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Analyzing labor supply behavior with latent job opportunity sets and institutional choice constraints

by

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Abstract

In this paper we discuss a general framework for analyzing labor supply behavior in the presence of complicated budget- and quantity constraints of which some may be unobservable. The point of departure is that an individual's labor supply decision can be considered as a choice from a set of discrete alternatives (jobs). These jobs are characterized by attributes such as hours of work, sector specific wages and other sector specific aspects of the jobs. We focus in particular on theoretical justification of functional form assumptions and properties of the random components of the model.

The paper also includes an empirical application based on Norwegian data, in which the labor supply of married women is estimated.

JEL classification: J22,C51.

Key words: Labor supply, non-convex budget sets, non-pecuniary job-attributes, sector-specific wages.

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1. Introduction

The purpose of our study is to develop a particular framework for modeling labor supply behavior in the presence of complicated budget sets, qualitative job attributes and restrictions on hours of work, and to apply this framework to analyze workers' observed choice of sector and hours in the labor market.

In the 1970s and 1980s labor supply studies applied the traditional textbook model for labor supply, extended to allow for convex and smooth tax functions (cf. contributions such as Rosen (1976), Wales and Woodland (1979), Nakamura and Nakamura (1981), Kohlase (1986) and Ransom (1987)). However, in most western countries the tax system and social benefit rules imply a non-convex budget set.[1] Fixed costs of working and tax deductions if working contribute to these non-convexities. Attempts to take the non-convexity properties of the tax structure into account include Burtless and Hausman (1978), Blomquist (1983), (1992), Arrufat and Zabalza (1986), Hausman (1980), (1981), (1985), and Hausman and Ruud (1984). In principle it is possible to apply the "Hausman approach" to account for nonlinear and non-convex budget sets. That approach, however, is rather cumbersome when there are more than one adult in the household or when complicated social benefit- and tax deduction rules are present. In contrast, the particular approach advocated in our paper, and which we shall describe in a moment, has the advantage that it becomes simple to handle complicated nonlinear tax and transfer systems. This is also the case for many-persons households.

In the studies mentioned above the mathematical structure of the modeling framework rests upon the assumption that the fundamental choice variables of the household in this context are "consumption" (composite) and "leisure" (hours of work), which can be chosen freely subject to the economic budget constraint. Yet, it seems apparent that hours of work and income are only two out of several job-related attributes which are important for individual behavior in the labor market. "Type of work", or "non-pecuniary job attributes", do often matter a great deal and may even be more important than hours of work. An extreme example of the latter phenomenon is found among scientists, artists and government bureaucrats for whom specific work activities represents major means for self-realization. Another characteristic of the labor market is that hours of work is fixed for many type of jobs. Thus, if an individual wishes to change his workload he would in this case have to change job.[2] This assumption is consistent with the findings of Altonji and Paxson (1988).

In view of the arguments above it may be more appropriate to consider labor supply behavior as the outcome of households choosing from a finite set of job "packages", each of which are characterized by an offered wage rate, offered hours of work, and non-pecuniary attributes. The individual specific choice sets of job opportunities may be thought of as being determined by employers or in negotiation between employers and unions. The qualitative job attributes are often unobservable, or at most only partly observable to the analyst. This is the point of departure taken in this paper. Specifically, the choice environment is assumed to consist of a latent, individual specific set of jobs. A job is characterized by a combination of fixed hours of work, wage rate and non-pecuniary job-attributes (such as type of work and working conditions). The notion of individual specific choice sets is important for our modeling of choice constraints. In our setup there are two sources of unobserved heterogeneity, namely unobserved heterogeneity in tastes and in opportunities.

In Dagsvik (1994) a general framework for modeling this type of settings was developed. Simplified versions of this framework have been applied by Anderson et al (1988) and Aaberge et al. (1990), (1995) and (1999) to analyze labor supply behavior. In contrast, this paper is more theoretical in that it focuses on a detailed discussion and interpretation of underlying assumptions of the framework in the context of modeling labor supply behavior, and relates the present approach to previous ones. In particular, we discuss how functional form and the probability law of unobservables can be justified from behavioral arguments.

Previous attempts to take (quantity) constraints on the choice set into account have been restricted solely to one job attribute, namely hours of work. Contributions by Ilmakunnas and Pudney (1990), Kapteyn et al. (1990), Dickens and Lundberg (1993), and van Soest (1994) emphasize the inability of standard empirical labor supply models to account for observed peaks in the hours of work distribution at part-time and full-time hours. They have discussed approaches to take account of this type of constraints in the econometric modeling of labor supply. These approaches are, however, different from the one developed in this paper.

Ilmakunnas and Pudney (1990) formulate a labor supply model which is a mixture of logit-type models across unobservable choice sets, where the choice sets consist of the alternatives “part time”, “full time” and “non-participation”.

Van Soest (1994) on the contrary, assumes that the choice set consists of a finite (given) number of hours of work options and he specifies a model which is a mixture of logit-type models across unobservable taste-shifters. He interprets the observed concentrations of hours of work as resulting from agents having strong preferences for “full-time” and/or “part-time” hours of work.

Dickens and Lundberg (1993) formulate a model which, similarly to Ilmakunnas and Pudney (1990), is a mixture of discrete choice models across unobservable and finite choice sets. Compared to Ilmakunnas and Pudney the assumptions they make about the choice sets are more general. Specifically, they assume that the number of job offers is generated by a binomial distribution with a fixed maximum (10). Moreover, each job has fixed and unobservable hours of work generated by a discrete distribution with a fixed maximum. Thus, while Ilmakunnas and Pudney, as well as Dickens and Lundberg, assume that the agents face choice constraints which rationalize the observed concentration of hours of work in the data, van Soest assumes that all agents face the same finite choice set. Van Soest thus assumes that the concentration of hours of work at “full-time” and “part-time” follow from preferences. In all of these recent labor supply contributions the individuals are assumed to have the same wage across jobs. Thus, in previous labor supply studies it is assumed that an individual has a fixed wage rate, and the possibility of job-specific wages are ignored. Recent labor market theories, like the theories of efficient wages and trade unions, suggest that wages may differ across jobs. And more important, wage dispersion among observationally identical workers seems to be a well established empirical fact, see for example Krueger and Summers (1988) and Edin and Zetterberg (1992). In the labor supply literature there are, however, approaches that allow offered wage rates to vary systematically with hours worked (see Moffitt, 1984).

A serious problem with most structural econometric models is the lack of theoretical support for the choice of functional forms and distributional assumptions of the unobservables. In this paper the distributional properties of the agents’ preferences, in the presence of latent choice sets of jobs, follow from an assumption; “Independence from Irrelevant Attributes”, proposed by Dagsvik (1994). This assumption is an extension of the familiar IIA assumption proposed by Luce (1959a). Under this assumption and a particular Poisson process representation of the distribution of the latent choice sets of jobs, the implied distribution of realized hours and wage rates turns out to be analogous to the continuous logit model introduced by Ben-Akiva et al. (1985). IIA is clearly a theoretical axiom which captures the notion of idealized probabilistic rationality in the following sense: Provided the alternatives are “properly” defined, only alternatives in the (current) choice set are relevant for the choice outcome. It is true that IIA is unrealistic in some situations, which it shares with many theoretical assumptions in economics. For example, it is well known that IIA may not hold if unobserved heterogeneity is not accounted for. In our model we have introduced a random effect, which means that IIA only is assumed to hold conditional on the random effect variable.

A similarly important challenge is to provide a justification for the choice of *functional form* of the deterministic components in the probability model of realized hours and wage rates. In this respect the attitude among economists seems to be a general resignation: It is believed to be a hopeless task to achieve useful results on a purely theoretical basis, that is from first principles. As a consequence, the functional form problem is “solved” by selecting a convenient mathematical structure and applying data and statistical methods to choose between competing candidates. Unfortunately, without theoretical principles almost any form is a priori possible and the correct one is difficult to determine because of the problem of unobserved variables and measurement errors. What seems to be little known among economists is that there is a tradition within the field of psychophysics to justify functional forms based on invariance principles. These principles are similar to certain invariance principles applied in physics, such as the laws of mechanics, which are required to be invariant under uniform translation and rotation of the coordinate system. In this paper we discuss how results in Falmagne (1985) apply in our context and lead to theoretically justified functional forms.

The empirical part of the paper deals with labor supply among married females in Norway in 1994, who can choose between working in the public and the private sector of the economy. Other authors that analyze agents’ choice of sector are Magnac (1991) and Heckman and Sedlacek (1990). Magnac also allows for rationing in the sense that workers face costs of entry into a sector. However, neither Heckman and Sedlacek nor Magnac consider workers’ choice of hours. The labor supply model developed here can easily be extended to deal with joint decisions of wife and husband and for the sake of completeness this extension is shown in Appendix A.

