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Realized portfolio selection in the euro area.

Claudio Morana (+)*

(+) Dipartimento di Scienze Economiche e Metodi Quantitativi
Università del Piemonte Orientale, Novara (Italy) and
International Centre for Economic Research (ICER, Torino)

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Abstract

A new approach to mean-variance efficient portfolio selection is introduced. The method is based on realized regression theory and the regression based portfolio selection approach of Britten-Jones (1999), yielding a conditional version of the Britten-Jones (1999) method. Application to euro area stock markets diversification, differently from other standard approaches, actually yields a balanced and stable allocation of wealth, free from the problem of corner solutions, suggesting that diversification among euro area stock markets is still be feasible and desirable. Evidence that the monetary union may have had a much less important impact on the integration of euro area equity markets, as well as that the latter is still in progress, is provided.

Key words: asset allocation, portfolio choice, stock market integration, international diversification, euro area, realized regression.

JEL classification: C13, C22, F30, G11

* **Address for correspondence**: Claudio Morana, Università del Piemonte Orientale, Facoltà di Economia, Dipartimento di Scienze Economiche e Metodi Quantitativi, Via Perrone 18, 28100, Novara, Italy. E-mail: claudio.morana@eco.unipmn.it.

1 Introduction

Recent contributions to optimal portfolio choice point to conditional implementation of the mean-variance approach. For instance, in Brandt and Santa-Clara (2003) and Brandt et al. (2004) time-varying portfolio weights are parametrically related to macroeconomic factors. Differently, a non parametric approach is followed in Brandt (1999) and Ait-Sahalia and Brandt (2001).

The paper contributes to the literature on conditional portfolio choice under two perspectives.

The first contribution is methodological, since a new portfolio selection approach is introduced. The proposed method is based on realized regression theory and the regression based portfolio selection approach of Britten-Jones (1999). As shown by Britten-Jones (1999), the weights of a sample efficient portfolio can be recovered by a regression approach implementing the minimization of the sum of the squared deviations between the excess return of a portfolio constructed using K risky assets and a target portfolio with unitary excess return and zero risk (standard deviation). In the paper this latter approach is generalized by means of realized regression theory, yielding a time-varying parameter, or conditional, version of the Britten-Jones (1999) approach.

Financial and economic integration in the current framework can be expected to lead to a sequence of return realized variance-covariance matrices increasingly affected by a reduced rank state, as only a small number of factors are expected to drive the excess returns over the integration process. Under complete market integration just one factor should explain the comovement in excess returns.[1] The time-varying approach should then not only allow to assess whether market integration has taken place, but also to monitor its progression over time. Moreover, the linkages between time-varying portfolio weights and macroeconomic factors can also be assessed by studying ex-post the dependence of the optimal weights on changing macroeconomic conditions. Econometric models can then be employed to forecast out of sample the optimal weights, yielding a dynamic dimension to the proposed approach. Relatively to the conditional approaches available in the literature, the proposed methodology has the advantages of simplicity and minimal information requirements in implementation.

The second contribution is empirical, as the proposed portfolio approach is employed to investigate the degree of equity markets integration and the feasibility of geographic diversification after the monetary union in the euro

[1] See for instance the theoretical framework in Morana and Beltratti (2008).

area. It is found that the monetary union may have had a much less important impact on the integration of euro area equity markets than what in general believed in the literature, and that the latter, after almost a decade, is still in progress. Hence, diversification among euro area stock markets may still be feasible and desirable. Yet, the strong correlation detected across markets should be taken into account, to prevent the drawbacks usually associated with ill-conditioning of the variance-covariance matrix, i.e. unstable mean-variance portfolio allocations, and corner solutions as well. Hence, appropriate corrections (see for instance Ledoit and Wolf, 2004) should be carried out. The findings however suggest that shrinking the return unconditional variance-covariance matrix may not guarantee a dramatic improvement over the standard Britten-Jones approach, as corner solutions seem to be equally likely. On the other hand, a clear-cut improvement is provided by the realized portfolio selection strategy proposed in the paper, particularly in its two-step version, where the ill-conditioning problem of the variance covariance matrix is fully solved. Albeit the proposed strategy is static, a forecasting exercise reveals that macroeconomic information can be usefully employed to forecast the optimal weights out of sample, yielding a dynamic dimension to the approach. Overall, the realized portfolio selection approach is effective and simple to implement also in the case of a very large number of assets, not suffering from the usual problems of unstable optimal weights sequences and corner solutions.

After this introduction, the paper is organized as follows. In section two the realized portfolio selection approach is presented. In sections three the empirical results are presented. Finally, in section four conclusions are drawn.

2 Realized portfolio selection

Following Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002), suppose that the log $M \times 1$ vector price process, p_t, follows a multivariate continuous-time stochastic volatility diffusion

$$dp_t = \mu_t dt + \Omega_t dW_t, \tag{1}$$

where W_t denotes a standard M-dimensional Brownian motion process, and both the processes for the $M \times M$ positive definite diffusion matrix Ω_t and the M-dimensional instantaneous drift μ_t are strictly stationary and jointly independent of the W_t process. Then, conditional on the sample path realization of Ω_t and μ_t, the distribution of the continuously compounded h-period return $r_{t+h,h} = p_{t+h} - p_t$ is

$$r_{t+h,h}|\sigma \left\{ \mu_{t+\tau}, \Omega_{t+\tau} \right\}_{\tau=0}^{h} \sim N(\int_0^h \mu_{t+\tau} d\tau, \int_0^h \Omega_{t+\tau} d\tau). \tag{2}$$

The integrated diffusion matrix

$$\int_0^h \Omega_{t+\tau} d\tau \tag{3}$$

can be employed as a measure of multivariate volatility.

By the theory of quadratic variation, under some weak regularity conditions,

$$\hat{\Omega}_{t+h} = \sum_{j=1,\dots,[h/\Delta]} r_{t+j\cdot\Delta,\Delta} r'_{t+j\cdot\Delta,\Delta} \xrightarrow{p} \int_0^h \Omega_{t+\tau} d\tau, \tag{4}$$

i.e. the realized variance covariance matrix estimator is a consistent estimator, in the frequency of sampling ($\Delta \rightarrow 0$), of the integrated variance covariance matrix.

Following Britten-Jones (1999), the weights of a sample efficient portfolio can be recovered by a regression approach implementing the minimization of the sum of the squared deviations between the excess return of a portfolio constructed using K risky assets and a target portfolio with unitary excess return and zero risk (standard deviation). In a time-varying framework the regression model can then be written as

$$i_t = \sum_{k=1}^{K} \beta_{k_t} er_{k_t} + \varepsilon_t, \tag{5}$$

where i is the unitary vector, $er_{k_t} = r_{k_t} - r_{f_t}$ is the time t excess return on the kth risky asset, where r_{f_t} is the time t return on the risk-free asset, and ε_t measures the deviation at time t of the portfolio return from unity.

The $(K \times K)$ realized variance-covariance matrix for the excess returns on the risky assets can be written as

$$\hat{\Omega}_{(K)_{t+h}} = \sum_{j=1,\dots,[h/\Delta]} er_{K_{t+j\cdot\Delta,\Delta}} er'_{K_{t+j\cdot\Delta,\Delta}} \xrightarrow{p} \int_0^h \Omega_{(K)_{t+\tau}} d\tau, \tag{6}$$

while the $(K \times 1)$ vector of covariances of the excess return on the target portfolio with the excess return on each of the risky assets can be denoted as

$$\hat{\Omega}_{(iK)_{t+h}} = \sum_{j=1,...,[h/\Delta]} i_{t+j\cdot\Delta,\Delta} er_{K_{t+j\cdot\Delta,\Delta}} \xrightarrow{p} \int_0^h \Omega_{(iK)_{t+\tau}} d\tau. \tag{7}$$

It then follows that

$$\hat{\beta}_{t,t+h} = \hat{\Omega}^{-1}_{(K)_{t+h}} \hat{\Omega}'_{(iK)_{t+h}} \tag{8}$$

is a consistent estimator, in the frequency of sampling $(\Delta \to 0)$, of $\beta_{t,t+h}$[2], i.e.

$$\hat{\beta}_{t,t+h} \xrightarrow{p} \beta_{t,t+h} = \left(\int_0^h \Omega_{(K)_{t+\tau}} d\tau \right)^{-1} \left(\int_0^h \Omega_{(iK)_{t+\tau}} d\tau \right)'. \tag{9}$$

It then finally follows that the vector of realized portfolio weights at time s can be computed as

$$\hat{\beta}_s = (X'X)^{-1}_s (X'i)_s \qquad s = 1, ..., S, \tag{10}$$

where the mnth element in the matrix $(X'X)_s$ is given by $x_{mn_s} = \sum_{h=1}^{H} er_{m_{s,h}} er_{n_{s,h}}$, $m, n = 1, ..., K$, and the mth element in the vector $(X'i)_s$ is given by $x_{im_s} = \sum_{h=1}^{H} er_{m_{s,h}} i_{s,h}$ $m = 1, ..., K$.[3]

Benefits in terms of stability of portfolio weights are expected from noise filtering. In fact, when realized weights are computed starting from daily, rather than high frequency (intra-daily) data, measurement error may affect the estimates. In the paper the cubic spline smoother is implemented following Kohn and Ansley (1989).

