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Non-Market Interaction

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ABSTRACT

A large body of recent research argues that social, or non-market, interactions can explain a wide range of puzzling phenomena from fashion cycles to stock market crashes. This paper attempts to connect the range of these papers with a general model and a broad empirical overview. We establish conditions for existence and uniqueness of equilibria in social interactions models. The existence of multiple equilibria requires sufficient non-linearity in social interactions and only moderate heterogeneity across agents—strategic complementarities are neither necesssary nor sufficient for multiple equilibria. We establish conditions for the existence of a social multiplier, which is the ratio of the aggregate outcome-input relationship to the individual outcome-input relationship. Models with multiple equilibria are empirically indistinguishable from models with significant social multipliers. Finally, we show the formal relationship between three known methods of empirically estimating social interactions, and suggests the plusses and minuses of these three approaches.

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1 Introduction

Why are there stock market crashes? Why is France overwhelmingly Christian and Thailand overwhelmingly Buddhist? Why did the great depression occur? Why do crime rates vary so much over time and space? Why did the adoption of hybrid corn follow an *s*-shaped curve? Why is there racial segregation? Why do mass cultural phenomena like the Hula Hoop and Harry Potter occur?

This bizarre juxtaposition of questions are bound together by one common element. Over the past 30 years, economists have suggested the answers to every one of these questions lie in models of social, and in most cases, non-market, interactions,[1] where non-market interactions are interactions between individuals, which are not regulated by the price mechanism.[2] Non-market interactions are important because they may create a *social multiplier* or multiple equilibria. In turn social multipliers or multiple equilibria create the excess variance of agreggate endogenous variables relative to fundamentals, that seem to characterize stock market crashes, religious differences, the great depression, wildly different crime rates and the Hula Hoop.

Many models of non-market interactions exhibit strategic complementarities, which are said to occur when the marginal utility to one person of undertaking an action is increasing with the number of his peers who are undertaking the same action. Consequently a change in fundamentals has a direct effect on behavior and an indirect effect of the same sign. Each person's actions change not only because of the direct change in fundamentals, but also because of the change in the behavior of their neighbors. The result of all these indirect effects is the social multiplier. When this social multiplier is large, we expect to see high levels of variance across space or time. Furthermore, for very large levels of social interaction, multiple equilibria can occur when there are different outcomes for exactly the same fundamentals. The existence of multiple equilibria also helps us to understand high levels of variance.

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[1]The connection between stock market crashes and social interaction models is discussed in Shiller [2000]. Becker and Murphy [to apppear] discusses social interactions and religion. Cooper and John [1986] connect non-market interactions and business cycles. Glaeser, Sacerdote and Scheinkman [1996] argue that social interactions can explain the high variance of crime rates over space. Ellison and Fudenberg [1993] think about technology adoption and local interactions. Schelling [1971,1972] pioneered social interaction models by discussing segregation. Bickchandani, Hirshleifer and Welch [1992] model fads and fashion.

[2]Non-market interactions are thus particular forms of externalities.

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But non-market interactions models don't just predict high levels of variance. When individuals can choose locations, the presence of these interactions often predicts segregation across space. Cities exist because of agglomeration economies which are likely to come from non-market complementarities[3]. Indeed, the selection of like types into the same neighborhoods is often what makes measuring social interactions quite difficult. In dynamic settings, social interactions can produce s-shaped curves which help to explain the observed time series patterns of phenomena as disparate as telephones and women in the workplace.

The potential power of these models explains the explosion of work in this area over the past few years. In this essay, we explore the common mathematical structure of these models. While there has been a great deal of work in this area, there has been less effort to understand the common elements between the disparate models. Instead of discussing the literature broadly, we will start with a particularly general social interactions model and examine the implications of this model. Several of the better known models in this area can be seen as special cases of this more general model.

In this general model, the utility function includes an individual's action (or consumption), the actions of flexibly defined peer or reference groups, personal characteristics (including income) and common prices. People are arranged by social proximity and the reference groups may include only one's closest neighbor, or the entire city. One controversy in the literature is whether social interactions are best thought of as being local (i.e. people are really affected only by their closest neighbor) or global (i.e. people are affected by the average behavior of people in a large community). Our framework is sufficiently flexible to examine these extremes.

Although there are several examples in the literature that deal with discrete choice, many others deal with variables such as education, which are more naturally thought of as continuous (*e.g.* Benabou [1993], [1996] and Durlauf [1996a], [1996b].) In addition, in certain contexts the variables that we observe (*e.g* out-of-wedlock births) are consequences of behavior that the agents - as opposed to the social scientist - are observing and mimicking, and this behavior may be more naturally modeled as a continuous variable. For this reason and because it is mathematically simpler we emphasize models with continuous choices.[4]

We first examine the conditions under which for each set of exogenous parameters we obtain a unique equilibrium. A sufficient condition for a

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[3]See *e.g.* Krugman [1991]

[4]Some results for a model with discrete action space are derived in section 2.4.

unique equilibrium is that the absolute value of the second derivative of utility with respect to one's own action is greater than the absolute value of the cross partial derivative between one's own action and the action of the peer group. We refer to this condition as the Moderate Social Influence (MSI) condition. One, perhaps surprising, fact is that uniqueness is guaranteed by this condition with no assumptions about the degree to which interactions are global or local.

We establish sufficient conditions for the existence of multiple equilibria. These conditions tend to be satisfied if there exists sufficient non-linearity in the effect of peer actions on an individual's own optimal action, and not too much heterogeneity among agents. Hence when sorting creates homogeneity, multiple equilibria are more likely.

As mentioned above the main ingredient used to generate multiple equilibria in models of non-market interactions has been strategic complementarities. We show that, because we consider a richer structure of reference groups, strategic complementarity is *not* necessary for multiple equilibria. We present an example where each agent belongs to one of two groups. Individuals in each group want to differentiate themselves from the average individual in the other group in the consumption of a good. No other social interactions are present. If the desire to differentiate is strong enough, there are at least three equilibria. In one, the symmetric equilibrium, both types consume the same amount of the good. In the other two equilibria, one group consumes more of the good than in the symmetric equilibrium, while the other group consumes less.

Even in cases with multiple equilibria, as long as there is some individual heterogeneity, (generically) there will not be a continuum of equilibria. Around each equilibrium there is an interval that contains no other equilibria. As the model does not tell us the size of this interval, the empirical relevance of this finding may be limited. However, it does suggest that empirical approaches to estimating multiple equilibria models should focus on situations where the different equilibria are sufficiently distinct.

Another result is that, if the MSI condition holds, and strategic complementarity prevails, there is always a well defined social multiplier.[5] This social multiplier can be thought of as the ratio of the response of an individual action to an exogenous parameter (that effects only that person) and the (per capita) response of the entire peer group to a change in the same parameter that effects the entire peer group. In the empirical framework section of the paper, we discuss estimating this multiplier. The presence of

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[5]When the MSI condition fails to hold, the social multiplier becomes unbounded.

this social multiplier implies that social interactions are generally connected with unusually high variances of agreggates. In fact, as we will argue later, it is very difficult to empirically distinguish between models with a unique equilibrium and a large social multiplier and models with multiple equilibria.

Although the results concerning uniqueness and the social multiplier are independent of the interaction structure, the same is not true for ergodicity. However we show that if MSI prevails, shocks are independent and identically distributed across individuals, and each agent is influenced by the average action of all other agents, then the equilibrium average action for a large population is independent of the actual shocks that occur.

We do not discuss the welfare properties of equilibria. Indeed, the presence of heterogeneous individuals in our benchmark model makes it more difficult to rank equilibria. One possibility is to proceed as Brock and Durlauf [1995] and examine *ex-ante* welfare. In general, the ranking of equilibria according to ex-ante welfare typically depends on finer aspects of the interaction structure. We have decided not to focus on these issues here.

In the third section of the paper, we present a linear quadratic version of the social interaction model which can be used for estimation. We then discuss how different approaches to estimating the extent of social interactions fit into this framework. There are three primary approaches to estimating social interactions.

First social interactions are estimated by looking at the covariance between individual outcomes and the average outcome of a peer group. Even in the best case scenario, ordinary least square coefficients based on these covariances do not yield unbiased coefficients of social interaction parameters. The problem occurs because peer outcomes are partially determined by the individual's outcome. Our framework suggests a correction in the univariate case for this bias. This correction will not work if unobserved individual attributes are correlated with peer attributes. This correlation can occur either because of omitted community level characteristics or because of differential sorting into neighborhoods. The standard correction for this mutual interdependence (following Case and Katz, 1991) is to use an instrumental variables approach which relies on the exogenous characteristics of peers. This approach will successfully solve problems due to omitted community attributes, but not if there is sorting into communities. Randomized experiments offer the best chance of identifying peer effects.

The second empirical approach uses the variance of aggregates. This approach is in a sense fundamentally equivalent to the first more standard approach. The basic advantage of the first approach is that it allows the econometrician to control for individual level characteristics appropriately.