This paper is organized as follows. In the next section we present the model which includes a characterization of the stochastic properties of the unobserved variables and the functional form of the deterministic part of the utility function of the agents. In Section 3 we demonstrate how some previous labor supply models follow as a special case from our model. In Section 4 we discuss an empirical application.

2. The modeling framework

As alluded to above, the choice environment of a worker is assumed to consist of a set of latent job- and non-market opportunities. Each job is characterized by fixed observed attribute variables that describe the contract (wage rate and hours of work) and unobserved attributes that describe the job-type. We shall first discuss the case where qualitative attributes are latent. Later, we extend the framework to accommodate sector-specific jobs (public sector versus private sector).

2.1. Preferences and choice sets

Let U(C,h,z) be the (ordinal) utility function of the household where C denotes household consumption, z indexes the market and non-market opportunities, or job-types, and h is the realized hours of work of the married female. Let positive indices, $z = 1,2,\ldots,$ refer to market opportunities (jobs) and non-positive ones refer to non-market opportunities. To a market opportunity z, there are associated fixed hours of work, H(z), and wage rate, W(z). The opportunity index z in the utility function accommodates the notion that workers may have preferences over job-types (which includes preferences for working in specific sectors of the economy) in addition to income and hours of work. For a given wage rate, w, the economic budget constraint is represented by

$$C = f(hw, I) \tag{2.1}$$

where I is non-labor income, which includes the given income of the husband, and f(·) is the function that transforms gross income into after-tax household income. The function f(·) will capture all details of the tax and benefit system. The price index for the composite good, C, is set equal to one. Our first assumption concerns the structure of the preferences.

Assumption 1

The utility function has the structure

$$U(C,h,z) = \mathrm{v}(C,h)e(z)$$

where v(·) *is a deterministic function and e(z) is a random taste-shifter.*

The random taste-shifter is assumed to account for the unobservable individual characteristics and non-pecuniary job-type attributes that affect utility. For notational simplicity we shall in the following use the notation

(2.2) $$\psi(h,w) \equiv v\big(f(hw, I), h\big).$$

In addition to (2.1), there are restrictions on the set of feasible market opportunities a specific worker faces because there are job-types for which the worker is not qualified and there may not be jobs available for which he is qualified.

We will next discuss the distribution of the hours and wages associated with market opportunities and the distribution of the associated taste-shifters. The taste-shifters $\{\varepsilon(z)\}$ may vary across opportunities as well as across agents because different agents may value a specific opportunity differently. The non-market opportunities have zero hours of work and zero wage rates. Thus, the agent's opportunity set can be represented by $\wp = \{(H(z), W(z), \varepsilon(z)); z = \ldots, -2, -1, 0, 1, 2, \ldots\}$.

Assumption 2

The triples in $\tilde{A}$ are realizations from a non-homogeneous Poisson process on $[0,\bar{h}]\times[0,\infty)\times(0,\infty)$, where $\bar{h}$ is an upper bound on hours of work. Moreover, different agents face different choice sets, which are realizations from independent copies of the Poisson process.

The rationale for using a Poisson process formulation as a point of departure is that the Poisson framework offers a very convenient and flexible representation of the distribution of independent points in some given space. Recall that the non-homogeneous Poisson process on $[0,\bar{h}]\times[0,\infty)\times(0,\infty)$, is completely analogous to the homogeneous one-dimensional case. The probability law of a Poisson process can be represented by the associated intensity measure, $d\lambda(h,w,\varepsilon)$. This means that the probability that there is a point of the process for which $H(z)\in(h, h+dh)$, $W(z)\in(w, w+dw)$, $\varepsilon(z)\in(\varepsilon, \varepsilon+d\varepsilon)$, equals $d\lambda(h,w,\varepsilon)$. Moreover, in a Poisson process the points of the process are independently distributed and the probability that there is more than one point within $(h, h+dh)\times(w, w+dw)\times(\varepsilon, \varepsilon+d\varepsilon)$ is negligible.

Assumption 3

The intensity measure $d\,l\,(h,w,e)$ *has the structure*

$$(2.3)\qquad d\,l\,(h,w,e) = \begin{cases} q_1\, g(h,w)e^{-2}\, dh\, dw\, de \;\; \text{when}\;\; h>0, w>0, e>0\,, \\ (1-q_1)e^{-2}\, de \;\; \text{when}\;\; h=w=0, e>0\,, \end{cases}$$

where g(h,w) is a probability density and $q_1 \in (0,1)$ *is a constant.*

A justification for the structure (2.3) is given in Dagsvik (1994). There it is demonstrated that this structure is consistent with a particular version of IIA (“independence from irrelevant attributes”). Recall that the underlying intuition of the IIA assumption is, loosely speaking, that the agent's ranking of job opportunities from a subset B (say), within the choice set of feasible jobs with the same level of hours of work and wage rate, does not change if the choice set of feasible jobs is altered. Recall also that the stochastic formulation of IIA means that this property only is claimed to hold *on average*. As mentioned in the introduction, we shall allow for a random effect in the empirical model specification which means that IIA only is assumed to hold conditional on the random effect.

The structure of the intensity measure means that the taste-shifters $\{\varepsilon(z)\}$ are independently distributed of $\{(H(z), W(z))\}$ (cf. Section 2.5 where we return briefly to this issue). The above formalism implies that the choice sets are allowed to vary randomly across observationally identical agents, because two different agents face independent realizations from the Poisson process. The term g(h,w)dhdw yields the probability that a market opportunity with $H(z) \in (h, h+dh)$ and $W(z) \in (w, w+dw)$ shall be feasible (cf. Dagsvik, 1994). The term θ_1 can be interpreted as the fraction of the feasible opportunities that are market opportunities. Hence $1-\theta_1$ is the corresponding fraction of feasible opportunities that are non-market opportunities. The density defined by $\theta_1\, g(h,w)$ when $h>0, w>0,$ and by $1-\theta_1$ when $h=w=0$, will be called the *opportunity density*. Note that although the non-market opportunities look the same to the analyst due to the fact that they have observable attributes (wages and hours) equal to zero, they are perceived as different by the agent since he may have preferences over qualitative unobservable attributes. Recall that the agents themselves are assumed to be perfectly certain about their opportunities, so the opportunity density is just an aggregate representation of unobserved heterogeneity in the set of opportunities from the econometrician’s point of view.

In Dagsvik (1994) it is demonstrated that the set of Poisson points for which the utilities lie above any given positive level is finite (with probability one). Thus the set of "interesting" feasible jobs is (almost surely) finite, and it varies from one agent to another.

2.2. A discrete/continuous choice model with heterogeneous opportunity sets

We are now ready to express the probability distribution of *realized* hours and wages, including the probability of not working. Let Φ(h,w) be the joint cumulative distribution of realized hours and wages that follow from utility maximizing behavior, i.e.,

(2.4) $$\Phi(h,w) \equiv P\left(\max_{\substack{z \\ H(z)\le h, W(z)\le w}} \left(\psi\left(H(z),W(z)\right)\varepsilon(z)\right) = \max_z \left(\psi\left(H(z),W(z)\right)\varepsilon(z)\right)\right).$$

Equation (2.4) defines the probability that the chosen opportunity (i.e. job) has hours of work less than or equal to h and wage rate less than or equal to w.

Theorem 1

Assume that Assumptions 1 to 3 hold. Then the probability density $j(h,w)$ *is given by*

(2.5) $$j(h,w) = \frac{y(h,w)\,g(h,w)q}{y(0,0) + q\int\int_D y(x,y)\,g(x,y)\,dx\,dy}$$

for $h > 0, w > 0$, *and*

(2.6) $$j(0,0) = \frac{y(0,0)}{y(0,0) + q\int\int_D y(x,y)\,g(x,y)\,dx\,dy},$$

for $h = w = 0$, *where* $q = q_1/(1-q_1)$, *and* $D = (0,\bar{h}] \times R_+$.

The proof of Theorem 1 follows from Dagsvik (1994), but to make the paper self-contained we have outlined the proof in Appendix B.

The parameter θ can be interpreted as the ratio of the mean number of feasible market opportunities to the mean number of feasible non-market opportunities. Note that ψ(h,w) and hence φ(h,w) also depend on I, but for notational simplicity this dependence is suppressed in the notation in (2.4)-(2.6).

Although we have assumed that the agent's taste-shifters are (stochastically) independent of the set of offered hours and wage rates, the *distribution function* of the preferences and the opportunity density are allowed to be dependent. In other words, the market forces that regulate the balance between supply and demand, be it a market clearing regime or not, are assumed to operate solely on an aggregate level. The opportunity density may depend on the production technology of the firms as well as of the contract and wage setting policies of the unions and the firms. It is beyond the scope of this paper to discuss how the opportunity density $\theta g(\cdot)$, through market equilibrium processes, depend on the systematic part of the utility function, $\psi(\cdot)$. This means of course that the estimated model only can be applied to simulate behavior conditional on the opportunity density. In Dagsvik (2000), it is suggested how an explicit equilibrium model version can be specified.

