than is observed among U.S. states. Nevertheless, the productivity required to obtain especially high population densities is considerably above estimates of the increases in productivity caused by such high density. As argued by Kim (1999) and Davis and Weinstein (2002), accounting for high crowdedness requires substantial differences in local "fundamentals." But as argued by Chatterjee (2003), much smaller exogenous differences in productivity can underpin large differences in density at low degrees of crowdedness. Hence realized density among sparsely populated cities may be especially history dependent.

The paper proceeds as follows. Section 2 describes in more detail the empirical variation in crowdedness across U.S. cities. Sections 3 and 4 lay out the model and discuss its calibration. Section 5 describes the model's numerical results, both for a baseline calibration and for several large perturbations to it. It then presents empirical results that confirm two of the model's key implications. For both wages and housing expenditures, positive correlations with density increase in magnitude as density increases. Section 6 compares the productivity differences required to sustain crowdedness with estimates of population density's agglomeration effect. A last section briefly concludes.

2 Empirical Motivation

That crowdedness varies hugely across cities is clear. By how much it does so is less clear. More specifically, a theoretical city corresponds to several possible geographic units. And depending on the size of the geographic unit, crowdedness can be measured in several possible ways.

From a theoretical perspective, a "city" is meant to be a geographic place where a given group of people both live and work. For the United States, metropolitan areas (which are defined by the Office of Management and Budget primarily based on Census Bureau data) best correspond to this. The raw density of metropolitan areas—that is, their population divided by land area—varies by a multiplicative factor of more than four hundred (Table 1, Panel A). The problem is that metropolitan areas are constructed as the combination of whole counties, large parts of which may be agricultural or unoccupied.

"Urbanized areas" (UAs) are an alternative empirical counterpart of a theoretical city. They are constructed by the Census Bureau to include only the densely-settled land area within metropolitan areas. Raw population density among urbanized areas varies by a multiplicative factor of eight (Panel B). The problem with UAs is that their raw population density greatly understates the population density experienced by millions of people living in the most crowded of them. For example, the New York City UA has a population of just under 18 million and a raw density of 5.3 (thousand persons per square mile). But for the 8 million people living in the actual municipality of New York City, population density is 26.4. For the 1.5 million people living in the borough of Manhattan, it is 66.9.

To address the shortcomings of the previous two alternatives, this paper's preferred measure of crowdedness is a population-weighted mean of metro-area subunit densities (Glaeser and Kahn, 2004). Whereas raw population density gives crowdedness as experienced by the average unit of land, population-weighted density gives crowdedness as experienced by the average person. More specifically, the Census Bureau partitions all U.S. counties into subdivisions. These subdivisions are further partitioned into the portions of any municipalities that lie within them (many municipalities span multiple subdivisions) along with any remaining unincorporated area. A population-weighted mean of the raw densities of such county-subdivision place/remainders suggests that metro area crowdedness in 2000 varied by a multiplicative factor of forty-nine (Panel C).[1]

A final measure of crowdedness is the raw population density of municipalities. It varies by a multiplicative factor of forty-five (Panel D). Of course, the disadvantage of using municipalities as an empirical counterpart of cities is that many people live in one municipality but work in another. But the similar range of variation to that of the weighted metro-area density reinforces the latter as a good measure of crowdedness.

3 Model

The model uses a static, open-city framework. The world is made up of two open economies, one small and one large. The small economy can be interpreted as a locality: a well-defined market for factors and goods. The large economy can be interpreted as the aggregate of numerous other localities. The size distinction reflects relative land areas. An important semantic point is that the small economy may be considerably more crowded than the large economy, in which case it might be interpreted as a "big city".

3.1 Firms

Within each economy ($i = s, l$), perfectly-competitive firms employ a constant-returns-to-scale production function that combines land, capital, and labor (D_i, K_i, and L_i) to produce a traded numeraire good and housing (X_i and H_i). Housing must be consumed in the economy

[1] A population weighted mean of the county subdivisions suggests that density varies by a multiplicative factor of sixteen hundred. A population weighted mean of census tracts, which are the next smallest unit below the county-subdivision place/remainders, suggests that density varies by a multiplicative factor of sixty-four.

in which is produced. Aggregate production within each economy is given by

$$X_i = \mathrm{A}_{X,i}\, D_{X,i}^{\alpha_{X,D}}\, K_{X,i}^{\alpha_{X,K}}\, L_{X,i}^{\alpha_{X,L}} \tag{1}$$

$$H_i = \mathrm{A}_{H,i}\left(\eta_{D,KL} D_{H,i}^{\frac{\sigma_{D,KL}-1}{\sigma_{D,KL}}} + \left(1-\eta_{D,KL}\right)\left(K_{H,i}^{\alpha_{H,K}} L_{H,i}^{\alpha_{H,L}}\right)^{\frac{\sigma_{D,KL}-1}{\sigma_{D,KL}}}\right)^{\frac{\sigma_{D,KL}}{\sigma_{D,KL}-1}} \tag{2}$$

Production of the traded good is Cobb-Douglas. The factor income share parameters are each assumed to be strictly positive with $\alpha_{X,D} + \alpha_{X,K} + \alpha_{X,L} = 1$. Production of housing is constant elasticity of substitution (CES) with respect to land and an implicit intermediate product of capital and labor. The elasticity of substitution between land and the capital-labor intermediate good is given by $\sigma_{D,KL}$. The weighting parameter $\eta_{D,KL}$, which lies strictly between 0 and 1, calibrates the relative share of factor income accruing to land. The capital-labor intermediate hybrid good is produced with constant returns to scale: $\alpha_{H,K} + \alpha_{H,L} = 1$. These coefficients determine the division of factor income between capital and labor.

Total factor productivity, $\mathrm{A}_{X,i}$ and $\mathrm{A}_{H,i}$, varies exogenously between the two economies. It thus serves as the only source of crowding. In contrast, the endogenous growth and new economic geography literatures typically assume that TFP increases with the scale of production. Section 6 below compares the productivity differences *required* to sustain crowding with estimates of the productivity differences such crowding may engender.

Profit maximization by perfectly competitive firms induces demand such that each of the factors is paid its marginal revenue product. Frictionless intersectoral mobility assures intersectoral factor price equalization within each economy. Let p_i give the price of housing in terms of the traded good. The economy-specific returns to land, capital, and labor are respectively given by

$$r_{D,i} = \partial X_i/\partial D_i = p_i\, \partial H_i/\partial D_i \tag{3}$$

$$r_{K,i} = \partial X_i/\partial K_i = p_i\, \partial H_i/\partial K_i \tag{4}$$

$$w_i = \partial X_i/\partial L_i = p_i\, \partial H_i/\partial L_i \tag{5}$$

Capital is additionally assumed to be perfectly mobile across economies. Hence its return must be the same in both economies. Because the present framework is static, this identical capital rent is taken as exogenous. In a dynamic neoclassical framework, it would equal the real interest rate plus the rate of capital depreciation.

3.2 Individuals

Individuals derive utility from consumption of the traded good and of housing:

$$U_i = \left(\eta_{x,h}\, x_i^{\frac{\sigma_{x,h}-1}{\sigma_{x,h}}} + \left(1 - \eta_{x,h}\right) h_i^{\frac{\sigma_{x,h}-1}{\sigma_{x,h}}} \right)^{\frac{\sigma_{x,h}}{\sigma_{x,h}-1}} \tag{6}$$

The parameter $\sigma_{x,h}$ measures the constant elasticity of substitution between the traded good and housing. The weighting parameter $\eta_{x,h}$ lies strictly between zero and one.

Optimizing behavior by individuals equates the ratio of marginal utility to price *within* each economy. Additionally, individuals' mobility equates utility levels *between* economies.

$$\partial U_i / \partial x_i = \frac{\partial U_i / \partial h_i}{p_i} \tag{7}$$

$$U_s = U_l \tag{8}$$

Individuals supply labor inelastically. They must each satisfy a budget constraint,

$$x_i + p_i h_i \ = \ w_i + nonwage \tag{9}$$

Under the base set of assumptions below, non-wage income is assumed to be zero. In this case, capital and land rents can be interpreted as being paid to absentee owners who reside outside of either economy. Under an alternative set of assumptions discussed in the sensitivity analysis, non-wage income is the per capita sum of all capital and land rents collected in both economies: $nonwage \ = \ \sum_i (r_K\, K_i \ + \ r_{D,i}\, D_i) / \sum_i L_i$. The variable L_i gives the population of each economy. Note that non-wage income is assumed to always be identical between the two economies.