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[2] This is proved by noting that $\hat{\Omega}_{(iK)_{t+h}} \xrightarrow{p} \int_0^h \Omega_{(iK)_{t+\tau}} d\tau$ and $\hat{\Omega}_{(K)_{t+h}} \xrightarrow{p} \int_0^h \Omega_{(K)_{t+\tau}} d\tau$

and that $\hat{\Omega}^{-1}_{(K)_{t+h}} \xrightarrow{p} \left(\int_0^h \Omega_{(K)_{t+\tau}} d\tau \right)^{-1}$ using also the continuous mapping theorem.

[3] See Barndorff-Nielsen and Shephard (2002) for additionald details on the asymptotic properties of the realized regression estimator.

Imposing short-sale constraints By construction, neither the estimated portfolio weights necessarily sum to unity, nor short-sale constraints are imposed. Following Brandt et al. (2005), both constraints, however, can be easily imposed after estimation by simply setting to zero the negative weights and normalizing the positive weights as $\hat{\beta}^{+}_{m,s} = \frac{\max[0,\hat{\beta}_{m,s}]}{\sum\limits_{j=1}^{K} \max[0,\hat{\beta}_{j,s}]}$. Standard errors for the estimated time-varying weights can be easily computed by bootstrapping.

On the other hand, more general forms of linear restrictions can be imposed by implementing the restricted version of the realized regression estimator. By writing the set of q linear restrictions at time s as $R_s\beta_s = r_s$, where R_s is the $q \times k$ selection matrix and r_s is a $q \times 1$ vector, the restricted realized estimator can be written as

$$\hat{\beta}^{*}_{s} = \hat{\beta}_s + (X'X)^{-1}_s R'_s [R_s (X'X)^{-1}_s R'_s]^{-1} \left(r_s - R_s \hat{\beta}_s \right) \qquad s = 1, ..., S, \quad (11)$$

by direct generalization of the restricted least squares estimator.

Multi-period portfolio optimization The realized regression approach allows for a straightforward solution of the single-period or myopic portfolio choice problem. Albeit not in general, there are however important cases in which the myopic approach can be considered optimal, as for instance when investment opportunities are constant (Nielsen and Vassalou, 2002) or unhedgeable (Brandt, 2004). Moreover, the multi-period portfolio choice problem can be solved optimally as a sequence of single-period portfolio optimizations in the case of logarithmic preferences. Finally, investing optimally may not come without costs, as the expected utility gains have to be traded-off with the expected utility losses determined by errors in the specification of the multi-period portfolio choice problem.

Yet, as the portfolio weights are computed from predictable quantities, particularly the variance-covariance matrix of returns, the same property can be expected to hold for the portfolio weights. This should then allow to handle the multi-period optimization problem under a different perspective, as a sequence of optimal weights can be parsimoniously forecasted out of sample by means of an econometric model, possibly augmented to control for state variables (macro or financial factors) related to the time variability of the portfolio weights.

Refinements The proposed approach may be refined in different ways. A first refinement consists of applying the shrinkage estimator of Ledoit and

Wolf (2004) to the variance-covariance matrix $(X'X)_s$, which is the key ingredient in the computation of the portfolio weights.[4] Several benefits are expected from shrinking, as for instance an increased stability in portfolio weights due to the reduction in the sampling error and enforcing positive definiteness. Yet, this latter approach suffers from arbitrary in the selection of the shrinkage target.

A two-step realized portfolio approach An alternative approach, not suffering from this latter drawback, consists of a two-step strategy. In the first step the principal components explaining the bulk of total variance, i.e. 90% or 95%, are extracted from the excess returns for the set of candidate multicollinear assets. In the second step the realized portfolio approach is then carried out using the principal components as regressors in the auxiliary realized regressions. The latter portfolios (principal components) by construction are orthogonal to each other and therefore invertibility and optimal conditioning of the sequence of realized variance-covariance matrices, used for the estimation of the sequence of optimal weights, are granted. Once the weights for the principal components portfolios are obtained, the optimal weights for the candidate assets can be obtained by taking into account both the latter weight estimates and the contribution of each asset in the linear combination determining the auxiliary portfolios. The procedure is detailed below.

First step Consider the vector of daily excess returns $\mathbf{er}_{h,s}$, $h = 1, ..., H$, $s = 1, ..., S$, on the K candidate assets, measured at day h of month s, and denote $\hat{\mathbf{\Sigma}}$ the variance-covariance matrix estimated over the full sample of daily data. Principal components analysis yields the decomposition

$$\hat{\mathbf{\Sigma}} = \hat{\mathbf{A}}\hat{\mathbf{\Lambda}}\hat{\mathbf{A}}^{'}, \tag{12}$$

where $\hat{\mathbf{\Lambda}}$ is the $K \times K$ diagonal matrix of the estimated eigenvalues of the $\hat{\mathbf{\Sigma}}$ matrix, and $\hat{\mathbf{A}}$ is the $K \times K$ matrix of the estimated associated orthogonal eigenvectors. The K principal components (auxiliary portfolios) at time period h,s are then computed as

$$\hat{\mathbf{c}}_{h,s} = \hat{\mathbf{A}}^{'}\mathbf{er}_{h,s}, \tag{13}$$

and the associated eigenvalues measure the variance of each principal component. If the $\hat{\mathbf{\Sigma}}$ matrix is of reduced rank $k < K$, then the number of principal

[4]This follows from the fact that daily stock returns show a zero expectation. Hence, the cross-product matrix coincides with the variance-covariance matrix.

components or auxiliary portfolios to be used in the second step will only be equal to k.

Second step The optimal realized weights for the auxiliary portfolios $\hat{\mathbf{c}}_s$, at time period s, can then be computed as

$$\hat{\boldsymbol{\gamma}}_s = (X'X)_s^{-1} (X'i)_s \qquad s = 1, ..., S, \tag{14}$$

where the mnth element in the matrix $(X'X)_s$ is given by $x_{mn_s} = \sum_{h=1}^{H} \hat{c}_{m_{h,s}} \hat{c}_{n_{h,s}}$, $m, n = 1, ..., k$, and the mth element in the vector $(X'i)_s$ is given by $x_{im_s} = \sum_{h=1}^{H} \hat{c}_{m_{h,s}} i_{h,s}$ $m = 1, ..., k$.

The vector of optimal weights for the involved monthly excess returns at time period s is then given by

$$\hat{\boldsymbol{\delta}}_s' = \hat{\boldsymbol{\gamma}}_s' \hat{\mathbf{A}}_k', \tag{15}$$

where $\hat{\mathbf{A}}_k$ is the $K \times k$ sub matrix composed of the first k columns of the matrix $\hat{\mathbf{A}}$. The weights $\hat{\boldsymbol{\delta}}_s$ can then be normalized in order to comply with the restriction that weights need to sum to unity, i.e.

$$\hat{\mathbf{w}}_s = \hat{\boldsymbol{\delta}}_s / (\hat{\boldsymbol{\delta}}_s' i_s) \qquad \hat{\mathbf{w}}_s' i_s = 1. \tag{16}$$