The third empirical approach uses the logic of the multiplier. In this approach, the relationship between exogenous variables and outcomes for individuals is compared with the relationship between exogenous variables and outcomes for groups. The ratio is a measure of the size of social interactions.

All three of these approaches offer different methods of capturing social interactions and in many cases, the estimates will be absolutely equivalent. However, all will suffer when there are omitted variables and sorting. The empirical hope is that randomization of people with peers (as in Sacerdote [2000]) will help us to break this sorting. However, this randomization is rare and if we estimate social interactions only in those cases where we have randomization, then we are likely to have only limited empirical scope for this type of work.

2   Theoretical Models of Non-Market Interactions

Economics has always been concerned with social interactions. Most often, economists have written about social interactions that are mediated by the market. In particular, economists have focused on the interaction that occurs when greater demand of person x for commodity z raises the price of that commodity and reduces the consumption by person y. This negative interaction lies at core of our discipline, and when this interaction appears in our models, it tends to predict "well-behaved" systems, with unique equilibria, monotonic convergence, etc. Although negative interactions can create cycles (high demand in one period raises prices the next period) as in the case of cobweb models, they tend not to create high levels of variability.

However, as economists tried to explain more puzzling phenomena, particularly large variances over time and space, they moved to positive interaction models. The most famous early example of this move is in Keynes, whose famous "beauty contest" description of the stock market suggested the possibility of positive market-mediated interactions. One person's demand for the stock market could conceivably induce others to also purchase shares. This type of model has only recently been formalized (e.g. Froot, Scharfstein and Stein [1992]). Several other authors have focused on different mechanisms where we see market-mediated positive social interactions (*e.g.* Murphy, Shleifer and Vishny [1989]). These models create the possibility of multiple equilibria or high levels of variability for a given set of fundamentals.

Our interest is fundamentally in non-market interactions. The literature on these interactions has paralleled the rise in market-mediated positive in-

teraction models and has many similarities. Schelling [1971,1972] pioneered the study of non-market interactions in economics. Following in Schelling's tradition, economists have examined the role of non-market interactions in a myriad of contexts.

Many of the recent papers on non-market interactions use random field models, also known as interactive particle systems, imported from statistical physics. In these models one typically postulates individual's interdependence and analyzes the macro behavior that emerges. Typical questions concern the existence and multiplicity of *macro phases* that are consistent with the postulated individual behavior. Follmer [1974] was the first paper in economics to use this framework. He modeled an economy with locally dependent preferences, and examined when randomness in individual preferences will affect the aggregate, even as the number of agents grows to infinity. Brock [1993] and Blume [1993] recognized the connection of a class of interactive particle models to the economic literature on discrete choice. Brock and Durlauf [1995r] develops many results on discrete choice in the presence of social interactions. Other models inspired in statistical physics start with a more explicit dynamic description on how agents choices at a point in time are made, conditional on other agents' previous choices, and discuss the evolution of the macro behavior over time. Bak, Chen, Scheinkman and Woodford [1993] (see also Scheinkman and Woodford [1995]) study the impact of independent sectoral shocks on aggregate fluctuations with a "sandpile" model that exhibits self-organized criticality and show that independent sectoral shocks may affect average behavior, even as the numbers of sectors grow large. Durlauf [1993] constructs a model based on local technological interactions to examine the possibility of productivity traps, in which low productivity techniques are used, because other producers are also using low productivity processes. Glaeser, Sacerdote and Scheinkman [1996] used the *voter* model[6] to analyze the distribution of crime across American cities. Topa [1997] examines the spatial distribution of unemployment with the aid of *contact processes*.[7]

A related literature, that we do not discuss here, studies social learning.[8] In these models agents typically learn from observing other agents and base decisions on the observed decisions of others.

Becker and Murphy (to appear) presents a particularly far ranging anal-

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[6] *e.g.* Ligget [1985]

[7] *e.g.* Ligget [1985]

[8] *e.g.* Arthur [1989], Banerjee [1992], Bickhchandani, Hirshleifer and Welch [1992], Ellison[1993], Ellison and Fudenberg [1994], Gul and Lundholm [1995], Kirman [1993] and Young [1993].

ysis of the social interactions in economics. This volume extends Becker's (1991) earlier analysis of restaurant pricing when there are social interactions in the demand for particular restaurants. This work is particularly important because of its combination of social interactions with maximizing behavior and classic price theory. In the same vein, the work of Pesendorfer [1995] on cycles in the fashion industry examines how a monopolist would exploit the presence of non-market interactions.

In the remainder of this section we develop a model of non-market interactions that will hopefully serve to clarify the mechanisms by which non-market interactions affect macro behavior. The model is self-consciously written in a fashion that makes it close to traditional economic models, though it can accommodate versions of the models that were inspired by statistical physics. We consider only a static model, although we also examine some *ad hoc* dynamics. On the other hand, our framework encompasses several of the examples in the literature, and is written so that special cases can be used to discuss issues in the empirical evaluation of non-market interactions (see Section 3).[9] In the spirit of the work of Pesendorfer [1995] or Becker and Murphy [to appear] we explicitly allow for the presence of prices, so that the model can be used to study the interplay between market and non market interactions.

We next write down the details of the model. In 2.2 we describe some examples in the literature that fit into our framework. We present results for the case of continuous actions in 2.3. These results concern sufficient conditions for uniqueness or multiplicity of equilibria, the presence of a *social multiplier*, the stability of equilibria, and ergodicity. Models with discrete actions and large populations are treated in 2.4. At the end of this section we give a short discussion of endogenous peer group formation.

2.1 A model of non-market interactions

We identify each agent with an integer $i = 1, \ldots, n$. Associated with each agent i are his peer, or reference, groups P_i^k, $k = 1, \ldots K$, each one a subset of $\{1, \ldots, n\}$ that does not contain i. We allow for multiple reference groups ($K > 1$) in order to accommodate some of the examples that have appeared in the literature,(Example 5 below.) Later we will discuss the case where the agent chooses his reference groups, but in most of what follows the P_i^k's

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[9]In particular, our framework is related to the pioneering work of Cooper and John[1988] on coordination failures. They study a particular version of our framework where each agent interacts with the average agent in the economy, and focus on models where there is no heterogeneity across agents.

are fixed. Each agent is subject to a "taste shock" θ_i, a random variable with support on a set Θ. Finally, each agent chooses an action $a \in \mathcal{A}$. Typically the set $\mathcal{A}$ will be a finite set, *the discrete choice case,* or an interval of the real line, *the continuous choice case.* The utility of agent i depends on the action chosen by him, a_i, and the actions chosen by all other agents in his peer group. More precisely we assume that:

$$U^i = U^i(a_i, A_i^1, \ldots, A_i^K, \theta_i, p), \text{ where} \tag{1}$$

$$A_i^k = \sum_{j=1}^{n} \gamma_{ij}^k a_j, \tag{2}$$

with $\gamma_{ij}^k \geq 0$, $\gamma_{ij}^k = 0$ if $j \notin P_i^k$, $\sum_{j=1}^n \gamma_{ij}^k = 1$, and $p \in \Pi$ is a vector of parameters.

In other words, the utility enjoyed by agent i depends on his own chosen action, on an weighted average of the actions chosen by agents in his reference groups, on his taste shock, and on a set of parameters. We allow the utility function to vary across agents because in some cases we want to identify variations in the parameters θ_i explicitly with variations on income or other parameters of the problem. We also assume that the maximization problem depends only on the agent being able to observe the relevant average action of peers. In some applications we will allow for $P_i^k = \emptyset$, for some of the agents. In this case we may set A_i^k to be an arbitrary element of $\mathcal{A}$, and U^i to be independent of A_i^k. In many examples each agent will have a unique reference group. In this case we drop the superscript k and write P_i instead of P_i^1 etc...

Typically p will be interpreted as the exogenous (per-unit) price of taking action.[10] In addition we will think of $\theta_i = (y_i, \zeta_i)$, where y_i represents income and ζ_i a shock to taste. In this case,

$$U^i(a_i, A_i^1, \ldots, A_i^K, \theta_i, p) = V(a_i, A_i^1, \ldots, A_i^K, \zeta_i, y_i - pa_i). \tag{3}$$

An equilibrium is defined in a straightforward way. For given vectors $\theta = (\theta_1, \ldots, \theta_n) \in \Theta^n$ and p, an equilibrium for (θ, p) is a vector $a = (a_1, \ldots, a_n)$ such that, for each i,

$$a_i \in \operatorname{argmax} U^i(a_i, \sum_{j=1}^{n} \gamma_{ij}^1 a_j, \ldots, \sum_{j=1}^{n} \gamma_{ij}^K a_j, \theta_i, p), \tag{4}$$

This definition of equilibrium requires that, when making a decision, agent i observes A_i^k - the summary statistics of other agents' actions that

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[10]One can also extend the notion of equilibrium to allow for an endogenous p.

affect his utility. As usual we can interpret this equilibrium as a steady state of a dynamical system in which at each point in time, agents make a choice based on the previous choices of other agents, though additional assumptions, such as those in Proposition 4 below are needed to guarantee that the dynamical system will converge to such a steady state.