In principle θ could take any positive value, but we will argue that it only makes sense that $\theta \in [0,1]$.[3] To realize this, note that when $\psi(h,w)$ is a constant for all $h > 0, w > 0$,?which means that utility is i.i.d. across job-opportunities, then by (2.6) the probability of working is given by; $1 - \varphi(0,0) = \theta/(1+\theta)$. If in this case the agent is *not constrained* in the labor market, the probability of working should be 0.5, which corresponds to $\theta = 1$. Since the probability of working is increasing in θ, then $\theta = 1$ is an upper bound, and consequently this state may be interpreted as a case where all (systematic) constraints in the labor market have been removed.

Thus, we have demonstrated that the formulation above allows for a particular type of quantity constraints which typically is rather difficult to account for by means of the econometric formulations used in previous labor supply studies.

Let $g_1(w|h)$ be the conditional density of offered wages given the level of offered hours, and $g_2(h)$ the unconditional density of offered hours. Assume for a moment that $v(C,h)$ is multiplicatively separable in hours and consumption, i.e., $v(C,h) = v_1(C)\,v_2(h)$, and that fixed cost of working is observed. Then Dagsvik and Strøm (1997) demonstrate that $v_1(C)$ and $g_1(w|h)$ are non-parametrically identified. A more difficult task is to separate $v_2(h)$ from $g_2(h)$. Due to the fact that

$$\psi(h,w;I)g(h,w) = v_1\left(f(hw,I)\right)v_2(h)g_1\left(w \mid h\right)g_2(h) \tag{2.7}$$

we can only identify $v_2(h)g_2(h)$ non-parametrically. Thus, to disentangle $v_2(h)$ from $g_2(h)$ one needs to make additional assumptions. Below we make functional form assumptions that ensures identification.[4] *However, it is important to be aware of the fact that if the purpose is to* *carry out policy simulations for which the distribution of offered hours is kept fixed, it is not necessary to identify $v_2(h)$ and $g_2(h)$, separately.*[5]

2.3. Functional form

Current quantitative economic research often suffers from the lack of theoretical principles on which assumptions about functional form can be made. While elaborate and sophisticated theoretical models of behavior exist, such models are often not detailed enough to be useful for purposes other than qualitative predictions. The standard approach in this case is either to "let the data determine" functional forms within ad hoc selected parametric classes, or to resort to semi-parametric methods. This is clearly unsatisfactory in the context of structural modeling. In the preceding sections we have insisted on a theoretical foundation for the stochastic properties of our model. These properties led to a particular representation of the labor supply choice probabilities ((2.5) and (2.6)) in terms of functions that represent preferences and opportunities. However, unless we are able to justify the choice of functional form of the systematic part of the utility function and the opportunity distribution, the implications may, as regards structural empirical analyses, be ambiguous. This is due to the fact that the class of a priori admissible opportunity distributions and utility functions is very large. In this section we shall discuss some interesting implications from the theory of psychophysical measurement and dimensionality analysis. The point of departure taken and exploited in some of the literature of psychophysical measurement is that numerical representations of sensory perceptions and physical stimuli can only be measured up to a scale[6]. For example, if the relevant stimuli are quantities or money, this type of variables are measured on a ratio scale. There is by now a considerable literature that addresses the issue of meaningfulness and dimensional invariance of scientific laws. It is of course beyond the scope of this paper to summarize this discussion; suffice to say here that "meaningfulness" concerns more precisely defined invariance properties with respect to the relevant scale types. A seminal contribution in this context is a paper by Luce (1959b), where he applies the above mentioned invariance principles to justify the power- and the logarithmic mappings linking stimuli and sensory response in psychophysical experiments. We shall now apply a more formalized version of this approach as presented in Falmagne (1985), to restrict the class of functional forms for the systematic part of the utility function.

To this end, consider now the particular case with an opportunity distribution that has all mass in two points (h, w_1) and (h, w_2), with probability mass equal to 0.5 in either point.

(Since preferences are assumed independent of opportunities, the analyst is for the sake of interpretation and theoretical analysis, free to select any opportunity distribution he finds suitable for a specific purpose while keeping the function v(·) unchanged.) Then from (2.5) it follows that

$$\frac{\psi(C_1,h_1)}{\psi(C_1,h_1)+\psi(C_2,h_2)} \equiv \tilde{\varphi}(C_1,L_1;C_2,L_2) \tag{2.8}$$

where $\tilde{\varphi}(C_1,L_1;C_2,L_2)$ is the probability that (C_1,L_1) is preferred to (C_2,L_2). It is understood that consumption in this context means disposable income minus subsistence expenditure and leisure means leisure minus subsistence leisure.

Assumption 4

Suppose C_1, C_2, C_1^*, C_2^*, L_1, L_2, are such that

$$\tilde{j}\,(C_1,L_1;C_2,L_2) \le \tilde{j}\,\left(C_1^*,L_1;C_2^*,L_2\right).$$

Then

$$\tilde{j}\,(rC_1,L_1;rC_2,L_2) \le \tilde{j}\,\left(rC_1^*,L_1;rC_2^*,L_2\right)$$

for any positive r.

Assumption 4 is a version of what is called dimensional invariance. It states that if the fraction of workers that prefer (C_1,L_1) to (C_2,L_2) is less than the fraction of workers that prefer $\left(C_1^*,L_1\right)$ to $\left(C_2^*,L_2\right)$, then the same is true when the respective consumption levels are scale transformations of the original levels. Note that it is *not* required that $\tilde{\varphi}(rC_1,L_1;rC_2,L_2)$ is independent of r.

There is a number of studies in experimental psychophysics that are concerned with the measurement of the utility of income. Stevens (1975) and his followers have found that a power function fits the data well,[7] where C means consumption in excess of a subsistence level, cf. Stevens (1975), p. 246. Similarly, one could argue that one should subtract a lower threshold from leisure, where this threshold may represent the necessary time for sleep and rest. Keeping this in mind, Assumption 4 captures the notion that one's basic needs (subsistence) are fulfilled then the absolute levels of quantities tend not to be essential, rather the individuals relate to relative consumption levels. Note, however, that Assumption 4 does not claim that $\tilde{\varphi}(rC_1, L_1; rC_2, L_2)$ is independent of r. It expresses instead that if the number of individuals that prefer (C_1^*, L_1) to (C_2^*, L_2) is greater than the number of individuals that prefer (C_1, L_1) to (C_2, L_2), this inequality remains true when consumption levels are increased or decreased by the same factor. For the sake of understanding the limitation of Assumption 4, we can think of two objections against this assumption. One objection is that the individual's perception about his personal subsistence level may be somewhat vague and may not be identified by a single fixed amount. Rather it may vary from one moment to the next according to fluctuations in his mood and state of mind. If satiation is present and rC_1^* and rC_2^* are close to satiation levels for sufficiently large r and $L_1 = L_2 = L$ (which means that the deterministic part of the utility approaches a constant), the second inequality in Assumption 4 may be reversed because $\tilde{\varphi}(rC_1^*, L; rC_2^*, L)$ will be close to 0.5, independent of the levels of C_1^*, C_2^* and L.

The notion that relative stimuli levels matter rather than absolute ones is supported by numerous stated preference experiments, see for example Stevens (1975).

Assumption 5

Suppose $L_1, L_2, L_1^*, L_2^*, C_1, C_2$, *are such that*

$$\tilde{J}(C_1, L_1; C_2, L_2) \le \tilde{J}(C_1, L_1^*; C_2, L_2^*).$$

Then

$$\tilde{J}(C_1, rL_1; C_2, rL_2) \le \tilde{J}(C_1, rL_1^*; C_2, rL_2^*)$$

for any positive r.

We realize that Assumption 5 is completely analogous to Assumption 4 and thus the motivation is similar.[8]

Theorem 2

If Assumptions 4 and 5 hold, then

$$\text{(2.10)} \qquad \log \mathrm{v}(C,h) = b_1 \frac{\left(C^{a_1}-1\right)}{a_1} + b_2\left(\frac{L^{a_2}-1}{a_2}\right) + b_3 \frac{\left(C^{a_1}-1\right)\left(L^{a_2}-1\right)}{a_1 a_2}$$

where $\{a_j\}$ *and* $\{b_j\}$ *are constants,* $a_j < 1$ *and* $b_j > 0$.

A proof of Theorem 2 in the separable case when $\mathrm{v}(\mathrm{C},\mathrm{h}) = \mathrm{v}_1(\mathrm{C})\mathrm{v}_2(\mathrm{h})$ is given in Appendix B. The proof in the general case is given in Dagsvik (2001).

If one imposes the stronger assumption that $\tilde{\varphi}(\mathrm{r}\,\mathrm{C}_1,\mathrm{L}_1;\mathrm{r}\,\mathrm{C}_2,\mathrm{L}_2)$ is *independent* of r, this implies that $\alpha_1 = 0$.