3.3 Closure

In addition to the profit and utility maximization conditions, several adding up constraints must be met. For each of the economies, the land and labor factor markets and the housing market must clear.

$$D_{X,i} + D_{H,i} \ = \ D_i \tag{10}$$

$$L_{X,i} + L_{H,i} \ = \ L_i \tag{11}$$

$$h_i\, L_i \ = \ H_i \tag{12}$$

Additionally, the sum of local populations must equal the exogenously given total population.

$$\sum_i L_i = L \tag{13}$$

The combined optimization conditions, individual budget constraints, local adding up constraints, and global adding up constraint can be reduced to a nonlinear system of eleven equations with eleven unknowns. The absence of any sort of increasing returns to scale combined with the fixed land supply and decreasing marginal utility suggests that any solution to this system will be unique.

4 Calibration

The primary purpose of the present paper is to gauge the magnitude of the variation in total factor productivity that is required to match the widely varying degree of crowdedness we observe across U.S. cities. In this spirit and to not imply a false level of precision, parameters are set to round values. The numerical results section includes an extensive sensitivity analysis.

To simplify the analysis, the small economy is henceforth assumed to have approximately zero land area. This shuts down any feedback from it to the large economy. Doing so is especially helpful when land and capital factor payments are assumed to be made to individuals within the two-economy system rather than to absentee owners. The large economy serves three functions. First is to calibrate the weighting parameters in the housing production and utility functions. Second is to determine the reservation level of utility that individuals in the small economy must attain. Third is to determine the level of non-wage income when factor payments are indeed recycled

4.1 Production

The calibration of production requires determining the large-economy factor income share accruing to each of land, capital, and labor in both the traded-good and housing sectors. For the housing sector, it additionally requires determining the elasticity of substitution between land and the capital/labor composite. Lastly, the rate-of-return determining capital intensity needs to be specified. Table 2 summarizes the base parameterization as well as alternative values that will be used in the sensitivity analysis.

The land share of factor income derived from the production of the traded good is assumed to be 1.6%. This value is a weighted average across a large number of industries using intermediate input shares estimated by Jorgenson, Ho, and Stiroh (2005).[2] It is nearly identical to the 1.5% land share Ciccone (2002) suggests is reasonable for the manufacturing sector. Sensitivity analysis is conducted for land factor shares equal to 0.4% and to 4.8%. One-third of remaining factor income is assumed to accrue to capital; two-thirds is assumed to accrue to labor (Gollin, 2002). Because traded-good production is Cobb Douglas, the assumed factor shares will hold in both economies.

Non-Cobb-Douglas production in the housing sector implies that factor income shares will differ between the two economies. Numerical results are somewhat sensitive to the assumed land share. Under the base parameterization, its large-economy value is set to 35%. This is below a recent estimate that land accounts for approximately 39% of the implicit factor income attributable to aggregate U.S. housing stock. (Davis and Heathcote, 2005).[3] Using micro data, several other researchers have found substantially lower land shares. Based on houses sold in the Knoxville metro area, Jackson, Johnson, and Kaserman (1984) estimate that land accounts for 27% of implicit factor income. Based on houses constructed in new subdivisions of the Portland Oregon metro area, Thorsnes (1997) estimates that it accounts for 17%. But Knoxville is among the least densely populated metro areas. And new subdivisions tend to be located at the metro fringes. In both cases, land prices are likely to be below average. If the production elasticity of substitution with land is below one, land's factor share would be below average as well. None of the empirical studies accounts for the especially high land intensity of services that are strongly complementary with housing such as residential streets, parks, schools, and municipal utilities. For the sensitivity analysis, the housing land factor share is assumed to equal 20% and 50%. As with traded-good production, one-third of remaining factor income is assumed to accrue to capital; two-thirds is assumed to accrue to labor.

The elasticity of substitution between land and non-land inputs in the production of

[2] The industry-specific intermediate input estimates, which are not included in the publication, were kindly provided by the authors.

[3] Davis and Heathcote find that between 1975 and 2004, land accounted for an average of 47% of the sales value of aggregate U.S. housing stock. Adjusting for the fact that structures depreciate but land does not using a 1.6% rate of structure depreciation as suggested by Davis and Heathcote and a 4% required real rate-of-return gives a 38.8% land factor share.

housing, $\sigma_{D,KL}$, is assumed to be 0.75. No clear consensus exists on an appropriate value. A survey by MacDonald (1981) reports preferred estimates from twelve different studies ranging from 0.36 to 1.13. Updating this research, Jackson, Johnson, and Kaserman (1984) estimate the elasticity to lie somewhere between 0.5 and 1. More recently, Thorsnes (1997) argues that a unitary elasticity of substitution cannot be rejected. For the sensitivity analysis, $\sigma_{D,KL}$ is assumed to equal 0.50 and 1.

Finally, the rent on the services of capital goods, r_K, is set to 0.08, which implicitly represents the sum of a required annual real return plus an annual allowance for depreciation. However, results are completely insensitive to the parameterization of r_K. This makes sense since the framework has no natural time context.

4.2 Utility

The calibration of the utility function, (6), requires parameterizing the elasticity of substitution between the traded good and housing as well as setting the weighting parameter that determines the large-economy share of consumption spent on housing.

The elasticity of substitution, $\sigma_{x,h}$, is assumed to equal 0.50. It is calibrated using cross-section data on housing prices and the share of consumption expenditure spent on housing. The dots in Figure 1 plot the latter against the former for 24 large metro areas.[4] The lines represent the expected housing expenditure share for each of three elasticities of substitution.[5] The line for $\sigma_{x,h}$ equal to 0.50 almost exactly overlays the fitted relationship from a linear regression. This baseline value is close to numerous estimates of the price elasticity of housing demand, the negative of which corresponds to $\sigma_{x,h}$ (Goodman, 1988,

[4]The housing price measure is an index of the rental price of apartments in professionally-managed properties with five or more units. It is constructed by Torto Wheaton Research based on quarterly surveys. The index adjusts for the number of bedrooms per unit and a property's age but not for other characteristics such as square footage, parking, and location. The inability to control for these implies that the index measures a hybrid of housing rental prices and housing rental expenditures. Because of substitution, expenditures understate variations in prices. An accurately-measured house price would likely result in a scatter more horizontal than depicted in Figure 1. An additional shortcoming of the present price index is that it fails to measure the price of owner-occupied housing. The resulting direction of bias is less clear.

[5]For each elasticity, the weighting parameter $\eta_{x,h}$ is chosen so that the expected expenditure share passes through the fitted expenditure share for Pittsburgh based on a linear regression. Pittsburgh's weighted density is close to the population median.

2002; Ermisch, Findlay, and Gibb, 1996; Ionnides and Zabel, 2003). As another source of comparison, some typical open-economy calibrations of the elasticity of substitution between traded and nontraded goods include 0.44 (Mendoza, 1995) and 0.74 (Tesar, 1995). For the sensitivity analysis, $\sigma_{x,h}$ is assumed to equal 0.25 and 0.75.

For a given traded-good-to-housing elasticity, $\sigma_{x,h}$, the weighting parameter $\eta_{x,h}$ is chosen such that large-economy individuals spend 18% of their consumption expenditures on housing. This approximately matches the aggregate U.S. value from 2001 to 2003. The sensitivity analysis alternatively assumes large-economy housing expenditure shares of 14% and 22%.

5 Numerical Results

The model's mechanics are straightforward. The large economy serves to calibrate the utility and production weighting parameters. It also determines the reservation level of utility that small-economy residents must attain and the amount of non-labor income they receive when all factor payments are recycled. As the small economy's total factor productivity exogenously increases, so too do the marginal products of labor, capital, and land. These attract complementary inflows of labor and capital. The resulting increase in aggregate housing demand puts additional upward pressure on land prices. In equilibrium, traded-good-denominated wages, house prices, and land prices must all rise. The equating of small-economy with large-economy utility comes via an increase in small-economy traded consumption but a decrease in small-economy housing consumption.

The first subsection below illustrates these mechanics under the base calibration. The required TFP to increase crowdedness is relatively small when the small economy is sparsely populated. But as the small economy becomes more crowded, the additional TFP required to achieve additional increases in crowdedness becomes large. The productivity increases are accompanied by similar sized increases in wages, somewhat larger increases in housing prices, and an order-of-magnitude larger increases in land prices.

A second subsection discusses the sensitivity of the model's quantitative results to the parameterization. Unsurprisingly, resistance to crowding is strongly increasing with respect to the implicit land factor content share of large-economy consumption.

A last subsection argues that the numerical results nicely match several empirical estimates. In particular, the model predicts that the induced elasticities of wages and housing expenditures with respect to population density will be increasing. Cross-sectional regressions find strong evidence that this is indeed the case.

5.1 Base Calibration

Numerical results from the base calibration are shown in Figure 2.

Panel A plots the small-economy relative total factor productivity required to attain a range of relative population densities. The vertically-plotted TFP variation should be interpreted as exogenous. It is assumed to hold for production of both the traded good and housing. The horizontally-plotted relative population density should be interpreted as an endogenous response. So, for example, a small-economy population density one-fourth that of the large economy follows from a small-economy TFP that is 0.92 that of the large economy. A small-economy population density four times that of the large economy follows from a small-economy TFP that is 1.14 times that of the large economy.[6] The slope of the locus measures the elasticity of required TFP with respect to relative population density, ϵ_{TFP}. The locus' positive second derivative implies that this elasticity increases with crowdedness. Specifically, ϵ_{TFP} increases from 0.05 to 0.07 to 0.12 as density increases from one-fourth to one to four.

