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Minding the Gap: Central Bank Estimates of the Unemployment Natural Rate

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MINDING THE GAP: CENTRAL BANK ESTIMATES OF THE UNEMPLOYMENT NATURAL RATE

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Abstract: A time-varying parameter framework is suggested for use with real-time multiperiod forecast data to estimate implied forecast equations. The framework is applied to historical briefing forecasts prepared for the Federal Open Market Committee to estimate the U.S. central bank's ex ante perceptions of the natural rate of unemployment. Relative to retrospective estimates, empirical results do not indicate severe underestimation of the natural rate of unemployment in the 1970s.

Keywords: FOMC Greenbook forecasts; the Great Inflation; time-varying natural rates.

JEL classification: E3, E5, N1

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1 Introduction

Natural rates are important, but infrequently discussed, components of macro models that describe equilibrium levels of economic activity and anchor agent expectations. Defined by Frisch (1936) as conditional equilibrium solutions of models in the absence of dynamic frictions, natural rates are often neglected background scenery in theoretical macro models where fixed natural rates are collected in equation intercepts and time-varying natural rates are accommodated by recasting variables in "gap" formats or deviations from natural rates. Even with such real-world experiences as the slowdown of trend growth and the acceleration of inflation in the 1970s, time-variation in natural rates could be ignored in theoretical models that assumed full information on the size and timing of structural breaks.

The assumption of full information continued to be prevalent with the development of New Keynesian (NK) policy models in the mid-1990s. These models generally contained an output gap, with deviations in output from a time-varying natural rate, and invariant natural rates for inflation and the real interest rate.[1]

More recent work has relaxed the assumption of common information generally associated with rational (model-consistent) expectations, and examined implications of natural rate measurement errors by agents in the private sector. Examples include errors in estimating the trend growth of labor productivity by wage earners in Ball and Mankiw (2002), and errors in estimating the central bank target for inflation by bond traders in Kozicki and Tinsley (2001a, 2001b).

A wide variety of empirical techniques have been used to estimate time-varying natural rates, including the Hodrick-Prescott filter used by Ball and Mankiw (2002), time trend polynomials, Beveridge-Nelson (1981) time series partitions, and Kalman filter estimates of unobserved states.[2] Kozicki and Tinsley (2001a, 2001b) suggest that the limits of long-horizon forecasts can often provide reasonable estimates of real-time perceptions of natural rate equilibria, and use surveys of expected inflation over 5-10 year horizons to

[1]This mixture of a time-varying natural rate for output and invariant natural rates for the remaining state variables is a staple of the models discussed in the influential volume edited by Taylor (1999).

[2]A partial list of Kalman filter estimates include Laubach and Wiliams (2002) and Clark and Kozicki (forthcoming) for the natural rate of real interest rates and Kozicki and Tinsley (forthcoming) for central bank and private sector views of the effective policy target for inflation.

illustrate variations in bond trader views of the central bank target for inflation.[3]

Interestingly, there is almost no empirical work to recover central bank real-time perceptions of natural rates.[4] Consequently, this paper will draw on the history of multiperiod forecasts presented to the Federal Open Market Committee (FOMC), the US monetary policy committee, to estimate the evolving perceptions of the central bank of the natural rate for unemployment, $\bar{u}_t$.

Estimates of the central bank perceptions of natural rates are interesting also because central bank measurement errors are central to a recent strand of historical policy interpretations. A series of influential papers by Orphanides (2003a, 2003b, forthcoming) suggest that the US central bank substantially overestimated the natural rate for output, $\bar{y}_t$, in the 1970s, mistakenly inducing low levels of the policy interest rate. Although this research has instigated useful work on consequences of real-time errors in estimates of the natural rate of output and trend productivity, the applicability to policy formation in the 1970s is conjectural. A major obstacle to confirming this interpretation of monetary policy in the 1970s is the lack of a continuous historical record of central bank estimates of the natural rate for output. In the absence of evidence that staff estimates of $\bar{y}_t$ were routinely reported to the FOMC in the 1970s, Orphanides (2003a) uses output natural rates presented in annual reports of the Council of Economic Advisers (CEA) as a real-time proxy. However, given representative specifications of aggregate pricing equations in the 1970s, it is more plausible that the FOMC gauged real resource slack by aggregate unemployment.[5]

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[3]The survey estimates are also used to confirm the relevance of a real-time learning algorithm that does not use the inflation survey information.

[4]A notable exception is the construction by Romer and Romer (2002) of central bank estimates of the natural rate for unemployment, which will be discussed in section 3.

[5]CEA natural rate estimates are infrequently cited in the FOMC *Memorandum of Discussion* (MOD) during the 1970s, and do not appear to have been supported by staff forecasts. Examples include: "(T)he potential GNP as estimated by the Council of Economic Advisers is based on a 3.8 per cent unemployment rate. That may well be too low an unemployment target for sustainable economic growth without inflation," Partee, FOMC Economist (MOD, 11/17/1970, p.31) and "Mr. Partee observed that the target for the unemployment rate referred to in the Annual Report of the Council of Economic Advisers already seemed to have been increased from 4 to 4-1/2 percent....according to the (Greenbook) projections, even a 5 per cent unemployment rate would be associated with considerable continuing inflation in the short run." (MOD, 3/19/1973, p.28) . Staff estimates of the "high-employment fiscal surplus or deficit" are reported in Greenbook forecasts since April 1970 to measure changes in discretionary fiscal policy, based on the methodology suggested in Okun and Teeters (1970), but estimates of high-employment GNP are not recorded in 1970s Greenbook forecasts or used to gauge inflationary pressure.

This paper provides measures of central bank real-time estimates of the natural rate for unemployment since 1970. The estimates are based on staff forecasts presented to the FOMC before policy meetings and recorded in the Greenbook (GB), the staff briefing document. Greenbook data provide several advantages over other real-time data sources. The multiperiod forecasts in a Greenbook provide repeated observations of predictions by the implicit forecast model of that Greenbook. Importantly, Greenbook forecasts provide measures of real-time central bank *perceptions* that are not evident in real-time data releases. Thus, use of Greenbook forecast data provides sufficient summary measures of the ex ante information of forecasters, reduces model identification issues, and eliminates estimation biases due to unanticipated disturbances in ex post data.

Despite the potential advantages, previous studies have limited their analysis to subsets of the Greenbook data. Data are typically excluded because the reporting format of the dataset presents several computational challenges, including variations in the number of Greenbooks per year, differing forecast horizons per Greenbook, and the influence of judgmental add factors on near-term forecasts. An advantage of the methodology outlined in this paper is that it allows examination of the entire data set rather than arbitrary selections of data subsets.

The next two sections of the paper lay the framework for subsequent empirical analysis. Section 2 sketches an NK model with time-varying natural rates. Determinants of the natural rates and an NK variant of Okun's Law are derived in the appendix. Section 3 introduces the organization of historical multiperiod forecasts presented to the FOMC, and discusses time series specifications used to estimate time-varying natural rates. Section 4 presents estimates of the central bank evolving perceptions of the natural rate of unemployment. Section 5 concludes.

2 Atheoretic and theory-based characterizations of natural rates

An obstacle to explicit consideration of natural rates is that equilibria of macroeconomic variables are not ordinarily observable. Consequently, a model is required to identify the dynamic equilibria of variables. Two types of dynamic equilibria are common in macroeconomic analysis.

In the case of anchoring long-run forecasts, such as implicit in modelling long-maturity bond rates, the *asymptotic* natural rate anchor for forecasts of the real interest rate, ρ_t, for example, is the infinite-horizon forecast, $\bar{\rho}_t = \lim_{k\to\infty} E_t\rho_{t+k}$.[6] Asymptotic natural rates are often measured by the limiting forecasts of atheoretic time series models, such as the natural rates constructed for VAR models in Cogley and Sargent (2005). In univariate analysis, such as the example of the real interest rate, the asymptotic natural rate of a stationary real rate is a constant, often approximated by a long-sample mean. Alternatively, if the real interest rate contains a unit root, the asymptotic natural rate will vary in each period and can be represented by a moving average of recent observations, which includes a random walk as a special case.[7]

In analyses of structural model dynamics, however, the concept often associated with natural rate deviations or "gaps," follows that of Frisch (1936) where *instantaneous* natural rates of macro models are, by construction, equilibria for a subset of variables and conditioned on the current values of the remaining states.[8] In this approach, even if the real interest rate is stationary, for example, the associated instantaneous natural rate may vary from period-to-period as it is conditioned on existing realizations of hidden or unobservable variables such as current capital, productivity growth, and preferences of agents. Another feature of instantaneous natural rates is that the relevant structural model will generally impose relationships among the natural rates of the state variables. Consequently, time variation in the natural rate of the real interest rate, for example, may not be independent of variations in the natural rate of output or employment.

Natural rates are a familiar feature of New Keynesian macro models, as in the following

[6] Kozicki and Tinsley (2001a, 2001b) associate the term "endpoints" with long-run asymptotic natural rates.

[7] Nonstationarity is not always due to unit roots, and Kozicki and Tinsley (2001a) explore time variation in asymptotic natural rates due to infrequent shifts of model parameters.

[8] A notable example is Woodford's (2003) revival of the Wicksellian natural rate of interest under flexible pricing.

representative example:

$$\begin{aligned} y_t - \bar{y}_t &= E_t\{y_{t+1} - \bar{y}_{t+1}\} - a_2 E_t\{\rho_t - \bar{\rho}_t\} + \epsilon_{y,t} && (1) \\ \pi_t - \bar{\pi}_t &= b_1 E_t\{\pi_{t+1} - \bar{\pi}_{t+1}\} + b_2(y_t - \bar{y}_t) + \epsilon_{\pi,t} && (2) \\ r_t &= \bar{\rho}_t + \bar{\pi}_t + c_2(\pi_t - \bar{\pi}_t) + c_3(y_t - \bar{y}_t) + \epsilon_{r,t} && (3) \\ y_t - \bar{y}_t &= -a'(u_t - \bar{u}_t) && (4) \end{aligned}$$

where the household demand for log output, y, and the inflation consequences of Calvo-type delayed price adjustments by monopolistic firms, π, are represented by equations (1) and (2).[9] Equation (3) is a description of the policy rate, r, controlled by the central bank. In standard formulations, all parameters are nonnegative, the expectations operator, $E_t\{\,.\}$, denotes rational (model consistent) expectations, and overbars denote natural rates.[10]

The natural rate convention is a convenient way to focus on short-run responses of the macro economy. But the cost of this simplification is that the natural rates, especially if conditioned on unobservable variables, are not likely to remain constant for more than short intervals. Anticipating examination of time-varying natural rates, the natural rates of output, $\bar{y}_t$, inflation, $\bar{\pi}_t$, and the real interest rate, $\bar{\rho}_t$, are assigned time subscripts in the model above. Theoretical determinants of the natural rates in a representative New Keynesian model are discussed in the appendix.

Equation (4) is a New Keynesian version of Okun's Law to translate output gaps to unemployment gaps. Use of this relationship results in equations explicitly involving the unemployment rate, u_t, and the unemployment natural rate, $\bar{u}_t$—variables more frequently emphasized in historical policy deliberations than the output gap. For example, variants of

[9]Microfoundations of IS and Phillips equations, such as (1) and (2), are discussed in Woodford (2003). Equation (2) is an approximation of an NK inflation equation when $\bar{\pi} > 0$, as shown in Kozicki and Tinsley (2002b).

