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Rolf Aaberge, Ugo Colombino and John E. Roemer

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Optimal Taxation According to Equality of Opportunity:
A Microeconometric Simulation Analysis

Rolf Aaberge[•] **Ugo Colombino**[**] **John E. Roemer**[***]

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Abstract
The purpose of this paper is to introduce and adopt a generalised version of Roemer's (1998) Equality of Opportunity (EOp) framework for analysing optimal income taxation. EOp optimal tax rules seek to equalise income differentials arising from factors beyond the control of the individual. Unlike the pure EOp criterion of Roemer (1998) the generalised EOp criterion allows for alternative weighting profiles in the treatment of income differentials between and within types when types are defined by circumstances that are beyond people's control. An empirical microeconometric model of labour supply in Italy is used to simulate and identify optimal tax rules within classes of two- and three-parameter tax rules. A rather striking result of the analysis is that the optimal tax rule turns out to be the pure lump-sum tax, under Roemer's pure EOp criterion as well as under the generalised EOp criterion with moderate degrees of aversion to within-type inequality. A high degree of within-type inequality aversion instead produces EOp-optimal rules with positive marginal tax rates. When the EOp-version of the Gini welfare function is adopted as Eop criterion, the optimal tax rule turns out to be close to the actual 1993 Italian tax system, if not for the important difference of prescribing a universal lump-sum positive transfer of 3,500,000 ITL, which has no comparable counterpart in the actual system. On the other hand, when using the conventional equality of outcome (EO) criterion, the pure lump-sum tax always turns out to be optimal, at least with respect to the classes of two- and three-parameter rules. We also compute second-best solutions, namely we exclude lump-sum taxes. Overall, the results do not conform to the perhaps common expectation that the EO criterion is more supportive of "interventionist" (redistributive) policies than an EOp approach.

Key words: Equality of opportunity, equality of outcome, labour supply, optimal income taxation
JEL classification: D19, D63, H21, H24, H31
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[•] Research Department, Statistics Norway, Oslo, Norway
[**] Department of Economics, Turin, Italy and Statistics Norway, Oslo, Norway
[***] Department of Political Science, Yale University, New Haven, USA

1. Introduction

This paper presents an empirical analysis of optimal taxation, adopting Equality of Opportunity (EOp) as the evaluation criterion.

The EOp-criterion is a computable concept of equality of opportunity developed by Roemer (1998). This concept is interesting from the policy point-of-view, since the majority of citizens in most industrialised countries, although not unfavourable to redistribution, seem sensitive to the way that a certain outcome has been attained. Redistribution is more likely to receive support if it is designed to correct circumstances that are beyond people's control. On the other hand, if a bad outcome is associated with a lack of effort, redistribution will be much less acceptable.

In a previous contribution that originated from an international research project, this concept has been applied to evaluate the EOp performance of income tax rules in various countries, using a relatively simple common model of labour supply behaviour with calibrated parameters[1]. This paper extends the previous study in several respects. First, to allow for alternative weighting profiles in the treatment of income differentials that arise from factors beyond the individuals' control, a generalised version of Roemer's (1998) EOp-criterion is introduced. Secondly, we employ a relatively sophisticated model of labour supply that provides a simultaneous treatment of partners' decisions and accounts for quantity constraints on the distribution of hours. Finally, while the previous study only concerned male heads of household's 25-40 years old this study deals with approximately the entire labour force. To our knowledge, this is the first tax evaluation based on models for both married couples and single individuals. Most tax evaluations are either based on representative agent models or micro-econometric models for single individuals or married females conditional on husbands' income.

In Section 2 we briefly discuss the justification and definition of the EOp-criterion and its relationship to more traditional concepts of social welfare, where the concern focuses upon the equality of outcome (EO) criterion rather than equality of opportunity. In the same section we also explain how the EOp-criterion can be generalised to take into account inequality in outcomes that do not arise from unequal opportunities.

In Section 3.1 we use a micro-econometric model of household labour supply, estimated on 1993 Italian data, to simulate the effects of various constant-revenue affine tax rules, i.e. the tax rules defined by a lump-sum transfer (positive or negative) and a constant marginal tax rate that produces the same revenue collected with the observed 1993 rule. These tax rules are evaluated and compared according to the generalised EOp-criterion. Furthermore, the EOp-optimal tax rule is also identified.

[1] See Roemer et al. (2001).

In Section 3.2 we perform a similar exercise as in Section 3.1, but looking at the class of tax-rules defined by a transfer and two tax rates (instead of one as for the affine rules). In Section 4 we compare the evaluation of tax rules according the EOp and EO criteria. Since it in many cases turn out that the optimal tax rule is the pure lump-sum tax, we have in Section 5 provided second-best optimal tax rules; i.e. tax rules that are optimal within the class of tax systems where lump-sum tax components are excluded. Section 6 summarises the main results. Appendixes A, B and C give essential information on the microeconometric model, on the dataset, and on the 1993 tax rule.

2. The EO and EOp criteria

The standard approach in evaluating tax systems is to employ a social objective (welfare) function as the basic evaluating instrument. This function is commonly used to summarise the changes in (adult-equivalent) incomes resulting from introducing various alternatives to the actual tax system in a country. The simplest way to summarise the changes that take place is to add up the income differentials, implying that individuals are given equal welfare weights independently of whether they are poor or rich. However, if besides total welfare we also care about the distributional consequences of a tax system, then an alternative to the linear additive welfare function is required. In this paper we rely on the rank-dependent social welfare functions that have their origin from Mehran (1976) and Yaari (1988)[2] and are defined by

(2.1) $$W_k = \int_0^1 p_k(t)F^{-1}(t)dt, \quad k = 1,2,...,$$

where F^{-1} is the left inverse of the cumulative distribution function of (adult-equivalent) income with mean μ, and $p_k(t)$ is a weight function defined by

(2.2) $$p_k(t) = \begin{cases} -\log t, & k = 1 \\ \frac{k}{k-1}\left(1 - t^{k-1}\right), & k = 2,3,.... \end{cases}$$

Note that the inequality aversion exhibited by W_k decreases with increasing k. As $k \to \infty, W_k$ approaches inequality neutrality and coincides with the linear additive welfare function defined by

(2.3) $$W_\infty = \int_0^1 F^{-1}(t)dt = \mu.$$

[2] Several other authors have discussed rationales for this approach, see e.g. Sen (1974), Hey and Lambert (1980), Donaldson and Weymark (1980, 1983), Weymark (1981), Ben Porath and Gilboa (1992) and Aaberge (2001).