2.2 Some examples

Example 1 *The discrete choice model of Brock and Durlauf [1995].*[11]
Here the set $\mathcal{A} = \{-1, 1\}$, and $\Theta = R$. Each agent has a single reference group, all other agents, and the weights $\gamma_{ij} \equiv 1/(n-1)$. This choice of reference group and weights is commonly referred to as global interactions. Let

$$U^i(a_i, A_i, \theta) = ha_i - J(A_i - a_i)^2 + \left(\frac{1 - a_i}{2}\right)\theta_i, \qquad (5)$$

where $h \in R$, $J > 0$. The θ_i's are assumed to be independently and identically distributed with

$$Prob\,(\theta_i \leq z) = \frac{1}{1 + \exp(-\nu z)},$$

for some $\nu > 0$. h measures the preference of the average agent for one of the actions, J the desire for conformity, and θ_i is a shock to the utility of taking the action $a_i = -1$. Brock and Durlauf also consider generalized versions of this model where the γ_{ij}'s vary, thus allowing each agent to have a distinct peer group.

Example 2 *Glaeser and Scheinkman [to appear].*
The utility functions are:

$$U_i(a_i, A_i, \theta_i, p) = -\frac{1 - \beta}{2}a_i^2 - \frac{\beta}{2}(a_i - A_i)^2 + (\theta_i - p)a_i.$$

Here $0 \leq \beta \leq 1$ measures the taste for conformity. In this case,

$$a_i = [\beta A_i + \theta_i - p]. \qquad (6)$$

Notice that when $p = 0$, $\beta = 1$ and the $A_i's$ are the average action of all other agents, this is a version of the Brock-Durlauf model with continuous actions. Unfortunately, this case is very special. Equilibria only exist if $\sum_i \theta_i = 0$, and in this case a continuum of equilibria would exist. The model is, as we will show below, much better behaved when $\beta < 1$.

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[11]A related example is in Aoki[1995]

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In Glaeser and Scheinkman [to appear] the objective was to admit both local and global interactions in the same model to try to distinguish empirically between them. This was done by allowing for two reference groups, and setting $P_i^1 = \{1, \ldots, n\} - i$, A_i^1 the average action of all other agents, $P_i^2 = \{i-1\}$ if $i > 1$, $P_1^2 = \{n\}$, and writing:

$$U^i(a_i, A_i^1, a_{i-1}, \theta_i, p) = -\frac{1 - \beta_1 - \beta_2}{2} a_i^2 - \frac{\beta_1}{2}(a_i - A_i^1)^2 - \frac{\beta_2}{2}(a_i - a_{i-1})^2 + (\theta_i - p)a_i.$$

Example 3 *The class of models of strategic complementarity discussed in Cooper and John [1988].*

Again the reference group of agent i is $P_i = \{1, \ldots, n\} - i$. The set $\mathcal{A}$ is an interval on the line and $A_i = \frac{1}{n-1} \sum a_{j \neq i}$. There is no heterogeneity and the utility of each agent is $U^i = U(a_i, A_i)$. Cooper and John[1988] examine symmetric equilibria. The classic externality example fits in this framework. Each agent chooses an effort a_i and the resulting output is $f(a_i, \bar{a})$. Each agent consumes his per-capita output and has a utility function $u(c_i, a_i)$. Write

$$U(a_i, A_i) = u(f(a_i, \frac{(n-1)A_i + a_i}{n}), a_i).$$

Example 4 *A simple version of the model of Diamond [1982] on trading externalities.*

Each agents draws an e_i which is his cost of production of an unit of the good. The e_i's are distributed independently across agents and with a distribution F with support in an interval $[c, d]$. We assume that F has a density $f > 0$. After a period in which the agent decides to produce or not, he is matched at random with a single other agent, and if they have both produced, they exchange the goods and each enjoys utility u. Otherwise, if the agent has produced, he obtains utility θ_i from the consumption of his own good. If the agent has not produced he obtains utility 0. A negative θ_i should be interpreted as the cost of disposing the produced good. We assume that all agents use a cut-off policy, a level $c \leq x_i \leq d$ such that the agent produces, if and only if $e_i \leq x_i$. We set

$$a_i = F(x_i),$$

the probability that agent i will produce. Here the reference group is again all $j \neq i$, and

$$A_i = E(a_j | j \neq i) \equiv \frac{\sum_{j \neq i} a_j}{n - 1}$$

Hence, if he uses policy a_i an agent has an expected utility that equals

$$U_i(a_i, A_i, \theta_i) = \int_c^{x_i} [uA_i + \theta_i - e]f(e)de.$$

Optimality requires that $x_i = \min\{uA_i + \theta_i, d\}$.

Suppose first that $\theta_i \equiv 0$. A symmetric equilibrium ($a_i \equiv a$) will exist whenever there is a solution to the equation

$$a = F(ua). \tag{7}$$

If F is the uniform distribution in $[0, u]$ then every a is a symmetric equilibrium. As we will show in Proposition 2 this situation is very special. For a fixed F, for almost every vector $\theta = (\theta_1, \dots, \theta_n)$, equilibria are isolated.

Example 5 *A matching example that requires multiple reference groups (Pesendorfer[1995]).*

In a simple version, there are two groups, leaders (L) and followers (F), with n_L and n_F members respectively. An individual can use one of two kinds of clothes. Buying the first one ($a = 0$) is free, buying the second ($a = 1$) costs p. Agents are matched randomly to other agents using the same clothes. Suppose the utility agent i, who is of type $t \in \{L, F\}$ and is matched to an agent of type t', is $V_i(t, t', a, p, \theta_i) = u(t, t') - ap + \theta_i a$, where θ_i is a parameter that shifts the preferences for the second kind of clothes. Assume that:

$$u(L, L) - u(L, F) > p > u(F, L) - u(F, F) > 0, \tag{8}$$

where we have abused notation by writing $u(L, L)$ instead of $u(t, t')$ with $t \in L$ and $t' \in L$ etc...In this example, each agent has two reference groups. If $i \in L$ then $P_i^1 = L - \{i\}$ and $P_i^2 = F$. On the other hand, if $i \in F$ then $P_i^1 = L$ and $P_i^2 = F - \{i\}$.

2.3 Equilibria with continuous actions

In this subsection we derive results concerning the existence, number of equilibria, stability and ergodicity of a basic continuous action model. We try not to rely on a specific structure of reference groups or to assume a specific weighting for each reference group. We assume that $\mathcal{A}$ is a (possibly unbounded) interval in the real line, that each U^i is at least twice continuously differentiable, and that $U_{11}^i < 0$. Each agent i has a single reference group P_i. The choice of a single peer group for each agent and of a scalar action is not crucial, but it substantially simplifies the notation.

We also assume that the optimal choices are interior and hence, since $i \notin P_i$, the first order condition may be written as:

$$U_1^i(a_i, A_i, \theta_i, p) = 0 \tag{9}$$

Since $U_{11}^i < 0$, then $a_i = g^i(A_i, \theta_i, p)$ is well defined and,

$$g_1^i(A_i, \theta_i, p) = -\frac{U_{12}^i(a_i, A_i, \theta_i, p)}{U_{11}^i(a_i, A_i, \theta_i, p)}. \tag{10}$$

Recall that for a given vectors $\theta = (\theta_1, \dots, \theta_n) \in \Theta^n$ and p, an equilibrium for (θ, p) is a vector $a(\theta, p) = (a_1(\theta, p), \dots, a_n(\theta, p))$ such that, for each i,

$$a_i(\theta, p) = g^i(A_i(a(\theta, p)), \theta_i, p). \tag{11}$$

Proposition 1 gives conditions for the existence of an equilibrium.

Proposition 1 *Given a pair $(\theta, p) \in \Theta^n \times \Pi$, suppose that I is a closed bounded interval such that, for each i, $g^i(A_i, \theta_i, p) \in I$, whenever $A_i \in I$. Then there exists at least one equilibrium $a(\theta, p) \in I^n$. In particular, an equilibrium exists if there exists an $m \in R$, with $[-m, m] \subset \overset{\circ}{\mathcal{A}}$, and such that, for any i and $A_i \in [-m, m]$, $U_1^i(-m, A_i, \theta_i, p) \geq 0$ and $U_1^i(m, A_i, \theta_i, p) \leq 0$.*

Proof: Let $G(a, \theta, p) : R^n \times \Theta^n \times \Pi \to R^n$, be given by:

$$G(a, \theta, p) = (g^1(A_1, \theta_1, p), \dots, g^n(A_n, \theta_n, p)).$$

If $a \in I^n$, since A_i is a convex combination of the entries of a, $A_i \in I$. Since $g^i(A_i, \theta_i, p) \in I$, whenever $A_i \in I$, the (continuous) function $G(\cdot, \theta, p)$ maps I^n into I^n, and therefore must have at least one fixed point. The second part of the proposition follows since $U_{11} < 0$ implies that $g^i(A_i, \theta_i, p) \in [-m, m]$, whenever $A_i \in [-m, m]$. QED

Proposition 1 gives us sufficient conditions for the existence of an equilibrium for a given (θ, p). The typical model however describes a process for generating the $\theta_i's$. In this case not all pairs (θ, p) are equally interesting. The process generating the θ_i's will impose a distribution on the vector θ, and we need only to check the assumptions of Proposition 1 on a set of θ's that has probability 1. For a fixed p, we define an **invariant interval** I as any interval such that there exists a set $\Lambda \subset \Theta^n$ with $\text{Prob}(\Lambda) = 1$, such that for each i, and for all $\theta \in \Lambda$, $g^i(A_i, \theta_i, p) \in I$, whenever $A_i \in I$. If multiple disjoint compact invariant intervals exist, multiple equilibria prevail with probability one.

