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Adoption of Standards under Uncertainty

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Abstract

The presence of noise in compliance times may have a critical impact on the selection of new technological standards. A technically superior standard is not necessarily viable because an arbitrarily small amount of noise may render coordination on that standard impossible. The criterion for the viability of a standard is that the sum of "support ratios" of all players must be smaller than one, where "support ratio" is defined as the ratio of the firm's per-period cost of supporting the standard to the per-period gross benefit that the firm receives after all players comply with the standard.

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1 Introduction

When a firm invests in Supply Chain Management (SCM) software, it hopes that by the time the system is deployed, its supply chain partners will have already deployed their SCM software. In a perfect world, all partners would pick a single date for their systems to go online, and it would be individually optimal for each firm to follow that schedule. In reality, however, even trains do not always come on time, and it is impossible to perfectly anticipate the exact time of SCM deployment—some firms will necessarily be later than others. When an individual firm schedules SCM software deployment, it does not take into account the expected negative externality of postponing its investment. Thus, deployment may happen inefficiently late or, if externalities are sufficiently large, never.

This problem arises whenever firms schedule complementary projects or investments. For another example, consider Bluetooth. Bluetooth is a technological standard that enables "wireless links between mobile computers, mobile phones, portable handheld devices, and connectivity to the Internet."[1] To take advantage of this technology, a firm needs to install a radio chip in its hardware, and write software integrating the chip with the rest of the system. This will be profitable only if by the time the design and manufacture of these products are complete, there are other Bluetooth-enabled devices from other firms to communicate with.

[1]http://www.bluetooth.com/

In this paper we study such synchronization issues, when firms want to make complementary business decisions which need advance planning. To focus our attention, we talk about the adoption of standards, but the insights are applicable in a broader context, from investments in complementary technologies to large-scale real estate development. We show that adding a stochastic component (noise) to adoption times may have a critical impact on the viability of a standard. Perhaps surprisingly, in a wide range of cases, a universally desired standard may not be viable in a sense that any amount of noise may render coordination on that standard impossible. As a result, given several alternatives, market participants may forgo a technically superior (Pareto dominant) standard in favor of an inferior one, if the former is not viable. Propositions 1 and 5 show that the viability of a standard depends only on "support ratios," where "support ratio" is defined as the ratio of the firm's per-period cost of supporting the standard to the per-period gross benefit that the firm receives when other firms comply with the standard.

Let us sketch a two-player example that illustrates the model considered herein. For simplicity, we assume that there are only two possible standards: the status quo and the new standard that Pareto dominates the status quo. Both firms simultaneously choose target dates for compliance with the new standard. The actual compliance time is uncertain, it is equal to the target time plus noise. As soon as a firm is compliant with the standard, it incurs a per-period cost of supporting the standard, c.[2] Complying with the new standard starts paying off only after the standard is adopted by both firms. When (and if) this happens, each firm starts receiving a stream of net benefits at the rate d (i.e. the gross per-period benefit is $c + d$).

Without noise in compliance times, the game has a continuum of pure strategy equilibria: any adoption time is an equilibrium as long as both players choose that time to comply with the standard.[3] Proposition 1 shows that this multiplicity of equilibria is a knife-edge result. If there is noise in adoption times, *at most* two equilibria survive. There is always a trivial equilibrium where neither player ever adopts the new standard. The equilibrium where the new standard is adopted may or may not exist. Proposition 1 also establishes a necessary condition for the viability of a standard. This condition becomes necessary and sufficient as the players' discount rate converges to one. Coordination on the new standard is impossible if the cost of maintaining it is more than half the gross benefit that a standard yields after adoption by both players. In other words, a standard is not viable if its support ratio, $\frac{c}{c+d}$, is greater than one half. This is true for any distribution of noise in the disturbance terms.

[2]The cost of supporting a standard may take many forms. For example, hardware manufacturers who were first to put Bluetooth communication technology into their products incurred a waiting cost, because including a Bluetooth chip increases the manufacturing cost and adds no value for customers unless other Bluetooth-equipped devices are available. In other situations, early adopters may incur inventory costs. All else being equal, a firm would prefer to invest later rather than sooner due to time value of money.

[3]We are assuming that the new standard is Pareto dominant if when adopted it yields a positive net benefit for all players.

Let us sketch the intuition behind this result. First, observe that the best outcome for both players is simultaneous adoption. From the ex-post perspective, a player who complies first "wishes" he had targeted a slightly later compliance time, since that would have saved him c per period. Similarly, a player who complies last wishes he had targeted a slightly earlier compliance time because that would have saved him the gross benefit from the standard minus the cost of maintaining the standard (which is exactly equal to c for a "borderline viable" standard). Thus, roughly speaking, a standard is not viable if the benefit to the second adopter from lowering his target compliance time is smaller than the cost to the first adopter from lowering his target compliance time. In this case, first order conditions imply that each player's best response is to try to be last with probability greater than one half—consequently the equilibrium where the new standard is adopted disappears.

The model of the standard adoption process presented in Section 3 is highly stylized—players only get benefits after everyone complies. It highlights the dramatic effects that an arbitrarily small amount of uncertainty has on the equilibrium standard selection. In a deterministic world only the net benefits from a standard matter (the gross benefits minus the costs of supporting a standard); in this case the support ratio is irrelevant. If, however, there is any amount of noise in compliance times, support ratios become important. These effects do not go away in a more sophisticated model where firms choose among several competing standards. Also, small amounts of noise continue to have a large impact on equilibrium selection in models where network externalities gradually increase in the number of adopters (See Section 6).[4]

2 Related Literature

The idea that adding noise to the model may reduce the set of equilibria has a long history in economics. Recently, it figures prominently in the work on global games, first introduced in Carlsson and van Damme (1993). In global games, agents receive noisy signals about the true economic fundamentals. This captures the lack of common knowledge about the true state of the economy (See Morris and Shin (2002) for the most recent survey of the global games literature).[5]

The strand of the global games literature closest to our results is the work on synchronization games with asynchronous clocks. This work was preceded by a paper by Halpern and Moses (1990), who show that asynchronous clocks may prevent synchronization because statements about timing never become

[4] It is worth mentioning that adding noise to other parameters of the model, such as the agents' discount rate, costs of compliance with a standard and benefits of standard adoption, does not pin down the equilibrium of the model. That is why a model of standard adoption should capture the uncertainty about compliance times and could neglect the uncertainty about other parameters of the model.

[5] Some equilibrium refinements are also based on the idea of perturbing a game. Trembling-hand perfection is one example. However, there is a significant difference between the logic behind equilibrium refinements and global games. Both this paper and the global games literature attempt to consider games that capture some features of the underlying economic reality that may play an important role in the selection of equilibrium. Unlike equilibrium refinements, we do not seek to improve the equilibrium concept, we seek to improve the model.

common knowledge. Abreu and Brunnermeier (2003) show that a bubble may persist despite the presence of rational arbitrageurs who learn about the existence of the bubble at different times; essentially the difficulty in coordinating an attack on an asset is due to arbitrageurs' clocks not being synchronized. Morris (1995) considers a synchronization problem faced by agents who decide when to start working. Each worker knows the time on his watch but watches are not perfectly synchronized. Morris shows that if clocks are not perfectly synchronized coordination may not be achieved.

