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Phillips Curve Instability and Optimal Monetary Policy

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Abstract: This paper assesses the implications for optimal discretionary monetary policy if the slope of the Phillips curve changes. The paper first derives a 'switching' Phillips curve from the optimal pricing decision of a monopolistic firm that faces a changing cost of price adjustment. Two states exists, a state with a high cost of price adjustment that generates a 'flat' Phillips curve and a low-cost state that generates a relatively 'steep' curve. The second aspect of the paper constructs a utility-based welfare criterion. A novel feature of this criterion is that it has a relative weight on output gap deviations that is state dependent, so it changes with the cost of price adjustment. Optimal monetary policy is computed subject to the switching-Phillips curve under both ad-hoc and utility-based welfare criteria. The utility-based criterion instructs monetary policy to disregard the slope of the Phillips curve and keep its systematic actions constant across different states. This stands in contrast to the prescription coming under the ad-hoc criterion, which advises monetary policy to change its systematic behavior according to the slope of the Phillips curve.

Keywords: Optimal monetary policy, Phillips curve, regime-switching

JEL classification: E52, E58, E61

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1. Introduction

The slope of the Phillips curve is an important parameter in the minds of policymakers. Empirical evidence suggests a 'flattening' of the Phillips curve in recent decades, indicating inflation has become less responsive to movements in measures of aggregate economic activity, such as the output gap.[1] Although this phenomenon appears using reduced-form estimation procedures, as in Atkeson and Ohanian (2001), it also appears using structural approaches to estimation, as in Smets and Wouters (2007). Competing explanations for the change in the slope of the Phillips curve include the following, possibly interrelated, set of factors : better conduct of monetary policy, globalization and changes in the price-setting behavior of firms.[2] Although each factor may contribute, this paper focuses on the last explanation and its implications for the conduct of optimal monetary policy.

In particular, this paper models changes in the degree of price stickiness at the level of a monopolistically competitive firm. The microfoundations of the firm's price-setting behavior are similar to Rotemberg (1982), except the term governing the magnitude of the cost of price adjustment is state-dependent and subject to change over time. The optimal pricing equation from this problem yields a nonlinear, or 'switching', Phillips curve. The switching-Phillips curve takes the same form as the forward-looking New Keynesian Phillips curve, except the coefficients on expected inflation and the output gap are state dependent.

A second aspect of the paper assesses the implications for optimal monetary policy under discretion subject to the switching-Phillips curve. An advantage of specifying the microfoundations of the firm's pricing problem is that it makes possible the construction of a utility-based welfare criterion to evaluate different monetary policies. An interesting feature of the utility-based measure is that it has a state-dependent relative weight on output gap deviations. The weight changes synchronously with changes in the degree of price stickiness, indicating higher losses arise due to inflation in states with relatively high costs of price

[1] For example, Atkeson and Ohanian (2001), Roberts (2006) and Williams (2006) document the flattening of the Phillips curve for the U.S. and Iakova (2007) does the same for the U.K.

[2] See Mishkin (2007) for an overview.

adjustment. In other words, inflation imposes higher costs on firms in states with relatively sticky prices, so it is precisely in these states that monetary policy increases the relative weight on inflation stabilization.

To derive the optimal monetary policy, the central bank optimizes its welfare criterion subject to the switching-Phillips curve, yielding the optimal targeting rule relating the output gap to inflation. As a basis for comparison, an ad-hoc welfare criterion is first used to derive the optimal targeting rule. This criterion uses the common assumption that the relative weight on output gap deviations is constant, as in Clarida, Gali, and Gertler (1999). The resulting optimal targeting rule directs the central bank to switch rules, or change its systemic response to inflation, depending on the state. In states with relatively flexible prices, the Phillips curve is steep and inflation is relatively less costly to firms. Consequently, the central bank adjusts the output gap relatively less aggressively to stabilize inflation. In states with a higher cost of price adjustment, inflation is more costly, so the central bank adjusts the output gap more aggressively to stabilize inflation. Thus, the systematic response of the central bank, under the ad-hoc criterion, varies with the state. In similar contexts, this result can also be found in Blake and Zampolli (2006), Moessner (2006), Zampolli (2006) and Svensson and Williams (2007). In contrast, the optimal targeting rule using the utility-based welfare criterion instructs monetary policy to have a *constant* systematic response to inflation. So the optimal targeting rule does not switch, but is invariant across the different states. This constant systematic response arises due to the offsetting effects of a changing relative weight in the welfare criterion, which is absent in the ad-hoc specification, and changing slope of the Phillips curve.

Empirical studies finding the flattening of the Phillips curve, such Lubik and Schorfheide (2004), Boivin and Giannoni (2006), and Smets and Wouters (2007), estimate variants of a structural DSGE model. Of course, changes in monetary policy regime can have an impact on the relationship between inflation and output. For example, Roberts (2006) documents that a change in monetary regime around 1980 is an important factor in understanding the change in the reduced-form relationship between output and inflation. However, structural estimates of the slope of the Phillips curve depend on private sector parameters and are therefore, independent of parameters describing monetary policy. Consequently, as Boivin and Giannoni (2006) and Smets and Wouters (2007) discuss, the change in the slope-coefficient in the structural Phillips curve can be due to parameters governing price-setting behavior.

Given that empirical evidence suggests a change in the slope of the Phillips curve, then simply postulating a Phillips curve relation with switching (i.e. state-dependent) coefficients may have appeal. However, incorporating elements of regime change after solving an optimization problem and linearizing does violence to the microfoundations upon which most modern macroeconomic models are based. Incorporating regime change into the original optimization problem, as in this paper, preserves the underlying foundations. In this sense, the structural relations describing private sector behavior in this paper are restricted relative to Svensson and Williams (2007), where all parameters in the linearized relations are subject to change.[3] The restrictions in this paper come from microfoundations that take a stand on which deep parameters change and how these changes manifest themself in the structural relationships.

In some respect, the 'flattening' of the Phillips curve due to greater price setting frictions seems perverse. More flexibility and competition in goods markets, along with improved technology for acquiring information and adjusting prices, should work in the opposite direction making prices more flexible. This does appear to present a puzzle. However, as Mishkin (2007) notes, environments with low and stable inflation may lead firms to conclude they can increase the average duration they leave their prices fixed with little cost. As a consequence, the slope of the Phillips curve would decline and inflation become less responsive to movements in the output gap as inflation in many countries has stabilized. This line of reasoning suggests a link between aggregate inflation and the price setting behavior of private firms. Indeed, Rubio-Ramirez and Villaverde (2007) estimate a DSGE model for the U.S. with time-varying structural parameters and Calvo (1983) price setting. They find that

[3] Although, incorporating state-dependent coefficients into linearized structural relationships is useful for modeling model uncertainty, as Svensson and Williams (2007) emphasize.

the average duration between when firms reoptimize their price increases as the trend of inflation declines, and vice versa. Galì and Gertler (1999) and Cogley and Sbordone (2005) estimate a Phillips curve relation, also using the Calvo price setting mechanism, across different subsamples. They too find longer average duration between price reoptimizations for more recent subsamples, a period with relatively low and stable inflation.[4] Ball, Mankiw, and Romer (1988) provide both cross-country and time series evidence that prices are more responsive to movements in aggregate demand when inflation is relatively high and volatile. Similarly, Caballero and Engel (1993) present evidence that the degree of price flexibility does vary with economic conditions and prices were more flexible in the U.S. during the high and volatile inflation of the 1970s. Outside of the U.S., Demery and Duck (2005) present evidence that the frequency of price adjustment increases in high inflation environments in the UK and Gagnon (2006) does the same for Mexico.

This evidence linking aggregate inflation rates to price-setting decisions of firms suggests a model where the cost, or frequency, of price adjustment is endogenous and depends on the aggregate inflation rate. Such a model is computationally feasible, but analytically intractable. Assessing the implications for optimal monetary policy in such a framework also posses considerable difficulty. This paper, as a first pass, uses analytic techniques from Davig and Leeper (2007) to solve rational expectation models with regime change and from Rotember and Woodford (1997) and Woodford (2003) to compute optimal policies. These tools allow for sharp analytic characterizations of equilibrium relationships and optimal policies. For these reasons, linking aggregate inflation and the price-setting behavior of firms in a serious way is left for future work.[5]

Optimal monetary policy in the presence of a switching-Phillips curve also differs from previous work focusing on the implication of switching policy rules, such as Andolfatto and Gomme (2003), Leeper and Zha (2003), Davig and Leeper (2007), and Chung, Davig

[4]Cogley and Sbordone (2005) note that formal statistical testing across subsamples, however, cannot reject a constant Calvo parameter.