[10]While approaches to modelling dynamics and expectations have evolved in recent decades, specifications of many equilibrium relationships in macro models have remained relatively stable since the 1970s, such as log-linear production functions and vertical long-run Phillips curves. By contrast, the NK model is an invention of the mid-1990s, so it may seem historically inaccurate to impose NK equation formats on real-time forecast data. However, intertemporal first-order conditions have been used in economics since Roos (1927); additional historical references and examples from the 1970s may be found in Tinsley (1970), Aoki (1976), and Holly, Rustem and Zarrop (1979).

the resulting unemployment-based Phillips curve,

$$\pi_t - \bar{\pi}_t \;\; = \;\; b_1 E_t\{\pi_{t+1} - \bar{\pi}_{t+1}\} - b_2 a'(u_t - \bar{u}_t) + \epsilon_{\pi,t} \tag{5}$$

are used in section 4 to identify central bank estimates of the natural rate for unemployment.

A versatile approach to estimating time-varying natural rates uses time-varying parameter (tvp) specifications. A simple atheoretic model of unemployment, often associated with asymptotic natural rate estimates, is the pth-order autoregression,

$$u_t \;\; = b_{1,t} + b_{2,t}u_{t-1} + \sum_{j=1}^{p-1} b_{j+2,t}\Delta u_{t-j} \tag{6}$$

where the natural rate can be identified by $\bar{u}_t = \frac{b_{1,t}}{1-b_{2,t}}$, for $0 \leq b_{2,t} < 1$.[11]

A time-varying structural model, associated with instantaneous natural rate estimates, is the unemployment gap variant of the pricing equation in equation (5)[12]

$$\pi_t \;\; = \;\; b_{1,t} + E_t\{\pi_{t+1}\} + b_{2,t}u_t \tag{7}$$

where the natural rate of unemployment can be identified by $\bar{u}_t = -\frac{b_{1,t}}{b_{2,t}}$ for $b_{2,t} \neq 0$.

Both models can be represented in the general specification,

$$y_t \;\; = x_t'\beta_t \tag{8}$$

where the following definitions

$$\begin{aligned} x_t' &\equiv [1, u_{t-1}, \Delta u_{t-1}, \ldots, \Delta u_{t-p+1}] \\ x_t' &\equiv [1, E_t\pi_{t+1}, u_t] \end{aligned} \tag{9}$$

are, respectively, the relevant x vectors for the autoregression equation (6) and the structural equation (7).

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[11]Note that (6) does not contain an explicit additive disturbance. In this paper, all variables on both sides of the equal sign are forecasts, not realizations. Consequently, variation in the intercept is attributed to the time-varying natural rate. Stochastic measurement errors are discussed below.

[12]Equation (7) is a simplified version, where $E_t\bar{\pi}_{t+1} = \bar{\pi}_t$ and the coefficient of expected inflation, b_1, in equation (5) is set to unity. The latter is is a reasonable approximation if the household discount factor, ß, is near one, see Table A1 in Kozicki and Tinsley (2002b).

3 Estimating time-varying natural rates in multiperiod forecast data

This section discusses an organization of the data set of real-time central bank predictions that takes advantage of the multiperiod forecast format, and reviews the models used to characterize time-variation in the perceived natural rate of unemployment.

The historical record of central bank forecasts has notable advantages for estimating the associated forecast model. Each forecast document has multiperiod forecasts, providing multiple observations on the predictions of the forecast model of that document. Also, forecasts are often available for both dependent variables and standard model regressors. Thus, regressors are free of simultaneous equation bias that is attributable to unobserved shocks in current and future periods.[13]

In addition to the bookkeeping complications inherent in any real-time analysis, the real-time data used here present certain computational challenges, such as variations in forecast horizons across policy meetings and different frequencies of policy meetings per year. Also, because the forecast data are assumed to be sufficient statistics for all information accessed by forecasters, they are likely to include effects of short-run judgemental "add factor" adjustments which are not systematically recorded in redacted FOMC documents.

Apart from theoretical reasons for anticipating time variation in natural rates, as illustrated in the appendix, there are several reasons to expect time-varying *perceptions* of natural rates in real-time forecast models, including expanding samples and rotations in staff forecast assignments. In the absence of strong ex ante priors on the nature of central bank perceptions, we adopt an agnostic approach to estimating time variation in the central bank perception of the natural rate, and use both atheoretic and structural model specifications. As will be demonstrated, structural equation estimates of the perceived natural rate of unemployment are sensitive to alternative specifications of the central bank views of inflation expectations of private agents.

[13]Of course, simultaneous equation bias can still occur if significant forecast model arguments that affect both dependent variables and explanatory regressors are omitted.

3.1 *Exploiting the multiperiod format of Greenbook forecasts*

An important advantage of Greenbook data is that they are sufficient summaries of information used by the central bank to generate its real-time forecast of current and future economic activity. Consequently, all data used in this paper are drawn from historical Greenbooks.

The Greenbook is a staff briefing document presented to FOMC members before a policy meeting of the FOMC. Part II contains background analyses of recent economic and financial data, and Part I presents the staff multiperiod forecast of economic activity. The baseline Greenbook forecast is a "judgemental" forecast. Components of the forecast are selected in a series of meetings by the senior staff and sectoral specialists, who prepare initial projections for their area of expertise. Although forecast preparation meetings include forecast inputs from economy-wide econometric models, such as the quarterly MPS model (used from the late-1960s through the mid-1990s),[14] the dominant inputs are sectoral forecasts from staff specialists. In addition to monitoring a broader set of high-frequency data releases than are incorporated in the economy-wide quarterly model, each specialist generally considers forecasts from a range of alternative sectoral models before formulating a sectoral forecast.

The baseline Greenbook forecast is considered the modal, or most-likely, outcome, given recent policy decisions and objectives. Forecast assumptions conditioned on perceived current policy and objectives include the senior staff's judgement of likely outcomes in financial markets over the forecast horizon, such as the behavior of intermediate- and long-term interest rates. As of mid-2005, these forecast conditioning assumptions have not been redacted by the FOMC Secretariat.

Because the Greenbook forecast model is judgemental, its equations are not formally documented. Thus, an econometric reconstruction of the implicit macro model underlying GB judgemental forecasts confronts many of the same problems as do estimations of macro models from government agency measurements of economic activity. There are some advantages, however, including real-time records of agency measurements and the

[14]Although forecasts by the staff quarterly model are referenced as a benchmark check on judgemental forecasts, the primary FOMC products of the quarterly model are simulations of alternative forecast scenarios and policy options that differ from the assumptions of the baseline GB forecast.

judgemental multiperiod forecasts generated for each Greenbook. For example, the Greenbook forecast generated in period t generally contains not only staff forecasts of π_t and u_t, but also the current forecast of next quarter's inflation, $E_t\pi_{t+1}$, as would be required by the structural equation (7).

The conditional information structure of the FOMC Greenbook briefings can be illustrated by more detailed time subscripting. The forecast of a variable in the current quarter, y_t, reported by a Greenbook generated in t_g is represented by

$$y_{t|t_g} = x'_{t|t_g}\beta_{t|t_g} \tag{10}$$

where the t_g date is contained in quarter t $(t-1 < t_g < t)$.[15] The $\beta_{t|t_g}$ vector contains the coefficients of a linear approximation of the forecast model consistent with the Greenbook forecasts in t_g.

As in (8), the $x_{t|t_g}$ vector contains the explanatory variables, which may contain a unit intercept and variables recorded in the current Greenbook. However, in contrast to the general definitions in (9), the measurements here are drawn only from the Greenbook in t_g, including estimates of activity for periods that lead or lag the dependent variable, $y_{t|t_g}$. Thus, the information structure of the x vectors for the autoregression equation (6) and the structural equation (7) is

$$\begin{aligned} x'_{t|t_g} &= [1, u_{t-1|t_g}, \Delta u_{t-1|t_g}, \ldots, \Delta u_{t-p+1|t_g}] \\ x'_{t|t_g} &= [1, \pi_{t+1|t_g}, u_{t|t_g}] \end{aligned} \tag{11}$$

where all variables referenced in the x vector denote Greenbook forecasts or real-time measurements, not retrospective measurements.

An advantage of access to contiguous multiperiod forecasts in each Greenbook is that iterative forecast functions are not required to generate future forecasts of dependent and explanatory variables. Rather, the forecast of the dependent variable in the $t+h$ period of the forecast horizon of the current Greenbook can be represented by

$$y_{t+h|t_g} = x'_{t+h|t_g}\beta_{t|t_g}, \qquad h = 0, \ldots, H_{t_g} \tag{12}$$

[15]Data on National Income and Product Accounts (NIPA) are forecast until published by the Bureau of Economic Analysis. The advance release on activity in a quarter is available roughly one month after the close of a quarter. Greenbooks often incorporate forecasts of NIPA revisions that occur with the subsequent preliminary and final releases of NIPA data.

where H_{t_g} indicates the forecast horizon for the Greenbook in t_g. Here, the $x_{t+h|t_g}$ vector contains the same explanatory variables as in $x_{t|t_g}$, except each variable is advanced by h periods. Note also that the coefficients of the forecast model, $\beta_{t|t_g}$, remain in a linear format, rather than the usual multiplicative form associated with forecast functions.

An important assumption of this paper, illustrated in equation (12), is that the same model is used to predict the multi-period forecasts in a given Greenbook, $\beta_{t+h|t_g} = \beta_{t|t_g}, h \geq 0$. In other words, the parameters of the linear forecast model, $\beta_{t|t_g}$, are assumed to be invariant across all forecast periods within the forecast horizon, $h = 0, 1, \ldots, H_{t_g}$. This is not always a tenable assumption, in part due to the potential for extensive judgemental adjustments of forecasts in initial periods of the forecast horizon, and an adjustment for this is discussed later.

However, the structure of the Greenbook forecast model, encapsulated in the parameter vector, $\beta_{t|t_g}$, may vary in succeeding Greenbooks or calendar quarters. An incomplete list of reasons for variation in the Greenbook forecast model includes: new developments in economic or econometric theories of macro modelling; recent forecast performances of competing sectoral models; rotation of staff forecast assignments; replacement of senior staff and forecast coordinators; and inquiries from members of the FOMC.

The sample used in this paper includes the 126 quarters from 1966Q3 through 1997Q4, a span that contains 315 Greenbooks. Greenbook forecasts were more frequent in early years of the sample, but the frequency has remained at eight per year since 1981. The average forecast horizon, $H + 1$, is 5.7 quarters. However, as shown in Figure 1, the forecast horizon was much shorter in the 1960s and early 1970s, sometimes including only the current quarter, $H = 0$. Typically, the forecast horizon is longest for Greenbooks prepared before the semi-annual congressional testimony of the Fed chairman, and then diminishes in the next few Greenbooks with the passing of each subsequent quarter.

Finally, as the originating date of a representative GB forecast is contained in quarter t, the conditioning subscript, t_g, is generally dropped in subsequent discussion to ease notation.

3.2 *Alternative time-varying parameter (tvp) specifications of natural rates*

Stacking the multiperiod forecasts associated with Greenbooks in period t gives the *measurement* equation,[16]

$$\begin{aligned} y_t &= \Xi_t \vec{\beta}_t + a_t \\ &= [\tilde{X}_t, X_t] \begin{bmatrix} \tilde{\beta}_t \\ \bar{\beta} \end{bmatrix} + a_t \end{aligned} \tag{13}$$

where a_t is a vector of measurement errors to account for transitory forecast arguments not recorded in the Greenbooks. The dependent variable, y_t, and the measurement error, a_t, are equal length vectors to account for the number of observations in period t. The matrix of regressors, $[\tilde{X}_t, X_t]$, conforms to the dimensions of vector y_t and the parameter vector, $\vec{\beta}_t$. The matrix X_t contains k regressors, including a unit vector. The $\vec{\beta}_t$ vector is partitioned into a $k \times 1$ fixed vector, $\bar{\beta}$, and a $\tilde{k} \times 1$ time-varying vector of deviations, $\tilde{\beta}_t$, whose unconditional mean is zero. The effective time-varying coefficients of the forecast model, β_t, are obtained by summing the fixed and time-varying deviation vectors

$$\beta_t \equiv \bar{\beta} + \begin{bmatrix} \tilde{\beta}_t \\ 0_{k-\tilde{k}} \end{bmatrix} \tag{14}$$

where $0_{k-\tilde{k}}$ is a $(k - \tilde{k}) \times 1$ zero vector. Note that $\tilde{k} < k$ if the last $k - \tilde{k}$ elements of β_t are invariant over time.[17] The measurement error is normally distributed, $a_t \sim N(0, R_t)$, where $R_t \equiv \sigma_a^2 I$.