2.4. Extension to several sectors

An essential motivation for the framework discussed in this paper is that it is particularly convenient for modeling workers' choice among jobs with observable non-pecuniary job attributes. In general, jobs in different sectors may differ with respect to job security (with the government sector at one extreme and export industries at the other) and the nature of the tasks to be performed.

In this section we shall outline how the model can be extended to a multisectoral setting. To this end, we now suppose that the agent can choose among m sectors. The utility function in this case is assumed to have the structure

$$\text{(2.12)} \qquad \mathrm{U}(\mathrm{C},\mathrm{h},\mathrm{j},\mathrm{z}) = \mathrm{v}(\mathrm{C},\mathrm{h})\mu_j \varepsilon_j(\mathrm{z})$$

where j indexes sector, $j = 0,1,...,m$, and $j = 0$ represents "not working", and $\mu_j > 0$ is a constant that represents the pure preference of working with sector j specific tasks. The extension of the intensity measure in (2.3) is given by

$$\text{(2.13)} \qquad d\lambda_j(h,w,\varepsilon) = \begin{cases} \theta_{1j} g_j(h,w)\varepsilon^{-2}, & \text{when} \quad h > 0,\ w > 0,\ \varepsilon > 0 \\ \left(1 - \sum_{k=1}^{m} \theta_{1k}\right)\varepsilon^{-2}, & \text{when} \quad h = w = 0,\ \varepsilon > 0 \end{cases}$$

where $d\lambda_j$ is the sector-specific intensity measure and $g_j(h,w)$ is the corresponding opportunity density. That is, $g_j(h,w)\,dh\,dw$ is the mean fraction of feasible jobs in sector j with offered hours of work and wage rates within $(h,h+dw)\times(w,w+dw)$. Let $\theta_j=\theta_{1j}\Big/\left(1-\sum_{k=1}^{m}\theta_{1k}\right)$, where θ_j is the mean fraction of market opportunities relative to non-market opportunities. Let $\varphi_j(h,w)\,dh\,dw$ denote the probability of choosing a job in sector j with hours of work and wage rate within $(h,h+dw)\times(w,w+dw)$. Similar to Theorem 2 it follows that

$$\varphi(h,w,j)=\frac{\psi(h,w)\mu_j\theta_j g_j(h,w)}{\psi(0,0)+\sum_{k=1}^{m}\mu_k\theta_k\iint_D\psi(x,y)g_k(x,y)\,dx\,dy} \tag{2.14}$$

for $h>0, w>0$, and

$$\varphi(0,0,0)=\frac{\psi(0,0)}{\psi(0,0)+\sum_{k=1}^{m}\mu_k\theta_k\iint_D\psi(x,y)g_k(x,y)\,dx\,dy}. \tag{2.15}$$

3. The relationship between the present framework and previous labor supply models

3.1. Relation to the Hausman approach

It is interesting to note that a specification of the labor supply model based on the Hausman type approach, follows as a special case of a random coefficient version of the present framework. To realize this, assume for expository simplicity and in accordance with the assumptions made in the Hausman type of models, that the wage rate is *fixed* for a given individual and that the opportunity distribution of hours is *uniform*. Suppose furthermore that the coefficients of $\psi(\cdot)$ are random and let the error term $\varepsilon(z)$ be replaced by $\varepsilon(z)^{\sigma}$, where σ is a constant. From (2.4) it now follows that the corresponding choice probability density of hours can be expressed as

$$\varphi^*(h|w)=E\left\{\frac{\psi(h,w)^{1/\sigma}}{\psi(0,0)^{1/\sigma}+\int\psi(x,w)^{1/\sigma}dx}\right\} \tag{3.1}$$

where the expectation operator now is taken with respect to the random coefficients of the utility function. Now let $\sigma\to 0$ in (3.1). In Dagsvik and Strøm (1997) it is then proved that

(3.2) $$\varphi^*(h|w) \xrightarrow[\sigma\to 0]{} E\,\delta(h,w)$$

where

$$\delta(h,w) = \begin{cases} 1 \text{ if } \max_x \psi(x,w) = \psi(h,w), \\ 0 \text{ otherwise}. \end{cases}$$

In other words, (3.2) expresses the density of a labor supply function derived by first maximizing $\psi(h,w)$ with respect to h for given w, and subsequently integrating out the random coefficients.

To illustrate this point further let us consider a typical specification that has been applied by Hausman and several other researchers. This specification is given by the utility function

(3.3) $$U^*(C,h) = \left(\frac{h-\alpha_1}{\alpha_2}\right)\exp\left(\frac{\alpha_2\left(C+\alpha_3\right)}{h-\alpha_1}\right)$$

where α_1 and α_3 are unknown parameters and $\alpha_2 \le 0$, is a random coefficient. The utility function (3.3) implies that the labor supply function is linear in the marginal wage rate and virtual income.[9] Eq. (3.3) implies that

(3.4) $$\psi(h,w)^{1/\sigma} = \left(\frac{h-\alpha_1}{\alpha_2}\right)^{1/\sigma} \exp\left(\frac{\alpha_2\left(C+\alpha_3\right)}{\sigma\left(h-\alpha_1\right)}\right).$$

From (3.4) it follows (under suitable identification conditions) that σ, α_1, α_3 and the distribution of α_2 can be estimated from a likelihood function based on (3.1). In other words, the Hausman type of models can be viewed as embedded in a particular random coefficient version of the framework developed in this paper.

3.2. Relation to studies with latent constraints on hours of work

For the sake of comparison with some recent studies in labor supply econometrics that discuss modeling strategies for dealing with constraints, consider for a moment the following setting: The agent has a utility function $\tilde{U}(C,h,e)$ where ε is a random taste-shifter (independent of (C,h)). The budget constraint is given by (2.1) and the offered wage rate is fixed for each agent. Assume that hours of work take values in a finite set B (say). Let

(3.5) $$V(h,w,\varepsilon) = \tilde{U}\left(f(hw,I),h,\varepsilon\right).$$

Then it follows that the probability density of hours, conditional on the wage rate and the set B, is given by

(3.6) $$\hat{\varphi}\left(h \mid w, B\right) \equiv P\left(V(h,w,\varepsilon) = \max_{x \in B} V(x,w,\varepsilon)\right).$$

Suppose now that B is unobserved by the analyst and can take any value in the set $\{B_1, B_2, \ldots, B_m\}$. For example B could consist of the options “full-time”, “part-time” and “not working”, or of “part-time” and “not working”. To account for this, assume that B is random. Let q_j be the probability that $B = B_j$. The unconditional probability density that corresponds to the data the analyst has at hand therefore equals[9]

(3.7) $$\hat{\varphi}\left(h \mid w\right) \equiv E_B\, \hat{\varphi}\left(h \mid w, B\right) = \sum_{B_j \supset h} q_j\, P\left(V(h,w,\varepsilon) = \max_{x \in B_j} V(x,w,\varepsilon)\right).$$

In (3.7) it is the quantity “hours of work” that is rationed, whereas in our model, presented in section 2 above, a latent choice variable, “job opportunity”, is introduced. Possible rationing of hours may occur because there may be few or no feasible jobs with the desired hours of work. The models developed by Ilmakunnas and Pudney (1990), and Dickens and Lundberg (1993) fall within the framework represented by (3.7).

Our notion of unobservable choice variables proposed in this paper has several attractive features. First, the framework with unobservable job opportunities introduced in section 2 allows for the interpretation that the outcome of an agent's labor supply decision is the result of the agent maximizing utility over “job-packages” with several attributes of which hours of work is only one of them. Second, the framework is convenient for dealing with latent opportunity sets, while the type of formulation represented by (3.7) is a mixture of multinomial logit type densities and becomes rapidly intractable when m increases.

4. An empirical application

4.1. Empirical specification

The present application does not exploit the full potential of the methodology: The only non-pecuniary attribute that is recorded in the data is which sector of the economy the jobs belong to. For simplicity, we only consider the case with two sectors, private and public sector, and for the following reasons. For women with higher level of education there are more job opportunities in the public than in the private sector. Moreover, in the public sector more emphasize has been put on facilitating combination of work and childcare, and thus it is more likely to find a job with a subsidized day-care center in the public than in the private sector. The public sector is more unionized than the private sector. Wages are more compressed and hours are more constrained. However, the job security tends to be higher in the public sector than in the private sector. Thus, there are important differences between the private and the public sector that could influence the labor supply decisions of married women. Some of these differences are observed (like wages) while others are not. The modeling framework appropriate for this application is the one outlined in subsection 2.4, where sector one is the public sector and sector two is the private sector and $m = 2$.

We will assume that offered hours and offered wages are independent, i.e.