It s relatively straightforward to construct models with multiple equilibria that are perturbations of models without heterogeneity. Suppose that Θ is an interval containing 0 and that $g(A, \theta)$ is a smooth function that is increasing in both coordinates.[12] Let x be a stable fixed point of $g(\cdot, 0)$ *i.e.* $g(x, 0) = 0$ and $g_1(x, 0) < 1$. If the interval Θ is small enough, there exists an invariant interval containing x. In particular, if a model without heterogeneity has multiple stable equilibria, the model with small noise will also have multiple equilibria. The condition on invariance must hold for almost all $\theta \in \Theta$. In particular if we have multiple *disjoint* invariant intervals and we shrink Θ, we must still have multiple disjoint invariant intervals. On the other hand if we expand Θ, we may loose a particular invariant interval and multiple equilibria are no longer assured. An implication of this reasoning is that when individuals are sorted into groups according to their θ's, and agents do not interact across groups, then multiple equilibria are more likely to prevail. In section 2.5 we discuss a model where agents sort on their θ's.

The next example shows that in contrast to the results of Cooper and John[1988], in our model, because we consider a richer structure of reference groups, strategic complementarity is *not* necessary for multiple equilibria.

Example 6 *This is an example to show that in contrast to the case of purely global interactions, strategic complementarity is not a necessary conditions for multiple equilibria. There are two sets of agents $\{S_1\}$ and $\{S_2\}$, and n agents in each set. For agents of a given set, the reference group consists of all the agents of the other set. If $i \in S_k$,*

$$A_i = \frac{1}{n} \sum_{j \in S_\ell} a_j,$$

$\ell \neq k$. There are two goods and the relative price is normalized to one. Each agent has an initial income of one unit and his objective is to maximize:

$$U_i = \log a_i + \log(1 - a_i) + \frac{\lambda}{2}(a_i - A_i)^2. \tag{12}$$

Only the first good exhibits social interactions, and agents of each set want to differentiate from the agents of the other set. Provided $\lambda < 8$ the utility function is strongly concave. However there is no strategic complementarity

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[12]The assumption that g is increasing in θ is only a normalization. In contrast, the assumption that g is increasing in A is equivalent to $U_{12} > 0$, *i.e.* an increase in the average action by the members of his reference group, increases the marginal utility of an agent's own action. This assumption was called strategic complementarity in Bulow, Geanokoplos and Kemplerer [1985].

- an increase in the action of others, (weakly) decreases the marginal utility of an agent's own action. We will look for equilibria with a_i constant within each set. An equilibrium of this type is described by a pair x, y of actions for each set of agents. In equilibrium we must have:

$$1 - 2x + \lambda x(1-x)(x-y) = 0, \qquad (13)$$
$$1 - 2y + \lambda y(1-y)(y-x) = 0. \qquad (14)$$

Clearly $x = y = 1/2$ is always an equilibrium. It is the unique equilibrium that is symmetric across groups. Provided $\lambda < 4$ the Jacobian associated with equations (13) and (14) is positive, what is compatible with uniqueness even if we consider asymmetric equilibria. However whenever $\lambda > 4$ the Jacobian becomes negative and other equilibria must appear. For instance if $\lambda = 4.04040404$, $x = .55$ and $y = .45$ is an equilibrium, and consequently so is $x = .45$ and $y = .55$. Hence at least three equilibria obtain, without strategic complementarity.

Proposition 1 gives existence conditions that are independent of the structure of the reference groups and of the weights γ'_{ij}s. Also, the existence of multiple invariant intervals is independent of the structure of interactions embedded in the P_i's and γ_{ij}'s, and is simply a result of the choice of an individual's action, given the "average action" of his reference group, the distribution of his taste shock, and the value of the exogenous parameter p.

In some social interaction models, such as the Diamond search model, (Example 4 above) there may exist a continuum of equilibria. The next proposition shows that these situations are exceptional.

Proposition 2 *Suppose Θ is an open subset of R^k and that there exists a coordinate j such that $\frac{\partial U_1^i}{\partial \theta_i^j} \neq 0$, that is θ_i^j has an effect in the marginal utility of the action. Then, for each fixed p, except for a subset of Θ^n of Lebesgue measure zero, the equilibria are isolated. In particular if the θ_i^j's are independently distributed with marginals that have a density with respect to the Lebesgue measure then, for each fixed p, except for a subset of Θ^n of zero probability, the equilibria are isolated*

Proof: For any p, consider the map $F(a, \theta) = a - G(a, \theta, p)$. The matrix of partial derivatives of F with respect to θ^j is a diagonal matrix with entry $d_{ii} \neq 0$, since $\frac{\partial U_1^i}{\partial \theta_i^j} \neq 0$. Hence for each fixed p, DF has rank n and it is a consequence of Sard's theorem,(see *e.g.* Mas-Colell page 320) that except perhaps for a subset of Θ^n of Lebesgue measure zero F_1 has rank n. The implicit function theorem yields the result. QED

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Notice that the search model discussed in Example 4 satisfies the assumptions of the Proposition. This justifies our earlier claim that the continuum of equilibria that exists when $\theta_i \equiv 0$ is exceptional. In the model discussed on Example 2, if $p = 0$, $\beta = 1$, and the reference group of each agent is made up by all other agents (with equal weights), then if $\sum \theta_i \neq 0$, there are no equilibria, while if $\sum \theta_i = 0$, there is a continuum. Again, the continuum of equilibria is exceptional. However if $\beta < 1$ there is a unique equilibrium for any vector θ. This situation is less discontinuous than it seems. In equilibrium,

$$\frac{\sum a_i}{n} = \frac{1}{1-\beta} \frac{\sum \theta_i}{n}.$$

Hence, if we fix $\sum \theta_i$ and drive β to one, the average action becomes unbounded.

Although Proposition 2 is stated using the $\theta_i's$ as parameters, it is also true that isolated equilibria become generic if there is heterogeneity across individuals' utility functions.

One occasionally proclaimed virtue of social interaction models is that they create the possibility that multiple equilibria might exist. Proposition 1 gives us sufficient conditions for there to be multiple equilibria in social interactions models. One way to insure uniqueness is this context is to place a bound on the effect of social interactions. We will say that *Moderate Social Influence* (henceforth MSI) prevails, if the marginal utility of an agent's own action is less affected (in absolute value) by a change in the average action of his peers than by a change on his own action. More precisely we say that MSI prevails if

$$\left| \frac{U_{12}^i(a_i, A_i, \theta_i, p)}{U_{11}^i(a_i, A_i, \theta_i, p)} \right| < 1. \tag{15}$$

From equation (10) the MSI condition implies:

$$|g_1^i(A_i, \theta_i, p)| < 1. \tag{16}$$

This last condition is, in fact, weaker than inequality (15), since it is equivalent to inequality (15) when a_i is optimal, given (A_i, θ_i, p). We use only inequality (16), and therefore we will refer to this term as the MSI condition.

The next proposition shows that if the MSI condition holds, there will be a unique equilibrium.[13]

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[13]Cooper and John [1988] had already remarked that this condition is sufficient for uniqueness in the context of their model

Proposition 3 *If for a fixed (θ, p), MSI holds (that is inequality (16) is verified for all i,) then there exists at most one equilibrium $a(\theta, p)$.*

Proof: Let $F(a, \theta, p) = a - G(a, \theta, p)$. The matrix of partial derivatives with respect to a, that we denote by $F_1(a, \theta, p)$, has diagonal elements equal to 1 and, using equation (10), off diagonal elements $d_{ij} = -g_1^i(A_i, \theta_i, p)\gamma_{ij}$. Also, for each i,

$$\sum_{j \neq i} |d_{ij}| = |g_1^i(A_i, \theta_i, p)| \sum_{j \neq i} \gamma_{ij} = |g_1^i(A_i, \theta_i, p)| < 1.$$

Hence $F_1(a, \theta, p)$ is a matrix with a positive dominant diagonal, and as a consequence, for each (θ, p), $F(a, \theta, p) = 0$ has a unique solution (Mckenzie [1960], or Gale and Nikaido [1963]). QED

Again MSI needs only to hold for a set of θ's with full probability. The assumption in Proposition 3 is independent of the structure of interactions embedded in the P_i's and the γ_{ij}'s. An example where MSI is satisfied is when $U(a_i, A_i, \theta_i, p) = u(a_i, \theta_i, p) + w(a_i - A_i, p)$, where $u_{11} < 0$, and, for each p, $w(\cdot, p)$ is concave.