The setting of Morris's paper is very similar to ours—in both models agents gain once everybody participates but "early arrival" is costly. However, Morris (1995) model is a global game, and the inability to coordinate is due to agents having private information and thus lacking common knowledge about timing. In contrast, in our model there is no issue of clock synchronization, our agents have no private information, and the common knowledge assumption is maintained. The difficulty in coordination is due to the inability to exactly control compliance times. Thus, our model is not a global game. Nevertheless, our results share some of the remarkable features often encountered in global games, namely (1) without noise there is a continuum of equilibria, adding nose to the model pins the equilibrium down; (2) there exists an equilibrium robust to noise.[6] For the discussion of the connection between the game considered herein and potential and global

[6]Loosely speaking, robustness to noise means that the same equilibria are pinned down by a small amount of noise, regardless of the exact form of the noise.

games see Appendix D.

Basu and Weibull (2002) also study synchronization, in the context of social norms. In their model, an individual may choose to be "punctual" or "tardy," and punctuality (or non-punctuality) may be just one of several equilibria, rather than a society's innate trait.

Our results show that in a wide range of cases a technically superior (Pareto dominant) standard may not be viable in a sense that any amount of noise may render coordination on that standard impossible. The failure of a useful standard to get adopted is a common result in the standards literature. There are many possible reasons why this may happen or why an inferior standard may prevail. They include ownership/sponsorship of standards, current technical superiority and acceptance vs. future/long-term superiority, and incompleteness of information (Katz and Shapiro 1985, 1986, 1994, Farrell and Saloner 1985, Besen and Farrell 1994, Liebowitz and Margolis 1994).

On the other hand, this is the opposite of Farrell and Saloner (1985) conclusion that if players make adoption decisions sequentially, "a somewhat surprising result emerges: if all firms would benefit from change [to a new standard] then all *will* change" (p. 71). Farrell and Saloner point out that in most cases players make adoption decisions simultaneously. In that case their model has multiple equilibria. However, they show that it is an equilibrium for players to switch to the Pareto dominant standard. This result hinges on the assumption that the adoption of a new standard by a firm is an instantaneous process, and thus there are no costs of imperfect coordination of adoption times. The result of Proposition 1 of our paper implies that in a simultaneous-move game the Pareto efficient equilibrium considered in Farrell and Saloner (1985) may disappear if any amount of uncertainty is present. Thus, the predictions of sequential- and simultaneous-move models are very different: the adoption of the Pareto dominant standard is the unique equilibrium of the sequential-move game. In contrast, in a simultaneous-move game the adoption of the Pareto dominant standard may be impossible if any amount of uncertainty is present. In Section 5 we reconcile the difference by adding a dynamic aspect to the game—we make the assumption that once a firm complies with a standard, others observe that and can begin to comply as well. Under this assumption, for a small average compliance time (i.e. as the expected compliance time goes to zero) the sequential-move game of Farrell and Saloner (1985) is a valid approximation, and the Pareto efficient outcome is an equilibrium. On the other hand, for a large average compliance time (i.e. as the expected compliance time goes to infinity) the simultaneous-move model considered herein is a valid approximation.

Therefore, average compliance time (or, equivalently, average time-to-build) plays an important role in the adoption of standards. Pacheco-de-Almeida and Zemsky (2003) find that time-to-build also has a significant impact on investment timing and the tradeoff between flexibility and commitment, firm heterogeneity, and the evolution of prices under demand uncertainty.

Finally, Section II of Farrell and Saloner (1986) presents a model related to ours, where agents can switch from an old standard to a new one. Each agent faces occasional switching opportunities, arriving randomly in an independent Poisson process. For some values of the costs and benefits of the standards (with or without other agents complying with them), agents do not switch to the new standard even if they unanimously favor the switch, because each prefers others to switch first. This effect, however, is driven by the technological infeasibility of the new standard in the absence of transfers between agents—an agent who switches first is in expectation worse off than he would be if nobody switched. In our setup, in contrast, the effect is driven by strategic considerations and the inability of agents to commit to their compliance times. If noise is small (i.e. the rate of the Poisson process is high), then in the setup of Farrell and Saloner (1986) technological frictions vanish and efficient standard gets adopted, whereas in our setup commitment problems remain and the result is unchanged.

3 The Model

We start with a simple model where complying with a standard is only profitable if all other firms comply as well. This simple model is sufficient to illustrate the importance of noise in the process of standard adoption and creation. In Section 6 we will consider a more general model of network externalities. The key assumption of our model is that each firm can select the target time by which it expects to become compliant with a new standard. The actual compliance time, however, is uncertain—it is equal to the target compliance time plus a disturbance term. The random disturbances are uncorrelated across firms, and thus perfect coordination is impossible—some firms are bound to comply earlier than others. While a firm is waiting for others to comply, it bears waiting costs. It only gets benefits after everyone (or, in a more general model, a sufficient number of other firms) complies.

More formally, suppose there are N firms that consider adopting a new standard. Each firm can choose a target compliance time $\mu_i \geq m_i$ at which it plans to comply with the standard (m_i is an exogenous constraint—for each firm there is some minimum time required to comply), or a firm can choose not to comply at all, which we denote by *out*. All firms select their target times simultaneously. If the firm decides not to comply, its payoff is 0. Otherwise, its actual compliance time t_i is equal to μ_i plus a random disturbance drawn from continuous probability distribution F_i independent of other firms' disturbances. As soon as a firm complies with the new standard, it has to pay a cost of supporting it of c_i per period. When (and if!) all firms adopt the standard, firm i starts getting a flow of net benefits d_i (i.e. the per-period gross benefit is $c_i + d_i$). The adoption time, i.e. the time when all firms comply, is denoted by $t_* = \max_i \{t_i\}$ (if one of the agents never adopts, we say that $t_* = \infty$). For simplicity, we assume that c_i and d_i do not change over time. The firm's payoff is a discounted flow of costs and benefits from the new standard: $\Pi_i = E[\int_{t_i}^{\infty} \beta^t \pi_i(t)dt]$, where $\pi_i(t)$ is the sum of cost and gross benefit accrued at time t,

$$\pi_i(t) = \begin{cases} 0 & \text{for } t \leq t_i \\ -c_i & \text{for } t_i < t \leq t_* \\ d_i & \text{for } t_* < t. \end{cases}$$

Assume that the discount factor, β, is strictly less than 1. We will refer to the game described above as $\Gamma(\beta)$.

