INTERNATIONAL CENTRE FOR ECONOMIC RESEARCH
ICER

WORKING PAPER SERIES

Elisa Luciano

CALIBRATING RISK-NEUTRAL DEFAULT CORRELATION

Working Paper no. 12/2005
May 2005

APPLIED MATHEMATICS
WORKING PAPER SERIES

Calibrating risk-neutral default correlation

Elisa Luciano*
University of Turin
and International Center for Economic Research, Turin

December 18, 2004

Abstract

The implementation of credit risk models has largely relied on the use of historical default dependence, as proxied by the correlation of equity returns. However, as is well known, credit derivative pricing requires risk-neutral dependence. Using the copula methodology, we infer risk neutral dependence from CDS data. We also provide a market application and explore its impact on the fees of some credit derivatives.

JEL classification number: G12

The assessment of the joint default probability of groups of obligors, as well as related notions, such as the probability that the n-th one of them defaults, is a crucial problem in credit derivatives pricing and hedging. In order to solve it, academics and practitioners have recently relied on copula methods, which allow to split any joint default probability into the marginal ones and a function, the copula itself, which represents only the dependence between defaults. The splitting up makes both default modelling and calibration much easier, since it permits separate fitting at the univariate and joint level.

Copula techniques require on the one side the choice of a specific dependence or copula function C, on the other side the selection of a level of the parameter/s which characterize the copula.

As for the copula choice, structural based models naturally lead to a so-called Gaussian or Student copula, while in intensity-based models the same copulas are very often introduced for analytical convenience, especially in high dimensions.

As for parameter calibration, it is fairly standard, both in structural and intensity-based models, to use the historical equity correlation as a proxy for

---

[*] A more technical version of the results in this paper has been presented at the XVII Annual Warwick Options Conference, September 24, 2004, under the title "Credit derivatives and counterparty risk pricing through copulas: recent developments". The current version has been prepared for presentation at the 2005 EFMA meeting, Milan, July 2005. Computational assistance by A. Fraquelli is gratefully acknowledged. Contact address: Prof. Elisa Luciano, University of Turin & ICER, Villa Gualino, V. S. Severo, 63 I-10133 Torino, Italy, tel.+39-011-6604828, fax + 39-011-6600082, luciano@econ.unito.it

asset correlation. In turn, the use of historical correlation is based on the assumption of no premium for default correlation. The assumption, which can be shown to be theoretically incorrect, was compulsory in the presence of illiquid and restricted credit derivatives markets. Growing markets for credit derivatives however allow us to calibrate risk neutral correlation from observed market prices of credit products, so as to avoid the restrictive hypothesis of no premium for default dependence.

Up to now, the technique of dependence calibration from credit derivatives has been extensively used with CDOs: since CDOs involve a huge number of obligors, while dependence measures are bivariate, default correlation has been assumed to be the same between all obligors in the pool or in one of its tranches. This restriction, which can be justified through one-factor models, has some drawbacks, such as the so-called correlation smile, of which practitioners are well aware. It can lead to misleading investment choices, as Mashal, Naldi and Tejwani (2004) point out.

The equality of pairwise correlations could be relaxed by using derivatives based on bivariate default, such as derivatives subject to counterpart risk, which are naturally priced in the copula framework (see for instance Cherubini, Luciano and Vecchiato (2004)).

Credit default swaps (CDS's) seem to be the natural choice, due to their high liquidity, once the possibility that the guarantor defaults is taken into account. In turn, counterpart risk can be consistent with the empirical studies on CDS's carried out so far: Blanco, Brennan and Marsh (2004) for instance find violations of the parity between bond yield spreads and CDS premia, by ignoring the vulnerability of the latter. Since CDS premia, once vulnerability is taken into account, can be greater than their non-vulnerable correspondents, the violations in Blanco et alii could disappear by taking counterpart defaults into account. As another example, consider the evidence reported in Ericsson, Jacobs and Oviedo-Helfenberger (2004): on their data, only 60% of the CDS premia is explained by theoretical variables, with no apparent room for a residual common factor. As Blanco et alii, they do not take into consideration the vulnerable nature of CDS's: counterpart risk therefore could enhance their results, especially the asymmetry between bid and ask R-squared.

From the theoretical point of view, vulnerable default swaps have been accurately priced: see for instance Turnbull (2004). This paper shows how to use these products in order to calibrate risk neutral default correlation and, at the same time, to select the "best" copula[1]. It applies the methodology to a small set of obligors and studies the impact on higher dimensional derivative pricing, such as CDOs.

---

[1] From the theoretical point of view, choosing a best-fit copula is equivalent to selecting a risk neutral measure among the several one which are consistent with market incompleteness.

1 Credit Default Swaps with a vulnerable issuer

Let us recall first of all that under fairly weak conditions the joint default probability at time $T, \Pr(\tau_i \leq T, \tau_j \leq T)$, where τ_i and τ_j are the times to default of obligors i and j, can written as a function, the copula indeed, of the default probabilities of the single obligors:

$$\Pr(\tau_i \leq T, \tau_j \leq T) = C(\Pr(\tau_1 \leq T), \Pr(\tau_2 \leq T))$$

Among the most renowned copulas, which are used in credit modelling (see for instance the Credit Metrics^{TM} documentation, as well as Frey, McNeil and Nyfeler (2001) or Mashal and Naldi M. (2003)), are the Gaussian and Student one. They are respectively represented as:

$$C^{Ga}(v, z) = \Phi_\rho \left(\Phi^{-1}(v), \Phi^{-1}(z)\right)$$

where Φ_ρ is the bivariate cumulative normal distribution with correlation coefficient ρ, while Φ^{-1} is the inverse of the univariate standard normal distribution, and

$$C_{\rho,v}(v, z) = t_{\rho,v} \left(t_v^{-1}(v), t_v^{-1}(z)\right)$$

where $t_{\rho,v}$ is the bivariate cumulative Student distribution with correlation coefficient ρ and v degrees of freedom (dof), while t_v is the corresponding univariate function.

