Rank dependent expected utility models of tax evasion

by

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ABSTRACT

In this paper the rank-dependent expected utility theory is substituted for the expected utility theory in models of tax evasion. It is demonstrated that the comparative statics results of the expected utility, portfolio choice model of tax evasion carry over to the more general rank dependent expected utility model.

Keywords: Tax evasion, rank dependent expected utility, dual theory.

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1. Introduction

The main purpose of this paper is to analyse – by applying a rank dependent expected utility $model^1$ – the effect on tax evasion of changes in the probability and severity of punishment, and also of changes in income and tax rates. There are several reasons for doing this. (i) The expected utility (EU) model, still dominant in economic analysis of uncertainty, has been seriously challenged in a number of studies, and it is therefore reasonable to explore the characteristics of non-expected utility models in various fields. (ii) The situation of a potential tax evader is in some ways similar to situations in laboratory experiments where the expected utility model has performed badly. (iii) The rank dependent expected utility (RDEU) model seems to be the best among the non-expected utility models as far as sharp comparative statics results are concerned.

In section 2 some of the relevant properties of the RDEU theory are presented, followed in section 3 by a discussion of certain issues in modelling tax evasion. The formal structure of a RDEU model of tax evasion is given in section 4. Section 5 examines the comparative statics of a RDEU portfolio choice model, whereas the special case of a dual model is analysed in section 6.

2. RDEU vs EU

The flourishing field of generalized expected utility theory has provided explanations of several phenomena that appear as paradoxes within the theory of expected utility.² Several of these phenomena seem to be related to the fact that marginal utility of wealth and attitude towards risk is merged in the expected utility model. This amalgamation makes the EU model particularly simple and tractable, but

¹ The model is also known as anticipated utility (AU), expected utility with rank-dependent preferences (EURDP), the μ - θ model, the dual theory of choice under uncertainty, and rank-dependent utility (RDU). The many names indicate that authors dealing with different problems have come up with essentially the same model.

² Quiggin (1993, 37-49) gives an overview of the many challenges to EU theory, i.a. the Allais paradox, the Ellsberg paradox, preference reversal, insurance and gambling jointly, and difficulties in empirical constructions of utility functions. He also (49-32) discusses, and challenges, some attempts to solve the various problems by introducing weighting of probabilities, in particular the prospect theory. Attempts to model this phenomenon by transforming probabilities have not been successful (Quiggin 1993, p. 63).

at the same time hampers a more profound study of the individual's attitude towards uncertainty. The EU concept of risk aversion is partly a property of attitudes to wealth, and not of attitudes to risk per se. By keeping the von Neumann-Morgenstern utility function, and at the same time allowing for transformations of probabilities, the RDEU model generalises the EU model.

Ellsberg (1961) provided an early demonstration of the importance of *ambiguity* in decision making, and showed that uncertainty is not totally captured by the concept of probability. Ambiguity is an intermediate state between ignorance, in which no distributions can be ruled out, and risk, in which all but one distribution is ruled out. Ambiguity results from the decision maker having limited or vague information and knowledge of the process generating outcomes.³ Empirical evidence indicates that people distinguish between risk and ambiguity.⁴ In a situation of risk the decision maker has objective or subjective probabilities of given outcomes. In a situation of ignorance the decision maker has no information concerning the likelihood of potential outcomes. Studies show that ambiguity aversion and risk aversion are not (highly) correlated, a correlation one would expect if they were just different designations of the same phenomena.⁵ Both ambiguity avoidance and ambiguity-seeking behaviour have been found in laboratory experiments.⁶

EU theory can neither explain the Ellsberg paradox nor some other phenomena obtained in various experiments. The theory does not capture important factors that characterise risky decision making: (i) The context in which the decision is taken can

³ Ellsberg (1961, 657) defined ambiguity as "a quality depending on the amount, type, reliability and "unanimity" of information, giving rise to one's degree of "confidence" in an estimate of relative "likelihoods" of future events.

⁴ Camerer (1999).

⁵ Cohen, Jaffray, and Said (1985), Hogharth and Einhorn (1990), and Schoemaker (1982).

