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Private Money as a Competing Medium of Exchange

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Abstract

Using a relatively mild restriction on the beliefs of the $\alpha-\text{MMEU}$ preference functional, in which the decision maker's degree of ambiguity and degree of pessimism are each parameterized, we present a rather general theory of religious choice in the decision theory tradition, one that can resolve dilemmas, address the many Gods objection, and address the inherent ambiguity. Using comparative static analysis, we are able to show how changes in either the degree of ambiguity or the degree of pessimism can lead a decision maker to "convert" from one religion to another. We illustrate the theory of religious choice using an example where the decision maker perceives three possible religious alternatives.

Private Money as a Competing Medium of Exchange[*]

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Short Abstract

When it is costly for a store (i.e., clearinghouse) to mediate trade, a speculative demand for money may arise. Trading money to another agent for a good, rather than trading good for good through a store, allows the money holder to obtain the good while avoiding the fee the store must charge to cover its costs. Under certain conditions, it is profitable for a store to issue money if agents are willing to accept it, and issuing money is necessary to maintain control of the mediation market, if the mediation market is contestable. In our monetary equilibrium, money is privately issued; it is fiat in character; and it circulates in a quantity determined by agent expectations and market forces. This modeling approach, which blends the Walrasian and search theoretic approaches, is relatively parsimonious and offers a useful new avenue for examining the role of money as it competes as a medium of exchange.

Long Abstract

We extend the search theoretic literature by presenting a model in which a determinant quantity of private money arises endogenously, with the quantity of money being dependent upon the confidence people have in it. We do this in a model where money is not needed to resolve the double coincidence of wants problem because there is a competing medium of exchange that can resolve it. In our model, money arises because it can increase expected utility by reducing expected transactions costs.

Our model is a blend of the Walrasian and search theoretic approaches. We start with the standard, decentralized search theoretic barter economy. If there is no money or other medium to facilitate trade, only paired agents achieving a double coincidence of wants will barter and earn utility. Our approach begins by assuming, after agents participate in the decentralized market, they may trade in a centralized market, through a clearing house we call a store. To maintain parsimony, the technology of the store is not endogenously derived, but rather is assumed purchasable at a cost. We show that, when the cost of the store technology is low enough, the introduction of a store is profitable. The assumption that the centralized market is contestable then allows us to define a "store equilibrium," where store profits are arbitraged away, the store is a Pareto improvement, and each agent's expected utility is as high as it can be, given the store's operation cost.

Our primary contribution is to show that a monetary innovation can upset the store equilibrium just described. The key insight is that a speculative demand for money may arise when it is costly for a central clearing house to mediate trade. Trading money to another agent for a good, rather than trading good for good through a store, allows the money holder to obtain the good while avoiding the fee the store must charge to cover its costs. Under certain conditions, it is profitable for a store to issue money if agents are willing to accept it, and issuing the money is necessary to maintain control of the contestable mediation market. In our monetary equilibrium, money is privately issued, fiat in character, circulating in a quantity determined by agent expectations and market forces.

This modeling approach is relatively parsimonious and offers a useful new avenue for examining the role of money as a competing medium of exchange.

1. Introduction

In reality, money is not the only medium of exchange. Rather, it coexists and competes with other mediums. For two competing mediums to coexist, each must have a relative advantage. What is its relative advantage of money?

In the model we develop here, money's relative advantage is it allows traders to avoid the markup a competing intermediary must charge to cover its costs. We call the competing intermediary a "store," and it is a centralized clearing house comparable to that typically associated with the Walrasian auctioneer. [1] The relative advantage of trading through the store is a transaction can be made with certainty, whereas seeking to trade through money is risky. A money holder foregoes consumption in the present, hoping to trade away the money for good in the next period. However, monetary trade only occurs when the money holder finds someone with a desired good and the trading partner is willing to accept money. In our monetary equilibrium, monetary transactions and trade through the store coexist because the transactions costs of store trade just balance the risk associated with holding money.