To analyze the equilibria of $\Gamma(\beta)$, we construct an approximation with no time discounting. To be able to do that, we renormalize payoffs, and for each i subtract the net benefit after universal adoption, d_i, from firm i's instantaneous payoff in every period, i.e.

$$\pi_i^1(t) = \begin{cases} -d_i & \text{for } t \leq t_i \\ -(c_i + d_i) & \text{for } t_i < t \leq t_* \\ 0 & \text{for } t_* < t. \end{cases}$$

More precisely, define $\Gamma(1)$ as follows. Action space and probability distributions of disturbances are the same as before, but payoffs are different. If a player chooses *out*, his payoff is u_{out}[7]. If player i chooses some target compliance time and another player chooses *out*, player i's payoff is $-\infty$. If all players choose to comply, the payoff of player i is given by the expected value of $-c_i(t_* - t_i) - d_i t_*$, where $t_* = \max_i \{t_i\}$, and vector t is equal to vector μ plus random vector ϵ of disturbances drawn from continuous prob-

[7] u_{out} is assumed to be "sufficiently" low. The exact definition will be made clear in the next section.

ability distribution $F_1 \times \cdots \times F_N$. We also define the support ratio of firm i, $s_i = \frac{c_i}{c_i+d_i}$.

To summarize the notation:

$\{out, [m_i, +\infty)\}$	action space of firm i
$\mu_i \geq m_i$	target compliance time of firm i if it decides to comply
t_i	actual compliance time of firm i
ϵ_i	firm i's disturbance, $t_i - \mu_i$
F_i	distribution of ϵ_i
c_i	per-period cost paid by firm i after it complies
d_i	per-period net benefit received by firm i after all firms comply
β	time discount factor
u_{out}	in $\Gamma(1)$, payoff of a firm which decides not to comply
t_*	$\max_i \{t_i\}$, i.e. the adoption time
s_i	support ratio of firm i, equals $\frac{c_i}{c_i+d_i}$.

4 The Viability of a Standard

The following proposition characterizes the equilibrium set of $\Gamma(\beta)$ as $\beta \to 1$. It gives a criterion of the viability of a standard, i.e. a necessary and sufficient condition for the existence of equilibrium where the standard is adopted provided that players are sufficiently patient. If the sum of support ratios of all players is less than one, a standard is viable. This condition *does not depend* on the distribution of noise—it only depends on the firms' support ratios. Also, it says that as β increases, equilibrium target compliance times decrease, i.e. as players become more patient, they adopt earlier.

Proposition 1 *If* $\sum_{i=1}^{N} s_i < 1$, *then*

(i) there exists $\beta_0 < 1$ *such that for any* $\beta_0 < \beta < 1$ *game* $\Gamma(\beta)$ *has exactly two equilibria—one in which all players choose to adopt, and one in which all players choose not to adopt,*[8]

(ii) $\lim_{\beta \to 1} \mu^*(\beta) = \mu^*(1)$, *where* $\mu^*(\cdot)$ *denotes the vector of target compliance times in the equilibrium where the standard is adopted,*

(iii) for any $\beta_0 < \beta_1 \leq \beta_2 \leq 1$, $\mu^*(\beta_1) \geq \mu^*(\beta_2)$.

(iv) If $\sum_{i=1}^{N} s_i > 1$, *then there exists* $\beta_0' < 1$ *such that for any* $\beta_0' < \beta < 1$ *game* $\Gamma(\beta)$ *has only one equilibrium, and in that equilibrium all players choose not to adopt.*

We prove this proposition in two steps. Step 1 is to characterize the equilibria of the game with no time discounting, $\Gamma(1)$—this is done in Proposition 2. Step 2 is to show that equilibria of $\Gamma(\beta)$ converge to those of $\Gamma(1)$ as $\beta \to 1$.

Step 1. First, we prove two auxiliary results.

Lemma 1 *Suppose players have distributions of disturbances* $\{F_i\}$. *Take any strictly positive numbers* $\{p_i\}$ *such that* $\sum p_i = 1$. *Then there exists a vector of target times such that each player* i *adopts last with probability* p_i.

Proof. See Appendix. ■

[8]Remarkably, there are no mixed equilibria.

Lemma 2 *Suppose players have distributions of disturbances* $\{F_i\}$*. Take any strictly positive numbers* $\{p_i\}$ *such that* $\sum p_i \leq 1$*. Take any numbers* $\{m_i\}$*. Then there exists a vector of target times,* μ*, such that (i) for all* i*,* $\mu_i \geq m_i$*, (ii) each player* i *adopts last with probability greater than or equal to* p_i*, and (iii) if* $\mu_i > m_i$*, player* i *adopts last with probability exactly equal to* p_i*. If* $\sum p_i < 1$*, such vector* μ *is unique.*

Proof. See Appendix. ■

Now we can state the necessary and sufficient condition for the existence of an equilibrium of $\Gamma(1)$ where firms choose to adopt. Such equilibrium exists if and only if the sum of the probabilities with which players want to be last is less than or equal to one.

Proposition 2 *For sufficiently low values of* u_{out}*, game* $\Gamma(1)$ *has a Nash equilibrium where players choose to participate if and only if*

$$\sum_{i=1}^{N} s_i \leq 1, \tag{1}$$

and when the above inequality is strict, such equilibrium is unique. Also, there is only one other equilibrium—all players choose out.[9]

Proof. First, notice that if in an equilibrium at least one player plays *out* with a positive probability, all of them have to play *out* with probability 1 (to get u_{out} instead of $-\infty$). Therefore, "all *out*" is an equilibrium and

[9] In particular, this proposition implies that if benefits are small relative to costs, e.g. $d_i = 0$ for all i, there is only one equilibrium—all players choose *out*.

in all other equilibria (if they exist) players have to mix among target times and never play *out*.

Suppose player i takes the distribution of adoption times of other players as given. Then, in his personal optimum, he will choose his adoption time μ_i in such a way that either $\mu_i = m_i$ and the probability of him being last is $q_i \geq s_i$ or $\mu_i > m_i$ and the probability of him being last is $q_i = s_i$. (To see that, suppose that $\mu_i > m_i$ and the probability of him being last is $q_i > c_i/(c_i + d_i)$. If instead he plans to adopt slightly earlier, at $\mu_i - \epsilon$, in expectation he gains $q_i d_i \epsilon + O(\epsilon^2)$ (when he is the last one to adopt) and loses $(1 - q_i)c_i\epsilon + O(\epsilon^2)$ (when he is not). For μ_i to be optimal, it has to be the case that $q_i d_i - (1 - q_i)c_i = 0 \Longrightarrow q_i = c_i/(c_i + d_i)$. Similar arguments apply to the case $q_i < c_i/(c_i + d_i)$.)