[5]For the model in this paper, an explicit link is made between the volatility of shocks and the cost of price adjustment. However, this link is to motivate the model specification and has no material impact on either the model dynamics or the optimal monetary policy under discretion.

and Leeper (2006). These papers posit monetary rules that change regimes exogenously, while keeping parameters in the relations describing private sector behavior constant. For example, Davig and Leeper (2007) assesses the implications of a switching 'simple' monetary rule, where an exogenous Markov-chain governs the switching. Private sector parameters and structural relationships are invariant to the monetary policy rule in place, although the switching policy process does imply decision rules and pricing functions have coefficients that switch with the monetary regime. In contrast, this paper posits a framework with parameters in the forward-looking Phillips curve that are subject to change. Any resulting changes in the parameters describing monetary policy reflect an optimal response to the changing structure of the economy.

This paper is organized as follows : section 2 derives the switching-Phillips curve under the assumption of switching quadratic costs of price adjustment for a monopolistically competitive firm, section 3 illustrates the implications of the switching Phillips curve in a DSGE model, section 4 solves for the optimal monetary policy under discretion using an ad-hoc welfare criterion, section 5 solves again the optimal discretionary policy, except using a utility-based criterion, and section 6 concludes.

2. A Switching Phillips Curve

This section embeds state-dependent parameters into the optimization problem of a monopolistically competitive firm. As in Rotemberg (1982), the firm faces quadratic costs of adjustment, except the term governing the magnitude of the cost of price adjustment is subject to change. Introducing changing costs of price adjustment results in a switching-Phillips curve relation, derived from explicit foundations, with state-dependent coefficients on the output gap and expected inflation.

2.1 Changing Costs of Price Adjustment

The fixed-regime approach, which keeps parameters constant, imposes a cost on monopolistic intermediate-goods producing firms for adjusting their price, given by

$$\frac{\varphi}{2}\left(\frac{P_t\left(j\right)}{\Pi P_{t-1}\left(j\right)}-1\right)^2 Y_t, \tag{1}$$

where $\varphi \geq 0$ is the magnitude of the price adjustment cost, Π denotes the gross steady state rate of inflation and $P_t\left(j\right)$ denotes the nominal price set by firm j.[6] The cost is measured in terms of the final good Y_t. The assumption of quadratic adjustment costs implies that firms change their price every period in the presence of shocks, but will adjust only partially towards the optimal price the firm would set in the absence of adjustment costs. As with any type of quadratic adjustment cost, a firm prefers a sequence of small adjustments to very large adjustments in a given period. Alternatively, these costs may vary according to a state, s_t, such as

$$\frac{\varphi\left(s_t\right)}{2}\left(\frac{P_t\left(j\right)}{\Pi P_{t-1}\left(j\right)}-1\right)^2 Y_t, \tag{2}$$

where firms face a state-dependent cost of price adjustment. For $s_t \in \{1,2\}$, the state evolves according to a two-state Markov chain with transition matrix

$$\Pi = \begin{bmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{bmatrix}, \tag{3}$$

with $p_{mn} = \Pr[S_t = n|S_{t-1} = m]$ for $m, n = 1, 2$.[7] Changes in the state governing the cost of price adjustment are exogenous, evolving according to a Markov-chain and are observed by both private agents and the central bank.

As previously discussed, a case exists that changes in the price-setting friction are linked to factors such as aggregate volatility and the average inflation rate. Monetary policy then plays a role determining the cost of price adjustment and can indirectly affect this cost by engaging in policies mitigating aggregate volatility. Boivin and Giannoni (2006) emphasize

[6] See Ireland (2004) for a detailed treatment of quadratic costs of price adjustment in a DSGE model.
[7] The assumption of two states, or regimes, is made for convenience and tractability, it can be replaced with an assumption concerning any finite number of states.

that monetary policy post-1980 has indeed been more effective in this respect. However, monetary policy cannot completely mitigate the effects of supply shocks on both inflation and output, so states with highly volatile supply shocks could still impact the cost of price adjustment. Thus, changes in $\varphi(s_t)$ can be linked to changes in aggregate supply volatility. In the next section, such a link is made explicit, where subsequent analysis then considers implications for optimal monetary policy when aggregate supply shock volatility and $\varphi(s_t)$ change.

The Rotemberg (1982) approach of costly price adjustment is used instead of the Calvo (1983) mechanism because the distribution of prices at time t under the Calvo mechanism is no longer a simple convex combination of the lagged aggregate price level and optimal relative price set at time t, since the average frequency of price adjustment evolves stochastically. Also, the firm's first-order condition under the Rotemberg mechanism lends itself naturally to a recursive formulation in the presence of switching coefficients. Under the Calvo mechanism with a changing frequency of repricing, the firm's first-order condition is an infinite sum embedding the changing coefficients and is not as easily mapped into a recursive form. A recursive formulation greatly simplifies the analysis in the presence of Markov-switching coefficients. In the standard fixed-regime setting, both approaches yield the same reduced-form forward-looking Phillips curve. Whether this is also true under regime switching is not clear, although it will likely be the case that regimes with a high frequency of repricing will have a steeper Phillips-curve than in states with a low frequency of repricing. In a sense, changes in the degree of the price adjustment cost (i.e. $\varphi(s_t)$) more broadly represent a reduced-form description of changes in the price setting friction.

2.2 The Optimal Pricing Problem

Each of the monopolistically competitive intermediate-goods producing firms seek to maximize the expected present-value of profits,

$$E_t \sum_{s=0}^{\infty} \beta^s \Delta_{t+s} \frac{D_{t+s}(j)}{P_{t+s}}, \tag{4}$$

where Δ_{t+s} is the representative household's stochastic discount factor, $D_t(j)$ are nominal profits of firm $j \in [0,1]$, and P_t is the nominal aggregate price level. Also, firm j produces good j. For given s_t, real profits are

$$\frac{D_t(j)}{P_t} = \frac{P_t(j)}{P_t} y_t(j) - \psi_t y_t(j) - \frac{\varphi(s_t)}{2}\left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1\right)^2 Y_t, \tag{5}$$

where ψ_t denotes real marginal cost and $y_t(j) = n_t(j)$ is the production of intermediate goods by firm j using labor input $n_t(j)$.

There exists a final-goods producing firm that purchases the intermediate inputs at nominal prices $P_t(j)$ and combines them into a final good using the following constant-returns-to-scale technology

$$Y_t = \left[\int_0^1 y(j)^{\frac{\theta_t-1}{\theta_t}} dj\right]^{\frac{\theta_t}{\theta_t-1}}, \tag{6}$$

where $\theta_t > 1 \,\forall\, t$ is the elasticity of substitution between goods. Variations in θ_t translate into markup shocks of the firm's price over its marginal cost. The profit-maximization problem for the final-goods producing firm yields a demand for each intermediate good given by

$$y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta_t} Y_t. \tag{7}$$

For a given s_t, substituting $(5)-(7)$ into (4) and differentiating with respect to $P_t(j)$ yields the first-order condition

$$\begin{aligned} 0 \;=\; & (1-\theta_t)\,\Delta_t\left(\frac{P_t(j)}{P_t}\right)^{-\theta_t}\left(\frac{Y_t}{P_t}\right) + \theta_t\Delta_t\psi_t\left(\frac{P_t(j)}{P_t}\right)^{-\theta_t-1}\left(\frac{Y_t}{P_t}\right) - \\ & \varphi(s_t)\,\Delta_t\left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1\right)\left(\frac{Y_t}{\Pi P_{t-1}(j)}\right) + \\ & \beta E_t\left[\varphi(s_{t+1})\,\Delta_{t+1}\left(\frac{P_{t+1}(j)}{\Pi P_t(j)} - 1\right)\left(\frac{P_{t+1}(j)\,Y_{t+1}}{\Pi P_t(j)^2}\right)\right]. \end{aligned} \tag{8}$$

which can be written as a system, where each equation represents the first-order condition, conditional on a particular state.

In a symmetric equilibrium, every firm faces the same ψ_t and Y_t, so the pricing decision is the same for all firms, implying $P_t(j) = P_t$. Also, steady-state inflation and output are constant across states. Steady state marginal costs are given by

$$\psi = \frac{\theta - 1}{\theta}, \tag{9}$$

and $\psi_t = \theta_t^{-1}(\theta_t - 1)$ are marginal costs in the flexible-price case where $\varphi(1) = \varphi(2) = 0$.

Conditional expectations of inflation are $E_t\pi_{t+1} = E[\pi_{t+1} |\Omega_t]$, where $\pi_t = \log(\Pi_t/\Pi)$ and Ω_t represents information available at time t. Using the approach in Davig and Leeper (2007), conditional expectations can be rewritten using a smaller information set excluding the current state, Ω_t^{-s}, where $\Omega_t = \Omega_t^{-s} \cup \{s_t\}$. Distributing probability mass over states at $t+1$ yields

$$E_t\pi_{t+1} = E[\pi_{t+1} \left| s_t = i, \Omega_t^{-s}\right] = p_{i1}E[\pi_{1t+1} \left|\Omega_t^{-s}\right] + p_{i2}E[\pi_{2t+1} \left|\Omega_t^{-s}\right], \tag{10}$$

which uses the state-contingent notation that defines $\pi_t = \pi_{it} \Leftrightarrow s_t = i$ for $i = 1, 2$. This notation simply indicates that inflation at t depends on the regime at t, and not directly on past regimes. When taking expectations of variables written in state-contingent notation, let $E_t\pi_{it+1} \equiv E[\pi_{it+1} \left|\Omega_t^{-s}\right]$.