These two choices, while apparently awkward, have proved to provide a very good fitting to hystorical asset and default data. In the sequel, while applying the copula formalism, we will adopt them too. We will need them, in particular, while writing down the payoff and valuation of CDS.

Let us denote as i the guarantor or insurance seller, who sells protection against default within time T of a reference credit, issued by j. In case default occurs to j, i should pay to the insurance buyer the so-called contingent leg of the contract, consisting in the loss given default on the reference bond, $Lgd^{\,j} = 1 - R^{\,j}$. However, in case of default both by the guarantor and the reference credit, only a fraction of the due amount, corresponding to the recovery rate of the guarantor, R^i, is paid. For the sake of simplicity[2], let us assume that, in case of default, the loss payment occurs at expiration of the contract, T. Assume also that the default-free interest rates and recovery rates are non-stochastic[3]. Denote as B_t the value at time 0 of a zcb with maturity t, unit face value.

According to the no-arbitrage evaluation principle, the contingent leg should then be priced as

$$B_T \left[(1 - R^{\,j}) \Pr(\tau_i > T, \tau_{\,j} \leq T) + R^{\,i}(1 - R^{\,j}) \Pr(\tau_{\,i} \leq T, \tau_{\,j} \leq T)\right]$$

---

[2] This assumption can be relaxed for more realistic calibration, and accrued interest considerations can be introduced. For a continuous time version see for instance Hull and White (2000).

[3] A straighforward extension consists in assuming them stochastic, but independent.

where Pr() is the risk-neutral probability of the event in parenthesis, and τ_i, τ_j are the times to default of the two obligors i and j.

Let $F_i(T)$ and $F_j(T)$ be the (risk-neutral) distributions of τ_i, τ_j, evaluated at $T : F_i(T) := \Pr(\tau_i \leq T)$. Using these distributions and a (risk-neutral) copula representation of the joint default probability, $\Pr(\tau_i \leq T, \tau_j \leq T) = C(F_i(T), F_j(T))$, the contingent leg becomes

$$B_T(1 - R^j)\left[F_j(T) - \left(1 - R^i\right) C(F_i(T), F_j(T))\right]$$

As for the fee leg, the protection buyer pays to i a periodic fee, s, if and only if both the guarantor and the reference credit survive. We then have the following fee leg value:

$$\sum_{t=0}^{T-1} sB_t\breve{C}(1 - F_i(t), 1 - F_j(t))$$

where $\breve{C}$ is the (risk-neutral) copula representing the joint survival probability of the two entities, $\Pr(\tau_i > T, \tau_j > T)$, which in turn is related to the copula C by the relationship

$$\breve{C}(1 - F_i(t), 1 - F_j(t)) = 1 - F_i(t) - F_j(t) + C(F_i(t), F_j(t))$$

The theoretical CDS fee is therefore

$$s = \frac{B_T(1 - R^j)\left[F_j(T) - \left(1 - R^i\right) C(F_i(T), F_j(T))\right]}{\sum_{t=0}^{T-1} B_t\breve{C}(1 - F_i(t), 1 - F_j(t))} \tag{1}$$

2 Calibration and risk neutral dependence

Let us consider CDS ask quotes, as offered by major investment banks: in the ask case, the bank acts as the agent i above[4]. At the same time, let us assume that we can infer the marginal (risk neutral) default probabilities of both the issuing bank and the reference credit from the bond market, either using an analytical model, such as an intensity-based one, or simply taking the empirical marginal default probabilities at the horizons $t = 0, 1, ...T$: in the sequel, we will use the spread-over-Treasury curves and infer from them the empirical margins.

For a given copula choice, such as the Gaussian, the actual ask quotes and their theoretical versions, given by (1) above, can be used in a straightforward way to infer the (implied) dependence measure of i and j, for instance a linear correlation coefficient, $\rho(\tau_i, \tau_j)$. As usual, the implied measure can be taken to be the one which minimizes the pricing errors, over a given period of time.

---

[4] We do not consider bid quotes, since they involve the marginal default probability of a generic counterpart, and its joint default with the quoting bank. At most, under an assumption of prudential, "worst" default dependency between the bidding bank and its counterpart, bid quotes could be used to infer the implicit default probability of the counterpart itself.

Repeating the minimization for different copula families and comparing the CDS pricing errors, one will also have a selection criterion for copulas.

As an example of the above calibration, we constructed the risk neutral dependence matrices for three names, used them as block matrices for bigger portfolios, and compared them, in terms of impact on some credit derivatives pricing, with the historical correlation one. The derivatives on which the impact is appreciated are a FTD swap on three names, a FTD on nine names and a CDO with nine homogeneous tranches.

Since the purpose of the whole calibration is purely illustrative, we work with a small sample of three names, which, for privacy purposes, we will denote as obligors 1, 2 and 3. The first two belong to the financial USA sector, the third to the telecommunication, EU one: at the moment of the sample construction, they belonged respectively, according to Standard & Poor's, to the rating classes AA, A and BBB. Their CDS 's are part of the I-Traxx (both series 1 & 2), in both cases being senior unsecured.

2.1 Marginal default probabilities

In order to calibrate the marginal risk neutral default probabilities, we took from Bloomberg the appropriate (by sector and rating) spreads-over-Treasury, for the maturities 1 to 5 years. We considered the spreads over the period August 2-October 22, 2004, and we divided the observation period into three subperiods, of 20 working days each.

We first obtained the average spread-over Treasury for each entity, in each subperiod. We got from it the empirical default probabilities[5], knowing that, if the spread $y_i(t)$ for the maturity t is observed, and the recovery rate on i is R_i, the corresponding default probability is

$$F_i(t) = \frac{1-\exp(-y_i(t)t)}{1-R_i} \tag{2}$$

While doing this, we used the same recovery rate of the joint level, namely 40%, due to the seniority of the CDS's under exam.