The required TFP increase as the small economy becomes more crowded causes a rise in traded-good-denominated wages (Panel B). Because of capital deepening, wages vary by slightly more than does TFP. As with TFP, the wage-to-density locus has a positive second derivative. In other words, the induced elasticity of wages with respect to density is increasing with crowdedness.

The remaining panels of Figure 2 plot the relationships between a number of other endogenous outcomes and population density. Increases in population density pull land out of traded good production into housing production (Panel C). As density increases from one-fourth to one to four, the percent of land devoted to housing production increases from 62 to 74 to 83. Relative land prices vary by an order of magnitude more than do wages (Panel

[6]The increase in population density as productivity increases is a numerical result rather than an analytical one. Increases in only traded-good productivity can actually lower density as land becomes too valuable to be used for housing production. But numerical results show such "crowding out" occurs only when labor's share of factor income is below 20%, which for most industries is unrealistically low.

D). They go from 0.16 to 7.3 as relative density goes from one-fourth to four. As the price of land increases, so too does its share of housing factor income (Panel E). But the actual land factor content of housing—that is, land per unit of housing—falls with density (not shown). At a one-fourth density, the quantity of land per housing unit is more than three times its large-economy level. At four-times density, land per unit housing is approximately one third its large-economy level.

The sharply rising price of land causes the price of housing to increase as well (Panel F). But the rise in house prices—from 0.60 to 2.1 as density rises from one-fourth to four—is considerably more moderate. Housing expenditures rise by even less (also Panel F). As with TFP and wages, the induced elasticities of house prices and house expenditures with respect to density increase with crowdedness. The share of expenditures devoted to housing increases with crowdedness as well(Panel G). But the actual quantity of housing consumed falls (Panel H). To achieve the large-economy reservation utility, traded-good consumption must rise with crowdedness (also Panel H). At a one-fourth density, relative traded and housing consumption are respectively 0.96 and 1.24. At a four-times density, relative traded and housing consumption are 1.08 and 0.75.

5.2 Sensitivity Analysis

The present model requires five key parameterization choices. Figure 3, Panels A through E, illustrate the dependence of required TFP on each of these. Table 3 provides a partial summary. Unsurprisingly, changes that increase the implicit land factor share of large economy consumption—either by explicitly increasing land's factor share in production or by increasing the expenditure share of land-intensive housing—increase resistance to crowding. Less obviously, decreasing the production and consumption elasticities increases resistance to crowding at high relative densities but decreases it at low relative densities.

Resistance to crowding depends closely on the importance of land in production (Panels A and B). As discussed above, the elasticity of required TFP with respect to density under the base calibration increases from 0.05 to 0.07 to 0.12 as relative density increases from one-quarter to one to four. Tripling land's factor share of traded-good production from its baseline value of 1.6%, ϵ_{TFP} at the same benchmark density levels rises from 0.08 to 0.10 to 0.14. Increasing land's factor share of housing production from its baseline value of 35% to

50%, ϵ_{TFP} rises from 0.06 to 0.10 to 0.19.

Variations in resistance to crowding imply different relative population densities for a given difference in productivity. Under the base calibration, a TFP deficit of 5% (i.e., a TFP level of 0.95) causes the small economy to have a relative density of 0.46. With land accounting for just 20% of housing production, the same 5% TFP deficit causes density to fall all the way to 0.28. With land accounting for 50% of housing production, a 5% TFP deficit causes population density to fall only to 0.58.

Correspondingly, variations in resistance to crowding are reflected in different required TFP levels to achieve a given relative density. Attaining a relative density of four requires a 14% TFP premium under the base calibration. With housing land factor shares of 20% and 50%, it requires respective premiums of 8% and 22%.

Resistance to crowding responds nonmonotonically to the housing-production elasticity of land (Panel C). At low relative densities, ϵ_{TFP} is increasing with $\sigma_{D,KL}$. At intermediate and high relative densities (including at a unitary density), it is decreasing with $\sigma_{D,KL}$. The latter relationship is easily visible and is the more intuitive. As crowding makes land more scarce, a high elasticity allows for easy substitution to the hybrid capital-labor input.

The nonmonotonic sensitivity to $\sigma_{D,KL}$ derives from the curvature of the CES production function. As the elasticity goes to zero, the housing production isoquant between land (on a horizontal axis) and the hybrid capital-labor input (on a vertical axis) becomes Leontief. In other words, at low levels of the land input, the isoquant is nearly vertical. At high levels, it is nearly horizontal. Land per unit of housing is inversely correlated with population density. At low densities, increased crowdedness is associated with leftward movement along the horizontal portion of the isoquant. The marginal cost of housing production increases only slightly as the land input decreases. But at high densities, increased crowdedness is associated with upward movement along the vertical portion of the isoquant. The marginal cost of production increases sharply with further decreases in the land input.

As might be expected, increasing housing's share of consumption expenditures increases resistance to crowding (Panel D). This is because production of housing is more intensive in land than is production of the traded good.

Similarly intuitive is the increased resistance to crowdedness at intermediate and high population densities that arises from decreasing the substitutability between the traded good and housing (Panel E). As $\sigma_{x,h}$ decreases, individuals become less willing to endure low housing consumption in return for high traded good consumption. But for the same reason as the nonmonotonic response to $\sigma_{D,KL}$, resistance to crowdedness is relatively insensitive to $\sigma_{x,h}$ at low population densities.

Panels F and G show resistance to crowding under low-resistance and high-resistance combinations of the parameters just discussed. In Panel F, parameters are chosen to minimize and maximize resistance to crowdedness *at high density levels.* The low resistance combination assumes weightings that implicitly minimize land's share of consumption (low land factor shares for both goods and a low housing consumption share) along with high elasticities of substitution ($\sigma_{D,KL}$ equal to 1 and $\sigma_{x,h}$ equal to 0.75). Under this low-land, high-elasticity combination, small differences in TFP lead to large changes in population density. Resistance, ϵ_{TFP}, remains between 0.02 and 0.03 even as density varies from well below one-fourth to well above four. Under the converse high-land, low-elasticity combination, extremely large changes in TFP are required to get large density changes. At one-fourth density, ϵ_{TFP} equals 0.09 and it then rapidly increases with crowdedness. In Panel G, parameters are chosen to minimize and maximize resistance to crowdedness *at low density levels.* In this case the low-resistance combination assumes low elasticities of substitution. At one-fourth density, ϵ_{TFP} is just 0.01. But it then rapidly ramps up as density increases above one. Under the high-land, high-elasticity combination, resistance remains fairly stiff at all densities.

The numerical results so far have assumed that TFP varies for both the traded good and housing. Resistance to crowdedness is stronger if TFP varies only for the traded good (Panel H, dashed-dotted line). But the larger variation in traded-good productivity required to achieve a given difference in crowdedness is slight at low and intermediate densities.

Another baseline assumption is that individuals receive only labor income. Allowing for capital income stiffens resistance to crowding (Panel H, dashed line). In this case, the alternative assumption is that individuals receive an identical capital payment regardless of where they live. As discussed in the theory section, its size is the per capita sum of factor payments to land and capital across both economies. Without capital income, real wages—that is, traded-good denominated wages deflated by a "true cost of living index" (Diewert, 1993)— must be equal across economies. With capital income, real wages must be higher in the more crowded economy. This is to compensate for the lower purchasing power of capital income where housing prices are high. The higher real wages in turn rely on even larger variations in TFP.

The parameterization and baseline assumptions obviously affect numerous endogenous outcomes in addition to the resistance to crowdedness. Table 3 shows the sensitivity of the main variables' elasticity with respect to crowdedness at a unitary density. Most of the qualitative results are intuitive.

For example, land and house prices rise more steeply with density and housing consumption falls more steeply with density as land's large-economy factor share of housing increases. For densities above one, the same holds true as $\sigma_{D,KL}$ decreases. Also for densities above one, decreasing $\sigma_{x,h}$ causes land and house prices to rise more steeply with density but housing consumption to fall less steeply with it. Less intuitive is that a higher traded-good land factor share dampens elasticities with respect to crowdedness. Recall from above that a higher traded-good land share increases ϵ_{TFP}. In contrast, the elasticities of wages, land prices, housing prices, and housing expenditures all fall. Increasing the land factor share of traded-good production increases the availability of land that can be pulled into the housing sector. Substituting away from land is more attractive in traded-good production than in housing production because of the former's assumed unitary elasticity of substitution.