Imposing symmetry and (9), a linear approximation to the firm's optimal price-setting equation can be written in terms of inflation using state-contingent notation as

$$\pi_{1t} = p_{11}\beta E_t[\pi_{1t+1}] + (1 - p_{11})\frac{\varphi_2}{\varphi_1}\beta E_t[\pi_{2t+1}] + \frac{\theta - 1}{\varphi_1}\widehat{\psi}_t + u_t, \tag{11}$$

and for $s_t = 2$ as

$$\pi_{2t} = p_{22}\beta E_t[\pi_{2t+1}] + (1 - p_{22})\frac{\varphi_1}{\varphi_2}\beta E_t[\pi_{1t+1}] + \frac{\theta - 1}{\varphi_2}\widehat{\psi}_t + u_t, \tag{12}$$

where $\varphi_i = \varphi(i)$ for $i = 1, 2$, $\widehat{\psi}_t = \log(\psi_t/\psi)$ and u_t is a markup, or aggregate supply, shock. Interpreting these relations as a Phillips curve with state-dependent parameters, a more general representation is

$$\pi_{it} = \varphi_i^{-1}\beta E_t[\varphi(s_{t+1})\pi_{t+1}] + \frac{\theta - 1}{\varphi_i}\psi_t + u_t, \tag{13}$$

for $i = 1, 2$, which reduces to the fixed-regime specification when either $\varphi_i = \varphi$ for all i or $p_{11} = p_{22} = 1$.[8] Equation (13) illustrates how changing costs of price adjustment manifest

[8] See Appendix A for detailed derivations of (11) and (12).

themself in the coefficients on marginal cost and expected inflation. Relatively sticky prices, due to costly price adjustment, results in a ‘flat’ Phillips curve, whereas less friction in price-setting results in a ‘steep’ Phillips curve. Thus, a flat Phillips curve implies that output gap movements have a relatively small effect on inflation and equilibrium adjustments to shocks occur more so through quantities than prices.

3. The Switching Phillips Curve in a DSGE Model

This section explores some implications of the switching Phillips curve in an otherwise baseline New Keynesian framework under a simple monetary rule. Analysis under a simple monetary rule is useful for providing intuition of how the switching-Phillips curve affects aggregate dynamics. In particular, analytic expressions are available in the case of serially uncorrelated shocks.

In addition to the switching-Phillips curve, the model contains a forward-looking IS equation that can be derived explicitly from a representative household’s optimization problem, as in Woodford (2003), where households have a period utility function of the form

$$U\left(C_t, N_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta}, \tag{14}$$

where C_t denotes the level of the composite consumption good and N_t is the composite of labor services. The complete model in state-contingent notation resembles the prototypical New Keynesian model, except for the coefficients in the Phillips curve, and is given by

$$x_{it} = E_t x_{t+1} - \sigma^{-1}\left(i - E_t \pi_{t+1}\right) + g_t, \tag{15}$$

$$\pi_{it} = \varphi_i^{-1} \beta E_t \left[\varphi\left(s_{t+1}\right)\pi_{t+1}\right] + \kappa_i x_{it} + u_t, \tag{16}$$

$$i_{it} = \alpha \pi_{it}, \tag{17}$$

where $\kappa_i = \varphi_i^{-1}\left(\theta - 1\right)\left(\sigma + \eta\right)$ for $i = 1, 2$. Disturbances to the intertemporal IS equation (15), g_t, are demand disturbances due to factors such as changes in government purchases.

The switching-Phillips curve uses a measure of the output gap, x_t, which is defined as the log deviation of output from it flexible price level, in place of the marginal cost term.[9]

The exogenous disturbances are autoregressive and mutually uncorrelated,

$$g_t = \phi g_{t-1} + \upsilon_t, \tag{18}$$

$$u_t = \rho u_{t-1} + \varepsilon_t, \tag{19}$$

where $|\phi| < 1$, $|\rho| < 1$, $\upsilon_t \sim N(0, \sigma_\upsilon(s_t)^2)$, $\varepsilon_t \sim N(0, \sigma_\varepsilon(s_t)^2)$ and $E[\upsilon_t \varepsilon_s] = 0$ for all t and s. The volatilities of the shocks, $\sigma_\upsilon(s_t)$ and $\sigma_\varepsilon(s_t)$, are state dependent and switch synchronously with changes in the term governing the cost of price adjustment. The relative magnitudes of the state-dependent parameters are $\sigma_\upsilon(1) > \sigma_\upsilon(2)$, $\sigma_\varepsilon(1) > \sigma_\varepsilon(2)$ and $\varphi(1) < \varphi(2)$. This pattern of inequalities associates the state with relatively more volatile shocks, $s_t = 1$, with the state having relatively lower costs of price adjustment.[10]

The switching volatility of the shocks does not have any implications for the first-order approximations, but are included simply to motivate the changes in the price adjustment parameter. Also, the assumption that $\sigma_\upsilon(s_t)$, $\sigma_\varepsilon(s_t)$ and $\varphi(s_t)$ all switch synchronously may appear restrictive. However, this assumption bears little significance, since each state-dependent parameter may switch independently without affecting the linear equilibrium relations given by (15) and (16).[11]

A change in the state governing adjustment costs, s_t, does not affect the steady state values for inflation and the output gap. Thus, in the steady state, a change in s_t will not generate any dynamics and firms simply adjust their price equal to steady state inflation.

Davig and Leeper (2007), using the approach in McCallum (1983), illustrate how to

[9] The relationship between the output gap and marginal cost term is given by $\widehat{\psi}_t = (\sigma + \eta) x_t$, where $x_t = \log(Y_t/Y^*)$. Y^* is the efficient steady state level of output under flexible prices, see Appendix D for details.

[10] Again, these assumptions reflects similar patterns in estimates from Lubik and Schorfheide (2004) and Smets and Wouters (2007).

[11] Nonsynchronous switching in the volatility of the shocks may have important implications for the full underlying nonlinear model. See Davig and Leeper (2006) for an example of a nonlinear model with shocks that have state-dependent volatilities switching nonsynchronously.

solve Markov-switching rational expectations models using the method of undetermined coefficients on the minimum set of state variables. Following this approach, solutions take the form

$$\pi_{it} = a_i^D g_t + a_i^S u_t, \tag{20}$$

$$x_{it} = b_i^D g_t + b_i^S u_t. \tag{21}$$

A simple case arises when $\phi = \rho = 0$, where the solution for $s_t = i$ is

$$\begin{aligned} a_i^D &= \frac{\sigma\kappa_i}{\sigma + \alpha\kappa_i}, \; a_i^S = \frac{\sigma}{\sigma + \alpha\kappa_i}, \\ b_i^D &= \frac{\sigma}{\sigma + \alpha\kappa_i}, \; b_i^S = -\frac{\alpha}{\sigma + \alpha\kappa_i}. \end{aligned}$$

Since there is no serial correlation in the shocks and no internal propagation mechanism, the impact of the switching slope of the Phillips curve is contemporaneous and solutions match their fixed-regime counterparts. A state with a higher cost of price adjustment implies a relatively small value for the slope of the Phillips curve, κ, resulting in output gap movements having a small impact on inflation. So as κ declines, the impact of demand shocks on inflation also declines. Supply shocks directly impact inflation, but are offset by output gap movements, where the extent of the offsetting effect increases as κ increases.

Similar intuition applies for serially correlated shocks, but convenient analytic expressions are not available.[12] To provide an example of the dynamics with serially correlated shocks, numerical values are chosen as follows : $\alpha = 1.5$, $\beta = .99$, $\sigma = 1$, $\theta = 10$ and $\phi = \rho = .75$. For values of the cost of adjustment parameters, one approach is to use estimates for the slope of the Phillips curve from models that split the sample pre- and post-1980. For example, Lubik and Schorfheide (2004) estimate a New Keynesian model using data from pre- and post-Volcker subsamples. Although, Lubik and Schorfheide estimate a model with prices adjusting according to the Calvo mechanism, the implications for aggregate inflation dynamics are the same as under quadratic price adjustment costs. Specifying a value for the steady state markup and using their estimates for the slope of the Phillips curve in the two subsamples,

[12] Appendix B provides details how to compute the numerical solution on the minimum set of state variables.

given by $\kappa_1 = .75$ and $\kappa_2 = .58$, implies values for the cost of adjustment parameters. Their estimates also indicate higher volatility in both aggregate supply and demand disturbances in the pre-Volcker era. Transition probabilities are set as $p_{11} = p_{22} = .95$, implying an average duration for each regime of 20 quarters.[13]

Figure 1 reports the response to a demand shock conditional on the two different states. Although the variances of the shocks are different across the two states, Figure 1 reports responses for a demand shock of the same magnitude to highlight the differences arising from Phillips curve specification.[14] For inflation, the responses in the two states are similar, though the state with larger slope-coefficient on the output gap (solid-line) exhibits a slightly stronger response, reflecting the lower cost of price adjustment. The impact on output is larger in the state with relatively high price adjustment costs (dashed-line), which also has the lower value for the slope-coefficient (i.e. $\kappa_2 = .58$). In this state, firms meet higher demand via the adjustment of quantities more so than their price.