(4.1) $$g_j(h,w) = g_{j1}(w)g_{j2}(h)$$

for $j = 1,2$. Although offered wages and hours vary across jobs, our assumption is that hours are *set* independent of wages. The justification for this assumption is that offered wages, in the unionized part of economy, are set in yearly wage settlements. Normal working hours, on the other side, is determined more infrequently, typically once or twice every decade. The density of offered hours, $g_{2j}(h)$ is assumed uniform except for peaks at full-time and part-time hours. Recall that uniformly distributed offered hours corresponds to the notion of a perfect competitive economy. Thus, the full-time peak in the hours distribution captures institutional restrictions and hence market imperfections in the economy. We allow the sizes of the full-time and part-time peaks to vary across sectors. The rationale is that the public sector is more regulated than the private sector, also because the private sector is more heterogeneous and less unionized. Thus we expect the full-time peaks associated with the public sector to be higher than the full-time peak associated with the private sector. Note also that normal working hours may vary across jobs according to how strenuous the jobs are considered to be. For example nurses, fire-workers and police officers have typically lower normal working hours than the average worker.

In the absence of random effects, it would have been possible to apply McFadden’s (1978) estimation procedure to estimate continuous logit models. McFadden’s estimation procedure replaces the integrals in the denominators of the densities in (2.14) and (2.15) by a sum over a small set of random points, where each term is adjusted by appropriate weights. In other words, the continuous logit model is replaced by a discrete logit version. McFadden has demonstrated that this method yields consistent and asymptotically normal parameter estimates. However, as will be discussed shortly, our empirical model will be modified to allow for particular random effects and this implies that the logit structure is lost and consequently, McFadden's procedure can no longer be applied.

To facilitate estimation, we have discretisized observations on hours of work. For each sector we have specified 7 hours of work intervals. The medians of the intervals range from 420 annual hours in the first interval to 2808 in the 7th interval. For each sector the full-time peak occurs in the 5th interval where the median is 1950 annual hours. The part-time peak is related to the 3rd interval with a median equal to 1040 annual hours. These intervals correspond to the most common agreements of what constitutes full time and half time annual hours of work.

In section 2.3 we postulated particular invariance properties that allowed us to characterize the functional form of the structural part of the utility function. Unfortunately, we have not been able to provide similar principle to characterize the functional form of $g_{j1}(w)$. Recall that $g_{j1}(w)$ is the subjective density of offered wage rates, as perceived by the agent. We shall therefore, in the present application, abandon the specification and estimation of $g_{j1}(w)$, which implies that we can only estimate the marginal density of chosen hours of work and sector.

Let $\bar{w}_j$ be the subjective mean in the offered wage rate distribution in sector j, i.e.,

$$\bar{w}_j = \int_{y>0} y g_j(y) dy . \tag{4.2}$$

By the mean value theorem we have that

$$\int_{y>0} \psi(h,y) g_{j1}(y) dy \cong \psi(h, \bar{w}_j) . \tag{4.3}$$

The approximation in (4.3) is good if the variance in the subjective opportunity density $g_{j1}(w)$ is small.

To allow for unobserved heterogeneity in the opportunity densities we assume that

$$\bar{w}_j = w_j^* \eta_j \tag{4.4}$$

for $j=1,2$, where $\{\eta_j\}$ are *random effects*. We assume that $\log \eta_j$, $j=1,2$, are independent and normally distributed, $N(0,\sigma_j)$.

The systematic term of the subjective mean wage rate, w_j^*, is assumed to vary across sectors and $\log w_j^*$ is assumed to be a linear function of length of schooling, work experience and work experiences squared.

Thus, when accounting for the unobserved heterogeneity in the opportunity densities, it follows from (2.14), (2.15) and (4.2) to (4.4) that the resulting choice probabilities that corresponds to our observations are

$$(4.5) \qquad \bar{\varphi}(h,j) \equiv \int_{y>0} \varphi(h,y,j) g_{j1}(y)dy = E\left[\frac{\psi(h, w_j^*\eta_j)g_{j2}(h)b_j}{\psi(0,0)+b_1\sum_{x>0}\psi(x,w_1^*\eta_1)g_{12}(x)+b_2\sum_{x>0}\psi(x,w_2^*\eta_2)g_{22}(x)}\right],$$

for $h>0$, $j=1,2$, and

$$(4.6) \qquad \varphi(0,0,0) = E\left[\frac{\psi(0,0)}{\psi(0,0)+b_1\sum_{x>0}\psi(x,w_1^*\eta_1)g_{12}(x)+b_2\sum_{x>0}\psi(x,w_2^*\eta_2)g_{22}(x)}\right],$$

where expectation is taken with respect to $\{\eta_1,\eta_2\}$, and $b_j = \mu_j\theta_j$.

For many reasons, most women are working in the service branch of the economy and thus for women there are more feasible jobs available in firms that provide services than elsewhere. In Norway, most of the services are provided by the public sector (health services, education etc) and many of the jobs here require higher education, while the services provided in the private sector say, in retail sale, are typically based on low-skill labor. Thus it is reasonable to assume that b_j may depend on education. We will expect that the higher the education is, the higher is the number of feasible jobs in the public sector. We have assumed that

$$(4.7) \qquad \log b_j = f_{j1} + f_{j2}S$$

where S is the length of education.

In accordance with Proposition 1 we have chosen $v(\cdot)$ to be a Box-Cox type function, separable in leisure and consumption. Specifically,

(4.8)
$$
\begin{aligned}
\log v(C,h) = {} & \alpha_2\left(\frac{10^{-5}(C-C_0)^{\alpha_1}-1}{\alpha_1}\right)+\left(\frac{L^{\alpha_3}-1}{\alpha_3}\right)\left(\alpha_4+\alpha_5\log A+\alpha_6(\log A)^2+\alpha_7 CU6+\alpha_8 CO6\right)\\
& +\alpha_9\left(\frac{10^{-5}(C-C_0)^{\alpha_1}-1}{\alpha_1}\right)\left(\frac{L^{\alpha_3}-1}{\alpha_3}\right)
\end{aligned}
$$

where A, is the age of the married woman, CU6 and CO6 are number of children less than 6 and above 6 years, C is given by $f(hw, I)$, L is leisure, defined as

(4.9)
$$L = 1 - h/3640,$$

and α_j, $j=1,2,...,8$, are unknown parameters. Observe that we have subtracted from total annual hours a "subsistence" level that allows for sleep and rest.

Consistent with psychophysical evidence, we have also introduced a subsistence threshold level, C_0 for consumption in the $v(\cdot)$ function. We have chosen C_0 to be close to the official estimate of a subsistence level in Norway (NOK 60 000). If $\alpha_1 < 1$, $\alpha_3 < 1$, $\alpha_2 > 0$, and

(4.10)
$$\alpha_4 + \alpha_5 \log A + \alpha_6 (\log A)^2 + \alpha_7 CU6 + \alpha_8 CO6 > 0$$

then $\log v(C,h)$ is increasing in C, decreasing in (h) for fixed C and strictly concave in (C,h).

To facilitate the estimation procedure we have estimated the wage equation (regressed log w_j^* against the observed covariates mentioned above) in a first step by applying a version of two stage Heckman approach to control for selectivity. Conditional on these estimates the remaining parameters of the model are estimated by the maximum likelihood procedure. To compute the expectations in (4.5) and (4.6) we have generated a large number of independent random variables: $\{\eta_{jk}, k=1,2,...,M\}$ for $j=1,2,$ and $\log \eta_{jk} \sim N(0,\sigma_j)$.

Hence, we can write

(4.11)
$$\bar{\varphi}(h,1) = \frac{1}{M^2}\sum_{k=1}^{M}\sum_{t=1}^{M}\frac{\psi(h, w_1^*\eta_{1k})g_{12}(h)b_1}{\psi(0,0)+b_1\sum_{x>0}\psi(x,w_1^*\eta_{1k})g_{12}(x)+b_2\sum_{x>0}\psi(x,w_2^*\eta_{2t})g_{22}(x)}$$

and similarly for $\bar{\varphi}(h,2)$ and $\bar{\varphi}(0,0,0)$.

4.2. Data

Data used in this study concerns the labor supply of married women in Norway 1994 and consists of a merged sample from "Survey of Income and Wealth, 1994", Statistics Norway (1994) and "Level of living conditions, 1995", Statistics Norway (1995). Data covers married couples as well as cohabiting couples with common children. The age of the spouses ranges from 25 to 64. None of the spouses are self-employed and none of them are on disability or other type of benefits. All taxes paid are observed and in the assessment of disposable income, at hours not observed, all details of the tax system are accounted for. Observed hours of work are related to main job as well as possible side jobs. The size of the sample used in estimating the labor supply model is 674. Wage rates above NOK 350 or below NOK 40 are not utilized when estimating the wage equations. The respondents in the survey report their hourly wage, but from register data we also observe annual wage income. Since we observe annual hours worked we can measure wage rates two ways. The wage rates are computed as the ratio of annual wage income to hours worked.