Example Suppose the vector of daily excess returns for the $K = 5$ collinear asset candidates is denoted as $x_{h,s} = (\ x_{1_{h,s}} \quad x_{2_{h,s}} \quad ... \quad x_{K_{h,s}}\ )'$, $h = 1, ..., H$, and that the excess return for a non collinear asset is denoted as $z_{1_{h,s}}$. Moreover, suppose that by applying principal components analysis it is found that just the first two principal components are enough to account for 95% of total variance, i.e. $k = 2$, yielding the auxiliary portfolios $\hat{c}_{1_{h,s}} = \hat{\alpha}_1 x_{1_{h,s}} + \hat{\alpha}_2 x_{2_{h,s}} - \hat{\alpha}_3 x_{3_{h,s}}$ and $\hat{c}_{2_{h,s}} = \hat{\beta}_1 x_{1_{h,s}} - \hat{\beta}_2 x_{4_{h,s}} - \hat{\beta}_3 x_{5_{h,s}}$, $h = 1, ..., H$, where $\hat{\alpha}_i$ and $\hat{\beta}_i$, $i = 1, ...,$ are all positive parameters, elements of the first two columns of the eigenvector matrix $\hat{A}$, respectively. Then, suppose that the application of the second step, i.e. the realized portfolio approach, yields, for time period s, the estimated weights $\hat{\gamma}_{1,s}$ for the auxiliary portfolio $\hat{c}_{1_{h,s}}$, $\hat{\gamma}_{2,s}$ for the auxiliary portfolio $\hat{c}_{2_{h,s}}$, and $\hat{\gamma}_{3,s}$ for the non collinear asset $z_{1_{h,s}}$. The optimal portfolio, at time period h, s, at the second step is then as follows

$$\hat{\gamma}_{1,s}(\hat{\alpha}_1 x_{1_{h,s}} + \hat{\alpha}_2 x_{2_{h,s}} - \hat{\alpha}_3 x_{3_{h,s}}) + \hat{\gamma}_{2,t}(\hat{\beta}_1 x_{1_{h,s}} - \hat{\beta}_2 x_{4_{h,s}} - \hat{\beta}_3 x_{5_{h,s}}) + \hat{\gamma}_{3,s} z_{1_{h,s}}$$

and, by rearranging,

$$\hat{\delta}_{1,s}x_{1_{h,s}} + \hat{\delta}_{2,s}x_{2_{h,s}} - \hat{\delta}_{3,s}x_{3_{h,s}} - \hat{\delta}_{4,s}x_{4_{h,s}} - \hat{\delta}_{5,s}x_{5_{h,s}} + \hat{\delta}_{6,s}z_{1_{h,s}},$$

with $\hat{\delta}_{1,s} = (\hat{\gamma}_{1,s}\hat{\alpha}_1 + \hat{\gamma}_{2,s}\hat{\beta}_1)$, $\hat{\delta}_{2,s} = \hat{\gamma}_{1,s}\hat{\alpha}_2$, $\hat{\delta}_{3,s} = \hat{\gamma}_{1,s}\hat{\alpha}_3$, $\hat{\delta}_{4,s} = \hat{\gamma}_{2,s}\hat{\beta}_2$, $\hat{\delta}_{5,s} = \hat{\gamma}_{2,s}\hat{\beta}_3$, $\hat{\delta}_{6,s} = \hat{\gamma}_{3,s}$.

The optimal weights can then be computed as $\hat{w}_{l,s} = \hat{\delta}_{l,s}/\sum_{j=1}^{K}\hat{\delta}_{j,s}$, $l = 1, ..., K$.

3 Optimal portfolio diversification in the euro area

Is international diversification still feasible in the euro area after the introduction of the euro? The response to the latter question is strictly related to the degree of international stock markets comovement, i.e. to the degree of market integration progressively attained over time in the euro area.

The available empirical evidence is mixed, overall suggesting that market integration has benefited from the monetary unification, i.e. the elimination of currency risk and monetary integration, in the aftermath of the introduction of the euro (Fratzscher et al., in press), but that the process is still in progress after about a decade. For instance, a decrease in relative volatility has been found for the most volatile markets of the euro zone, i.e. Italy and Spain, relative to the least volatile ones, i.e. France and Germany (Morana and Beltratti, 2002), as well as a reduction in the equity home bias for portfolios owned by European institutional investors (Adam et al., 2002). Yet, market participation of households across euro zone countries is still heterogeneous (Guiso et al., 2003). Moreover, the degree of market integration, relative to the 1990s (the convergence period), could have even stabilized, if not reduced, in the last few years. Some studies have in fact documented an increase in markets comovements over the 1993-1998 period (Baele et al., 2004; Hardouvelis et al., 1999; Ehrman et al., 2004). Yet, the evidence on the decrease in geographic dispersion in euro area stock returns is not compelling.[5]

Differently from previous studies, in this paper the assessment of the degree of euro area stock markets integration is carried out in the realized regression framework. The analysis is focused on the rank properties of the

[5] Results of Adjaoute and Danthine (2003) and Baele et al. (2004) that since 2001 the cross-sectional dispersion of returns is larger at the sectorial level than at the country level are in fact sensitive to the inclusion of the ICT bubble period in the sample, and therefore could be a sample artifact.

sequence of realized variance-covariance matrices for euro area excess stock returns $(X'X)_s$, $s = 1980{:}1, ..., 2007{:}1$, measured at the quarterly frequency, which is employed in the computation of the realized portfolio weights as well. In particular, as stock markets integration progresses over time, the degree of comovement in stock returns should increase, yielding a reduced rank return realized variance-covariance matrix. The intensity of the phenomenon, may then be evaluated by means of principal components analysis. The latter involves the assessment of the relative size of the eigenvalues of the quarterly sequence of realized variance-covariance matrices, allowing to measure how the proportion of total variance associated with each eigenvalue (explained by each principal component) varies over time, as well as the proportion of the variance of each return series accounted by each principal component.[6] The proportion of variance explained by the largest eigenvalue of the realized variance-covariance matrix at any point in time can then be taken as a direct measure of the degree of stock market integration since, under perfect or complete stock market integration, just one factor should explain total return variance.

Albeit, the above market integration analysis is of interest on his own, it is however also preliminary to the portfolio selection strategy proposed in the paper, i.e. it is required for the estimation of the auxiliary portfolios employed in the two-step strategy.

3.1 Stock market integration and auxiliary portfolios

The data set is composed of the MSCI price indexes for eleven out of the thirteen euro area members, i.e. Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal, Spain[7], and for some other major European, i.e. Switzerland, the UK, Denmark, Sweden, Norway, and world stock markets, i.e. the US, Japan, and Pacific Basin countries ex Japan. Daily data are available for most of the series from January 1, 1980, apart from Finland, Ireland and Portugal (January 1, 1988) and Greece (January 1, 2001). The three-month US Treasury Bill rate has been employed as a measure of the risk-free rate. For all the series the sample ends in March 31,

[6] According to the notation used in the paper, the proportion of total variance accounted by the p-th principal component is then $\hat{\mathbf{\Lambda}}_{s,p}/\sum_j \hat{\mathbf{\Lambda}}_{s,j}$, where $\hat{\mathbf{\Lambda}}_{s,p}$ is the p-th element on the main diagonal of the matrix $\hat{\mathbf{\Lambda}}_s$. Moreover, the proportion of variance of the i-th excess return accounted by the j-th principal component can be computed as $\hat{a}^2_{s,ij}\hat{\mathbf{\Lambda}}_{s,j}/(\sum_j \hat{a}^2_{s,ij}\hat{\mathbf{\Lambda}}_{s,j})$, where $\hat{a}_{s,ij}$ is the i,j-th elements in the $\hat{\mathbf{A}}_s$ matrix.

[7] MSCI price index data for Luxembourg and Croatia are not available. The latter are however minor countries within the monetary union. Moreover, accession of Croatia has taken place only in January 2007.

2007. Data in local currency have been used in the analysis, relying on the assumption that investors are able to hedge, at least in part, their foreign exchange risk. In order to control for not fully overlapping trading periods two-day returns have been employed, yielding a total of 109 quarterly observations.

The analysis has considered different samples, according to data availability. The first sample, i.e. EA-7, is composed of the stock return indexes for the seven largest countries in the euro area, i.e. Italy, Germany, France, Spain, the Netherlands, Austria, and Belgium, for which data are available since January 1980. The second sample, i.e. EA-10, in addition to the previous countries, also considers Finland, Portugal and Ireland, for which data are available since January 1988. Finally, the third sample, i.e. EA-11, also includes the return series for Greece, for which data are available since June 2001. The results for the euro area have been contrasted with those obtained from a sample including the five European countries which are not part of the monetary union, i.e. the UK, Switzerland, Norway, Sweden, and Denmark (E-5).

In addition to the quarter by quarter analysis, the assessment of the integration process has been carried out also over three time intervals, i.e. the pre-convergence period (1980:1-1993:3), the convergence period (1993:4-1998:4) and the monetary union period (1999:1-2007:1).[8]

3.1.1 Stock market integration in the euro area: results

Since both global and regional factors may affect the comovement in euro area stock markets, the impact of the monetary union on euro area stock markets integration has been assessed conditional to global factors. The implementation has been twofold. Firstly, stock market dynamics have been assessed with and without including the global/US market in the sample.[9] Secondly, the analysis has been repeated on data filtered for the linear influence of the global/US market. This second approach allows for a full control of global factors when assessing the contribution of regional factors, assuming orthogonality between global and regional factors.[10]

[8] The watermark between the pre-convergence and the convergence period is the ratification of the Maastricht Treaty, which took place in November 1993.

[9] The analysis has also been repeated including, in addition to the US index, the Japanese index, and the Pacific Basin ex-Japan index. The findings are numerically very similar to the case in which only the US index is employed and are available from the author upon request.