5.3 Empirical Match

The model predicts that the elasticities of wages and of housing expenditures with respect to population density are each increasing. Cross-sectional regressions using aggregate data find strong support for both predictions at low population densities but not at high ones. From a quantitative perspective, the estimated wage elasticities are considerably higher than is predicted by the baseline parameterization of the model. Two of the model's simplifying assumptions are likely to account for a portion of the discrepancy between the estimated and predicted wage elasticities.

Table 4 shows results from regressing the log of the median wage among non-Hispanic white males on the linear and quadratic logs of relative population density. The regressions are descriptive only and are not meant to imply any sense of causality. Columns 2 through 5 use metropolitan areas as the geographic unit of observation. Density is calculated using a population-weighting of county-subdivision place/remainders as in Table 1 Panel C. The denominator for measured relative density is the population median among metro residents.

The coefficient on linear log density corresponds to the wage elasticity at the population median. Not controlling for anything, this elasticity is estimated to be 0.20 (column 2). Controlling for educational attainment, it falls to 0.15 (column 3). Including the educational controls is the more sensible specification since it better corresponds to the model's assumption of homogenous agents. The same rationale underlies the limiting of the sample population to non-Hispanic white males. With the education controls, a positive coefficient on quadratic relative population density differs from zero at the 0.05 level. In other words, the wage elasticity is indeed estimated to be increasing with respect to density.

Columns 4 and 5 show results from a slightly more flexible specification. It allows the coefficients on quadratic density to differ depending on whether density is above or below the population median. Doing so increases the leverage of high-density observations, which are vastly outnumbered by low-density ones. Allowing for such a spline on the quadratic term shows that the increase in wage elasticity holds only for densities below the population median.[7]

Column 6 shows results using urbanized areas as the geographic unit of observation and density measured simply by total population divided by total land rather than by a population-weighting of subunits. Similar to the case of metro areas, the elasticity of wages with respect to density is found to be increasing for population densities below the population median. The corresponding coefficient on quadratic density differs from zero only at the 0.10 level. But the equality of the two quadratic coefficients (for observations below and above the population median) can not be rejected. Constraining them to be equal yields a small positive coefficient on quadratic density that is significant at the 0.01 level (not shown).

A similar pattern of an increasing elasticity for observations with below-median density also holds for housing expenditures (Table 5). In columns 2 and 3, expenditures are measured by monthly rent. Both for metro and urbanized areas, a positive coefficient on below-median quadratic density statistically differs from zero, though for metro areas it only does so at the 0.10 level. Constraining the metro-area specification to a single quadratic term yields a positive corresponding coefficient that differs from zero at the 0.05 level (not shown). A disadvantage of measuring expenditures by rent is that high-quantity housing units are often available only for purchase. In columns 4 and 5, expenditures are measured by owners' estimates of their unit's sales value. The corresponding elasticities with respect to density

[7]Maximum likelihood estimation suggests that an increasing elasticity may hold up to densities considerably above the population median.

prove to be even more sharply increasing for below-median observations.

In addition to testing the model's prediction that the wage and housing expenditures are increasing with density, it is also possible to quantitatively compare predictions with estimates. The predictions are considerably lower. In Table 4, Column 1 shows the predicted wage elasticity under the model's base specification at densities one half, one, and two. Over this interval, it rises from 0.06 to 0.08 to 0.11. The bottom rows of columns 2 through 6 show estimated wage elasticities at these same densities relative to the population median. Under the preferred empirical specification (column 5), the estimated elasticity rises from 0.11 to 0.20 and then falls to 0.15.[8]

There are several possible explanations for the lower predicted compared to estimated elasticity of wages with respect to density. One is the model's simplifying assumption that the small economy's land area is fixed. As higher productivity attracts population to the small economy, its land area is likely to increase. Indeed the elasticity of population-weighted density with respect to population is only 0.34, which suggests that two-thirds of any increase in population gets dissipated by an increase in land area (Table 1 Panel C). Allowing for such changes in land area would increase the variation in productivity—and hence in wages—required to achieve a given variation in population density.

A second possible explanation for the lower predicted wage elasticity is the model's assumption of homogeneous individuals. Of course, people differ in innumerable ways. For present purposes, the most relevant is that some people may have more human capital than others. The empirical results suggest that individuals in more dense cities may have higher human capital. For example, controlling for education lowers the estimated wage elasticity. Similarly, other unobserved attributes are likely to account for a portion of the positive empirical correlation (Combes, Duranton, and Gobillon, 2003; Lee, 2005).

[8] An important caveat is that the observed population median density is only a rough proxy for the modeled large-economy density. For example, large-economy density might alternatively correspond to the population-weighted mean density or—equivalently—to population-weighted aggregate density. However, the 18-percent base calibration of the large-economy housing share more closely corresponds to the observed shares of metro areas with density near the population median rather than to those with density near the population mean. More meaningful than the point comparisons is the lack of overlap between the modeled and estimated elasticities. Metro areas with density one-half to twice the metro-are population median contain 64 percent of the metro-area sample population. The interval includes the population mean, which is 1.6 times the population median.

Still another explanation is that the model is better parameterized by a combination that results in higher predicted wage elasticities. For example, the high-land, low-substitutability combination predicts wage elasticities similar to the estimated range.

The predicted housing expenditure elasticities more closely match estimates. The predicted elasticity range (0.22 to 0.33) does exceed the preferred rent-based estimate (0.10 to 0.16) but is right in the center of the preferred value-based estimate (-.13 to 0.41).

The model predicts numerous other elasticities and shares. Most are considerably difficult to measure empirically. The model does approximately match the relationship between density and the housing share of consumption. But the calibration of $\sigma_{x,h}$ was based, at least in part, to make this so. The especially high elasticity of the price of land with respect to density seems reasonable. As density rises from one quarter to four, the price of land rises by a factor of 45. Tax assessments of land in central business districts of large U.S. cities vary by approximately the same degree.[9]

Similarly reasonable is that housing's share of land will increase with density. Landscaped corporate parks are simply not affordable in crowded places. From a quantitative perspective, the base parameterization predicts that 74 percent of large-economy land will be used for housing. This compares with a USDA estimate that 55 percent for all urban land was used for residential purposes in 1997 (Vesterby and Krupa, 1997). However, the USDA estimate is based on an extremely expansive definition of "urban" to include all places with just 2,500 people. Many such urban places are sparsely populated and so are predicted by the model to have low residential land shares.

Another comparison concerns whether the magnitude of the productivity differences required to sustain observed differences in crowdedness are plausible. Here the answer is almost certainly yes. Under the baseline calibration, a small economy with relative density 6.8 (corresponding to New York City) requires TFP 1.37 times that of a small economy with relative density 0.4 (corresponding to Dothan Alabama). In other words, a 37 percent productivity difference suffices to underpin the observed forty-nine fold difference in density. No good

[9]Cities and approximate assessed value per square foot circa 2005 are Columbus ($230), Dallas ($60), Milwaukee ($70), Sacramento ($220), San Francisco ($2,300), Seattle ($500), Tampa ($50), and Washington D.C. ($1,600). Values represent highest assessment among central business district parcels of at least five thousand square feet. Central business district locations were chosen to include the most expensive commercial property near what appear on maps to be downtown areas.

estimates of city-level productivity exist with which to compare this. But the much larger productivity differences that are observed across U.S. states and across developed nations suggest that the required variation in city-level productivity is easily plausible. Specifically, Ciccone and Hall (1996) estimate that the most productive U.S. state (New Jersey) had output per worker in 1988 that was 1.70 times that of the least productive U.S. state (South Dakota).[10] Hall and Jones (1999) estimate that U.S. TFP in 1988 was 41 percent above that of Ireland and Denmark and 52 percent above that of Japan. Among the non-former communist members of the European Union, the ratio of the highest to lowest TFP in 1988 (corresponding to Italy and Greece, respectively) was 1.79.

Overall, the model appears to be a reasonably good match to empirical estimates of correlations with density. This suggests that it can serve to evaluate estimates of the agglomeration effects of density.

6 Increasing Returns

The modeling so far has taken total factor productivity to be exogenous. In fact, it is unlikely to be so. Indeed the very existence of cities is often premised on the existence of a productivity advantage via some sort of increasing returns to scale (Marshall, 1890; Jacobs, 1969). This section compares the productivity required to sustain crowdedness as predicted by the model with recent estimates of the increases in productivity attributable to such crowdedness. The required TFP is generally larger. Increasing returns in productivity appears insufficient to account for the wide range of population density across U.S. cities. This conclusion is especially true in accounting for the most crowded cities.

Many studies have tried to measure the increase in productivity attributable to city size. Estimates of the elasticity of TFP with respect to employment tend to fall between 0.03 and 0.08 (Rosenthal and Strange, 2004). Only a few studies have sought to measure the increase in productivity attributable to *density*. Using U.S. state and county data, Ciccone and Hall (1996) estimate the elasticity of TFP with respect to employment density, υ_{TFP}, to be approximately 0.04. Using European data, Ciccone (2002) estiamtes υ_{TFP} to be 0.05.