It follows by straightforward calculations that $W_k \le \mu$ for all j and that W_k is equal to the mean μ for finite k if and only if F is the egalitarian distribution. Thus, W_k can be interpreted as the equally distributed (equivalent) level of equivalent income. As recognised by Yaari (1988) this property suggests that I_k, defined by

(2.4) $$I_k = 1 - \frac{W_k}{\mu}, \; k = 1, 2, ...$$

can be used as a summary measure of inequality and moreover is a member of the "illfare-ranked single-series Ginis" class introduced by Donaldson and Weymark (1980). As noted by Aaberge (2000), I_1 is actually equivalent to a measure of inequality that was proposed by Bonferroni (1930), whilst I_2 is the Gini coefficient.[3]

Note that each of the welfare functions W_1, W_2, and W_3 and the corresponding measures of inequality (I_1, I_2, and I_3) exhibit aversion to inequality and thus obey the Pigou-Dalton principle of transfers. The essential difference between these welfare functions and the corresponding measures of inequality is revealed by their transfer sensitivity properties. When the transfer sensitivity is judged according to Kolm's principle of diminishing transfers it follows from Aaberge (2000) that I_1, I_2, and I_3 assign more weight to transfers between persons with a given income difference if these incomes are lower than if they are higher only for income distributions that are members of certain families of distributions. The I-coefficient satisfies the principle of diminishing transfers for log-concave distribution functions, whilst I_2 and I_3 satisfy this transfer principle for distributions (F) that are strictly concave and F^2 is strictly concave, respectively. By contrast, when we rely on Mehran's principle of positional transfer sensitivity rather than on the principle of diminishing transfers the results of Aaberge (2000) show that I_1 satisfies this principle for all distribution functions, whereas I_2 and I_3 do not. Note that the principle of positional transfer sensitivity differs from the principle of diminishing transfers by requiring a fixed difference in ranks rather than a fixed difference in incomes. In this case the Gini coefficient (I_2) attaches an equal weight to a given transfer irrespective of whether it takes place at the lower, the middle or the upper part of the income distribution, whilst I_3 assigns more weight to transfers at the upper than at the middle and the lower part of the income distribution. Roughly speaking, this means that I_1 exhibits very high downside inequality aversion and is particularly sensitive to changes that concern the poor part of the population, whilst I_2 normally pays more attention to changes that take place in the middle part of the income distribution. The I_3-

[3] For further discussion of the family $\{I_k : k=1, 2, ...\}$ of inequality measures we refer to Mehran (1976), Donaldson and Weymark (1980, 1983), Bossert (1990) and Aaberge (2000, 2001).

coefficient exhibits upside inequality aversion and is thus particularly sensitive to changes that occur in the upper part of the income distribution.

For a given sum of incomes the welfare functions W_1, W_2, and W_3 take their maximum value when everyone receives the same income and may thus be interpreted as EO-criteria (equality of outcome) when employed as a measure for judging between tax systems.

However, as indicated by Roemer (1998) the EO-criterion is controversial and suffers from the drawback of receiving little support among citizens in a nation.[4] This is due to the fact that differences in outcomes resulting from differences in efforts are, by many, considered ethically acceptable and thus should not be the target of a redistribution policy. An egalitarian redistribution policy should instead seek to equalise those income differentials arising from factors beyond the control of the individual. Thus, not only the outcome, but its origin and how it was obtained, matters. This is the essential idea behind Roemer's (1998) theory of equality of opportunity, where people are supposed to differ with respect to *circumstances*, which are attributes of the environment of the individual that influence her earning potential, and which are "beyond her control".

This study defines circumstances by family background, and classifies the individuals into three types according to father's years of education:

- less than 5 years (Type 1),
- 5-8 years (Type 2), and
- more than 8 years (Type 3).

Assume that $F_j^{-1}(t)$ is the income level of the individual located at the t^{th} quantile of the income distribution (F_j) of type j. The differences in incomes within each type are assumed to be due to different degrees of effort for which the individual is to be held responsible, whereas income differences that may be traced back to family background are considered to be beyond the control of the individual. As indicated by Roemer (1998) this suggests that we may measure a person's effort by the quantile of the income distribution where he is located. Next, Roemer declares that two individuals in different types have expended the same degree of effort if they have identical positions (rank) in the income distribution of their type. Thus, an EOp (Equality of Opportunity) tax policy should aim at designing a tax system such that $\min F_j^{-1}(t)$ is maximised for each quantile t. However, since this criterion is rather demanding and in most cases will not produce a complete ordering of the tax systems under consideration a weaker ranking criterion is required. To this end Roemer (1998) proposes to employ as the social objective the average of the lowest income at each quantile,

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[4] See also Dworkin (1981a, 1981b), Arneson (1989, 1990), Cohen (1989) and Roemer (1993).

(2.5) $$\tilde{W}_{\infty} = \int_0^1 \min_j F_j^{-1}(t)dt$$

Thus, $\tilde{W}_{\infty}$ ignores income differences *within* types and is solely concerned about differences that arise from differential circumstances. By contrast, the EO criteria defined by (2.1) does not distinguish between the different sources that contribute to income inequality. As an alternative to (2.1) and (2.5) we introduce the following extended family of EOp welfare functions,

(2.6) $$\tilde{W}_k = \int_0^1 p_k(t) \min_j F_j^{-1}(t)dt, \ \ k = 1,2,...,$$

where $p_k(t)$ is defined by (2.2).

The essential difference between $\tilde{W}_k$ and $\tilde{W}_{\infty}$ is that $\tilde{W}_k$ gives increasing weight to the welfare of lower quantiles in the type-distributions. Thus, in this respect $\tilde{W}_k$ captures also an aspect of inequality within types. As explained above, the concern for within type inequality is greatest for the most disadvantaged type, i.e. for the type that forms the largest segment(s) of $\left\{\min_j F_j^{-1}(t) : t \in [0,1]\right\}$.

Note that $\min_i F_i^{-1}(t)$ defines the inverse of the following cumulative distribution function $\left(\tilde{F}\right)$

(2.7) $$\tilde{F}(x) = \Pr\left(\tilde{F}^{-1}(T) \le x\right) = \Pr\left(\min_i F_i^{-1}(T) \le x\right) = 1 - \prod_i \left(1 - F_i(x)\right),$$

where T is a random variable with uniform distribution function (defined on [0,1]). Thus, we may decompose the EOp welfare functions $\tilde{W}_k$ as we did for the EOp welfare functions W_k. Accordingly, we have that

(2.8) $$\tilde{W}_k = \tilde{W}_{\infty}\left(1 - \tilde{I}_k\right), \ \ k = 1,2,...$$

where $\tilde{I}_k$, defined by

(2.9) $$\tilde{I}_k = 1 - \frac{\tilde{W}_k}{\tilde{W}_{\infty}}, \ \ k = 1,2,...$$

is a summary measure of inequality for the mixture distribution $\tilde{F}$.