Figure 2 reports the responses to a supply shock of equal magnitude conditional on each state. Since supply shocks move inflation and output in opposite directions, an adverse supply shock directly increases inflation, but is offset to some extent by the downward movement in output. The degree to which the decline in output attenuates the affect of a supply shock on inflation depends on the slope of the Phillips curve. For $\kappa_2 = .58$, the state with relatively high costs of price adjustment, the offsetting effect on inflation from the decline in output is less than in the state with lower costs of price adjustment. Thus, a positive aggregate supply disturbances generates relatively more inflation despite firms having a higher cost of price adjustment. As Figure 2 illustrates, these factors imply that the volatility of both inflation and output rises in response to aggregate supply shocks as the Phillips curve flattens.

[13]Since Lubik and Schorfheide (2004) do not estimate a Markov-switching model, there is little guidance on specifying the transition probabilities.

[14]The size of the shock is equal to an across-regime average of a two standard-deviation demand shock from Lubik and Schorfheide (2004).

4. Optimal Discretionary Policy with an Ad-hoc Loss

Short-run inflation dynamics have an important impact on the appropriate conduct of monetary policy. Optimal policy under discretion, such as in Clarida, Gali, and Gertler (1999), instructs policy to 'lean against the wind', meaning that the central bank should contract aggregate demand when inflation rises. The extent of the response depends on two factors: the slope of the Phillips curve and the weight policymakers assign to output gap deviations. A Phillips curve with a steep slope allows the central bank to exert considerable influence over inflation by contracting aggregate demand, which is tempered by concerns over output gap stability. If the slope of the Phillips curve changes, implying the influence output gap movements exert on inflation also changes, then should the central bank vary how aggressively it 'leans against the wind'? The answer to this question is sensitive to the assumptions made concerning the central bank's loss function. Similar to Blake and Zampolli (2006), Moessner (2006), Zampolli (2006) and Svensson and Williams (2007), the answer given in this section, under an ad-hoc loss, is 'yes' - the central bank should vary the systematic response of the output gap to movements in inflation. However, this result is sensitive to the ad-hoc specification for the loss function, as will be evident in the following section.

4.1 State-Contingent Targeting Rules

The central bank's ad-hoc loss function is

$$L_t = -\frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda x_{t+i}^2 \right), \tag{22}$$

where λ is the relative weight on output deviations. Rotember and Woodford (1997) and Woodford (2003) derive a loss function with the same form as (22) using a second-order Taylor series expansion to the representative household's expected utility function. An advantage of this approach is that it yields a utility-based value for λ that depends on structural parameters of the model, one of which is the slope-coefficient on the marginal cost term in the Phillips curve. Given this parameter is subject to change, the current assumption that

λ is constant is most-likely to be misleading concerning optimal monetary policy. The next section derives the relevant utility-based welfare criterion when the term governing the cost of price adjustment is subject to change. However, using the ad-hoc loss above is useful as a starting benchmark.

The optimal discretionary policy minimizes (22) subject to the switching-Phillips curve

$$\pi_{it} = \varphi_i^{-1} \beta E_t \left[\varphi \left(s_{t+1} \right) \pi_{t+1} \right] + \kappa_i x_{it} + u_t, \tag{23}$$

for $i = 1, 2$, under the assumption that policy actions do not affect private agents' expectations. Since the optimization problem is static, the central bank only needs to be concerned with setting policy based on the current state and does not need to take into account how states evolve going forward. A first-order condition exists for each state, summarized by the optimal state-contingent targeting rules

$$x_{it} = -\frac{\kappa_i}{\lambda} \pi_{it}, \tag{24}$$

for $i = 1, 2$, indicating the central bank should optimally vary how aggressively it acts to offset aggregate supply disturbances depending on the state. In states with relatively low costs of price adjustment, such as $s_t = 1$, inflation is relatively responsive to changes in the output gap. The optimal targeting rule for $s_t = 1$ instructs policy to use this leverage and adjust the output gap by a greater amount, relative to $s_t = 2$, in response to a given value for inflation. So with $\kappa_1 > \kappa_2$, the central bank sets policy to adjust aggregate demand more aggressively when $s_t = 1$ than when $s_t = 2$.

Although the optimal policy is under discretion, the central bank has committed to behave in a certain way in each state. The more aggressive policy in the state with lower costs of price adjustment works to control expectations of future inflation, mitigating the impact of shocks on inflation in the state with higher costs of price adjustment. In the U.S., the Volcker disinflation represents an episode where rather large output losses were tolerated to reduce inflation. To the extent this episode remains embedded in expectations, the optimal discretionary solution suggests how this episode has benefitted subsequent policymakers.

Leeper and Zha (2003) refer to these effects, arising from the potential of future regime change, as *expectation formations effects*. If private expectations anticipate a regime with very aggressive monetary policy, these actions control expected inflation and consequently, current inflation.

4.2 Conditional Efficiency Frontiers

Taylor (1979) demonstrates that aggregate supply shocks force upon policymakers a trade-off between inflation and output volatility. The position of the optimal trade-off frontier, or efficiency frontier, depends on the variance of the underlying aggregate supply shocks and structural parameters of the model. The weight policy makers place on output gap stabilization determines the point on the frontier minimizing the ad-hoc loss function. In the current framework with changing structural parameters, there exist conditional frontiers depending on the current state. For example, Figure 3 reports the frontiers using the parameterization in the previous section, except closing the model with optimal discretionary policy under the assumption $\lambda = .25$.[15] The variance of the underlying aggregate supply disturbance is temporarily assumed to be constant across states to isolate the effects of the switching slope-coefficient in the Phillips curve.

The frontier conditional on the κ_1 state is more favorable compared to the κ_2 state with the smaller slope-coefficient on the output gap. To understand the more favorable trade-off for κ_1, it is useful to consider a central bank that strictly targets inflation, where $\lambda = 0$. In this case, the central bank adjusts the output gap to any extent necessary to achieve zero inflation in both states. However, output volatility will differ across states if $\kappa_1 \neq \kappa_2$. If $\lambda = 0$, then the output gap response to an aggregate supply shock is $a_i = \kappa_i^{-1}$, indicating the central bank adjusts the output gap inversely to the slope-coefficient in the Phillips curve. For κ_1, firms face lower costs when adjusting prices so output gap movements are relatively effective at stabilizing inflation. For κ_2, inflation is less responsive to output gap movements, so the central bank must induce larger movements to attain the same magnitude of inflation

[15] See Appendix C for details.

volatility. So, output volatility for $s_t = 1$ is less than for $s_t = 2$, even though inflation volatility is zero in each state.

Given some concern over output gap stability, so $\lambda > 0$, the central bank will still adjust the output gap relatively more for a given supply shock under κ_2, but permits some inflation volatility. The concern over output gap stability results in relatively more inflation and output volatility under κ_2, causing the efficiency frontier for $s_t = 2$ to lie outside of the frontier for $s_t = 1$.[16]

Figure 3 may appear paradoxical since the frontier for κ_1, roughly representing the pre-Volcker period, lies inside the frontier with the smaller κ_2. Evidence supporting the Great Moderation, such as McConnell and Perez-Quiros (2000) and Stock and Watson (2003), indicates the pre-Volcker period was more volatile than the post-1982 period, seeming to suggest a reversal of the relative position of the two frontiers. The apparent paradox arises for two reasons : 1) the frontiers represent the volatility trade-off under optimal discretionary policy, which is unlikely to be an accurate characterization of U.S. monetary policy in the 1970s and 2) empirical evidence suggest the volatility of exogenous shocks is lower in the post-1982 period.

Substantial empirical evidence suggests monetary policy was systematically less aggressive in the 1970s than afterward.[17] Using the ad-hoc loss, the optimal policy under discretion advises exactly the opposite. In states with a large slope-coefficient on the output gap, as in the 1970s, monetary policy should react systematically more aggressively to inflation. Due to the likely non-optimal monetary policy in the 1970s, the economy was operating well away from its optimal frontier in the pre-Volcker era.