In Table 1 we report the summary statistics for the sample used in estimating the labor supply model.

Table 1. Summary statistics for married women, Norway 1994

	Not working		Public sector		Private sector	
	Mean	Std.	Mean	Std.	Mean	Std
Age in years	41.17	11.10	43.25	8.59	41.78	9.00
Education in years	10.66	1.63	11.96	2.31	10.70	1.57
No of children, 0-6	0.77	0.98	0.38	0.72	0.38	0.69
No of children, 7-17	0.71	0.91	0.61	0.81	0.60	0.77
Annual hours of work	0	0	1520	518	1411	597
Disposable household income, NOK per year	286 949	227 787	326 034	119 530	327 146	131 592
Wage rate, NOK per hours			104.30	28.52	100.56	30.46
Number of observations	97		266		311	
Fractions	0.144		0.395		0.461	

4.3. Estimation results for the wage equations

In this section we report estimates of the wage equation and the structural model.

Table 2. Estimates of wage equations. Married women, Norway 1994

Variables	Public sector		Private sector	
	Estimates	t-values	Estimates	t-values
Constant	3.37	13.5	3.70	25.2
Experience in years/100	3.21	6.0	2.55	5.1
(Experience in years)^2 /100	-4.75	-5.3	-3.80	-4.2
Education in years/100	5.57	4.9	5.26	4.2
Log (Probability of working in the chosen sector)	-0.12	-2.0	0.06	0.9
No of observations	691		580	
R^2	0.14		0.08	

In the wage equations, the logarithm of observed wage rates, $\log W_k, k=1,2$, is regressed against working experience, working experience squared, education level and a term capturing possible selectivity. It can be demonstrated that one can control for selectivity bias by applying $\log P_j$ as an additional independent variable where P_j is a reduced form trinomial logit model for being in sector j, $j=0,1,2$, (where $j=0$ means not working). The explanatory variables in the trinomial logit are age, education level, no of children and income of the spouse. Estimates of the wage equations are given in Table 2, and we observe that on the margin workers get slightly better paid for experience and education in the public sector than in the private sector. However, the differences in returns across sectors are not significant. On a much larger sample Barth and Røed (2001) reports similar results for 1995.

Judged by R^2 the explanatory power of the wage equations is low. Thus, it seems important to account for unobservables when estimating the structural model. In Section 4.1 we have explained how we account for the unobservables in the wage equations. This is done by introducing random effects when we estimate the structural model.

4.4 Estimates of labor supply probabilities

Estimates of the parameters in the structural choice model are given in Table 3. The estimates imply that the utility function is quasi-concave. We observe that the marginal utility of leisure is driven by the presence of children in the household. Thus, women only seem to value leisure (outside full-time and part-time hours) when they have children. We also note that the parameter associated with the interaction term between consumption and leisure is not significantly different from zero. The estimates of the opportunity density confirm the conjecture that there are more jobs available in the public sector for higher educated women than for women with little education. In the private sector education does not affect the set of feasible job opportunities. This means that if length of schooling is increased while wage rates are kept fixed, participation in the public sector will increase. Moreover, the full-time peak, and to some extent also the part-time peak, is more distinct in the public sector than in the private. As mentioned above this may be due to the fact that the public sector is more unionized than the private one.

Table 3. Estimation results for the parameters of the labor supply probabilities

Variables	Parameters	Estimates	t-values
Preferences:			
Consumption:			
Exponent	α_1	0.79	6.9
Scale · 10^{-5}	α_2	0.78	3.0
Leisure:			
Exponent	α_3	0.08	0.4
Constant	α_4	43.36	1.1
Log age	α_5	–23.45	–1.1
$(\log \text{age})^2$	α_6	3.58	1.2
No of children 0-6	α_7	0.82	3.0
No of children 7-17	α_8	0.88	3.7
Consumption and Leisure, interaction	α_9	0.01	0.1
The parameters b_1 and b_2; $\log b_j = f_{j1} + f_{j2}S$			
Constant public sector (sector 1)	f_{11}	–4.77	–5.3
Constant private sector (sector 2)	f_{21}	–0.56	–0.6
Education public sector	f_{12}	0.29	3.7
Education private sector	f_{22}	–0.10	–1.1
Opportunity density of offered hours, $g_{k2}(h)$, k=1,2			
Full-time peak, public sector (sector 1)*	$\log\left(g_{12}(h_{Full})/g_{12}(h_0)\right)$	1.86	10.6
Full-time peak, private sector (sector 2)	$\log\left(g_{22}(h_{Full})/g_{22}(h_0)\right)$	1.31	8.3
Part-time peak, public sector	$\log\left(g_{12}(h_{Part})/g_{12}(h_0)\right)$	0.88	5.2
Part-time peak, private sector	$\log\left(g_{22}(h_{Part})/g_{22}(h_0)\right)$	0.68	4.4
Log likelihood		–1111.5	

* The notation h_0 refers to an arbitrary level of hours of work different from full-time and part-time hours.

The number of random draws (M) used in the simulations used to compute the likelihood function equals 50.

Table 4 compares observed and predicted aggregates, and we note that the model predicts these aggregates pretty well.

Table 4. Observed and predicted aggregates. Married women, Norway 1994.

Variables	Not working		Public sector		Private sector	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
Choice probabilities	0.144	0.143	0.395	0.391	0.461	0.466
Annual hours	0	0	1520	1447	1411	1471
McFadden's ρ^2	0.39					

As noted above, we can only identify the product $v_2(h)g_2(h)$ non-parametrically. To disentangle $v_2(h)$ from $g_2(h)$ we have assumed that the clustering of hours of work at part-time and full-time work is due to technological organizational constraints and/or regulation of hours introduced by unions and/or the government. The term $g_2(h)$ is meant to capture this aspect of the labor market in the highly unionized Norwegian economy. Thus, through parametric identification our model implies that observed concentration of hours of work around part-time and full-time work arise because there are institutional constraints in the labor market rather than because individuals have strong preferences for full-time and part-time hours of work. If the parameters of the utility function are robust with respect to our assumption, then our empirical model may be applied also to simulate the impact of a change in the institutional constraints on available working hours in the market.

To this end we have therefore reestimated the model under the assumption that *offered* hours are uniformly distributed. It goes without saying that the parameters attached to the leisure term will be affected when we force the clustering of hours to be explained solely by preferences. The estimates are reported in Table 5. The exponent related to leisure drops from 0.08 to -0.69 and becomes significantly different from zero. As before, for women without children the leisure term is not precisely determined. The estimates of the parameters related to the effect of children on leisure are not significantly changed.

Of greater interest is how the estimates of the other parameters of the utility function is affected when offered hours are assumed to be uniformly distributed. As shown below the other parameters are very little affected.

Table 5

		Uniformly distributed offered hours except at part-time and full-time hours		Uniformly distributed offered hours	
		Estimates	t-values	Estimates	t-values
Consumption:					
Exponent		0.79	6.9	0.76	7.4
Scale · 10^{-5}		0.78	3.0	0.90	3.4
Consumption and leisure, interaction		0.01	0.1	-0.01	-0.4
b_1 and b_2:					
Constant, sector 1	f_{11}	-4.77	-5.3	-5.03	-5.8
Constant, sector 2	f_{21}	-0.56	-0.6	-0.97	-1.0
Education, sector 1	f_{12}	0.29	3.7	0.28	3.7
Education, sector 2	f_{22}	-0.10	-1.1	-0.12	-1.4

4.5. Wage elasticities

The mean utility, $\psi_j(.)$, is the utility concept that comes closest to the one often used by others in the calculation of elasticities. To calculate these elasticities one has to assume that the labor supply of the mean sample household can be simulated by maximizing the deterministic part of the utility function under the constraint represented by a linearized version of the budget constraint. Of course, this approach is rather crude since it implies that the stochastic structure of the model is ignored.

Another set of elasticities arises when we consider how the mean in the *distribution* of labor supply is affected by changes in say, wage levels. We denote these elasticities as *aggregate* ones since they take into account unobserved and observed heterogeneity in the population. Moreover, they also account for the non-convexity of the budget constraint due to taxation and hours restrictions, and are thus consistent with the structure of the model.

In Table 6 we report what we have called aggregate uncompensated elasticities. They are calculated as follows: The model is used to simulate the labor supply for each female under the current regime and when the wage rates in each sector, and in both sectors, respectively, are increased by one per cent. The aggregate elasticity of female labor supply is obtained by calculating the relative change in the mean female labor supply (over all females in the sample) that results from a one percent wage increase for the females, ceteris paribus. The “estimates” in Table 6 are based on 10 sets of simulations.