⁶ See i.a. Becker and Brownson (1964), Yates and Zukowski (1976) and Einhorn and Hogarth (1986). Surveying several experimental studies Edwards (1992, p. 5) makes the following comment: "Currim and Sarin (1989, 1990) compared experimental subjects' assessed expected utility models with their prospect theory, weighted utility, and lottery dependent utility models; and Daniels and Keller (1990) assessed expected utility models in predicting choices on a hold-out sample of paired comparison choices, even when the problems were structured to induce expected utility property violations. However, the potential for improved predictive performance by generalised expected utility models may still be achieved. For example, Daniels and Keller (1992) have explored a choice-based assessment mechanism in which lottery dependent expected utility appears to perform better than expected utility. Also, Shafir et al. (1989) proposed an advantage model of choice that outperformed two special cases of expected utility".

change the evaluation of risk; (ii) the character of the uncertainty that people encounter in real-world situations is different from the risk encountered in gambling; and (iii) the payoffs can affect the weights given to uncertainty.

In this paper the rank-dependent expected utility theory will be applied to study to what extent the comparative statics results obtained by use of the EU model carry over to the RDEU model.⁷ Among the host of non-expected utility models with different preference functionals that have been proposed in order to tackle various theoretical and empirical problems raised in studies of individual behaviour under uncertainty, the RDEU model is chosen for various reasons⁸. According to Quiggin (1993, p. 72) relation (1) and (2) below is the only possible generalisation of the EU theory that is separable in outcomes and probabilities, and in which the requirements of first stochastic dominance, transistivity and continuity are satisfied.⁹ Separability makes the model simple, and is crucial for some of the sharp comparative statics results of this theory. It also performs quite well in experiments where various utility theories have been compared.¹⁰

As shown by Quiggin (1993, p. 92) the RDEU model is able to accommodate for a majority of the observed violations of EU predictions, while retaining enough structure to preserve the standard comparative statics results. He also asserts (p. 93)

⁷ Quiggin (1993) has given the most comprehensive treatment of the RDEU model. The present paper can be seen as a response to his suggestion of investigating to what extent the many nice results obtained in various fields by assuming EU carry over to RDEU models. Another paper responding to his suggestion is Eide (1995).

⁸Different types of models have been developed to explain and predict behaviour under ambiguity. These include (i) models based on the idea of anchor probability (Einhorn and Hogarth (1986)), models which represent ambiguity as a second order probability (Marshak (1975) and Bernasconi (1997)), and models in which the probability of events is not additive (Gilboa and Schmeidler (1989)). The present paper is an example of the latter. Prospect theory has also been developed in order to accommodate such characteristics, but this theory, at variance with ambiguity theory, concerns gambles with well-defined probabilities.

⁹ E.g. the theory of Honda (1977) and Kahneman and Tversky (1979) violate dominance requirements.

¹⁰ Hey and Orme (1994) conclude such an investigation thus (p. 1321): "Expected utility theory (and its special case risk neutrality) emerges from this analysis fairly intact. For possibly 39% of the subjects ... EU appears to fit no worse than any of the other models ... For the 61% of the subjects, one or more of the eight "top-level" functionals ... fits significantly better in statistical terms, though often the economic significance is not all that great. Of the eight "top-level" functionals it would appear that the two rank dependent functionals and the quadratic utility model emerge as strongest contenders (with the Quiggin weighting function having a modest lead over its power weighting rival)." Harless and Carmerer (1994), using as much as 23 data sets, conclude, however, that our choice of preference functionals must depend on the researchers' preference for fit and parsimony. No functional is to be preferred on both accounts.

that the comparative statics results do not depend on the special assumptions of the EU theory.

Whereas risk aversion in the EU theory corresponds to a simple condition on the utility function, the RDEU model implies a fundamental distinction between attitudes to probabilities and attitudes to outcomes, cfr. Quiggin (1993, p. 76):

First there is outcome risk aversion, associated with the idea that marginal utility of wealth is declining. This is the standard notion of risk aversion from EU theory defined by concavity of the utility function. Second, there are attitudes specific to probability preferences.¹¹ An obvious ground for risk aversion in probability weighting arises for people characterized by pessimism, that is, those who adopt a set of decision weights that yields an expected value for a transformed risky prospect lower than the mathematical expectation. This yields a natural generalization of the basic definition or risk aversion to the RDEU model.

It is worth noticing that Allais (1988) in his axiomatisation of the main ideas in his 1953 article comes up with the RDEU model. Discussing the independent works of Quiggin (1982), Yaari (1987) and Segal (1987) he states: "It is *very significant* that, starting from entirely different premises, all three authors have been led to a *mathematical formulation* that is analogous to my own" (emphasis in original).