The marginal default probabilities so obtained are collected, for each maturity (horizon, from 1 to 5 years) and each subperiod of observation, in the following table:

---

[5] We preferred to use raw implied marginal probabilities, instead of fitting a marginal default probability model, such as a constant intensity for each name or even a stochastic one, in order to avoid marginal model risk.

	obligor #1			obligor #2			obligor #3		
horizon	08/02/04-08/27/04	08/30/04-09/24/04	09/27/04-10/22/04	08/02/04-08/27/04	08/30/04-09/24/04	09/27/04-10/22/04	08/02/04-08/27/04	08/30/04-09/24/04	09/27/04-10/22/04
1	0.33%	0.47%	0.47%	0.53%	0.53%	0.53%	0.31%	0.37%	0.37%
2	1.63%	1.42%	1.42%	1.92%	1.76%	1.76%	0.85%	0.94%	0.94%
3	3.00%	2.84%	2.84%	3.16%	3.14%	3.14%	1.49%	1.77%	1.77%
4	3.95%	3.56%	3.56%	4.19%	4.18%	4.18%	1.89%	2.15%	2.15%
5	5.17%	4.95%	4.95%	5.57%	5.44%	5.44%	3.61%	4.07%	4.07%

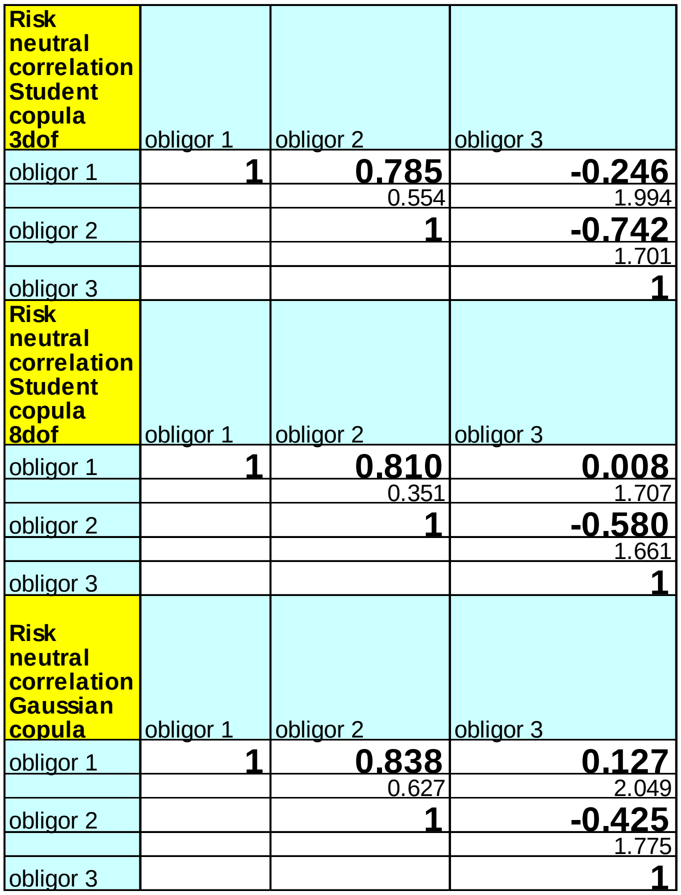
2.2 Joint default probabilities

At the joint level, we calibrated the dependence parameter for three different copulas: the Gaussian and the Student-t, with two different dof, namely 3 and 8. Both the Gaussian and Student copulas, which are the most extensively used ones in practice, include as dependence measures the linear correlation coefficients between the obligors. The Gaussian, in particular, is "the limit", as the number of degrees of freedom diverges, of the Student copula. The levels of dof have been chosen according the levels previously ascertained for stock returns.

In order to work out the joint risk neutral default probabilities and the corresponding correlation matrices, we collected the 5 year CDS ask quotes from two of the obligors, over the period mentioned above (August 2-October 22, 2004). We selected the 5 year maturity because of the greater liquidity. As mentioned above, we used a flat recovery of 40%, considering the seniority of the CDS's, and the appropriate swap curve for riskless discounting (USA or Euro). In the swap case we used daily data from Bloomberg.

For each copula choice, each subperiod and each couple of obligors, we estimated the linear correlation coefficient between the survival times, $\rho(\tau_i, \tau_j)$, so as to minimize the sum of the squared CDS pricing errors. To end up with, we took the average correlation over the whole period under exam.

The correlation matrices so obtained are reported in the table below, in bold, together with their average (minimized) pricing error, in basis points:



Risk neutral correlation Student copula 3dof	obligor 1	obligor 2	obligor 3
obligor 1	**1**	**0.785**	**-0.246**
		0.554	1.994
obligor 2		**1**	**-0.742**
			1.701
obligor 3			**1**

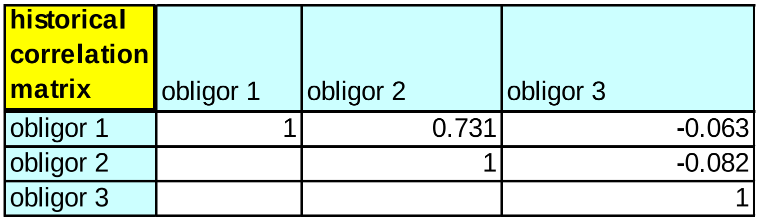
Risk neutral correlation Student copula 8dof	obligor 1	obligor 2	obligor 3
obligor 1	**1**	**0.810**	**0.008**
		0.351	1.707
obligor 2		**1**	**-0.580**
			1.661
obligor 3			**1**

Risk neutral correlation Gaussian copula	obligor 1	obligor 2	obligor 3
obligor 1	**1**	**0.838**	**0.127**
		0.627	2.049
obligor 2		**1**	**-0.425**
			1.775
obligor 3			**1**

Implied Correlation coefficients, averages over the three observation periods (aug-oct 2004),
for different copula functions (in bold),
together with their minimized pricing errors (plain)

As the number of degrees of freedom increases, our correlation coefficients increase too, as expected, since low dof compensate for low correlation. The natural selection procedure for copulas representing the link between the three couples of obligors consists, as said above, in choosing as "best" copula the one which minimizes the pricing error: in the present case, despite the fact that the errors are very close, the best fit is given by the Student copula with eight degrees of freedom. However, in the sequel, while appreciating the impact of risk neutrality on some derivatives pricing, we will use all the copulas explored so far, so as to appreciate the potential effects of a non-best copula selection.