Allowing the variance of the supply shock to vary across states, as the original model specification indicates, can reverse the relative position of the two frontiers in Figure 3. Using the estimates of aggregate supply volatility from Lubik and Schorfheide (2004) for the

[16] Although the frontier for κ_1 always lies inside of κ_2, assuming a constant variance of supply shocks across states and $\kappa_1 > \kappa_2$, output volatility for κ_1 will eventually exceed that for κ_2 for a high enough value of λ.

[17] For example, see Clarida, Gali, and Gertler (2000).

pre- and post-Volcker periods, setting $\sigma_\varepsilon(1) = 1.16$ and $\sigma_\varepsilon(2) = .64$ yields the conditional efficiency frontiers in Figure 4.

The flattening of the Phillips curve in the context of the Great Moderation raises an interesting implication. As Figure 3 indicates, which keeps the volatility of shocks constant across states, a flatter Phillips curve implies higher inflation and output volatility in the post-1982 period. Given this contradicts the empirical evidence of the Great Moderation, the implication is that the volatility of aggregate supply shocks had to decline. In other words, if the Phillips curve flattens, then better conduct of monetary policy by itself cannot bring about a moderation in both inflation and output volatility - there must also be a decline in the volatility of the underlying aggregate supply shocks. However, if monetary policy is clearly sub-optimal, a distinct possibility in the pre-1982 period, then a shift to a regime more closely resembling optimal policy can also bring about a decline in both inflation and output.

5. A Utility-Based Welfare Criterion

A loss function in squared deviations of the output gap and inflation from their steady state values is a common specification, such as Clarida, Gali, and Gertler (1999). Woodford (2003), however, shows how a second-order approximation to the expected utility of the consumer under the assumption of staggered price-setting as in Calvo (1983) gives rise to a loss function of this form, where the weight on the output gap term is a function of the frequency of price adjustment. Eusepi (2005) derives the utility-based welfare function for price adjustment subject to quadratic costs, as in Rotemberg (1982), and shows how the weight on the output gap term depends on the parameter governing the cost of price adjustment. In a setting where this cost can change, this section shows how the weight on the output gap also changes with the cost of price adjustment and how this affects the optimal policy under discretion.

The appendix derives the following approximated utility of the representative household

$$L_t = -\Omega_i \left[\pi_t^2 + \lambda_i x_t^2\right], \tag{25}$$

where $-\Omega_i = -.5\varphi_i$ scales the loss according to the cost of price adjustment and

$$\lambda_i = \frac{\eta + \sigma}{\varphi_i}, \tag{26}$$

indicating that the weight on output gap deviations depends on the state governing the cost of price adjustment. If the utility function has log consumption and is linear in labor, so $\sigma = 1$ and $\eta = 0$, then (26) is simply $\lambda_i = \varphi_i^{-1}$. Thus, the utility-based welfare criteria is a loss function featuring a state-dependent weight on the output gap term.

In a state with a relatively low cost of price adjustment, deviations in inflation create a small loss, so the weight on the output gap is relatively high. Conversely, in a state with a high cost of price adjustment, deviations in inflation are costly, so the central bank should place less emphasis on output stabilization. This intuition is similar to that from the utility-based welfare criteria derived under the Calvo mechanism of price adjustment, as in Woodford (2003). When the price adjustment is infrequent, losses arise from price dispersion, so the central bank should place low weight on output stabilization relative to the case when price adjustment occurs more frequently.

Minimizing the central bank's utility-based loss function subject to the switching-Phillips curve under the assumption that policy actions do not affect private agents' expectations results in the first-order conditions for inflation and the output gap yields

$$x_{it} = -\frac{\kappa_i}{\lambda_i}\pi_{it} \tag{27}$$

or after substituting for λ_i and κ_i, which reduce to

$$x_{it} = (1 - \theta)\,\pi_{it}, \tag{28}$$

indicating the central bank should *not* optimally vary how aggressively it acts to offset aggregate supply disturbances. The optimal targeting rule is a constant relation between output and inflation, independent of the state, depending only upon the elasticity of substitution between goods. This result differs from the optimal discretionary policy under an ad-hoc loss, where the optimal discretionary policy instructs the central bank to switch policies in accordance with the structure of the economy.

In the state with relatively high costs of price adjustment, both the weight attached to output gap stabilization and the slope-coefficient in the Phillips curve are relatively small. Under an ad-hoc loss, a low slope-coefficient directs policy to reduce the systematic output gap response to inflation deviations precisely because such movements are less effective at stabilizing inflation. However, it is in states with a low-slope coefficient, or high costs of price adjustment, when inflation volatility is more costly to firms. The utility-based welfare criterion reflects this higher cost of inflation volatility by down-weighting the emphasis on output gap stabilization.

Thus, in the high-cost state, two opposing forces exactly offset to bring about the invariant policy response : 1) a lower slope-coefficient on the output gap, which directs policy to reduce output gap movements to stabilize inflation and 2) a lower weight on the output gap, which directs policy to increase output gap movements to stabilize inflation. These two effects exactly offset under the assumption of switching quadratic costs of adjustment.[18] The difference in comparison to the optimal policy under the ad-hoc loss function is that it only accounts for the first factor, the change in the slope of the Phillips curve, and ignores the welfare implications of inflation in the different states.

6. Conclusion

This paper shows that changing costs of price adjustment can generate instability in a forward-looking Phillips curve relation. In particular, the coefficients on both expected inflation and marginal cost, or the output gap, are subject to change in coordination with changes in the state governing the cost of adjusting prices. In addition, Phillips curve

[18] Analogous reasoning applies to the low-cost state.

instability has implications for optimal monetary policy. Under an ad-hoc welfare criterion, the optimal policy adjusts the systematic component of policy along with changes in the state. However, since the microfoundations of the firm's optimization problem are explicitly stated, it is possible to derive a utility-based welfare metric. A novel feature of this metric is that it has a state-dependent weight on the output gap term. The weight depends inversely on the cost of price adjustment, so in the low cost state, relatively more weight is placed on output stabilization. The implication for optimal monetary policy under discretion is that policy should not vary the systematic component of policy. This result stands in contrast to the prescription coming under the ad-hoc criterion, which recommends the systematic component of policy change with the state.

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APPENDIX

A. Deriving the Switching Phillips Curve

For $s_t = 1$, the conditional first-order condition after distributing the $\varphi(s_{t+1})$ term is

$$\begin{aligned} 0 \;=\; & (1-\theta_t)\,\Delta_t \left(\frac{P_t\,(j)}{P_t}\right)^{-\theta_t}\left(\frac{Y_t}{P_t}\right) + \theta_t \Delta_t \psi_t \left(\frac{P_t\,(j)}{P_t}\right)^{-\theta_t - 1}\left(\frac{Y_t}{P_t}\right) - \\ & \varphi\,(1)\,\Delta_t \left(\frac{P_t\,(j)}{\Pi P_{t-1}\,(j)} - 1\right)\left(\frac{Y_t}{\Pi P_{t-1}\,(j)}\right) + \\ & \beta p_{11} \varphi\,(1)\, E_t \left[\Delta_{t+1}\left(\frac{P_{t+1}\,(1,j)}{\Pi P_t\,(j)} - 1\right)\left(\frac{P_{t+1}\,(1,j)\,Y_{t+1}\,(1)}{\Pi P_t\,(j)^2}\right)\right] + \\ & \beta\,(1-p_{11})\,\varphi\,(2)\, E_t \left[\Delta_{t+1}\left(\frac{P_{t+1}\,(2,j)}{\Pi P_t\,(j)} - 1\right)\left(\frac{P_{t+1}\,(2,j)\,Y_{t+1}\,(2)}{\Pi P_t\,(j)^2}\right)\right], \end{aligned} \tag{A-1}$$

where $P_{t+1}\,(i,j)$ represents the nominal price for firm j when $s_{t+1} = i$ and $Y_{t+1}(i)$ represents final output when $s_{t+1} = i$. An analogous first-order condition exists for $s_t = 2$, except p_{11} is replaced with $(1 - p_{22})$ and $(1 - p_{11})$ is replaced with p_{22}. Using $(A - 1)$, the firm's optimal pricing condition for $s_t = 1$, after imposing $P_t\,(j) = P_t$, is given by