If in addition to MSI we assume strategic complementarity $(U_{12} > 0)$, we can derive stronger results. Suppose p has a component, say p^1, such that each g^i has a positive partial derivative with respect to p^1. In equilibrium, we have

$$\frac{\partial a}{\partial p^1} = (F_1)^{-1}(a, \theta, p)\frac{\partial g}{\partial p^1}. \tag{17}$$

Since F_1 has a dominant diagonal that is equal to one, we may use the Neumann expansion to write:

$$(F_1)^{-1} = I + (I - F_1) + (I - F_1)^2 + \dots . \tag{18}$$

Recall that all diagonal elements of $(I - F_1)$ are zero and that the off-diagonal elements are $-d_{ij} = g_1^i(A_i, \theta_i, p)\gamma_{ij} > 0$. Hence each of the terms in this infinite series is a matrix with non-negative entries, and

$$\frac{\partial a}{\partial p^1} = (I + H)\frac{\partial g}{\partial p^1}, \tag{19}$$

where H is a matrix with non-negative elements. The non-negativity of the matrix H, means that there is a **social multiplier** (as in Becker and Murphy [to appear])[14]. An increase in p^1, holding all a_j's $j \neq i$ constant,

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[14]Cooper and John[1988] define a similar multiplier by considering symmetric equilibria of a game.

leads to a change

$$da_i = \frac{\partial g^i(A_i, \theta_i, p)}{\partial p^1} dp^1,$$

while, in equilibrium, that change equals

$$\left[\frac{\partial g^i(A_i, \theta_i, p)}{\partial p^1} + \sum_j H_{ij} \frac{\partial g^j(A_j, \theta_j, p)}{\partial p^1} \right] dp^1.$$

The effect of a change in p^1 on the average

$$\bar{A} \equiv \frac{\sum_i a_i}{n}$$

is, in turn:

$$d\bar{A} = \frac{1}{n} \left[\sum_i \frac{(\partial g^i(A_i, \theta_i, p)}{\partial p^1} + \sum_{i,j} H_{ij} \frac{\partial g^j(A_j, \theta_j, p)}{\partial p^1} \right] dp^1$$

This same multiplier also impacts the effect of the shocks θ_i. Differences in the sample realizations of the θ_i's are amplified through the social multiplier effect.

The size of the social multiplier depends on the value of $g_1^i \equiv \frac{\partial g}{\partial A_i}$. If these numbers are bounded away from one, one can bound the social multiplier. However, as these numbers approach unity the social multiplier effect gets arbitrarily large. In this case, two populations with slightly distinct realizations of the θ_i's could exhibit very different average values of the actions. In the presence of unobserved heterogeneity it may be impossible to distinguish between a large multiplier (that is g_1 is near unity) and multiple equilibria.

Propositions (1) and (3) give us conditions for multiplicity or uniqueness. At this level of generality it is impossible to refine these conditions. It is easy to construct examples where $g_1 > 1$ in some range, but still only one equilibrium exits.

One common way to introduce *ad-hoc* dynamics in social interaction models is to simply assume that in period t each agent chooses his action based on the choices of the agents in his reference group at time $t-1$.[15] Such processes are not guaranteed to converge, but the next proposition shows that when MSI prevails convergence occurs.

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[15]In social interaction models, *ad hoc* dynamics is frequently used to select among equilibria as in Young [1993, 1998]or Blume and Durlauf[1999].

Let $a^t(\theta, p, a^0)$ be the solution to the difference equation

$$a^{t+1} = G(a^t, \theta, p),$$

with initial value a^0.

Proposition 4 *If for a fixed (θ, p), $|g_1^i(\cdot, \theta_i, p)| < 1$, for all i, then*

$$\lim_{t \to \infty} a^t(\theta, p, a^0) = a(\theta, p).$$

Proof: For any matrix M, let $\|M\| = \max_i \sum_j |M_{ij}|$, be the matrix norm. Then, $\max_i |a_i^{t+1} - a_i(\theta, p)| \leq \sup_y \|G_1(y, \theta, p)\| (\max_i |a_i^t - a_i(\theta, p)|) \leq \max_i |a_i^t - a_i(\theta, p)|$. Hence the vectors a^t stay in a bounded set B and, by assumption, $\sup_{y \in B} \|G_1(y, \theta, p)\| < 1$. Hence $\lim_{t \to \infty} a^t(\theta, p, a^0) = a(\theta, p)$.

One intriguing feature of social interaction models is that in some of these models, individual shocks can determine aggregate outcomes for large groups. In contrast to the results presented earlier, which are independent of the particular interaction structure, ergodicity depends on a more detailed description of the interactions. For instance, consider the model in Example 2 above with $p = 0$, the θ_i's i.i.d., $P_1 = \emptyset$ and $P_i = \{1\}$ for each $i > 1$. That is, agent 1 is a "leader" that is followed by everyone. Then $a_1 = \theta_1$ and $a_i = \theta_i + \beta a_1$. Hence the average action, even as $n \to \infty$ depends on the realization of θ_1, even though the assumption of Proposition 3 holds. Our next proposition shows that when MSI holds, shocks are i.i.d., and individuals' utility functions depend only on their own actions and the average action of their peer group, then, under mild technical conditions, the average action of a large population is independent of the particular realization of the shocks.

Proposition 5 *Suppose that*

1. θ_i *is identically and independently distributed.*

2. U^i *(and hence g^i) is independent of i.*

3. $P_i = \{1, \ldots, i-1, i+1, \ldots, n\}$.

4. $\gamma_{i,j} \equiv \frac{1}{n-1}$.

5. $\mathcal{A}$ *is bounded*

6. *MSI holds uniformly, that is:*

$$\sup_{A_i, \theta_i} |g_1(A_i, \theta_i, p)| < 1$$

Let $a^n(\theta, p)$ denote the equilibrium when n agents are present and agent i receives shock θ_i. Then there exists an $\bar{A}(p)$ such that with probability one,

$$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{a_i^n(\theta, p)}{n} = \bar{A}(p) \qquad (20)$$

Proof: We omit the argument p from the proof. Let $A^n(\theta) = \sum_{i=1}^{n} \frac{a_i^n(\theta)}{n}$. The boundedness of $\mathcal{A}$ insures that there are convergent subsequences $A^{n_k}(\theta)$. Suppose the limit of one such convergent subsequence is $A(\theta)$. Notice that $|A_i^{n_k}(\theta) - A^{n_k}(\theta)| \leq b/n_k$, for some constant b. Hence, for any $\epsilon > 0$, we can find K such that if $k \geq K$,

$$
\begin{aligned}
|\sum_{i=1}^{n_k} \frac{a_i^{n_k}(\theta)}{n_k} - \sum_{i=1}^{n_k} \frac{g(A(\theta), \theta_i)}{n_k}| &= |\sum_{i=1}^{n_k} \frac{g(A_i^{n_k}, \theta_i)}{n_k} - \sum_{i=1}^{n_k} \frac{g(A(\theta), \theta_i)}{n_k}| \\
&\leq \epsilon \qquad (21)
\end{aligned}
$$

Furthermore, since the θ_i are i.i.d. and g_1 is uniformly bounded, there exists a set of probability one, that can be chosen independent of A, such that,

$$\sum_{i=1}^{n} \frac{g(A, \theta_i)}{n} \to \int_{\Theta} g(A, y) dF(y),$$

where F is the distribution of each θ_i. Hence, given any $\epsilon > 0$, if k is sufficiently large,

$$|A^{n_k}(\theta) - \int_{\Theta} g(A(\theta), y) dF(y)| \leq \epsilon,$$

or

$$A(\theta) = \int_{\Theta} g(A(\theta), y) dF(y).$$

in the hypothesis of the proposition guarantees that $g(\cdot, \theta_i)$ is a contraction and, as a consequence, this last equation has at most one solution, $\bar{A}$. In particular, all convergent subsequences of the bounded sequence $A^n(\theta)$ converge to $\bar{A}$ and hence, $A^n(\theta) \to \bar{A}$.Q.E.D.

The assumptions in the proposition are sufficient, but not necessary, for ergodicity. In general, models in which shocks are i.i.d. and interactions are local tend to display ergodic behavior.

2.4 "Mean field" models with large populations and discrete actions

In this subsection we will examine models with discrete action spaces (actually two possible actions), in which the utility function of the agents depends on their own action and the average action taken by the population. Much of our framework and results are inspired by the treatment by Brock and Durlauf [1995] of Example 1 described above. The action space of individuals is $\{0, 1\}$. As in Brock and Durlauf we will assume that

$$U^i = U(a_i, A, p) + (1 - a_i)\theta_i,$$

that is the shock θ_i is the extra utility an agent obtains from taking action 0. We will assume that $U(a_i, \cdot, \cdot)$ is smooth and that the θ_i's are i.i.d. with a c.d.f. F with continuous density f. Agents do not internalize the effect that their action has on the average action.