[10] The second approach is also based on the assumption, coherent with the available empirical evidence, that linkages between the US and euro area stock markets are unidirectional, with the US market affecting the euro area market, but not the other way

The results of the principal components analysis for the three selected time intervals are reported in Table 1, while in Figure 1 the sequence of the (smoothed) proportion of variance explained by the first largest eigenvalue of the sequence of realized variance-covariance matrix $\{X'X\}_s$ is plotted for the various samples.

As is shown in Figure 1, integration dynamics are similar across groups of countries both over time periods and across samples. In fact, for both the euro area and European groups a similar increasing trend in the sequence of proportion of variance explained by the first largest eigenvalue of the sequence of realized variance-covariance matrices can be noted. However, the dynamics are not monotonic, as three cyclical phases of integration can be noted since the 1980s, with timing fairly consistent with the three sub periods selected, i.e. the pre-convergence (1980-1993:2), convergence (1993:3-1998:4), and monetary union (1999:1-2007:1) phase.

As far as actual data are concerned, for the EA-7 group the first phase covers the period 1985 to 1991, pointing to an increase in the proportion of explained variance from about 40% to 60%. After a temporary slowdown, possibly due to the ERM system crisis in 1992, a second growth phase can be noted over the period 1994-1998, with the proportion of explained variance increasing from about 55% to 70%. Since this latter period coincides with the "convergence phase" for the euro area member countries, the observed increase in market comovement may be possibly related to the economic coordination determined by the Maastricht Treaty convergence provisions. Finally, the last phase starts in 1999 and it appears to be still in progress. Over this latter phase, i.e. the monetary union phase, the process of integration appears to have further accelerated, as the proportion of variance associated with the largest eigenvalue of the realized variance covariance matrix not only has got close to about 80%, but is also still following an increasing trend. A similar dynamic pattern can also be found for the EA-10 and EA-11 groups, as well as for the E-5 group, albeit the strength of market comovement in these latter cases is lower than for the EA-7 group.

The above picture is however only partially confirmed by the analysis carried out on filtered data, as sizeable differences in the integration path for the actual and filtered data can be noted towards the end of the convergence period and over the monetary union period, with filtering causing a 10% to 20% downwards shift in its level, depending on the sample investigated. Moreover, the increased pace of integration detected for the EA-7 group over the convergence period, as well as since 2005, can be actually associated with the effects of global dynamics, as filtered data point to a slow down in

around. See Morana (in press) for recent supporting empirical evidence.

the pace of integration, recovering in the aftermath of the monetary union only, and stabilizing since 2004. Actually, the analysis carried out on the larger EA-10 and EA-11 samples even reveals a reversal in stock market comovement over the period 2003-2005, which would have however recovered since 2006. Similar findings hold for the E-5 group as well, as filtered data point to a stabilization in the integration process towards the end of the convergence period, rather than to steady growth over time. Only since 2002 the gap between the estimated paths using the actual and filtered data would have started decreasing, being currently close to 5%. Finally, by comparing filtered EA-7/10/11 and E-5 data, it is possible to note that the degree of integration within the euro area would have got stronger than for the E-5 group only since 1997 for the EA-7 group and since 1999 for the EA-10 group, although for the latter a reversal would have occurred since 2004, with stock market comovement being currently slightly stronger for the E-5 group than for the EA. This latter finding is even more evident for the EA-11 sample, as the gap relatively to the E-5 sample is larger, and close to about 5%.

Additional insights are provided by the assessment of the proportion of variance explained by the first principal component for each of the actual (filtered) stock return series (Table 1). Over the monetary union period, the latter explains a proportion of US return variance in the range 0.4 to 0.5, similar to the smallest euro area stock markets (Austria, Belgium, Portugal, Ireland and Greece) and Norway and Denmark, while for the largest euro area markets (Germany, France, Italy, the Netherlands) the proportion of explained variance is in the range 0.70 to 0.90. Interestingly, the degree of stock market comovement for the UK, Switzerland and Sweden (E-3) is unambiguously lower than the one characterizing Germany, France, Italy, and the Netherlands (EA-4) only for the monetary union period. On the other hand, for the pre-convergence and convergence periods, the proportion of explained variance by the first largest eigenvalue is similar, in the range 0.3 to 0.5 (0.19 to 0.45) and 0.5 to 0.7 (0.40 to 0.63) over the pre-convergence and converge period, respectively. Overall, Austria, Greece, Ireland, Portugal, Norway and Denmark are sources of heterogeneity over the full time span investigated, while Spain, Belgium and Finland have progressively shown more coordinated fluctuations with the EA-4 group over time.

3.1.2 Implications and estimation of the auxiliary portfolios

Some important conclusions can be drawn from the above evidence.

Firstly, monetary unification does not seem to have had an important impact on euro area stock markets integration, possibly apart from the aftermath of the introduction of the euro, as stock market fluctuations tend to be more coordinated within sub groups of euro area countries (EA-7) rather than within the whole set of countries (EA-11). Moreover, the degree of integration within the euro area has got stronger than for the E-5 group well in advance the introduction of the euro for the EA-7 group, while for the EA-10 group a reversal would have occurred since 2004, with stock markets comovement being currently slightly stronger for the E-5 group than for the EA-10 group. Even worse, the degree of integration for the E-5 group has been stronger than for the EA-11 group over the whole monetary union period. The heterogeneity contributed by some of the markets, i.e. Austria, Greece, Ireland, and Portugal, should be noted as part of the explanation of the finding.

Secondly, while global (US) factors do contribute to the explanation of common stock market dynamics for euro area and European countries, regional factors are potentially an even more important determinant. By focusing on the monetary union period, it can be concluded that about 80% of the integration process in the EA-11 group is determined by regional factors and 20% by global factors. The contribution of regional factors increases to 91% if the EA-7 group is considered. On the other hand, figures for the E-5 group yields 89% and 11%, for regional and global factors, respectively. Yet, the same figures can be found for the convergence period for the EA-7 and E-5 groups.[11]

Finally, while the process is not complete, the degree of integration of euro area stock markets is strong, as about 80% of total return variance can be associated with a single common factor. Hence, the degree of comovement across euro area and European stock markets is strong and likely to expose standard mean-variance analysis to important drawbacks, i.e. ambiguity and instability. The former refers to the fact that a large number of resampled efficient frontiers may be statistically equivalent to a given efficient frontier. The latter refers to the fact that portfolio weights are strongly sensitive to small changes in the information set. Moreover, many negative weights, inconsistent with the no short-sale constraint, may be obtained from unconstrained optimization, while constrained optimization would lead to corner solutions, i.e. to the exclusion of many assets and to unreasonably large weights for few assets. Both extreme sensitivity to the information set and corner solutions may be directly related to ill-conditioning (near non invertibility) of the variance-covariance matrix of the candidate assets excess returns.

Hence, as the implementation of standard mean-variance based selection

[11] These figures are computed by the ratio of the proportion of total variance explained by the first largest eigenvalue for the actual and filtered data, and its complement to one.

strategies for the computation of the optimal weights would not work properly in the presence of strongly collinear asset returns, the implementation of optimal portfolio diversification in the euro area, should at least make use of shrinkage-based corrections, to lessen the ill-conditioning of the variance-covariance matrix (see for instance Ledoit and Wolf, 2004, for a recent contribution). The two-step procedure proposed in the paper can be seen as an effective (and superior) alternative to variance-covariance matrix shrinkage.

Two different exercises have been carried out. The first exercise concerns the selection of a mean-variance optimal portfolio based on euro area stock returns only (EA-7, EA-10, EA-11). The second exercise extends the set of portfolio candidates to all the European stock return indexes (EA-11+E-5) and the US, Japan and the Pacific Basin ex Japan, for a total of ninenteen markets (G-19). Concerning the determination of the number of auxiliary portfolios (principal components), the cut-off level for the proportion of explained total variance has been set to 95%. As shown in Table 2, the number of principal components necessary to account for the selected cut-off level of total variance is in the range 6 to 12. Moreover, on average, the cumulated proportion of variance explained for each series is close to 95%, pointing to a satisfactory reconstruction of the return process in all the cases.

3.2 Portfolio diversification

In Figure 2 the sequence of optimal weights estimated by means of the two-step strategy for the European portfolios are plotted[12], while in Table 3, Panels A and B the mean estimated weights are compared with those obtained by applying the standard one-step Britten-Jones (1999) approach, the one-step realized portfolio approach, and a modified version of the Britten-Jones (1999) approach, implementing the shrinkage estimator of Ledoit and Wolf (2004). In all cases the no short-sale constraint is imposed ex-post, by setting to zero the negative weights and renormalizing the positive weights.

