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Term Structure Transmission of Monetary Policy (Why Bond Traders Are Paid More Than Central Bankers)

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TERM STRUCTURE TRANSMISSION OF MONETARY POLICY
(WHY BOND TRADERS ARE PAID MORE THAN CENTRAL BANKERS)

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Abstract: The sensitivity of bond rates to macro variables appears to vary both over time and over forecast horizons. The latter may be due to differences in forward rate term premiums and in bond trader perceptions of anticipated policy responses at different forecast horizons. Determinacy of policy transmission through bond rates requires a lower bound on the average responsiveness of term premiums and anticipated policy responses to inflation.

Keywords: Asymmetric information; no-arbitrage term structure; the Great Inflation; the Taylor Principle.

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1 Introduction

> "Monetary policy works largely through indirect channels–in particular, by influencing private-sector expectations and thus long-term interest rates." Bernanke (2004)
>
> "Financial markets are the channel through which our policy affects the economy, and asset prices contain valuable information about investors' expectations for the course of policy, economic activity, and inflation, as well as the risks about those expectations." Kohn (2005)

Bond rates are essential conduits for the transmission of monetary policy. But bond rates contain bond trader expectations of future policy rates, not recent policy rates. Thus, monetary policy depends on the policy perceptions of the bond market, and the connection of these perceptions to announced or recently observed policy is not fully understood.

The yield to maturity of a zero coupon bond is the average of forward rates over the maturity of the bond. If the bond rate is the principal policy transmission channel, what matters for stabilizing policy is that the bond rate average of the forward rates displays an elastic response to expected inflation. Consequently, perceived inelastic responses by the policy rate to inflation in the short-run may be counterbalanced by elastic responses by forward rates in the remaining periods encompassed by the bond.

Indeed, as noted later, long-run responses of nominal bond rates to inflation appear to have been greater than unity since the mid-1960s, both for samples before and after 1980. As bond rates contain averages of expected policy rates, this seems inconsistent with empirical evidence in a number of papers, such as Clarida, Gali, and Gertler (2000) and Lubik and Schorfheide (2004), that a general feature of US monetary policy in the 1960s and 1970s was an inelastic response of the policy rate with respect to inflation.

One explanation of the different inflation sensitivity of the policy rate and of bond rates may be that forward rate term premiums are also responsive to macro variables, including inflation. If this is so, term premiums demanded by traders may compensate for modestly unstable short-run policy.

An additional possible explanation is that bond traders in the 1960s and 1970s expected stable responses in the longer run. But, at a minimum, the concurrence of inelastic policy rate adjustments and elastic bond rate responses suggests that the connection between anticipated and recent policy may be more tenuous than commonly recognized by extrapolations of recently observed policy rate responses.

The central roles of bond rates and the perceptions of bond traders in the transmission of policy are discussed in remaining sections of the paper. Section 2 examines the recent use of indeterminacy analysis in

interpretations of US monetary policy during the 1960s and 1970s and indicates that an alternative condition for determinacy is required if the bond rate is the policy transmission channel. Section 3 demonstrates that frictions in adjusting real expenditures are essential to the transmission role of bond rates and, consequently, bond rates play no role in the dynamics of the standard New Keynesian (NK) model. This section also compares the responsiveness of historical bond rates to macro variables since the mid-1960s. Section 4 briefly sketches a no-arbitrage model of the term structure with term premiums that reflect time-varying compensation for macroeconomic uncertainty, and extends this theoretical model to allow for the possibility of horizon-dependent expectations by bond traders. Section 5 presents estimated responses of forward rates to forecasts of macro variables, and section 6 concludes.

2 Determinate and indeterminate interpretations of the 1970s

Although a number of studies have suggested that US monetary policy was an important contributor to the size and persistence of inflation in the 1970s, there remains considerable disagreement as to the major flaw in the design of historical policy. This section uses a simple model to illustrate dynamic implications for inflation of two prominent interpretations of U.S. monetary policy, one suggesting the policy rate was influenced by erroneous estimates of the natural rate for economic activity and the other indicating the policy rate did not keep pace with inflation. The model is then extended to show the dynamic implications for inflation if the bond rate is the principal conduit of policy.

Orphanides (2003) suggests that U.S. monetary policy in the 1970s was consistent with a Taylor rule but the central bank severely overestimated the trend of potential output, leading to policy rates that were below those consistent with the Taylor benchmark policy.

The dynamic implications of this *natural rate error* interpretation are equivalent to those of a policy rate feedback response where the effective inflation target is larger than that originally intended by the central bank. Suppose, as suggested by Orphanides, the intended policy rate response was generated by the classic Taylor (1993) rule

$$r_t \quad = \quad \bar{\rho} + \pi_t + .5(\pi_t - \bar{\pi}^{cb}) + .5(y_t - \bar{y}_t^{cb}), \tag{1}$$

where r_t denotes the nominal policy rate, $\bar{\rho}$ is the equilibrium real rate, $\bar{\pi}^{cb}$ is the intended inflation target, and $\bar{y}_t^{cb}$ is the central bank perception of the natural rate for output which overstates the true natural rate, $\bar{y}_t$. The policy rule in equation (1) is equivalent to a policy description where the policy rate responds to the

output deviation from the true natural rate of output,

$$r_t \quad = \quad \bar{\rho} + \pi_t + .5(\pi_t - \bar{\pi}_t^*) + .5(y_t - \bar{y}_t). \tag{2}$$

but the *effective* inflation target is increased by precisely the overestimate of the output natural rate,

$$\bar{\pi}_t^* \quad = \quad \bar{\pi}^{cb} + (\bar{y}_t^{cb} - \bar{y}_t).$$

Because the nominal policy rate response to inflation in the Taylor benchmark rule exceeds unity, this interpretation implies that the transitional period of rising inflation following the shift in the effective inflation target is self-terminating. That is, in well-behaved models of the economy with an elastic response of the policy rate to inflation, the solution for inflation is determinate and will eventually converge to the new inflation target. Thus, under the natural rate error interpretation, rising inflation in the 1970s is due to the dynamic transition of inflation from the neighborhood of the intended inflation target, $\bar{\pi}^{cb}$, to the neighborhood of the effective inflation target, $\bar{\pi}_t^*$. Ceteris paribus, the dynamic transition of inflation will be accompanied by positive output "gaps," $y_t - \bar{y}_t > 0$, induced by the inadvertent reductions in real interest rates.

An alternative interpretation of the 1970s is proposed by Clarida, Gali, Gertler (2000) where the response of the policy rate is inelastic with respect to expected inflation. Under this *passive policy* interpretation, the economy is vulnerable to self-fulfilling expectations of higher inflation induced by agent forecasts of inflationary shocks (exogenous sunspots). Unlike the natural rate error interpretation, the sensitivity to arbitrary sunspots is not self-terminating and will continue as long as the policy response is passive. The solution for inflation is indeterminate, and there is no mechanism for inflation to remain in the neighborhood of the central bank target for inflation.

2.1 A model of (in)determinate inflation

A rudimentary model is used to illustrate differences in dynamic behavior under these alternative interpretations of monetary policy.

$$\begin{aligned} \tilde{y}_t &= -aE_t(\tilde{r}_t - \tilde{\pi}_{t+1}) + e_t, \\ \tilde{\pi}_t &= E_t\tilde{\pi}_{t+1} + b\tilde{y}_t, \\ \tilde{r}_t &= cE_t\tilde{\pi}_{t+1}, \end{aligned} \tag{3}$$

where $\tilde{x}_t$ denotes the equilibrium deviation of variable x_t. The first equation in (3) indicates that deviations in output, $\tilde{y}_t$ are determined by equilibrium deviations in the one-period ex ante real interest rate, $E_t\{\tilde{r}_t - \tilde{\pi}_{t+1}\}$, and by a stochastic disturbance, e_t. In the second equation, equilibrium deviations in the inflation rate, $\tilde{\pi}_t$, are determined by a standard New Keynesian (NK) pricing equation, and the third equation in (3) describes a forward-looking policy response by equilibrium deviations in the nominal policy rate, $\tilde{r}_t$. All coefficients are positive, and the output equation disturbance is a first-order autoregression, $e_t = \gamma e_{t-1} + \epsilon_t$; thus, there is a single structural i.i.d. shock, ϵ.

Eliminating the policy rate by substituting the third equation of (3) into the first

$$\tilde{y}_t \quad = \quad -aE_t(c-1)\tilde{\pi}_{t+1} + e_t, \tag{4}$$

and using this to eliminate output from the second equation in (3), gives a first-order equation for inflation

$$\tilde{\pi}_t \quad = \quad (1 - ab(c-1))E_t\tilde{\pi}_{t+1} + be_t. \tag{5}$$

Consistent with the natural rate error conjecture, a determinate solution for inflation will exist if the policy response lies in the interval $1 < c < 1 + \frac{2}{ab}$.[1] Defining $\mu \equiv 1 - ab(c-1)$, inflation dynamics in this rudimentary model are driven solely by the autoregressive demand shock,

$$\begin{aligned} \tilde{\pi}_t \quad &= \quad E_t \sum_{i=0}^{\infty} \mu^i b e_{t+i}, \\ &= \quad \frac{be_t}{1-\gamma\mu}, \end{aligned} \tag{6}$$

and the one-period forecast error of inflation is $\eta_{t+1} \equiv \tilde{\pi}_{t+1} - E_t\tilde{\pi}_{t+1} = \frac{b\epsilon_{t+1}}{1-\gamma\mu}$.[2]

By contrast, consistent with the passive policy conjecture, the empirical estimates of U.S. monetary policy in Clarida, Gali, Gertler (2000) indicate that policy rate responses to inflation in the 1960s and 1970s were inelastic, $c < 1$. In the current example, this implies $\mu > 1$; consequently, stable solutions to equation (5) are not unique and can be represented by

$$\tilde{\pi}_t \quad = \quad \frac{1}{\mu}\tilde{\pi}_{t-1} - \frac{b}{\mu}e_{t-1} + \eta_t, \tag{7}$$

[1] Typically, the product ab is a small positive fraction. Representative estimates of the product of output and inflation equation slopes include .014 in Rudebusch and Svensson (1999) and .15 in Rotemberg and Woodford (1999).

[2] As with standard NK models, there are no intrinsic dynamics associated with equation (5). Thus, a lengthy dynamic transition to a new equilibrium under the natural error interpretation requires additional sources of lagged adjustment. If the revision in the effective inflation target is not known to agents under the natural rate error conjecture, lengthy learning lags can be associated with statistical analysis of observed inflation, primarily due to the time required for observations of the new policy regime, vid. Kozicki and Tinsley (2001a).

The forecast error, η_t, is an arbitrary martingale difference, which may or may not be correlated with the structural disturbance. As in Lubik and Schorfeide (2004), the forecast error can be represented as

$$\eta_t = m\epsilon_t + s_t, \tag{8}$$

where s_t is an exogenous "sunspot" shock. Thus, in contrast to the solution in (6), rational expectations does not limit the forecast error, η_t, to be a unique function of the structural shock, ϵ_t.[3]

The current period responses to the sunspot shock, s_t are

$$\begin{aligned} \Delta_s\tilde{\pi}_t &= s_t, \\ \Delta_s\tilde{r}_t &= \frac{c}{1+ab(1-c)}s_t, \\ \Delta_s\tilde{y}_t &= \frac{a(1-c)}{1+ab(1-c)}s_t, \end{aligned} \tag{9}$$

where $\Delta_s x_t \equiv x_t(s_t) - x_t(0)$, and responses are largest for inflation. Similar to the consequences of an overestimate of the natural rate of output or, equivalently an increase in the effective central bank target for inflation, the contribution of a positive sunspot is to increase both output and inflation.

Thus, neither an inadvertent increase in the central bank target for inflation nor the sunspot interpretation is alone able to explain the general tendency towards stagflation in the 1970s. The twelve-month moving average of inflation and the negative of the monthly unemployment "gap" deviation are displayed in Figure 1 for a 1960-1990 sample. Although the positive association between inflation and the negative unemployment gap is reasonably close in the 1960s, the two series diverge in the 1970s, with inflation continuing to rise and the negative unemployment gap trending toward or below zero.[4]

Kozicki and Tinsley (2005a) suggest an alternative interpretation of the 1970s, where U.S. monetary policy was directed at achieving designated growth rates of the narrow money supply. The policy of money growth targeting is confirmed by transcripts of the Federal Open Market Committee (FOMC) and by empirical estimates of time-varying responses by policy rates to real-time briefing forecasts.

The historical conduct of money growth rate targeting supports two disparate implications of the natural rate error and passive policy interpretations of the 1970s. First, as in the natural rate error interpretation,

[3]Alternative solutions to (5) and the additional degree of freedom introduced under indeterminacy are demonstrated in the appendix.

[4]Lubik and Schorfeide (2004) provide posterior estimates for two types of sunspots in a 1960Q1-1979Q2 sample of US data. In the case of orthogonal sunspots, $m = 0$, sunspot contributions to the variances of inflation and the nominal policy rate are about 70%, with nearly all of the variance of output attributed to demand shocks. In the case of non-orthogonal sunspots, the sunspot contributions to the variances of inflation and the nominal interest rate are modest, about 10%.

intermediate targeting of the money supply in the 1970s induced an increase in the inflation target. Under money growth targeting, the effective inflation target is determined by the natural rate version of the equation of exchange

$$\bar{\pi}_t \quad = \quad \Delta\bar{m}_t - \Delta\bar{y}_t + \Delta\bar{v}_t, \tag{10}$$

where $\Delta\bar{m}_t$ denotes the FOMC target for money supply growth and $\Delta\bar{v}_t$ is the trend growth of velocity. As in the natural rate error interpretation, an unexpected reduction in the growth rate of trend output in the 1970s, $\Delta\bar{y}_t \downarrow$, induced an increase in the effective inflation target. However, the money growth targeting policy induced even larger increases in the effective inflation target due to sizeable unexpected positive shifts in the trend growth of velocity, $\Delta\bar{v}_t \uparrow$.

Second, empirical estimates of policy rate responses in Kozicki and Tinsley (2005a) also support the description of passive monetary policy in the 1970s. As documented in Kozicki and Tinsley (2005a), FOMC transcripts in the 1970s indicate that the nominal policy rate settings were directed at reversing projected deviations of money supply growth from target growth rates. However, empirical results indicate that long-run responses of the policy rate were inelastic with respect to the growth rate of nominal aggregates.

2.2 Inflation determinacy under bond rate transmission

The general presumption in the literature is that evidence of passive monetary policy in the 1960s and 1970s implies that the central bank and the private sector may have been influenced by exogenous inflationary sunspots. However, if the principal transmission of monetary policy is through the responses of private sector borrowing rates to the policy rate, then the susceptibility of the economy to sunspots depends on the perceptions of bond traders regarding the passivity of anticipated policy. Thus, the sunspot interpretation is based on two untested assumptions: First, that bond traders can infer, in real time, that the central bank policy is passive.[5] And second, that the passivity of monetary policy is expected to persist over lengthy forecast horizons. With regard to the first assumption, Kozicki and Tinsley (2001a, 2001b) indicate that the mean lag of adjustment of the perceived inflation target consistent with Treasury bond rates exceeded 5 years in the 1970s and 1980s. In addition, FOMC announcements in the 1970s regarding explicit policy

[5] In contrast to the regression analysis in Clarida, Gali and Gertler (2000), empirical analysis in Kozicki and Tinsley (2005a) uses the retrospective advantage of access to the central bank real-time forecasts of explanatory variables. If external observers are not privy to central bank information, Beyer and Farmer (2004) illustrate that it is not always possible for the observers to discriminate between determinate and indeterminate policies.

targets were limited to one-year horizons. It is not obvious that bond traders would extrapolate difficulties in reaching one-year objectives to policy failure in the long run.