The format of the *transition* equation is

$$\vec{\beta}_t = \Phi \vec{\beta}_{t-1} + e_t \tag{15}$$

where the partitions of the transition matrix and the transition shock vector are

$$\Phi = \begin{bmatrix} \tilde{\Phi} & 0 \\ 0 & I_k \end{bmatrix}, \text{ and } e_t = \begin{bmatrix} \tilde{e}_t \\ 0 \end{bmatrix}. \tag{16}$$

[16]Each calendar quarter contains more than one Greenbook. To provide some smoothing of β_t estimates and to facilitate reporting at a fixed frequency, forecast observations from the Greenbooks of a single calendar quarter are stacked in quarterly measurement vectors or matrices.

[17]The matrix $\tilde{X}_t$ is a subset of X_t when $\tilde{k} < k$. Also, the t subscript is generally reserved for either Greenbook periods, $t = 1, \ldots, T_g$ or calendar quarters, $t = 1, \ldots, T_q$. In some instances, it is useful to refer to components of a single forecast in the stack of all Greenbook forecasts by the τ subscript, $\tau = 1, \ldots, T$, where $T = T_g + \sum_{t_g=1}^{T_g} H_{t_g}$.

The nonzero transition shocks are also normally distributed, $\tilde{e}_t \sim N(0, \tilde{Q})$.

We consider three different specifications of time-varying regression parameters that have appeared in the macro literature. Each amounts to different restrictions on the dimension of the time-varying partition, $\tilde{\beta}_t$, and on the eigenvalues of the associated transition matrix, $\tilde{\Phi}$. The competing specifications make different assumptions about the scope and persistence of time variation in the structure of the forecast model. As we did not initially have strong priors over these alternatives, all three specifications are estimated in each application.

random walk intercept (RWI)

A widely-used specification of time-varying natural rates in recent macro papers rests on the assumption that the intercept follows a random walk, $\beta_{1,t} = \beta_{1,t-1} + e_{1,t}$.[18] As noted by Stock and Watson (1998), if the variance of the random walk transition shock, $\sigma^2_{e_1}$, is small, the maximum likelihood estimate may be biased toward zero. Consequently, we use the Stock and Watson median-unbiased estimator of the variance of the transition shock, $\sigma^2_{e_1} = \hat{\nu}^2 \sigma^2_u$, where u denotes residuals of the fixed-coefficient regression, $y_\tau = X_\tau \bar{\beta}$, and ν is a function of a changepoint test, $supF_{\tau'}$, for intercept shifts over the middle (70%) observations of the sample, $\tau_{15\%} < \tau' < \tau_{85\%}$.[19] After obtaining the median-unbiased estimate of the transition variance, the remaining parameters, such as the time-varying intercept and fixed slope coefficients, are estimated by recursive filtering and smoothing equations.[20].

Although means and sampling errors are estimated for the remaining regression coefficients, $\bar{\beta}_i, i = 2, \ldots, k$, the fixed partition of the random walk intercept is the initial condition, $\beta_{1,t_0} = \bar{\beta}_1$. To provide an approximate comparison with estimates of mean coefficients from alternative specifications, the finite sample average of the random walk

[18]Applications that employ random walk intercepts to estimate a time-varying natural rate for unemployment include Staiger, Stock and Watson (1997), Gordon (1997), and Orphanides and Williams (2002). Time-varying intercepts have also been applied to estimation of the natural rate of the real interest rate, such as Laubach and Williams (2002), Kozicki (2004), and Clark and Kozicki (forthcoming).

[19]Following Stock and Watson (1998), $\nu = \frac{\lambda}{T}$, where the probability of a zero pileup by maximum likelihood moves inversely with the local-to-zero parameter, λ, vid. Stock and Watson (1998, Table 1).

[20]Reviews of Kalman filters include Harvey (1989) and Shumway and Stoffer (2000). In imitating real-time forecasting procedures, it is not uncommon to use filtered estimates, such as $\bar{u}_{t|t}$, so as to not overstate the information available to forecasters. However, the information available to Greenbook forecasters is fixed in the recorded forecasts, and we use smoothed natural rate estimates, $\bar{u}_{t|T_q}$ to reduce the inference errors of the constructions.

intercept estimates is reported as the estimated mean, $\hat{\bar{\beta}}_1 \equiv \frac{1}{T_q}\sum_{t=1}^{T_q}\hat{\beta}_{1,t}$, along with the estimated standard deviation of this finite sample average.[21]

random walk coefficients (RWC)

In the examples of autoregressive equations, such as (6), or structural equations, such as (7), recovery of the natural rate requires transforming the estimated intercept by one or more slope coefficients of regressors. However, even modest variations in slope coefficients can imply large changes in natural rates. To allow for the possibility of economically meaningful variations in slope coefficients, the second approach extends the random walk specification to the slope coefficients of regressors, $\beta_{k,t}, k = 2, \ldots, \tilde{k}$.

Estimation is similar to that for the random walk intercept case, with appropriate modifications for the larger dimension of the $\tilde{\beta}_t$ vector. The transition matrix is an identity matrix, $\tilde{\Phi} = I_{\tilde{k}}$. Following Boivin (forthcoming), the $\tilde{k} \times \tilde{k}$ variance matrix of the transition shocks is pre-estimated using a median-unbiased estimator, $\tilde{Q} = (\hat{\nu})^2(\frac{1}{T}\tilde{X}'\tilde{X})^{-1}\Omega(\frac{1}{T}\tilde{X}'\tilde{X})^{-1}$ where $\tilde{X}$ denotes the full sample column stack of $\tilde{X}_t$. As the format of potential heteroskedasticity in the residuals due to time-varying coefficients is not known prior to model estimation, a heteroskedasticity-consistent estimator is used, $\Omega = \frac{1}{T}\tilde{X}'D\tilde{X}$, where the nonzero elements of the $T \times T$ diagonal matrix are: $D_{\tau,\tau} = \frac{T}{T-\tilde{k}}\hat{u}_\tau^2$. Again, ν is determined by the $supF_{\tau'}$ test statistic, formulated for shifts in $\tilde{k}$ elements of the coefficient vector, $\bar{\beta}$.

stationary coefficients (SC)

There are several pragmatic advantages to random walk parameter specifications, including parsimonious identification of the transition matrix, $\tilde{\Phi}$. However, this is only a tractable approximation with possibly unrealistic implications for model parameters. In the current context, reservations extend to the assumption that all changes in coefficients are permanent and to the assumption that coefficients can evolve over time without finite bounds.[22] If the evolution of empirical macro models is broadly viewed as an example of statistical learning of

[21]For the random walk specification, $b_t = b_{t-1} + e_t$, denote the sample average as, $\bar{b} = \frac{1}{T_q}\sum_{t=1}^{T_q} b_t$. For a known initial condition, b_0, the variance of the sample average is $\sigma_{\bar{b}}^2 = \frac{1}{T_q^2}\sum_{i=1}^{T_q} i^2\sigma_e^2 = \frac{2T_q^2+3T_q+1}{6T_q}\sigma_e^2$, which can be large in lengthy samples.

[22]Postwar aggregate unemployment and inflation rates in the US have remained within relatively narrow ranges, and empirical tests supporting unit root behavior appear to be sensitive to the choice of sample.

stable underlying relationships, it seems reasonable to allow for the possibility that forecast model parameters may be approximated by autoregressions about a central tendency.

Principal differences in the stationary coefficient specification are that both the transition matrix, $\tilde{\Phi}$, and the covariance matrix of transition shocks, $\tilde{Q}$, are estimated by maximum likelihood.[23] Access to maximum likelihood estimation also permits likelihood tests of competing specifications with stationary coefficients.

Finally, in evaluating effects of stochastic parameter variation, it is useful to gauge the relative economic importance of estimated movements in a parameter. Graphs of the estimated trajectories of slope coefficients are not always informative because, as noted earlier, the effective contributions of slope parameters depend on regressor scales. In the case of stationary parameters, the steady-state variance of the dependent variable due to variation in β is

$$\mathrm{var}(y) \quad = \quad \tilde{X}V(\beta)\tilde{X}'$$

where elements of the $\tilde{k} \times \tilde{k}$ steady-state covariance of the stationary parameters, $V(\beta)$, can be recovered from the column stack

$$\mathrm{vec}V(\beta) \quad = \quad [I_{\tilde{k}^2} - \tilde{\Phi} \otimes \tilde{\Phi}]^{-1}\mathrm{vec}\tilde{Q}.$$

A steady-state variance decomposition for the stationary coefficient specification

$$\mathrm{vardecom}(\beta_j) \quad \equiv \quad \frac{100}{\mathrm{var}(y)}[\tilde{X}_j^2 V_{jj} + \frac{1}{2}\sum_{i \neq j} \tilde{X}_i \tilde{X}_j V_{ij}] \tag{17}$$

is reported in the tables of the next section.[24]

4 Estimates of the natural rate for unemployment

This section briefly reviews several influential estimates of the natural rate of unemployment and inferences drawn from these constructions. These estimates are then compared with

[23] To initialize maximum likelihood estimation in the stationary coefficients specification, each diagonal element of $\tilde{\Phi}_0$ is set to .8 and $\tilde{Q}_0$ is set to the median-unbiased estimator used in the random walk coefficients specifications.

[24] Equation (17) assigns half of the covariance, V_{ij}, to β_i and β_j, following Swamy and Tinsley (1980). Consequently, some elements of the variance decomposition may be negative under this convention.

alternative estimates of $\bar{u}_t$ based on historical Greenbook forecasts using tvp regression models.

4.1 *Representative estimates of the natural rate of unemployment*

A real-time estimate of the quarterly unemployment rate, u_t, and several estimates of the natural rate for unemployment, $\bar{u}_t$, are charted in Figure 2.[25] Natural rate estimates based on retrospective samples of historical data are represented by Staiger, Stock and Watson (1997)[26] and the Congressional Budget Office (2004). Both peak in the 1970s, with the CBO estimates averaging a half-point below the Staiger et al. estimates over the twenty-year sample shown.

As shown in Figure 2, there is much less agreement among real-time estimates of the behavior of the natural rate in the 1970s. The HP filter estimate is applied to the real-time estimates of the unemployment rate, and rises from about 4 percentage points to the level of the Staiger et al. retrospective estimate by the mid-1970s.[27] The smoother short series ending in 1976 is from Perry (1977). The remaining short series, ending in 1972, 1975, 1976, and 1978, are estimates of the natural rate of unemployment obtained by reverse engineering the real-time output gaps in Orphanides (2003a) using Okun's Law.[28]

These real-time estimates of the natural rate for unemployment support the view that monetary policy could have mistakenly eased in the 1970s, but with considerable variation in the size of the implied errors. Using the CBO retrospective estimate as an estimate of the "true" unemployment natural rate, underestimates of the natural rate range from 3

[25] Annual dates on the horizontal axis denote the start of a year. As with other real-time estimates of historical variables used in this paper, the real-time estimate of the unemployment rate in period t, u_t, is drawn from the last Greenbook of the second following quarter, $t+2$.

[26] Interpolated from first-quarter estimates provided on http://www.wws.princeton.edu/ mwatson/.