As shown in Table 3 and Figure 2, it is possible to note that the two-step realized portfolio approach does not suffer from the corner solution problem, as, on average, the selected portfolios are fairly balanced in all cases, ensuring an effective diversification across markets. As shown in Figure 2, by inspecting the dynamic path of the optimal raw weights over time, it is possible to note the exclusion from the portfolio of some of the assets at a given point in time, but in none of the cases the proportion of excluded assets is large. For instance, for the EA-10 and EA-11 samples in only 4% of the cases the

[12] For reasons of space, in Figure 2 only the weights for the EA-10 case are plotted. Additional results are available upon request from the author.

proportion of excluded assets is larger than 50%, while for the EA-7 sample the figure is 14%.

Moreover, when the smoothed path is assessed, none of the assets is excluded from the optimal portfolio. In addition, while the estimated sequence of optimal weights shows some variability over time, once the smoothed (noise filtered) weights are employed, i.e. observational noise is accounted for, the estimated paths only point to slowly evolving dynamics, describing the trend evolution in the optimal portfolio selection strategy. Coherent with the different information sets considered, the estimated paths for the optimal weights tend to differ across sets of assets, particularly when the EA-7 set is contrasted with the EA-10 and EA-11 sets. Enlarging the information set leads in general to more balanced portfolios, avoiding the concentration of wealth in just one or few assets. For instance, the moderation in the quota of wealth which should be invested in the Italian market is sizeable (in the range 50% to 60%) when the findings for the EA-7 set is contrasted with the findings for the EA-10 or EA-11 sets.

The smoothed allocation paths do reveal interesting information, suggesting that current portfolios should slightly favour Finland and Greece over the other EA-11 markets (25% of wealth in total). Similarly, Finland should be favoured when the set of candidate assets is augmented to account also for non-euro area European countries and other leading markets, i.e. the US, Japan, and the Pacific Basin ex-Japan, as 10% of wealth should be allocated to the latter market. Slightly larger quotas than average should also be allocated to Norway, Sweden, Ireland and Greece, for a total of 37% of wealth.

By comparing the results for the 2-step realized regression approach with the 1-step realized regression approach, it is possible to note that also in this latter case no evidence of corner solutions can be found. Yet, by comparing the standard deviations of the cross sectional distribution of the time average allocations it can be concluded that a less balanced allocation is achieved in the 1-step approach than in the 2-step approach (the mean standard deviation is 0.028 for the 1-step approach and 0.022 for the 2-step approach, with a 30% increase in dispersion), due to the collinearity among asset returns. Moreover, the sequence of estimated weights over time also shows much more variability relatively to the 2-step case, with the mean of the cross-sectional distribution of standard deviations being 0.140 and 0.071, respectively, i.e. 100% larger for the 1-step case than for the 2-step case.

Differently, for all the sets of assets investigated, the standard Britten-Jones approach leads to the exclusion of about 50% of the assets from the optimal portfolios, as well as to much less balanced allocations and to much more estimation uncertainty relatively to the two-step realized portfolio approach (even without accounting for observational noise). For instance, as far as the global (G-19) portfolio is concerned, about 62% of wealth would be invested in three stock index, i.e. Spain (26%), Pacific Basin ex Japan (19%) and Belgium (19%). An additional 13% would be invested in the US index, and 7% and 10% in the indexes for the Netherlands and Norway, respectively, for a total of 92% of wealth. On the other hand, the two-step realized portfolio approach, on average, would only allocate about 14% of wealth in the former three markets and an additional 13% in the latter three markets, for a total of 27% of wealth. Similarly, for the euro area-11 (EA-11) portfolio, the standard Britten-Jones (1999) approach would lead to the concentration of about 87% of wealth in three indexes, i.e. Austria, Belgium and Spain, while the two-step realized portfolio, on average would only allocate about 24% of wealth in the latter three markets. On the other hand, Figures for the EA-10 sample are 82% (Austria, France, Spain, Belgium) and 38%, for the Britten-Jones and (on average) the two-step realized portfolio approach, respectively. For the EA-7 sample figures are 100% (Italy, France, Spain and the Netherlands) and 58%, respectively. Hence, in all the cases the Britten-Jones approach leads to the selection of much less balanced portfolios.

Applying the shrinkage estimator of Ledoit and Wolf (2004) in the framework of the standard Britten-Jones approach does not lead to dramatic improvements. Since the findings may be sensitive to the choice of the shrinkage target, two targets have been employed. i.e. the constant correlation matrix and the diagonal matrix. In all the cases the estimated shrinkage intensity is fairly low (in the range 0.03-0.15), albeit increasing with the degree of multicollinearity affecting the sample variance-covariance matrix. As is shown in Table 2, Panel B, the improvement over the standard Britten-Jones approach for the G-19 sample is not dramatic, as the estimates still point to fairly unbalanced portfolio weights and the exclusion of over 50% of the assets from the optimal portfolios, as well as the allocation of over 60% of wealth in only three assets. Moreover, the selection of the assets is not robust to the shrinkage target employed. In fact, while for the standard Britten-Jones approach Pacific Basin ex-Japan, Spain and Belgium are the selected key assets, when the constant correlation matrix is used as target, Ireland, Switzerland and Denmark get the largest weight. Finally, when the diagonal matrix is employed Pacific Basin ex Japan, Austria and Spain are the selected countries. Similar instability in the selected portfolio components can be found for the EA-11 case, as the only asset always selected is the Spanish market. On the other hand, results for the EA-7 and EA-10 samples are much more robust, since both the excluded assets and the weights received by the included assets are fairly similar across the three approaches. The results are then strongly affected by the degree of multicollinearity characterizing the samples, which is stronger for the G-19 and EA-11 samples than for the EA-10 and EA-7 samples, also due to the fact that a much shorter sample is employed for the construction of the variance-covariance matrix in the former cases (23 observations). The ill-conditioning of the variance-covariance matrix then implies a stronger estimated shrinkage intensity, i.e. close to 0.15, and therefore a stronger influence of the target matrix on the estimated weights. Shrinking the variance-covariance matrix however does not seem to yield a real improvement relatively to the standard Britten-Jones approach, i.e. a more balanced portfolio selection than in the standard case.

Overall the above findings suggest that geographic diversification across euro area or European stock markets is still feasible and desirable, albeit the strong comovement across equity markets should be properly accounted for. The proposed approach, differently from standard static approaches, does account for the latter feature, yielding balanced and stable allocations over time.

3.2.1 Explaining time-varying weights

An important advantage of the proposed approach over other conditional approaches as Brandt and Santa-Clara (2003) and Brandt et al. (2004) is the straightforward implementation, requiring only the excess returns on the candidate assets, as estimation is not conditional to macroeconomic or financial information. However, macroeconomic and financial data can be employed ex-post for forecasting the optimal weights out of sample, yielding a dynamic dimension to the realized portfolio approach.

Forecasting can be based on dynamic econometrics models, i.e. reduced form equations, which can be handled either in the multivariate or in the univariate framework. Since the portfolio weights sum to one, multivariate analysis requires dropping one weight equation from the system, in order to avoid singularity in the error variance covariance matrix. Then, estimation by Maximum Likelihood ensures invariance of the results to the weight equations dropped from the system. Yet, this latter approach is unlikely to be feasible in practical applications where the number of equations is large. Alternatively, each of the weight equations can be estimated separately and, after estimation, the fitted or forecasted weights can be normalized in order to ensure compatibility with the adding-up constraint.

Since the macroeconomic/financial information set to be used for forecasting potentially may be very large, drawing on the recent literature on factor vector autoregressive models, in order to avoid overfitting, conditioning can be made relatively to common factors extracted from the whole set or homogeneous subsets of macroeconomic variables (see, for instance, Stock and Watson, 2005 and references therein).

In the application provided, dynamic weight equations for the G-18 sample (EA-10 + E5+ US +JP + PB) have been estimated and forecasted.[13] Concerning the information set employed, year on year real GDP growth rates and ten-year nominal Government bonds rates for seventeen out of eighteen countries have been employed (macroeconomic data for Pacific Basin ex Japan countries has been neglected). Principal components analysis has been employed to extract a reduced number of orthogonal factors from the two sets of macroeconomic variables. Concerning the interest rates series, only the first principal component has been extracted, since the latter accounts for about 95% of total variance. Differently, for GDP growth rates the first three principal components, jointly accounting for 75% of total variance, i.e. 55%, 12% and 8%, respectively, have been employed. Lagged values only for the four factors have then been employed as regressors in the weight reduced form equations. Moreover, in order to assess the information content of the macroeconomic factors for the portfolio weights, no lagged values for the smoothed weights have been included in the specification.