To illustrate the difference in conditions for indeterminacy if the transmission role of the bond rate is made explicit, we return to the simple model used earlier but replace the one-period interest rate in the output equation by a two-period real bond rate.

$$\begin{aligned} \tilde{y}_t &= -\frac{a}{2}E_t(\tilde{r}_t - \tilde{\pi}_{t+1} + \tilde{r}_{t+1} - \tilde{\pi}_{t+2}) + e_t, \\ \tilde{\pi}_t &= E_t\tilde{\pi}_{t+1} + b\tilde{y}_t, \\ \tilde{r}_t &= c_1 E_t\tilde{\pi}_{t+1}, \\ E_t\tilde{r}_{t+1} &= c_2 E_t\tilde{\pi}_{t+2}, \end{aligned} \tag{11}$$

The fourth equation represents bond traders' expectation in t of the policy rate in $t+1$, where the perception of the future policy response, c_2, is not restricted to be identical to the response perceived in the current period, c_1.

Substituting the first, third and fourth equations in (11) into the second equation gives a second-order equation for inflation

$$\tilde{\pi}_t = (1 - \frac{ab(c_1 - 1)}{2})E_t\tilde{\pi}_{t+1} - \frac{ab(c_2 - 1)}{2}E_t\tilde{\pi}_{t+2} + be_t, \tag{12}$$

The solution for inflation has the form

$$\tilde{\pi}_t = E_t\{\frac{be_t}{(1 - \lambda_1^{-1}F)(1 - \lambda_2^{-1}F)}\}, \tag{13}$$

where the roots of the characteristic equation are determined by[6]

$$\begin{aligned} \lambda_1\lambda_2 &= \frac{2}{ab(c_2 - 1)}, \\ \lambda_1 + \lambda_2 &= \frac{2 - ab(c_1 - 1)}{ab(c_2 - 1)}. \end{aligned} \tag{14}$$

The perceptions of the bond traders must satisfy three conditions for determinacy, vid. Woodford (2003)

$$\begin{aligned} \lambda_1\lambda_2 > 1 &\Rightarrow c_2 < 1 + \frac{2}{ab}, \\ (1 + \lambda_1)(1 + \lambda_2) > 0 &\Rightarrow c_1 < c_2 + \frac{4}{ab}, \\ (1 - \lambda_1)(1 - \lambda_2) > 0 &\Rightarrow \frac{c_1 + c_2}{2} > 1. \end{aligned} \tag{15}$$

[6]The roots of the associated companion form system for (12) are derived in the appendix.

The first two conditions in (15) establish upper bounds for the policy responses and will depend, in general, on the particular specifications of the model.

The third requirement for determinacy in (15) is not dependent on other model parameters and provides a lower bound for the average perceived policy rate response. Thus, even if the current period response is passive, $c_1 < 1$, the average perceived response may satisfy the lower bound requirement for determinacy. This is a generalization of the Taylor Principle for one-period interest rates, where the nominal bond rate response to anticipated inflation over the maturity of the bond should exceed unity.[7]

The next section examines the theoretical basis of the fundamental assumption that bond rates provide the principal transmission channel for monetary policy, and presents summary measures of historical bond rate responses to inflation.

3 The role of bond rates in the transmission of monetary policy

The key to an essential role for the term structure in the transmission of monetary policy is the presence of real-world frictions or dynamic adjustment costs. The level of the capital stock, and the level of the associated output or consumption, is determined by the level of bond rates. In a frictionless world, the choice between consumption today or consumption tomorrow is a function only of the one-period interest rate, which is equivalent (under risk neutrality) to one-period holding returns to bonds.

By contrast, when expenditure decisions are subject to adjustment frictions, dynamic adjustment of consumption is a function of the full term structure of interest rates. The next two subsections illustrate the transmission role of bond rates for two examples of frictions: adjustment costs for investment in capital goods, and the presence of habit in household utility specifications.

3.1 Bond rates in the demand for quasi-fixed investment

The implication of transactions costs for the policy transmission role of bond rates is illustrated using a marginal q model of investment.

[7] Here, as elsewhere in this paper, conditions for determinacy can be extended to include policy responses to equilibrium deviations in real activity. Strictly speaking, conditions for determinacy are system properties and not just limited to the inflation responsiveness of current and anticipated real policy rates, such as models where nominal interest rates may play an important stabilizing role, vid. Beyer and Farmer (2004). To simplify exposition, discussion in this paper assumes real variables, such as output, are responsive only to real interest rates, consistent with responses by households and firms in conventional NK models.

The owner of a depreciable capital asset, with a physical half-life of $\frac{1}{\delta_s}$, chooses the level of capital, K_s, and rate of investment, I_s, to maximize the present value of rents in a competitive market

$$\max_{K_s, I_s} E_t \sum_{t=0}^{\infty} \beta^t U_{c,t}(R^k_{s,t} K_{s,t} - P^I_{s,t} I_{s,t}), \tag{16}$$

subject to a dynamic constraint on the evolution of installed capital

$$K_{s,t+1} \leq (1-\delta_s)K_{s,t} + I_{s,t} - c(I_{s,t}/I_{s,t-1})I_{s,t},$$

where β is the household discount factor, $U_{c,t}$ is the owner (representative household) marginal utility of consumption, R^k_s is the real return on a unit of capital, P^I_s the relative price of replacement capital, δ_s is the rate of depreciation, and $c(I_{s,t}/I_{s,t-1})$ is the cost of adjustment for investment.

The first-order conditions for optimal capital and investment are, respectively,

$$\begin{aligned}
0 &= E_t\{\beta^{t+1} U_{c,t+1} R^k_{s,t+1} - \beta^t U_{c,t} P^K_{s,t} + \beta^{t+1} U_{c,t+1} P^K_{s,t+1}(1-\delta_s)\}, \\
0 &= E_t\{-\beta^t U_{c,t} P^I_{s,t} + \beta^t U_{c,t} P^K_{s,t} - \beta^t U_{c,t} P^K_{s,t} c_1'(I_{s,t}/I_{s,t-1}) \frac{I_{s,t}}{I_{s,t-1}} \\
&\quad -\beta^t U_{c,t} P^K_{s,t} c(I_{s,t}/I_{s,t-1}) + \beta^{t+1} U_{c,t+1} P^K_{s,t+1} c_2'(I_{s,t+1}/I_{s,t}) \left(\frac{I_{s,t+1}}{I_{s,t}}\right)^2\},
\end{aligned} \tag{17}$$

where P^K, the Lagrangian multiplier of the equation for the evolution of capital, is the relative price of installed capital; and $c_i'(x_1/x_2)$ denotes the partial derivative of the adjustment cost function with respect to the ith argument.

Normalizing the first equation in (17) on the price of installed capital gives

$$\begin{aligned}
P^K_{s,t} &= E_t\{\beta \frac{U_{c,t+1}}{U_{c,t}}(R^k_{s,t+1} + (1-\delta_s)P^K_{s,t+1})\}, \\
&= E_t\{R^{-1}_{e,t}(R^k_{s,t+1} + (1-\delta_s)P^K_{s,t+1})\}, \\
&= E_t\{\sum_{j=1}^{\infty}[\prod_{i=0}^{j-1} R^{-1}_{e,t+i}](1-\delta_s)^{j-1} R^k_{t+j}\}.
\end{aligned} \tag{18}$$

The second line in (18) uses an implication of a standard household Euler equation for consumption that the real rate of interest is $R^{-1}_{e,t} = \beta \frac{U_{c,t+1}}{U_{c,t}}$. The third line indicates that the return to capital in the nth period of the forecast horizon, $(1-\delta_s)^{n-1} R^k_{t+n}$, is discounted by the gross real yield-to-maturity on an n-period bond.

Log-linearizing the second equation in (17) about equilibria gives a second-order difference equation

for investment expenditures[8]

$$ln\tilde{I}_{s,t} = \frac{\beta}{1+\beta}E_t ln\tilde{I}_{s,t+1} + \frac{1}{1+\beta}ln\tilde{I}_{s,t-1} + \frac{1}{(1+\beta)c''}E_t\tilde{q}_t, \quad (19)$$

where q_t denotes the log of marginal q or the ratio of the price of installed capital to the price of replacement capital, $exp(q_t) = \frac{P^K_{s,t}}{P^I_{s,t}}$.

By inspection, the eigenvalues of the characteristic equation are $(1, \frac{1}{\beta})$. Thus, the solution for the level of investment expenditure

$$ln\tilde{I}_{s,t} = ln\tilde{I}_{s,t-1} + \frac{1}{c''}\sum_{i=0}^{\infty}\beta^i E_t\tilde{q}_{t+i}, \quad (20)$$

requires forecasts of marginal q_{t+i}, and the associated term structure of interest rates as illustrated in (18), over the indefinite future.

3.2 Bond rates in NK equations for output

The preceding subsection indicates that the demand for total expenditures, aggregated over goods with disparate durability and adjustment costs, will be a function of the full term structure of interest rates. By contrast, the convention in NK models is to include only a single short-term or long-term interest rate in the output equation.[9] This subsection illustrates that the dynamic transmission role of interest rates is significantly altered by introducing habit preferences into the utility function of the representative household.

In the standard NK model, the output equation is based on the household Euler equation for consumption. An infinitely-lived representative household aims to maximize the present value of utility, U_t,

$$\max_{B,C,N} E_t\sum_{t=0}^{\infty}\beta^t U_t, \quad (21)$$

subject to the flow budget constraint

$$C_{t+i} + B_{t+i} \leq w_{t+i}N_{t+i} + R_{e,t+i}B_{t+i-1}.$$

where C denotes consumption; B is the stock of real bonds which earn the one-period gross real return, R_e; and household employment, N, is paid the real wage, w.

[8] As in Christiano, Eichenbaum, and Evans (2005), the level and first derivative of the adjustment cost function, $c(.)$, are zero in equilibrium.

[9] The output equation is a explicit function of the one-period real interest rate in Rotemberg and Woodford (1999), a four-quarter average of real interest rates in Rudebusch and Svensson (1999), a two-year real bond rate in Coenen and Wieland (2000), and a ten-year real rate in Fuhrer and Moore (1995). Estimation of output equations with either a quarterly real funds rate, a 1-year real bond rate, or a ten-year real bond rate is examined in Fuhrer and Rudebusch (2004).

Given the additively separable utility function[10]

$$U(C_{t+i},.) \equiv \frac{C_{t+i}^{1-\alpha}-1}{1-\alpha} \tag{22}$$

where α is the CRRA curvature parameter, $\alpha > 0$, the first-order conditions for consumption, C_t, and bond holdings, B_t, are, respectively,

$$\begin{aligned} 0 &= C_t^{-\alpha} - \Lambda_t, \\ 0 &= -\Lambda_t + E_t\{\beta R_{e,t+1}\Lambda_{t+1}\}, \end{aligned} \tag{23}$$

where Λ is the Lagrangian multiplier of the budget constraint.

Substituting the first equation into the second equation of (23), imposing the market clearing condition, $C_t = Y_t$, and taking log deviations from equilibria gives the standard NK output equation, such as developed in Woodford (2003),

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1} - \frac{1}{\alpha}\tilde{\rho}_t\}. \tag{24}$$

The NK output equation appears to indicate that the relevant interest rate for current consumption decisions is the one-period real interest rate, ρ_t. However, successively eliminating the forward consumption term indicates that the current level of consumption is a function of the future path of expected equilibrium deviations in one-period real rates or, approximately, the equilibrium deviation in the expected real yield-to-maturity on an n-period bond, $\tilde{\rho}_{n,t}$.

$$\begin{aligned} \tilde{y}_t &= -\frac{1}{\alpha}E_t\sum_{i=0}^{\infty}\tilde{\rho}_{t+i}, \\ &\simeq -\frac{n}{\alpha}E_t\tilde{\rho}_{n,t}. \end{aligned} \tag{25}$$

In both equations (24) and (25), adjustments of current output to perceived equilibrium deviations of interest rates are instantaneous because there are no frictions or impediments to dynamic adjustments of real expenditures. In this sense, standard NK output equations are analogous to the present value price of installed capital, shown earlier in (18). In other words, in the absence of frictions, just as the neoclassical q theory provides a theory of capital demand but not a theory of investment, the standard NK output equation reflects a theory of consumption demand but not a well-defined theory of dynamic adjustment in real expenditures.

[10]To abbreviate notation, the remaining arguments of the separable utility function are not shown.

As an alternative to introducing explicit costs of adjusting the flow of consumption, we consider the policy transmission role of interest rates in output equations when multiplicative 'habits" or reference paths for consumption, Γ_t, are introduced into household utility,

$$U(C_t, .) \quad = \quad \frac{(C_{t+i}\Gamma_t^{-\nu})^{1-\alpha} - 1}{1-\alpha}. \tag{26}$$

In the case of multiplicative exogenous habit, such as "keeping up with the Jones, " vid. Abel (1990), the reference level for each household is a function of aggregate consumption. Alternatively, the habit reference path may be endogenous and refer to the past consumption level of the household, vid. Fuhrer (2000).

Log habit is expressed as a distributed lag of past log consumption,

$$ln\Gamma_t \quad = \quad \nu(L) ln C_{t-1}, \tag{27}$$

where $\nu(L)$ is a polynomial in the lag operator, L. The output equation associated with second-order endogenous habit, $\nu(L) = \nu_1 + \nu_2 L$, is derived in the appendix. Here, we focus on a one-lag exogenous habit specification.

For a one-lag exogenous habit specification, $\Gamma_t = C_{t-1}$, the first-order equation for consumption is

$$0 \quad = \quad C_t^{\alpha} C_{t-1}^{\nu(1-\alpha)} - \Lambda_t, \tag{28}$$

replacing the first equation in (23). Combining (28) with the second equation in (23) and log-linearizing about equilibria gives the output equation under exogenous habit

$$\tilde{y}_t \quad = \quad (1-\lambda) E_t \tilde{y}_{t+1} + \lambda \tilde{y}_{t-1} - \frac{1}{\alpha + \nu(\alpha - 1)} E_t \tilde{\rho}_t, \tag{29}$$

where $\lambda \equiv \frac{\nu(\alpha-1)}{\alpha+\nu(\alpha-1)}$.

By inspection, the two eigenvalues of the characteristic equation are $(1, \frac{\lambda}{1-\lambda})$. There are two solution formats: If $\lambda > .5$, the non-unit root is largest, $\frac{\lambda}{1-\lambda} > 1$, and the solution is

$$\tilde{y}_t \quad = \quad \tilde{y}_{t-1} - \frac{1}{\alpha + \nu(\alpha - 1)} \sum_{i=0}^{\infty} \left(\frac{1-\lambda}{\lambda} \right)^i E_t \tilde{\rho}_{t+i}. \tag{30}$$

By contrast, if $\lambda \leq .5$, the unit root is largest, $\frac{\lambda}{1-\lambda} \leq 1$, and the solution is

$$\begin{aligned} \tilde{y}_t \quad &= \quad \frac{\lambda}{1-\lambda} \tilde{y}_{t-1} - \frac{1}{\alpha} \sum_{i=0}^{\infty} E_t \tilde{\rho}_{t+i}, \\ &\simeq \quad \frac{\lambda}{1-\lambda} \tilde{y}_{t-1} - \frac{n}{\alpha} E_t \tilde{\rho}_{n,t+i}, \end{aligned} \tag{31}$$

where the last line in (31) provides a transparent example of the policy transmission role of the n-period bond rate under habit.

As noted earlier, specifications of interest rates in the output equations of empirical macro models vary widely, with some models using one-period rates and others using long-term rates. Models with explicit one-period rates do not necessarily imply that bond rates are unimportant in policy transmission. Linear Euler equations can always be reformulated as functions of the one-period interest rate, although the implied dynamic restrictions may not be as straight-forward as those in output equations with explicit bond rates.[11]

The drawback of formulating output equations as functions of the one-period rate is the usual problem of asymmetric information. Given that theory indicates forecasts of long-horizon returns in bond markets are important determinants of private sector expenditures, the information set of bond traders is more pertinent, if not larger, than that of a macro modeller. Unless the modeller ensures that the averages of forward rates generated by the model are equivalent to observed bond rates, the model description of policy transmission will reflect the modeller's priors regarding long-horizon forecasts, which may differ markedly from the long-horizon forecasts contained in bond market observations.[12]

Interestingly, there is almost no empirical literature exploring competing specifications of short-term and long-term interest rates in output equations. If frictions in adjusting real expenditures are important, the theoretical examples in this section suggest that long-term ex ante real interest rates should dominate competing short-term real rate regressors in reduced-form regressions. A sequence of bivariate tests is reported in Kozicki and Tinsley (2002), where U.S. manufacturing utilization, a proxy for the output gap, is regressed on competing short-term and long-term ex ante real interest rates over a 1967m1 - 1997m7 sample. The tests confirm that spreads between the long-term and short-term interest rates are statistically insignificant when regressions are conditioned on the long-term rates and, conversely, long-short spreads are significant when regressions are conditioned on the short-term interest rates.[13]

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[11]For example, compare equation (73) to equation (74) in the appendix.