As concerns the historical (linear) correlation matrices, we obtained them using the daily log returns over the last year (June 2003 to August 2004) from Datastream. Since the data are liable to represent fat-tailed distributions, we decided not to estimate directly the linear correlation coefficients, but to work through the Kendall's taus. The historical correlation matrix so obtained is:



historical correlation matrix	obligor 1	obligor 2	obligor 3
obligor 1	1	0.731	-0.063
obligor 2		1	-0.082
obligor 3			1

Historical correlation matrix, 2003-4

In order to discuss the impact of risk-nuetral versus historical correlation, and therefore model risk, we use them to price multivariate credit derivatives. Pricing is done using the marginal default probabilites and implied correlation matrices of the last subperiod (not reported here for brevity), together with the last day observations for the riskless rates.

3 Impact of risk neutral correlation on FTDs

This section explores the impact of moving the correlation matrix from the historical to the risk neutral ones, in pricing a FTD on the three names under exam and a FTD on a nine-asset portfolio built from them, both with maturity 5 years.

It is known that the no-arbitrage fee of a FTD is the one which equates the fee and default leg.

As for the default leg, let us assume that the FTD pays at the end of year t, in case default has occurred during it. With equal recovery on all the underlying credits, as in our example, the default leg expected present value is

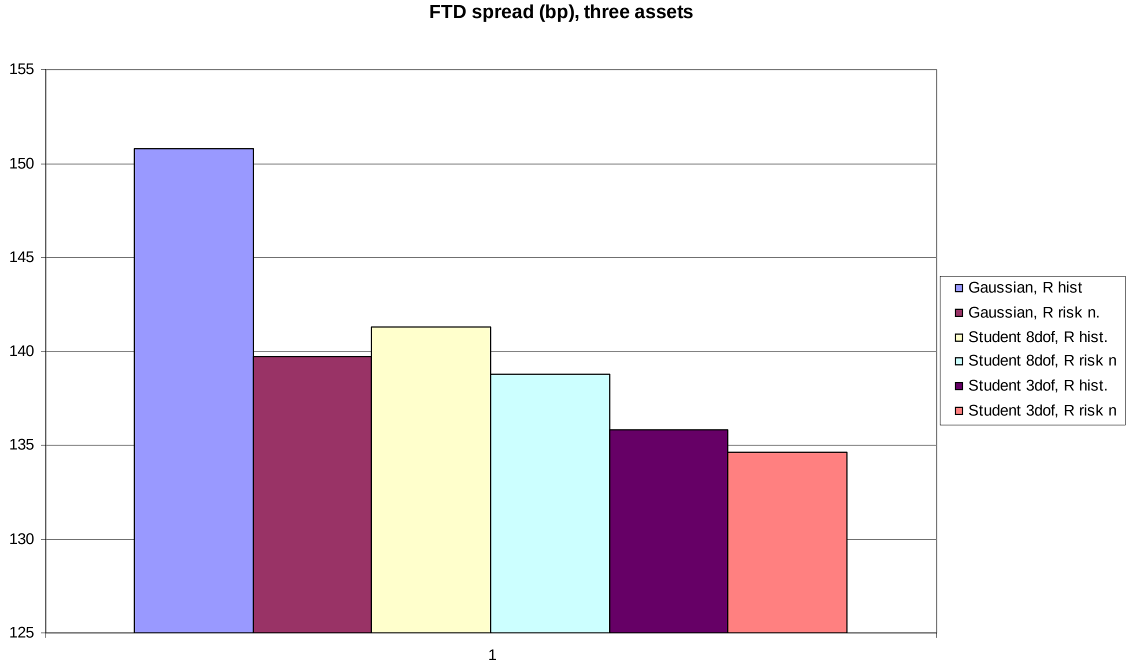
$$(1-R)\sum_{t=1}^{T} B_t \left[\breve{C}^{123}(t-1)-\breve{C}^{123}(t)\right]$$

where the copula $\breve{C}^{123}(t) := \breve{C}^{123}(1-F_1(t), 1-F_2(t), 1-F_3(t))$ is obtained from the bivariate one, as in Cherubini, Luciano, Vecchiato (2004).

The fee leg, assuming an yearly fee w, has expected present value

$$w\sum_{t=0}^{T-1} B_t \breve{C}^{123}(t)$$

Maintaining both the recovery rate and the (average) marginal survival probabilities above, we have the no-arbitrage fees presented in the following picture:



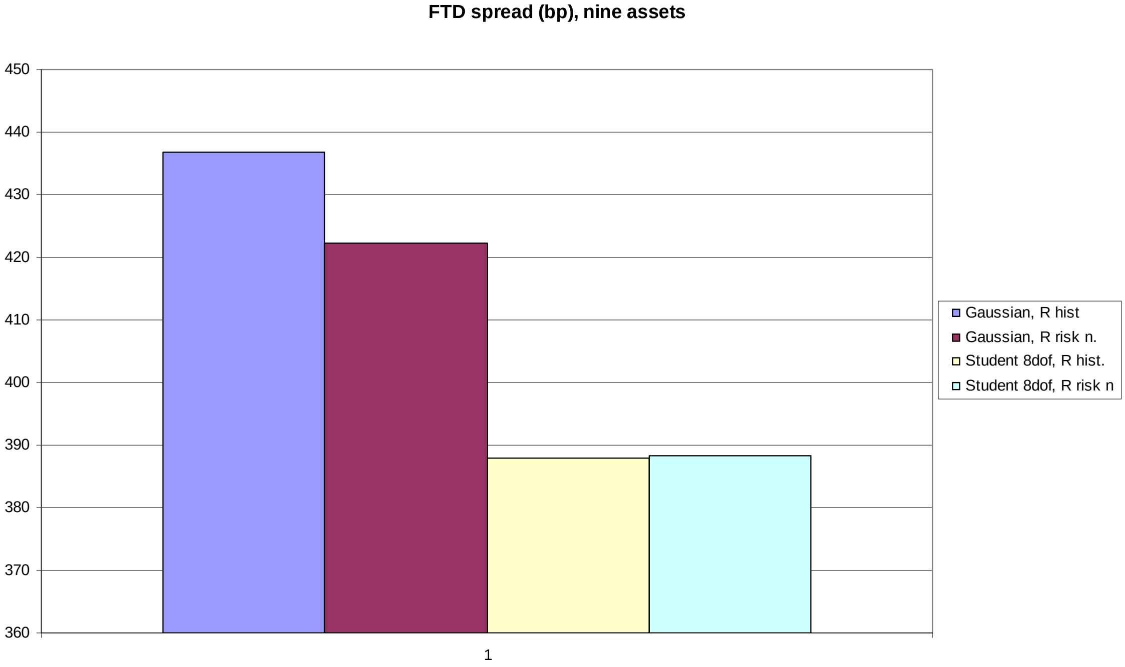
Fees of the FTD, three assets, in bp, for different copula functions and correlation matrices

The reader can notice that - with the exception of the Gaussian case, on which we will comment further below - the impact of the copula choice seems to be higher than the correlation matrix one, for a given copula. It is therefore crucial to be able to select the best fit copula (in our case, the Student with 8 dof), something which makes sense under risk neutrality, not under the historical measure.

In order to explore whether copula selection plays even a more pronounced role with more obligors, let us construct a portfolio of nine names, as follows: the first three have the same marginal distribution as obligor 1, the second three as 2, the last three as 3. As for the correlation structure, the first group has the same cross-correlation as 1 and 2, the second as 2 and 3, the last as 1 and 3. This implies, in terms of correlation matrix, building a 9×9 block matrix from each of the previous ones[6]. We disregard the Student t case with three dof, since the block correlation matrix is not positive definite. With this new portfolio, we repeat the FTD fee calculation. The results are reported in the figure below:

---

[6] As an illustrative example, the correlation matrix corresponding to the historical measure



Fees of the FTD, nine assets, in bp, for different copula functions and correlation matrices (R)

The phenomenon that we observed for the three asset case seems to be even more pronounced in this synthetic, nine-asset one: the copula choice (i.e. the selection of "the" risk neutral measure) seems to be much more relevant, in terms of the impact on FTD prices, than using the equity historical correlation instead of a time-to-default, risk-neutral one. Changing the correlation (from historical to risk neutral) while keeping the copula fixed produces a change in the fee of less than 0.06 percentage points in the Student case, less than 3.5 points in the Gaussian case. Opposite to this, changing the copula from Gaussian to Student increases the fee by 12.5% under the historical measure, by 9% under the risk neutral one. In terms of basis points, as the reader can notice from the

---

is:

1	.731	.731						
	1	.731						
		1						
			1	-0.063	-0.063			
				1	-0.063			
					1			
						1	-0.082	-0.082
							1	-0.082
								1

picture, the order of magnitude of the change goes from around ten to around fifty bp.

The anomaly observed above for the three asset case disappears: we can explain this with the fact that a Gaussian dependence structure, as is well known, fails to capture the actual default behavior - this is confirmed by the pricing errors reported above). However, this failure is weaker when the number of assets increases.

4 Impact of risk neutral correlation on CDOs

In order to appreciate further the effect of using a risk-neutral time-to-default correlation matrix instead of an equity historical one, we investigated a synthetic collateralized debt obligation (CDO) case. Each CDOs has a reference portfolio, is divided into tranches, specified by the percentage of portfolio losses they cover, and presents again a loss and a premium leg.

The loss leg for each tranche consists in loss refunding up to the upper bound of the tranche, L^+, with a deductible equal to the lower bound, L^-. If m obligors, denoted as $k_1, k_2, ...k_m$, belong to the tranche (L^+, L^-), the expected value of the refund for the tranche , $E\left(L^+, L^-, t\right)$, is:

$$E\left(L^+, L^-, t\right) =$$

$$\sum_{i=1}^{m} \max\left\{\min\left\{L^- + \frac{i}{n}\left(L^+ - L^-\right), L^+\right\} - L^-, 0\right\} P(M(t) = k_i)$$

where $P(M(t) = k_i)$ is the risk-neutral probability of having a number of defaulted firms at time t, $M(t)$, equal to k_i. This probability can be calculated from the survival copula of the n obligors, via Montecarlo simulation. Assuming a maturity of T years, and refunding evenly distributed over the year, one can evaluate the loss leg of the tranche (L^+, L^-) as