Expression (2.8) demonstrates that the EOp welfare functions $\tilde{W}_k$ for $k < \infty$ take into account value judgements about the trade-off between the mean income and the inequality in the distribution of income for the most EOp disadvantaged people. Thus, $\tilde{W}_k$ may be considered as an inequality within type adjusted version of the pure EOp welfare function that was introduced by Roemer (1998). As explained above, the concern for within type inequality is greatest for the most disadvantaged type, i.e. for the type that forms the largest segment(s) of the mixture distribution $\tilde{F}$. Alternatively, $\tilde{W}_k$ for $k < \infty$ may be interpreted as an EOp welfare function that, in contrast to $\tilde{W}_\infty$, gives increasing weight to individuals who occupy low effort quantiles.

Note that the EOp criterion was originally interpreted as more acceptable—from the point of view of individualistic-conservative societies. Our extended EOp welfare functions can be considered as a mixture of the EO welfare functions and the pure EOp welfare function; they are concerned about inequality between types as well as inequality within the worst-off distribution defined by (2.7). EOp looks at what happens to the distribution formed by the most disadvantaged segments of the intersecting type-specific distributions (defined by 2.7). Moreover, the pure version of the criterion only looks at the mean of the worst-off distribution. By contrast, EO takes into account the whole income distribution. For a given sum of incomes, EO will consider equality of income (everyone receives the same income) as the most desirable income distribution. The pure EOp will instead consider equality in mean incomes across types as the ultimate goal. Since the extended EOp combines these two criteria, transfers that reduce the differences in the mean incomes between types as well as the income differentials between the individuals within the worst-off distribution are considered equalising by the extended EOp. Thus, in the case of a fixed total income also the extended EOp will consider equality of income as the most desirable distribution. However, by transferring money from the most advantaged type to the most disadvantaged type, EOp inequality may be reduced although transfers may be conflicting with the Pigou-Dalton transfer principle. Whether it is more "efficient" to reduce inequality between or within types depends on the specific situation. When labour supply responses to taxation are taken into account the composition of types in the worst-off distribution will change and depend on the chosen welfare function $\left(\tilde{W}_k\right)$ as well as on the considered tax rule. Thus, the large heterogeneity in labour supply responses to tax changes that is captured by our model(s) makes it impossible to state anything on EOp- or EO-optimality before the simulation exercises have been completed.

3. Micro-econometric simulation of tax reforms

The purpose of this section is to make an EOp evaluation of the 1993 Italian tax system and various alternative two-parameter and three-parameter tax rules under the constraint of fixed tax revenue. To this end we employ the labour supply model(s) and simulation framework explained in Appendix A to simulate the labour supply behavior of single females, single males, and couples that are between 18 and 54 years old. To capture the heterogeneity in preferences we have estimated three separate models of labour supply: one for single females, one for single males and one for couples. Note, however, that the condition of tax revenue neutrality concerns the total population of individuals at the ages between 18 and 54. Thus, the tax paid by each of the groups is treated as an endogenous variable in the simulation exercises. The main features of the 1993 tax rule are briefly illustrated in Appendix C.

The tax reform simulations consist of five main steps:

1. The tax rule is applied to individual earners' gross incomes in order to obtain disposable incomes. New labour supply responses in view of a new tax rule are taken into account by the household labour supply models for singles and couples briefly described in Appendix A. Note that the utility functions (and choice sets) of the underlying micro-econometric model(s) are stochastic. Thus, we use stochastic simulation to find, for each individual/couple, the optimal choice given a tax-transfer rule. The simulations are made under the conditions of unchanging total tax revenue and non-negative disposable household incomes.
2. To each decision making individual between 18 and 54 years old, an *equivalent income* is imputed, computed as total disposable household income divided by the square root of the number of household members.
3. We then build the individual equivalent income distributions F_1, F_2 and F_3 for the types defined according to parental (actually father's) education: less than 5 years (type 1), 5-8 years (type 2) and more than 8 years (type 3).
4. Finally, we compute $\tilde{W}_k$ for k =1, 2, 3 and ∞.
5. Optimization is performed by iterating the above steps, in order to find the tax rule that produces the highest value of $\tilde{W}_k$ for each value of k under the constraint of unchanged tax revenue, provided that the tax rule is a member of certain sets of two- and three-parameter tax rules.

3.1. EOp-evaluation of alternative two-parameter tax rules

The alternative two-parameter tax rules are of the following type: $x = c + (1 - t)y$, where

y = gross income,

x = disposable income,

c = lump-sum transfer (positive or negative), and

t = constant marginal tax rate.

Note that the income and tax figures below are measured in 1000 ITL. The results of the two-parameter tax reform simulations are summarised in Tables 1 and 2 and in Figure 1.

Table 1. Optimal two-parameter tax systems under various EOp social objective criteria $\left(\tilde{\mathbf{W}}_{\mathbf{k}}\right)$

k	1	2	3	∞
marginal tax rate, t	.774	.637	0	0
lump-sum tax/transfer, c	11,500	9,500	-5,790	-5,790

Table 1 presents the EOp-optimal affine tax rules for different values of k, i.e. for different degrees of concern for within type inequality. Recall that the higher is k, the lower is the concern for within type inequality.

As demonstrated by Table 1 the optimal policy is very sensitive to the value of k. For $k \geq 3$, the EOp-optimal tax rule is the pure lump-sum tax (i.e. $t = 0$ and $c < 0$) whereas for $k \leq 2$ the optimal tax rule consists of a very high marginal tax rate and a positive lump-sum transfer. An implication is that the concern for the equality of opportunity by itself does not imply high marginal tax rates. Only if we also account for within type inequality, does the optimal policy entail high marginal tax rates.