Because agents hold money with the hope of finding a good deal on purchasing good, the demand for money is speculative. We derive this speculative demand using a parsimonious extension of the Kiyotaki and Wright (1993) search theoretic money model. [2] This is of interest because the demand for money in such models typically arises from the transactions motive, for a double coincidence of wants problem is naturally present. Our model is unique in that money arises from a speculative motive, even though money resolves the double coincidence of wants

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[1] We use the term store for our mediator primarily to avoid confusing our mediation conception with other conceptions in the literature, including trading post, middleman, and clearing house.

[2] Thus, our monetary theory is one with a microfoundation, as defined by Shi (2006, p. 646): That which "derives the role of money from the trading environment."

problem when it is used to complete transactions. This novel result stems from introducing a competing medium of exchange.

Starr (2003, p. 456) has suggested that "the focus on the absence of double coincidence of wants – as distinct from transaction costs – as an explanation for the monetization of trade may miss a significant part of the underlying causal mechanism."[3] He presents a general equilibrium model, where there is no double coincidence of wants problem, and shows that a form of commodity money arises when transactions costs motivate traders to trade through one particular good as they seek other goods. Because our model includes both the decentralized market of the search theoretic context and the centralized market of the general equilibrium context, we can relate money's role as a medium of exchange to its role as a reducer of transactions costs. Our model suggests that resolving the double coincidence of wants problem is not the fundamental role of money, for the competing intermediary also resolves that problem. Rather, our model suggests that money's fundamental role money is to reduce transactions costs.

Our model is also relatively unique in that money arises endogenously from the microfoundation of the model. It is not public money helicopter dropped by government. Rather, it is private money issued by the existing intermediary. Under particular conditions, the store can earn a windfall profit by creating money and issuing it to traders. This motivates a money supply. A demand for money arises when traders expect as much utility from trading through money than from trading through the store. Our equilibrium existence theorem delineates the environmental conditions under which money can be profitably introduced and

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[3] Starr's model captures Tobin's (1980) suggestion that "The use of a particular language or a particular money by one individual increases its value to other actual or potential users." Starr further uses his model to explain the circulation of fiat money by imposing the assumption that only the government sponsored money can be used to pay taxes. When trading volume reduces the average transactions cost, the volume of trade necessitated by the payment of taxes can give the government issued, intrinsically useless asset a low enough transactions cost that it becomes the economy's money.

circulate as a medium of exchange that competes with the clearing house intermediation provided by the store.

In our model, the money is fiat in character. Shi (2006, p. 644), referring to Wallace (1980), defines fiat money as a circulating object that is (a) intrinsically useless and (b) not backed by government policy. The money in our model is fiat money by this definition.   The circulating money medium is an intrinsically useless IOU issued by the store. While the store "backs" the money by agreeing to exchange good for money whenever a trader so desires, no trader will ever accept money expecting to return it to the store, for doing so is not rational. Rather, money is only held with the expectation of trading it to another agent for good in the decentralized market.

As in the typical fiat money model, if agents do not have enough confidence that others will accept money, then no monetary equilibrium will arise because holding money is too risky. What distinguishes our model is what happens when the threshold level of confidence is reached. In the typical model, the confidence agents have in money must be consistent with the amount of money issued by government. However, because money is issued privately in our model, according to the profitability of doing so, the quantity of private money adjusts to the confidence agents have in money. Consequently, the equilibrium quantity of money in our model is parameterized by the confidence agents have in money, so the quantity of money circulating in equilibrium is higher when confidence in money is higher.