[10] The model suggests that real output per worker will slightly understate differences in TFP because of land scarcity. Nominal output per worker—that is, failing to control for differences in housing prices—will moderately overstate differences in TFP.

Combes, Duranton, and Gobillon (2003) argue that both of these estimates are likely to be biased upward because they fail to control for unobserved individual attributes. Allowing for individual fixed effects in a large panel of French workers, their preferred estimate suggests that υ_{TFP} is no higher than 0.02.

The model's baseline parameterization predicts that the elasticity of required TFP with respect to density, ϵ_{TFP}, overlaps the upper end of these estimates. As population density rises from one sixteenth to one fourth, ϵ_{TFP} rises from 0.036 to 0.049 (Table 3). But at higher relative densities, the required increases in TFP to sustain increases in crowdedness become much larger. As population density rises from one to four, ϵ_{TFP} rises from 0.073 to 0.123.

Small-economy relative TFP can be thought of as combining an endogenous agglomeration component that is increasing in density and an exogenous component that is unrelated to density. The estimated agglomeration elasticity, υ_{TFP}, characterizes the endogenous component. The exogenous component can be measured as combined relative TFP at the large-economy density. Equivalently, the endogenous component of TFP is assumed to equal zero at a unitary relative density.

Figure 4 compares predicted and estimated elasticities. In each of the panels, the dashed line shows the small-economy productivity required to sustain a given small-economy population density under the baseline calibration. The solid lines represent loci of small-economy productivity and density under different assumed combinations of υ_{TFP} and of the small-economy exogenous productivity component. In Panels A and C, small-economy exogenous TFP is assumed to be one. In other words, the small economy is assumed to have TFP equal to that of the large economy when its density equals that of the large economy. In Panels B and D, small-economy exogenous TFP is allowed to differ from one. In Panels A and B, υ_{TFP} is assumed to equal a "low" value of 0.02. In Panels C and D, it is assumed to equal a "high" value of 0.05.

With exogenous TFP equal to one and a low υ_{TFP}, the unique equilibrium is for small-economy density to equal that of the large economy (Panel A). As small-economy density increases above one, the TFP gained due to agglomeration is less than the TFP required to sustain the higher density. Conversely, as small-economy density decreases below one, the TFP lost due to negative agglomeration is less than the decrease in TFP needed to sustain the lower density.

More generally, a low υ_{TFP} implies that the exogenous TFP component uniquely determines small-economy density. To be sure, agglomeration magnifies variations in the exogenous productivity component; but only by a relatively small amount. In Panel B, the top solid line represents a small economy with an equilibrium relative density of four. The *combined* TFP to sustain this assumed equilibrium is 1.14. The prerequisite *exogenous* TFP to do so is 1.11. Agglomeration thus accounts for 22 percent of the total required TFP differential (1-0.11/0.14). The bottom solid line represents a small economy with an assumed one-fourth equilibrium relative density. In this case, the required combined TFP is 0.92 and the prerequisite exogenous TFP is 0.946. Here, agglomeration accounts for 32 percent of the total required TFP differential. Agglomeration's higher share in sustaining the lower density equilibrium reflects the increase in ϵ_{TFP} with density.

Increasing the agglomeration elasticity introduces the possibility of multiple equilibria. In Panel C, the exogenous component of small-economy productivity is again assumed to equal one. At a unitary density, ϵ_{TFP} exceeds υ_{TFP}. Hence there is a stable equilibrium in which the two economies have equal density. But at densities below one, the required locus (dashed line) lies only slightly below the endogenous locus (solid line). Moreover, the required locus is flattening as density decreases. Although it is not shown, the two loci intersect at a one-sixteenth density. In other words, the endogenous loss in productivity from having a density of one sixteenth rather than one equals the lower density required to sustain a small-economy density of one sixteenth. However, this second equilibrium is not stable. With slightly higher "initial" density, actual productivity is above what is required. In a dynamic setting, presumably the small economy would grow until it attained a unitary density. With initial density slightly below one sixteenth, actual productivity is below what is required. Presumably the small economy would lose population until it no longer existed.

A high agglomeration elasticity combined with an exogenous productivity component slightly below one implies a virtual continuum of low-density equilibria. In Panel D, the lower solid line represents an economy that is formally assumed to have equilibrium density of one fourth. The required combined small-economy TFP is 0.920 and the prerequisite exogenous TFP component is 0.986. Agglomeration thus accounts for 83 percent of the required productivity differential. As is visually clear, however, the agglomeration and required loci are virtually congruent for relative densities from one half down to one eighth. Any small-economy density in this range is approximately consistent with the assumed small-economy exogenous productivity. Which below-average density outcome is actually realized may depend more on history and other idiosyncratic factors than on static fundamentals.

In contrast, *high-density* equilibria remain unique notwithstanding the high estimated agglomeration elasticity. But increasing returns does greatly magnify variations in exogenous productivity. The top solid line in Panel D represents an economy with a four-times equilibrium density. Agglomeration accounts for 54 percent of the required combined higher productivity. To achieve an eight-times relative crowdedness, agglomeration can account for 42 percent of the required combined higher productivity. As is clear visually, the steeper slope of the required productivity line implies a unique high-density equilibria. For there to be multiple high-density equilibria, ϵ_{TFP} would need to remain close to or below 0.05 as density increases above one. Variations to the base calibration that achieve this include lowering land's share of housing production to 20 percent or increasing the housing-production elasticity of substitution with land to one (Table 3; Figure 3 Panels B and C).

The shortfall of increasing returns in accounting for high density cities is much more robust under the low estimate of the agglomeration elasticity. The only parameterization under which it can do so combines an implicit low land factor share of consumption with high elasticities of substitution (Figure 3, Panel F). In this case, ϵ_{TFP} remains below 0.026 all the way up to a relative density of eight. But under all of the other parameterizations tested in the sensitivity analysis, the required increases in productivity to achieve high-density outcomes far exceed estimates of υ_{TFP} in the range of 0.02.

Overall, a comparison of the model's numerical results with the agglomeration estimates places some upper bounds on the role of increasing returns to scale in accounting for the variation in population density across cities. Even if actual agglomeration is at the high end of estimates, it falls well short of being able to account for above-average population densities. If it is at the low end of estimates, it also falls well short of being able to account for below-average densities. Such conclusions are reinforced by the possibility that the model understates required productivity differences by failing to account for the endogenous expansion in land size with population increases.

7 Conclusions

Crowdedness varies hugely across U.S. cities. A simple, static general equilibrium model suggests that moderate-sized differences in cities' total factor productivity can account for such variation. Nevertheless, the productivity differences required to sustain high levels of crowdedness considerably exceed estimates of the higher productivity such crowdedness causes.

What, then, compensates for the shortfall of increasing returns in accounting for high crowdedness? Economic theory suggests two main possibilities. One is variations in productivity unrelated to density. For example, productivity may depend on locational fundamentals such as easy access to raw materials, navigable waterways, seaports, and other transportation infrastructure (Wright, 1990; Sokoloff, 1988; Rappaport and Sachs, 2003). Or it may depend on government policies such as regulation, taxes, and service provision (Holmes, 1998).

Second, the difference between required and endogenous productivity might be compensated for by high consumption amenities (Rappaport, 2004). The most obvious such amenity is nice weather. U.S. residents have been crowding into warmer-winter, cooler-summer weather cities throughout most of the twentieth century (Rappaport 2006). Individuals are similarly likely to be willing to endure crowded conditions in return for the chance to enjoy nearby beaches, mountains, lakes, and other natural recreational opportunities; or they may be willing to do so in order to obtain desired government policies such as the efficient provision of low pollution, low crime, and good schools (Roback, 1982; Blomquist et. al., 1988; Gyourko and Tracy, 1989, 1991; Kahn, 2000). Alternatively, consumption amenities may arise endogenously due to the wide product variety and cultural amenities that high density can support (Glaeser, Kolko, Saiz, 2001).

In contrast to its inability of increasing returns to underpin high levels of crowdedness, increasing returns to scale may be able to account for variations in crowdedness at low population densities. If so, history rather than fundamentals can account for the extreme sparsity of population of some cities compared to the merely below-average density of others. While increasing returns can not underpin New York City's crowdedness, it may be enough to separate outcomes such as those of Houston from Beaumont/Port-Arthur Texas (2.4 versus 1.2 thousand persons per square mile), Atlanta from Athens Georgia (1.8 versus 0.7), and Des Moines from Waterloo/Cedar Falls Iowa (2.0 versus 1.0).