Figure 1. Distributions of observed equivalent income by type. 1000 ITL

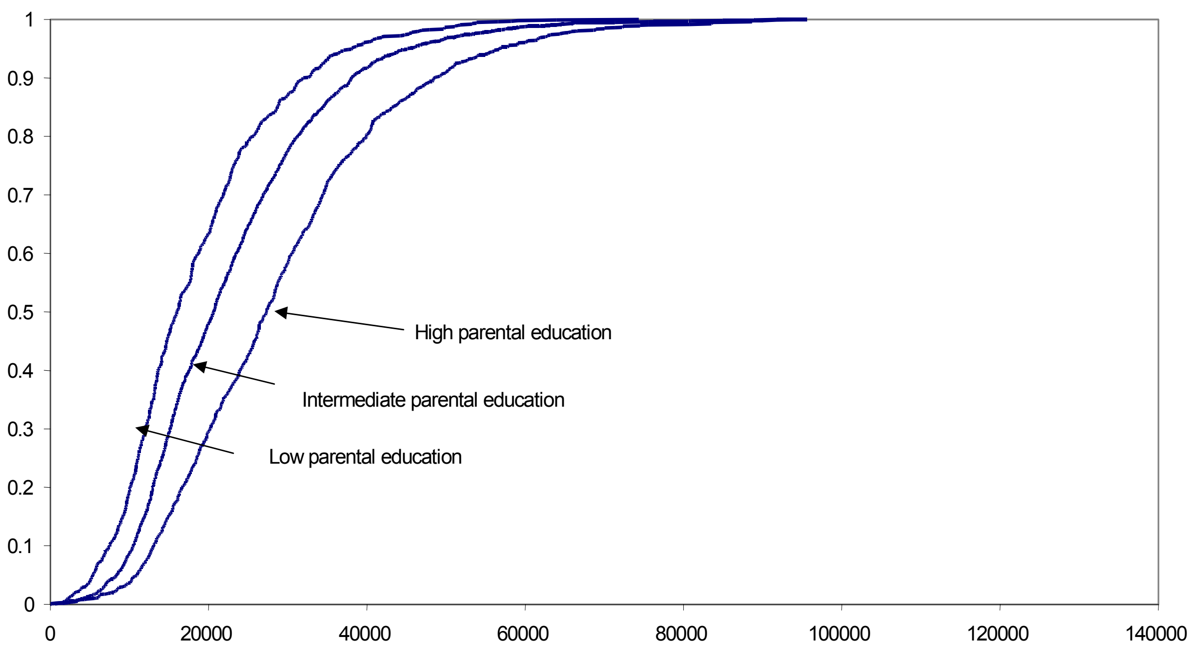

Figure 2. Distributions of individual equivalent income by type under the EOp2(1) and EOp2(3) tax systems. 1000 ITL

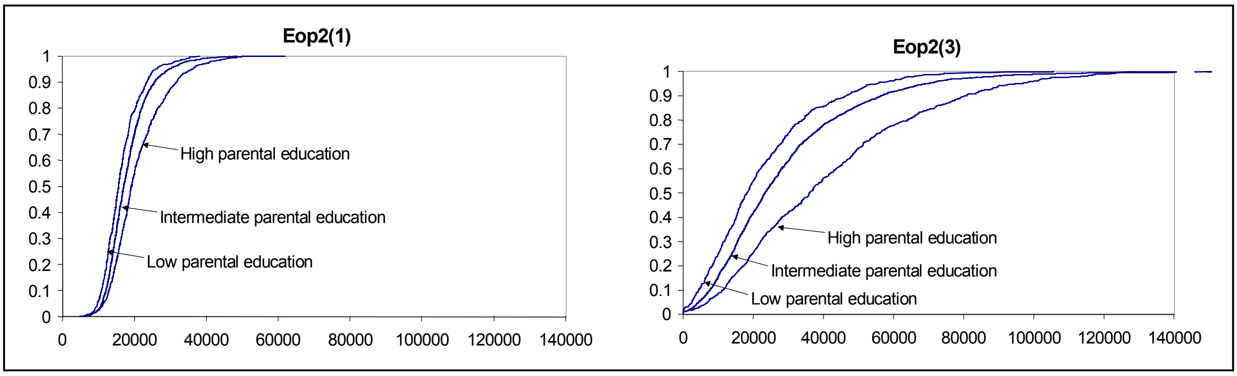

Table 2 and Figures 1 and 2 give more details. The graphs illustrate the equivalent income distributions under the actual 1993 tax rule (Figure 1) and under the EOp-optimal rules for k=1 and k≥3 (Figure 2). Table 2 reports the value of the EOp criterion for different tax rules. In particular, we focus on the comparison between the observed rule (1993), the pure flat tax (a theoretical benchmark), and the three linear rules that are EOp optimal under different values of k. In each column (i.e. for each k) the bold figure is the maximised value of the EOp criterion, i.e. it corresponds to the EOp-optimal tax rule. EOp2(r) denotes the EOp-optimal affine tax rule when k = r.

Table 2. EOp-performance $\left(\tilde{W}_k\right)$ of the 1993 tax system, a flat tax system and three different EOp-optimal two-parameter tax systems

Tax system \ Social objective function $\left(\tilde{W}_k\right)$		k: 1	2	3	∞
1993 tax system		10,523	12,797	13,893	18,323
Flat tax	$\begin{pmatrix} t = .181 \\ c = 0 \end{pmatrix}$	10,834	13,496	14,823	20,449
EOp2 (1)	$\begin{pmatrix} t = .774 \\ c = 11,500 \end{pmatrix}$	**12,661**	13,652	14,077	15,641
EOp2 (2)	$\begin{pmatrix} t = .637 \\ c = 9,500 \end{pmatrix}$	12,406	**13,660**	14,237	16,486
EOp2 (3)	$\begin{pmatrix} t = 0 \\ c = -5,790 \end{pmatrix}$	9,942	13,270	**14,992**	**22,231**

Table 2 enables us to compare the EOp performance of the various rules for a given k (note that the comparison only makes sense between elements of the same column). We can see that although the flat tax is never EOp-optimal, for any value of k, it improves upon the observed 1993 rule. More generally, one can always find an affine tax rule that is EOp-preferred to the observed 1993 one. However, the direction along which one can find EOp-optimal tax rules depends crucially on the value of k. If $k = 1$ one has to move towards very high marginal tax rates (coupled with high transfers). If k is greater than 1, then the EOp-optimal tax rules require lower marginal tax rates – and more revenue collected through the lump-sum part of the tax. These aspects are further illustrated by Figure 3, where we draw the curve – in the (c,t) plane – of the revenue-constant affine tax rules, and for any k we indicate the sets of tax rules with a lower or with a higher EOp performance with respect to the observed rule. As k increases the graphs in Figure 1 demonstrate that the more we reduce the marginal tax rate – and the more revenue we collect through lump-sum taxation – the better is the EOp-performance.

The fact that the optimal tax rule is the pure lump-sum tax, provided that we do not put too much weight on within type inequality, is a somewhat striking result in itself. After all, EOp is an egalitarian criterion, and one would expect it to favour greater marginal taxation. How can we explain this apparently counter-intuitive result? A possible explanation lies with the relatively high labour supply response of the least advantaged individuals. Since the EOp-criterion requires the maximisation of a weighted average of the incomes of the least advantaged type, and since the labour supply of these individuals turns out to be very responsive to higher net wage rates, it follows that lower marginal tax rates (or, in the limit, a marginal tax rate equal to 0) can in fact improve substantially the welfare of this group. However, this effect may be counterbalanced if we are given enough weight (low value of k) to low effort individuals. Table 4 gives some support to this argument by illustrating the labour supply response of the different types when facing alternative tax rules. When the pure lump-sum tax is applied, the labour supply (and therefore the available income) of type 1 (the most disadvantaged group) increases much more (as percentage variation) than labour supply of types 2 or 3. To be sure, a bias in favour of the lump-sum tax might be due to the fact that we equate income and welfare. When accounting for the value of leisure (object of on-going research), the policy prescriptions might change.