Money in our economy may or may not be essential. [4]   This is a standard finding in search models of money because holding money precludes production by assumption. Consequently, holding

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[4] Shi (2006, p. 644) defines money as being "essential" if it "improves the efficiency of resource allocations relative to an economy without money." He argues essentiality is a property to be sought in a model of money because (a) we want to understand how much money can improve welfare, and (b) a model with non-essential money is not likely representative because it would not likely stand the test of time.

money generates a negative externality that is not internalized: The economy's output level is reduced and monetary trade becomes less likely. Consistent with standard money models, when barter is easy enough in our model, or when agents don't discount the future enough, more money only reduces welfare. Alternatively, when barter is difficult enough or agents discount the future enough, the effect of money on welfare in our model depends upon the operating cost level of the store. At a low store operating cost, more money only increases welfare. At a high store operating cost, more money only decreases welfare. At intermediate store cost levels, there is a welfare maximizing quantity of money. This optimal quantity of money would arise if and only if the willingness of agents to accept money happens to be just right. In summary, for money to be essential in our model, it is necessary that the store have a relatively low operating cost, and either barter must be relatively difficult or agents must discount the future at a relatively high rate.

The organization of the paper is as follows. Section 2 reviews the relevant literature. Section 3 presents our economy's microfoundation, which we label the Barter Economy. Section 4 reviews how the introduction of fiat money into this Barter Economy by government can facilitate trade and enhance welfare, if agents are sufficiently willing to trade for and hold fiat money. In section 5, we return to the Barter Economy and introduce a store that we assume can mediate trade when barter fails, which provides the clearing house trading environment we need for section 6. In section 6, we develop a model where the existing store in a contestable mediation market recognizes the need to issue a form of fiat money in order to maintain control of the mediation market. We define an equilibrium for this money-store economy, provide an existence theorem, and a stability theorem. In section 7, we examine welfare issues, and we conclude with some discussion in section 8.

2. Related Literature

Our model is an extension of the Kiyotaki and Wright (1993) search theoretic model,[5] most closely related in form to the presentation of Ljungqvist and Sargent (2000, pp. 602-604). In this respect, our work is less related to that of Starr (2003) mentioned in the introduction, and more related to Ritter (1995), Williamson (1999), Cavalcanti et al (1999), and Berentsen (2006). Ritter (1995, p. 140) claims the KW model is incomplete because it does not explain where money comes from. He shows that if a coalition of KW agents is large enough, it will have the credibility and incentive to successfully issue fiat money.[6] Using a variant of Ritter's model, Berentsen (2006) demonstrates the relevance of public knowledge, and the ability to punish the money issuer, for supporting the circulation of money when it is being issued by a private, revenue-maximizing monopolist. Williamson (1999) develops a model where banks issue bank notes to mitigate a mismatch between the receipt of investment payoffs and the desired timing of consumption. Cavalcanti, et al. (1999) create a structure where banks are motivated to issue bank notes to obtain profit from float; the bank can consume when the note is issued, but does not have to pay for the consumption until the note is redeemed.[7] While these explanations for the introduction of money are innovative, none of them is based upon the idea that money exists and circulates because it effectively competes with another medium of exchange, which is the idea behind our model.

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[5] Kiyotaki and Wright (1989) pioneered this approach to monetary theory, delineating the circumstances under which commodity money would endogenously arise to ameliorate the trading friction naturally present in the trading environment.

[6] In more detail, Ritter (1995) first shows no individual KW agent has sufficient incentive to issue money, even though there would be individual benefit if the social contrivance could arise. However, when a coalition of agents can bond together, with the objective of maximizing the utility of the average member in the coalition, the incentive problem can be overcome. The coalition's optimization problem recognizes the over issuance of money hurts the long run utility of money, whereas the individual optimization problem does not. The size of the coalition determines its credibility in issuing the fiat money and, as in the KW model, credibility must reach a threshold for money to circulate. Thus, the ability for fiat money to arise and circulate ultimately depends upon the coalition being large enough. The coalition need not be a government, but it is reasonably interpreted as such.

[7] Cavalcanti, et al. (1999) model banks as being required to have deposit credits on hold with a clearing house that are debited when traders redeem the notes issued by the bank. They show that reserve requirement keeps banks from over-issuing the private money notes, which would cause a break down of the banking system.