If the sum of these "desired" probabilities s_i is greater than one, then, since each player wants to adopt last with at least his "desired" probability, no μ can satisfy these conditions. When $\sum s_i \leq 1$, by Lemma 2, such μ exists and hence it is an equilibrium provided that each player's expected payoff is greater than u_{out}. When the inequality is strict, uniqueness also follows directly from Lemma 2. ■

Step 2. The proof that equilibria of $\Gamma(\beta)$ converge to those of $\Gamma(1)$ as $\beta \to 1$ is rather technical, and we present it in the Appendix. This completes the proof of Proposition 1. ■

Note that it is clear from the proof that if inequality (1) is strict, then in the equilibrium where the standard is adopted some firms comply as soon as they can, while for the rest the probability of being last is equal to the support ratio.

An interesting question is what happens if we relax the assumption of costs and benefits being constant over time. The results change little if costs are decreasing and benefits are increasing in time—a standard can be adopted if and only if the sum of *limit* support ratios is less than one. If, however, costs and benefits vary less regularly over time, increasing over some intervals and decreasing over others, the criteria for adoptability become more complicated.

5 The Observability of Compliance Times

In Section 4 we showed that in a wide range of cases a technically superior standard may not be viable. This is the opposite of Farrell and Saloner (1985) conclusion that if players make adoption decisions sequentially, then a Pareto superior standard gets adopted in equilibrium. In this section we reconcile the difference by adding a dynamic aspect to the game—we make an assumption that once a firm complies with a standard, others observe that and can begin to comply as well. Proposition 3 shows that for small average compliance times (i.e. as the expected compliance time goes to zero) the sequential-move game of Farrell and Saloner is a valid approximation, and the Pareto efficient outcome is the only equilibrium. On the other hand, for large average compliance times (i.e. as the expected compliance time goes to infinity), the simultaneous-move model of Section 3 is a valid approximation (Proposition 4), and therefore a Pareto optimal standard may be impossible to implement.

Assume that at each time t a firm can initiate the compliance process if it has not already done so. Once initiated, the process takes an uncertain amount of time. Let T_i denote the expected amount of time it takes firm i to comply; the actual compliance time (if the firm initiated the process at time t) is thus $t+T_i+\epsilon_i$, where ϵ_i is a random deviation. We assume that distributions of random deviations are bounded for all players and independent of each other. Define $T_{\min} = \min_i\{T_i + \underline{\epsilon_i}\}$ and $T_{\max} = \max_i\{T_i + \overline{\epsilon_i}\}$, where $\underline{\epsilon_i}$ and $\overline{\epsilon_i}$ are the lower and the upper bounds of stochastic deviations ϵ_i of firm i. In other words, $T_{\min}$ is the shortest amount of time it takes any firm to comply, and $T_{\max}$ is the longest amount of time it takes any firm to comply once it has initiated the compliance process. Once a firm has initiated the process, it cannot reverse it or influence the time it is going to take.

Each firm observes when others comply, i.e. at time t each firm knows who has complied prior to time t and when they did it. However, it does not know who has *initiated* the compliance process. We also assume that the discount rate $\beta < 1$ is held constant. Then the following propositions hold.

Proposition 3 *If the new standard is strictly Pareto optimal, then as* $T_{\max} \to 0$*, equilibrium payoffs of the players approach the payoffs they would obtain if each firm immediately decided to comply with the standard.*[10]

[10]Notice that $T_{\max} \to 0$ implies that for each player both average compliance time and

Proof. See Appendix. ■

Now consider a family of games with observable compliance times where all players' expected initiation-to-compliance times T_i are increased by the same $x \geq 0$, while holding the distributions of disturbances the same. Notice that without observable compliance times all these games are identical, up to multiplying all players' payoffs by β^x, and thus we can without ambiguity talk about the corresponding simultaneous-move game without observable compliance times.

Proposition 4 *As $x \to \infty$, the game with observable compliance times has an equilibrium where the standard is adopted if and only if there exists an equilibrium of the corresponding simultaneous-move game where the standard is adopted.*

Proof. See Appendix. ■

6 Network Externalities

Up to this point we have assumed a very specific form of network externalities. We now show how our results can be extended to network externalities of a general form. We continue to assume that there are N players who choose their target compliance times and whose actual compliance times are independent stochastic deviations from their targets. We also assume that

the amount of noise go to zero.

all players are identical. Firms bear per-period cost c after they comply. The per-period net benefit of a firm that has complied with the standard is now $d(k)$ (i.e. the gross benefit is $c + d(k)$), where k is the number of firms that have complied up to that moment, including itself; $k \in \{1, 2, \ldots, N\}$. We assume that $d(N) > 0$, i.e. the new standard is profitable if everyone adopts it, and that $d(k)$ is weakly increasing in k.[11]

Then the following result holds.

Proposition 5 *For sufficiently patient players, there exists an equilibrium where the new standard is adopted if and only if* $\frac{1}{N}\sum_k \frac{1}{s(k)} > 1$, *where* $s(k) = \frac{c}{c+d(k)}$.[12]

Proof. We omit the approximation part of the proof, since it is completely analogous to Step 2 of the proof of Proposition 1, and we go directly to the case with no discounting. Notice that since players are identical, in the equilibrium where they comply they have to target the same time. Therefore, for a given firm, its probability of being the kth firm to comply is equal to $\frac{1}{N}$ for any k. Therefore, its expected net benefit from delaying its compliance by a small amount of time is proportional to $c - \frac{1}{N}\sum_k(c + d(k))$, which is negative if and only if $\frac{1}{N}\sum_k \frac{1}{s(k)} > 1$.

We have to be careful with the case $\frac{1}{N}\sum_k \frac{1}{s(k)} = 1$. Then in the game with no discounting players do not want to deviate if all target the same

[11] This assumption says that externalities from technology adoption are positive. Fudenberg and Tirole (1985, 1986) study the timing of technology adoption under the opposite conditions, where players impose negative externalities on each other.

[12] Of course, $s(k)$ can be greater than one, since $d(k)$ is the *net* benefit and can be negative.

compliance times. In the presence of any nontrivial discounting, however, a player's higher benefits are discounted at a higher rate, since they on average happen when he complies later, and therefore he would be strictly better off by deviating by a small amount. ■

The proposition states that a standard can be adopted in equilibrium if and only if the average of inverse support ratios ($\frac{1}{N}\sum_k \frac{1}{s(k)}$) is greater than one. From symmetry, it follows that in equilibrium each player has the same probability of complying first, last, or anything in between. Thus, the condition simply states that in expectation, the flow of benefits at the time of compliance is greater than the flow of costs of maintaining a standard. If that were false, a player would prefer to comply later.