[12]The sensitivity of long-horizon forecasts to alternative modelling assumptions regarding time-variation in conditional equilibria is illustrated in Kozicki and Tinsley (2001a, 2001b) and in section 5 below.

[13]The explanatory role of credit risk premiums in private borrowing rates is also empirically supported in Kozicki and Tinsley (2002) but ignored in the current paper.

3.3 The responsiveness of historical bond rates to macro variables

If perceptions of bond traders are an essential link in the transmission of monetary policy then section 2 indicates that the determinacy of policy rests on the sensitivity of nominal bond rates to inflation.

The long-run responses of nominal bond rates to inflation, the level of unemployment, and the difference of unemployment are reported in Table 1. Inflation is measured by the annualized first-difference of the log deflator for personal consumption expenditures (pce), and unemployment by the civilian unemployment rate. The bond rates are the nominal rates on 1-year, 3-year, 5-year and 10-year zero-coupon bonds from McCulloch and Kwon (1993). Regressions are reported for two monthly samples.

Regressions reported in the bottom panel of Table 1 span the period after the abandonment of nonborrowed reserves targeting to the end of the FOMC chairmanship of Paul Volcker, 1982m1 - 1987m7. The results are consistent with bond trader forecasts of aggressive policy responses to inflation. Long-run mean responses by bond rates to inflation are well above unity for all maturities. Significant long-run mean responses are also indicated for the change in unemployment by 1-year and 3-year bond rates. No mean responses to the level of unemployment are significant.

The top panel in Table 1 reports on regressions for a 1966-79 sample, ending just prior to the announcement of the well-known shift in operational policy in October 1979.[14] The second column in the top panel indicates that the mean long-run response to inflation is above unity for bond rates of all maturities. Perhaps consistent with bond trader perceptions of the emphasis of 1970s central bank policy on money growth targeting, the mean long-run response to the change of the unemployment rate is also significant for all maturities.

As the results of Table 1 are consistent with elastic bond rate responses to inflation in both samples, it appears that the lower bound condition for determinacy of monetary policy derived in section 2 is satisfied in the 1960s and 1970s, as well as in the 1980s. However, bond trader perceptions that, on average, monetary policy was consistent with stable policy responses to inflation appears to contradict the evidence of passive policy rate responses in the 1960s and 1970s in such studies as Clarida, Gali, and Gertler (2000) and Lubik and Schorfeide (2004). Possible resolutions of this puzzle are explored in the remaining sections of this paper.

[14]The federal funds market was not well-developed prior to 1966, vid. Tinsley et al. (1982) and Fuhrer (1996).

4 No-arbitrage bond pricing under horizon-dependent perceptions

The lower bound for determinacy of monetary policy derived in section 2 requires that equilibrium deviations in nominal bond rates should be elastic with respect to equilibrium deviations in expected inflation over the maturities of the bonds or, equivalently, that equilibrium deviations in ex ante real interest rates should respond positively to deviations in expected inflation.

Examples in preceding sections have implicitly assumed that term premiums of bond rates are constant over time and drop out of equilibrium deviations. An alternative is to allow for term premiums that may vary systematically with macro variables.

To establish notation, the nominal interest rate on an n-period zero coupon bond is denoted

$$\begin{aligned} r_{n,t} &= \frac{1}{n}\sum_{i=0}^{n-1} f_{t,i}, \\ &= \frac{1}{n}\sum_{i=0}^{n-1}(E_t r_{t,i} + \psi_{t,i}), \\ &= \frac{1}{n}\sum_{i=0}^{n-1} E_t r_{t,i} + \Psi_{n,t}. \end{aligned} \tag{32}$$

The first line of (32) indicates that the nominal bond rate is the average of forward rates, $f_{t,i}$, over the lifetime of the bond. The second line shows that the forward rate in the ith period of the n-period forecast horizon is equal to the expected policy rate in the ith period, $E_t r_{t,i}$, plus a possibly time-varying forward rate term premium, $\psi_{t,i}$. The last line shows that the term premium of the n-period bond rate, $\Psi_{t,n}$, is equal to the average of the forward rate term premiums.[15]

As bond rates consist not only of expected policy rate averages but averages of possibly time-varying forward rate term premiums, one potential explanation of elastic responses of nominal bond rates to inflation in the 1960s and 1970s is that time-varying term premiums may operate as automatic stabilizers, reducing the effective lower bound required for determinate policy rate responses. This explanation rests on systematic positive responses of forward rate term premiums to expected inflation.

The next two subsections sketch a class of no-arbitrage models of bond rate pricing, developed by Duffee (2002), that provides a tractable formulation of term premium responses to macro determinants of policy rates, such as inflation.[16]This model is extended in the remaining subsections to allow for horizon-dependent

[15]In the terminology of Shiller (1990), $\Psi_{n,t}$ is the rollover term premium.

[16]Recent examples of empirical estimates of the term structure exploring macro variable determinants of term premiums include Ang and Piazzesi (2003), Rudebusch and Wu (2004a, 2004b), Ang, Dong, and Piazzesi (2005), Duffee (forthcoming), and Dewachter and Lyrio (forthcoming(a) and forthcoming(b)).

expectations.

4.1 *Macro variable dynamics*

Explicit macro variables are introduced to characterize bond trader perceptions of expected policy rate responses. Bond traders may also price unobserved variables. Latent factors associated with additional determinants of bond prices are assumed to be orthogonal to the observed macro factors and irrelevant for the current analysis.

The dynamics of macro variables are described by a first-order companion form system

$$X_t = \phi + \Phi X_{t-1} + \varepsilon_t, \tag{33}$$

where the stochastic driving vector, ε_t, is denoted by

$$\varepsilon_t \equiv \Sigma\epsilon_t, \qquad \epsilon_t \sim N(0, I).$$

The X vector contains observations on observable macro determinants of the policy rate, such as unemployment, u, and inflation, π.[17] Each macro variable is partitioned into its perceived equilibrium attractor or "natural rate," such as $(\bar{\pi}_t, \bar{u}_t)$, and deviations from these natural rates.[18]

$$\begin{aligned} \pi_t &\equiv \bar{\pi}_t + \tilde{\pi}_t, \\ u_t &\equiv \bar{u}_t + \tilde{u}_t, \end{aligned} \tag{34}$$

Variation over time in the natural rate perceptions of bond traders is captured by constant-gain learning equations,

$$\begin{aligned} \bar{\pi}_t &= \gamma_{\bar{\pi}}\pi_{t-1} + (1-\gamma_{\bar{\pi}})\bar{\pi}_{t-1} + \varepsilon_{\bar{\pi},t}, \\ \bar{u}_t &= \gamma_{\bar{u}}u_{t-1} + (1-\gamma_{\bar{u}})\bar{u}_{t-1} + \varepsilon_{\bar{u},t}. \end{aligned} \tag{35}$$

Time-variation in the natural rate deviations of inflation and unemployment is captured by a pth-order, bivariate vector autoregression,

$$\begin{aligned} \tilde{\pi}_t &= a_{11,1}\tilde{\pi}_{t-1} + \sum_{i=1}^{p-1} a_{11,i+1}\Delta\tilde{\pi}_{t-i} + a_{12,1}\tilde{u}_{t-1} + \sum_{i=1}^{p-1} a_{12,i+1}\Delta\tilde{u}_{t-i} + \varepsilon_{\tilde{\pi},t}, \\ \tilde{u}_t &= a_{21,1}\tilde{\pi}_{t-1} + \sum_{i=1}^{p-1} a_{21,i+1}\Delta\tilde{\pi}_{t-i} + a_{22,1}\tilde{u}_{t-1} + \sum_{i=1}^{p-1} a_{22,i+1}\Delta\tilde{u}_{t-i} + \varepsilon_{\tilde{u},t}., \end{aligned} \tag{36}$$

[17] Additional macro determinants might include observable indicators of disturbances in credit markets.

[18] In a slight abuse of conventional terminology, it is convenient to refer to the central bank target for inflation perceived by bond traders, $\bar{\pi}_t$, as the "natural rate" for inflation.

If the macro system is stable, each macro variable reverts to its natural rate in the long run.

4.2 *No-arbitrage bond pricing under invariant policy responses*

As noted earlier, elastic responses of U.S. Treasury bond rates to inflation in the 1960s and 1970s seem difficult to square with evidence from studies indicating the passivity of US monetary policy in those decades. One possible explanation is that bond term premiums may be sufficiently responsive to inflation to overcome a modest inelasticity in policy rate responses.

The policy rate in period $t+h$, anticipated by bond traders in period t, $E_t r_{t,h}$, is assumed to be a linear function of macro variables anticipated for $t+h$,

$$E_t r_{t,h} \quad = \quad \delta_0 + \delta_1' E_t X_{t+h}. \tag{37}$$

The vector of response parameters, δ_1, is fixed, both over time and over forecast horizons.

In the absence of arbitrage, the price of a multiperiod asset that does not pay dividends is determined by the expected product of stochastic discount factors, M_{t+i}, over the lifetime of the asset. In the case of a zero-coupon, n-period nominal bond paying \$1 at maturity, the current price is

$$\begin{aligned} P_{n,t} &= E_t\{M_{t+1} M_{t+2} \dots M_{t+n}\}, \\ &= E_t\{M_{t+1} P_{n-1,t+1}\}, \end{aligned} \tag{38}$$

where the last line in (38) follows by the law of iterated expectations.

If investors are risk neutral, the one-period stochastic discount factor reduces to

$$M_{t+1} \quad = \quad exp(-r_t), \tag{39}$$

where r_t is the policy rate, known in t. However, if investors are risk sensitive, the stochastic discount factor is

$$M_{t+1} \quad = \quad exp(-r_t)\xi_{t+1}, \tag{40}$$

where ξ_{t+1} is the Radon-Nikodym derivative that translates the distribution of the discounted asset price to a martingale by removing predictable drift due to bond risk premiums.[19]

In the case of log-normal bond prices, the relevant Radon-Nikodym derivative takes the form

$$\xi_{t+1}(\lambda_t) \quad = \quad exp(-\lambda_t' \epsilon_{t+1} - \frac{1}{2}\lambda_t'\lambda_t), \tag{41}$$

[19] Change of drift under the Girsanov theorem is discussed in Duffie (1996).

where λ denotes the vector of market prices of uncertainty associated with the stochastic determinants of the asset price.[20] The vector of market prices is assumed to be the essentially affine formulation suggested by Duffee (2002),

$$\lambda_{t+i} \quad = \quad \lambda_0 + \lambda_1' X_{t+i}. \tag{42}$$

Given nontrivial risk pricing, the bond traders' risk-adjusted view of the dynamics of the macro economy is denoted by

$$X_t \quad = \quad \breve{\phi} + \breve{\Phi} X_{t-1} + \Sigma \breve{\epsilon}_t, \tag{43}$$

which differs from the empirical (aka physical) dynamics in (33). The difference is demonstrated by deriving the bond price valuation under the risk-neutral probability measure, using the risk-adjusted macro model in (43).

As in Campbell, Lo and MacKinlay (1997), discrete-time bond prices in this Gaussian affine model of the term structure can be represented by

$$P_{n,t} \quad = \quad exp(-A_n - B_n' X_t). \tag{44}$$

As noted above, in the absence of arbitrage, the price of an n-maturity bond is determined by

$$\begin{aligned} P_{n,t} \quad &= \quad E_t(e^{-r_t} \xi_{t+1} P_{n-1,t+1}), \\ &= \quad E_t^{rn}(e^{-r_t} P_{n-1,t+1}), \end{aligned} \tag{45}$$

where the expectation in the first line of (45) is under the physical probability measure, and the expectation in the second line is under the risk-neutral probability measure.

Substitute from (44) and (43) into the second line of (45) and take expectations under the risk-neutral probability measure to give,

$$exp(-A_n - B_n' X_t) \quad = \quad exp(-r_t - A_{n-1} - B_{n-1}'(\breve{\phi} + \breve{\Phi} X_t) + \frac{1}{2} B_{n-1}' \Sigma \Sigma' B_{n-1}).$$

Finally, substitute for the policy rate from the perceived policy response equation, (37), and take logs to give

$$A_n + B_n' X_t \quad = \quad A_{n-1} + B_{n-1}'(\breve{\phi} + \breve{\Phi} X_t) - \frac{1}{2} B_{n-1}' \Sigma \Sigma' B_{n-1} + \delta_0 + \delta_1' X_t. \tag{46}$$

[20] If the probability density of the macro shocks, $f(\epsilon_{t+1})$, is Gaussian, the translated density is also Gaussian, $\xi_{t+1}(\lambda_t) f(\epsilon_{t+1}) = f(\epsilon_{t+1} + \lambda_t)$, where the means are shifted by λ_t.

Equating constant terms and slope coefficients on both sides of the equal sign in (46) provides recursions for the bond pricing coefficients under risk-neutral dynamics

$$\begin{aligned} A_n &= A_{n-1} + B_{n-1}'\breve{\phi} - \frac{1}{2}B_{n-1}'\Sigma\Sigma' B_{n-1} + \delta_0, \\ B_n' &= B_{n-1}'\breve{\Phi} + \delta_1'. \end{aligned} \tag{47}$$

The initial conditions are $A_1 = \delta_0$ and $B_1' = \delta_1'$.

Alternatively, bond pricing recursions using the physical dynamics in (33) can be recovered by adjusting the stochastic disturbances, ϵ. Recall that the effect of the Radon-Nikodym differential in (45) is to shift the mean of the disturbances, ϵ_t, by the relevant pricing vector, λ_{t-1}. The risk-neutral disturbances from this density translation are

$$\breve{\epsilon}_t = \epsilon_t + \lambda_{t-1}. \tag{48}$$

Substituting (48) into the empirical macro system (33) and using the essentially affine definition of the pricing vector, λ, in (42) illustrates the risk-adjusted macro perceptions of the bond traders

$$\begin{aligned} X_t &= \phi + \Phi X_{t-1} + \Sigma\epsilon_t, \\ &= \phi + \Phi X_{t-1} + \Sigma(\breve{\epsilon}_t - \lambda_{t-1}), \\ &= (\phi - \Sigma\lambda_0) + (\Phi - \Sigma\lambda_1')X_{t-1} + \Sigma\breve{\epsilon}_t, \\ &= \breve{\phi} + \breve{\Phi}X_{t-1} + \Sigma\breve{\epsilon}_t. \end{aligned} \tag{49}$$

Thus, the bond price recursions in (47) can be restated using the explicit risk-adjusted parameters of the empirical macro model, where $\breve{\phi} = \phi - \Sigma\lambda_0$ and $\breve{\Phi} = \Phi - \Sigma\lambda_1'$.