We also assume strategic complementarity, which in this context we take to be: $U_2(1, A, p) - U_2(0, A, p) > 0$, that is, an increase in the average action increases the difference in utility between action 1 and action 0.

Given A, agent i will take action 1, if and only if, $\theta_i \leq U(1, A, p) - U(0, A, p)$. In a large population a fraction $F(U(1, A, p) - U(0, A, p))$ will take action 1, the remainder will take action 0.

A *mean-field equilibrium*, thereafter MFE, is an average action $\bar{A}$ such that

$$F(U(1, \bar{A}, p) - U(0, \bar{A}, p)) - \bar{A} = 0. \tag{22}$$

This definition of an MFE is exactly as in the Brock and Durlauf treatment of Example 1 above. The next proposition correspond to their results concerning equilibria in that example.

Proposition 6 *An MFE always exists. If $0 < \bar{A} < 1$ is an equilibrium where*

$$f(U(1, \bar{A}, p) - U(0, \bar{A}, p))[U_2(1, \bar{A}, p) - U_2(0, \bar{A}, p)] > 1, \tag{23}$$

then there are also at least two other MFE's, one on each side of $\bar{A}$. On the other hand, if at every MFE, $f(U(1, \bar{A}, p) - U(0, \bar{A}, p))[U_2(1, \bar{A}, p) - U_2(0, \bar{A}, p)] < 1$, there exists a single MFE.

Proof: $H(A) = F(U(1, A, p) - U(0, A, p)) - A$ satisfies $H(0) \geq 0$, and $H(1) \leq 0$ and is continuous. If inequality (23) holds, then $H(\bar{A}) = 0$ and, $H'(\bar{A}) > 0$. QED.

The first term on the left hand side of inequality (23) is the density of agents that are indifferent between the two actions, when the average action is $\bar{A}$. The second term is the marginal impact of the average action on the preference for action 1 over action zero, which, by our assumption of strategic complementarity, is always > 0. This second term corresponds exactly to the intensity of social influence that played a pivoting role in determining the uniqueness of equilibrium in the model with a continuum of actions.

If there is a unique equilibrium,[16] then the social multiplier will equal:

$$\frac{\partial \bar{A}}{\partial p} = \frac{f(U(1, \bar{A}, p) - U(0, \bar{A}, p))[U_3(1, \bar{A}, p) - U_3(0, \bar{A}, p)]}{1 - f(U(1, \bar{A}, p) - U(0, \bar{A}, p))[U_2(1, \bar{A}, p) - U_2(0, \bar{A}, p)]}. \qquad (24)$$

The numerator in this expression is exactly the average change in action, when p changes, and agents consider that the average action remains constant. The denominator is, if uniqueness prevails, positive.

As we emphasized in the model with continuous actions, there is a continuity in the multiplier effect. As the parameters of the model (U and F) approach the region of multiple equilibria, the effect of a change in p on the equilibrium average action approaches infinity.

In many examples the distribution F satisfies:

1. Symmetry: ($f(z) = h(|z|)$)

2. Monotonicity (h is decreasing.)

If in addition the model is unbiased ($U(1, 1/2, p) = U(0, 1/2, p)$), then $A = 1/2$ is an MFE. The fulfillment of inequality (23) now depends on the value of $f(0)$. This illustrates the role of homogeneity of the population in producing multiple equilibria. If we consider a parameterized family of models in which the random variable $\theta_i = \sigma x_i$ where $\sigma > 0$, then $f^\sigma(0) = \frac{1}{\sigma} f^1(0)$. As $\sigma \to 0$ ($\sigma \to \infty$) inequality (23) must hold (*resp.* must reverse). In particular if the population is homogeneous enough, multiple equilibria must prevail in the unbiased case.

These reasoning can be extended to biased models, if we assume that $[U_2(1, \cdot, p) - U_2(0, \cdot, p)]$ is bounded and bounded away from zero, and that

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[16]In here and in what follows we require strict uniqueness that is, the left hand side of inequality (23) is less than one.

the density f^1 is continuous and positive.[17] For, in this case, for σ large,

$$\sup_{A}\{f^\sigma(U(1, A, p) - U(0, A, p))[U_2(1, A, p) - U_2(0, A, p)]\} < 1. \qquad (25)$$

Hence equilibrium will be unique, if the population displays sufficient heterogeneity. On the other hand as $\sigma \to 0$, inequality (25) is reversed and multiple equilibria appear.

We can derive more detailed properties if we assume,that in addition to the symmetry and monotonicity properties of f, that $U_{22}(1, A, p) - U_{22}(0, A, p) \leq 0$, that is the average action A has a diminishing marginal impact on the preference for the high action. In that case, it is easy to show that there are at most three equilibria.

2.5   Choice of peer group

The mathematical structure and the empirical description of peer or reference groups varies from model to model. In several models (*e.g* Gabszewicz and Thisse [1996],Benabou [1993], Glaeser, Sacerdote and Scheinkman [1996] or Mobius [1999]) the reference group is formed by geographical neighbors. To obtain more precise results one must further specify the mathematical structure of the peer group relationship - typically assuming either that all fellow members of a given geographical unit form a reference group or that each agent's reference group is formed by a set of near-neighbors. Mobius [1999] shows that, in the context that generalizes Schelling's [1972] tipping model, the persistence of segregation depends on the particular form of the near-neighbor relationship. Glaeser, Sacerdote and Scheinkman [1996] show that the variance of crime-rates across neighborhoods or cities would be a function of the form of the near-neighbor relationship.

Kirman [1983], Kirman, Oddou and Weber [1986], and Ioannides [1990] use random graph theory to treat the peer group relationship as random. This approach is particularly useful in deriving properties of the probable peer groups as a function of the original probability of connections. Another literature deals with individual incentives for the formation of networks (*e.g.* Boorman [1975], Jackson and Wolinsky [1996], Bala and Goyal [to appear])[18]

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[17]An example that satisfies these conditions is the model of Brock and Durlauf described in Example 1 above. Brock and Durlauf use a slightly different state space, but once the proper translations are made, $U_2(1, A, p) - U_2(0, A, p) = kJ$ for a positive constant k and $0 < f^1(z) \leq \nu$.

[18]A related problem is the formation of coalition in games *e.g.* Myerson [1991].

One way to model peer group choice is to consider a set of neighborhoods indexed by $\ell = 1, \ldots, m$ each with n_ℓ slots with $\sum_\ell n_\ell \geq n$.[19] Every agent chooses a neighborhood to join after the realization of the $\theta_i's$. To join neighborhood P^ℓ one must pay q_ℓ. The peer group of agent i, if he joins neighborhood ℓ, consists of all other agents j that joined ℓ with $\gamma_{ij} = \gamma_{ij'}$ for all peers j and $j\prime$. We will denote by A^ℓ, the average action taken by all agents in neighborhood ℓ. Our equilibrium notion, in this case will parallel Tiebout's equilibrium (see *e.g.* Bewley [1981].)

For given vectors $\theta = (\theta_1, \ldots, \theta_n) \in \Theta^n$ and p, an equilibrium will be a set of prices $(q_1, \ldots, q_m)$, an assignment of agents to neighborhoods, and a vector of actions $a = (a_1, \ldots, a_n)$, that is an equilibrium given the peer groups implied by the assignment, such that, if agent i is assigned to neighborhood ℓ, there is no neighborhood ℓ' such that

$$\sup_{a_i} U^i(a_i, A^{\ell'}, \theta_i, p) - q_{\ell'} > \sup_{a_i} U^i(a_i, A_i, \theta_i, p) - q_\ell \qquad (26)$$

In other words, in an equilibrium with endogenous peer groups we add the additional restriction that no agent prefers to move.

To examine the structure of the peer groups that arise in equilibrium we assume, for simplicity, that the U^i's are independent of i, that is that all heterogeneity is represented in the θ_i's. If an individual with a higher θ gains more utility from an increase of the average action than an individual with a lower θ, then segregation obtains in equilibrium. More precisely if Θ is an interval $[t_o, t^o]$ of the line, and

$$V(A, \theta, p) \equiv \sup_{a_i} U(a_i, A, \theta, p),$$

satisfies:

$$V(A, \theta, p) - V(A', \theta, p) > V(A, \theta', p) - V(A', \theta', p)$$

whenever, $A > A'$ and $\theta > \theta'$, there exist points $t_0 = t_0 < t_1, \ldots, < t_m = t^0$ such that, agent i chooses neighborhood ℓ if and only if $\theta_i \in [t_{\ell-1}, t_\ell]$. (*e.g.* Benabou [1993], Glaeser and Scheinkman [to appear]). Though other equilibria exist, these are the only "stable" ones.

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[19]This treatment of peer group formation is used in Benabou [1993] and Glaeser and Scheinkman[to appear]. However, in several cases, peer groups have no explicit fees for entry. Mailath, Samuelson and Shaked [1996] examine the formation of peer groups when agents are matched to others from the same peer group.