Shi (2006) identifies three ways researchers have introduced mediums of exchange that can compete with money in the KW framework: mechanism design, bilateral credit, and middlemen. [8]   Our store is tangentially related to each. Mechanism design involves creating a resource allocation mechanism, compatible with the incentives of the agents, a form of public record keeping. Using this approach, Kocherlakota (1998) shows money can play an essential role only if the public record keeping device is imperfect. Bilateral credit involves giving an agent the ability to issue a nontransferable IOU to complete a trade when there is only a single coincidence of wants[9]. Credit is an imperfect substitute for money because the creditor must monitor the debtor, and not consume, until the debtor is in a position to repay the IOU. This allows bilateral credit and money coexist in equilibrium, and Shi (1996) shows the introduction of bilateral credit enhances efficiency.   Adding middlemen has involved either assuming middlemen can improve information about quality of the good being purchased by making an investment, [10]   or assuming middlemen improve the likelihood of the match by investing to acquire capacity to store more types of goods.[11]   With either of these approaches, money and trade through middlemen can coexist when the investment cost for middlemen is not too high.

When we allow our agents participate in both a decentralized and centralized market, we are adopting an approach initiated by Lagos and Wright (2005). While the microfoundational workings of the decentralized market are explained, those of the centralized market are not. This modeling choice is naturally open to the criticism that the model does not have a complete microfoundation. However, the

[8] Howitt (2005) presents yet another approach. He uses a creative modification of the Starr (2003) framework, where profit seeking firms create shops that reduce agent search costs by easily located. This approach can be viewed as providing a microfoundation for the trading post story of the type introduced by Shapley and Shubik (1977). Because Howitt's shops have fixed costs of operation and can only mediate the trade of a limited number of goods, money can arise as an additional medium of exchange, for firms must accept money in exchange for goods in order to generate enough trading volume to cover the fixed costs.

[9] This discussion of bilateral credit is based upon Shi (1996). See Li (2001) and Corbae and Ritter (2002) for other examples.

[10] See Li (1999).

[11] See Johri and Leach (2002) and Schevchenko (2004).

tradeoff is greater parsimony. [12] The parsimony not only keeps our primary results from being obscured, but it also leaves open a variety of interesting extensions that would not otherwise be possible. We discuss possible extensions in our concluding remarks.

3. The Barter Economy

The Barter Economy presented here is a discrete time version of the Kiyotaki and Wright's (1993) model, very closely related to that described by Ljungqvist and Sargent (2000, pp. 602-604). There is a continuum of infinitely lived agents and a continuum of goods, each normalized to one. Agents are specialized in consumption. The exogenous variable x denotes the proportion of goods that can provide utility to a given agent, and x also equals the proportion of agents that can obtain utility from consuming any particular good. The utility u of consuming y units of a consumable good is given by

$u(y) = y$.[13]

Agents are also specialized in production. Production is a random draw from the continuum of goods, which yields one unit of a particular good to the producing agent. The produced good does not provide utility to the agent, but can be stored without cost. All agents initially produce, so all begin the first period with good. Production may only occur at the moment when one time period proceeds to the next.

In each period, there is a social interaction modeled as a bilateral random matching process. The probability that one agent is matched with another is θ, where

(1) $\quad 0 < \theta \le 1.$

Because no good has any special characteristics, the probability that a good will be accepted in exchange is independent of the good held by an agent. Thus, x is the probability that any given trader will want any given good in exchange (i.e., the probability of a single coincidence of wants), and x^2 is the probability an agent in a match will experience a double coincidence of wants. It is assumed

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[12] He, Huang, and Wright (2005, p. 639) similarly contend that introducing a centralized market is a useful simplification, one that allows them to more easily develop and examine a banking sector in the KW framework.
[13] This utility function is beyond what is needed here, where there is no technology available for dividing goods, but it will useful once a store technology is introduced that allows the store to divide goods.

(2) $0 < x < 1$,

so a double coincidence of wants may or may not occur when two agents are matched.