Table 3. Decomposition of EOp social welfare $\left(\tilde{W}_k\right)$ under the 1993 tax system, a flat tax system and various EOp-optimal two-parameter tax systems

Tax system	$\tilde{W}_\infty$	Measure of inequality		
		$\tilde{I}_1$	$\tilde{I}_2$	$\tilde{I}_3$
1993 tax system	18,323	.426	.302	.242
Flat tax $\begin{pmatrix} t=.181 \\ c=0 \end{pmatrix}$	20,449	.470	.340	.275
EOp2 (1) $\begin{pmatrix} t=.774 \\ c=11,500 \end{pmatrix}$	15,642	.191	.127	.100
EOp2 (2) $\begin{pmatrix} t=.637 \\ c=9,500 \end{pmatrix}$	16,486	.247	.171	.136
EOp2 (3) $\begin{pmatrix} t=0 \\ c=-5,790 \end{pmatrix}$	22,231	.553	.403	.326

Figure 3. Sets of revenue constant affine tax systems under different EOp welfare criteria $\left(\tilde{W}_k\right)$

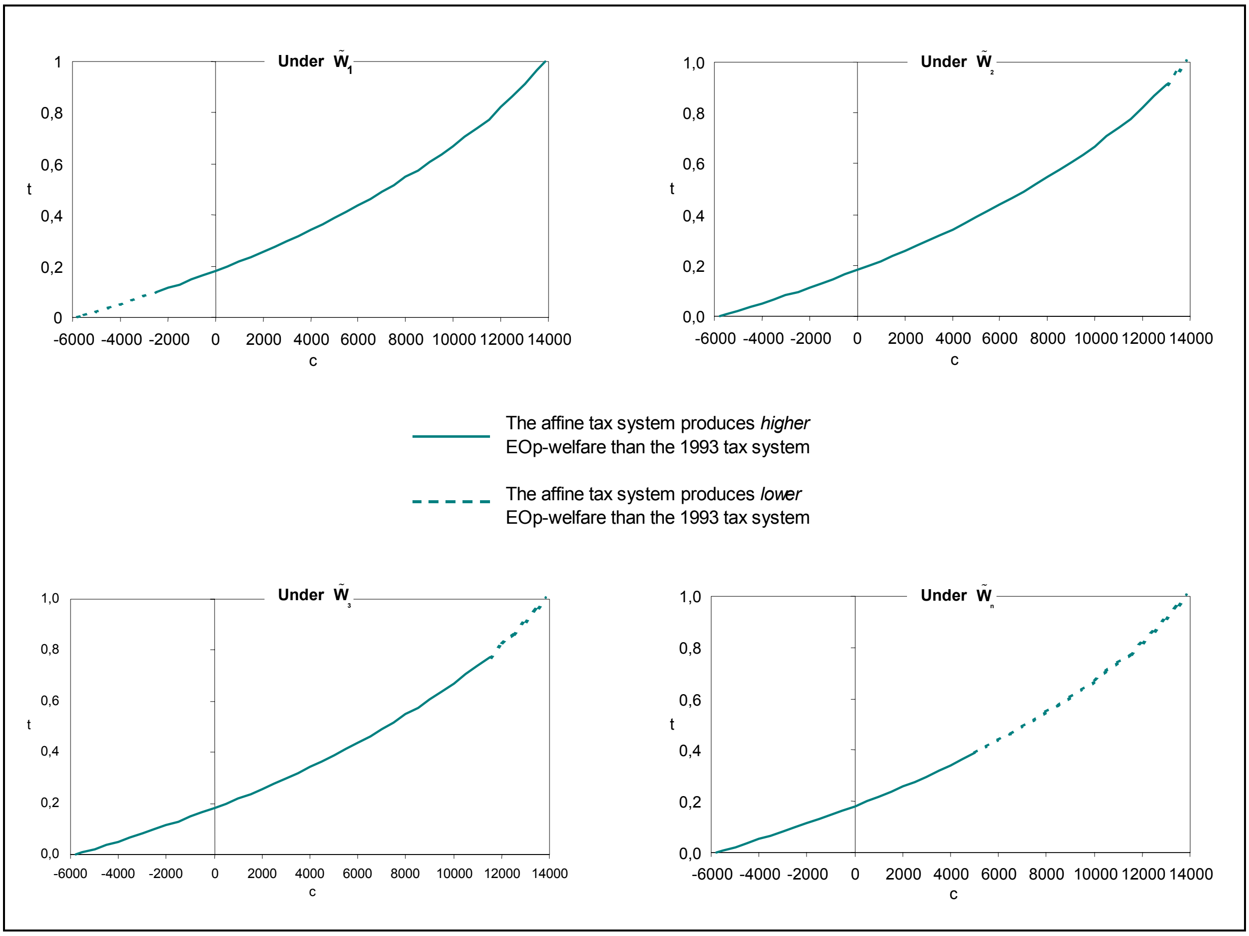

Table 4. Labour supply by types under different tax systems*

Tax system	All	Type		
		1	2	3
1993 tax system	1383	1279	1383	1469
Flat tax $\begin{pmatrix} t=.181 \\ c=0 \end{pmatrix}$	1391 *(+0.58)*	1369 *(+7.04)*	1362 *(+1.52)*	1471 *(+0.14)*
EOp2 (1) $\begin{pmatrix} t=.774 \\ c=11{,}500 \end{pmatrix}$	1095 *(-20.82)*	1109 *(-13.29)*	1087 *(-21.40)*	*1100 (-25.12)*
EOp2 (2) $\begin{pmatrix} t=.637 \\ c=9{,}500 \end{pmatrix}$	1160 *(-16.12)*	1142 *(-10.71)*	1148 *(-16.99)*	*1200 (-18.31)*
EOp2 (3) $\begin{pmatrix} t=0 \\ c=-5{,}790 \end{pmatrix}$	1487 *(+7.52)*	1450 *(+13.37)*	1459 *(+5.50)*	1578 *(+7.42)*

*Percentage changes relative to the labour supply under the 1993 tax system in parentheses.