3 Empirical approaches to social interactions

The theoretical models of social interaction models discussed above are, we believe, helpful in understanding a wide variety of important empirical regularities. In principle, large differences in outcomes between seemingly homogeneous populations, radical shifts in aggregate patterns of behavior, and spatial concentration and segregation can be understood through social interaction models. But these models are not only helpful in understanding stylized facts, they can also serve as the basis for more rigorous empirical work. In this section, we outline the empirical approaches that can be and have been used to actually measure the magnitude of social interactions.

For simplicity, in this empirical section we focus on the linear-quadratic version of the model discussed in Example 2. Our decision to focus on the linear-quadratic model means that we ignore some of the more important questions in social interactions. For example, the case for place-based support to impoverished areas often hinges upon a presumption that social interactions have a concave effect on outcome. Thus, if impoverished neighborhoods can be improved slightly by an exogenous program, then the social impact of this program (the social multiplier of the program) will be greater than if the program had been enacted in a more advantaged neighborhood. The case for desegregation also tends to hinge on concavity of social interactions. Classic desegregation might involve switching low human capital people from a disadvantaged neighborhood and high human capital people from a successful neighborhood. This switch will be socially advantageous if moving the low human capital people damages the skilled area less than moving the high human capital people helps the less skilled area. This will occur when social interactions operate in a concave manner.

As important as the concavity or convexity of social interactions have been, most of the work in this area has focused on estimating linear effects.[20]

Since our aim is not to survey[21], but rather to highlight certain issues that arise in the empirical analysis, we make many simplifying assumptions that help us focus on the relevant problems. We will use the linear model in Example (2). We assume we can observe data on C, equally sized[22], groups.

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[20]Crane (1991) is a notable exception. He searches for non-linearities across a rich range of variables and finds some evidence for concavity in the social interactions involved in out-of-wedlock births. Reagon, Weinberg and Yankow (2000) similarly explore non-linearities in research on work behavior and find evidence for concavity.

[21]A recent survey of the econometrics of a class of interaction based binary choice models, and a review of the empirical literature, can be found in Brock and Durlauf [to appear].

[22]The assumption of equally sized groups is made only to save on notation.

All interactions occur within a group.

Rewriting equation (6) for the optimal action, to absorb p in the θ_i' we have:

$$a_i = \beta A_i + \theta_i. \tag{27}$$

We will examine here a simple form of global interactions. If agent i belongs to group ℓ,

$$A_i = \frac{1}{n-1} \sum_{j \neq i} a_j,$$

where the sum is over the agents j in group ℓ, and n is the size of a group. We will also assume that $\theta_i = \lambda_\ell + \varepsilon_i$, where the ε_i's are assumed to be i.i.d., with mean zero λ_ℓ is a place specific variable (perhaps price) that affects everyone in the group, while ε_i is an idiosyncratic shock that is assumed to be independent across people.

The average action within a group is:

$$\frac{\sum_i a_i}{n} = \frac{\lambda_\ell}{1-\beta} + \frac{\sum_i \varepsilon_i}{n(1-\beta)}. \tag{28}$$

The optimal action of agent i is then:

$$a_i = \frac{\lambda_\ell}{1-\beta} + \frac{(n-1-\beta n + 2\beta)\varepsilon_i}{(n-1+\beta)(1-\beta)} + \frac{\beta \sum_{j \neq i} \varepsilon_j}{(n-1+\beta)(1-\beta)}. \tag{29}$$

The variance of actions on the whole population, is

$$\text{Var}(a_i) = \frac{\sigma_\lambda^2}{(1-\beta)^2} + \sigma_\varepsilon^2 \left(1 + \left(\frac{\beta}{1-\beta}\right)^2 \frac{3(n-1) - 2\beta(n-2) - \beta^2}{(n-1+\beta)^2}\right). \tag{30}$$

As $n \to \infty$ this converges to $\frac{\sigma_\lambda^2}{(1-\beta)^2} + \sigma_\varepsilon^2$. In this case, and in the cases that are to follow, even moderate levels of n ($n = 30+$) yield results that are quite close to the asymptotic result. For example, if $n = 40$, and $\beta \leq .5$, then the bias is at most $-.05\sigma_\varepsilon^2$. Higher values of β are associated with more severe negative biases, but when $n = 100$, a value of $\beta = .75$ (which we think of as being quite high) is associated with a bias of only $-.135\sigma_\varepsilon^2$.

3.1 Variances Across Space

The simplest, although hardly the most common, method of measuring the size of social interactions is to use the variance of a group average. The intuition of this approach stems from early work on social interactions and

multiple equilibria (see, e.g. Schelling [1978], or Becker [1989], or Sah [1991]). These papers all use different social interaction models to generate multiple equilibria for a single set of parameter values. One interpretation of the connection between social interactions and multiple equilibria is that social interactions are associated with large differences across time and space that are too large to be justified by fundamentals. Glaeser, Sacerdote and Scheinkman [1996] use this intuition to create a model where social interactions are associated with a high degree of variance across space without multiple equilibria. In principle, it is difficult to separate out extremely high variances from multiple equilibria models, but Glaeser and Scheinkman [to appear] argue that for many variables high variance models with a single equilibrium are a more parsimonious means of describing the data.

Suppose we obtain $m \leq n$ observations of members of a group. The sum of the observed actions, normalized by dividing by the square root of the number of observations, will have variance:

$$Var\left(\frac{\sum_i a_i}{\sqrt{m}}\right) = \frac{m\sigma_\lambda^2}{(1-\beta)^2} + \frac{\sigma_\varepsilon^2}{(1-\beta)^2} + (n-m)\sigma_\varepsilon^2 \frac{\beta^2(n-2) - 2\beta(n-1)}{(1-\beta)^2(n-1+\beta)^2}.$$

(31)

When $m = n$, (31) reduces to: $\frac{n\sigma_\lambda^2}{(1-\beta)^2} + \frac{\sigma_\varepsilon^2}{(1-\beta)^2}$, which is similar to the variance formula in Glaeser, Sacerdote and Scheinkman [1996] or Glaeser and Scheinkman[to appear]. Thus, if $m = n$ and $\sigma_\lambda^2 = 0$, as $n \to \infty$ the ratio of the variance of this normalized aggregate to the variance of individual actions converges to $\frac{1}{(1-\beta)^2}$. Alternatively, if m is fixed, then as n grows large, the aggregate variance converges to

$$\frac{m\sigma_\lambda^2}{(1-\beta)^2} + \sigma_\varepsilon^2,$$

and the ratio of the aggregate variance to the individual variance (when $\sigma_\lambda^2 = 0$) converges to one.

The practicality of this approach hinges on the extent to which σ_λ^2 is either close to zero or known.[23] As discussed above, λ_ℓ may be non-zero either because of correlation of background factors or because there are place specific characteristics that jointly determine the outcomes of neighbors. In some cases, researchers may know that neighbors are randomly assigned and that omitted place specific factors are likely to be small. For example,

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[23]In principle we could use variations in n across groups, and the fact that when $m = n$ the variance of the aggregates is an affine function of m to try to separately estimate σ_λ and σ_ε.

Sacerdote [2000] looks at the case of Dartmouth freshman year roommates who are randomly assigned to one another. He finds significant evidence for social interaction effects. In other contexts (see Glaeser, Sacerdote and Scheinkman [1996]) there may be methods of putting an upper bound on σ_λ^2 which allows the variance methodology to work. Our work found extremely high aggregate variances that seem hard to reconcile with no social interactions for reasonable levels of σ_λ^2. In particular, we estimated high levels of social interactions for petty crimes and crimes of the young. We found lower levels of social interactions for more serious crimes.

3.2 Regressing Individual Outcomes on Group Averages

The most common methodology for estimating the size of social interactions is to regress an individual outcome on the group average. Crane [1991], discussed above is an early example of this approach. Case and Katz [1991] is another early paper implementing this methodology (and pioneering the instrumental variables approach discussed below). Since these papers, there has been a torrent of later work using this approach and it is the standard method of trying to measure social interactions.