It is also interesting to note that Gilboa (1987) and Schmeidler (1989) independently seem to have discovered the RDEU model in studies of ambiguity, i.e. in studies where objective probabilities are absent. Here, the decision weights are interpreted as non-additive subjective probabilities. In the standard RDEU model developed by Quiggin (1993) objective probabilities are assumed to be known, and these are transformed into non-additive decision weights.¹²

¹¹ It is arguable that the term 'risk aversion' is more properly applied to preferences over probabilities than to preferences over outcomes. [Quiggins footnote]

¹² Quiggin argues that the difference in interpretation could be seen as only a difference in authors' tastes.

3. Issues in modelling tax evasion

According to Allingham and Sandmo's (1972) portfolio choice approach to income tax evasion¹³, a risk-averse taxpayer, with a von Neumann-Morgenstern utility function, will under-report his income whenever the expected gain minus expected punishment of evasion is positive. Intuition as well as empirical evidence seems to contradict this conclusion. For the more common types of tax evasion the sanctions in many countries consist of fines less (or not much higher) than the amount evaded, whereas the probabilities of tax returns being audited are of the order of a few percent. In general one would therefore expect most individuals to be tax evaders, a result that is not supported by empirical evidence, - quite a few seem to comply. Some explanations of why people are more law abiding than perhaps expected are related to social norms, stigma, or moral sentiments. In this paper another explanation is considered: Behaviour is in accordance with the theory of rank dependent expected utility.

Using a result by Segal and Spivak (1990), Bernasconi (1998) presents a related explanation. Assuming that the preference functional is not differentiable near certainty (no tax evasion), Bernasconi finds that individuals sometimes prefer not to cheat even when the expected return of evasion is positive. The approach accommodates for a high degree of risk aversion near the certainty point. Bernasconi's result presupposes the use of non-expected utility models. He demonstrates that not all such models can be used to solve the apparent puzzle of tax compliance. The RDEU model, however, has the appropriate characteristics. The present exploration of the RDEU model can be considered as a supplement to Bernasconi's article.

Some of the problems appearing as paradoxes within the EU theory (in particular those of Allais and Ellsberg) seem to be related to *low* probability events. Since the probabilities of being audited are quite low in many countries, one might expect the RDEU model to have a better chance than the EU model to represent the behaviour of the tax payers. Furthermore, the probability of being sanctioned can only vaguely be known by the tax payer, a situation of ambiguity that calls for an RDEU representation.

¹³ For summaries of the literature on tax evasion see Erhard and Feinstein (1994), Franzoni (2000) or Eide (2000).

Alm (1988) and Beck and Jung (1989) have extended previous tax compliance research by developing models in which taxpayers are uncertain of their taxable incomes and associated tax liabilities (due to such things as the complexity of the tax law and the uncertainty of audit outcomes). Alm (1988) found that increased uncertainty had a substantial impact on a number of taxpayers' decisions including investing in tax shelters, receiving compensation in wage or non-wage forms, spending on tax deductible items, and under-reporting one's income. Beck and Jung (1989) concluded that the effects of uncertainty on taxpayer compliance can differ depending on risk-taking attitudes, the likelihood of audit and the magnitude of penalties. When the magnitude of penalties and the perceived likelihood of audit are high, increasing uncertainty increases compliance regardless whether taxpayers are risk-averse or risk-neutral. However, when audit probabilities and penalty rates are low (and closer to the values that would be expected to occur naturally), risk-neutral taxpayers are shown to have incentives to reduce compliance. For risk-averse taxpayers, the effects of increasing uncertainty depend on the degree of risk aversion.¹⁴

These results encourage exploring RDEU models of tax evasion.