The following corollary of Proposition 5 reflects the fact that the free rider problem does not become more severe if the number of players in the standard adoption game is increased.[13]

Corollary 1 *Consider two games with different numbers of players but identical costs c and identical network externalities $d(k)$. If a standard can be adopted in an equilibrium of a game that has N players, then it can be adopted in a game that has more than N players.*

Throughout the paper we refer to players as firms, because the results of Section 4 are relevant for standard adoption games where the number

[13] Suppose that in the game with N players a new standard is viable. Then if one more player is added to the game, each player would like to target a compliance time that is earlier or the same as in the N player game. Also, if one of the players changes his compliance time to an earlier date, none of the players would want to increase their compliance times.

of participants is small and each participant may be pivotal (players are a handful of corporations). The results of the present section are also relevant for standards that can only succeed if adopted by millions of consumers or other small players, even though there is no longer any uncertainty about the share of players who have complied at any given moment. The following corollary makes this claim formal. Assume that there is a continuum of identical players. Let $D(\alpha)$ denote the per-period net benefit to a player who is compliant with the new standard at the time when *share* α of the population of players is compliant; as before, the support ratio is $S(\alpha) = \frac{c}{c+D(\alpha)}$.

Corollary 2 *For sufficiently patient players, there exists an equilibrium where the new standard is adopted if and only if* $\int_0^1 \frac{1}{S(\alpha)} d\alpha > 1$.

7 Overcoming Coordination Failure

Just like the presence of adverse selection does not necessarily imply that markets collapse, synchronization problems do not necessarily imply that standards will not be adopted. Rather, institutions may arise to overcome these failures, and understanding the problems helps us better understand the institutions.

The most straightforward way to overcome synchronization failure is to write enforceable contracts, specifying penalties for late compliance. Ostrovsky and Schwarz (2002) characterize the socially optimal target compliance times and present incentive mechanisms that would induce players to target these times in equilibrium. In practice, however, compliance times may be noncontractible. Partially enforceable contracts may go a long way towards overcoming the synchronization failure, and figuring out what kinds of enforceability are sufficient is an interesting area for future research.

Another way to achieve coordination is to "discretize" time and thus eliminate the possibility of being "slightly late." Perhaps unintentionally, annual industry trade shows may accomplish that. A trade show provides wide exposure to new products, and missing one may result in a year of lost profits from the new standard. Sometimes, discretization is natural—Christmas only comes once a year, and that's when many consumer goods manufacturers sell most of their merchandize. A videogame producer cannot afford to be slightly late with the new release for a game console.

It may also be easier to adopt a standard if the compliance process is gradual, with a firm's costs and benefits increasing as its degree of compliance, e.g. the number of compliant products, goes up. Alternatively, it may help if the intermediate stages of the firm's compliance process are observable to outsiders—for example, a beta-version of a software package may serve as such a signal. Also, when representatives of interested parties work together on specifying and improving a standard, constant communication allows the tracking of other firms' progress and thus helps alleviate synchronization issues. This is, in fact, how Internet standards are developed by the World Wide Web Consortium (W3C).[14] The consortium includes more than 500 entities: all major software firms, many universities, publishers, and even the Library of Congress. When a need for a new standard is identified (e.g. MathML—a way of displaying mathematical equations on web pages, XML Query—a way of efficiently exchanging data between web pages and databases, and so on), a working group of interested parties' engineers is created to develop the standard. These engineers communicate with each other as they work out technical specifications and documentation, and also work with their firms' developers on implementing the standards. Thus, by the time a version of the standard is finalized and publicly released, there is already a critical mass of adopters.

Assuming costs and benefits add up when firms merge, the support ratio of a merged firm is lower than the sum of support ratios of its components. Thus, mergers reduce the sum of support ratios of market participants and help make standards viable. Once the standard is adopted, we may see spinoffs. This prediction sounds far-fetched, but this is in fact what happens quite often in large-scale real estate projects, when a single developer builds up a piece of land and then sells or leases the parts off. Celebration, Florida[15] and Santana Row in San Jose, California[16] are just two recent examples of towns built by a single developer from scratch. Residential and commercial property in these areas was subsequently sold or leased.

[14]http://www.w3.org/
[15]http://www.celebrationfl.com/
[16]http://www.santanarow.com/

8 Concluding Remarks

Our results imply some interesting corollaries. First, they say that a Pareto improving standard is not necessarily viable. The following quotation from the Court's Findings of Fact in the U.S. v. Microsfot case gives a very similar argument:

> 41. In deciding whether to develop an application for a new operating system, an [Independent Software Vendor's] first consideration is the number of users it expects the operating system to attract. Out of this focus arises a collective-action problem: Each ISV realizes that the new operating system could attract a significant number of users if enough ISVs developed applications for it; but few ISVs want to sink resources into developing for the system until it becomes established. Since everyone is waiting for everyone else to bear the risk of early adoption, the new operating system has difficulty attracting enough applications to generate a positive feedback loop.[17]

Another setting where our results apply is the creation of standards by various industry groups. We can view this process as a two-stage game.

[17]The document goes on to say that "the vendor of a new operating system cannot effectively solve this problem by paying the necessary number of ISVs to write for its operating system, because the cost of doing so would dwarf the expected return." We disagree with this claim—in our opinion, the reason for the operating system vendor's inability to pay the ISVs has to do with complications inherent in writing and enforcing the necessary contracts.

First, an industry consortium develops and recommends a single standard out of a large universe of technically feasible standards. Then each player decides if and when to adopt a standard recommendation. The subgame is modeled as a standard adoption game considered earlier. The objective of the consortium is to select a Pareto improving standard that maximizes the total payoff of the industry participants.[18] This objective implies that the consortium will always choose to recommend a viable standard, whenever a viable standard is available. Thus, the equilibrium recommendation of the consortium may be Pareto dominated by some technologically feasible standard.

A disclaimer is in order: it is not our contention that noise in adoption terms determines the outcome of a battle among competing standards. However, looking at support ratios may offer an insight into competition among standards: for a standard to survive and be backed by some coalition of players it has to be the case that its support ratios are sufficiently low.

[18]Nothing would change if the objective function of the consortium were to maximize some objective function that is increasing in the profit of each player.

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A Proofs of Section 4 Lemmas

A.1 Proof of Lemma 1

Let $f(\mu) = \sum |q_i - p_i|$, where μ is a vector of target compliance times and q_i is the probability that player i complies last. Take μ^* which minimizes f. Suppose $f(\mu) > 0$. Then there exists i such that $q_i^* > p_i$. Reduce μ_i slightly (call the new vector μ') so that the new q_i' is between p_i and q_i^*. $f(\mu') = |q_i' - p_i| + \sum_{j \neq i} |q_j' - p_j| = f(\mu^*) - |q_i^* - q_i'| + \sum_{j \neq i}(|q_j' - p_j| - |q_j^* - p_j|)$. Notice that for all $j \neq i$, $q_j' \geq q_j^*$ and for at least one j, $p_j \geq q_j'$. Hence, $f(\mu') < f(\mu^*) - |q_i^* - q_i'| + \sum_{j \neq i} |q_j' - q_j^*| = f(\mu^*)$. Contradiction.