$$\begin{aligned} 0 \;=\; & (1-\theta)\,\Delta_t\left(\frac{Y_t}{P_t}\right) + \theta\Delta_t\psi_t\left(\frac{Y_t}{P_t}\right) - \varphi\,(1)\,\Delta_t\left(\frac{P_t}{\Pi P_{t-1}} - 1\right)\left(\frac{Y_t}{\Pi P_{t-1}}\right) + \\ & \beta p_{11}\varphi\,(1)\,E_t\left[\Delta_{t+1}\,(1)\left(\frac{P_{t+1}\,(1)}{\Pi P_t} - 1\right)\left(\frac{P_{t+1}\,(1)\,Y_{t+1}\,(1)}{\Pi P_t^2}\right)\right] + \\ & \beta\,(1-p_{11})\,\varphi\,(2)\,E_t\left[\Delta_{t+1}\,(2)\left(\frac{P_{t+1}\,(2)}{\Pi P_t} - 1\right)\left(\frac{P_{t+1}\,(2)\,Y_{t+1}\,(2)}{\Pi P_t^2}\right)\right], \end{aligned} \tag{A-2}$$

where substituting in $P_t/P_{t-1} = \Pi_t$ yields

$$\begin{aligned} 0 \;=\; & (1-\theta_t)\,\Delta_t + \theta_t\Delta_t\psi_t - \varphi\,(1)\,\Delta_t\left(\frac{\Pi_t}{\Pi} - 1\right)\left(\frac{\Pi_t}{\Pi}\right) + \\ & \beta p_{11}\varphi\,(1)\,E_t\left[\Delta_{t+1}\,(1)\left(\frac{\Pi_{t+1}\,(1)}{\Pi} - 1\right)\left(\frac{\Pi_{t+1}\,(1)\,Y_{t+1}\,(1)}{\Pi y_t}\right)\right] + \\ & \beta\,(1-p_{11})\,\varphi\,(2)\,E_t\left[\Delta_{t+1}\,(2)\left(\frac{\Pi_{t+1}\,(2)}{\Pi} - 1\right)\left(\frac{\Pi_{t+1}\,(2)\,Y_{t+1}\,(2)}{\Pi Y_t}\right)\right]. \end{aligned} \tag{A-3}$$

Log-linearizing around the constant steady state yields

$$
\begin{aligned}
0 \;=\;& \left(1-\theta\left(1+\widehat{\theta}_t\right)\right)\Delta\left(1+\widehat{\Delta}_t\right)+\theta\left(1+\widehat{\theta}_t\right)\Delta\left(1+\widehat{\Delta}_t\right)\psi\left(1+\widehat{\psi}_t\right)- \\
& \varphi(1)\,\Delta\left(1+\widehat{\Delta}_t\right)\pi_t\left(1+\pi_t\right)+ \\
& \beta p_{11}\varphi(1)\,E_t\left[\Delta\left(1+\widehat{\Delta}_{t+1}(1)\right)\pi_{t+1}(1)\left(1+\pi_{t+1}(1)\right)\left(1+Y_{t+1}(1)\right)\left(1-Y_t\right)\right]+ \\
& \beta\left(1-p_{11}\right)\varphi(2)\,E_t\left[\Delta\left(1+\widehat{\Delta}_{t+1}(2)\right)\pi_{t+1}(2)\left(1+\pi_{t+1}(2)\right)\left(1+Y_{t+1}(2)\right)\left(1-Y_t\right)\right],
\end{aligned}
\tag{A-4}
$$

where $\pi_t = \log\left(\Pi_t/\Pi\right)$, $\widehat{\Delta}_t = \log\left(\Delta_t/\Delta\right)$, $\widehat{\psi}_t = \log\left(\psi_t/\psi\right)$ and $\widehat{\theta}_t = \log\left(\theta_t/\theta\right)$. Values without a time subscript are steady state values. Eliminating higher-order terms and using $\psi = \theta^{-1}\left(\theta-1\right)$ yields

$$
\pi_t = \beta p_{11} E_t\left[\pi_{1,t+1}\right] + \left(1-p_{11}\right)\beta\frac{\varphi_2}{\varphi_1}E_t\left[\pi_{2,t+2}\right] + \frac{(\theta-1)}{\varphi_1}\widehat{\psi}_t + u_t, \tag{A-5}
$$

where $u_t = -\widehat{\theta}_t$. The same approach is taken for $s_t = 2$, where the general representation can be rewritten as (13).

B. Solving the NK Model with the Switching-Phillips Curve

The posited solutions $(20)-(21)$ can be substituted into the structural equations $(15)-(16)$ to yield systems that relate the structural parameters to the solution coefficients. For supply shocks, the system is

$$
\begin{bmatrix}
\alpha - p_{11}\rho & -\left(1-p_{11}\right)\rho & 1-p_{11}\rho & -\left(1-p_{11}\right)\rho \\
-\left(1-p_{22}\right)\rho & \alpha - p_{22}\rho & -\left(1-p_{22}\right)\rho & 1-p_{22}\rho \\
1-\beta p_{11}\rho & -\beta\left(1-p_{11}\right)\frac{\varphi_2}{\varphi_1}\rho & -\kappa_1 & 0 \\
-\beta\left(1-p_{22}\right)\frac{\varphi_1}{\varphi_2}\rho & 1-\beta p_{22}\rho & 0 & -\kappa_2
\end{bmatrix}
\begin{bmatrix}
a^S(1) \\ a^S(2) \\ b^S(1) \\ b^S(2)
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 1 \\ 1
\end{bmatrix}
$$

and for demand shocks

$$\left[\begin{array}{cccc} \alpha - p_{11}\phi & -(1-p_{11})\phi & 1-p_{11}\phi & -(1-p_{11})\phi \\ -(1-p_{22})\phi & \alpha - p_{22}\phi & -(1-p_{22})\phi & 1-p_{22}\phi \\ 1-\beta p_{11}\phi & -\beta(1-p_{11})\frac{\varphi_2}{\varphi_1}\phi & -\kappa_1 & 0 \\ -\beta(1-p_{22})\frac{\varphi_1}{\varphi_2}\phi & 1-\beta p_{22}\phi & 0 & -\kappa_2 \end{array}\right]\left[\begin{array}{c} a^D(1) \\ a^D(2) \\ b^D(1) \\ b^D(2) \end{array}\right] = \left[\begin{array}{c} 1 \\ 1 \\ 0 \\ 0 \end{array}\right].$$

C. Solving for Dynamics and Computing Efficiency Frontiers Under Optimal Discretionary Policy

To compute the efficiency frontiers under the ad-hoc loss function, first solve for the dynamics of output and inflation under the discretionary policy. Use the system of targeting rules in (24) , which in state-contingent notation is

$$\left[\begin{array}{cc} \lambda p_{11} & \lambda(1-p_{11}) \\ \lambda(1-p_{22}) & \lambda p_{22} \end{array}\right]\left[\begin{array}{c} E_t x_{1t+1} \\ E x_{2t+1} \end{array}\right] = \left[\begin{array}{cc} -\kappa_1 p_{11} & -\kappa_2(1-p_{11}) \\ -\kappa_1(1-p_{22}) & -\kappa_2 p_{22} \end{array}\right]\left[\begin{array}{c} E\pi_{1t+1} \\ E\pi_{2t+1} \end{array}\right],$$

Solving in terms of the expected conditional output gaps yields

$$Ex_{1t+1} = -\frac{\kappa_1}{\lambda}E\pi_{1t+1}, \tag{A-6}$$

$$Ex_{2t+1} = -\frac{\kappa_2}{\lambda}E\pi_{2t+1}. \tag{A-7}$$

Substituting $(A-6)-(A-7)$ into the switching-Phillips curve (23) yields a dynamic system in x_{1t} and x_{2t}

$$\left[\begin{array}{c} x_{1t} \\ x_{2t} \end{array}\right] = \left[\begin{array}{cc} p_{11}\gamma_1 & (1-p_{11})\gamma_1\frac{\kappa_1\varphi_2}{\kappa_2\varphi_1} \\ (1-p_{22})\gamma_2\frac{\kappa_2\varphi_1}{\kappa_1\varphi_2} & p_{22}\gamma_2 \end{array}\right]\left[\begin{array}{c} Ex_{1t+1} \\ Ex_{2t+1} \end{array}\right] - \left[\begin{array}{c} 1 \\ 1 \end{array}\right]u_t, \tag{A-8}$$

where $\gamma_i = \lambda\beta\left(\lambda + \kappa_i^2\right)^{-1}$ and $i = 1, 2$.