[27] The smoothing parameter for the filter of quarterly observations is 1600. Of course, the two-sided HP filter uses forward information not accessible in real time.

[28] We are indebted to Athanosios Orphanides for supplying the real-time output gap data. The estimates in Figure 2 are obtained by $\bar{u}_t = u_t + \frac{1}{a'}(y_t - \bar{y}_t)$, using an Okun's Law coefficient of $a' = 2.2$. Although Okun (1962) initially suggested estimates of a' around 3.2, Tatom (1978) indicates an estimate around 2.2 is consistent with 1955Q1-1969Q4 and 1955Q1-1977Q3 samples.

percentage points for the real-time estimates ending in 1976 to 1 percentage point for the real-time estimates ending in 1978. These errors are huge; a 3 percentage point error for a retrospective unemployment natural rate around 6 percent results in an underestimate of 50%. However, the output gaps used in the reverse engineering undoubtedly reflect also real-time errors in measuring GNP.

The conventional view of the natural rate for unemployment in the 1960s was 4 percent so an upper bound for natural rate errors in the first-half of the 1970s is about 2 percentage points (a 33% error). Assuming Perry (1977) represents a conventional estimate of the natural rate by the mid-1970s, an upper bound for underestimates in the second-half of the 1970s is 1 percentage point (a 17% error).[29] This suggests that if the FOMC had been using a stable Taylor rule, as empirically supported by Orphanides (2002, forthcoming), errors in estimating the natural rate of unemployment could have mistakenly reduced the policy rate by 100 to 200 basis points.[30]

As noted earlier, standard models for predicting inflation in the 1970s were variations of a Phillips curve with the unemployment gap, $u_t - \bar{u}_t$, as an explanatory regressor. Romer and Romer (2002) have suggested that the "very low estimates of normal unemployment that characterized the economic beliefs of policymakers in the 1960s and 1970s would naturally tend to lead policymakers to systematically underpredict inflation." The forecast errors of Greenbook current-quarter predictions of GNP inflation are charted in Figure 3.[31] The mean of the inflation prediction errors is near zero, 0.02%, over a 1969Q1 - 1997Q2 sample. The largest inflation underpredictions in the 1970s follow crude oil price shocks in 1973-4 but precede oil shocks in 1979-80. The mean prediction error is positive, 0.83%, in the first-half of the 1970s but negative, -0.14%, in the second-half.

[29] The Perry (1977) estimate of $\bar{u}_t$ reaches 4.9 in 1976Q4. This likely overstates the error for many real-time estimates of the unemployment natural rate in the mid-1970s. Whereas Hall(1970) indicates "most economists agree that this is somewhere between 4 and 5 percent unemployment," Hall (1974) suggests that the natural rate is about 5.5 percentage points.

[30] An Okun's Law coefficient of around two implies the coefficient of a Taylor rule response to the unemployment gap is one. By contrast, Orphanides (2003a, Figure 5; forthcoming, Figure 4) suggests shortfalls of the funds rate, due to central bank real-time measurement errors, approach 600 basis points during intervals in the 1970s.

[31] Current-quarter prediction errors of real-time inflation measurements are measured by the average of the forecast error of the last Greenbook in the current quarter, t and in the preceding quarter, $t-1$.

If the inflation bias conjecture is correct, a reduction in the measurement error of the natural rate of unemployment might explain the smaller inflation forecast errors in the last-half of the 1970s. Romer and Romer suggest -.125 as an estimate of the slope of the Phillips curve. Assuming the measurement error of conventional estimates of $\bar{u}$ fell by one percentage point in the 1970s, as suggested above, this would explain only 13% of the net reversal of the mean forecast error of inflation in the first and second halves of the 1970s.

However, Romer and Romer (2002) do not rely on conventional estimates of $\bar{u}_t$, and present an innovative use of Greenbook forecasts to support their position that the central bank made large errors in measuring the natural rate of unemployment. Their constructions are based on inverting a standard "accelerationist" Phillips curve to give

$$\hat{\bar{u}}_t = \frac{1}{3}\sum_{h=0}^{2}(u_{t+h} + \frac{1}{.125}\Delta\pi_{t+h}) \tag{18}$$

where, using our earlier notation, all variables on the rhs of (18) are forecasts from the Greenbook in t.

Even though the Romer and Romer (2002) constructions in (18) use averages of the three initial forecasts in the Greenbook forecast horizon, $h = 0, 1, 2$, their estimates of $\bar{u}_t$ are quite volatile over time. Romer and Romer (2002) list averages over subperiods, which are reproduced in the second column of Table 1.[32] When contrasted with the CBO estimates of $\bar{u}_t$ in the first column of the table, these estimates suggest a more dramatic interpretation of central bank perceptions in the 1970s. In contrast to the underestimation by conventional estimates of $\bar{u}$ of about 2 percentage points in the first-half and 1 percentage point in the second-half of the 1970s, the Romer and Romer estimates imply the natural rate estimates implicit in Greenbook forecasts underestimated $\bar{u}_t$ by nearly 3 percentage points in the first-half, and overestimated by 0.9 percentage points in the subsequent sixteen quarters ending with the Miller tenure, a swing of nearly 4 percentage points in Greenbook revisions of the natural rate for unemployment.

For an initial comparison with real-time estimates, the last two columns of Table 1 list common "real-time" constructions of the unemployment natural rate based on expanding

[32]The subperiods in Table 1 correspond to the tenures of FOMC Chairmen, which included William McChesney Martin, Arthur Burns, G. William Miller, Paul Volcker, and Alan Greenspan.

sample means.[33] In each of the last two columns, the civilian unemployment rate at the start of the sample is around 4 percent, the conventional estimate of the natural rate in early postwar periods.[34] In the first two subsamples shown in Table 1, $Martin$ and $Burns_1$, the expanding sample estimates are sensitive to the sample used. For the first half of the 1970s, $Burns_1$, the underestimate of the natural rate is 1 percentage point in the third column, (A), and 1.7 percentage points in the fourth column, (B). Neither of the expanding sample estimates is able to generate the underestimate of nearly 3 percentage points indicated in the second column.

4.2 *Implicit Greenbook estimates of $\bar{u}_t$ from tvp autoregressions*

Atheoretic (asymptotic) estimates of natural rates of unemployment implied by Greenbook forecasts are provided by fourth-order tvp autoregressions, using the format in equation (6).

$$u_{t+h} = \beta_{1,t} + \beta_{2,t} u_{t+h-1} + \sum_{j=1}^{3} \beta_{j+2,t} \Delta u_{t+h-j} + a_{t+h}, \qquad h = 0, 1, \ldots, H. \tag{19}$$

In the applications summarized in the top panel of Table 2, the tvp autoregression is fit to the multiperiod forecasts of all 315 Greenbooks in the 1966Q3 - 1997Q4 sample, a total of 1784 observations.[35] After examining a number of tvp applications, our experience is that the means of the time-varying coefficients, the maximum and minimum of the implied natural rates, and the variance decompositions provide useful summary contrasts among alternative specifications. Where relevant, these statistics are shown for the three tvp specifications: the random walk intercept (RWI) model; the random walk coefficients (RWC) format; and the stationary coefficients (SC) specification.

Both mean coefficients and bounds of the natural rates are similar across the three tvp specifications in the top panel of Table 2. This is born out in Figure 4, which shows that the

[33] Hall (1999, p. 433) suggests: "There is a robust estimator of the natural rate available....As Friedman pointed, the unemployment rate fluctuates around the natural rate irrespective of the monetary regime. Hence the average of the unemployment rate is a good estimate of the natural rate."

[34] Retrospective CBO (2004) estimates of the natural rate of unemployment are 5.4 percent points for 1956Q1 and 5.7 percent for 1966Q1.

[35] Change-point tests for the unemployment autoregression imply $\lambda = 16.2$. For this level of the local-to-zero parameter, Table 1 in Stock and Watson (1998) suggests the probability of a zero pileup by maximum likelihood is less than 13%.

natural rates of the three specifications move closely together. The 70% confidence interval shown is for the stationary coefficients specification.[36] This interval is large, containing not only all three tvp specifications, but also the HP filter and CBO estimates of the natural rate for most of the sample. The SC autoregression remains stable throughout the sample, with the coefficient of the lagged level, $\beta_{2,t}$, remaining well below unity. Some of the remaining slope coefficients were rather volatile, such as $\beta_{3,t}$ which oscillated between values of .07 to .77 over the sample. However, the unemployment effects of these movements are not necessarily quantitatively important. A more informative measurement is the variance decomposition shown in the last line of the top panel of Table 2. This decomposition shows that the largest stochastic unemployment effects are those associated with movements in the two determinants of the natural rate: the intercept, $\beta_{1,t}$, and the coefficient of the lagged level, $\beta_{2,t}$.

An interesting property of the tvp natural rates shown in Figure 4 is that turning points of these constructions tend to lead those of the two-sided HP filter estimate, whose turning points are coincident with those of the real-time unemployment rate. This suggests that conventional estimators of natural rates, fit to historical data, may significantly underestimate the ability of judgemental forecasters to anticipate movements in natural rates.[37] One difference is that the tvp estimators used here are fit to multiperiod judgemental forecasts of future activity.

A second feature of the tvp natural rate estimates in Figure 4 is that they are considerably more volatile than the retrospective CBO estimate and almost as volatile as the HP filter estimate. One reason is that forecasts in the initial quarters of the forecast horizon, $h = 0, 1$, are often heavily adjusted by judgemental "add factors" to take account of information that is not contained in Greenbook records, such as high-frequency data releases or judgement about the persistence of recent forecast errors.[38] This is less of a problem for more distant

[36] The confidence interval of the natural rate is constructed by the delta method, using smoothed estimates of the covariance matrices of the coefficient vector, β_t.

[37] In analyses of incomplete or asymmetric information, statistical learning models generally impose lengthy learning lags.

[38] Although the term "add factors" suggests intercept corrections, judgemental adjustments may also reflect time-varying weights placed on competing models. Use of disparate information sources in the Greenbook forecast is discussed in Kalchbrenner and Tinsley (1977).

autoregressive forecasts in the forecast horizon because the influence of initial transitory add factors dissipate and, by construction, persistent adjustments of the forecast are captured either in the time-varying coefficients of the forecast model or in the forward forecast regressors that replace lagged real-time measurements of past behavior as h advances.

To reduce the influence of transitory adjustments in initial quarters of the forecast horizon on estimated natural rates, the lower panel in Table 2 reports the results of tvp autoregressions where the first three forecasts in a Greenbook, $h = 0, 1, 2$, are dropped from the sample. As can be seen from the earlier charting of forecast horizons in Figure 1, this requires dropping Greenbooks with short forecast horizons, $H_{t_g} < 3$, from the sample. To provide a sample with contiguous quarters, the sample span is now the 116 quarters from 1969Q1 to 1997Q4, containing 261 Greenbooks. The total number of observations for forecasts that contain the fourth quarter of the forecast horizon or later, $h \geq 3$, is 879, a reduction of about 50%.

Differences in the tvp specifications of the shorter stack of forecasts are evident in the lower panel of Table 2. The bounds of the natural rate constructions are closer together, indicating that the shorter stack reduces the influence of transient judgemental forecast adjustments.[39] Reduced effects of idiosyncratic add factors are also evident in the variance decomposition of the stationary coefficients specification, where the relative importance of intercept variations has been reduced by nearly 60%. Although not shown, the unemployment natural rate implied by the reduced stack of Greenbook forecasts is smoother and less subject to large amplitude movements than the natural rate constructions implied by the full stack of forecasts shown in Figure 4. Regardless of the forecast samples used, as indicated in the last two columns of Table 3, the natural rate constructions of either of the SC autoregressions do not support the substantial underestimates of $\bar{u}_t$ in the 1970s reported by Romer and Romer (2002), shown in the second column.