In Figure 3 the actual and predicted values have been plotted for the eighteen optimal smoothed weights. As is shown in the plots, macroeconomic information can be usefully employed to describe the sequence of optimal weights selected by the realized portfolio approach. In all of the cases the actual and fitted values are fairly close, with coefficients of determination falling in the range 0.85 to 0.98 (Table 3). In none of the cases overfitting is a problem, as in all of the cases the final econometric models is very pasimonious, containing on average 7 lagged values among the four sets of orthogonal factors.[14] Additional evidence is provided by standard Granger causality tests, computed conditional to the inclusion of lagged information on the optimal actual weights equation by equation. As is shown in Table 4 and Figure 3, the coefficients of determination computed on the actual weights, rather than on the smoothed weights, still point to a sizeable proportion of explained variance, albeit in general observational noise accounts for a larger proportion of total variability. In all of the cases neglecting macroeconomic information leads to a loss of information as the BIC criterion for the pure time series models is larger than for the augmented models. Moreover, not only macroeconomic information is useful to account for the in sample variability of the actual optimal weights, but also to forecast them out of sample. In fact, the 1-step forecast tests reported in Table 4 point to forecasting power for

[13] Data for Greece have been neglected in order to benefit from a larger temporal dimension, otherwise not available.

[14] For reasons of space detailed results are not reported. They are however available from the author upon request.

the augmented models in all of the cases. The evaluation of the forecasting ability of the model, performed by means of a Bonferroni bounds test jointly considering the 72 1-step forecast available, coherently would never allow to reject the null of accurate forecasting at any standard significance level.

4 Conclusions

In the paper a new approach to efficient portfolio selection is proposed. The approach generalizes the regression based portfolio selection approach of Britten-Jones (1999) in the framework of realized regression theory, allowing to estimate the full sequence of optimal time-varying portfolios weights. Economic and financial integration in the current framework can be expected to lead to a sequence of realized variance-covariance matrices increasingly affected by a reduced rank state, as only a small number of factors is expected to drive the excess returns over the integration process. Hence, the proposed approach should not only allow to assess whether market integration has taken place, but also to monitor the way it is progressing over time. As a linkage between time-varying portfolio weights and macroeconomic factors can be established ex-post, not only it is possible to justify the optimality of portfolio rebalancing in the light of changing macroeconomic conditions, but also to accurately forecast the future optimal wealth allocation, yielding a dynamic dimension to the proposed approach. Relatively to other conditional approaches, the proposed method has the advantage of simplicity in implementation, as well as of minimal information requirement in estimation.

The proposed method, particularly in its two-step version, has been found to provide a clear-cut improvement over the standard Britten-Jones (1999) approach, even when a shrinkage estimator of the variance-covariance matrix, as the one proposed by Ledoit and Wolf (2004), is employed. The findings suggest that the proposed method neither suffers from instability nor from corner solutions, as well as accurate out of sample optimal weights forecasting can be achieved.

The results also point out that albeit the degree of comovement across euro area stock markets is strong, full integration at the regional or global level would not seem to have occurred yet. Hence, diversification among European or euro area stock markets is still be feasible and desirable, albeit appropriate tools for implementation are required.

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Table 1: Principal components analysis
Proportion of explained variance for each return series (average)

Actual data

EA+US	1980:1 − 2007:1			1988:1 − 2007:1			2001:3 − 2007:1	E-5 + US	1980:1 − 2007:1		
	EA-7			EA-10			EA-11		E-5		
	PC	C	MU	PC	C	MU	MU		PC	C	MU
TOT	.45	.62	.74	.43	.55	.67	.65	TOT	.48	.63	.65
US	.24	.33	.49	.22	.32	.39	.47	US	.25	.34	.47
IT	.42	.59	.75	.33	.51	.65	.74	UK	.31	.57	.72
GE	.40	.64	.85	.46	.55	.74	.85	CH	.36	.57	.64
AT	.27	.39	.33	.40	.30	.24	.36	NW	.44	.47	.43
FR	.36	.68	.87	.40	.57	.79	.87	SW	.31	.62	.70
SP	.26	.63	.76	.34	.55	.66	.76	DK	.21	.43	.42
BE	.25	.49	.64	.30	.49	.51	.69				
NE	.39	.64	.79	.37	.60	.71	.79				
FI				.34	.55	.72	.68				
PT				.20	.28	.35	.34				
IR				.40	.34	.30	.44				
GR							.34				

Filtered data

EA	1980:1 − 2007:1			1988:1 − 2007:1			2001:3 − 2007:1	E-5	1980:1 − 2007:1		
	EA-7			EA-10			EA-11		E-5		
	PC	C	MU	PC	C	MU	C		PC	C	MU
TOT	.45	.58	.67	.41	.49	.60	.53	TOT	.49	.59	.60
IT	.47	.64	.68	.31	.47	.52	.58	UK	.19	.40	.48
GE	.33	.53	.75	.41	.50	.57	.68	CH	.30	.51	.43
AT	.28	.33	.29	.41	.29	.16	.28	NW	.63	.60	.50
FR	.30	.57	.81	.31	.46	.66	.74	SW	.45	.63	.71
SP	.28	.54	.67	.27	.47	.52	.59	DK	.26	.42	.39
BE	.23	.37	.56	.23	.37	.39	.54				
NE	.29	.51	.70	.27	.50	.56	.65				
FI				.32	.48	.68	.60				
PT				.22	.23	.27	.26				
IR				.33	.28	.22	.33				
GR							.30				

In the Table the time average of the proportion of total variance (TOT) and of the variance of each series (λ_i) explained by the first largest eigenvalue of the sequence of realized variance-covariance matrices for the actual and filtered data is reported for various samples and time spans. Three time spans, i.e. the pre-convergence period (PC), the convergence period (C) and the monetary union period (MU), and two datasets for the euro area (EA-7, EA-10, EA-11) and Europe (E-5), including the United States, have been investigated. The countries included are Italy (IT), Germany (GE), Austria (AT), France (FR), Spain (SP), Belgium (BE), the Netherlands (NE), Finland (FI), Portugal (PT), Ireland (IR), Greece (GR), the United Kingdom (UK), Switzerland (CH), Norway (NW), Sweden (SW), Denmark (DK), and the United States (US).

Table 2: Principal components analysis, auxiliary portfolios estimation
Cumulated proportion of explained variance for each return series

	G-19	EA-11	EA-10	EA-7
TOT	0.96	0.96	0.96	0.96
#	12	6	8	6
US	0.96			
JP	0.99			
PB	0.64			
IT	0.89	0.88	1.00	1.00
GE	0.93	0.96	0.93	0.96
AT	0.99	0.93	0.99	0.99
FR	0.95	0.95	0.85	0.99
SP	0.92	0.92	0.99	1.00
BE	0.92	0.89	0.96	0.95
NE	0.93	0.93	0.84	0.80
FI	1.00	1.00	1.00	
PT	0.98	0.92	1.00	
IR	1.00	0.98	1.00	
GR	1.00	0.97		
UK	0.87			
CH	0.86			
NW	0.99			
SW	0.99			
DK	1.00			

In the Table the cumulated proportion of the variance of each series ($\sum_{i=1}^{\#} \lambda_i$), explained by the first # largest eigenvalues of the estimated variance-covariance matrix of the actual return series, necessary to account for 95% of total variance (TOT), over the full time horizon available, is reported for four sets of assets, i.e. the EA-7 (Italy (IT), Germany (GE), Austria (AT), France (FR), Spain (SP), Belgium (BE), the Netherlands (NE)), the EA-10 (EA-7, Finland (FI), Portugal (PT), Ireland (IR)), the EA-11 (EA-10, Greece (GR)), the G-19 (EA-11, the United Kingdom (UK), Switzerland (CH), Norway (NW), Sweden (SW), Denmark (DK), the United States (US), Japan (JP), Pacific Basin ex Japan (PB)).

Table 3, Panel A: Mean-variance optimal portfolio weights

Realized regression: 2-step approach

	$E[\hat{\beta}^{+}]$				$SD[\hat{\beta}^{+}]$			
	G-19	EA-11	EA-10	EA-7	G-19	EA-11	EA-10	EA-7
US	.038				.033			
JP	.046				.050			
PB	.033				.023			
IT	.034	.075	.100	.191	.025	.029	.079	.186
GE	.059	.097	.107	.137	.033	.039	.038	.079
AT	.057	.071	.092	.160	.048	.056	.094	.152
FR	.046	.085	.095	.113	.028	.035	.043	.073
SP	.046	.091	.110	.164	.029	.027	.085	.150
BE	.043	.082	.079	.123	.030	.031	.047	.105
NE	.046	.090	.088	.111	.032	.039	.046	.077
FI	.105	.130	.157		.097	.132	.131	
PT	.054	.087	.101		.048	.071	.092	
IR	.067	.072	.069		.057	.071	.063	
GR	.061	.120			.050	.089		
UK	.034				.022			
CH	.038				.025			
NW	.064				.046			
SW	.073				.046			
DK	.054				.045			