Dynamics for the above system can be solved as before, using the method of undetermined coefficients on the minimum set of state variables. Monetary policy completely offsets disturbances to the IS equation, so the state variables are s_t and u_t. Decision rules then have the form

$$x_{it} = a_i u_t, \tag{A-9}$$

for $i = 1, 2$. Using (24), inflation is then given by

$$\pi_{it} = -\frac{\lambda a_i}{\kappa_i} u_t, \tag{A-10}$$

for $i = 1, 2$. Substituting the decision rules $(A-9)$ with unknown a_i into $(A-8)$ yields

$$\begin{aligned} \frac{\lambda}{\kappa_1} a_1 u_t &= p_{11} \frac{\lambda}{\kappa_1} \beta a_1 \rho u_t + (1 - p_{11}) \frac{\lambda}{\kappa_2} \frac{\varphi_2}{\varphi_1} \beta a_2 \rho u_t - \kappa_1 a_1 u_t - u_t, && \text{(A-11)} \\ \frac{\lambda}{\kappa_2} a_2 u_t &= p_{22} \frac{\lambda}{\kappa_2} \beta a_2 \rho u_t + (1 - p_{22}) \frac{\lambda}{\kappa_1} \frac{\varphi_1}{\varphi_2} \beta a_1 \rho u_t - \kappa_2 a_2 u_t - u_t, && \text{(A-12)} \end{aligned}$$

implying the following relationships between parameters

$$\begin{bmatrix} \left(\frac{\lambda}{\kappa_1} - p_{11} \frac{\lambda}{\kappa_1} \beta \rho + \kappa_1 \right) & -(1 - p_{11}) \frac{\lambda}{\kappa_2} \frac{\varphi_2}{\varphi_1} \beta \rho \\ -(1 - p_{22}) \frac{\lambda}{\kappa_1} \frac{\varphi_1}{\varphi_2} \beta \rho & \left(\frac{\lambda}{\kappa_2} - p_{22} \frac{\lambda}{\kappa_2} \beta \rho + \kappa_2 \right) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}. \tag{A-13}$$

The locus of points describing the efficiency frontier are then given by computing the unconditional variance of x_{it} and π_{it} using $(A-9)$ and $(A-10)$, conditional on i, for a grid of values over λ. Under the utility-based approach, dynamics and the corresponding efficiency frontiers can be derived in an analogous way.

D. Deriving the Utility-Based Welfare Criterion Under Switching Costs of Price Adjustment

The representative household's period utility function is

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \tag{A-14}$$

where C_t is the composite good and N_t is time spent working. Firm level production function is

$$y_t(j) = n_t(j), \tag{A-15}$$

where aggregate labor services are

$$N_t = \int_0^1 n_t(j)dj. \tag{A-16}$$

In the absence of fiscal subsidies, the first-order condition for labor is

$$\frac{W_t}{P_t} = \frac{1}{\mu} = \frac{N_t^\eta}{C_t^{-\sigma}}. \tag{A-17}$$

However, a constant employment subsidy exists that is proportional to the households labor income, which offsets the inefficiently low level of production in the steady state arising from monopolistic distortions. Under perfectly flexible prices, firms set their relative price equal to a markup $\mu > 1$ that exceeds their marginal cost of production. The employment subsidy given to households, financed by lump-sum taxes on households, results in an efficient steady state level of production. Monetary policy then focuses on stabilization policies, versus policies to undo the monopolistic distortions. In the deterministic steady state, the monopolistic firm is not adjusting its price, so the changing parameter governing the costs of price adjustment does not create any distortions. Adding the fiscal subsidy results in

$$(1+s)\frac{W_t}{P_t} = \frac{N_t^\eta}{C_t^{-\sigma}} = 1, \tag{A-18}$$

where the subsidy is $(1+s) = \mu$.

The aggregate resource constraint is

$$Y_t = C_t + \frac{\varphi(s_t)}{2}(\Pi_t - 1)^2, \tag{A-19}$$

where steady state inflation is set to zero. Also, $Y_t = N_t$. Substituting $(A-19)$ into $(A-14)$ yields

$$U(Y_t, \Pi_t, s_t) = \frac{\left(Y_t - \frac{\varphi(s_t)}{2}(\Pi_t - 1)^2\right)^{1-\sigma}}{1-\sigma} - \frac{Y_t^{1+\eta}}{1+\eta}. \tag{A-20}$$

The second-order approximation to the first term of the representative agent's period utility function is given by

$$\frac{\left(Y_t - \frac{\varphi(s_t)}{2}(\Pi_t - 1)^2\right)^{1-\sigma}}{1-\sigma} \approx \frac{Y^{1-\sigma}}{1-\sigma} + Y^{-\sigma}\widetilde{Y}_t - \frac{1}{2}\sigma Y^{-\sigma-1}\widetilde{Y}_t^2 - \frac{\varphi(s_t)}{2}Y^{-\sigma}\widetilde{\pi}_t^2 \tag{A-21}$$

where $\varphi_i = \varphi(s_t)$ for $s_t = i$, $\widetilde{Y}_t = Y_t - Y$ and $\widetilde{\pi}_t = \Pi_t - \Pi$. Using the following approximations,

$$\widetilde{Y}_t \approx Y\left(Y_t + \frac{1}{2}Y_t^2\right) \tag{A-22}$$

$$\widetilde{\pi}_t \approx \left(\pi_t + \frac{1}{2}\pi_t^2\right) \tag{A-23}$$

yields

$$\begin{aligned}\frac{\left(Y_t - \frac{\varphi(s_t)}{2}(\Pi_t - 1)^2\right)^{1-\sigma}}{1-\sigma} \approx\ & \frac{Y^{1-\sigma}}{1-\sigma} + Y^{1-\sigma}\left(Y_t + \frac{1}{2}Y_t^2\right) - \\ & \frac{1}{2}\sigma Y^{1-\sigma}\left(Y_t + \frac{1}{2}Y_t^2\right)^2 - \frac{\varphi(s_t)}{2}Y^{-\sigma}\left(\pi_t + \frac{1}{2}\pi_t^2\right)^2.\end{aligned}$$

Removing the higher order terms and terms independent of policy $(t.i.p.)$ yields

$$\frac{\left(Y_t - \frac{\varphi(s_t)}{2}(\pi_t - 1)^2\right)^{1-\sigma}}{1-\sigma} \approx Y^{1-\sigma}Y_t + \frac{1}{2}Y^{1-\sigma}Y_t^2 - \frac{1}{2}\sigma Y^{1-\sigma}Y_t^2 \tag{A-24}$$

$$-\frac{1}{2}\varphi(s_t)\,Y^{-\sigma}\pi_t^2 + t.i.p. \tag{A-25}$$

The second-order approximation to the second argument of the utility function is

$$\frac{Y_t^{1+\eta}}{1+\eta} \approx Y^{1+\eta}Y_t + \frac{1}{2}Y^{1+\eta}(1+\eta)\,Y_t^2 + t.i.p. \tag{A-26}$$

Combining both components of the utility function and removing *t.i.p.* yields

$$U(Y_t, \pi_t, s_t) \approx Y^{1-\sigma} Y_t + \frac{1}{2} Y^{1-\sigma} Y_t^2 - \frac{1}{2}\sigma Y^{1-\sigma} Y_t^2 - \frac{1}{2}\varphi(s_t) Y^{-\sigma}\pi_t^2 - Y^{1+\eta}\left(Y_t + \frac{1}{2}(1+\eta) Y_t^2\right),$$

and rearranging terms yields

$$\begin{aligned} U(Y_t, \pi_t, s_t) &= Y^{1-\sigma} Y_t + \frac{1}{2} Y^{1-\sigma} Y_t^2 - \frac{1}{2}\sigma Y^{1-\sigma} Y_t^2 - \frac{1}{2}\varphi(s_t) Y^{-\sigma}\pi_t^2 \\ &\quad - (1-\Phi) Y^{1-\sigma}\left(Y_t + \frac{1}{2}(1+\eta) Y_t^2\right), \\ &= \frac{1}{2} Y^{1-\sigma}\left((1-\sigma-(1-\Phi)(1+\eta)) Y_t^2 + 2\Phi Y_t\right) - \frac{1}{2}\varphi(s_t) Y^{-\sigma}\pi_t^2, \\ &= -\frac{1}{2} Y^{1-\sigma}(\sigma+\eta)\left(Y_t^2 - 2\Phi(\sigma+\eta)^{-1} Y_t\right) + \frac{1}{2} Y^{1-\sigma}\Phi(1+\eta) Y_t^2 - \frac{1}{2}\varphi(s_t) Y^{-\sigma}\pi_t^2. \end{aligned}$$

Removing the distortion creating the inefficiently low steady state level of output with the subsidy, so $\Phi = 0$, yields

$$U(Y_t, \pi_t, s_t) = -\frac{1}{2}(\sigma+\eta) Y_t^2 - \frac{1}{2}\varphi(s_t)\pi_t^2, \quad \text{(A-27)}$$

$$U(Y_t, \pi_t, s_t) = -\frac{1}{2}\varphi(s_t)\left(\pi_t^2 + \frac{\sigma+\eta}{\varphi(s_t)} x_t^2\right), \quad \text{(A-28)}$$

where $x_t = Y_t - Y^*$, where $Y^* = 1$ from $(A-18)$ represents the efficient level of output in the steady state and is independent of monetary policy.