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Table 1: Variations in Population Density

Rankings by population density (thousand persons per square mile) in 2000 of continental U.S. local areas with population of at least 100,000

A. Metropolitan Areas (2003 OMB definitions; raw density)

Rank	Metropolitan Area	Density	
1	New York-Nrthrn New Jersey-Long Island, NY-NJ-PA	2.7	
2	Los Angeles-Long Beach-Santa Ana, CA	2.6	
3	San Francisco-Oakland-Fremont, CA	1.7	
4	Trenton-Ewing, NJ	1.6	
5	Bridgeport-Stamford-Norwalk, CT	1.4	
6	New Haven-Milford, CT	1.4	
7	Chicago-Naperville-Joliet, IL-IN-WI	1.3	**5.8**
8	Boston-Cambridge-Quincy, MA-NH	1.3	**times**
9	Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	1.2	
10	Detroit-Warren-Livonia, MI	1.1	
:	:	:	
:	:	:	
49	Minneapolis-St. Paul-Bloomington, MN-WI	0.5	
50	**population median** (Vallejo-Fairfield, CA)	0.5	
51	Orlando-Kissimmee, FL	0.5	
:	:	:	**437**
:	:	:	**times**
330	Prescott, AZ	0.0	
331	Rapid City, SD	0.0	
332	Flagstaff, AZ	0.0	

share of continental U.S. population: 82.0%

share of continental U.S. land area: 27.7%

elasticity with respect to population: $\varepsilon = 0.53$ (0.04); $R^2 = 0.40$

B. Urbanized Areas (Census 2000 definitions; UA's are essentially the densely settled portions of metropolitan areas.)

Rank	Urbanized Area	Density	
1	Los Angeles--Long Beach--Santa Ana, CA	7.1	
2	San Francisco--Oakland, CA	7.0	
3	San Jose, CA	5.9	
4	New York--Newark, NY--NJ--CT	5.3	
5	New Orleans, LA	5.1	
6	Vallejo, CA	4.7	
7	Las Vegas, NV	4.6	**2.4**
8	Oxnard, CA	4.5	**times**
9	Miami, FL	4.4	
10	Fairfield, CA	4.4	
:	:	:	
:	:	:	
49	Dallas--Fort Worth--Arlington, TX	2.9	
50	**population median** (Lancaster--Palmdale, CA)	2.9	
51	Trenton, NJ	2.9	
:	:	:	**8.3**
:	:	:	**times**
253	Spartanburg, SC	1.1	
254	Hickory, NC	0.9	
255	Barnstable Town, MA	0.9	

share of continental U.S. population: 63.6%

share of continental U.S. land area: 2.2%

elasticity with respect to population: $\varepsilon = 0.16$ (0.02); $R^2 = 0.21$

C. Metropolitan Areas (2003 OMB definitions; population-weighted mean of county--subdivision--place/remainder densities)

Rank	Metropolitan Area	Density	
1	New York-Nrthrn New Jersey-Long Island, NY-NJ-PA	18.9	
2	Los Angeles-Long Beach-Santa Ana, CA	7.8	
3	San Francisco-Oakland-Fremont, CA	7.2	
4	Chicago-Naperville-Joliet, IL-IN-WI	6.7	
5	Miami-Fort Lauderdale-Miami Beach, FL	5.8	
6	Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	5.2	
7	San Jose-Sunnyvale-Santa Clara, CA	5.1	**6.8**
8	Boston-Cambridge-Quincy, MA-NH	5.0	**times**
9	Salinas, CA	4.7	
10	Washington-Arlington-Alexandria, DC-VA-MD-WV	4.5	
:	:	:	
:	:	:	
49	Pittsburgh, PA	2.8	
50	**population median** (Omaha-Council Bluffs, NE-IA)	2.8	
51	Lincoln, NE	2.7	**49**
:	:	:	**times**
:	:	:	
330	Ocala, FL	0.5	
331	Bangor, ME	0.4	
332	Dothan, AL	0.4	

share of continental U.S. population: 82.0%

share of continental U.S. land area: 27.7%

elasticity with respect to population: $\varepsilon = 0.34$ (0.02); $R^2 = 0.39$

D. Municipalities (Census 2000 land areas)

Rank	Municipality	Density	
1	New York, NY	26.4	
2	Paterson, NJ	17.7	
3	San Francisco, CA	16.6	
4	Jersey City, NJ	16.1	
5	Cambridge, MA	15.8	
6	Daly City, CA	13.7	
7	Chicago, IL	12.8	**7.0**
8	Santa Ana, CA	12.5	**times**
9	Inglewood, CA	12.3	
10	Boston, MA	12.2	
:	:	:	
:	:	:	
89	Tacoma, WA	3.9	
90	**population median** (Garland, TX)	3.8	
91	San Diego, CA	3.8	**45**
:	:	:	**times**
:	:	:	
235	Peoria, AZ	0.8	
236	Augusta-Richmond, GA	0.8	
237	Chesapeake, VA	0.6	

share of continental U.S. population: 26.6%

share of continental U.S. land area: 0.7%

elasticity with respect to population: $\varepsilon = 0.17$ (0.06); $R^2 = 0.04$

Table 2: Base and Alternative Calibrations

Parameter	Base	Low Resistance* ("Loose")	High Resistance* ("Tight")
Factor Income Shares (large economy)			
Traded Good: Land, Capital, Labor	**1.6%, 32.8%, 65.6%**	0.4%, 33.2%, 66.4%	4.8%, 31.7%, 63.5%
Housing: Land, Capital, Labor	**35%, 21.7%, 43.3%**	20%, 26.7%, 53.3%	50%, 16.7%, 33.3%
Housing Production CES ($\sigma_{D,KL}$)	**0.75**	1	0.50
Housing Share of Consumption Expenditure (large economy)	**18%**	14%	22%
Utility CES ($\sigma_{x,h}$)	**0.50**	0.75	0.25

*Note: The CES substitution parameters ($\sigma_{D,KL}$, and $\sigma_{x,h}$) have an asymmetric effect on resistance. The "loose" values above are those for which resistance is lower at a relative density of one and above.

Table 3: Sensitivity of Elasticities

Endogenous Elasticity → (with respect to density)	**Required TFP**				**w**	**r_D**	**p**	**p·h**	**x**	**h**
at rel. density →	**1/16**	**1/4**	**1**	**4**	**1**					
↓ Parameterization ↓										
Baseline	0.036	0.049	0.073	0.123	0.079	1.367	0.438	0.258	0.039	-0.180
Traded-Good Factor Shares										
D=0.4%, K=33.2%, L=66.4%	0.023	0.037	0.065	0.122	0.089	1.487	0.493	0.291	0.044	-0.202
D=4.8%, K=31.7%, L=63.5%	0.067	0.079	0.099	0.137	0.064	1.222	0.355	0.210	0.032	-0.146
Housing Factor Shares										
D=20%, K=26.7%, L=53.3%	0.028	0.034	0.046	0.068	0.040	1.247	0.224	0.132	0.020	-0.092
D=50%, K=16.7%, L=33.3%	0.042	0.062	0.103	0.190	0.121	1.480	0.676	0.399	0.061	-0.277
Housing Production CES										
$\sigma_{D,KL}$ = 1	0.047	0.054	0.063	0.075	0.068	1.179	0.379	0.223	0.034	-0.155
$\sigma_{D,KL}$ = 0.50	0.026	0.041	0.087	0.218	0.093	1.626	0.519	0.306	0.047	-0.213
Housing Expenditure Share										
$p_l h_l/(x_l+p_l h_l)$ = 0.14	0.031	0.041	0.061	0.099	0.060	1.326	0.429	0.244	0.030	-0.184
$p_l h_l/(x_l+p_l h_l)$ = 0.22	0.040	0.056	0.086	0.145	0.098	1.400	0.445	0.271	0.049	-0.174
Utility CES, traded and housing										
$\sigma_{x,h}$ = 0.75	0.039	0.051	0.069	0.096	0.074	1.285	0.412	0.158	0.056	-0.253
$\sigma_{x,h}$ = 0.25	0.032	0.046	0.078	0.163	0.084	1.460	0.468	0.372	0.021	-0.096
Combination Parameterizations:										
low land, high σ	0.021	0.022	0.023	0.025	0.029	1.065	0.205	0.073	0.022	-0.132
high land, low σ	0.063	0.088	0.174	0.474	0.151	1.629	0.688	0.553	0.038	-0.134
low land, low σ	0.007	0.014	0.040	0.190	0.049	1.840	0.351	0.276	0.012	-0.076
high land, high σ	0.106	0.115	0.125	0.136	0.108	1.164	0.494	0.205	0.081	-0.289
Alternative Assumptions:										
Only Traded TFP Varies	0.037	0.053	0.086	0.172	0.096	1.419	0.537	0.317	0.048	-0.220
With Capital Income	0.050	0.071	0.108	0.169	0.129	1.417	0.444	0.262	0.040	-0.182