Notice that we assumed that the minimizing μ^* exists. When all distributions are bounded, this assumption is justified by the fact that we can restrict μ to, say, a set of vectors in which $\mu_1 = 0$ and all other μ_j are bounded by the sum of the sizes of supports of all N distributions of disturbances. When some distributions are not bounded, we make use of our assumption that all p_i are positive. We can choose $\epsilon > 0$, $2\epsilon < \min(p_i)$ and approximate unbounded distributions by bounded ones so that for all μ the difference between $f(\mu)$ for the unbounded distributions and their bounded approximations is less than ϵ. Then there exists μ^{**} such that $f_{\text{bounded approximation}}(\mu^{**}) = 0$, and then $\arg\min f(\mu)$ exists and belongs to the compact set $\{\mu | \mu_1 = 0,\ f_{\text{bounded approximation}}(\mu) \leq 2\epsilon\}$. ■

A.2 Proof of Lemma 2

Consider set $T = \{\mu | \forall i, \mu_i \geq m_i, q_i(\mu) \geq p_i\}$, where $q_i(\mu)$ is the probability that player i complies last given that players choose target times μ. Take $\mu^* \in T$ which minimizes $\sum_i \mu_i$ in T. Then μ^* satisfies the conditions of the lemma. Indeed, for all i, $q_i \geq p_i$, and we only need to show that for all i $\mu_i > m_i$ implies $q_i = p_i$. Suppose that is not so. Take i such that $q_i > p_i$ and $\mu_i > m_i$. We can slightly decrease μ_i so that it is still greater than m_i and q_i is still greater than p_i, i.e. the modified μ is still in T. But we decreased $\sum_i \mu_i$—contradiction!

Of course, it is necessary to prove that such minimizing μ exists. To show that, first notice that set T is not empty as, according to Lemma 1, there exists μ^{**} such that $q_1(\mu^{**}) = 1 - p_2 - \cdots - p_N$ and $q_i(\mu^{**}) = p_i$ for $i > 1$. Second, notice that we can search for μ^* in the intersection of sets T and $\{\mu | \mu_i \geq m_i, \sum \mu_i \leq \sum \mu_i^{**}\}$. The latter set is compact, the former is closed, and so their intersection is compact and, since function $\sum \mu_i$ is continuous in μ, there exist μ^* in that set which minimizes this function.

When $\sum p_i < 1$, such vector has to be unique: if there are two vectors (μ^1, μ^2) satisfying the conditions, take player i with the biggest increase in μ_i from μ^1 to μ^2; then both $q_i(\mu^2) > p_i$ and $\mu_i^2 > m_i$—contradiction. ■

B Proof of Proposition 1—Step 2

For simplicity, assume that distributions of disturbances F_i are bounded.

(i) Clearly, the strategy vector where nobody complies is an equilibrium. Let's show that for β sufficiently close to 1, there exists exactly one other pure equilibrium, and no mixed ones. The proof is similar to the proof of Lemma 2.

Take a Nash Equilibrium in which player i chooses compliance time μ_i with positive probability. For convenience, if player j chooses not to comply, let $t_j = \infty$ and $t_j > t_i$. By the same "marginal delay" reasoning as in Proposition 2, Player i's FOC for choosing μ_i is

$$c_i \int_{-\infty}^{\infty} \beta^{t_i} Prob(i \neq last|t_i) f_i(t_i - \mu_i) dt_i = d_i \int_{-\infty}^{\infty} \beta^{t_i} Prob(i = last|t_i) f_i(t_i - \mu_i) dt_i$$

if $\mu_i > m_i$ and

$$c_i \int_{-\infty}^{\infty} \beta^{t_i} Prob(i \neq last|t_i) f_i(t_i - \mu_i) dt_i \leq d_i \int_{-\infty}^{\infty} \beta^{t_i} Prob(i = last|t_i) f_i(t_i - \mu_i) dt_i$$

if $\mu_i = m_i$.

By adding $c_i \int_{-\infty}^{\infty} \beta^{t_i} Prob(i = last|t_i) f_i(t_i - \mu_i) dt_i$ to both sides, we get the equivalent FOC

$$c_i \int_{-\infty}^{\infty} \beta^{t_i} f_i(t_i - \mu_i) dt_i = (c_i + d_i) \int_{-\infty}^{\infty} \beta^{t_i} Prob(t_i \geq t_j \forall j|t_i) f_i(t_i - \mu_i) dt_i$$

if $\mu_i > m_i$ and

$$c_i \int_{-\infty}^{\infty} \beta^{t_i} f_i(t_i - \mu_i) dt_i \leq (c_i + d_i) \int_{-\infty}^{\infty} \beta^{t_i} Prob(t_i \geq t_j \forall j|t_i) f_i(t_i - \mu_i) dt_i$$

if $\mu_i = m_i$.

Crucially, the ratio of the right-hand side over the left-hand side goes up if μ_i goes up, unless all other players choose not to comply, and so one and only one point on the real line can satisfy this condition. This rules out mixing among compliance times.

Let $\hat{q}_i(\mu, \beta)$ be equal to

$$\frac{\int_{-\infty}^{\infty} \beta^{t_i} Prob(t_i \geq t_j \forall j | t_i) f_i(t_i - \mu_i) dt_i}{\int_{-\infty}^{\infty} \beta^{t_i} f_i(t_i - \mu_i) dt_i}.$$

Consider set $T(\beta) = \{\mu | \forall i \mu_i \geq m_i, \hat{q}_i(\mu, \beta) \geq \frac{c_i}{c_i+d_i}\}$. It follows from Lemma 2 that there exists $\mu_1 \geq m$ such that $q_i(\mu_1) = \hat{q}_i(\mu, 1) = Prob(t_i \geq t_j \forall j | \mu_1) > \frac{c_i}{c_i+d_i}$. $\lim_{\beta \to 1} \hat{q}_i(\mu_1, \beta) = q_i(\mu_1)$, and so there exists β_0 such that $\forall \beta > \beta_0$ we have $\hat{q}_i(\mu_1, \beta) > \frac{c_i}{c_i+d_i}$. Thus, $T(\beta)$ is nonempty. Take $\mu^* \in T$ which minimizes $\sum_i \mu_i$. It satisfies the FOC above and is a Nash Equilibrium. Let's show that there are no other equilibria.

First, let's show that there is no mixing between complying and not complying. Suppose player i is indifferent between the two, and his optimal compliance time is μ_i. $\int_t^{\infty} \beta^{\tau} d\tau = (1/\ln \beta)\beta^t$. Thus, $0 = 0 \ln \beta = \ln \beta E[-c_i(\int_{t_i}^{\infty} \beta^{\tau} d\tau) + (c_i + d_i)(\int_{t_{last}}^{\infty} \beta^{\tau} d\tau)] = -c_i E[\beta^{t_i}] + (c_i + d_i)E[\beta^{t_{last}}]$. But $(c_i + d_i)E[\beta^{t_{last}}] = (c_i + d_i)E[\beta^{t_i} Prob(i = last) + \beta^{t_{last}} Prob(i \neq last)] > (c_i + d_i)E[\beta^{t_i} Prob(i = last)] \geq$ (by FOC) $c_i E[\beta^{t_i}]$—contradiction. Therefore, there are no mixed equilibria.