We will illustrate the approach considering a univariate regression where an individual outcome is regressed on the average outcome in that individual's peer group (not including himself). In almost all cases, researchers control for other characteristics of the subjects, but these controls would add little but complication to the formulae. The univariate ordinary least squares coefficient for a regression of an individual action on the action of his peer is

$$
\frac{Cov\left(a_i, \sum_{j \neq i} a_j / (m-1)\right)}{Var\left(\sum_{j \neq i} a_j / (m-1)\right)}. \tag{32}
$$

The denominator is a transformation of (31), where m-1 replaces $\sqrt{m}$:

$$
\begin{aligned}
Var\left(\frac{\sum_{j \neq i} a_j}{m-1}\right) &= \frac{\sigma_\lambda^2}{(1-\beta)^2} + \\
&\quad m\sigma_\varepsilon^2 \frac{[(n-1+\beta) - \beta(n-m)]^2 + \beta^2 m(n-m)}{(m-1)^2(1-\beta)^2(n-1+\beta)^2}.
\end{aligned} \tag{33}
$$

The numerator is:

$$
Cov\left(a_i, \frac{\sum_{j \neq i} a_j}{m-1}\right) = \frac{\sigma_\lambda^2}{(1-\beta)^2} + \beta\sigma_\varepsilon^2 \frac{(2n-2-\beta n + 2\beta)}{(1-\beta)^2(n-1+\beta)^2} . \tag{34}
$$

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When $\sigma_\lambda = 0$ then the coefficient reduces to:

$$Coeff = \frac{(m-1)^2}{m} \frac{2\beta(n-1) - \beta^2(n-2)}{(n-1+\beta)^2 - (n-m)[2\beta(n-1) - \beta^2(n-2)]}. \quad (35)$$

When $m = n$:

$$Coeff = 2\beta \frac{(n-1)^2}{n(n-1+\beta)} - \beta^2 \frac{(n-1)^2}{(n-1+\beta)^2}. \quad (36)$$

Hence as $n \to \infty$, the coefficient converges to $2\beta - \beta^2$. Importantly, because of the reflection across individuals, the regression of an individual outcome on a group average cannot be thought of as an unbiased estimate of β. However, under some conditions ($m = n$, large, $\sigma_\lambda^2 = 0$), the ordinary least squares coefficient does have an interpretation as a simple function of β.

Again, the primary complication with this methodology is the presence of correlated error terms across individuals. Some of this problem is corrected by controlling for observable individual characteristics. Indeed, the strength of this approach relative to the variance approach is that it is possible to control for observable individual attributes. However, in most cases, the unobservable characteristics are likely to be at least as important as the observable ones and are likely to have strong correlations across individuals within a given locale. Again, this correlation may also be the result of place-specific factors that affect all members of the community.

One approach to this problem is the use of randomized experiments that allocate persons into different neighborhoods. The Gautreaux experiment was an early example of a program that used government money to move people across neighborhoods. Unfortunately, the rules used to allocate people across neighborhoods are sufficiently opaque that it is hard to believe that this program really randomized neighborhoods.

The Moving to Opportunity experiment contains more explicit randomization. In that experiment, funded by the department of Housing and Urban Development, individuals from high poverty areas were selected into three groups: a control group and two different treatment groups. Both treatment groups were given money for housing which they used to move into low poverty areas. By comparing the treatment and control groups, Katz, Kling and Liebman [2000] are able to estimate the effects of neighborhood poverty without fear that the sorting of people into neighborhoods is contaminating their results. Unfortunately, they can't tell whether their effects are the results of peers or other neighborhood attributes. As such, this work is currently the apex of work on neighborhood effects but it cannot

really tell us about the contribution of peers vs. other place based factors. Sacerdote [2000] also uses a randomized experiment. He is able to compare people who are living in the same building but who have different randomly assigned roommates. This work is therefore a somewhat cleaner test of peer effects.

3.3 Instrumental Variables

Before randomized experiments became available, the most accepted approach for dealing with cases where $\sigma_\lambda^2 \neq 0$ was to use peer group background characteristics as instruments for peer group outcomes. Case and Katz (1991) pioneered this approach and under some circumstances it yields valid estimates of β. To illustrate this approach, we assume that there is a parameter (x) which can be observed for all people and which is part of the individual error term, i.e. $\epsilon_i = \gamma x_i + \mu_i$. Thus, the error term can be decomposed into a term that is idiosyncratic and unobservable and a term that is directly observable. Under the assumptions that both components of ϵ_i are orthogonal to λ_ℓ and to each other, using the formula for an instrumental variables estimator we find that:

$$\frac{Cov\left(a_i, \sum_{j \neq i} x_j/(m-1)\right)}{Cov\left(\sum_{j \neq i} a_j/(m-1), \sum_{j \neq i} x_j/(m-1)\right)} = \frac{\beta}{\beta + (1-\beta)\frac{n-1}{m-1}} \qquad (37)$$

When $m = n$, this reduces to β. Thus, in principle, the instrumental variables estimator can yield unbiased estimates of the social interaction term of interest.

However, as Manski (1993) stresses, the assumptions needed for this methodology may be untenable. First, the sorting of individuals across communities may mean that $Cov(x_i, \mu_j) \neq 0$ for two individuals i and j living in the same community. For example, individuals who live in high education communities may have omitted characteristics that are unusual. Equation (37) is no longer valid in that case, and in general the instrumental variables estimator will overstate social interactions when there is sorting of this kind. Second, sorting may also mean that $Cov(x_i, \lambda_\ell) \neq 0$. Communities with people who have high schooling levels, for example, may also have better public high schools or other important community level characteristics.

Third, the background characteristic of individual j may directly influence the outcome of person i, as well as influencing this outcome through

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the outcome of individual j. Many researchers consider this problem to be less important, because the existence of this bias only occurs when there is some level of social interaction (i.e. the background characteristic of person j influencing person i). While this point is to some extent correct, it is also true that even a small amount of direct influence of x_j on a_i can lead to wildly inflated estimates of β , when the basic correlation of x_j and a_j is low. (Indeed, when this correlation is low, sorting can also lead to extremely high estimates of social interaction.) Because of this problem, instrumental variables estimates can often be less accurate than ordinary least squares estimates and need to be considered quite carefully, especially when the instruments are weak.

3.4   Social Multipliers

A final approach to measuring social interactions is discussed in Glaeser and Scheinkman [to appear] and Glaeser, Laibson and Sacerdote [2000], but to our knowledge has never been really utilized. This approach is derived from a lengthier literature on social multipliers where these multipliers are discussed in theory, but not in practice (see Schelling [1978].) The basic idea is that when social interactions exist, the impact of an exogenous increase in a variable can be quite high if this increase impacts everyone simultaneously. The effect of the increase includes not only the direct effect on individual outcomes, but also the indirect effect that works through peer influence. Thus, the impact on aggregate outcomes of an increase in an aggregate variable may be much higher than the impact on an individual outcome of an increase in an individual variable.

This idea has been used to explain how the pill may have had an extremely large effect on the amount of female education (see Goldin and Katz [2000]). Goldin and Katz [2000] argue that there is a positive complementarity across women who delay marriage that occurs because when one women decides to delay marriage, her prospective spouse remains in the marriage market longer and is also available to marry other women. Thus, one women's delaying marriage may increase the incentives for other women to delay marriage and this can create a social multiplier. Berman [to appear] discusses social multipliers and how they might explain how government programs appear to have massive effects on labor practices among orthodox Jews in Israel. In principle, social multipliers might explain phenomena such as the fact that there is a much stronger connection between out-of-wedlock births and crime at the aggregate level than at the individual level (see Glaeser and Sacerdote [1999]).

30

In this section, we detail how social multipliers can be used in practice to estimate the size of social interactions. Again, we assume that the individual disturbance term can be decomposed into $\epsilon_i = \gamma x_i + \mu_i$, and that $m = n$. When we estimate the micro regression of individual outcomes on characteristic x, when x is orthogonal to all other error terms, the estimated coefficient is:

$$\text{Individual Coeff} = \gamma \frac{(1-\beta)n + (2\beta-1)}{(1-\beta)n - (1-\beta)^2}. \tag{38}$$

Correlation between x_i and μ_i will always lead to coefficients that are nearly impossible to interpret so orthogonality between these two terms must be assumed. However, if x_i is correlated with μ_i, then use of place specific fixed estimate can potentially address this issue. Expression (38) approaches γ as n becomes large, and for even quite modest levels of n (n=20), this expression will be quite close to γ.

An ordinary least squares regression of aggregate outcomes on aggregate x variables leads to quite a different expression. Again, assuming that the x_i terms are orthogonal to both the λ_ℓ and μ_i terms, then the coefficient from the aggregate regression is $\frac{\gamma}{1-\beta}$.

The ratio of the individual to the aggregate coefficient is therefore:

$$\text{Ratio} = \frac{(1-\beta)n + 2\beta - 1}{n - 1 + \beta} \tag{39}$$

As n grows large, this term converges to $1 - \beta$, which provides us with yet another means of estimating the degree of social interactions. Again, this estimate hinges critically on the orthogonality of the error terms, which generally means an absence of sorting. It also requires (as did the instrumental variables estimators above), the assumption that the background characteristics of peers have no direct effect on outcomes.

3.5 Reconciling the Three Approaches

While we have put forward the three approaches as distinct ways to measure social interactions, in fact they are identical in some cases. In general, the micro-regression approach of regressing individual outcomes on peer outcomes (either instrumented or not) requires the most data. The primary advantage of this approach is that it creates the best opportunity to control for background characteristics. The variance approach is the least data intensive, as it generally only requires an aggregate and an individual variance. In the case of a binary variable, it requires only an aggregate variance. Of

course, as Glaeser, Sacerdote and Scheinkman [1996] illustrate, this crude measure can be improved upon with more information. The social multiplier approach lies in the middle. This approach is closest to the instrumental variable approach using micro data.

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