Table 4: Correlation of Wages and Density

	(1) MODEL (baseline)	(2)	(3)	(4)	(5)	(6)
Geographic Unit →		Metro	Metro	Metro	Metro	UA
RHS Variables:						
Education Controls		no	yes	no	yes	yes
log(rel density)		***0.20***	***0.15***	***0.26***	***0.20***	***0.20***
		(0.03)	***(0.02)***	***(0.03)***	***(0.02)***	***(0.07)***
(log(rel density))^2:						
all obs		*0.04*	***0.03***			
		(0.02)	***(0.01)***			
dens < pop median				***0.08***	***0.06***	*0.13*
				(0.02)	***(0.02)***	*(0.08)*
dens ≥ pop median				*-0.05*	-0.04	0.07
				(0.03)	(0.02)	(0.13)
Observations		332	332	332	332	255
Independent Var		2	6	3	7	7
R-sqrd		0.45	0.57	0.46	0.58	0.42
Marg R-sqrd			0.18		0.19	0.09
R-sqrd, no controls			0.45		0.46	0.19
P-val (equal quad term)				0.00	0.01	0.75
elasticty @ 1/2 dens	**0.06**	0.15	0.11	0.16	0.11	0.03
		(0.01)	(0.01)	(0.01)	(0.01)	(0.05)
elasticity @ 1x density	**0.08**	0.20	0.15	0.26	0.20	0.20
		(0.03)	(0.02)	(0.03)	(0.02)	(0.07)
elasticty @ 2x dens	**0.11**	0.25	0.19	0.20	0.15	0.29
		(0.05)	(0.04)	(0.03)	(0.02)	(0.14)

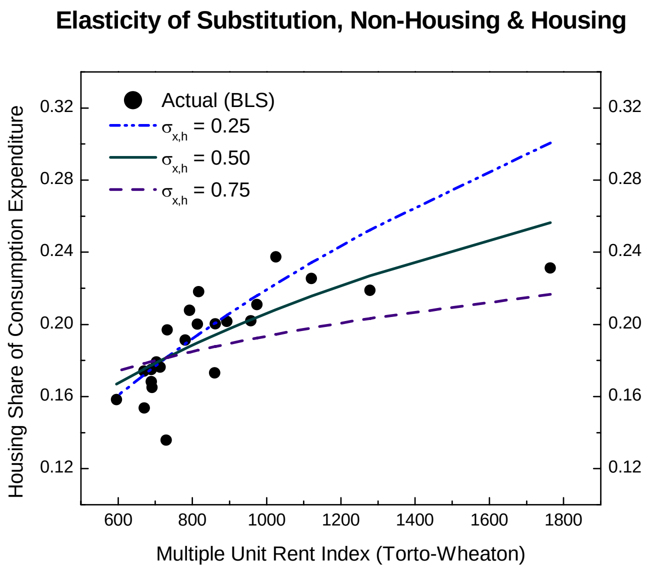
Dependent variable is the log of median annual labor income in 1999 for white, non-Hispanic males 16 years or over who worked full time, year-round in 1999. All regressions include a constant. Education controls are the percentage of the white, non-Hispanic males 25 and older with each of a high school, associate, bachelors, and graduate degree. Coefficient standard errors (in parentheses) are robust to heteroskedasticity. Bold type signifies coefficients that differ from zero at the 0.05 level. Italic type signifies coefficients that do so at the 0.10 level. For derived elasticities, standard errors are calculated using the "delta method" (Goldberger 1991). Geographic observations and densities correspond to those listed in Table 1. Metro area density is a population weighted mean of county-subdivision-place/remainders. Marginal R-squared is the increase in R-squared compared to a regression on only the educational controls. P-value is the level at which an F test rejects that the coefficients on quadratic density are the same for observations below and above the population median.

Table 5: Correlation of Housing Expenditure and Density

	(1) MODEL (baseline)	(2)	(3)	(4)	(5)
Dependent Var →		rent	rent	value	value
Geographic Unit →		Metro	UA	Metro	UA
RHS Variables:					
Education Controls		yes	yes	yes	yes
log(rel. density)		***0.16***	***0.25***	***0.41***	***0.59***
		(0.03)	***(0.08)***	***(0.07)***	***(0.14)***
(log(rel. density))^2:					
dens < pop median		*0.04*	***0.25***	***0.21***	***0.63***
		(0.02)	***(0.09)***	***(0.04)***	***(0.15)***
dens ≥ pop median		0.00	0.17	-0.07	0.27
		(0.03)	(0.17)	(0.06)	(0.28)
Observations		332	255	332	255
Independent Var		7	7	7	7
R-sqrd		0.64	0.46	0.59	0.47
Marg R-sqrd		0.08	0.09	0.10	0.12
R-sqrd, no controls		0.35	0.23	0.36	0.25
P-val (equal quad term)		0.46	0.77	0.00	0.36
elasticty @ 1/2 dens	**0.22**	0.10	-0.09	0.13	-0.28
		(0.01)	(0.06)	(0.03)	(0.10)
elasticity @ 1x density	**0.27**	0.16	0.25	0.41	0.59
		(0.03)	(0.08)	(0.07)	(0.14)
elasticty @ 2x dens	**0.33**	0.16	0.49	0.31	0.96
		(0.03)	(0.19)	(0.06)	(0.30)

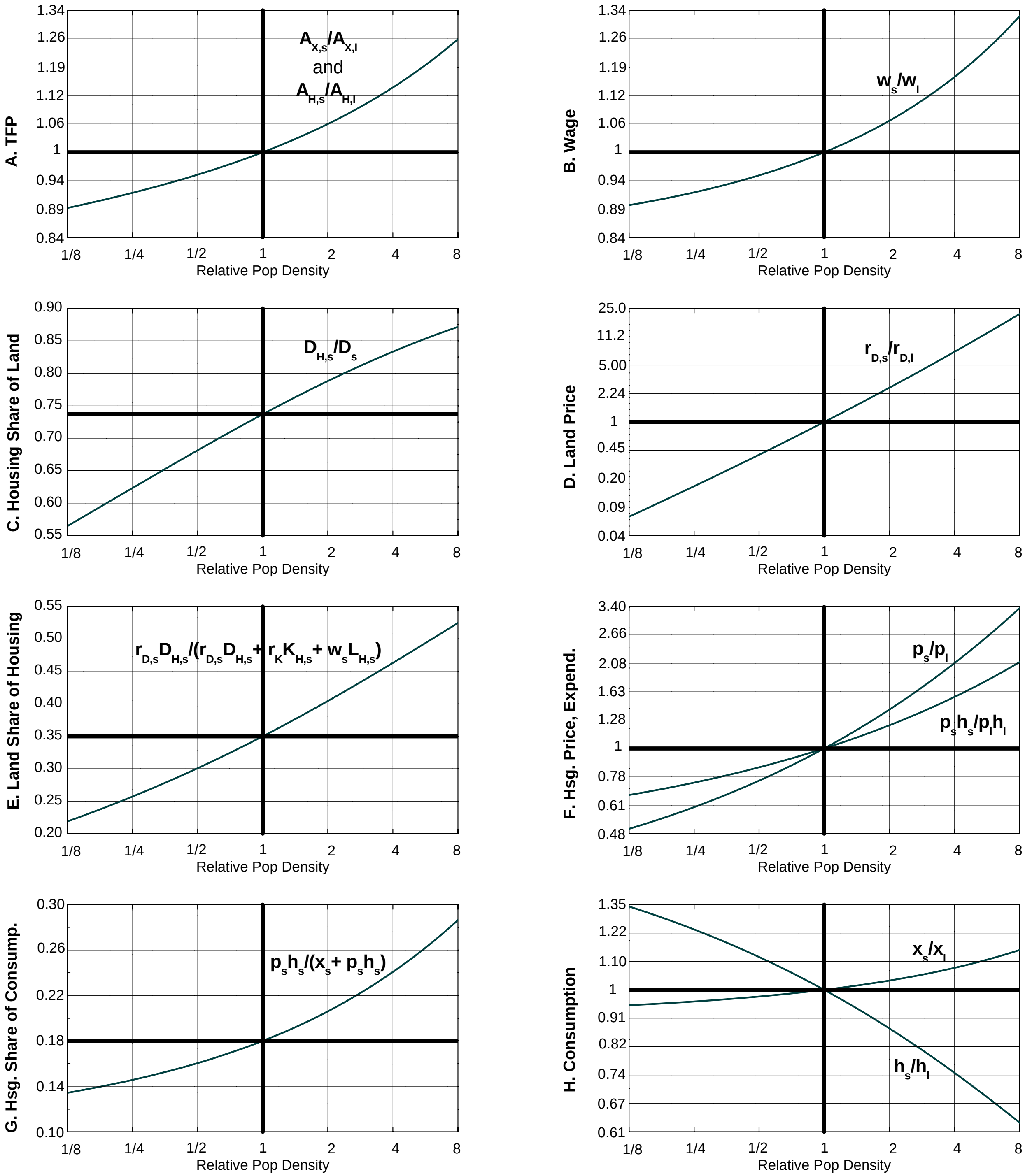
For columns 2 to 5, dependent variable is the log of the median monthly gross rent in 2000 for renter-occupied housing units with a white, non-Hispanic householder. For columns 6 to 9, dependent variable is the log of the estimated value in 2000 for owner-occupied housing units with a white, non-Hispanic householder. All regressions include a constant and controls for the percentage of the population with each of a high school, associate, bachelors, and graduate degree. For coefficients, standard errors in parentheses are robust to heteroskedasticity. For derived elasticities, standard errors in parentheses are calculated using the "delta method" (Goldberger 1991). Geographic observations and densities correspond to those listed in Table 1. Marginal R-squared is the increase in R-squared compared to a regression on only the educational controls. P-value is the level at which an F test rejects that the coefficients on quadratic density are the same for observations below and above the population median.