The proof that there can not be two equilibrium compliance time vectors

μ^1, μ^2 is the same as before—if there are, take player i with the biggest increase in μ_i from μ^1 to μ^2; then both $\hat{q}_i(\mu, \beta) > c_i/(c_i + d_i)$ and $\mu_i^2 > m_i$—contradiction.

(ii) Suppose $\mu^*(\beta)$ does not go to $\mu^*(1)$ as β goes to 1. Then there exists a subsequence $\{\beta^n\}$ converging to 1 such that $\mu^*(\beta^n)$ converges to some $\tilde{\mu} \neq \mu^*(1)$ (set of $\mu^*(\beta)$ is bounded as $\beta \to 1$). Then by continuity, $\tilde{\mu}$ satisfies the FOC with $\beta = 1$ and is therefore an equilibrium of game $\Gamma(1)$. But we know that $\Gamma(1)$ has only one equilibrium with compliance, equal to $\mu^*(1)$.

(iii) Take $\beta_1 < \beta_2$, and suppose for some i, $\mu_1 = \mu_i^*(\beta_1) < \mu_2 = \mu_i^*(\beta_2)$. Without loss of generality, assume that $i = \arg\max_j \{\mu_j^*(\beta_2) - \mu_j^*(\beta_1)\}$. By FOC,

$$(c_i + d_i) \int \beta_1^{t_i} Prob(t_i = last) f(t_i - \mu_1) dt_i \geq c_i \int \beta_1^{t_i} f(t_i - \mu_1) dt_i.$$

Since $\mu_1 < \mu_2$,

$$(c_i + d_i) \int \beta_1^{t_i} Prob(t_i = last) f(t_i - \mu_2) dt_i > c_i \int \beta_1^{t_i} f(t_i - \mu_2) dt_i.$$

$$\int \beta_1^{t_i}((c_i + d_i) Prob(t_i = last) - c_i) f(t_i - \mu_2) dt_i > 0.$$

Let t_i^* be such that $(c_i + d_i) Prob(t_i^* = last) - c_i = 0$. The integrand is negative for $t_i < t_i^*$ and positive for $t_i > t_i^*$. $\beta_2 > \beta_1$, and so $\left(\frac{\beta_2}{\beta_1}\right)^t$ is an increasing function. Therefore,

$$\int \beta_2^{t_i}((c_i + d_i)Prob(t_i = last) - c_i)f(t_i - \mu_2)dt_i \geq$$

$$\left(\frac{\beta_2}{\beta_1}\right)^{t_i^*} \int \beta_1^{t_i}((c_i + d_i)Prob(t_i = last) - c_i)f(t_i - \mu_2)dt_i > 0.$$

But this, together with $\mu_2 > \mu_1 \geq m_i$, is a violation of the FOC for an equilibrium.

(iv) To prove the last statement, assume the opposite. Then there is a sequence $\{\beta_n\}$ converging to 1 from above such that for each β_n there is an equilibrium where players choose to comply. Then there is a subsequence $\{\beta_k\}$ such that $\mu^*(\beta^k)$ converges to some $\tilde{\mu}$. But then by continuity, $\tilde{\mu}$ satisfies the FOC with $\beta = 1$ and is therefore an equilibrium of game $\Gamma(1)$. But we know that $\Gamma(1)$ does not have an equilibrium where players comply.

C Proofs of Section 5 Propositions

C.1 Proof of Proposition 3

We prove the statement by induction on N, the number of players. For $N = 1$ the statement is obvious. Suppose it holds for $N = k$, let's show that it also holds for $N = k+1$. Suppose there is a sequence of equilibria for $T_{\max} \to 0$ in which the payoffs of players converge to something other than the payoffs of the Pareto-efficient outcome (i.e. the immediate adoption of the standard).

Take any player i whose equilibrium payoff in the limit is strictly less than his payoff under the immediate adoption. If he deviates from his equilibrium strategy, and initiates the compliance process immediately, then others will observe that he has complied at most after $T_{\max}$. After that we are back in the game with $N-1$ players, which, by the assumption of induction, has equilibria payoffs arbitrarily close to Pareto optimal ones as $T_{\max}$ goes to 0. But then player i's payoff from deviating is less than the Pareto payoff by at most the costs and foregone profits up to $T_{\max}$ plus the costs and foregone profits while the $(N-1)$-player subgame takes place. But each of these two components goes to zero as $T_{\max}$ goes to zero, and so player i's payoff from deviating goes to his Pareto payoff—thus any equilibrium payoff has to approach the Pareto payoff as well.

C.2 Proof of Proposition 4

Suppose the simultaneous-move game has an equilibrium where the standard is adopted. Let x be such that $\min_i\{T_i + x + \underline{\epsilon_i}\}$ is greater than $\max_i\{T_i + \overline{\epsilon_i}\}$. Then the equilibrium with adoption of the simultaneous-move game remains an equilibrium of the game with observable compliance time, since the optimal target compliance times are such that players want to initiate the compliance process before they could have possibly observed other players' compliance.

On the other hand, suppose the simultaneous-move game does not have an equilibrium where the standard is adopted, but for arbitrarily large x the game with observable compliance times does. Notice that in an equilibrium with adoption, for a large enough x, each player (say, player 1) has to initiate the compliance process before observing others comply with a positive probability (otherwise the payoff of at least one of the other players is negative, as player 1 always complies too late). Let $\underline{\mu_i}(x)$ denote the earliest target compliance time of player i in an equilibrium with delay x; subtract $\underline{\mu_1}(x)$ from all $\underline{\mu_i}(x)$s to normalize. Now consider the sequence of vectors $\underline{\mu}(x)$ as x goes to infinity. This sequence has to be bounded—otherwise the player with the lowest $\underline{\mu_i}(x)$ would find it profitable to deviate and not comply at all. Thus, it has to converge to some vector $\underline{\mu}$. By assumption, there were no equilibria with compliance in the simultaneous-move game, and thus there is at least one player who would find it strictly profitable to slightly increase his target compliance time if everyone targeted $\underline{\mu}$. But then, for a large enough x, this player would also find it profitable to do that in the observable-compliance game with delay x.

D Robustness to Noise

Frankel, Morris, and Pauzner (2002) show that in global games, different equilibria may be pinned down by vanishingly small noise. They also show that a sufficient condition for an equilibrium in a global game to be robust to the structure of noise is to be a *weighted potential maximizer*, provided that the payoffs are *own-action quasiconcave*. These concepts are defined in

Section 6 of FMP as follows.