The notation convention is that negative entries of λ_1 contribute towards positive risk premiums. Consequently, as can be seen in the third line of (49), negative elements in λ_1 will increase the sensitivity of bond rates to variations in the macro variables, X_t.[21]

[21] In the absence of an explicit specification of investor utility functions, no theoretical restrictions are imposed on the λ_1' matrix. As the dimensions of the pricing matrix can be large, empirical investigations of essentially affine formulations of asset pricing often impose zero restrictions on elements of the λ_1' matrix, such as Ang and Piazzesi (2003), Ang, Dong, and Piazzesi (2005), and Dewachter and Lyrio (forthcoming(a) and forthcoming(b)). Kim and Orphanides (2005) suggest fewer zero restrictions are required if measurements include both bond rate data and surveys of interest rate forecasts over short and long horizons. Depending on the structure of the λ_1 matrix, term premium variation linked to a variable may not reflect uncertainty in that variable. For example, suppose $\Sigma\lambda_1' X_t$ is a 2×1 vector,

$$\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{21} \end{bmatrix} \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} = \begin{bmatrix} s_{11}\lambda_{11} + s_{12}\lambda_{21} & s_{11}\lambda_{12} + s_{12}\lambda_{22} \\ s_{21}\lambda_{11} + s_{22}\lambda_{21} & s_{21}\lambda_{12} + s_{22}\lambda_{22} \end{bmatrix} \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix}.$$

Note that responses of the term premium to movements in $x_{2,t}$ may not be related to the scale of the x_2 shock, s_{22}.

The adjustments to the slope coefficients of X_t will also alter the dynamics of the risk-adjusted macro model. To see this, denote the risk-adjusted transition matrix as

$$\begin{aligned} \breve{\Phi} &= \Phi - \Sigma\lambda_1', \\ &\equiv \Phi + \Phi^*. \end{aligned}$$

If both the Φ and Φ^* matrices were diagonal, it is easy to see that positive elements of Φ^* would increase the eigenvalues of the matrix sum, $\breve{\Phi}$. Or if the trace of Φ^* is positive, the sum of the eigenvalues of $\breve{\Phi}$ is increased. Although it is difficult to generalize the dynamic effects of risk pricing for unrestricted matrices, it can be shown[22] that upper bounds for singular values of $\breve{\Phi}$ are

$$\sigma_{i+j-1}(\breve{\Phi}) \leq \sigma_i(\Phi) + \sigma_j(\Phi^*), \qquad i+j < q+1, \tag{50}$$

where q denotes the minimum rank of (Φ, Φ^*), and $\sigma_k(A)$ denotes the singular values of the matrix A, in decreasing order, $\sigma_1 > \sigma_2 > \ldots > \sigma_q$. For example, when $i = j = 1$, the sum of the largest singular values of Φ and Φ^* is an upper bound for the largest singular value of $\breve{\Phi}$.

4.3 *Denoting horizon-dependent perceptions*

A second potential explanation of elastic bond rate responses to inflation is that bond trader perceptions of anticipated policy responses may vary over forecast horizons. In other words, even if current policy is perceived to be insufficiently responsive to inflation, bond traders may assume that future policy will be more attentive.

The principal difference in specification is that the coefficients of the perceived policy response may vary over the forecast horizon,

$$E_t r_{t,h} = \delta_{0,h} + \delta_{1,h}' E_t X_{t+h}, \tag{51}$$

where (51) replaces the policy rate specification shown earlier in (37).

Again, the dynamics of macro variables are described by a first-order companion form system, but it will ease notation considerably to include a constant regressor in the X vector,

$$X_t = \Phi X_{t-1} + \Sigma\epsilon_t,$$

[22] Horn and Johnson (1991). For a general matrix A, singular values are nonnegative square roots of the nonzero eigenvalues of A^*A or AA^*. Thus, the inequalities in (50) hold for eigenvalues if the relevant matrices are positive semidefinite.

This altered notation allows horizon-dependent responses to be compactly represented as

$$E_t r_{t,h} = \delta'_{h+1} E_t X_{t+h}, \tag{52}$$

where h indexes periods in the forecast horizon, $h = 0, \ldots, H$.

The pricing kernel is

$$M_{t+i} = exp[-\delta'_i X_{t+i-1} - \lambda'_{t+i-1}\epsilon_{t+i} - \frac{1}{2}\lambda'_{t+i-1}\lambda_{t+i-1}], \quad i = 1, 2, \ldots, \tag{53}$$

and the price of risk is the essentially affine formulation

$$\lambda_{t+i} = \lambda_0 + \lambda'_1 X_{t+i}.$$

The nonrecursive formulation of no-arbitrage bond prices is

$$\begin{aligned} P_{n,t} &= E_t(\prod_{i=1}^{n} M_{t+i}), \\ &= exp(-A_n - B'_n X_t). \end{aligned} \tag{54}$$

Due to the horizon-dependent structure of the policy rate in (52), the no-arbitrage coefficient sequence, $(A_i, B_i,\ i = 1, \ldots, n)$, is altered from that shown in (47). The revised structure of bond price coefficients is illustrated by the following examples.

Under the assumption that X_t is observable in t, the price of a one-period bond is

$$\begin{aligned} P_{1,t} &= E_t M_{t+1}, \\ &= E_t exp[-\delta'_1 X_t - \frac{1}{2}\lambda'_t\lambda_t - \lambda'_t\epsilon_{t+1}], \\ &= exp[-\delta'_1 X_t - \frac{1}{2}\lambda'_t\lambda_t + \frac{1}{2}Var([\lambda_0 + \lambda'_1 X_t]'\epsilon_{t+1})], \\ &= exp[-\delta'_1 X_t], \\ &= exp[-A_1 - B'_1 X_t]. \end{aligned} \tag{55}$$

Consequently,

$$\begin{aligned} A_1 &= 0 \\ B_1 &= \delta_1 \end{aligned}$$

The price of a two-period bond is

$$P_{2,t} = E_t(M_{t+1}M_{t+2}),$$

$$
\begin{aligned}
&= E_t exp[-\delta_1' X_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1} - \delta_2' X_{t+1} - \frac{1}{2}\lambda_{t+1}'\lambda_{t+1} - \lambda_{t+1}'\epsilon_{t+2}], \\
&= exp[-\delta_1' X_t - \frac{1}{2}\lambda_t'\lambda_t - \delta_2'\Phi X_t - \frac{1}{2}\lambda_{t+1}'\lambda_{t+1} \\
&\qquad + \frac{1}{2}Var([\delta_2'\Sigma + \lambda_t']\epsilon_{t+1} + \lambda_{t+1}'\epsilon_{t+2})], \\
&= exp[\frac{1}{2}\delta_2'\Sigma\Sigma'\delta_2 + \delta_2'\Sigma\lambda_0 - (\delta_1' + \delta_2'(\Phi - \Sigma\lambda_1'))X_t], \\
&= exp[-A_2 - B_2' X_t], \qquad (56)
\end{aligned}
$$

where

$$
\begin{aligned}
A_2 &= A_1 - \frac{1}{2}\delta_2'\Sigma\Sigma'\delta_2 - \delta_2'\Sigma\lambda_0, \\
B_2' &\equiv B_1' + \delta_2'[\Phi - \Sigma\lambda_1']. \qquad (57)
\end{aligned}
$$

Note that the new terms are all functions of the policy responses in the second period of the forecast horizon, δ_2. This pattern persists in future periods.

More general non-recursive expressions for A_n and B_n in (54) can be derived as follows:

$$
\begin{aligned}
P_{n,t} &= E_t \ exp\{-\sum_{i=1}^{n}[\delta_i' X_{t+i-1} + \frac{1}{2}\lambda_{t+i-1}'\lambda_{t+i-1} + \lambda_{t+i-1}'\epsilon_{t+i}]\}, \\
(iterated\ expectations) &= E_t \ \cdots \ E_{t+n-1} \ exp\{-\sum_{i=1}^{n}[\delta_i' X_{t+i-1} + \frac{1}{2}\lambda_{t+i-1}'\lambda_{t+i-1} + \lambda_{t+i-1}'\epsilon_{t+i}]\} \\
(decrement\ sum) &= E_t \ \cdots E_{t+n-1} \ exp\{-\sum_{i=1}^{n-1}[\delta_i' X_{t+i-1} + \frac{1}{2}\lambda_{t+i-1}'\lambda_{t+i-1} + \lambda_{t+i-1}'\epsilon_{t+i}] \\
&\qquad -\delta_n' X_{t+n-1} - \frac{1}{2}\lambda_{t+n-1}\lambda_{t+n-1} - \lambda_{t+n-1}\epsilon_{t+n}\} \\
(conditional\ E_{t-n-1}) &= E_t \ \cdots E_{t+n-2} \ exp\{-\sum_{i=1}^{n-1}[\delta_i' X_{t+i-1} + \frac{1}{2}\lambda_{t+i-1}'\lambda_{t+i-1} + \lambda_{t+i-1}'\epsilon_{t+i}] - \delta_n' X_{t+n-1}\} \\
(decrement\ sum) &= E_t \ \cdots E_{t+n-2} \ exp\{-\sum_{i=1}^{n-2}[\delta_i' X_{t+i-1} + \frac{1}{2}\lambda_{t+i-1}'\lambda_{t+i-1} + \lambda_{t+i-1}'\epsilon_{t+i}] \\
&\qquad -\delta_n'(\Phi X_{t+n-2} + \Sigma\epsilon_{t+n-1}) - \delta_{n-1}' X_{t+n-2} - \frac{1}{2}\lambda_{t+n-2}'\lambda_{t+n-2} - \lambda_{t+n-2}'\epsilon_{t+n-1}\} \\
(conditional\ E_{t+n-2}) &= E_t \ \cdots E_{t+n-3} \ exp\{-\sum_{i=1}^{n-2}[\delta_i' X_{t+i-1} + \frac{1}{2}\lambda_{t+i-1}'\lambda_{t+i-1} + \lambda_{t+i-1}'\epsilon_{t+i}] \\
&\qquad -(\delta_n'\Phi + \delta_{n-1}')X_{t+n-2} - \frac{1}{2}\lambda_{t+n-2}'\lambda_{t+n-2} + \frac{1}{2}Var_{t+n-2}[(\lambda_{t+n-2}' + \delta_n'\Sigma)\epsilon_{t+n-1}]\} \\
&= E_t \ \cdots E_{t+n-3} \ exp\{-\sum_{i=1}^{n-2}[\delta_i' X_{t+i-1} + \frac{1}{2}\lambda_{t+i-1}'\lambda_{t+i-1} + \lambda_{t+i-1}'\epsilon_{t+i}] \\
&\qquad -(\delta_n'\Phi + \delta_{n-1}')X_{t+n-2} + \frac{1}{2}\delta_n'\Sigma\Sigma'\delta_n + \delta_n'\Sigma\lambda_0 + \delta_n'\Sigma\lambda_1' X_{t+n-2}\}
\end{aligned}
$$

$$
\begin{aligned}
&= E_t \cdots E_{t+n-3}\, exp\{-\sum_{i=1}^{n-2}[\delta_i' X_{t+i-1} + \frac{1}{2}\lambda_{t+i-1}'\lambda_{t+i-1} + \lambda_{t+i-1}'\epsilon_{t+i}] \\
&\quad -(\delta_n'(\Phi - \Sigma\lambda_1') + \delta_{n-1}')X_{t+n-2} + \frac{1}{2}\delta_n'\Sigma\Sigma'\delta_n + \delta_n'\Sigma\lambda_0\} \\
(decrement\ sum) &= E_t \cdots E_{t+n-3}\, exp\{-\sum_{i=1}^{n-3}[\delta_i' X_{t+i-1} + \frac{1}{2}\lambda_{t+i-1}'\lambda_{t+i-1} + \lambda_{t+i-1}'\epsilon_{t+i}] \\
&\quad +\frac{1}{2}\delta_n'\Sigma\Sigma'\delta_n + \delta_n'\Sigma\lambda_0 \\
&\quad -(\delta_n'(\Phi - \Sigma\lambda_1') + \delta_{n-1}')X_{t+n-2} - \delta_{n-2}'X_{t+n-3} - \frac{1}{2}\lambda_{t+n-3}'\lambda_{t+n-3} - \lambda_{t+n-3}'\epsilon_{t+n-2}\} \\
(conditional\ E_{t+n-3}) &= E_t \cdots E_{t+n-4}\, exp\{-\sum_{i=1}^{n-3}[\delta_i' X_{t+i-1} + \frac{1}{2}\lambda_{t+i-1}'\lambda_{t+i-1} + \lambda_{t+i-1}'\epsilon_{t+i}] \\
&\quad +\frac{1}{2}\delta_n'\Sigma\Sigma'\delta_n + \delta_n'\Sigma\lambda_0 \\
&\quad -((\delta_n'(\Phi - \Sigma\lambda_1') + \delta_{n-1}')\Phi + \delta_{n-2}')X_{t+n-3} - \frac{1}{2}\lambda_{t+n-3}'\lambda_{t+n-3} \\
&\quad +\frac{1}{2}Var_{t+n-3}[(\lambda_{t+n-3}' + (\delta_n'(\Phi - \Sigma\lambda_1') + \delta_{n-1}')\Sigma)\epsilon_{t+n-2}] \\
&= E_t \cdots E_{t+n-4}\, exp\{-\sum_{i=1}^{n-3}[\delta_i' X_{t+i-1} + \frac{1}{2}\lambda_{t+i-1}'\lambda_{t+i-1} + \lambda_{t+i-1}'\epsilon_{t+i}] \\
&\quad -(\delta_n'(\Phi - \Sigma\lambda_1')^2 + \delta_{n-1}'(\Phi - \Sigma\lambda_1') + \delta_{n-2}')X_{t+n-3} \\
&\quad +\delta_n'\Sigma\lambda_0 + (\delta_n'(\Phi - \Sigma\lambda_1') + \delta_{n-1}')\Sigma\lambda_0 \\
&\quad +\frac{1}{2}\delta_n'\Sigma\Sigma'\delta_n + \frac{1}{2}(\delta_n'(\Phi - \Sigma\lambda_1') + \delta_{n-1}')\Sigma\Sigma'(\delta_n'(\Phi - \Sigma\lambda_1') + \delta_{n-1}')'\} \\
&= \ldots. \qquad (58)
\end{aligned}
$$

Thus,

$$
\begin{aligned}
A_n &\equiv -\sum_{i=2}^{n}[(\sum_{j=i}^{n}\delta_j'(\Phi - \Sigma\lambda_1')^{j-i})\Sigma[\Sigma'(\sum_{j=i}^{n}\frac{1}{2}\delta_j'(\Phi - \Sigma\lambda_1')^{j-i})' + \lambda_0]], \\
B_n' &\equiv \sum_{i=1}^{n}\delta_i'(\Phi - \Sigma\lambda_1')^{i-1} \\
&= B_{n-1}' + \delta_n'(\Phi - \Sigma\lambda_1')^{n-1}. \qquad (59)
\end{aligned}
$$

4.4 *No-arbitrage formulations of forward rates*

The formulations in (59) are particularly convenient for analysis of forward rates. The forward rate, f_{n-1}, is

$$
f_{t,n-1} = E_t r_{t,n-1} + \psi_{t,n-1},
$$

$$
\begin{aligned}
&= p_{n-1,t} - p_{n,t} \\
&= A_n - A_{n-1} + (B_n - B_{n-1})' X_t \\
&= C_n + \delta_n'(\Phi - \Sigma\lambda_1')^{n-1} X_t, \qquad (60)
\end{aligned}
$$

where the intercept in the last line of (60) is

$$
\begin{aligned}
C_n \equiv & -\sum_{i=2}^{n} [\delta_n'(\Phi - \Sigma\lambda_1')^{n-i}\Sigma[\Sigma'(\sum_{j=i}^{n} \frac{1}{2}\delta_j'(\Phi - \Sigma\lambda_1')^{j-i})' + \lambda_0] \\
& + (\sum_{j=i}^{n-1} \delta_j'(\Phi - \Sigma\lambda_1')^{j-i})\Sigma\Sigma'(\frac{1}{2}\delta_n'(\Phi - \Sigma\lambda_1')^{n-i})'].
\end{aligned}
$$

Thus, with horizon-dependent perceptions of policy responses, the effective coefficient vector of the X_t regressors for the forward rate, $f_{t,h-1}$, is

$$
\delta_h'(\Phi - \Sigma\lambda_1')^{h-1} \equiv \delta_h'(I - \Sigma\lambda_1'\Phi^{-1})^{h-1}\Phi^{h-1},
$$

permitting exploration of "Taylor rule" regressions for forward rates with possibly horizon-varying coefficients, $\delta_h',\ h = 1, 2, \ldots$. In the absence of term premiums, the forward rate regression reduces to

$$
\begin{aligned}
f_{t,h-1} &= \delta_h'\Phi^{h-1} X_t, \\
&= \delta_h' E_t X_{t+h-1}.
\end{aligned}
$$

Thus, forward rate term premiums are defined by

$$
\psi_{t,h} = C_{h+1} + \delta_{h+1}'(\Phi - \Sigma\lambda_1')^h X_t - \delta_{h+1}'\Phi^h X_t. \qquad (61)
$$

Recalling the discussion of the singular values of the risk-adjusted transition matrix in (50), if the effect of the risk pricing matrix, λ_1', is such that $(\Phi - \Sigma\lambda_1')^h$ is slower to decay than Φ^h, the slope contributions of forward rate term premiums will increase over the forecast horizon with h.[23]

5 Empirical responses of forward rates

The possibility of time-varying forward rate term premiums and horizon-dependent expectations of future policy rate responses suggests that forward rate responses to expectations of future macro variables may differ from the current-period policy rate responses estimated by Taylor rules.