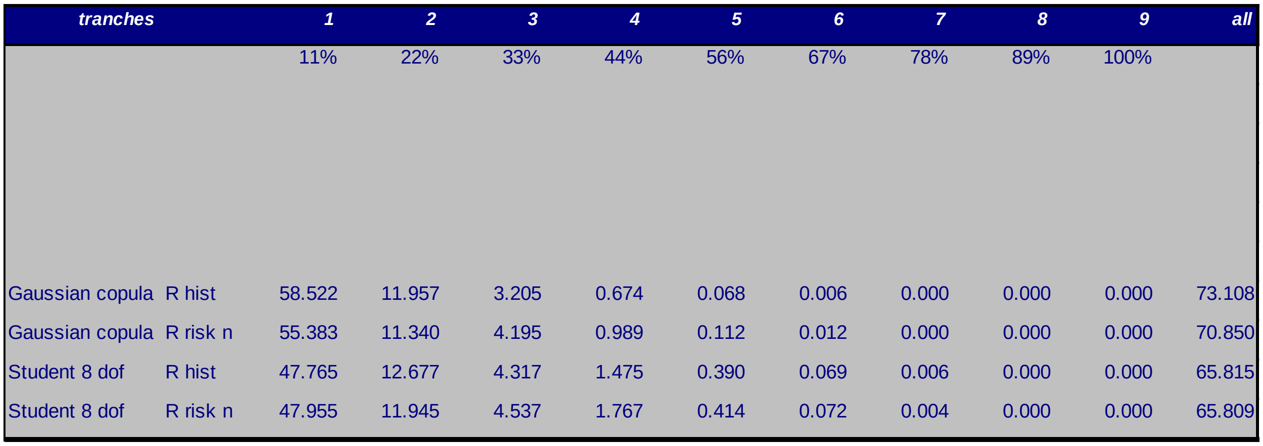
$$\sum_{t=1}^{T} B_{t-0.5}\left[E\left(L^+, L^-, t\right) - E\left(L^+, L^-, t-1\right)\right]$$

As for the fee leg of the tranche, the premium is generally proportional to the non-defaulted tranche amount. Assuming that the loss grows linearly during the year, and denoting as W the percentage fee, one has the fee leg

$$W\sum_{t=0}^{T-1} \frac{1}{2} B_t \left[1 - \frac{E\left(L^+, L^-, t+1\right)}{L^+ - L^-} + 1 - \frac{E\left(L^+, L^-, t\right)}{L^+ - L^-}\right]$$

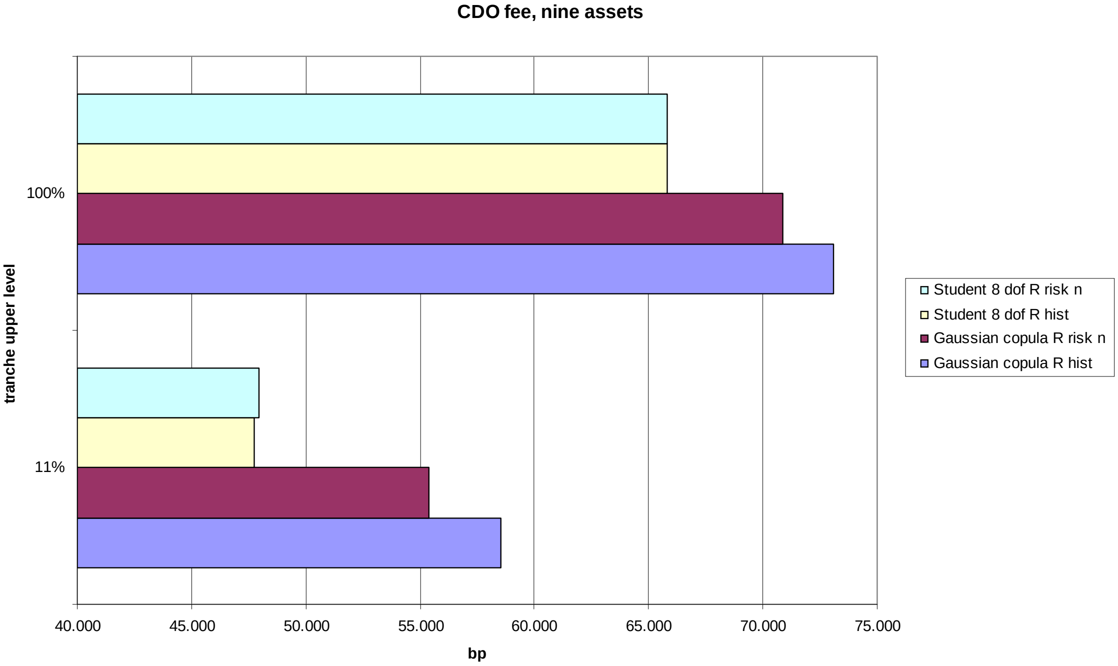
By equating the two legs, we get as usually the no-arbitrage fee.

We implemented the computation of this fee assuming $T = 5$, with the same swap term structure of the FTD, for the portfolio of nine assets described in the previous section, using the marginal distributions described there and the different risk-neutral correlation matrices, as well as the equity-historical one. We made each tranche collapse in a single asset ($m = 1$), for simplicity, so as to have 9 tranches, each covering 1/9 of the losses. The Montecarlo simulations were run using Gauss, with 1 m illion runs for each case. The results for the no-arbitrage W and the corresponding graph for the junior tranche and the whole CDO are reported in figures 3 and 4 respectively:



tranches		***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***all***
		11%	22%	33%	44%	56%	67%	78%	89%	100%	
Gaussian copula	R hist	58.522	11.957	3.205	0.674	0.068	0.006	0.000	0.000	0.000	73.108
Gaussian copula	R risk n	55.383	11.340	4.195	0.989	0.112	0.012	0.000	0.000	0.000	70.850
Student 8 dof	R hist	47.765	12.677	4.317	1.475	0.390	0.069	0.006	0.000	0.000	65.815
Student 8 dof	R risk n	47.955	11.945	4.537	1.767	0.414	0.072	0.004	0.000	0.000	65.809

CDO fee, nine assets, all tranches (separately and, in the last column, together),
for different copula functions and correlation matrices (R)



CDO fee, nine assets, junior tranche and whole CDO,
for different copula functions and correlation matrices (R)

As in the FTD case, the copula choice seems to affect the no-arbitrage fee much more than the correlation structure: if one considers all the tranches, for instance, switching from the Gaussian to the Student copula while keeping the correlation matrix fixed changes the fee by 10% with the historical correlation, by 7% under the risk-neutral one. In basis points, the change amounts to 8 and 5 bp. Switching from the historical to the risk neutral matrix while keeping the copula fixed changes the corresponding fee only by 3% under the Gaussian copula. With the Student copula the figure furtherly drops to almost zero. Most of the effect is perceived on the first tranche (up to 11%), whose protection fee changes by 11 bp while going from the Gaussian to the Student copula and adopting the historical correlations, by 10 under the implied measure. The corresponding bp changes under the same copula and different correlations are either negligible (Student case) or close to 3 bp (Gaussian case).