[39] As noted above, maintaining a contiguous sample for the shorter stack requires dropping the Greenbooks in 1966-68. The result is a modest increase in the minimum bound for $\bar{u}_t$. For example, when the full stack of Greenbooks is estimated over the shorter 1969Q1-1997Q4 sample, the minimum of $\bar{u}_t$ for the RWI specification increases from 5.0 to 5.3 percentage points, reducing the bounds spread from 3.5 percentage points to 3.2 percentage points. Consequently, after accounting for the shorter sample, about 85% of the reduction in the bounds spread is due to eliminating the first three forecasts of each Greenbook in the shorter stack.

4.3 *Implicit Greenbook estimates of $\bar{u}_t$ from tvp structural specifications*

This subsection explores time-varying natural rates of unemployment implied by tvp specifications of structural pricing equations. To reduce distortions associated with near-term judgemental add factors, the short stack of forecasts is used in estimation, where $h_{t_g} = 3, \ldots, H_{t_g}$.[40]

<u>*backward-looking specifications*</u>

The equation described in the top panel of Table 4 is the same as the Romer and Romer specification noted in equation (18), except that the mean slope of the Phillips equation, $\bar{\beta}_2$, is freely estimated and, in the case of the RWC and SC specifications, both the intercept and slope coefficients can vary over time. The backward-looking equation described in the bottom panel relaxes the assumption that expected inflation is a random walk and uses an $AR(2)$ autoregressive predictor of expected inflation but maintains the assumption that there is no long-run tradeoff between inflation and unemployment.[41] The mean coefficient, $\hat{\beta}_3$, of the additional lag in inflation, π_{t+h-2}, is significant with zero p-values in both the RWI and SC specifications.

Both panels of Table 4 indicate that the Romer and Romer restriction on the mean slope of the pricing equation, $\bar{\beta}_2 = -.125$, is contained within 95% confidence intervals. However, this masks large sample movements in the time-varying estimates of $\beta_{2,t}$. Evidence of this movement is seen in the variance decompositions of both panels, where movements in the slope parameter dominate the variance of the unemployment rate. Although not shown, both the RWC and SC estimates of $\beta_{2,t}$ remain around -.10 in the 1970s and then rise sharply and level off around -.05 by 1984. The sizeable reduction in the slope of the pricing equation after 1979 was a major contributor to the average increase of about 1 percentage point in constructions of the unemployment natural rate.

As shown in third column of Table 6, the backward-looking $AR(2)$ specification of the pricing equation supports underestimation of the natural rate for unemployment in the

[40]For comparability, all tvp structural pricing equations use the local-to-zero parameter: $\lambda = 8.6$.

[41]According to Enzler and Pierce (1974), macro data samples after 1971 supported a unit sum for the estimated coefficients of lagged inflation in empirical Phillips curves.

Greenbooks of the 1970s, with $\bar{u}_t$ rising from 4 percent in the early Burns years, to 4.3 percent in the Miller era, and to about 5.5 percent in the remainder of the sample. Of the equations estimated, the unemployment natural rate implied by the pricing equation with $AR(2)$ inflation expectations provides a lower-bound estimate of central bank perceptions of the unemployment natural rate in the 1970s.

forward-looking specifications

A risk of theory-based specifications is that they may not capture the undocumented relationships used by historical Greenbook forecasters. Inspection of representative staff Phillips curves from the 1970s suggests that additional regressors usually included one or more lags in the first-difference of the unemployment rate, sometimes called "speed effects," and additional lags of inflation, e.g. Enzler and Pierce (1974). One parsimonious way to capture the forecast implications of additional regressors and unknown dynamic specifications is to include forecast leads of these variables.

The tvp specifications presented in the top panel of Table 5 use the forward-looking specification

$$\pi_{t+h} = \beta_{1,t} + \beta_{2,t} u_{t+h} + \beta_{4,t} \Delta u_{t+h+1} + \beta_{5,t} \pi_{t+h+1} + (1 - \beta_{5,t}) \pi_{t+h-1} + a_{t+h} \qquad (20)$$

where $\beta_{4,t}$ captures forecasts of the forward change in unemployment, and $\beta_{5,t}$ is the weight on the forward forecast of inflation. Thus, this equation is similar to hybrid modifications of the NK pricing equation.

For all three tvp specifications in the top panel of Table 5, the mean coefficient of the unemployment rate, $\bar{\beta}_2$, is smaller in absolute value, indicating that the slope of the forward-looking Phillips curve is flatter than the backward-looking equations in Table 3. Although not shown, the coefficient of forward inflation, $\beta_{5,t}$, rises from around 0.4 in the early 1970s to around 0.6 in the mid-1980s, indicating more weight on the forward inflation forecast regressor in the 1980s and 1990s. The largest slope coefficient movements are associated with the coefficient of the forward change in the unemployment rate, $\beta_{4,t}$, which moves from negative values of around -.30 in the 1970s to positive values of around .15 by the mid-1980s. The zero crossing in 1984 also explains why the estimated mean, $\bar{\beta}_4$, is not significantly different from zero.[42]

Although the maximum and minimum bounds in Table 5 for the natural rate constructions of the RWI and SC equations are consistent with reasonable estimates of the unemployment natural rate, the bounds for the RWC specification imply implausible values. This is due to differences in the estimated behavior of the coefficient of the unemployment rate, $\beta_{2,t}$, in the random walk coefficients specification, RWC, and in the stationary coefficients specification, SC. As charted in Figure 5, the SC estimate rises towards zero, similar to the motion observed for this coefficient in the backward-looking equations. However, the RWC estimate of $\beta_{2,t}$ continues to climb and crosses zero in 1984, where the RWC equation cannot identify the unemployment natural rate. When $\beta_{2,t}$ is in the neighborhood of zero, the RWC specification generates very large positive and negative constructions of $\bar{u}_t$.

Using the stationary coefficients, SC, specifications for comparison, the lower bound for the unemployment natural rate of the backward-looking equation in the bottom panel of Table 4 is 3.8 percent whereas the lower bound in the forward-looking equation of the top panel of Table 5 is 5.5 percent. The likelihood ratio of the two equations is 4.1, favoring the forward-looking specification. Assuming uniform priors across the two equations, one way to proceed would be to construct a weighted average of the two natural rate constructions, with approximate posterior weights of 0.8 on the forward-looking estimate and 0.2 on the backward-looking estimate.

As an alternative to fixed-weight averaging, suppose both the backward-looking and forward-looking models are considered by the judgemental forecaster in a given period, where the probability of choosing the forward-looking specification is α_t, and the probability of choosing the backward-looking specification is $1-\alpha_t$. Estimates of the forecast model implied by this time-varying averaging are presented in the bottom panel of Table 5. Although realizations of α_t are not identified, most of the mean coefficients of the combined equation move in directions consistent with the likelihood ratio. For example, the mean slope of the

[42]Note that the mean of a time-varying parameter can be zero and yet changes in the parameter can significantly contribute to the variation of the dependent variable. The variance decomposition indicates that this is not the case here but we have retained the first-difference of the unemployment rate because of the prior evidence that staff Phillips equations used this regressor.

Phillips curve, $\bar{\beta}_2$, is closer to the mean slope of the forward-looking specification, and the mean coefficient on the lagged difference of inflation, $\bar{\beta}_3$, is about one-third the size of the mean estimate of the backward-looking specification. As with the forward-looking equation, the natural rate bounds implied by the RWC specification are unrealistically large due to the same trending behavior in the estimate of $\beta_{2,t}$.

The unemployment natural rate estimates implied by the tvp averaging equation from Table 5 and by the $AR(2)$ expectations equation from Table 4, are graphed in Figure 6. In contrast to the unemployment natural rate estimates of the atheoretic autoregressions shown earlier in Figure 4, the amplitude of each natural rate estimate from the structural equations is less than the HP filter estimate. From the early 1970s through the early 1980s, both estimates are relatively flat with the natural rate of the $AR(2)$ equation averaging around 4.3 percent and the natural rate estimate of the tvp averaging equation a percentage point higher. Both estimates then rise above the CBO retrospective estimate in the late 1980s, before turning down in the mid-1990s.[43]

Subsample averages of the two structural equation estimates of the natural rate for unemployment are listed in the last two columns of Table 6. Relative to the CBO (2004) retrospective estimates, both of the structural equation estimates confirm central bank underestimation of the natural rate for unemployment in the 1970s. The average underestimate in the 1970s of the $AR(2)$ equation is about 2 percentage points (about a 33% error), smaller than the nearly 3 percentage point (a 50% error) underestimation by the Romer and Romer (2002) natural rate estimates in the first-half of the 1970s.

However, the statistical evidence supports the estimates provided by the tvp averaging equation. As indicated by the last column of Table 6, the natural rate estimation errors of the tvp averaging equation are comparatively modest, with underestimations of about 1 percentage point (a 17% error) in the 1970s and about a 0.5 percentage point (an 8% error) during the tenure of Chairman Volcker.

Interestingly, the tvp averaging estimates in the last column of table 6 are not substantially different from the simple expanding sample mean estimates (A) shown earlier

[43]Whereas the CBO (2004) retrospective estimates of the unemployment natural rate are below 6 percentage points by the early 1990s, the estimates of the tvp averaging equation remain above, similar to the natural rate estimates reported in real-time by Weiner (1993).

in Table 1. However, the two series diverge in the mid-1990s, a period marked by well-known downward revisions in the central bank real-time estimates of the natural rate of unemployment. In contrast to the absence of reductions in the expanding sample estimates in Table 1, both Greenbook-based estimates fall markedly after the mid-1990s, as shown in Table 6. It is encouraging to note that the tvp averaging natural rate estimate of 5.6 percent reported for the $Greenspan_2$ subsample in Table 6, covering 1996Q1-1997Q4, is identical to the unemployment natural rate assumed in the Greenbook for the February 1997 FOMC meeting, reported in Svensson and Tetlow (2005).

5 Concluding remarks

Although natural rates are important arguments in short-run macro models, they are not ordinarily observable by either private agents or central banks. To complement a growing literature that warns against the design of monetary policies that reference deviations of variables from real-time estimates of natural rates, this paper explores central bank historical perceptions of the natural rate for unemployment.

As discussed in section 2, the concept of natural rates is not unambiguous and depends on the context of use. A distinction is drawn between *asymptotic* natural rates that anchor long-horizon expectations and *instantaneous* natural rates that are associated with conditional equilibrium deviations of selected macro variables. The former are often estimated by autoregressive time series models and the latter by structural equation models.

A number of papers have suggested that the US central bank severely overestimated potential output or, equivalently, substantially underestimated the natural rate of unemployment in the 1970s, but there appears to be little direct empirical evidence for this claim. This paper estimates the natural rate of unemployment that was implied by the real-time, multiperiod macro forecasts presented to the FOMC, the policy committee of the US central bank.

As noted in section 3, fitting models to real-time, multiperiod forecasts has several advantages, including multiple observations on predictions generated by the effective forecast model used in each period and the elimination of estimation biases associated with unobserved future shocks over the forecast horizon. A tractable framework for organizing the multiperiod forecast data is suggested that is amenable to time-varying parameter estimation of changes in the implied forecast model over time.

Relative to retrospective estimates of the natural rate of unemployment by CBO (2004), the central bank perceptions of the unemployment natural rate estimated by atheoretic tvp autoregressions show no evidence of systematic underestimation. The natural rate estimates implied by tvp structural equations are sensitive to specification of the central bank characterization of private agent inflation expectations. The structural equation format best supported by the data indicates central bank underestimation of the unemployment natural rate in the 1970s was modest, generally not more than one percentage point, with a retrospective natural rate error about one-third the size of that suggested in previous literature.[44]

[44] Greenbook-based estimates of the natural rate of unemployment are used in a companion paper on the evolution of FOMC policy and of the effective central bank target for inflation, Kozicki and Tinsley (2005).