Table 6. Aggregate uncompensated wage elasticities

Variables	Choice probabilities and mean hours before wage changes	Elasticities with respect to changes in wage rates		
		Public sector	Private sector	Both sectors
Participation probabilities:				
Working	0.857	0.18	0.23	0.37
Working in public sector	0.391	1.53	-1.27	0.20
Working in private sector	0.466	-0.96	1.49	0.52
Mean annual hours of work, conditional on sector:				
Public sector	1447	0.34	0.01	0.35
Private sector	1471	0.01	0.34	0.35
Mean total annual hours of work:				
Conditional on job in any sector	1460	0.17	0.22	0.35
Unconditional	1251	0.35	0.46	0.74

Elasticities are numerically small with the exception of how sector specific participation probabilities respond when wage levels are changed. The reasons why overall responses are small are high overall labor market participation among married Norwegian women and a regulated and rigid market for hours. However, we note that when the wage level in the public sector is increased by 1 per cent, the participation probability in this sector increases by as much as 1.53 per cent. Most of this increase comes at the expense of a decrease in the participation probability in the private sector of 0.96 per cent. Overall participation is stimulated by a minor 0.18 per cent. A similar pattern emerges when the wage level in the private sector is raised by 1 per cent. The probability of working in the private sector increases by 1.49 per cent and as in the preceding case, most of the increase comes from a reduction of the probability of working in the other sector, the public sector (1.27 per cent). A similar pattern emerges for annual hours of work, conditional on sector, but the impact is smaller. From the last row in Table 5 we notice that an overall increase of 1 per cent is estimated to raise supplied hours in the total population of married females by 0.74 per cent, with an almost equal split on increased participation and increased supply of hours, conditional on working.

5. Conclusion

In this paper we have discussed a particular approach for labor supply modeling, with special reference to the inadequacy of traditional econometric approaches to deal with;

(i) The notion that households have preferences over jobs, characterized by job- and sector-specific non-pecuniary attributes, hours of work, and wage rates, including convenient representations of the set of feasible job attributes,

(ii) exact representation of complicated and non-convex budget constraints,

(iii) justification of the functional form of the utility function and the distribution of unobserved variables.

We have demonstrated that the framework presented proves to be practical for dealing with (i) and (ii). A more fundamental theoretical issue is the problem of characterizing the functional form of the empirical model on the basis of theoretical principles. By drawing on the recent literature in measurement theory and theoretical psychophysics, we have shown that it is possible to apply invariance principles to constrain and justify the class of admissible functional forms.

An empirical version of the model has been estimated on a recent sample of Norwegian married women. The estimated model turns out to reproduce the data quite well. It is demonstrated that it is of empirical importance to distinguish between job opportunities across sectors of the economy. The estimated model is used in stochastic simulations to calculate sector specific as well as overall elasticities of labor supply with respect to wage levels. Apparently, weak responses in overall female labor supply shadow for much stronger inter-sector mobility.

Appendix A

Extension of the model to two-person households (married couples)

Let $U(C, h_F, h_M, z)$ denote the household's utility function where h_F and h_M denote the wife's and the husband's hours of work, respectively, and $z = (z_F, z_M)$ indexes the market and non-market opportunities of the wife and the husband. Similarly to Assumption 1 we assume that

(A.1) $$U(C, h_F, h_M, z) = v(C, h_F, h_M)\varepsilon(z)$$

where v(·) is a deterministic term and ε(z) is a random taste-shifter. For given wage rates, w_F and w_M, the economic budget constraint can be written as

(A.2) $$C = f(h_F w_F, h_M w_M, I)$$

where f(·) is the function on R_+^3 that transforms wage- and non-wage incomes of the household to household income after taxes. Let

(A.3) $$\psi(h_F, h_M, w_F, w_M) = v\left(f(h_F w_F, h_M w_M, I), h_F, h_M\right).$$

The household's opportunity set can be represented by

$$\wp = \left\{(H_F(z), H_M(z), W_F(z), W_M(z), \varepsilon(z)); z = ..-2,-1,0,1,2...\right\}$$

where $H_F(z) = H_M(z) = W_F(z) = W_M(z) = 0$ for $z \le 0$. The five-tuple in $\wp$ are realizations from a non-homogeneous Poisson process on $[0, \bar{h}] \times [0, \bar{h}] \times [0, \infty) \times [0, \infty) \times [0, \infty)$. The intensity measure of the Poisson process is given by

(A.4) $$d\lambda(h_F, h_M, w_F, w_M, \varepsilon) = \begin{cases} \theta_{11}\, g_F(h_F, w_F)\, g_M(h_M, w_M)\, \varepsilon^{-2}\, dh_F\, dh_M\, dw_F\, dw_M\, d\varepsilon, \\ \text{when } h_F > 0, w_F > 0, h_M > 0, w_M > 0, \varepsilon > 0, \\ \theta_{01}\, g_M(h_M, w_M)\, \varepsilon^{-2}\, dh_M\, dw_M\, d\varepsilon, \\ \text{when } h_M > 0, w_M > 0, h_F = w_F = 0, \varepsilon > 0. \end{cases}$$

For notational simplicity the case where the husband does not work is ruled out in this presentation. The interpretation of $g_F(\cdot)$ and $g_M(\cdot)$ is completely similar to the interpretation in the case of single person households. In the formulation in (A.4) it is assumed that (conditional on observed household characteristics) the offered hours and wage rates of the husband are independent of the offered hours and wage rates of the wife. This assumption can easily be relaxed at the cost of difficult identification problems. However, the market opportunities of the wife and the husband (represented by θ_{11} and θ_{01}) may be dependent. The parameter θ_{11} is the fraction of the feasible opportunities that are market opportunities to the household, while θ_{01} is the fraction of the feasible opportunities that are non-market opportunities to the wife and market opportunities to the husband. As in Section 2, it follows that the joint density of realized hours and wages, $\varphi(h_F, h_M, w_F, w_M)$, equals

(A.5) $$\varphi(h_F, h_M, w_F, w_M) = \frac{\theta\,\psi(h_F, h_M, w_F, w_M)\, g_F(h_F, w_F)\, g_M(h_M, w_M)}{K}$$

for $h_F > 0, h_M > 0, w_F > 0, w_M > 0,$ and

(A.6) $$\varphi(0, h_M, 0, w_M) = \frac{\psi(0, h_M, 0, w_M)\, g_M(h_M, w_M)}{K}$$

when $h_F = w_F = 0$, where $\theta = \theta_{11}/\theta_{01}$, and

(A.7) $$\begin{aligned} K = {} & \theta \int_0^{\bar{h}} \int_0^{\bar{h}} \int_0^{\infty} \int_0^{\infty} \psi(x_1, x_2, y_1, y_2)\, g_F(x_1, y_1)\, g_M(x_2, y_2)\, dx_1\, dx_2\, dy_1\, dy_2 \\ & + \int_0^{\bar{h}} \int_0^{\infty} \psi(0, x_2, 0, y_2)\, g_M(x_2, y_2)\, dx_2\, dy_2 \,. \end{aligned}$$

Appendix B

Proof of Theorem 1

By Assumption 1 and (2.2)

(B.1) $$U(z) = v\big(H(z), W(z)\big)\varepsilon(z).$$

The proof is completely analogous to the proof of Theorem 7 in Dagsvik (1994), but for the sake of completeness we outline the proof here.

Assume that {(H(z), W(z), ε(z)), z=1,2,…} are realizations of a Poisson process on $\left[0,\bar{h}\right]\times R_+^2$ with intensity measure as in (2.3).

Let

(B.2) $$d\mu(h,w) = \begin{cases} \theta_1 g(h,w)\,dh\,dw & \text{when} \quad h>0,\, w>0 \\ 1-\theta_1 & \text{when} \quad h=w=0. \end{cases}$$

Let A be a Borel set on $\left[0,\bar{h}\right]\times R_+$, and define

(B.3) $$U_A = \max_z \big(\psi\big(H(z), W(z)\big)\varepsilon(z)\big) \quad \text{subject to} \quad \big(H(z), W(z)\big)\in A.$$

U_A is the highest utility the agent can attain, subject to (H(z), W(z))∈A. We shall now derive the c.d.f. of U_A. Let

(B.4) $$B = \left\{(h,w,\varepsilon): v(h,w)\varepsilon > u,\, h,w\in A, (h,w,\varepsilon)\in\left[0,\bar{h}\right]\times R_+^2\right\},$$

and let N(B) be the number of Poisson points within B. By the Poisson law

(B.5) $$P\big(N(B)=n\big) = \frac{\Lambda(B)^n}{n!}\exp\big(-\Lambda(B)\big)$$

where $\Lambda(B) = EN(B)$, and is given by

(B.6) $$\Lambda(B) = \int_B d\lambda(h,w,\varepsilon) = \int_{(x,y)\in A,}\int_{\psi(x,y)\varepsilon>u} d\mu(x,y)\varepsilon^{-2}d\varepsilon = \frac{1}{u}\int_A \psi(x,y)\,d\mu(x,y)$$

Now it follows from (B.5)

(B.7) $$\begin{aligned} P\big(U_A\le u\big) &= P\big(\text{There are no points of the Poisson process in B}\big) \\ &= P\big(N(B)=0\big) = \exp\big(-\Lambda(B)\big) = \exp\left(-\frac{1}{u}\int_A \psi(x,y)\,d\mu(x,y)\right) \end{aligned}$$

Eq. (B.7) proves that U_A is type I extreme value distributed.[11]

Let $\bar{A}$ be the complement of A. Since the Poisson realizations are independently distributed, it follows that U_A and $U_{\bar{A}}$ are independent and type I extreme value distributed.