4. The RDEU model

In presenting the rank dependent expected utility model I follow Quiggin (1993, p. 57) and his notation. Let **x** be a vector of n outcomes with the probability vector **p**, and $U(\mathbf{x})$ a primitive utility function. The characteristic feature of this model is a probability weighting function $q:[0,1] \rightarrow [0,1]$, which is applied, not to the probabilities of individual events, but to the cumulative distribution function $F(\mathbf{x})$. The RDEU functional to be maximised is

$$V(\{(x_1, x_2, ..., x_n); (p_1, p_2, ..., p_n)\}) = U(x_i)h_i(p_1, p_2, ..., p_n),$$

where

¹⁴ See Sawyers (1990) for a survey of results in this field.

$$h_i(p_1, p_2, ..., p_n) = q(\sum_{j=1}^i p_j) = q(F(x_i)) - q(F(x_{i-1}))$$

In the case of two outcomes (punished or not punished), we have

$$h_1(p_1, p_2) = q(F(x_1)) - q(F(x_0)) = q(p_1),$$
(1)

$$h_2(p_1, p_2) = q(F(x_2)) - q(F(x_1)) = q(1) - q(p_1) = 1 - q(p_1).$$
(2)

That is, q defines the weight on the worst outcome (unsuccessful evasion) and 1q defines the weight on the better outcome (successful evasion).

5. An RDEU portfolio choice model of tax evasion

Consider a tax payer with exogenous income W_0 , unknown to the tax authorities. Declared income X, which is the taxpayer's decision variable, is taxed by a flat rate q. The probability that the tax authority becomes aware of evasion is P. If evasion is disclosed, the taxpayer will be punished in proportion to the tax evaded. Evaded tax is $q(W_0-X)$, and the additional payment is $pq(W_0-X)$, where p (>1) may be called the penalty rate. We assume that the individual's utility of income can be represented by a von Neumann-Morgenstern utility function U(W) with U'>0 and U''<0. The RDEU functional to be maximised is then

$$V(\{W_0 - \boldsymbol{q}X - \boldsymbol{p}\boldsymbol{q}(W_0 - X), W_0 - \boldsymbol{q}X; P, (1 - P)\}) = U_U(W_U)q(P) + U_S(W_S)(1 - q(P))$$

where

$$W_U = W_0 - \boldsymbol{q}X - \boldsymbol{p}\boldsymbol{q}(W_0 - X) \tag{3}$$

is income if tax evasion is unsuccessful (from the taxpayer's point of view), and

$$W_s = W_0 - \boldsymbol{q}X \tag{4}$$

is income if tax evasion is successful.

Derivation w.r.t. X gives the 1. order necessary condition for maximum:

$$\mathbf{p}U'(W_U)q(P) - U'(W_S)(1 - q(P)) = 0$$
(5)

The 2. order condition for maximum:

$$\frac{\partial^2 V}{\partial X^2} = q^2 \left[(p - 1)^2 U''(W_U) q(P) + U''(W_S) (1 - q(P)) \right] < 0$$

is satisfied, and (5) then determines the optimal value of declared income, X^* . The taxpayer's reactions to changes in the parameters are found by differentiation of (5), see appendix.

Assuming q'>0, an increase in the probability of punishment leads to an increase in income declared:

$$\frac{\partial X^*}{\partial P} = -\frac{\boldsymbol{q}}{D} \Big[(\boldsymbol{p} - 1) U'(W_U) + U'(W_S) \Big] \boldsymbol{q}'(P) > 0.$$

This qualitative result is the same as that obtained by Allingham and Sandmo (1972) in their expected utility model. The quantitative effect depends of course on the probability weighting function.

The effect of a change in exogenous income is given by

$$\frac{\partial X^*}{\partial W_0} = \frac{\boldsymbol{q}}{D} U'(W_S) [(1 - \boldsymbol{p} \boldsymbol{q}) R_A(W_U) - R_A(W_S)] (1 - \boldsymbol{q}(P)).$$

Assuming decreasing absolute risk aversion, $R_A(W_U) > R_A(W_S)$. The bracket is then definitely negative only if $\pi \theta \ge 1$. Only in that case can we be certain that our taxpayer will increase declared income when exogenous income increases. This qualitative result is also the same as that obtained by Allingham and Sandmo.

The effect on the *proportion* of income declared by an exogenous change in income is given by

$$\frac{\partial(\frac{X^{*}}{W_{0}})}{\partial W_{0}} = -\frac{1}{DW_{0}^{2}} \boldsymbol{q} U'(W_{s}) [R_{R}(W_{U}) - R_{R}(W_{s})](1 - q(P))$$

It is seen that when exogenous income increases, the proportion of income declared increases, stays constant, or decreases according to whether relative risk aversion is an increasing, constant or decreasing function of income. The same qualitative conclusion was obtained by Allingham and Sandmo.