5 Summary and conclusions

Default correlation is an important feature of credit derivatives pricing and hedging. The current practice consists in relying on asset correlation, as proxied by the stock returns one, in order to assess default correlation. However, the correlation which is needed is a risk-neutral, not the historical one given by stock behavior.

In this paper, we showed how to use the vulnerability feature of CDS's to infer risk neutral default correlation. We provided a calibration example, in order to appreciate possible effects of using - as customary - the historical correlations instead of the market-implied, risk neutral ones. On the example, the effect of using the best-fit risk neutral correlation matrix instead of the corresponding historical one (which presumes no price of default risk), is lower than the impact of using a non-best copula, while keeping the linear correlation coefficients of the times to default fixed. In a sense, this is good news: the literature so far in fact has used the historical correlation and tried to evaluate the copula-choice effect, although under the wrong measure.

Nonetheless, any general statement needs further investigation.

References

[1] Blanco R. - Brennan S. - Marsh I.W. (2004) "An empirical analysis of the dynamic relationship between investment grade bonds and CDS", working paper, Bank of England and Banco de Espana.

[2] Cherubini U. - Luciano E. - Vecchiato W. (2004) *Copulas for Finance*, J. Wiley.

[3] Ericsson J. - Jacobs K. - Oviedo-Helfenberger R. (2004) "The determinants of CDS Premia", working paper, McGill University.

[4] Frey R. - McNeil A. - Nyfeler M. (2001) "Copulas and Credit Models", *Risk*, October, pp. 111-114

[5] Hull J. - White A. (2000) "Valuing credit default swaps II: modeling default correlations", University of Toronto, working Paper.

[6] Mashal R. - Naldi M. (2003) "Extreme events and Deafult Baskets", in *"Credit Risk Modelling"*, ed. by Gordy, M., London: Risk Books, pp. 243-50.

[7] Mashal R. - Naldi M. - Tejwani G. (2004) "The implications of implied correlations", Lehman Brothers Quantitative Credit Research, July.

[8] Turnbull S. M. (2004) "Counterparty Risk and the Effects on P&L", working paper, University of Houston

APPLIED MATHEMATICS WORKING PAPER SERIES

1. Luigi Montrucchio and Fabio Privileggi, “On Fragility of Bubbles in Equilibrium Asset Pricing Models of Lucas-Type,” *Journal of Economic Theory* 101, 158-188, 2001 (ICER WP 2001/5).
2. Massimo Marinacci, “Probabilistic Sophistication and Multiple Priors,” *Econometrica* 70, 755-764, 2002 (ICER WP 2001/8).
3. Massimo Marinacci and Luigi Montrucchio, “Subcalculus for Set Functions and Cores of TU Games,” *Journal of Mathematical Economics* 39, 1-25, 2003 (ICER WP 2001/9).
4. Juan Dubra, Fabio Maccheroni, and Efe Ok, “Expected Utility Theory without the Completeness Axiom,” *Journal of Economic Theory* 115, 118-133, 2004 (ICER WP 2001/11).
5. Adriana Castaldo and Massimo Marinacci, “Random Correspondences as Bundles of Random Variables,” April 2001 (ICER WP 2001/12).
6. Paolo Ghirardato, Fabio Maccheroni, Massimo Marinacci, and Marciano Siniscalchi, “A Subjective Spin on Roulette Wheels,” *Econometrica* 71, 1897-1908, 2003 (ICER WP 2001/17).
7. Domenico Menicucci, “Optimal Two-Object Auctions with Synergies,” July 2001 (ICER WP 2001/18).
8. Paolo Ghirardato and Massimo Marinacci, “Risk, Ambiguity, and the Separation of Tastes and Beliefs,” *Mathematics of Operations Research* 26, 864-890, 2001 (ICER WP 2001/21).
9. Andrea Roncoroni, “Change of Numeraire for Affine Arbitrage Pricing Models Driven By Multifactor Market Point Processes,” September 2001 (ICER WP 2001/22).
10. Maitreesh Ghatak, Massimo Morelli, and Tomas Sjoström, “Credit Rationing, Wealth Inequality, and Allocation of Talent”, September 2001 (ICER WP 2001/23).
11. Fabio Maccheroni and William H. Ruckle, “BV as a Dual Space,” *Rendiconti del Seminario Matematico dell'Università di Padova*, 107, 101-109, 2002 (ICER WP 2001/29).
12. Fabio Maccheroni, “Yaari Dual Theory without the Completeness Axiom,” *Economic Theory* 23, 701-714, 2004 (ICER WP 2001/30).
13. Umberto Cherubini and Elisa Luciano, “Multivariate Option Pricing with Copulas,” January 2002 (ICER WP 2002/5).
14. Umberto Cherubini and Elisa Luciano, “Pricing Vulnerable Options with Copulas,” January 2002 (ICER WP 2002/6).
15. Steven Haberman and Elena Vigna, “Optimal Investment Strategies and Risk Measures in Defined Contribution Pension Schemes,” *Insurance: Mathematics and Economics* 31, 35-69, 2002 (ICER WP 2002/10).
16. Enrico Diecidue and Fabio Maccheroni, “Coherence without Additivity,” *Journal of Mathematical Psychology*, forthcoming (ICER WP 2002/11).
17. Paolo Ghirardato, Fabio Maccheroni, and Massimo Marinacci, “Ambiguity from the Differential Viewpoint,” *Journal of Economic Theory* 118, 133-173, 2004,

published under the title “Differentiating ambiguity and ambiguity attitude” (ICER WP 2002/17).