Realized regression: 1-step approach

	$E[\hat{\beta}^{+}]$				$SD[\hat{\beta}^{+}]$			
	G-19	EA-11	EA-10	EA-7	G-19	EA-11	EA-10	EA-7
US	.040				.053			
JP	.019				.045			
PB	.064				.075			
IT	.061	.103	.094	.123	.092	.144	.145	.191
GE	.085	.111	.115	.147	.155	.209	.176	.227
AT	.072	.103	.092	.098	.092	.105	.134	.153
FR	.096	.161	.147	.189	.114	.191	.200	.239
SP	.059	.104	.115	.146	.069	.122	.146	.191
BE	.057	.116	.132	.163	.104	.181	.178	.234
NE	.041	.067	.090	.134	.065	.120	.153	.196
FI	.023	.044	.068		.042	.074	.106	
PT	.053	.082	.080		.074	.110	.116	
IR	.041	.069	.067		.064	.095	.113	
GR	.024	.041			.047	.065		
UK	.079				.113			
CH	.055				.089			
NW	.046				.062			
SW	.041				.053			
DK	.044				.068			

Panel A reports the time average estimated optimal weights ($E[\hat{\beta}^{+}]$), with standard deviation ($SD[\hat{\beta}^{+}]$), obtained by means of the two-step and 1-step realized regression approach. Four sets of assets have been considered, i.e. the EA-7 (Italy (IT), Germany (GE), Austria (AT), France (FR), Spain (SP), Belgium (BE), the Netherlands (NE)), the EA-10 (EA-7, Finland (FI), Portugal (PT), Ireland (IR)), the EA-11 (EA-10, Greece (GR)), the G-19 (EA-11, the United Kingdom (UK), Switzerland (CH), Norway (NW), Sweden (SW), Denmark (DK), the United States (US), Japan (JP), Pacific Basin ex Japan (PB)).

Table 3, Panel B: Mean-variance optimal portfolio weights

Britten-Jones

	$\hat{\beta}^{+}$				$\hat{\sigma}_{\hat{\beta}^{+}}$			
	G-19	EA-11	EA-10	EA-7	G-19	EA-11	EA-10	EA-7
US	.125				.196			
JP	.021				.124			
PB	.187				.101			
IT	.000	.000	.000	.275	.000	.000	.000	.398
GE	.000	.109	.000	.000	.000	.257	.000	.000
AT	.054	.339	.187	.004	.070	.093	.237	.393
FR	.002	.000	.188	.178	.254	.000	.462	.556
SP	.258	.286	.183	.212	.158	.190	.351	.425
BE	.165	.236	.091	.000	.156	.202	.333	.000
NE	.074	.000	.260	.331	.155	.000	.492	.749
FI	.000	.029	.091		.000	.064	.162	
PT	.000	.000	.000		.000	.000	.000	
IR	.000	.000	.000		.000	.000	.000	
GR	.000	.000			.000	.000		
UK	.000				.000			
CH	.000				.000			
NW	.099				.117			
SW	.012				.140			
DK	.004				.125			

Shrinked Britten-Jones

	$\hat{\beta}^{+}$ (constant correlation)				$\hat{\beta}^{+}$ (diagonal)			
	G-19	EA-11	EA-10	EA-7	G-19	EA-11	EA-10	EA-7
US	.000				.000			
JP	.000				.000			
PB	.065				.240			
IT	.000	.163	.000	.246	.000	.000	.000	.310
GE	.012	.000	.000	.000	.000	.000	.000	.000
AT	.000	.000	.185	.019	.353	.662	.187	.000
FR	.012	.000	.185	.171	.000	.000	.194	.182
SP	.000	.544	.182	.182	.110	.181	.178	.250
BE	.000	.071	.068	.000	.042	.091	.112	.000
NE	.082	.000	.296	.382	.000	.000	.233	.259
FI	.000	.000	.085		.000	.000	.096	
PT	.000	.092	.000		.000	.000	.000	
IR	.188	.000	.000		.000	.000	.000	
GR	.000	.130			.036	.067		
UK	.053				.000			
CH	.134				.027			
NW	.000				.072			
SW	.075				.037			
DK	.380				.084			

Panel B reports the estimated optimal weights ($\hat{\beta}^{+}$) for the standard and shrinked Britten-Jones (1999) approach. The estimated standard errors for the weights ($\hat{\sigma}_{\hat{\beta}^{+}}$) are also reported. Four sets of assets have been considered, i.e. the EA-7 (Italy (IT), Germany (GE), Austria (AT), France (FR), Spain (SP), Belgium (BE), the Netherlands (NE)), the EA-10 (EA-7, Finland (FI), Portugal (PT), Ireland (IR)), the EA-11 (EA-10, Greece (GR)), the G-19 (EA-11, the United Kingdom (UK), Switzerland (CH), Norway (NW), Sweden (SW), Denmark (DK), the United States (US), Japan (JP), Pacific Basin ex Japan (PB)).

Table 4: G-18 portfolio optimal weights, estimation and forecasting analysis

	R_s^2	R_a^2	GC	BIC	BIC_m	FT
IT	.949	.273	.011	-2.606	-2.503	.327
GE	.949	.369	.010	-3.877	-3.853	.772
AT	.850	.541	.003	-2.667	-2.540	.279
FR	.979	.468	.006	-4.400	-4.368	.350
SP	.884	.444	.007	-3.236	-3.211	.335
BE	.984	.507	.001	-3.921	-3.715	.643
NE	.984	.425	.001	-4.578	-4.342	.804
FI	.936	.344	.004	-1.786	-1.703	.106
PT	.916	.600	.000	-2.836	-2.604	.385
IR	.929	.292	.007	-3.026	-2.923	.824
UK	.864	.351	.006	-4.869	-4.753	.732
CH	.946	.452	.002	-4.764	-4.656	.677
NW	.962	.545	.000	-2.767	-2.377	.979
SW	.870	.431	.000	-3.296	-3.034	.351
DK	.946	.475	.000	-4.060	-3.807	.547
US	.947	.723	.000	-4.421	-3.799	.128
JP	.845	.425	.001	-3.387	-3.138	.313
PB	.984	.392	.012	-3.067	-3.056	.523

In the Table the coefficient of determination of the regressions of the smoothed (R_s^2) and actual (R_a^2) optimal weights on their lags and on the lags of some key macroeoconomic variables are reported. The p-value of the test for the null of no Granger causality from the macroeconomic variables to the optimal weights is also reported (GC), as well as the BIC information criterion for the final econometric model (BIC) and the final econometric model without macroeconomic variables (BIC$_m$). Finally, the p-value of the test for the equality between the sequence of four 1-step out of sample predictions and their actual values is reported (FT). The markets considered are Italy (IT), Germany (GE), Austria (AT), France (FR), Spain (SP), Belgium (BE), the Netherlands (NE)), Finland (FI), Portugal (PT), Ireland (IR)), the United Kingdom (UK), Switzerland (CH), Norway (NW), Sweden (SW), Denmark (DK), the United States (US), Japan (JP), and Pacific Basin ex Japan (PB).

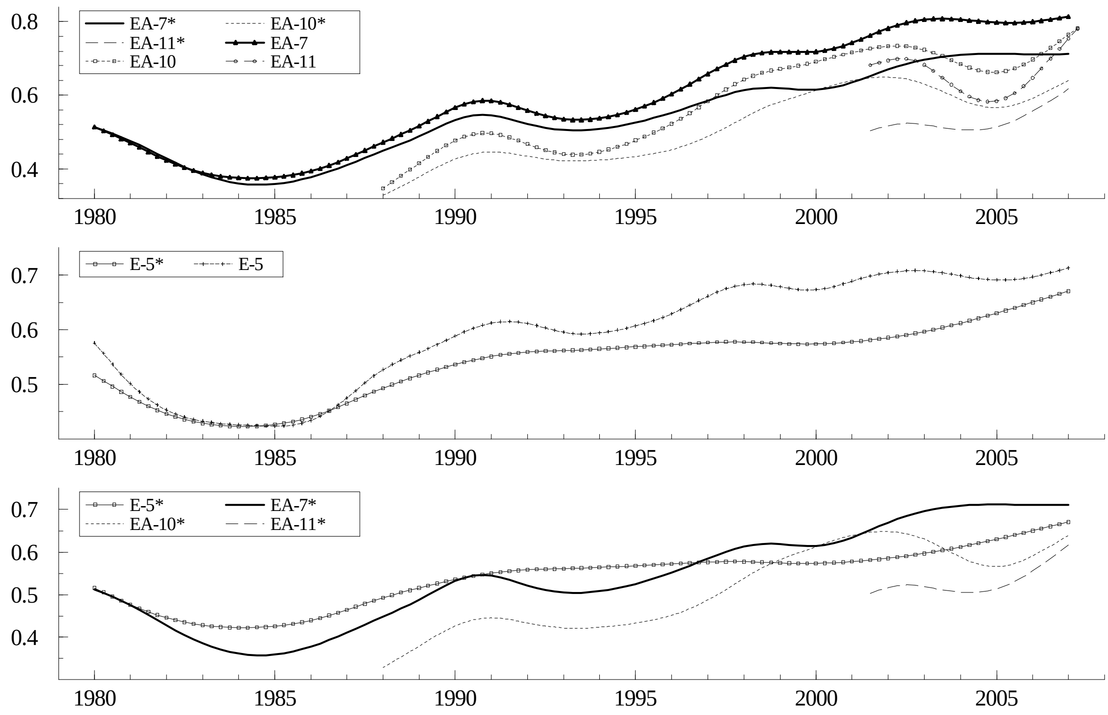

Figure 1: Stock market integration process: explained proportion of total variance by the largest eigenvalue of the realized variance-covariance matrix for various samples of actual and US effects filtered (*) returns.