This section discusses empirical estimates of forward rate regressions for monthly samples before and after the major shift in US monetary policy in October 1979.

[23]The Jordan form of a matrix, $A = P\Lambda P^{-1}$, implies $A^i = P\Lambda^i P^{-1}$.

5.1 *Forecast model and learning rate assumptions*

A direct test of the combined effects of term premium and expected policy rate responses is to estimate forward rate response equations over different forward horizons. For monthly observations, the instantaneous forward rates at twelve month intervals in the forecast horizon are represented by

$$f_{t,12h} \quad = \quad (1-\rho_1)f^*_{t,12h} + \rho_1 f_{t,12(h-1)} + \rho_2 \Delta^{(k)} f_{t,12(h-1)} + a_{t,12h}, \tag{62}$$

where the forward rate associated with the bond trader expectation of the policy rate in the absence of policy lag adjustments is

$$f^*_{t,12h} \quad = \quad E_t\{c_0 + c_1\bar{\pi}_t + c_2\tilde{\pi}^{(\bar{k})}_{t,12h} + c_3\tilde{u}_{t,12h} + c_4\Delta^{(k)}u_{t,12h}\}.$$

As before, $\bar{\pi}_t$ denotes the bond trader perception in period t of the central bank target for inflation; $\tilde{\pi}_{t,12h}$ is the projected deviation of inflation in the $12h^{th}$ month of the forecast horizon; $\tilde{u}_{t,12h}$ is the projected deviation of unemployment from bond trader perceptions of the unemployment natural rate, $\bar{u}_t$; and the superscripts, $(\bar{k})$ and (k), denote k-period averages and k-period summations, respectively.[24] For monthly data, $k = 12$.

As the specification in (62) is amenable to direct regression, it is straight-forward to check if estimates of combined responses, such as c_2, are consistent with stable bond rate responses, and if responses vary over different partitions of the forecast horizon. The regressions reported here do not impose the cross-equation restrictions implied by no-arbitrage, as derived in section 4, on the forward rate regressions of different horizons. Consequently, it is not possible to determine what proportions of combined responses are due to forward rate term premium responses, $\Sigma\lambda_1'$, or to expected policy rate responses, δ_h. There is one exception: under the physical probability measure, the expected response to the perceived inflation target is unity, $c_1 = 1$. Thus, significant deviations from $c_1 = 1$ indicate time-variation in forward rate term premiums due to a time-varying inflation target, $\bar{\pi}_t$.

Monthly predictions of expected inflation, $E_t\pi_{t,12h}$, and unemployment, $E_t u_{t,12h}$, are generated by a 12th-order VAR in inflation and unemployment, whose format was shown in (36).

Kozicki and Tinsley (2001a, 2001b) demonstrate that long-horizon predictions from VAR models are sensitive to specifications regarding the conditional equilibria of state variables. To illustrate differences in long-horizon forecasts, alternative estimates of ex ante real bond rates for 1-year, 5-year, and 10-year zero

[24]The k-period summation of the first-difference operator is the k-period difference, $\Delta^{(k)} = \Delta + \Delta L + \ldots + \Delta L^{k-1} = 1 - L^k$.

coupon bonds are shown in the three panels of Figure 2, along with the real federal funds rate, ρ_t. The multiperiod predictions of inflation used to adjust the nominal bond rates are generated by the alternative monthly VARs discussed in Kozicki and Tinsley (2001a).

The perceived equilibrium or central bank target for inflation is fixed at the sample mean in the top panel of Figure 2, $\bar{\pi}$. Under this specification, the 5-year and 10-year real bond rates appear to be trending up in the 1970s. In the middle panel of Figure 2, the perceived conditional equilibrium for inflation, $\bar{\pi}_t$, is similar to a Beveridge-Nelson unit root trend and closely tracks recent inflation. Consequently, ex ante real bond rates in the middle panel are much more volatile than those in the other two panels and fall sharply below zero in the first half of the 1970s. Inflation forecasts incorporated in the real rates shown in the bottom panel of Figure 2 reflect a change-point learning model of shifts in the conditional equilibrium, $\bar{\pi}_t$. Here, the ex ante real rates on 5-year and 10-year bonds appear to be without much of a discernible trend in the 1970s.

For the forward rate regressions reported below, the time-varying perceptions of bond traders for the central bank target for inflation, $\bar{\pi}_t$, and the natural rate for unemployment, $\bar{u}_t$, are represented by the constant-gain learning equations shown earlier in (35). Kozicki and Tinsley (2001b) indicate that a monthly gain of $\gamma_{\bar{\pi}} = .015$ provides an average approximation of private sector long-horizon forecasts of inflation in the 1980s. This benchmark constant-gain proxy for bond trader perceptions of the central bank target for inflation, $\bar{\pi}_t$, is shown in Figure 3, along with the 12-month moving average of pce inflation and the Hoey real-time survey of 5-10 year predictions of CPI inflation. As pce inflation tends to be somewhat less volatile than CPI inflation, the proxy is about one percentage point lower in early 1980 but then tracks the survey closely in the remainder of the 1980s. Reductions in survey predictions of long-horizon inflation and in the perceived inflation target lag considerably the fall of inflation in the early 1980s.

The same benchmark learning rate is assumed for bond trader perceptions of the natural rate of unemployment, $\gamma_{\bar{u}} = .015$. The associated constant-gain proxy for the natural rate of unemployment is shown in Figure 4, along with the historical unemployment rate and the Congressional Budget Office (2004) retrospective estimate of the natural rate. As with many real-time estimates of the natural rate of unemployment, the constant-gain proxy tracks below the retrospective CBO estimate in the 1970s, with an average underestimation error of about 1.25 percentage points in the first half of the 1970s before the error sharply diminishes in the remainder of the 1970s.

However, results in Kozicki and Tinsley (2001a, 2005b) indicate that learning rates need not be constant over time. Faster learning rates are more likely if agents perceive larger forecast errors for observable

variables and can reduce the real consequences of perception errors in episodes with a time-varying inflation target. But, faster constant-gain learning rates are inefficient in more tranquil periods, as larger responses to transient disturbances increase the dispersion of the ergodic distribution of perceived inflation targets about a fixed central bank target. Given the sensitivity of long-horizon forecasts to the specification of the perceived central bank target for inflation, as illustrated in Figure 2, we examine the effects of three constant-gain learning rates. In the case of a fixed inflation target, the learning rate is set to zero, $\gamma_{\bar{\pi}} = 0.0$, and the perceived inflation target is set to the sample mean. The benchmark perception of the central bank inflation target, shown in Figure 3, uses the constant-gain learning rate, $\gamma_{\bar{\pi}} = .015$, which implies a mean learning lag of about 5.5 years. Finally, a faster learning rate is also examined for perceptions of the central bank target for inflation, $\gamma_{\bar{\pi}} = .03$, with a mean learning lag of about 2.8 years.

5.2 *Forward rate regressions*

Forward rates at 1-year, 3-year, 5-year and 10-year horizons are shown in Figure 5. Although the forward rates generally move together, large differences can emerge, such as the spread of about 2.3 percentage points between the 10-year forward rate and the 1-year forward rate in December 1976. As forward rates at neighboring horizons tend to move closely, the forward rate regressions are grouped into three horizon partitions: 1-3 years, 4-6 years, and 7-10 years. This grouping assumes that perceived policy rate responses do not vary significantly within a partition. Because the term premium responses within a partition are not likely to be identical, unless they are zero, the regression residuals will be heteroskedastic reflecting deviations from estimated average responses.

Forward rate regressions for the three partitions are presented in Tables 2a and 2b for a pre-Volcker sample, 1966 m1 - 1979 m7.[25] Emphasis shadings of rows in the tables are judgemental, based in part on assumptions regarding the reasonableness of the estimated signs. In addition to requiring asymptotic t-ratios of 1.7 or higher, roughly corresponding to p-values less than .10, estimates of positive mean responses to unemployment or negative mean responses to inflation are not shaded. Although the signs of term premium responses may be opposite to the signs of expected policy rate responses, it is unlikely the combined response would overturn the direction of the expected policy rate response.[26]

[25]During this interval, the FOMC was chaired by William McChesney Martin, Jr, Arthur Burns, and G. William Miller.

[26]Buraschi and Jiltsov (2005) report positive term premiums for inflation, with larger premiums for longer maturities. Dewachter and Lyrio (forthcoming(b)) estimate positive term premiums for inflation that rise with maturity and negligible term premiums for GDP gaps. Positive term premiums are also estimated for a time-varying central tendency for inflation, similar to $\bar{\pi}_t$; these premiums also rise with maturity and are nearly triple the size of the term premiums for inflation. By contrast, Duffee (forthcoming) presents

Forecasts of inflation and unemployment regressors for results in Table 2a are generated by a VAR fit to the 1960-79 sample under the assumption of the benchmark learning rates, $\gamma_{\bar{\pi}} = \gamma_{\bar{u}} = .015$. The mean long-run response to the equilibrium deviation in inflation, $\tilde{\pi}_t$, is statistically insignificant for forward rates in the 1-3 year partition; is not statistically different from unity for forward rates in the 4-6 year partition; and is greater than unity for forward rates in the 7-10 year partition. Thus, the pattern of increasing responses over the forecast horizon is consistent with elastic responses to expected inflation by intermediate-maturity bond rates in the 1960s and 1970s.

The estimated forward rate responses to the perceived inflation target, $\bar{\pi}$, in Table 2a are around one in the 4-6 year partition but significantly less than unity in the 1-3 year and 7-10 year partitions. Under the physical probability measure, the expected coefficient of the inflation target is one, so this suggests forward rate term premiums either did not respond, or moved inversely, to the perceived inflation target during the 1960s and 1970s. Although mean responses to the forecast level or difference of the unemployment rate are negative, mean responses are not significant in the 1-3 and 4-6 year partitions.

Forward rate responses to expected inflation under alternative learning rates are examined in Table 2b for the pre-Volcker sample. The pattern of increasing inflation responses over the forecast horizon is relatively insensitive to variation in the assumed learning rate. However, determinacy of bond rate responses to expected inflation is better supported for perceptions of a fixed inflation target, $\gamma_{\bar{\pi}} = 0.0$, or the time-varying inflation target generated by the benchmark learning rate, $\gamma_{\bar{\pi}} = .015$.

Tables 3a and 3b present forward rate regressions for the sample, 1982 m1 - 1987 m7, a period that encompasses the last six and one-half years of the FOMC chairmanship of Paul Volcker but excludes the unusual interest rate volatility in 1979-81, during the experiment with nonborrowed reserves as the operating policy instrument. In Table 3a, the forward rate response to expected inflation, c_2 is not statistically different from unity in the 1-3 year partition, and greater than unity in the 4-6 year partition, although not significantly so. The mean inflation response in the 7-10 year partition is greater than unity but the associated p-value is marginally larger than .10.

As with the earlier sample, forward rates do not appear to consistently respond to forecasts of unemployment. However, in contrast to results in the pre-Volcker sample, the mean responses to the

evidence of negative term premium responses to inflation in a pre-Volcker sample. Note that negative term premium responses to inflation could conceivably reverse the historical roles of the inflation responses by the central bank and bond traders suggested in section 2. That is, system indeterminacy could occur if an elastic policy rate response to inflation is accompanied by inelastic bond rate responses to expected inflation.

perceived inflation target, c_1, in Table 3a are significantly greater than unity in the 1-3 year and 4-6 year partitions. This suggests forward rate term premiums responded positively to the perceived inflation target in the 1980s.

Forward rate responses to expected inflation in the 1980s under alternative learning rates are examined in Table 3b. Here, the pattern of increasing responses to expected inflation over the forecast horizon is statistically supported when the inflation target learning rate is equal to or exceeds the benchmark learning rate. Positive responses of forward rate term premiums to the perceived inflation target, $c_1 > 1$, are also indicated for the benchmark learning rate, $\gamma_{\bar{\pi}} = .015$, and the faster learning rate, $\gamma_{\bar{\pi}} = .03$.

Although the forward rate regressions provide only rough approximations of combined forward rate responses to macro variables, two results appear to be common to the pre-Volcker sample and the 1980s sample. First, forward rate responses to equilibrium deviations in inflation are generally larger at more distant horizons and often greater than one, consistent with elastic bond rate responses to inflation. Second, there is little evidence of systematic responses by forward rates to the level or difference of unemployment. A notable difference in sample results is that positive responses by forward rate term premiums to a time-varying inflation target are supported in the 1980s sample but not in the pre-Volcker sample.

6 Concluding remarks

Central bankers propose monetary policies but bond traders dispose, through expectations of future policy rates and term premiums that price uncertain outcomes. This paper examines several implications of bond rate transmission of monetary policy.

As discussed in section 2, some interpretations of the Great Inflation have focused on the stability of a Taylor rule description of the policy rate or on central bank assumptions regarding natural rates. If the bond rate is the transmission channel for monetary policy, these possible shortcomings in policy are not sufficient to assess the stability of the economy. The section shows that conditions for determinate equilibrium require a lower bound on bond rate responses to expected inflation.

The bond rate plays no essential dynamic role in models that exclude frictions on dynamic adjustment of real variables, such as the standard New Keynesian model. Section 3 illustrates the role of bond rates in dynamic output equations when endogenous or exogenous habits are introduced into household utility functions. Although dynamic expenditure equations are often formulated as functions only of the one-period

interest rate, the elimination of market bond rate observations substitutes the information set of the modeller for the more relevant information set of bond traders in long-horizon forecasts.

It is also noted that nominal bond rate responses to inflation are generally elastic with respect to inflation, even in the 1960s and 1970s when studies have demonstrated that the policy rate did not keep pace with inflation. This is puzzling as the forward rates of bond rates contain bond trader expectations of future policy rates. One possible explanation is that forward rate term premiums that price the uncertainty of expected inflation may operate as automatic stabilizers, reducing the lower bound requirement for expected policy rate responses. Another possibility is that bond trader expectations are horizon-dependent, so that longer-run expectations of policy are not cloned replications of current-period policy responses.

To accommodate these possibilities, the essentially affine model of no-arbitrage bond pricing is extended in section 4 to allow for horizon-dependent expectations of policy rate responses.

Forward rate regressions, presented in section 5, provide empirical support for the conjecture that forward rate responses at more distant horizons display larger long-run responses to equilibrium deviations in expected inflation. The regressions also suggest forward rate term premiums responded positively to perceptions of a time-varying inflation target in the 1980s but not in the 1960s and 1970s. Isolating the separate contributions of time-varying term premiums and horizon-dependent expectations of future policy rates will require imposing no-arbitrage cross-equation restrictions on the forward rate regressions.

If horizon-dependent perceptions are confirmed in future work, it would be useful to explore possible reasons for horizon dependency in expectations. If long-horizon expectations are merely inertial, that inertia can partially insulate the economy from poor monetary policies, as may have occurred in the 1960s and 1970s, but may also attenuate responses to new monetary policies.

Appendix

Determinate and indeterminate solutions for equation (5).