What happens to specific groups of people under the EOp-optimal rules and in particular under the pure lump-sum policy? Table 5 presents, for various sub-samples, their composition in terms of EOp-types, the average net observed income in 1993, and the change in average income when the lump-sum rule is applied. The results in Table 5 gives a more vivid understanding of the effects of the "reform" from the viewpoints of efficiency and equality. All the sub-samples on average gain in the sense that they get more income. If we look at the gains across types, we see that types 2 or 3 almost always gain proportionately more than type 1. However this is not relevant from the point of view of the EOp criterion, according to which we only care about what happens to the worst-off type for each quantile (in our case, in practice, this is type 1): under the lump-sum rule, type 1 gains more than under the alternative rules; it does not matter if type 2 and 3 gain even more. Where do these gains come from? Clearly there are two (interdependent) channels, higher net wages (in fact an agent gets the whole gross wage under the lump-sum rule) and higher labour supply. The labour supply response is documented in Table 4. For example, we can compute from Table 5 that overall average income increases by 54 per cent *gross* of the lump-sum tax of 5,790,000 ITL. Since the overall increase in labour supply amounts to 7.5 per cent (from Table 4), we have a 46.5 per cent gain attributable to the increase in net wage and to the interaction between wage and labour supply across the sample. We have seen that the lump-sum rule is outcome disequalizing (Table 3). However we know that the generalised EOp index is only affected by the inequality among the individuals belonging to the worst-off type. If we look at what is going on more generally in the whole sample, the effect upon distribution is less clear-cut. For example, the poor gain much more than the non-poor.

Table 5. Relative proportions, mean observed individual (disposable equivalent) income $\left(\mathbf{W}_{\mathbf{obs}}\right)$ and changes in mean individual income $\left(\mathbf{W}_{\infty}-\mathbf{W}_{\mathbf{obs}}\right)$ when the tax regime is changed to lump-sum taxation by gender, family status, economic status (poverty) and family background (type). In 1000 LIT

Individual and household characteristics		Household type (by family background) 1	2	3	All
All	Proportion (per cent)	20.3	54.7	25.0	100
	Mean income	21,107	22,831	29,312	23,540
	Changes in mean income	3,907	5,794	12,011	6,969
Single males	Proportion (per cent)	19.9	51.7	28.4	100
	Mean income	22,369	28,480	34,046	28,843
	Changes in mean income	3,210	7,013	7,343	6,350
Single females	Proportion (per cent)	15.8	51.7	32.6	100
	Mean income	18,076	20,110	26,085	21,734
	Changes in mean income	3,134	2,568	4,412	3,258
Two person households	Proportion (per cent)	15.3	51.2	33.5	100
	Mean income	24,377	28,613	33,913	29,741
	Changes in mean income	7,153	9,781	14,909	11,097
Three person households	Proportion (per cent)	16.5	55.0	28.5	100
	Mean income	20,091	24,795	29,050	25,235
	Changes in mean income	4,678	5,066	14,333	7,648
Households with more than three persons	Proportion (per cent)	23.5	55.8	20.7	100
	Mean income	16,848	20,516	27,349	21,064
	Changes in mean income	3,022	5,153	9,785	5,608
Poor individuals	Proportion (per cent)	39.2	50.4	10.4	100
	Mean income	7,235	7,720	7,424	7,500
	Changes in mean income	5,276	7,487	13,174	7,216
Non-poor individuals	Proportion (per cent)	18.0	55.2	26.8	100
	Mean income	21,320	24,541	30,368	25,528
	Changes in mean income	3,537	5,603	11,955	6,939

3.2. EOp-evaluation of alternative three-parameter tax rules

One might suspect that the results – in particular the EOp-optimality of a pure lump-sum tax for k = 3 or greater – are somewhat forced by the fact that we restrict the simulation to a two-dimensional class of tax rules. Since the disadvantaged individuals are more responsive – in terms of labour supply – than the rich and/or advantaged individuals, we should be able to improve upon the pure lump-sum tax or upon the high marginal rate rules, by adopting a two-dimensional tax rule. Here we explore this policy direction. The class of tax rules considered is defined as follows:

$$x = \begin{cases} c + (1 - t_1)y & \text{if } y \le \bar{y} \\ c + (1 - t_1)\bar{y} + (1 - t_2)(y - \bar{y}) & \text{if } y > \bar{y} \end{cases}$$

where

x = disposable income,

y = gross income,

$\bar{y}$ = average individual gross income in Italy on the survey year (1993).

With the revenue-neutral constraint the three-parameters $(\mathrm{c}, \mathrm{t}_1, \mathrm{t}_2)$ define a two-dimensional class of rules.

Table 6 reports the optimal three-parameter rules for different values of k. For example, for k=1 the optimal rule is defined by a transfer c = 12,500, a first marginal tax rate t_1 = 0.856 and a second marginal tax rate t_2 = 0.776. By comparing Table 6 with Table 1, we see that the EOp-optimal rules differ significantly depending on whether one considers a two-parameter (Table 1) or a three-parameter rule (Table 6). When k = 1, the three-parameter EOp-optimal rule gives two very high and slightly regressive tax rates[5] complemented by a large positive transfer, inducing a net-vs-gross income profile close to the ones implied by the Negative Income Tax schemes. The most marked differences with respect to the two-parameter case are found when using the k = 2. While the two-parameter case called for tax rate over 60 per cent coupled with a positive transfer of 9,500,000 ITL, the three-parameter case entails two very different tax rates with a marked progressive structure (from 25 per cent to 53 per cent) and a much lower transfer (3,500,000 ITL). For any k ≥ 3, the two-parameter case chooses the pure lump-sum tax as the EOp-optimal policy. When we use a three-parameter rule, with k = 3, we still have a positive tax rate (17 per cent) for the higher incomes,

[5] Regressive in the sense that the marginal tax rate decreases with income.

coupled with a 3,500,000 ITL lump-sum tax. However, when we employ the pure EOp-welfare function ($k = \infty$), we are back to the EOp-optimality of the pure lump-sum tax.

It is worth mentioning that when the EOp-version of the Gini welfare function is adopted, the optimal tax rule is close to the actual one (see Appendix C), if not for the important difference of prescribing a universal lump-sum positive transfer of 3,500,000 ITL, which has no comparable counterpart in the actual system.

Table 7 is the analogue of Table 3 for the three-parameter rule. It shows the decomposition of the EOp social welfare function for different values of k and different tax rules, that is, the current 1993 rule and the four EOp-optimal rules of Table 6, with EOp3(r) denoting the EOp-optimal three-parameter tax rule when k = r.