Appendix: Determinants of natural rates in an NK model

The household demand for output, equation (1), and the inflation implications of pricing by firms, equation (2), are consistent with the following assumptions:

The representative household is infinitely lived and, in each period, consumes C_t, an index of differentiated commodities using a CES aggregator. Purchases of consumption, bonds, and equity are financed by household income from financial assets and compensations for supply of labor, N, and labor utilization, X, to firms, where the range of labor supply is $0 \leq N \leq 1$. The allocation of income and the supplies of labor and utilization are determined by maximizing a discounted sum of expected utility,

$$E_t\{\Upsilon_t\} \quad = \quad E_t\{\sum_{i=0}^{\infty} ß^i \upsilon_{t+i}\} \tag{21}$$

given the fractional discount factor, ß. Household utility in t is represented by a CRRA specification,

$$\upsilon(Q_{t+i}, C_{t+i}, N_{t+i}, X_{t+i}) \quad = \quad \frac{Q_{t+i}C_{t+i}^{1-\alpha} - 1}{1-\alpha} - \frac{N_{t+i}^{1+\gamma}}{1+\gamma} - \frac{X_{t+i}^{1+\delta}}{1+\delta}$$

where Q is a preference shock, and the parameters $[\alpha, \gamma, \delta]$ are nonnegative.

In the business sector, the production function of the ith firm is

$$Y_t(i) \quad = Z_t N_t^a(i) X_t^b(i). \tag{22}$$

Each firm has access to a common labor-augmenting productivity process, Z. The nominal cost of production for the ith firm is $W_t N_t(i) + V_t X_t(i)$, where the compensation rates for labor, W_t, and labor utilization, V_t, are also the same for all firms.

Conditioned on its labor input, the cost-minimizing labor utilization demanded by the ith firm is[45]

$$X_t(i) \quad = (\tfrac{b}{a})^{\frac{1}{1+\delta}} N_t^{\frac{1+\gamma}{1+\delta}}(i). \tag{23}$$

Using (23) to eliminate utilization from (22), the effective production function of the ith firm is represented as

$$Y_t(i) \quad = Z_t K N_t^{\hat{a}}(i) \tag{24}$$

[45] As the discussion is focused on equilibrium relationships, we ignore friction specifications such as convex costs of adjusting the labor input.

where $K = (\frac{a}{b})^{\frac{b}{1+\delta}}$ and $\hat{a} = a + b\frac{1+\gamma}{1+\delta}$. Thus, conditioning only on household labor suggests that the effective short-run labor elasticity of production, $\hat{a}$, may exceed unity if the supply of labor utilization is substantially more elastic than the supply of labor, $\delta \ll \gamma$.

Consistent with short-run real output effects of monetary policy, prices of differentiated goods in monopolistic product markets are sticky. Interpretations of the New Keynesian supply equation, (2), include quadratic costs or Calvo-delays in adjusting price levels. However, in a flexible-price equilibrium, the relative price set by the ith firm will be

$$\frac{\bar{P}_t(i)}{\bar{P}_t} = \bar{\mu}\bar{\Psi}_t(i), \qquad \bar{\mu} \equiv \frac{\theta}{\theta-1} \tag{25}$$

where $\bar{\mu}$ denotes the equilibrium monopolistic price markup, θ is the price elasticity of demand, and $\bar{\Psi}(i)$ is the equilibrium real marginal cost of the ith firm,[46]

$$\bar{\Psi}_t(i) = (aQ_t)^{-1}\bar{Y}_t^{\alpha}\bar{N}_t^{\gamma}\bar{Y}_t^{-1}(i)\bar{N}_t(i) \tag{26}$$

where $\bar{Y}$ and $\bar{N}$ denote indexes of aggregate equilibrium output and labor.

In a symmetric equilibrium, $P_t(i) = P_t, N_t(i) = N_t$, and $Y_t(i) = Y_t$. Consequently, equation (25) implies that the natural rate for output in a flexible-price equilibrium is

$$\bar{Y}_t = (\tfrac{a}{\bar{\mu}})^{k_2}K^{k_3}Q_t^{k_2}Z_t^{k_3} \tag{27}$$

or, in logs, the time-varying natural rate of log output, $\bar{y}_t = log\bar{Y}_t$, is a linear function of the log preference shock, $q_t = logQ_t$ and log productivity, $z_t = logZ_t$,

$$\bar{y}_t = k_1 + k_2 q_t + k_3 z_t \tag{28}$$

where k_1 is the log of the constant terms in (27), $k_2 = \frac{\hat{a}}{1+\gamma+\hat{a}(\alpha-1)}$, and $k_3 = \frac{1+\gamma}{1+\gamma+\hat{a}(\alpha-1)}$.

In NK models, the natural rate for the real interest rate may also be a time-varying function of preference shocks and the growth rate of productivity, vid. Woodford (2003). For example, evaluating the household first-order conditions underlying equation (1) at

[46] First-order conditions for the household require $\frac{W_t}{P_t} = Q_t^{-1}C_t^{\alpha}N_t^{\gamma}$; and the cost-minimizing conditions for the ith firm require $\frac{W_t}{P_t} = a\Psi_t(i)\frac{Y_t(i)}{N_t(i)}$. Using the market clearing condition, $C_t = Y_t$, and eliminating the real wage yields the expression for real marginal cost in equation (26).

equilibrium gives[47]

$$\bar{\rho}_t = k_4 + \frac{k_3}{a_2} E_t \Delta z_{t+1} + \frac{k_2}{a_2} E_t \Delta q_{t+1}. \tag{29}$$

As noted in section 1, macro analyses of production resource constraints throughout the 1970s and 1980s were dominated by unemployment "gap" approximations of marginal cost, rather than by output gap representations. The log of the production function indicates that the natural rate of labor is

$$\bar{n}_t = -\tfrac{1}{\hat{a}} logK - \tfrac{1}{\hat{a}} z_t + \tfrac{1}{\hat{a}} \bar{y}_t$$

where $n_t = logN_t$. The natural rate of unemployment is

$$\bar{u}_t = -\omega \bar{n}_t \tag{30}$$

where the response of measured unemployment can be less than unity, $0 < \omega \leq 1$, due to employment entry and exit by household members not classified as actively seeking employment, vid. Blanchard and Diamond (1990). Combining the log of the production function with equation (30) indicates the natural rate of unemployment in the NK model is

$$\bar{u}_t = k_0 - \frac{\omega(k_3 - 1)}{\hat{a}} z_t - \frac{\omega k_2}{\hat{a}} q_t \tag{31}$$

where $k_0 = \frac{\omega}{\hat{a}}(logK - k_1)$. By equation (31), the natural rate for unemployment is invariant to shifts in productivity if $k_3 = 1$, which will occur if the utility of consumption is logarithmic $(\alpha = 1)$.[48]

Finally, combining equation (30) with the log of the production function provides a New Keynesian variant of Okun's Law, equation (4),

$$y_t - \bar{y}_t = -a'(u_t - \bar{u}_t) \tag{32}$$

where $a' = \frac{\hat{a}}{\omega}$.

[47]The intercept, k_4, in equation (29) is a function of the expected covariation of the real rate and the stochastic discount factor, vid. Kozicki and Tinsley (2002a). Estimations of time-varying $\bar{\rho}_t$ using specifications similar to equation (29) are discussed in Laubach and Williams (2002) and Clark and Kozicki (forthcoming).

[48]Comovements of labor productivity and of actual and perceived natural rates of unemployment under lagged learning are explored in Ball and Mankiw (2002) and Reis (2003).

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Table 1: Unemployment natural rates (%)

policy regime	natural rate source: CBO (2004)	Romer & Romer (2002)	expanding [1] sample means (A)	expanding [2] sample means (B)
Martin 67Q4-69Q4	5.8	2.5	5.0	3.7
Burns_1 70Q1-75Q2	6.0	3.1	5.0	4.3
Burns_2 75Q3-78Q1	6.2	8.2	5.3	5.2
Miller 78Q2-79Q2	6.3	4.6	5.4	5.5
Volcker 79Q3-87Q2	6.1	8.0	5.8	6.0
Greenspan_1 87Q3-96Q4	5.7	6.7	6.0	6.3
Greenspan_2 96Q1-97Q4	5.2	n.a.	6.0	6.2

1. Expanding samples begin in 1956Q1 with an initial unemployment rate, $u_1 = 4.0$.
2. Expanding samples begin in 1966Q1 with an initial unemployment rate, $u_1 = 3.9$.

Table 2: Unemployment autoregressions[1]

$$u_{t+h} = \beta_{1,t} + \beta_{2,t}u_{t+h-1} + \beta_{3,t}\Delta u_{t+h-1} + \beta_{4,t}\Delta u_{t+h-2} + \beta_{5,t}\Delta u_{t+h-3} + a_{t+h}.$$
$$\bar{u}_t = \beta_{1,t}/(1-\beta_{2,t}).$$

tvp format	estimated $\bar{\beta}_i$ [2]					estimated $\bar{u}_t$	
	GB horizon forecasts, $h = 0, \ldots, H$ [3]						
	$\bar{\beta}_1$	$\bar{\beta}_2$	$\bar{\beta}_3$	$\bar{\beta}_4$	$\bar{\beta}_5$	max	min
random walk intercept	.487 (.052)	.926 (.004)	.478 (.017)	.022 (.017)	.050 (.014)	8.5	5.0
random walk coefficients	.499 (.221)	.924 (.035)	.386 (.368)	.034 (.268)	.078 (.190)	8.7	4.9
stationary coefficients	.476 (.052)	.924 (.007)	.314 (.052)	.082 (.043)	.093 (.021)	8.5	4.9
(var decomp %)	61	38	1	0	0		
	GB horizon forecasts, $h = 3, \ldots, H$ [4]						
	$\bar{\beta}_1$	$\bar{\beta}_2$	$\bar{\beta}_3$	$\bar{\beta}_4$	$\bar{\beta}_5$	max	min
random walk intercept	.217 (.018)	.966 (.003)	.511 (.027)	.145 (.026)	-.011 (.014)	7.4	5.8
random walk coefficients	.247 (.100)	.961 (.016)	.466 (.237)	.133 (.188)	.002 (.142)	7.4	5.8
stationary coefficients	.223 (.035)	.965 (.006)	.455 (.088)	.141 (.064)	.006 (.027)	7.1	6.0
(var decomp %)	26	73	1	0	0		

1. $u_{t+h}-$ GB forecast civilian unemployment, $h \geq 0$.
2. (.) - std error; $\bar{\beta}_i-$ sample average of $\beta_{i,t}$ for random walk specifications.
3. sample 1966Q3-1997Q4.
4. sample 1969Q1-1997Q4.