The first-order equation

$$\tilde{\pi}_t \;=\; \mu E_t \tilde{\pi}_{t+1} + b e_t,$$

is cast into a first-order companion form

$$A_1 y_{t+1} \;=\; A_0 y_t + a_e e_t + a_\eta \eta_t,$$

where $y'_{t+1} = [\tilde{\pi}_t, E_t\tilde{\pi}_{t+1}]$,

$$A_1 \;=\; \begin{bmatrix} 1 & -\mu \\ 1 & 0 \end{bmatrix},\; A_0 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},\; a_e = \begin{bmatrix} b \\ 0 \end{bmatrix},\; \text{and } a_\eta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Using

$$A_1^{-1} \;=\; \begin{bmatrix} 0 & 1 \\ -\frac{1}{\mu} & \frac{1}{\mu} \end{bmatrix},$$

the reduced form is

$$y_{t+1} \;=\; B_0 y_t + b_e e_t + b_\eta \eta_t, \tag{63}$$

where

$$B_0 \;=\; \begin{bmatrix} 0 & 1 \\ 0 & \frac{1}{\mu} \end{bmatrix},\; b_e = \begin{bmatrix} 0 \\ -\frac{b}{\mu} \end{bmatrix},\; \text{and } b_\eta = \begin{bmatrix} 1 \\ \frac{1}{\mu} \end{bmatrix}.$$

By inspection, the eigenvalues of B_0 are $(0, \frac{1}{\mu})$, and the Jordan decomposition of B_0 is

$$\begin{aligned} B_0 &= P\Lambda P^{-1}, \\ &= \begin{bmatrix} 1 & 1 \\ 0 & \frac{1}{\mu} \end{bmatrix} \begin{bmatrix} 0 & \\ & \frac{1}{\mu} \end{bmatrix} \begin{bmatrix} 1 & -\mu \\ 0 & \mu \end{bmatrix}. \end{aligned}$$

Multiplying equation (63) by P^{-1} expresses the reduced form as an explicit function of the system eigenvalues,

$$x_t \;=\; \Lambda x_{t-1} + g_t + f_t, \tag{64}$$

where

$$x_t \;=\; \begin{bmatrix} \tilde{\pi}_t - \mu E_t \tilde{\pi}_{t+1} \\ \mu E_t \tilde{\pi}_{t+1} \end{bmatrix},\; g_t = \begin{bmatrix} b e_t \\ -b e_t \end{bmatrix},\; \text{and } f_t = \begin{bmatrix} 0 \\ \eta_t \end{bmatrix}.$$

Accounting for the zero elements of Λ and the two forcing terms, g_t and f_t reduces the two equations in (64) to

$$\begin{aligned} x_{1,t} &= g_{1,t}, \\ x_{2,t} &= \frac{1}{\mu} x_{2,t-1} + g_{2,t} + f_{2,t}. \end{aligned} \tag{65}$$

case 1: $|\mu| < 1$

In this instance, the second equation in (65) is unstable. The solution of the first equation is

$$\begin{aligned} \tilde{\pi}_t &= bE_t \sum_{i=0}^{\infty} \mu^i e_{t+i}, \\ &= \frac{be_t}{1-\gamma\mu}, \end{aligned}$$

which implies $E_t\tilde{\pi}_{t+1} = \frac{b\gamma e_t}{1-\gamma\mu}$. Substituting this solution for expected inflation into the second equation of (65) gives

$$\begin{aligned} \eta_t &= \mu E_t\tilde{\pi}_{t+1} - E_t\tilde{\pi}_t + be_t, \\ &= \frac{\mu b\gamma e_t}{1-\gamma\mu} - \frac{b\gamma be_{t-1}}{1-\gamma\mu} + be_t, \\ &= \frac{b(e_t - \gamma e_{t-1})}{1-\gamma\mu}, \\ &= \frac{b\epsilon_t}{1-\gamma\mu}. \end{aligned}$$

Thus, when the nonzero root, $\frac{1}{\mu}$ is unstable, the inflation solution is unique and requires that rational forecast errors, η_t, are determined solely by the structural errors, ϵ_t.

case 2: $|\mu| > 1$

In this instance, the nonzero root, $\frac{1}{\mu}$, is stable and the second equation in (65) provides a solution for expected inflation,

$$E_t\tilde{\pi}_{t+1} = \frac{1}{\mu} E_{t-1}\tilde{\pi}_t - \frac{b}{\mu} e_t + + \frac{1}{\mu}\eta_t.$$

Substituting this solution into the first equation of (65) provides the solution for inflation, equation (7) in the text,

$$\tilde{\pi}_t = \frac{1}{\mu}\tilde{\pi}_{t-1} - \frac{b}{\mu} e_{t-1} + \eta_t. \tag{66}$$

Other than requiring that η_t be a martingale difference, the assumption of rational expectations does not impose additional restrictions on the forecast errors. In addition to containing an arbitrary exogenous

(sunspot) shock, s_t, the forecast error may also be correlated with the structural disturbance,

$$\eta_t \quad = \quad m\epsilon_t + s_t.$$

Thus, the reduced form generates multiple solutions associated with arbitrary sunspots, s_t. Realizations of inflation will depend on what mechanisms agents use to coordinate on particular sunspot shocks.[27]

Eigenvalues of the companion form for equation (12).

The second-order inflation equation

$$\tilde{\pi}_t \quad = \quad (1 - \frac{ab(c_1 - 1)}{2})E_t\tilde{\pi}_{t+1} - \frac{ab(c_2 - 1)}{2}E_t\tilde{\pi}_{t+2} + be_t,$$

is restated in the first-order companion form

$$A_1 y_{t+1} \quad = \quad A_0 y_t + a_e e_t + a_\eta \eta_t,$$

where $y'_{t+1} = [\tilde{\pi}_{t+1}, E_t\tilde{\pi}_{t+2}]$,

$$A_1 \quad = \quad \begin{bmatrix} 0 & -\frac{ab(c_2-1)}{2} \\ 1 & 0 \end{bmatrix},\ A_0 = \begin{bmatrix} 1 & -\frac{(2-ab(c_1-1))}{2} \\ 0 & 1 \end{bmatrix},\ a_e = \begin{bmatrix} b \\ 0 \end{bmatrix},\ \text{and } a_\eta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Using

$$A_1^{-1} \quad = \quad \begin{bmatrix} 0 & 1 \\ -\frac{2}{ab(c_2-1)} & 0 \end{bmatrix},$$

the reduced form is

$$y_{t+1} \quad = \quad B_0 y_t + b_e e_t + b_\eta \eta_t,$$

where

$$B_0 \quad = \quad \begin{bmatrix} 0 & 1 \\ -\frac{2}{ab(c_2-1)} & \frac{2-ab(c_1-1)}{ab(c_2-1)} \end{bmatrix}.$$

The text equations for the eigenvalues (λ_1, λ_2) in (14) are provided by the trace and determinant of B_0,

$$\begin{aligned} \lambda_1\lambda_2 \quad &= \quad \frac{2}{ab(c_2 - 1)}, \\ \lambda_1 + \lambda_2 \quad &= \quad \frac{2 - ab(c_1 - 1)}{ab(c_2 - 1)}. \end{aligned}$$

[27] Equation (5) is a scalar example of the NK class of forward-looking rational expectation models. In contrast to Clarida, Gali and Gertler (2000), who suggest that the 1970s inflation may be explained by sunspot equilibria under passive monetary policy, Honkapohja and Mitra (2004) indicate that sunspot equilibria generated by NK models with an unstable Taylor rule cannot be learned by adaptive regressions.

Bond rate transmission under second-order endogenous habit (vid. equation (27)).

To illustrate the dynamic adjustments associated with more complex specifications of endogenous habit, consider the case where log habit is a distributed lag of past log consumption,

$$ln\Gamma_t \;=\; \nu(L)lnC_{t-1}, \tag{67}$$

and $\nu(L)$ is a polynomial in the lag operator, L.[28] For 2nd-order habit, $\nu(L) = \nu_1 + \nu_2 L$, the utility function can be written as

$$U(C_t,.) \;=\; \frac{(C_t^{1-\nu_1-\nu_2}(\frac{C_t}{C_{t-1}})^{\nu_1}(\frac{C_t}{C_{t-1}}\frac{C_{t-1}}{C_{t-2}})^{\nu_2})^{1-\alpha}-1}{1-\alpha}, \tag{68}$$

which illustrates that the effect of higher-order endogenous habit in concave utility functions is to smooth higher-order differences in consumption.[29]

Using the second-order specification of endogenous habit, the first-order condition for consumption is

$$0 \;=\; C_t^{\alpha}\Gamma_t^{-(1-\alpha)} - \nu_1\beta C_{t+1}^{1-\alpha}\Gamma_{t+1}^{-(1-\alpha)}C_t^{-1} - \nu_2\beta^2 C_{t+2}^{1-\alpha}\Gamma_{t+2}^{-(1-\alpha)}C_t^{-1} - \Lambda_t. \tag{69}$$

Substituting in the definition of endogenous habit and log-linearizing about equilibria gives the linear Euler equation for output under endogenous habit

$$\tilde{y}_t \;=\; E_t\{[a_1(L+\beta F) + a_2(L^2+\beta^2F^2)]\tilde{y}_t - a_3\tilde{\lambda}_t\}, \tag{70}$$

where L and F are the lag and lead time series operators, respectively. The coefficients of (70) are identified by

$$a_1 \;=\; d^{-1}v_1(\alpha-1)(1-v_2\beta); \quad a_2 \;=\; d^{-1}v_2(\alpha-1); \quad a_3 \;=\; d^{-1}(1-v_1\beta-v_2\beta^2), \tag{71}$$

where $d \equiv \alpha + \beta v_1(v_1(\alpha-1)-1) + \beta^2 v_2(v_2(\alpha-1)-1)$.

The regressor, $\tilde{\lambda}_t$, in the last term of the linearized Euler equation, (70), is the equilibrium deviation of the log of the Lagrangian multiplier, Λ_t. Log equilibrium deviations of the second equation in (23) indicate that

$$\begin{aligned}\tilde{\lambda}_t \;&=\; E_t\{\tilde{\lambda}_{t+1} + \tilde{\rho}_t\},\\ &=\; E_t\sum_{i=0}^{\infty}\tilde{\rho}_{t+i},\\ &\simeq\; nE_t\tilde{\rho}_{n,t}\end{aligned} \tag{72}$$

[28] To simplify notation, we assume the sum of the polynomial weights in (67) is equal to the exponent of inverse habit in the (26) description of utility, $\nu(1) \equiv \nu$. In contrast to the logarithmic weighted average in (67), originally proposed in Kozicki and Tinsley (2002), a linear weighted average is suggested in Fuhrer (2000). Corrado and Holly (2004) demonstrate that the linear weighted average specification can violate desirable properties of utility functions.

[29] See discussion and earlier references to polynomial characterizations of frictions in Kozicki and Tinsley (2002).

where $\tilde{\lambda}_t$ is the equilibrium deviation of the log of the Langrange multiplier and $\tilde{\rho}_t$ is the real one-period interest rate deviation. For large n, the last line indicates that the infinite forward sum is approximated by the equilibrium deviation of the expected yield-to-maturity on an n-period bond, $\tilde{\rho}_{n,t}$.

Substituting from (72) into the output equation (70), gives

$$\begin{aligned} \tilde{y}_t &= E_t\{[a_1(L+\beta F)+a_2(L^2+\beta^2F^2)]\tilde{y}_t - a_3\sum_{i=0}^{\infty}\tilde{\rho}_{t+i}\}, \\ &\simeq E_t\{[a_1(L+\beta F)+a_2(L^2+\beta^2F^2)]\tilde{y}_t - na_3\tilde{\rho}_{n,t}\}, \end{aligned} \tag{73}$$

where the second line in (73) is a transparent illustration of the policy transmission role of real rates on long-maturity bonds in dynamic adjustments of output demand under endogenous habit.

In the absence of habit, $\nu_1 = \nu_2 = 0$. Using the definitions in (71), it is easy to verify that without habit, the Euler equation coefficients are $a_1 = a_2 = 0; a_3 = \frac{1}{\alpha}$, and the last line in (73) reverts to the last line in (25).

As noted earlier, linear Euler equations can always be reformulated as functions of the one-period interest rate, although the implied dynamic restrictions may not be as straight-forward as those in output equations involving bond rates, such as (31) and (73).

To illustrate, substitute the log-linearized equation under endogenous habit (70) into the first line of (72). The result can be rearranged as

$$\Delta\tilde{y}_t = E_t\{\tilde{y}_{t+1} - \tilde{y}_{t-1} + a_1'(\beta\Delta\tilde{y}_{t+2} + \tilde{y}_{t+1} - \tilde{y}_{t-1}) + a_2'(\beta^2\Delta\tilde{y}_{t+3} + \Delta\tilde{y}_{t-1}) - a_3'\tilde{\rho}_t\}, \tag{74}$$

where the coefficients are rescaled versions of the coefficients defined in (71),

$$a_1' = \frac{-a_1}{1+a_1}; \quad a_2' = \frac{-a_2}{1+a_1}; \quad a_3' = \frac{a_3}{1+a_1}. \tag{75}$$

Again, it is straight-forward to verify that the absence of endogenous habit, $\nu_1 = \nu_2 = 0$, implies that the equation displaying the one-period interest rate, (74), collapses to the standard NK output equation, shown in (24).

References

Abel, A., 1990. "Asset Prices under Habit Formation and Catching Up with the Jones." *American Economic Review*, 80(2), 38-42.

Ang, A. and M. Piazzesi, 2003. "A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables." *Journal of Monetary Economics*, 50, 4, 745-87.

Ang, A., S. Dong, and M. Piazzesi, 2005. "No-Arbitrage Taylor Rules." University of Chicago working paper, August.

Bernanke, B., 2004. "The Logic of Monetary Policy." Speech, December 2. http://www.federalreserve.gov/boarddocs/speeches/2004/

Beyer, A. and R. Farmer, 2004. "On the Indeterminacy of New-Keynesian Economics." European Central Bank working paper 323, February.

Buraschi, A. and A. Jiltsov, 2005. "Inflation Risk Premia and the Expectations Hypothesis." *Journal of Financial Economics*, 75, 429-90.

Campbell, J., A. Lo, and C. MacKinlay, 1977. *The Econometrics of Financial Markets.* Princeton, NJ: Princeton University Press.

Christiano, L, M. Eichenbaum, and C. Evans, 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy*, 13(10), February, 1-45.

Clarida, R., J. Gali, and M. Gertler, 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *Quarterly Journal of Economics*, February, 147-80.

Coenen, G. and V. Wieland, 2000. "A Small Estimated Euro Area Model with Rational Expectations and Nominal Rigidities." *European Economic Review*, 49, 1081-104.

Congressional Budget Office, 2004. *The Budget and Economic Outlook: Fiscal Years 2005-2014*, January.

Corrado, L. and S. Holly, 2004, "Habit Formation and Interest Rate Smoothing," Centre for Dynamic Macroeconomic Analysis working paper, University of St. Andrews, September.

Dewachter, H. and M. Lyrio, forthcoming(a). "Macro Factors and the Term Structure of Interest Rates." *Journal of Money, Credit and Banking*.

Dewachter, H. and M. Lyrio, forthcoming(b). "A Joint Model for the Term Structure of Interest Rates and the Macroeconomy." *Journal of Applied Econometrics*.

Duffee, G., 2002. "Term Premia and Interest Rate Forecasts in Affine Models." *Journal of Finance*, 57, 405-43.

Duffee, G., forthcoming. "Term Structure Estimation Without Using Latent Factors." *Journal of Financial Economics*.

Duffie, D., 1996. *Dynamic Asset Pricing Theory*, Princeton: Princeton University Press.

Fuhrer, J., 1996. "Monetary Policy Shifts and Long-Term Interest Rates." *Quarterly Journal of Economics*, November, 1183-1209.

Fuhrer, J., 2000. "Habit Formation in Consumption and Its Implications for Monetary-Policy Models." *American Economic Review*, 90(3), June, 367-90.

Fuhrer, J. and G. Moore, 1995. "Monetary Policy Trade-offs and the Correlation between Nominal Interest Rates and Real Output." *American Economic Review*, 85(1), March, 219-39.