Table 6. Optimal three-parameter tax systems under various EOp social objective criteria $\left(\tilde{W}_k\right)$

k	1	2	3	∞
t_1	.856	.251	0	0
t_2	.776	.531	.168	0
c	12,500	3,500	-3,500	-5,790

Table 7. Decomposition of EOp social welfare $\left(\tilde{W}_k\right)$ under various three-parameter tax systems

Tax system	$\tilde{W}_\infty$	Measure of inequality		
		$\tilde{I}_1$	$\tilde{I}_2$	$\tilde{I}_3$
1993 tax system	18,323	.426	.302	.242
EOp3 (1) $\begin{pmatrix} t_1 = .856 \\ t_2 = .776 \\ c = 12{,}500 \end{pmatrix}$	15,393	.176	.116	.091
EOp3 (2) $\begin{pmatrix} t_1 = .251 \\ t_2 = .531 \\ c = 3{,}500 \end{pmatrix}$	18,508	.364	.253	.201
EOp3 (3) $\begin{pmatrix} t_1 = 0 \\ t_2 = .168 \\ c = -3{,}500 \end{pmatrix}$	21,156	.497	.355	.285
EOp3 (∞) $\begin{pmatrix} t_1 = t_2 = 0 \\ c = -5{,}790 \end{pmatrix}$	22,231	.553	.403	.326

4. Comparison of empirical results based on EOp and EO criteria

In this section we focus upon the evaluation of the EOp-optimal policies (illustrated in section 3) using the more traditional evaluation criterion of equality of outcome (EO criterion, see section 2). Table 8 reports the EO-performance, that is, the level of the EO social welfare function (defined in section 2) of five policies discussed above for various values of k. The policies are the observed 1993 tax rule, and the four EOp-optimal three-parameter rules for $k = 1, 2, 3$ and ∞. The Table shows the decomposition of the EO-criterion into the efficiency and the inequality terms. More generally, we have also searched for the EO-optimal rule within the whole classes of the two-parameter and three-parameter tax rules, and it always turns out that the pure lump-sum tax is optimal whatever the value of k. Thus, if we do not explicitly account for inequality between types in the EOp-manner, the optimal policy always consists in a zero marginal tax rate (coupled with a positive lump-sum tax), *whatever the degree of inequality aversion*. Table 8 clarifies that this result is due to very large efficiency effects of the lump-tax rule, large enough to over-compensate the also large inequality effects.

Table 8. Decomposition of the EO social welfare $(\mathbf{W_k})$ with respect to mean and income inequality under different tax systems

Tax system	Mean income	Measure of inequality		
		I_1	I_2	I_3
1993 tax system	23,540	.416	.295	.237
EOp3 (1) $\begin{pmatrix} t_1 = .856 \\ t_2 = .776 \\ c = 12,500 \end{pmatrix}$	16,560	.193	.130	.104
EOp3 (2) $\begin{pmatrix} t_1 = .251 \\ t_2 = .531 \\ c = 3,500 \end{pmatrix}$	21,477	.364	.255	.203
EOp3 (3) $\begin{pmatrix} t_1 = 0 \\ t_2 = .168 \\ c = -3,500 \end{pmatrix}$	27,573	.499	.363	.294
EOp3 (∞) $\begin{pmatrix} t_1 = t_2 = 0 \\ c = -5,790 \end{pmatrix}$	30,510	.544	.402	.327

5. Second-best policies

As we have seen in previous sections, in many cases it turns out the socially optimal tax rule is the pure lump-sum tax. However it is common to argue the lump-sum taxes are not realistic, for example because their implementation would not be politically acceptable. Therefore we also computed second-best optimal tax rules, i.e. we exclude rules that have a lump-sum tax component (however we admit lump-sum transfers). The results are summarised in Tables 9 and 10 respectively for the EOp and the EO criterion. As it is the case with first best policies, the second best policies are the same under EOp and under EO when $k = \infty$, with the second-best rule being a regressive one: no transfers, a 31.3% marginal tax rate on the first segment and a 0% marginal tax rate on the second segment. This same rule remains the second-best under EO for $k = 3$ and $k = 2$. For the same values of k, the EOp criterion prescribes instead a progressive rules (for $k = 2$, it also requires a positive transfer). For $k =1$ the two criteria diverge again: EOp prescribes a very large transfer together with very high (slightly regressive) marginal rates, while EO prescribes a modest transfer and much lower (regressive) marginal rates. Overall, as was also the case with the first-best policies, the EOp criterion seems to require more redistribution than the EO criterion.

Table 9. Second best three-parameter tax systems under various EOp social objective criteria $\left(\tilde{\mathbf{W}}_{\mathbf{k}}\right)$

k	1	2	3	∞
t_1	.856	.251	0.106	.313
t_2	.776	.531	0.346	0
c	12,500	3,500	0	0

Table 10. Second best three-parameter tax systems under various EO social objective criteria $\left(\tilde{\mathbf{W}}_{\mathbf{k}}\right)$

k	1	2	3	∞
t_1	.298	.313	.313	.313
t_2	.178	0	0	0
c	2000	0	0	0

6. Conclusion

We have used a micro-econometric model of household labour supply in Italy in order to simulate and identify optimal tax rules within classes of two- and three-parameter tax rules when the criterion of Equality of Opportunity as developed by Roemer (1998) defines the evaluation function. We have also offered a generalisation of the EOp criterion that permits us to complement the pure EOp criterion with a variable degree of aversion to inequality within types. A rather striking result is that the optimal tax rule turns out to be the pure lump-sum tax, under the pure EOp criterion or with moderate degrees of aversion to inequality within types (k greater than 2). The result seems to depend on a relatively high labour supply response from the most disadvantaged type: the labour supply incentives – and the efficiency effects for the most disadvantaged – generated by the pure lump-sum tax are large enough to overcome the disequalising effects of lump-sum taxation. A high degree of inequality aversion (k less than 2) instead produces EOp-optimal rules with positive marginal tax rates. It is worth mentioning that when the EOp-version of the Gini welfare function is adopted as EOp criterion, the optimal tax rule is close to the actual one (see Appendix C), if not for the important difference of prescribing a universal lump-sum positive transfer of 3,500,000 ITL, which has no comparable counterpart in the actual system.

On the other hand, when using the equality of outcome (EO) criterion, the pure lump-sum tax always turns out to be optimal, at least with respect to the classes of two- and three-parameter rules. Overall, the results do not conform with the perhaps common expectation that the EO criterion is more supportive of "interventionist" (redistributive) policies than an EOp approach. On the contrary, our data and our model indicate that EO never calls for redistribution, and only if an EOp criterion is introduced may redistributive intervention (through increasing marginal tax rates and/or positive transfers) be optimal depending on the degree of social aversion to inequality. To be sure, the policy prescription might change if we include the value of leisure in the measurement of individual welfare. For example, since under the pure lump-sum tax people work (and earn) a lot more, it might be the case that, when account is taken of their reduced leisure, the lump-sum tax is not so desirable. Including the value of leisure will be pursued in future work, which is also needed to explore the performance of tax rules that are more complex than the two- or three-parameter rules.

It also bears mentioning that we have constrained ourselves to announcing the same tax rule to *all* citizens. If we allowed ourselves to offer different tax rules to different types, then we conjecture that EOp optimization would require positive and high marginal tax rates for advantaged types and lump-sum taxes (or transfers) for disadvantaged types. This type-differentiation of taxation is only very imperfectly approximated by our three-parameter family of tax rules.