Table 3: Unemployment natural rates (%),
with tvp autoregressions

	natural rate source			
policy regime	CBO (2004)	Romer & Romer (2002)	$AR(4)$ [1] $h = 0, \ldots, H$	$AR(4)$ [1] $h = 3, \ldots, H$
Martin 67Q4-69Q4	5.8	2.5	5.3	n.a.
Burns_1 70Q1-75Q2	6.0	3.1	6.2	6.6
Burns_2 75Q3-78Q1	6.2	8.2	6.6	6.5
Miller 78Q2-79Q2	6.3	4.6	7.1	6.9
Volcker 79Q3-87Q2	6.1	8.0	7.4	6.6
Greenspan_1 87Q3-96Q4	5.7	6.7	6.2	6.1
Greenspan_2 96Q1-97Q4	5.2	n.a.	5.5	6.0

1. Based on the stationary coefficient variants of the fourth-order autoregressions in Table 2.

Table 4: Pricing equation
w/ autoregressive expected inflation[1]

$$\pi_{t+h} = \beta_{1,t} + \beta_{2,t}u_{t+h} + E_t\pi_{t+h+1} + a_{t+h}.$$
$$E_t\pi_{t+h+1} = \pi_{t+h-1} + \beta_{3,t}\Delta\pi_{t+h-1}.$$
$$\bar{u}_t = -\beta_{1,t}/\beta_{2,t}.$$

tvp format	estimated $\bar{\beta}_i$ [2]			estimated $\bar{u}_t$	
	random walk expected inflation				
	$\bar{\beta}_1$	$\bar{\beta}_2$	$\bar{\beta}_3$	max	min
random walk intercept	.421 (.095)	-.083 (.016)		6.9	3.3
random walk coefficients	.396 (.459)	-.078 (.068)		7.1	3.1
stationary coefficients	.408 (.142)	-.085 (.029)		7.0	3.9
(var decomp %)	20	80			
	AR(2) expected inflation				
	$\bar{\beta}_1$	$\bar{\beta}_2$	$\bar{\beta}_3$	max	min
random walk intercept	.478 (.092)	-.097 (.016)	-.213 (.022)	6.9	3.0
random walk coefficients	.431 (.434)	-.087 (.065)	-.212 (.175)	7.3	2.6
stationary coefficients	.506 (.130)	-.109 (.026)	-.260 (.035)	7.0	3.8
(var decomp %)	7	93	0		

1. sample 1969Q1-1997Q4; $\pi_{t+h}-$ GB forecast GNP/GDP deflator inflation; $u_{t+h}-$ GB forecast civilian unemployment, $h = 3, \ldots, H$.
2. (.) - std error; $\bar{\beta}_i-$ sample average of $\beta_{i,t}$ for random walk specifications.

Table 5: Pricing equation
w/ Greenbook expected inflation[1]

$$\begin{aligned}\pi_{t+h} &= \beta_{1,t} + \beta_{2,t} u_{t+h} + \beta_{3,t} \Delta\pi_{t+h-1} \\ &\quad + \beta_{4,t} \Delta u_{t+h+1} + \beta_{5,t}(\pi_{t+h+1} - \pi_{t+h-1}) + \pi_{t+h-1} + a_{t+h}. \\ \bar{u}_t &= -\beta_{1,t}/\beta_{2,t}.\end{aligned}$$

tvp format	estimated $\bar{\beta}_i$ [2]					estimated $\bar{u}_t$	
	Greenbook expected inflation						
	$\bar{\beta}_1$	$\bar{\beta}_2$	$\bar{\beta}_3$	$\bar{\beta}_4$	$\bar{\beta}_5$	max	min
random walk	.117	-.019		-.077	.573	9.3	4.0
intercept	(.068)	(.013)		(.103)	(.021)		
random walk	.082	-.015		-.032	.582	947	-88.3
coefficients	(.383)	(.058)		(.644)	(.153)		
stationary	.171	-.029		-.087	.567	7.1	5.5
coefficients	(.099)	(.016)		(.188)	(.052)		
(var decomp %)	46	54		0	0		
	tvp averaging of Greenbook and autoregressive expected inflation						
	$\bar{\beta}_1$	$\bar{\beta}_2$	$\bar{\beta}_3$	$\bar{\beta}_4$	$\bar{\beta}_5$	max	min
random walk	.173	-.030	-.072	-.138	.543	6.9	4.5
intercept	(.067)	(.014)	(.018)	(.103)	(.022)		
random walk	.107	-.022	-.072	-.083	.558	1746	-11.9
coefficients	(.371)	(.057)	(.134)	(.649)	(.137)		
stationary	.209	-.038	-.075	-.156	.538	6.7	4.8
coefficients	(.099)	(.016)	(.030)	(.203)	(.046)		
(var decomp %)	42	58	0	0	0		

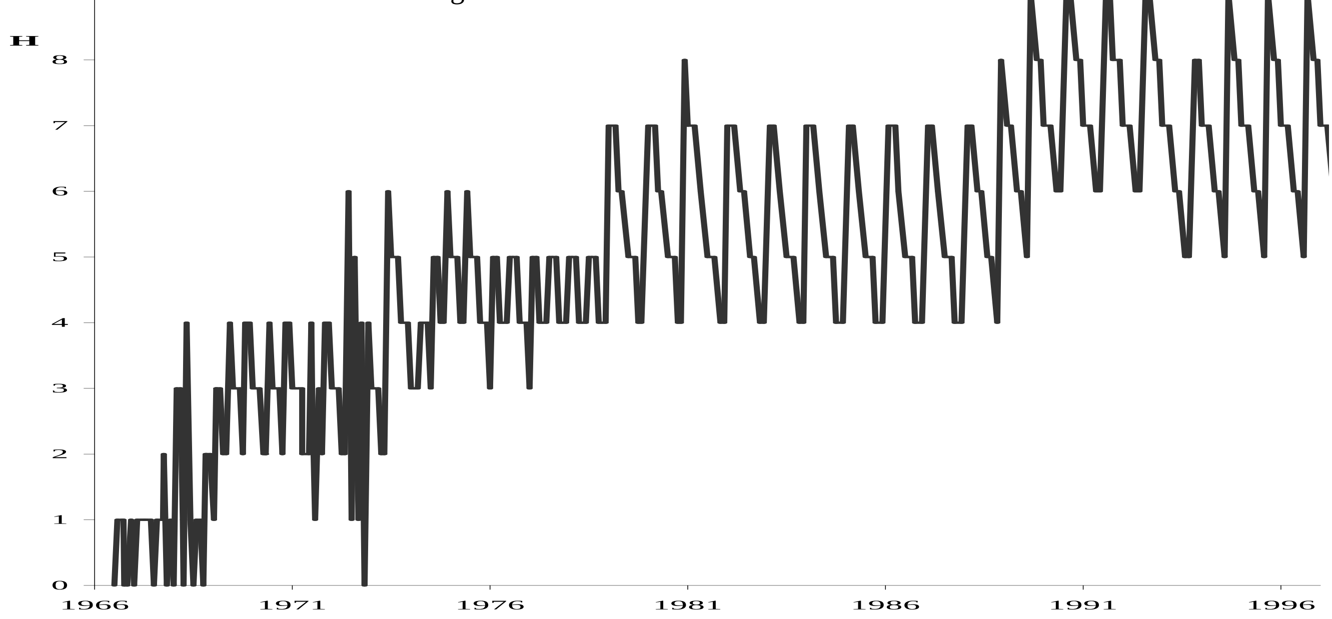
1. sample 1969Q1-1997Q4; $\pi_{t+h}-$ GB forecast GNP/GDP deflator inflation, $u_{t+h}-$ GB forecast civilian unemployment, $h = 3, \ldots, H$.
2. (.) - std error; $\bar{\beta}_i-$ sample average of $\beta_{i,t}$ for random walk specifications.

Table 6: Unemployment natural rates (%),
given alternative expected inflation specifications

policy regime	natural rate source			
	CBO (2004)	Romer & Romer (2002)	AR(2)[1] expectations	tvp averaging[2] expectations
Burns_1 70Q1-75Q2	6.0	3.1	3.9	5.2
Burns_2 75Q3-78Q1	6.2	8.2	4.3	5.3
Miller 78Q2-79Q2	6.3	4.6	4.3	5.3
Volcker 79Q3-87Q2	6.1	8.0	5.4	5.6
Greenspan_1 87Q3-96Q4	5.7	6.7	5.7	6.2
Greenspan_2 96Q1-97Q4	5.2	n.a.	5.0	5.6

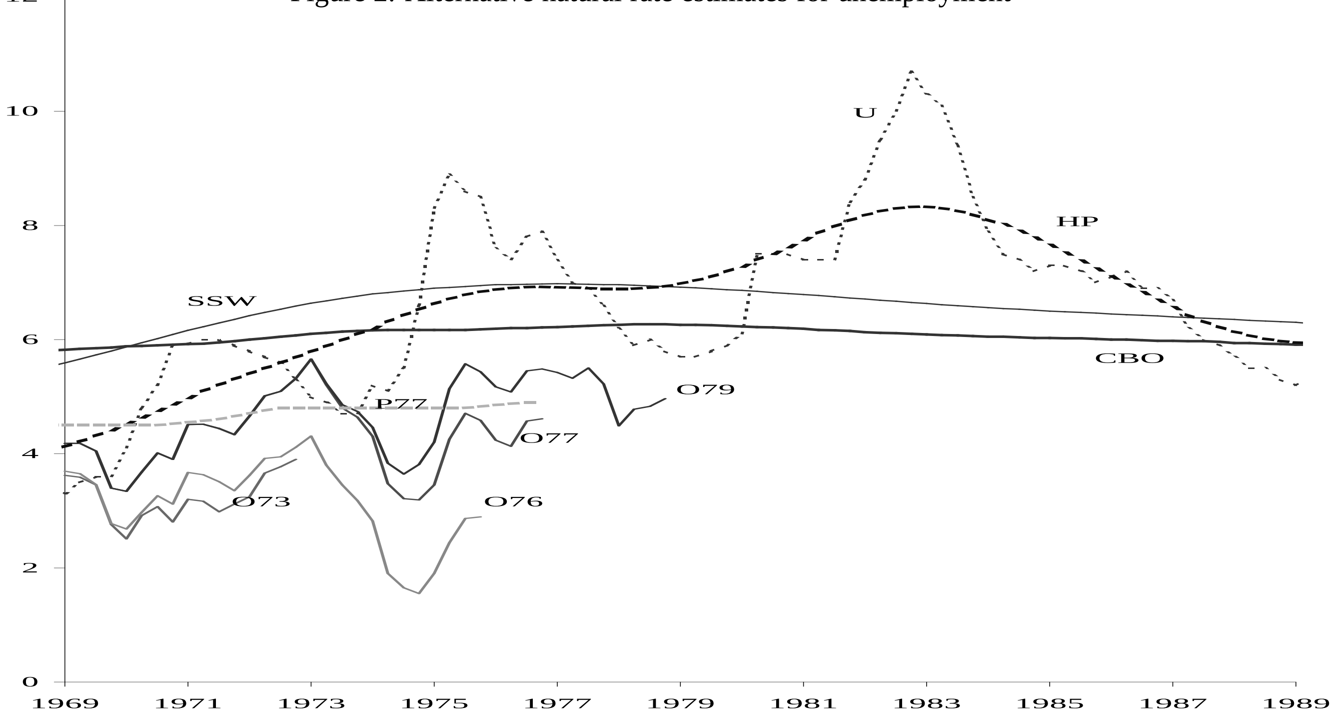
1. Based on stationary coefficients equation, bottom panel of Table 4.
2. Based on stationary coefficients equation, bottom panel of Table 5.

Figure 1: Greenbook forecast horizons [1]



1. H - horizon of Greenbook forecast, $h = 0, 1, \ldots, H$.

Figure 2: Alternative natural rate estimates for unemployment [1]



1. Real-time civilian unemployment: U.
Retrospective natural rates: CBO, Congressional Budget Office (2004); SSW, Staiger, Stock, & Watson (1977).
Real-time natural rates: HP, Hodrick-Prescott filter; P77, Perry (1977); O73, O76, O77, O79, calculated from Orphanides (2003a), see text.