The effect of a change in the penalty rate is given by

$$\frac{\partial X^*}{\partial \boldsymbol{p}} = -\frac{\boldsymbol{q}}{D} \left[U'(W_U) - (\boldsymbol{p} - 1)\boldsymbol{q}(W_0 - X)U''(W_U) \right] \boldsymbol{q}(P) \ .$$

Both elements in the bracket are positive, and an increase in the penalty rate increases the amount of income declared. Allingham and Sandmo obtained the same result.

Finally, the effect of changes in the tax rate is given by

$$\frac{\partial X^*}{\partial q} = -\frac{q}{D} U'(W_S)(1-q(P)) \Big\{ X^* \big[R_A(W_U) - R_A(W_U) \big] + p(W_0 - X^*) R_A(W_U) \Big\},\$$

which shows that when decreasing absolute risk aversion is assumed, our taxpayer will declare more of the income if the flat tax rate is increased. This result is the same as obtained by Yitzhaki (1974). The conclusion in Allingham and Sandmo (1972) is less sharp, because they, at variance with both Yitzhaki's and the present

model, assumed that the penalty was proportional to the income evaded (and not the tax evaded).

6. The dual model

The EU model and the dual model of Yaari (1987) are special cases of the RDEU model. The EU model is linear in the probabilities, whereas the dual model is linear in the preferences. According to the dual model individuals maximize an expected value with weighting of probabilities. This model might be more relevant for firms than for individuals. For our tax evasion problem the preference functional to be maximized is

$$Z = W_U q(P) + W_S (1 - q(P)) = (1 - pq)q(P)W_0 + (pq(P) - 1)X,$$

which shows that

X=W₀ if $\pi q(P) > 1$ (full compliance) X=0 if $\pi q(P) < 1$ (no declaration)

X is undetermined if $\pi q(P)=1$.

An exogenous increase in either sanction rate or in the probability of punishment will eventually change the taxpayer's behaviour from no declaration to full compliance. Changes in exogenous income or tax rate will not change the taxpayer's attitude to compliance.

7. Conclusion

It has been demonstrated that the comparative statics results of the portfolio choice model of tax evasion carry over to a related rank dependent expected utility model. The generalisation of expected utility to rank dependent expected utility has not changed the qualitative results obtained by Allingham/Sandmo/Yitzhaki. The same is true for the dual model. The quantitative effects depend of course on the weighting function.

Appendix

The taxpayer's reaction to changes in the parameters is found by differentiation of the 1. order condition (5) for obtaining a maximum value of the preference functional:

$$d[q(p-1)U'(W_U)q(P) - qU'(W_S)(1 - q(P))]$$

$$= q^{2}[(p-1)^{2}U''(W_U)q(P) - U''(W_S)(1 - q(P))]dX$$

$$+ q[(p-1)(U'(W_U) + U'(W_S)]q'(P)dP$$

$$+ q[(p-1)(1 - pq)U''(W_U)q(P) - U''(W_S)(1 - q(P))]dW_0$$

$$+ q[(U'(W_U) - q(p-1)U''(W_U)(W_0 - X)]q(P)dp$$

$$+ \{ [(p-1)U'(W_U)q(P) - U'(W_S)(1 - q(P)]]$$

$$+ q[-(p-1)U''(W_U)(X + p(W_0 - X)q(P) + XU''(W_S)(1 - q(P))]\}d\theta.$$

Probability of punishment:

$$\frac{\partial X^*}{\partial P} = -\frac{\boldsymbol{q}}{D} \left[(\boldsymbol{p} - 1) U'(W_U) + U'(W_S) \right] \boldsymbol{q}'(P) \,.$$

Exogenous income:

$$\frac{\partial X^*}{\partial W_0} = \frac{\boldsymbol{q}}{D} \left[(\boldsymbol{p} - 1)(1 - \boldsymbol{p} \boldsymbol{q}) U''(W_U) q(P) - U''(W_S)(1 - q(P)) \right].$$

Substituting the Arrow-Pratt absolute risk aversion, $R_A = -\frac{U''}{U'}$, gives

$$\frac{\partial X^*}{dW_0} = \frac{\boldsymbol{q}}{D} \Big[-(\boldsymbol{p}-1)(1-\boldsymbol{p}\boldsymbol{q})U'(W_U)R_A(W_U)q(P) + U'(W_S)R_A(W_S)(1-q(P)) \Big].$$