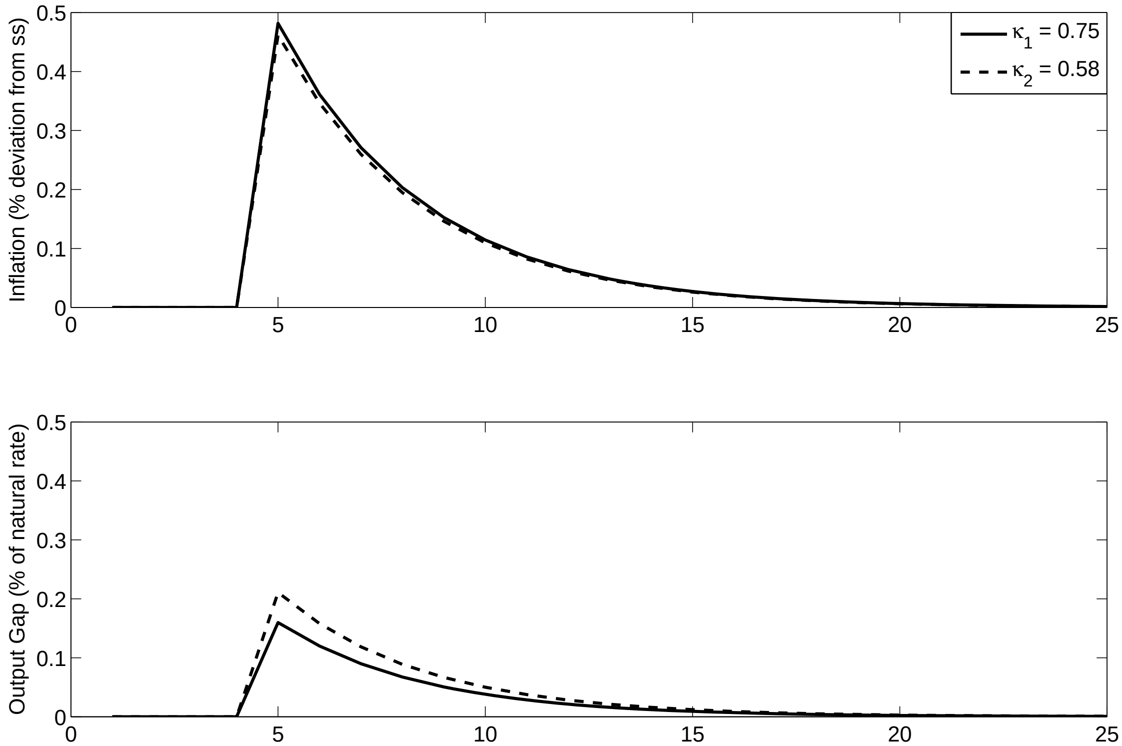

Figure 1: Conditional impulse responses to an aggregate demand shock under a simple monetary rule.

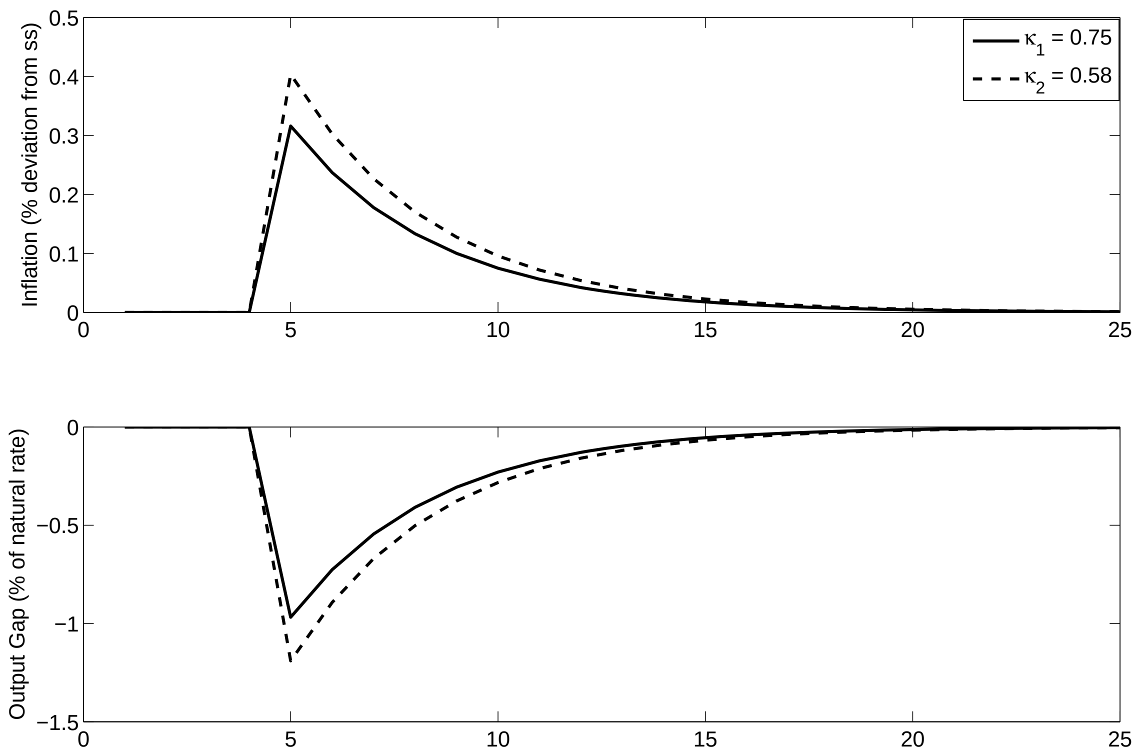

Figure 2: Conditional impulse responses to an aggregate supply shock under a simple monetary rule.

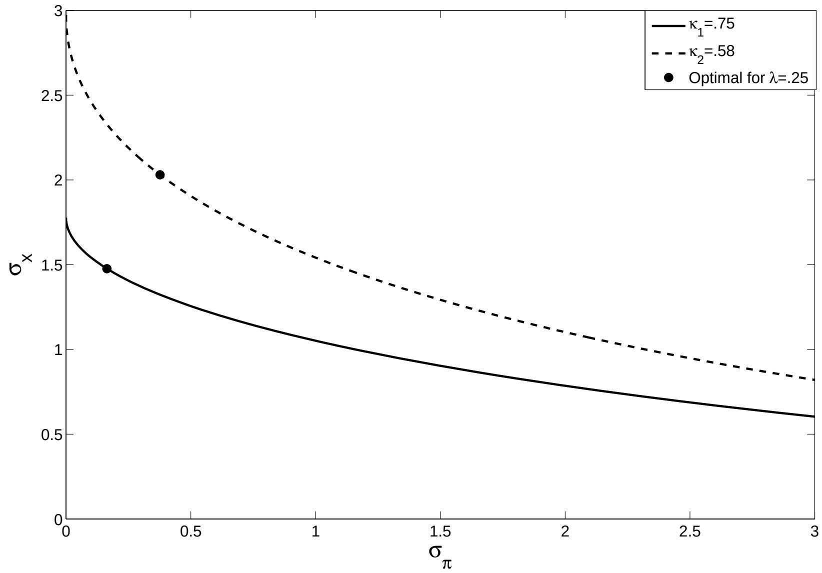

Figure 3: Conditional inflation-output volatility tradeoffs (Constant supply shock variance).

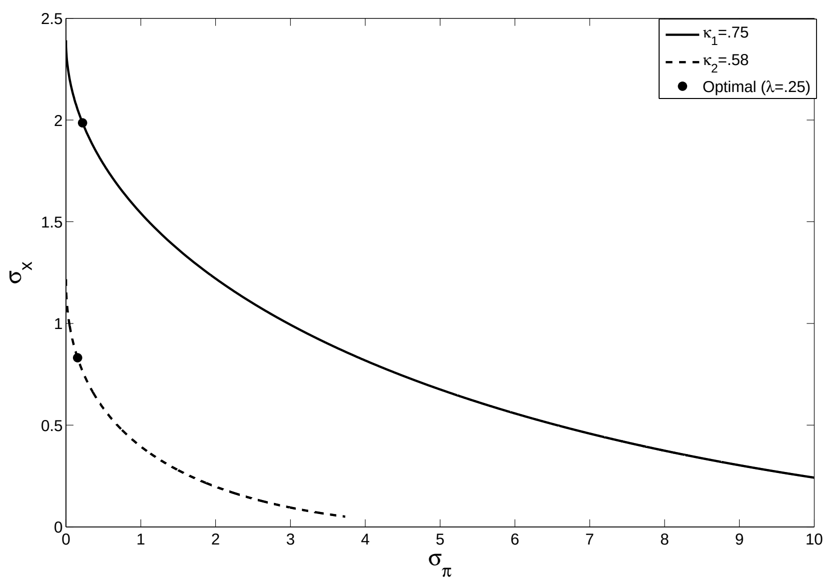

Figure 4: Conditional inflation-output volatility tradeoffs (State-dependent supply shock variances set to Lubik and Schorfheide (2004) estimates).