Let $\hat{z}$ be the index of the alternative that maximizes utility, i.e.,

(B.8) $$\psi\left(H(\hat{z}),W(\hat{z})\right)\varepsilon(\hat{z}) = \max_z\left(\psi\left(H(z),W(z)\right)\varepsilon(z)\right).$$

Obviously

(B.9) $$P\left(\left(H(\hat{z}),W(\hat{z})\right)\in A\right) = P\left(U_A = \max_z\left(\psi(H(z),W(z))\varepsilon(z)\right)\right) = P\left(U_A > U_{\bar{A}}\right)$$

From (B.7) it follows by straight forward calculus that

(B.10) $$P\left(U_A > U_{\bar{A}}\right) = \frac{\int_A d\mu(x,y)}{\int_{R_+^2} d\mu(x,y)}.$$

Hence, with $A = (0,h]\times(0,w]$, we get from (B.10) that

(B.11) $$\Phi(h,w)P\left(H(\hat{z})\le h, W(\hat{z})\le w\right) = \frac{\int_0^h\int_0^w \psi(x,y)d\mu(x,y)}{\int_{R_+^2}\psi(x,y)d\mu(x,y)}.$$

From (B.11) it follows that for $h>0, w>0$

(B.12) $$\varphi(h,w) = \frac{\theta_1\psi(h,w)g(h,w)}{\int_{R_+^2}\psi(x,y)d\mu(x,y)}$$

which yields (2.5).

Q.E.D.

Proof of Theorem 2 in the separable case

Note first that when $v(C,h) = v_1(C)v_2(h)$, then under Assumption 4 the choice probability, $\tilde{\varphi}(C_1,L;C_2,L) = \tilde{\varphi}(C_1,C_2)$, and it can be written as

(B.13) $$\tilde{\varphi}(C_1,C_2) = F\left(v_1(C_1)/v_1(C_2)\right)$$

where

(B.14) $$F(y) = \frac{y}{1+y}$$

for $y > 0$. Recall also that the input stimuli (consumption C), is measured on a ratio scale. Hence, Theorem 14.19 in Falmagne, p. 338, (see also his discussion on an application following the theorem) implies that[7]

(B.15) $$\tilde{\varphi}(C_1, C_2) = F^*\left(\frac{\beta_1 \left(C_1^\alpha - 1\right)}{\alpha} - \frac{\beta_2 \left(C_2^\alpha - 1\right)}{\alpha}\right)$$

where $\beta_1 > 0$, $\beta_2 > 0$, and α are constants, and F^* is a strictly increasing continuous function.[12] Since C_1 and C_2 can attain any positive value and can vary independently, it follows that the domain of F^* must be R. Also the balance condition

$$\tilde{\varphi}(C_1, C_2) + \tilde{\varphi}(C_2, C_1) = 1$$

must hold. In particular with $C_1 = C_2 = C$, we obtain that $\tilde{\varphi}(C, C) = 0.5$, for all C, which by (B.15) implies that $\beta_1 = \beta_2 \equiv \beta$. Let $M(x) = F^{-1}\left(F^*(x)\right)$. Since F^* is continuous, and F is continuous and strictly increasing, it follows that M(x) is continuous. Thus, from (B.14) and (B.15) we get that

(B.16) $$M\left(\frac{\beta\left(C_1^\alpha - 1\right)}{\alpha} - \frac{\beta\left(C_2^\alpha - 1\right)}{\alpha}\right) = \frac{v_1(C_1)}{v_1(C_2)}.$$

In particular, with $C_2 = 1$, we obtain

(B.17) $$v_1(C_1) = v_1(1)\, M\left(\frac{\beta\left(C_1^\alpha - 1\right)}{\alpha}\right).$$

By letting

$$x = \frac{\beta\left(C_1^\alpha - 1\right)}{\alpha} - \frac{\beta\left(C_2^\alpha - 1\right)}{\alpha}$$

and

$$y = \frac{\beta\left(C_2^\alpha - 1\right)}{\alpha},$$

(B.16) and (B.17) lead to the following functional equation

(B.18) $$M(x)\,M(y) = M(x+y)$$

for $x, y \in R$. But (2.15) is the well-known Cauchy equation which solution is the exponential function, cf. Theorem 3.2 and Remark 3.3 in Falmagne (1985), p. 82. Consequently, it follows from (B.18) that (apart from an additive constant)

(B.19) $$\log v_1(C) = \beta\left(C^{\alpha} - 1\right)\big/\alpha\,.$$

Q.E.D.

Footnotes

[1] In recent years the tax and benefit system has been simplified in many countries. Most budget sets are, however, still non-convex.

[2] Alternatively, the worker may have to change the content of his current job.

[3] Of course, this would not be true in “slavery” economies.

[4] In most of the previous work in labor supply modeling, for instance in the various applications of the Hausman type approach referred to in the introduction or in Van Soest (1994), it is (tacitly) assumed that the observed clustering of hours around so called “part-time” and “full-time” work is due to preferences. There are no restrictions on offered hours, which in the context of our framework can be interpreted as the distribution of offered hours being uniform. This can be justified if data have been generated in a free market economy with no regulation of offered hours. If offered hours are uniformly distributed, then in our framework $g_2(h)$ is equal to a constant and $v_2(h)$ is identified up to a multiplicative constant. However, it can be questioned whether the assumption of uniformly distributed offered hours is a good approximation to the conditions in the unionized and government regulated labor market in many west European countries. It seems more reasonable to assume that offered hours are determined to a large extent by institutional regulations and/or negotiations at an aggregate level and, of course, by the technology of firms. These characteristics of an unionized industrial society will typically imply that jobs with “part-time” and “full-time” hours of work are more frequently available in the labor market than jobs with other hours of work.

Some researchers, see for example Van Soest (1994) argue that one may assume that the peak at full time hours are due to preferences, since possible constraints on hours are unobserved. This argument will in general be flawed, because if in fact there are restrictions on hours of work then this may have important implications for the structural model. That it is not evident how one should deal with choice constraints is illustrated by the fact that a number of authors have demonstrated considerably ingenuity to deal with different approaches to rationing, cf. Section 2.4.

[5] The observed concentrations of hours of work at “part-time” and “full-time” hours may be due to both preferences and choice constraints. See Hamermesh (1986) for a theoretical motivation for this.

[6] Recall that the scale types are: Ordinal scale, Ratio scale, Interval scale and Logarithmic interval scale, cf. Falmagne (1985).

[7] Stevens and others have observed the power law in innumerable experiments. Sinn (1983) has compressed the content of Stevens' Psychophysical power law into the following statement: “Equal relative changes in stimulus intensity bring about equal relative changes in sensation intensity”.

[8] Although Luce (1959b) derived the power law as the functional relation between subjective continua and physical continua from the assumption of dimensional invariance, his approach nor Steven's empirical method do not apply directly to aggregate relations. Recall that the challenge faced here is to characterize choice probabilities, or equivalently, the mathematical and stochastic structure of a random utility function. If only the approach discussed by Luce (1959) was available then we would not be able to discriminate between specifications such as for example $v_1(C) = \beta C^\alpha$ and $v_1(C) = m\left(C^\alpha\right)$, where $m(\cdot)$ is an increasing function since in our context, utility, U(C,h,z), is ordinal and only determined up to a monotone transform. Thanks to the approach developed by Falmagne and Narens (cf. Falmagne, 1985, ch. 14) we are, however, able to get rather sharp results as demonstrated above.

[9] The labor-supply function is $h = (\alpha_1 + \alpha_3) + \alpha_1\alpha_2 w^* - \alpha_2 I^*$, where w^* is the marginal wage rate and I^* is virtual income. Unfortunately, the functional form of (3.3) cannot be justified by theoretical arguments of the type given in Section 2.4.

[10] The notation $B_j \supset h$, means that the summation takes place across all j for which $B_j \supset h$.

[11] Recall that the standard type I distribution function has the form exp(-1/y), y>0, cf. Resnick (1987). There is, however, some confusion in the literature, since other authors call this distribution type III.

[12] From Falmagne's Theorem 14.19 it follows that $\alpha \geq 0$. It is, however, easy to verify that the proof of the theorem also applies when α is negative.

Note that the above results do not depend on the particular structure of the function $F(\cdot)$ given by (2.11). It is sufficient that this function is strictly increasing and continuous and (2.10) holds.

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