Figure 1: Calibration of Consumption Elasticity



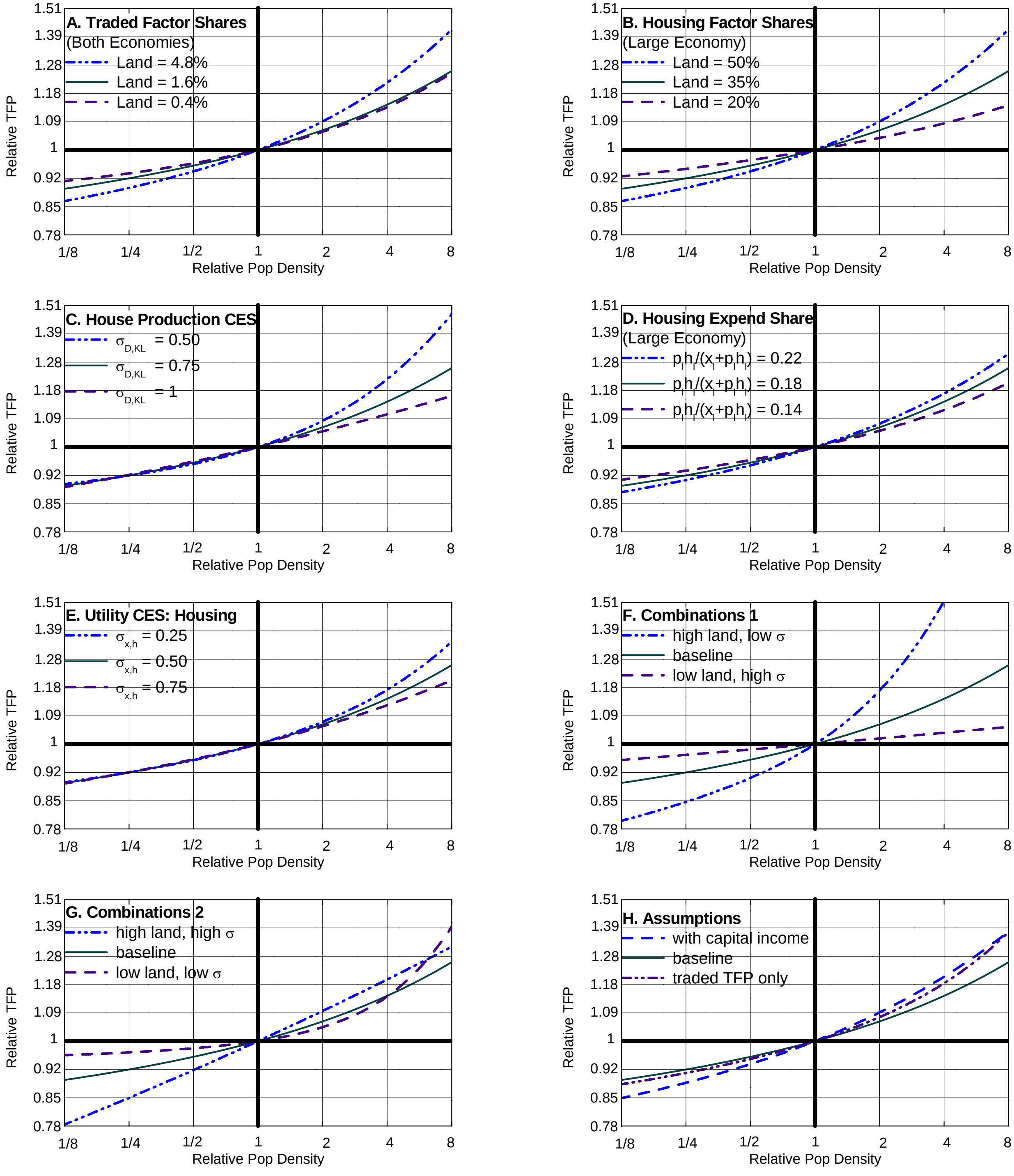
Dots plot aggregate share of consumption devoted to shelter in each of 24 large metro areas (BLS Consumer Expenditure Survey, 1997-to-2002 average) against Torto-Wheaton multi-unit rental price index (1997-to-2002 average). Lines represent expected housing shares as a function of the price index for each of three elasticity parameters.

Figure 2: Productivity-Driven Crowding



Panel A shows the required small-economy to large-economy ratio of tfp (in both the traded-good and housing sectors) to achieve different relative densities under the base calibration. Remaining panels show implied ratios of various endogenous variables. Horizontal axes are plotted using a log scale. Vertical axes are also plotted using a log scale except for share variables.

Figure 3: Required TFP Sensitivity Analysis



Panels show the required small-economy to large-economy ratio of tfp (in both the traded-good and housing sectors, except in Panel H) to achieve different relative densities under various perturbations to the base calibration. Axes are plotted using a log scale.

Figure 4: Increasing Returns to Scale

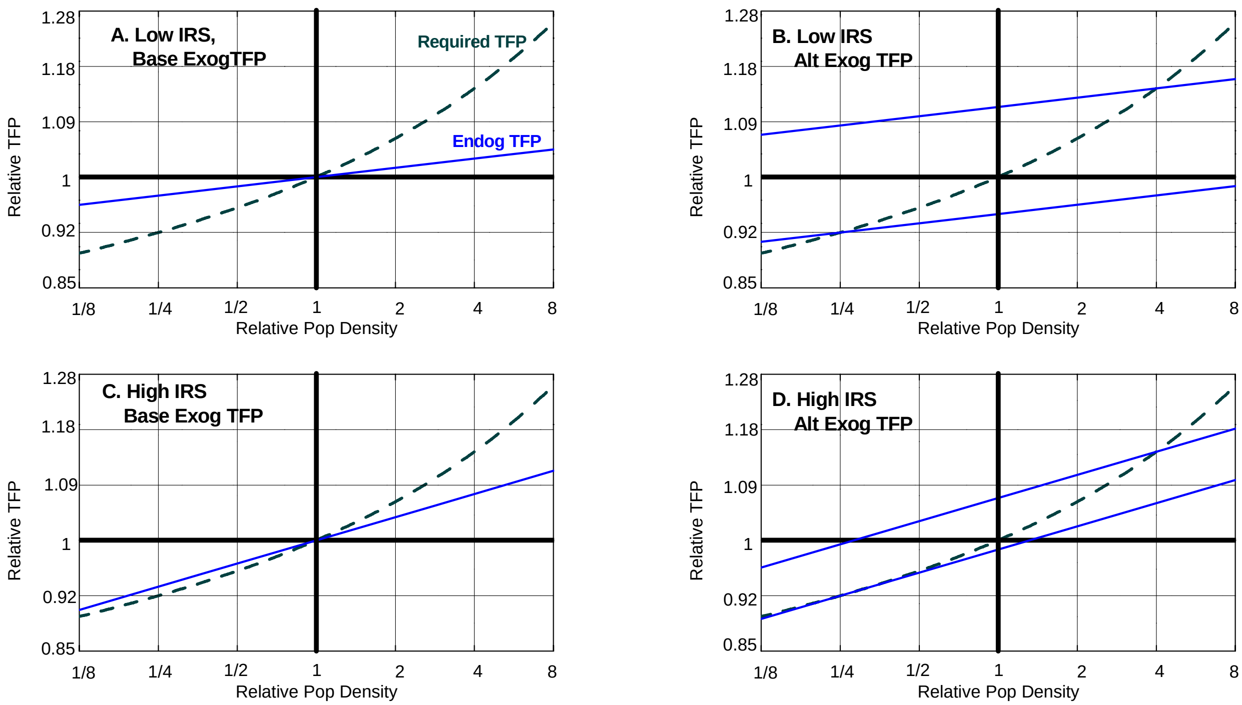


Figure compares TFP levels required to sustain various population densities with estimates of the increase in TFPassociated with higher density. The dashed lines show the required TFP to sustain relative population density under the baseline calibration. The solid lines show actual TFP under different combinations of an increasing-returns component and an exogenous component. In panels A and B, the agglomeration elasticity of TFP with respect to density is assumed to be 0.02. In panels C and D, it is assumed to be 0.05. In panels A and C, the exogenous component of small economy TFP is assumed to be identical to that of the large economy. In panels B and D, the the exogenous component is assumed at the levels that are required for equilibrium relative population density to be one-fourth and four.