18. Massimo Marinacci and Luigi Montrucchio, “A Characterization of the Core of Convex Games through Gateaux Derivatives,” *Journal of Economic Theory* 116, 229-248, 2004 (ICER WP 2002/18).
19. Fabio Maccheroni and Massimo Marinacci, “How to Cut a Pizza Fairly: Fair Division with Decreasing Marginal Evaluations,” *Social Choice and Welfare*, 20, 457-465, 2003 (ICER WP 2002/23).
20. Erio Castagnoli, Fabio Maccheroni and Massimo Marinacci, “Insurance Premia Consistent with the Market,” *Insurance: Mathematics and Economics* 31, 267-284, 2002 (ICER WP 2002/24).
21. Fabio Privileggi and Guido Cozzi, “Wealth Polarization and Pulverization in Fractal Societies,” September 2002 (ICER WP 2002/39).
22. Paolo Ghirardato, Fabio Maccheroni, and Massimo Marinacci, “Certainty Independence and the Separation of Utility and Beliefs,” *Journal of Economic Theory*, 120, 129-136, 2005 (ICER WP 2002/40).
23. Salvatore Modica and Marco Scarsini, “The Convexity-Cone Approach to Comparative Risk and Downside Risk”, January 2003 (ICER WP 2003/1).
24. Claudio Mattalia, “Existence of Solutions and Asset Pricing Bubbles in General Equilibrium Models”, January 2003 (ICER WP 2003/2).
25. Massimo Marinacci and Luigi Montrucchio, “Cores and Stable Sets of Finite Dimensional Games”, March 2003 (ICER WP 2003/7).
26. Jerome Renault, Sergio Scarlatti, and Marco Scarsini, “A Folk Theorem for Minority Games”, April 2003 (ICER WP 2003/10).
27. Peter Klibanoff, Massimo Marinacci, and Sujoy Mukerji, “A Smooth Model of Decision Making under Ambiguity”, *Econometrica*, forthcoming (ICER WP 2003/11).
28. Massimo Marinacci and Luigi Montrucchio, “Ultramodular Functions”, *Mathematics of Operations Research*, forthcoming (ICER WP 2003/13).
29. Erio Castagnoli, Fabio Maccheroni, and Massimo Marinacci, “Choquet Insurance Pricing: a Caveat”, *Mathematical Finance* 14, 481-485, 2004 (ICER WP 2003/14).
30. Thibault Gajdos and Eric Maurin, “Unequal Uncertainties and Uncertain Inequalities: an Axiomatic Approach, June 2003 (ICER WP 2003/15).
31. Thibault Gajdos and John A. Weymark, “Multidimensional Generalized Gini Indices”, June 2003 (ICER WP 2003/16).
32. Thibault Gajdos, Jean-Marc Tallon, and Jean-Christophe Vergnaud, “Decision Making with Imprecise Probabilistic Information”, June 2003 (ICER WP 2003/18).
33. Alfred Müller and Marco Scarsini, “Archimedean Copulae and Positive Dependence”, July 2003 (ICER WP 2003/25).
34. Bruno Bassan, Olivier Gossner, Marco Scarsini, and Shmuel Zamir, “Positive Value of Information in Games”, *International Journal of Game Theory*, forthcoming (ICER WP 2003/26).
35. Marco Dall’Aglio and Marco Scarsini, “Zonoids, Linear Dependence, and Size-Biased Distributions on the Simplex”, *Advances in Applied Probability* forthcoming (ICER WP 2003/27).
36. Taizhong Hu, Alfred Muller, and Marco Scarsini, “Some Counterexamples in Positive Dependence”, *Journal of Statistical Planning and Inference*, forthcoming (ICER WP 2003/28).

37. Massimo Marinacci, Fabio Maccheroni, Alain Chateauneuf, and Jean-Marc Tallon, “Monotone Continuous Multiple Priors”, *Economic Theory*, forthcoming (ICER WP 2003/30).
38. Massimo Marinacci and Luigi Montrucchio, “Introduction to the Mathematics of Ambiguity”, October 2003 (ICER WP 2003/34).
39. Fabio Maccheroni, Massimo Marinacci and Aldo Rustichini, “Variational representation of preferences under ambiguity”, June 2004 (ICER WP 2004/5).
40. Antonio Lijoi, Ramsés H. Mena, and Igor Prünster, “Hierarchical mixture modelling with normalized Gaussian priors”, November 2004 (ICER WP 2004/12).
41. Stephen G. Walker, Antonio Lijoi, and Igor Prünster, “Contributions to the understanding of Bayesian consistency”, November 2004 (ICER WP 2004/13).
42. Antonio Lijoi, Igor Prünster, and Stephen G. Walker, “On consistency of nonparametric normal mixtures”, October 2004 (ICER WP 2004/23).
43. Stephen G. Walker, Antonio Lijoi, and Igor Prünster, “On rates of convergence for posterior distributions”, October 2004 (ICER WP 2004/24).
44. Fabio Maccheroni, Massimo Marinacci, Aldo Rustichini and Marco Taboga, “Portfolio selection with monotone mean-variance preferences”, December 2004 (ICER WP 2004/27).
45. Fabio Maccheroni and Massimo Marinacci, “A strong law of large numbers for capacities”, *Annals of Probability*, forthcoming (ICER WP 2004/28).
46. Elisa Luciano and Elena Vigna, “A note on stochastic survival probabilities and their calibration”, January 2005 (ICER WP 2005/1).
47. Massimo Marinacci and Luigi Montrucchio, “On concavity and supermodularity”, March 2005 (ICER WP 2005/3).
48. Elisa Luciano and Elena Vigna, “Non mean reverting affine processes for stochastic mortality”, April 2005 (ICER WP 2005/4).
49. Lancelot F. James, Antonio Lijoi, and Igor Prünster, “Bayesian inference via classes of normalized random measures”, April 2005 (ICER WP 2005/5).
50. Elisa Luciano and Wim Schoutens, “A multivariate jump-driven financial asset model”, April 2005 (ICER WP 2005/6).
51. Elisa Luciano, “Calibrating risk-neutral default correlation”, May 2005 (ICER WP 2005/12).