Figure 3: Greenbook one-quarter forecast errors of inflation

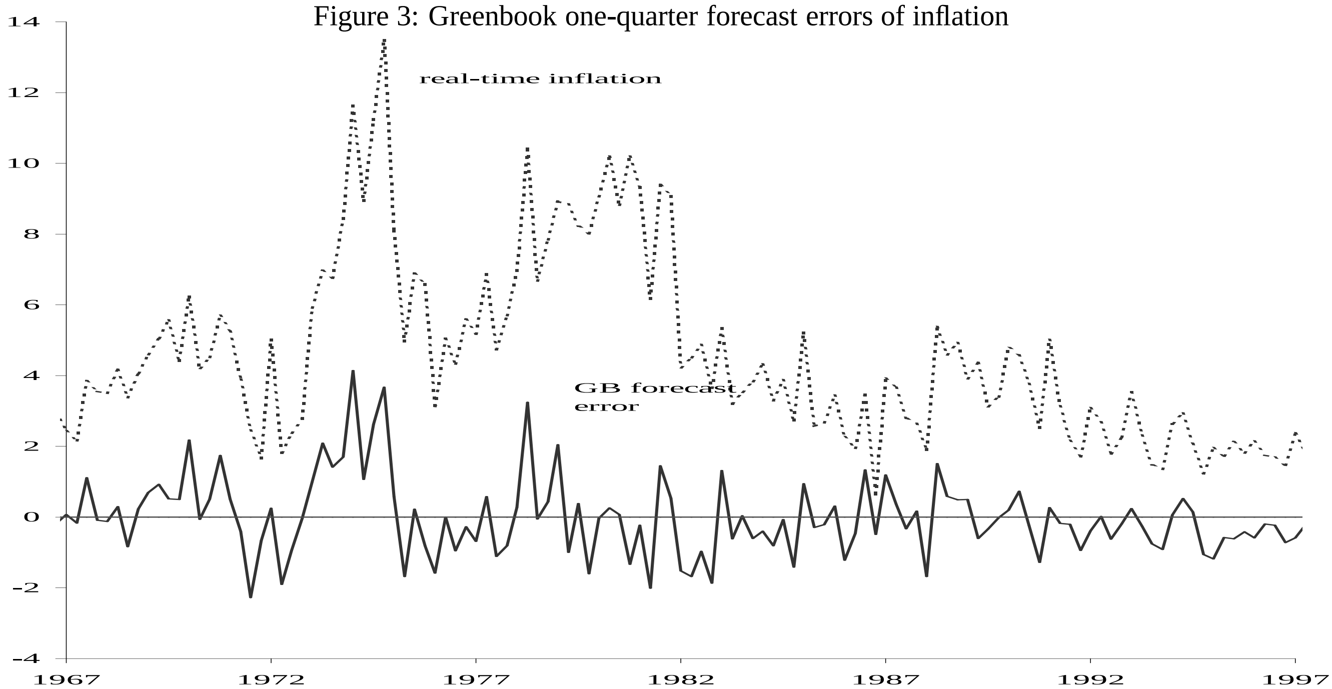
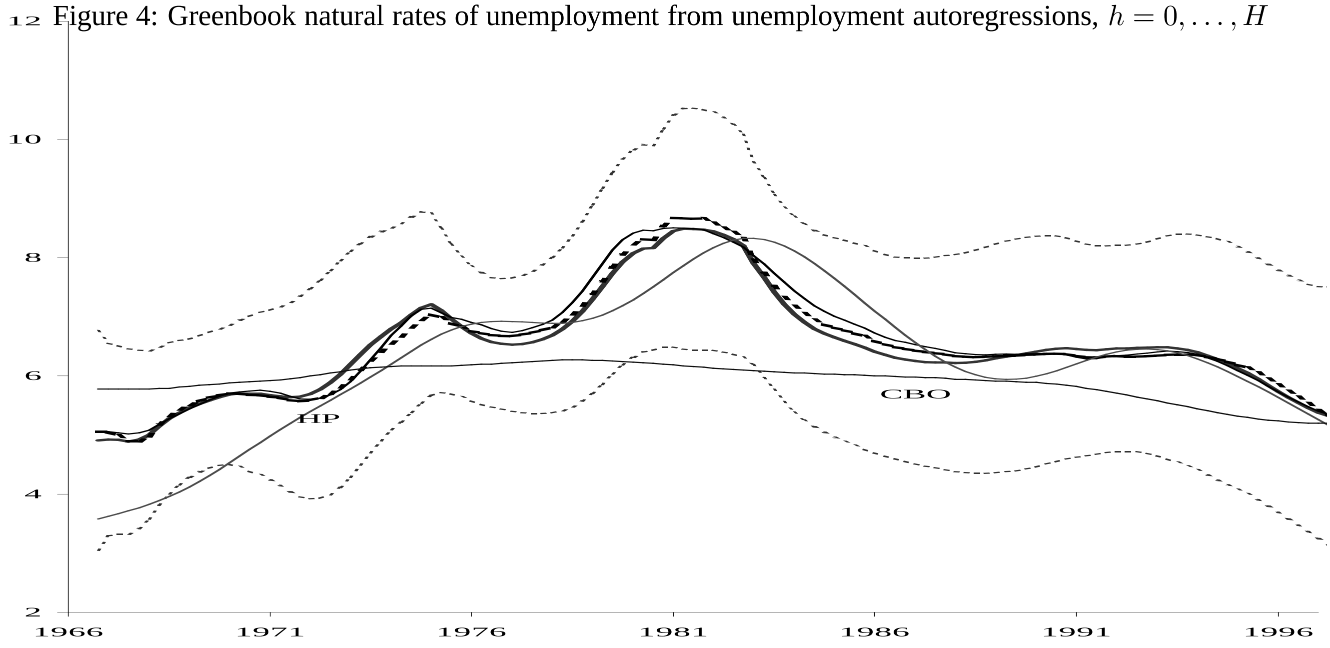
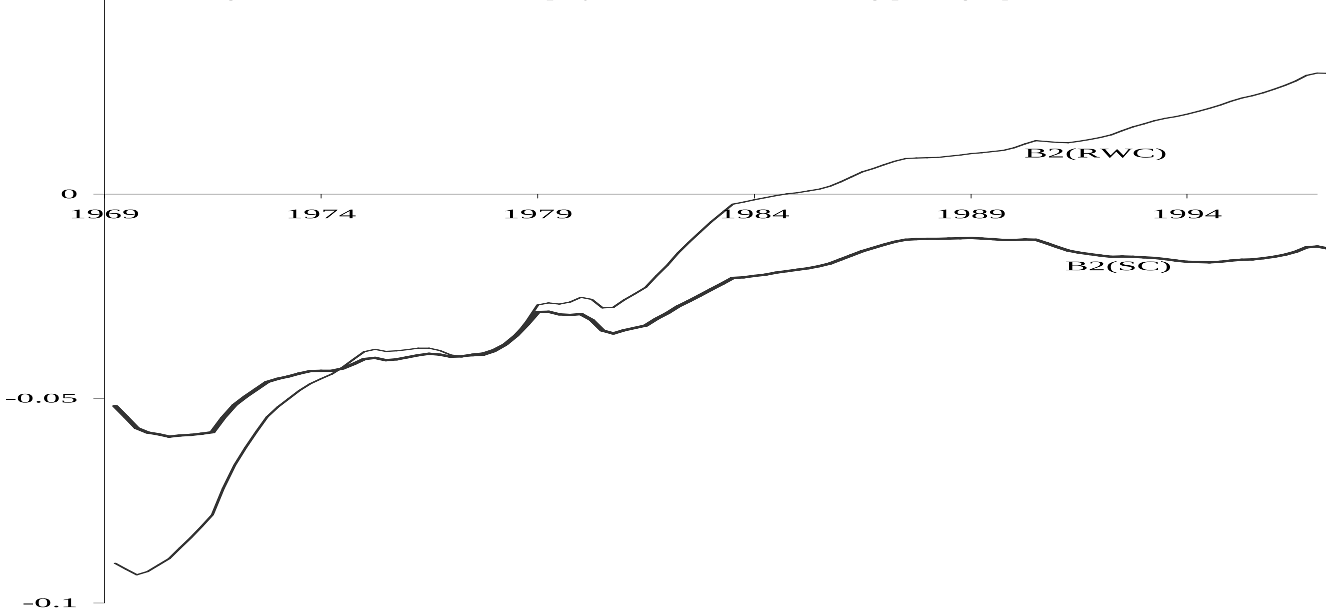


Figure 4: Greenbook natural rates of unemployment from unemployment autoregressions, $h = 0, \ldots, H$



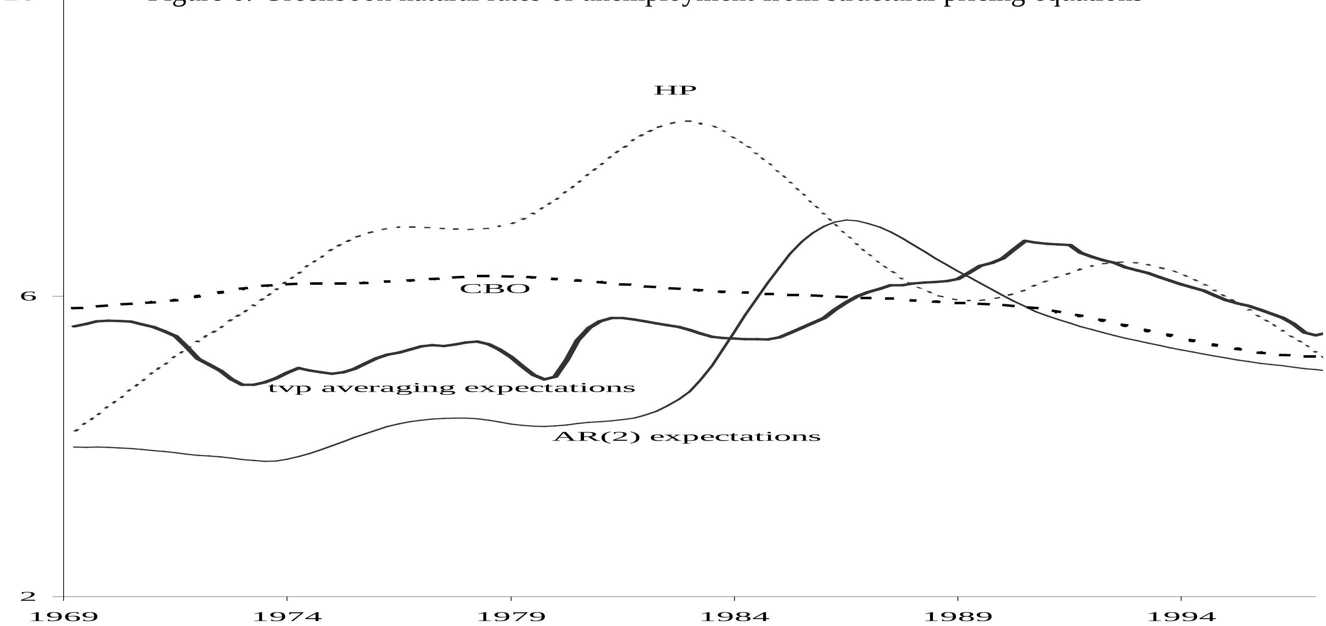
1. Unemployment autoregressions described in top panel of Table 3.
Thin solid line: random walk intercept specification, RWI;
thick dotted line: tvp random walk coefficients specification, RWC;
thick solid line: tvp stationary coefficients specification, SC.
Thin dotted lines are 70% confidence intervals of SC estimator.
CBO and HP filter estimates of natural rates described in the text.

Figure 5: Coefficient of unemployment in forward-looking pricing equations [1]



1. Coefficients from specifications in the top panel of Table 5.
   B2(RWC) - $\beta_{2,t}$ from random walk coefficients specification, RWC.
   B2(SC) - $\beta_{2,t}$ from stationary coefficients specification, SC.

Figure 6: Greenbook natural rates of unemployment from structural pricing equations [1]



1. AR(2) expectations equation in bottom panel of Table 4;
   Tvp averaging expectations equation in bottom panel of Table 5.
   HP filter and CBO natural rate estimates described in the text.