Substitution from (3) gives

$$\frac{\partial X^*}{dW_0} = \frac{q}{D} U'(W_S) [(1 - pq)R_A(W_U) - R_A(W_S)] (1 - q(P)).$$

Proportion of income declared:

$$\frac{\partial(\frac{X^{*}}{W_{0}})}{\partial W_{0}} = \frac{1}{W_{0}^{2}} \left(\frac{\partial X^{*}}{\partial W_{0}}W_{0} - X^{*}\right)$$

$$= \frac{1}{DW_{0}^{2}} \left[\boldsymbol{q} \left(\boldsymbol{p} - 1 \right) (1 - \boldsymbol{p} \boldsymbol{q}) U^{\prime\prime}(W_{U}) \boldsymbol{q}(P) W_{0} - \boldsymbol{q} U^{\prime\prime}(W_{S}) (1 - \boldsymbol{q}(P)) W_{0} \right]$$

$$+ \frac{1}{DW_{0}^{2}} \left[\boldsymbol{q}^{2} \left(\boldsymbol{p} - 1 \right)^{2} U^{\prime\prime}(W_{U}) \boldsymbol{q}(P) X^{*} + \boldsymbol{q}^{2} U^{\prime\prime}(W_{S}) (1 - \boldsymbol{q}(P)) X^{*} \right]$$

$$= \frac{1}{DW_{0}^{2}} \left[\boldsymbol{q} \left(\boldsymbol{p} - 1 \right) U^{\prime\prime}(W_{U}) \boldsymbol{q}(P) \left[(1 - \boldsymbol{p} \boldsymbol{q}) W_{0} + \boldsymbol{q}(\boldsymbol{p} - 1) X^{*} \right] - \boldsymbol{q} U^{\prime\prime}(W_{S}) (1 - \boldsymbol{q}(P)) (W_{0} - \boldsymbol{q} X^{*} \right]$$

By substitution from (3) and (4):

•

$$\frac{\partial(\frac{X^*}{W_0})}{\partial W_0} = \frac{1}{DW_0^2} \left[\boldsymbol{q} (\boldsymbol{p} - 1)U''(W_U) \boldsymbol{q}(P) W_U - \boldsymbol{q}U''(W_S)(1 - \boldsymbol{q}(P)) W_S \right]$$
$$= \frac{1}{DW_0^2} \left[-\boldsymbol{q} (\boldsymbol{p} - 1)U'(W_U) R_A(W_U) \boldsymbol{q}(P) W_U + \boldsymbol{q}U'(W_S) R_A(W_S)(1 - \boldsymbol{q}(P)) W_S \right].$$

Substitution of the Arrow-Pratt relative risk aversion, $R_R = -\frac{U''W}{U'}$, gives

$$\frac{\partial(\frac{X^*}{W_0})}{\partial W_0} = \frac{1}{DW_0^2} \left[-\boldsymbol{q} (\boldsymbol{p} - 1)U'(W_U) R_R(W_U) q(P) + \boldsymbol{q}U'(W_S) R_R(W_S) (1 - q(P)) \right]$$
$$= -\frac{1}{DW_0^2} \boldsymbol{q}U'(W_S) \left[R_R(W_U) - R_R(W_S) \right] (1 - q(P)) .$$

Penalty rate:

$$\frac{\partial X^*}{\partial \boldsymbol{p}} = -\frac{\boldsymbol{q}}{D} \left[U'(W_U) - (\boldsymbol{p} - 1)\boldsymbol{q}(W_0 - X^*)U''(W_U) \right] q(P).$$

Tax rate:

$$\frac{\partial X^*}{\partial \boldsymbol{q}} = (\boldsymbol{p}-1) \left[U'(W_U) - \boldsymbol{q} (X^* + \boldsymbol{p} (W_0 - X^*)) U''(W_U) \right] q(P) - \left[U'(W_S) + \boldsymbol{q} X^* U''(W_S) \right] (1 - q(P))$$

By substitution from the first order condition:

$$\frac{\partial X^*}{\partial q} = -\frac{q}{D}(p-1) \Big[-q(X^* + p(W_0 - X^*))U''(W_U)q(P) + X^*U''(W_S)(1 - q(P)) \Big] \\ = -\frac{q}{D}U'(W_S)(1 - q(P)) \Big\{ X^* \Big[R_A(W_U) - R_A(W_U) \Big] + p(W_0 - X^*)R_A(W_U) \Big\}.$$

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