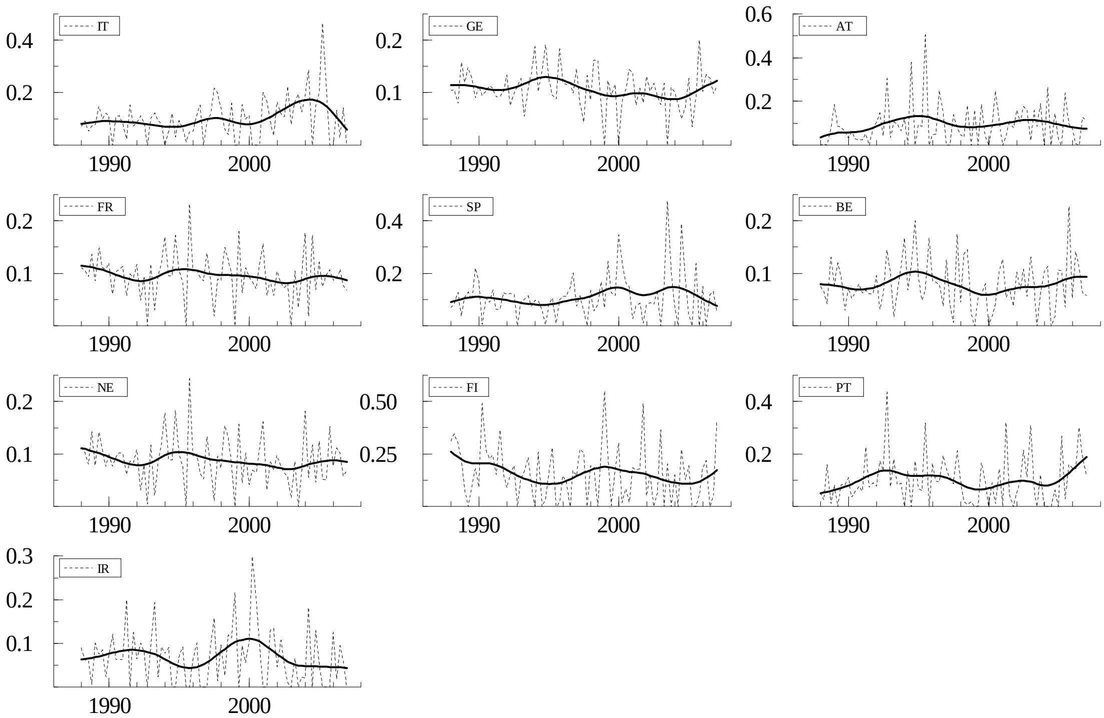

Figure 2: EA-10, Mean-variance optimal actual and smoothed portfolio weights.

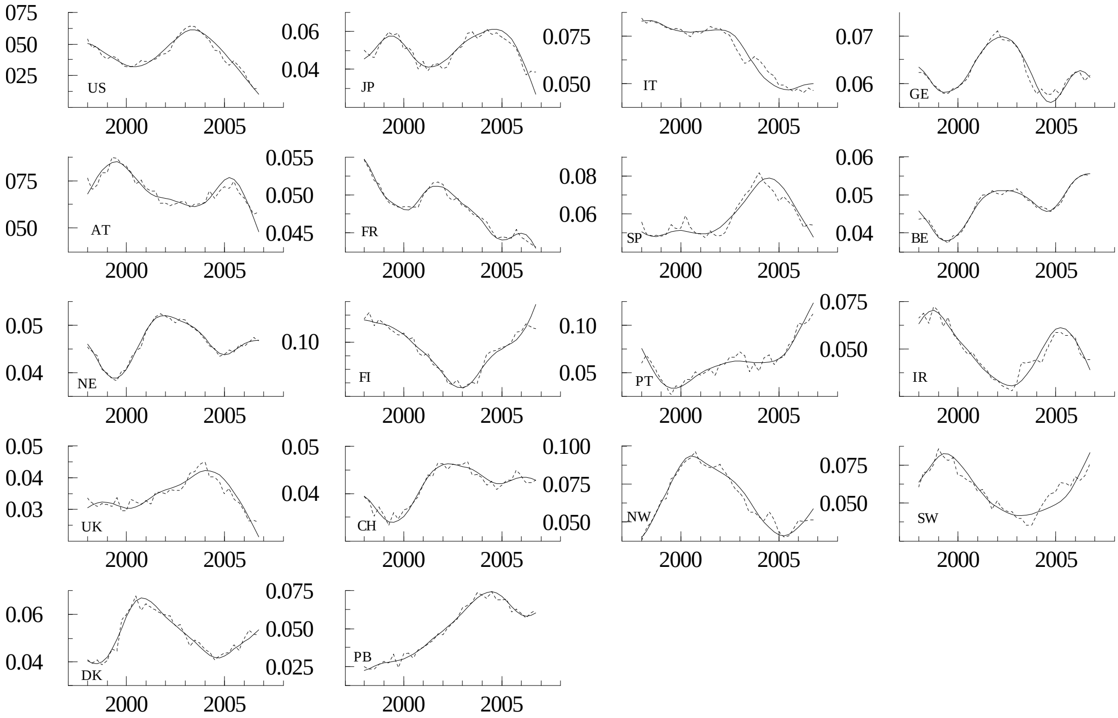

Figure 3: Smoothed (solid line) and predicted (dottet line) optimal weights for the G-18 portfolio.

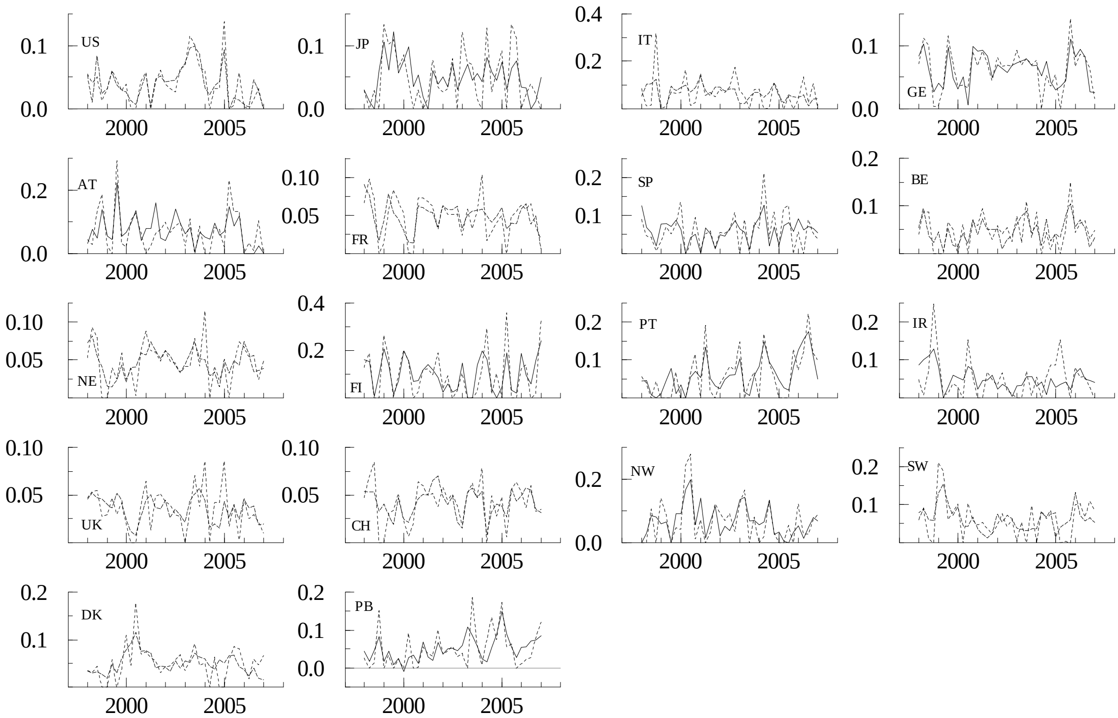

Figure 4: Actual (dotted line) and predicted (solid line) optimal weights for the G-18 portfolio.