Fuhrer, J. and G. Rudebusch, 2004. "Estimating the Euler Equation for Output." *Journal of Monetary Economics*, 51(6), September, 1133-53

Honkapohja, S. and K. Mitra. 2004. "Are Non-Fundamental Equilibria Learnable in Models of Monetary Policy?" *Journal of Monetary Economics*, 51, 1743-70.

Horn, R. and C. Johnson, 1991. *Topics in Matrix Analysis*, Cambridge, UK: Cambridge University Press.

Kim, D. and A. Orphanides, 2005. "Term Structure Estimation with Survey Data on Interest Rate Forecasts." Federal Reserve Board FEDS working paper, 2005-48, October.

Kohn, D., 2005. "Monetary Perspectives on Risk Premiums in Financial Markets." Speech, July 21.

http://www.federalreserve.gov/boarddocs/speeches/2005/

Kozicki, S. and P. Tinsley, 2001a. "Term Structure Views of Monetary Policy under Alternative Models of Agent Expectations." *Journal of Economic Dynamics & Control*, January, 149-84.

Kozicki, S. and P. Tinsley, 2001b. "Shifting Endpoints in the Term Structure of Interest Rates." *Journal of Monetary Economics*, 47, June, 613-52.

Kozicki, S. and P. Tinsley, 2002. "Dynamic Specifications in Optimizing Trend-Deviation Macro Models." *Journal of Economic Dynamics & Control*, 26, August, 1585-1611.

Kozicki, S. and P. Tinsley, 2005a. "Perhaps the FOMC Did What It Said It Did: An Alternative Interpretation of the Great Inflation." Federal Reserve Bank of Kansas City Working Paper RWP 05-04, October.

Kozicki, S. and P. Tinsley, 2005b. "Permanent and Transitory Policy Shocks in an Empirical Model with Asymmetric Information." *Journal of Economic Dynamics & Control*, November, 1985-2015.

Lubik, T. and F. Schorfeide, 2004. "Testing for Indeterminacy: An Application to U.S. Monetary Policy." *American Economic Review*, 94(1), March, 190-217.

Orphanides, O., 2003. "The Quest for Prosperity Without Inflation." *Journal of Monetary Economics*, 50, 633-63.

Rotemberg, J. and M. Woodford, 1999. "Interest Rates Rules in an Estimated Sticky Price Model." in J. Taylor (ed.) *Monetary Policy Rules*, Chicago: University of Chicago Press, 57-119.

Rudebusch, G. and L. Svensson, 1999. "Policy Rules for Inflation Targeting." in J. Taylor (ed.) *Monetary Policy Rules*, Chicago: University of Chicago Press, 203-46.

Rudebusch, G. and T. Wu, 2004a. "The Recent Shift in Term Structure Behavior from a No-Arbitrage Macro-Finance Perspective." Federal Reserve Bank of San Francisco working paper.

Rudebusch, G. and T. Wu, 2004b. "A Macro-Finance Model of the Term Structure, Monetary Policy, and the Economy." Federal Reserve Bank of San Francisco working paper.

Shiller, R., 1990. "The Term Structure of Interest Rates." in B. Friedman and F. Hahn (eds.) *Handbook of Monetary Economics, vol I*, Amsterdam: North-Holland, 627-72.

Taylor, J. 1993. "Discretion versus Policy Rules in Practice." in A. Meltzer and C. Plosser (eds.) *Carnegie-Rochester Conference Series on Public Policy,* Amsterdam: North-Holland, 195-214.

Tinsley, P., H. Farr, G. Fries, B. Garrett, and P. von zur Muehlen, 1982. "Policy Robustness: Specification and Simulation of a Monthly Money Market Model." *Journal of Money, Credit and Banking*, November, 829-56.

Woodford, M., 2003. *Interest and Prices*. Princeton, N.J.: Princeton University Press.

Table 1: Bond Rate Responsiveness to Macro Variables [1.]

$$r^*_{t,12h} = b_0 + b_1\pi_{t-1} + b_{11}(L)\Delta\pi_{t-1} + b_2 u_{t-1} + b_{22}(L)\Delta u_{t-1}.$$

$$r_{t,12h} = b_3 r_{t-1,12h} + b_{33}(L)\Delta r_{t-1,12} + (1-b_3)r^*_{t,12h} + a_{t,12h}.$$

bond rate	π b_1	u b_2	Δu $b_{22}(1)$	b_3	test $b_1 > 1$ [2]
	sample: 1966 m1 - 1979 m9				
h = 1	1.37	-1.28	-1.94	.880	.77
	(2.8)	(-2.0)	(-3.1)	(12)	
h = 3	1.06	-.889	-1.44	.864	.58
	(3.8)	(-2.3)	(-3.2)	(13)	
h = 5	1.05	-.853	-1.15	.864	.58
	(3.6)	(-2.1)	(-3.2)	(15)	
h = 10	1.12	-.904	-.812	.898	.64
	(3.4)	(-1.8)	(-3.0)	(20)	
	sample: 1982 m1 - 1987 m7				
h = 1	1.96	.068	-3.92	.704	.99
	(4.9)	(.19)	(-2.8)	(7.8)	
h = 3	1.95	.214	-2.97	.757	.96
	(3.7)	(.50)	(-2.1)	(10)	
h = 5	1.73	.338	-2.42	.774	.87
	(2.7)	(.71)	(-1.5)	(11)	
h = 10	1.20	.551	-2.42	.795	.60
	(1.6)	(1.0)	(-1.5)	(12)	

1. $b_{ii}(L)$ are 11th-order polynomials in L. Parentheses contain ratios of coefficients to asymptotic standard errors.
2. p-values.

Table 2a: Forward rate regressions, pre-Volcker 1966 - 1979,
with benchmark learning rates [1.]

$$f^*_{t,12h} = E_t\left\{c_0 + c_1\bar{\pi}_t + c_2\tilde{\pi}^{(\bar{k})}_{t,12h} + c_3\tilde{u}_{t,12h} + c_4\Delta^{(k)}u_{t,12h}\right\}.$$

$$f_{t,12h} = \rho_1 f_{t,12(h-1)} + \rho_2\Delta^{(k)}f_{t,12(h-1)} + (1-\rho_1)f^*_{t,12h} + a_{t,12h}.$$

forward rate	$\bar{\pi}$ c_1	$\tilde{\pi}^{(k)}$ c_2	$\tilde{u}$ c_3	$\Delta^{(k)}u$ c_4	ρ_1	ρ_2
	time-varying perceived natural rates, $\gamma_{\bar{\pi}} = \gamma_{\bar{u}} = 0.015$					
h = 1-3	.588	-.042	–	–	.513	-.009
	(9.2)	(-0.4)			(16)	(-0.2)
	.581	-.042	-.023	–	.509	-.005
	(9.1)	(-0.4)	(-0.3)		(15)	(-0.1)
	.581	.056	–	-.172	.518	-.017
	(9.0)	(0.3)		(-0.6)	(16)	(-0.4)
h = 4-6	.904	.661	–	–	.877	.219
	(10)	(2.5)			(38)	(2.5)
	.861	.375	-.564	–	.873	.211
	(8.5)	(1.5)	(-1.5)		(39)	(2.4)
	.860	2.92	–	-2.65	.873	.211
	(8.4)	(1.8)		(-1.5)	(39)	(2.4)
h = 7-10	.796	2.70	–	–	.967	.877
	(25)	(3.9)			(178)	(20)
	.813	1.32	-2.86	–	.967	.873
	(25)	(1.8)	(-3.7)		(179)	(20)
	.813	13.8	–	-13.1	.967	.873
	(25)	(4.3)		(-3.7)	(179)	(20)

1. Superscript $(\bar{k})$ denotes k-period averages. For monthly data, k = 12.
Parentheses contain ratios of coefficients to HAC standard errors.
VAR forecast model sample: 1960 m1 - 1979 m7.

Table 2b: Forward rate regressions, pre-Volcker 1966 - 1979,
with alternative learning rates [1.]

$$f^*_{t,12h} = E_t\left\{c_0 + c_1\bar{\pi}_t + c_2\tilde{\pi}^{(\bar{k})}_{t,12h} + c_3\tilde{u}_{t,12h} + c_4\Delta^{(k)}u_{t,12h}\right\}.$$

$$f_{t,12h} = \rho_1 f_{t,12(h-1)} + \rho_2\Delta^{(k)}f_{t,12(h-1)} + (1-\rho_1)f^*_{t,12h} + a_{t,12h}.$$

forward rate	$\bar{\pi}$ c_1	$\tilde{\pi}^{(k)}$ c_2	$\tilde{u}$ c_3	$\Delta^{(k)}u$ c_4	ρ_1	ρ_2
	fixed perceived inflation target, $\gamma_{\bar{\pi}} = \gamma_{\bar{u}} = 0.0$					
h = 1-3	1.0	.320 (5.1)	–	–	.540 (14)	-.037 (-0.7)
h = 4-6	1.0	.929 (1.8)	–	–	.970 (54)	.312 (3.1)
h = 7-10	1.0	1.59 (13)	–	–	.981 (346)	.870 (20)
	time-varying perceived inflation target, $\gamma_{\bar{\pi}} = \gamma_{\bar{u}} = 0.015$					
h = 1-3	.588 (9.2)	-.042 (-0.4)	–	–	.513 (16)	-.009 (-0.2)
h = 4-6	.904 (10)	.661 (2.5)	–	–	.877 (38)	.219 (2.5)
h = 7-10	.796 (25)	2.70 (3.9)	–	–	.967 (178)	.877 (20)
	time-varying perceived inflation target, $\gamma_{\bar{\pi}} = 0.03; \gamma_{\bar{u}} = .015$					
h = 1-3	.499 (10)	-.012 (-0.2)	–	–	.503 (17)	-.001 (-0.0)
h = 4-6	.670 (9.9)	.422 (2.1)	–	–	.866 (36)	.216 (2.3)
h = 7-10	.694 (26)	1.24 (4.0)	–	–	.965 (164)	.879 (20)

1. Superscript $(\bar{k})$ denotes k-period averages. For monthly data, k = 12.
Parentheses contain ratios of coefficients to HAC standard errors.
VAR forecast model sample: 1960 m1 - 1979 m7.

Table 3a: Forward Rate Regressions, Volcker 1982 - 1987
with benchmark learning rates [1.]

$$f^*_{t,12h} = E_t\left\{c_0 + c_1\bar{\pi}_t + c_2\tilde{\pi}^{(\bar{k})}_{t,12h} + c_3\tilde{u}_{t,12h} + c_4\Delta^{(k)}u_{t,12h}\right\}.$$

$$f_{t,12h} = \rho_1 f_{t,12(h-1)} + \rho_2\Delta^{(k)}f_{t,12(h-1)} + (1-\rho_1)f^*_{t,12h} + a_{t,12h}.$$

forward rate	$\bar{\pi}$ c_1	$\tilde{\pi}^{(k)}$ c_2	$\tilde{u}$ c_3	$\Delta^{(k)}u$ c_4	ρ_1	ρ_2
	time-varying perceived natural rates: $\gamma_{\bar{\pi}} = \gamma_{\bar{u}} = 0.015$					
h = 1-3	3.63	.732	–	–	0.574	-.155
	(8.4)	(2.6)			(9.9)	(-3.1)
	3.62	.731	.007	–	.574	-.155
	(9.5)	(2.8)	(0.0)		(10)	(-3.1)
	3.85	.429	–	.759	.529	-.120
	(9.9)	(1.3)		(1.9)	(9.1)	(-2.4)
h = 4-6	3.28	1.12	–	–	.918	-.232
	(3.6)	(1.8)			(24)	(-2.3)
	.754	9.18	-20.2	–	.955	-.265
	(0.2)	(1.1)	(-1.0)		(26)	(-2.7)
	.226	-1.50	–	13.3	.958	-.268
	(0.1)	(-0.6)		(1.0)	(27)	(-2.8)
h = 7-10	-.686	6.64	–	–	.957	.450
	(-0.4)	(1.6)			(60)	(5.4)
	1.62	-76.1	119	–	.942	.485
	(2.0)	(-2.6)	(2.7)		(65)	(6.4)
	2.15	-7.67	–	-28.6	.934	.467
	(2.7)	(-1.9)		(-3.2)	(60)	(6.2)

1. Superscript $(\bar{k})$ denotes k-period averages. For monthly data, k = 12.
Parentheses contain ratios of coefficients to HAC standard errors.
VAR forecast model sample: 1982 m1 - 1987 m7.

Table 3b: Forward rate regressions, Volcker 1982 - 1987,
with alternative learning rates [1.]

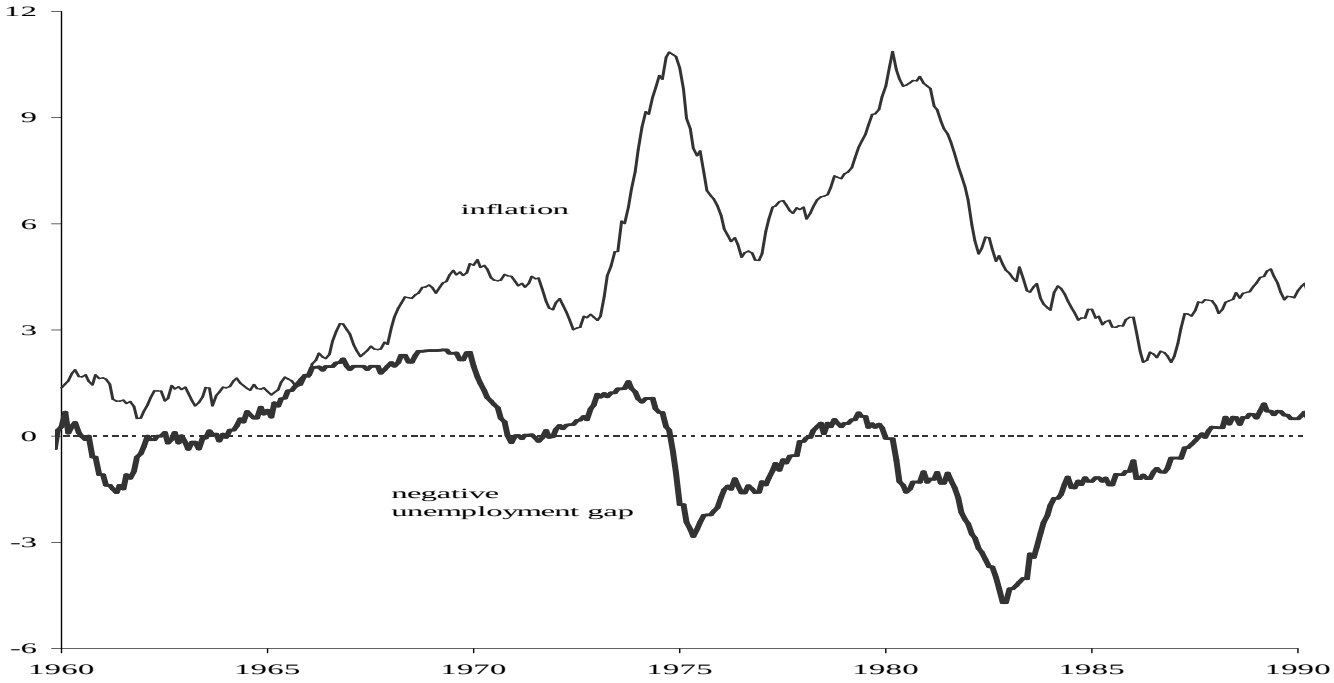
$$f^*_{t,12h} = E_t\left\{c_0 + c_1\bar{\pi}_t + c_2\tilde{\pi}^{(\bar{k})}_{t,12h} + c_3\tilde{u}_{t,12h} + c_4\Delta^{(k)}u_{t,12h}\right\}.$$

$$f_{t,12h} = \rho_1 f_{t,12(h-1)} + \rho_2\Delta^{(k)}f_{t,12(h-1)} + (1-\rho_1)f^*_{t,12h} + a_{t,12h}.$$

forward rate	$\bar{\pi}$ c_1	$\tilde{\pi}^{(k)}$ c_2	$\tilde{u}$ c_3	$\Delta^{(k)}u$ c_4	ρ_1	ρ_2
	fixed perceived inflation target, $\gamma_{\bar{\pi}} = \gamma_{\bar{u}} = 0.0$					
h = 1-3	1.0	4.37 (2.1)	–	–	.850 (24)	-.262 (-4.8)
h = 4-6	1.0	75.7 (1.1)	–	–	.978 (53)	-.285 (-2.9)
h = 7-10	1.0	36.2 (0.9)	–	–	.936 (80)	.412 (4.6)
	time-varying perceived inflation target, $\gamma_{\bar{\pi}} = \gamma_{\bar{u}} = 0.015$					
h = 1-3	3.63 (8.4)	.732 (2.6)	–	–	.574 (9.9)	-.155 (-3.1)
h = 4-6	3.28 (3.6)	1.12 (1.8)	–	–	.918 (24)	-.232 (-2.3)
h = 7-10	-.686 (-0.4)	6.64 (1.6)	–	–	.957 (60)	.450 (5.4)
	time-varying perceived inflation target, $\gamma_{\bar{\pi}} = 0.03; \gamma_{\bar{u}} = .015$					
h = 1-3	2.22 (5.7)	.774 (2.3)	–	–	.617 (10)	-.171 (-3.3)
h = 4-6	2.04 (2.8)	1.38 (1.7)	–	–	.933 (26)	-.240 (-2.3)
h = 7-10	1.40 (3.1)	4.01 (3.2)	–	–	.934 (61)	.441 (5.6)