Definition *A complete information game g is own-action quasiconcave if for all i and opposing action profiles $a_{-i} \in A_{-i}$ and for all constants c, the set $\{a_i : g_i(a_i, a_{-i}) \geq c\}$ is convex.*

Definition *Action profile a^* is a weighted potential maximizer (WP-maximizer) of g if there exists a vector $\xi \in \mathbb{R}^I_+$ and a weighted potential function $v : A \to \mathbb{R}$ with $v(a^*) > v(a)$ for all $a \neq a^*$, such that for all i, a_i, $a'_i \in A_i$ and $a_{-i} \in A_{-i}$,*

$$v(a_i, a_{-i}) - v(a'_i, a_{-i}) = \xi_i[g_i(a_i, a_{-i}) - g_i(a'_i, a_{-i})].$$

The results of Frankel, Morris, and Pauzner (2002) have parallels in our setting. Namely, the games presented herein indeed have weighted potential maximizers, and are own-action quasiconcave. On the other hand, changing our setting in such a way that the game no longer has a potential leads to the dependence of equilibrium on the structure of noise.

We focus our attention on the case with no discounting; the results do not change if we consider $\beta < 1$, but presentation gets more complicated.

D.1 Potentiality of "Noiseless" $\Gamma(1)$

Consider a "noiseless" version of game $\Gamma(1)$, where $t_i = \mu_i$.

$$\begin{aligned} \Pi_i(t_i) &= -c_i(t_* - t_i) - d_i t_* \\ \Pi_i'(t_i) &= c_i - (c_i + d_i)\chi(\text{player } i \text{ is last}), \end{aligned}$$

and therefore payoffs Π_i are own-action quasiconcave.

To show that this is also a weighted potential game, let $v(t) = \sum s_i t_i - t_*$, where s_i is the support ratio of player i. Let $\xi_i = \frac{1}{c_i + d_i}$. Then

$$\begin{aligned} v(t_i, t_{-i}) - v(t_i', t_{-i}) &= [s_i t_i - t_*] - [s_i t_i' - t_*'] \\ &= \frac{[c_i t_i - (c_i + d_i)t_*] - [c_i t_i' - (c_i + d_i)t_*']}{c_i + d_i} \\ &= \xi_i \left(\Pi_i(t_i, t_{-i}) - \Pi_i(t_i', t_{-i})\right). \end{aligned}$$

Thus, $v(t) = [\sum s_i t_i - t_*]$ is a weighted potential function of "noiseless" $\Gamma(1)$. If $\sum s_i < 1$, this function is maximized at a certain value of t (since we assume that target arrival times are bounded from below), and "noisy" $\Gamma(1)$ has a unique equilibrium with adoption. When $\sum s_i > 1$, v is unbounded (adding the same constant τ to all t_i increases v by $(\sum s_i - 1)\,\tau$), and $\Gamma(1)$ has no equilibrium with adoption. When $\sum s_i = 1$, there is a continuum of values of t maximizing function v (since adding the same constant to all target arrival times t_i leaves function v unchanged), and $\Gamma(1)$ has a continuum of equilibria with adoption.

D.2 Potentiality of the Adoption Game with Gradual Network Externalities and Identical Players

We now show that the game with identical players and gradual network externalities considered in Section 6 also has a potential function (the proof of its own-action quasiconcavity is very similar to the proof of $\Gamma(1)$'s own-action quasiconcavity, and is thus omitted). Namely, let $v(t) = -[d(1)t^1 + d(2)t^2 + \cdots + d(N)t^N]$, where t^1 is the actual compliance time of the earliest adopter, t^2 is the actual compliance time of the next adopter, and so on. Then

$$\begin{aligned}\frac{\partial v}{\partial t_i} &= -d(1)\chi(\text{player } i \text{ is first}) \\ &\quad -d(2)\chi(\text{player } i \text{ is second}) \\ &\quad -\cdots \\ &\quad -d(N)\chi(\text{player } i \text{ is last}).\end{aligned}$$

On the other hand, the expected net benefit of player i from delaying his compliance time by an infinitesimal amount of time, $\frac{\partial Pi_i}{\partial t_i}$, is also equal to $-d(1)\chi(\text{player } i \text{ is first}) - d(2)\chi(\text{player } i \text{ is second}) - \cdots - d(N)\chi(\text{player } i \text{ is last})$, and so $v(t_i, t_{-i}) - v(t_i', t_{-i}) = \Pi_i(t_i, t_{-i}) - \Pi_i(t_i', t_{-i})$. Therefore, v is a potential function of the game with gradual network externalities and identical players. Just like in the game $\Gamma(1)$, one can verify that this potential function has a (unique) maximizer if and only if the corresponding game has a (unique) equilibrium where players adopt the standard.

D.3 The Game with Gradual Network Externalities and Different Players

A general game with gradual network externalities and different players no longer has a weighted potential function (this can be checked by comparing cross-derivatives $\frac{\partial \Pi_i}{\partial \mu_j}$), and so we do not necessarily expect equilibria to be robust to the form of noise. Indeed, the following counterexample presents a game with gradual network externalities and different payoffs, in which the existence of equilibrium depends on the structure of noise.

There are three players. The net payoff of player i when he arrives kth is equal to $d_i(k)$, given in the table below.

$i \setminus k$	1	2	3
1	-1	0	1
2	-1	0	1
3	-1	-1	2

In the first case, suppose that each player's noise is distributed uniformly on $[0, \epsilon]$. Then it is an equilibrium for all players to target the same arrival time—the expected marginal benefit from deviating by an infinitesimal amount (we will call this "the expected first-day payoff") is zero for each player (proportional to $(-1+0+1)/3 = 0$ for players 1 and 2 and to $(-1-1+2)/3 = 0$ for player 3).

In the second case, consider the following form of noise. Players 1 and 2 either arrive very early ($[0, \epsilon]$), or very late ($[100\epsilon, 101\epsilon]$), with equal probabilities. Player 3's noise, on the other hand, is still uniform on $[0, \epsilon]$. Then this game has no equilibrium.

Indeed, suppose there is an equilibrium. It has to be in pure strategies, since, holding other players' strategies constant, a player's expected payoff is concave in his target arrival time. If players 1 and 2 target the same compliance time, pick any one of them; otherwise, pick the one who targets the earlier time. Without loss of generality, assume player 1 is picked. Normalize his earliest possible arrival compliance time to 0. Then it cannot be the case that player 3 ever arrives later than 100ϵ (because that would imply that player 3's target compliance time is greater than 99ϵ and he never arrives before ϵ. That, in turn, would imply that player 1 arrives first more often than he arrives last, making his expected first-day payoff negative, which is impossible in equilibrium). Thus, player 3 always arrives before time 100ϵ. Therefore, for him to be the last one it has to be the case that both players 1 and 2 comply in the earlier of the two intervals, which happens with probability .25. But then player 3's first-day payoff is no more than $.25 * 2 - .75 * 1 < 0$, which is impossible in equilibrium.