Appendix A

The microeconometric model and the simulation procedure

The model used draws upon the framework introduced by Dagsvik (1994). The agents choose among jobs, each job being defined by a wage rate *w,* hours of work *h* and other characteristics *j*. For expository simplicity we consider in what follows a single person household, although the model we estimate considers both singles and couples. The problem solved by the agent looks like the following:

(A.1) $$\max_{(x,h,j)\in B} U\left(x,h,j\right)$$

under the budget constraint $x = f\left(wh, m\right)$, where

h = hours of work
w = gross wage rate
j = other job and/or household characteristics
m = gross exogenous income
x = disposable income
f(.,.) = tax rule that transforms gross incomes (wh,m) into net income x.

The set B is the opportunity set, i.e. it contains all the opportunities available to the household. For generality we also include non-market opportunities into B; a non-market opportunity is a "job" with $\mathrm{w} = 0$ and $\mathrm{h} = 0$. Agents can differ not only in their preferences and in their wage (as in the traditional model) but also in the number of available jobs of different type. Note that for the same agent, wage rates (unlike in the traditional model) can differ from job to job. As analysts we do not know exactly what opportunities are contained in *B*. Therefore we use a probability density function to represent *B*. Let us denote by $p(h,w)$ the density of jobs of type (h,w). By specifying a probability density function on *B* we can for example allow for the fact that jobs with hours of work in a certain range are more or less likely to be found, possibly depending on agents' characteristics; or for the fact that for different agents the relative number of market opportunities may differ. From expression (A.1) it is clear that what we adopt is a choice model; choice, however, is constrained by the number and the characteristics of jobs in the opportunity set. Therefore the model is also compatible with the case of involuntary unemployment, i.e. an opportunity set that does not contain any market opportunity. Besides this extreme case, the number and the characteristics of market (and non-market) opportunities in general vary from individual to individual. Even if the set of market opportunities is not empty, in some cases it might contain very few elements and/or elements with bad characteristics.

We assume that the utility function can be factorized as

(A.2) $$U\left(f(wh,m),h,j\right)=V\left(f(wh,m),h\right)\varepsilon(h,w,j)$$

where V and ε are the systematic and the stochastic component respectively, and ε is i.i.d. according to:

(A.3) $$\Pr\left(\varepsilon \le u\right)=\exp\left(-u^{-1}\right)$$

The term ε is a random taste-shifter which accounts for the effect on utility of all the characteristics of the household-job match which are observed by the household but not by us. We observe the chosen *h* and *w*. Therefore we can specify the probability that the agent chooses a job with observed characteristics (*h,w*). It can be shown that under the assumptions (A.1), (A.2) and (A.3) we can write the probability density function of a choice (*h*,*w*) as follows[6]:

(A.4) $$\varphi(h,w)=\frac{V\left(f(wh,m),h\right)p(h,w)}{\iint\limits_{qz} V\left(f(zq,m),q\right)p(q,z)dqdz}$$

Expression (A.4) is analogous to the continuous multinomial logit developed in the transportation and location analysis literature. The intuition behind expression (A.4) is that the probability of a choice (*h*,*w*) can be expressed as the relative attractiveness – weighted by a measure of "availability" *p(h,w)* – of jobs of type (*h*, *w*). More details on the derivation of (A.4) can be found in Aaberge et al. (1999).

From (A.4) we also see that this approach does not suffer from the complexity of the tax rule *f*. The tax rule, however complex, enters the expression as it is, and there is no need to simplify it in order to make it differentiable or manageable as in the traditional approach. The crucial difference is that in the traditional approach the functions representing household behavior are derived on the basis of a comparison of marginal variations of utility, while in the approach that we follow a comparison of levels of utility is directly involved.

In order to estimate the model we choose convenient but still flexible parametric forms for *V* and *p(h,w)*. The parameters are estimated by maximum likelihood. The likelihood function is the

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6 See Aaberge et al. (1999).

product of the choice densities (A.4) for every household in the sample. We refer to Aaberge et al. (1998, 2002) for the estimated parameters[7].

Once the parameters have been estimated, we can simulate the effects of different tax rules. Then we can evaluate the effect of a new rule f^* by solving the new problem:

$$\max_{(h,w,j)\in B} V\left(f^*\left(wh,m\right),h,j\right)\varepsilon\left(h,w,j\right). \tag{A.5}$$

As a practical matter, the simulation procedure works as follows. First, for each household we simulate the opportunity set with 200 points: one is the chosen alternative, the other 199 are built by drawing from the estimated p(h,w) density. Second, for each household and each point in the opportunity set we draw a value ε from the distribution (A.3). Third, for each household we solve problem (A.5).

For further details on the empirical specification and the estimation results we refer to Aaberge et al. (1998, 1999, 2002).

[7] In Aaberge et al. (1998) the model is estimated on a sample containing only couples. The estimates for singles have been expressly produced for the present paper, and can be obtained upon request.

Appendix B

The dataset

The estimation and the simulation of the model is based on data from the 1993 Survey of Household Income and Wealth (SHIW93). This survey is conducted every two years by the Bank of Italy and, besides household and individual socio-demographic characteristics, contains detailed information on labour, income and wealth of each household component.

The sample that we select contains 4827 individuals (2160 couples, 310 single females and 206 single males). Singles and couples with income from self-employment are excluded from the sample: this is because their decision process may be substantially different from wage employees' and typically involves a permanent element of uncertainty.

We have restricted the ages of the individuals to be between 18 and 54 in order to minimize the inclusion in the sample of individuals who in principle are eligible for retirement, since the current version of the model does not take the retirement decision into account.

Due to the above selection rules, the estimates and the simulations should be interpreted as conditional upon the decisions not to be self-employed and not to retire.

The labour incomes measured by the survey are net of social security contributions and of taxes on personal income. Therefore, in order to compute gross incomes we have to apply the "inverse" tax code. In turn, the "direct" tax code has to be applied to every point in each household's choice set to compute disposable income associated with that point. Hourly wage rates are obtained by dividing gross annual wage income by observed hours.

Appendix C

The Italian 1993 tax rule

Here we summarise the main features of the personal income tax system in 1993. The unit of taxation is the individual. To the individual total taxable income, the following marginal tax rates are applied:

Income (1000 LIT)	Marginal tax rate (per cent)
Up to 7,200	10
7,200 - 14,400	22
14,400 – 30,000	27
30,000 – 60,000	34
60,000 – 150,000	41
150,000 – 300,000	46
Over 300,000	51

Some expenditures (such as medical or insurance) can be deducted from income before applying taxes. Child allowances and dependent spouse allowances – up to the amount of the gross tax – can be subtracted from the tax. Conditional on the number of household members and household total income, the head of the household receives family benefits.

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