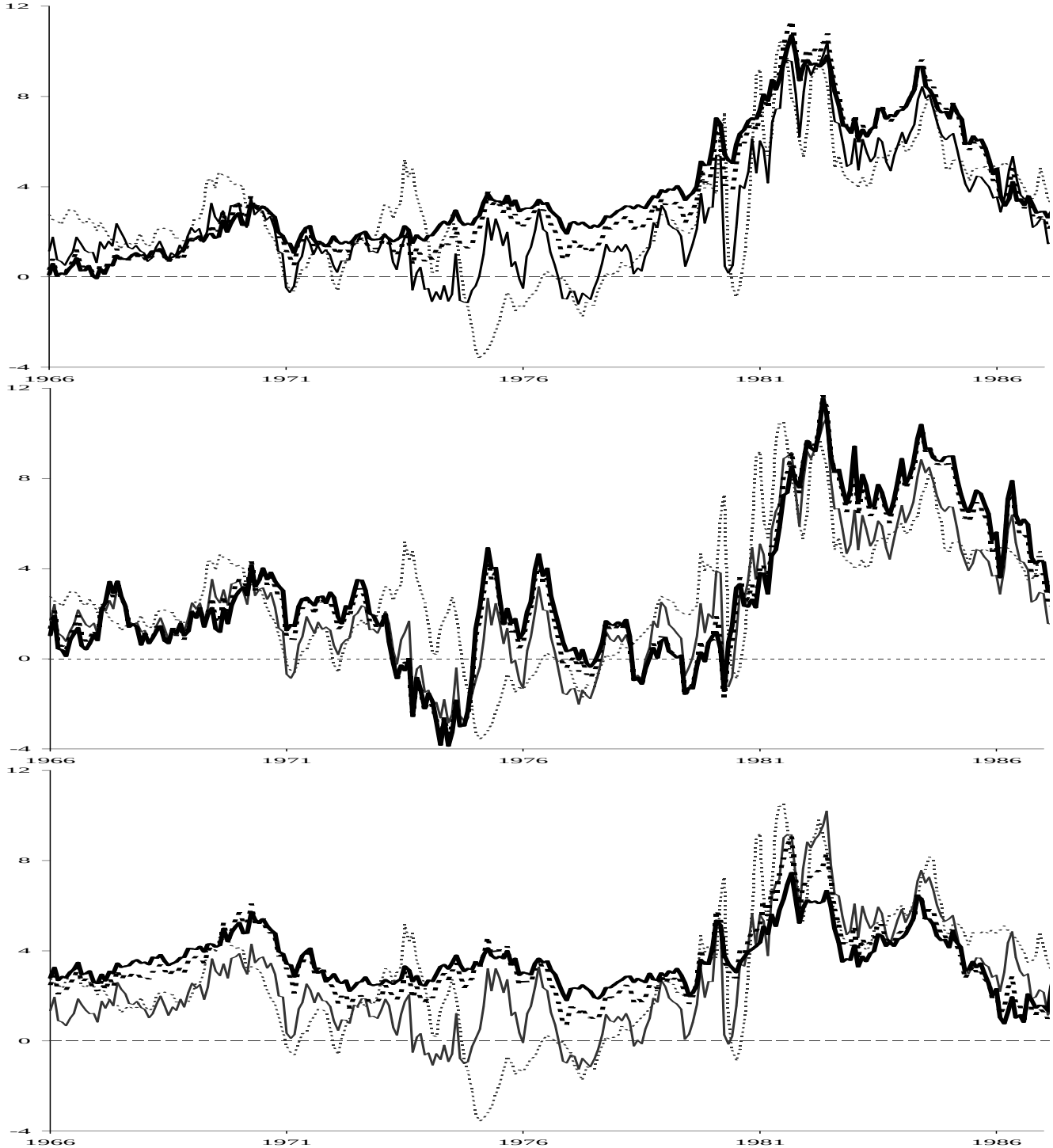
1. Superscript $(\bar{k})$ denotes k-period averages. For monthly data, k = 12.
Parentheses contain ratios of coefficients to HAC standard errors.
VAR forecast model sample: 1982 m1 - 1987 m7.

Figure 1: Inflation and the negative of natural rate deviations in unemployment [1]



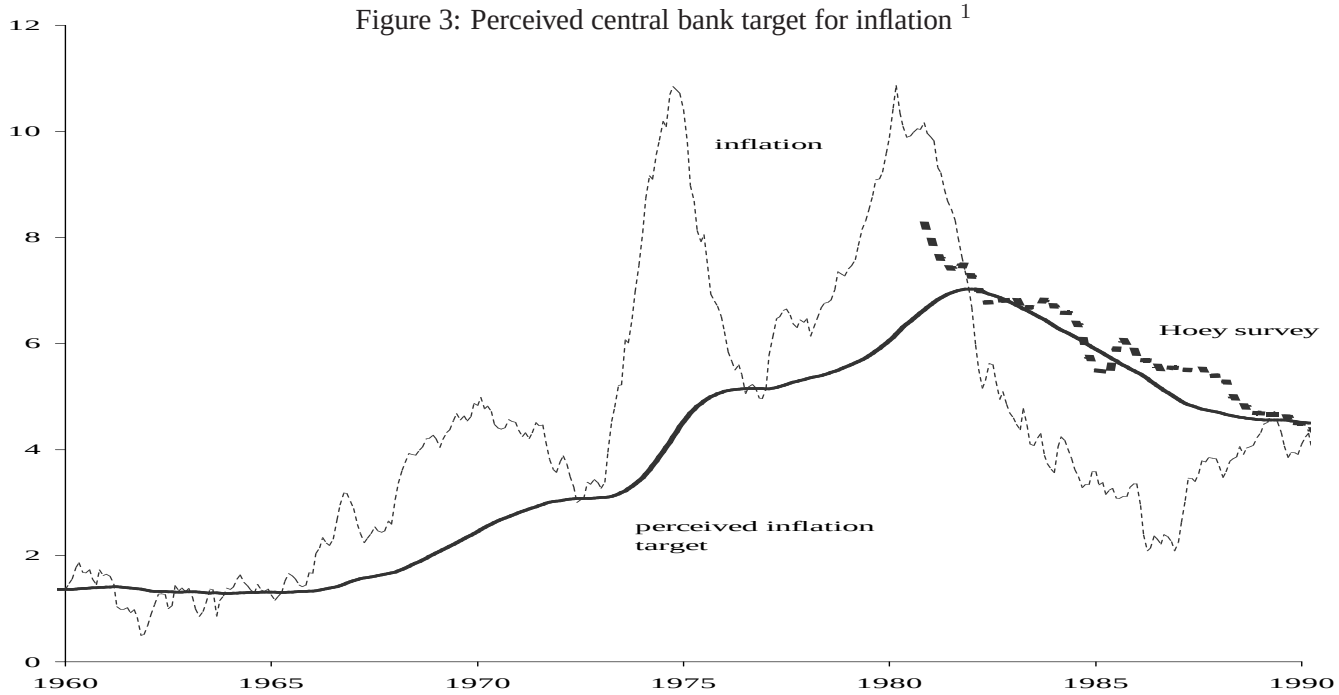
1. Twelve-month moving average of annualized inflation in the deflator for personal consumption expenditures (pce), and the negative of deviations in the civilian unemployment rate from the Congressional Budget Office (2004) estimate of the natural rate for unemployment.

Figure 2: Sensitivity of real bond rates to inflation target perceptions [1]



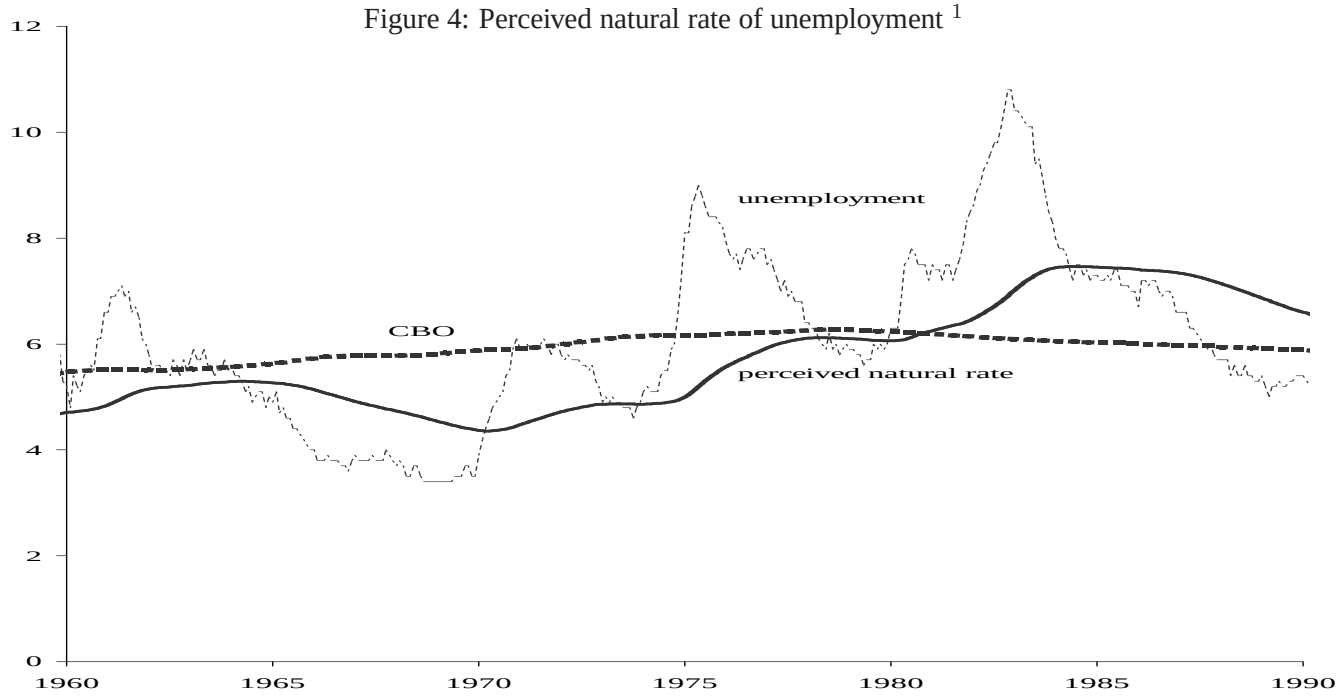
1. The central bank target for inflation perceived by bond traders, $\bar{\pi}_t$, is fixed in the top panel, difference-stationary in the middle panel, and based on change-point learning in the bottom panel (see text). Thin dotted line is real federal funds rate; thin solid line is real rate on a 1-year zero coupon bond (ZCB); thick dotted line is real rate on a 5-year ZCB; and thick solid line is real rate on a 10-year ZCB.

Figure 3: Perceived central bank target for inflation [1]



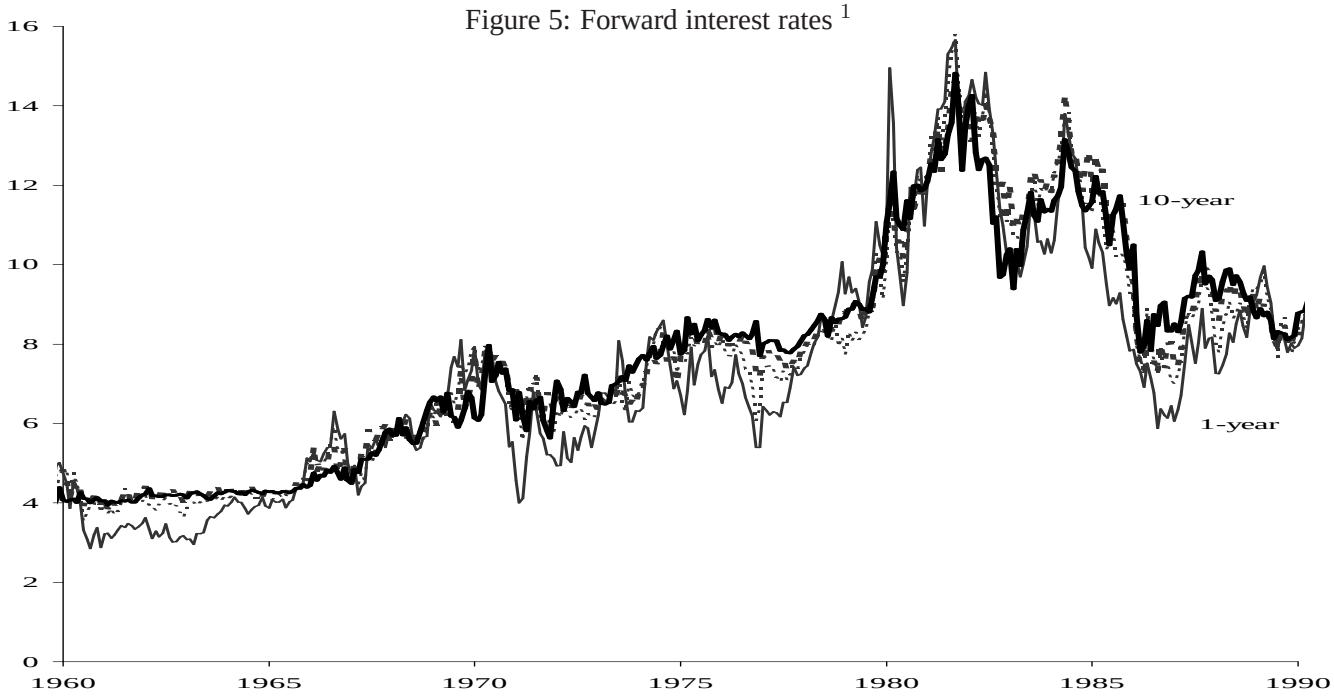
1. solid line: perceived inflation target with learning gain, $\gamma_{\bar{\pi}} = .015$ (see text).
dashed: Hoey survey of expected 5-10 year inflation.
dotted: inflation in personal consumption expenditure (pce) deflator, 12-month average.

Figure 4: Perceived natural rate of unemployment [1]



1. solid line: perceived natural rate of unemployment with learning gain $\gamma_{\bar{u}} = .015$ (see text).
dashed: Congressional Budget Office (2004) estimate of natural rate for unemployment.
dotted: civilian unemployment rate.

Figure 5: Forward interest rates [1]



1. thin solid line: 1-year forward rates; thin dotted line: 3-year forward rates; thick dotted line: 5-year forward rates; thick solid line: 10-year forward rates. Forward interest rates from McCulloch-Kwon (1993).