Each agent experiences the disutility ε whenever a good is accepted in trade, and

(3) $0 < \varepsilon < 1$.

This positive transactions cost rules out the formation of commodity money. Trading for a non-consumable good would generate the cost ε without positively affecting future trading or consumption opportunities. Thus, an agent will only trade for a consumable good. Further, an agent will trade and consume if a double coincidence of wants occurs because the agent is allowed produce after consumption and enter the next period with a unit of good. Because no agent can divide a good, each agent trades one unit of good for one unit of good in a barter transaction and experiences the net utility $u - \varepsilon = 1 - \varepsilon$.

Each agent discounts the future at the same constant rate. The discount factor β denotes the current period value of one unit of utility received in the next period. Agents discount the future to some extent, but not entirely, implying

(4) $0 < \beta < 1$.

Looking forward from any time period, the value function for an agent in the Barter Economy is

$V_G^{\mathbf{B}} = \theta x^2 (1 - \varepsilon) + \beta V_G^{\mathbf{B}}$, which implies each agent expects a lifetime discounted expected utility of

(5) $V_G^{\mathbf{B}} = \dfrac{\theta x^2 (1 - \varepsilon)}{1 - \beta}$.

4. The Kiyotaki-Wright Fiat Money Economy

Kiyotaki and Wright (1993) show that, if agents have enough confidence that fiat money will be accepted in exchange for good, then the circulation of fiat money can be supported, transforming the Barter Economy into a Fiat Money Economy and improving welfare when barter is difficult enough. Following the presentation of Ljungqvist and Sargent (2000), we present a Fiat Money Economy here that we can use for comparison purposes below.

Fiat money is introduced to the Barter Economy from an outside source; e.g. government. A

fraction $\overline{M} \in [0,1)$ of agents are offered one unit of a durable item with no intrinsic value. Agents who

accept the fiat money cannot produce. This implies agents within a given period can be divided into two

types: Good holders and money holders.   No agent is forced to hold money, but rather must choose to

hold it. For a Fiat Money Economy to arise, a fraction of agents must choose to hold money.

Let V_G^M and V_M^M denote the values associated with beginning a period with good and money,

respectively.   The Bellman equations for these two states can be written

(6) $V_G^M = \theta[1-M]x^2[1-\varepsilon+\beta V_G^M] + \theta Mx \max_p[p\beta V_M^M + (1-p)\beta V_G^M] + [1-\theta(1-M)x^2 - \theta Mx]\beta V_G^M$

(7) $V_M^M = \theta[1-M]xP[1-\varepsilon+\beta V_G^M] + [1-\theta(1-M)xP]\beta V_M^M$.

A good holder barters, or exchanges good for money, or holds over good to the next period, which is why

there are three terms on the right side of equation (6). The first term is associated with barter, which

occurs with probability $\theta(1-M)x^2$, when the good holder is matched with another good holder and a

double coincidence of wants occurs.   In this event, $1-\varepsilon$ units of utility are obtained from current

consumption and βV_G^M units of utility are obtained from all future transactions, associated with

producing and starting the next period as a good holder. The second term is associated with the

possibility of trading good for money, which occurs with probability θMx when the good holder is

matched with a money holder and the money holder wants the good held by the good holder. The good

holder chooses the probability $p \in [0,1]$ of accepting money that maximizes the expected future utility of

holding money, recognizing that accepting money will yield βV_M^M while holding over good will yield

βV_G^M .   If no barter and no trade for money occur, then the good holder holds the good over to the next

period, which yields βV_G^M with probability $1-\theta(1-M)x^2 - \theta Mx$.

A money holder trades the money for good or holds the money over to the next period, which is

why two terms appear on the right side of equation (7). To be able to trade money for good, the money

holder must be matched with a good holder and the good holder must be holding a consumable good.

This occurs with probability $\theta(1-M)x$. However, the good holder may not accept the money. Let

$P \in [0,1]$ denote the probability that a randomly chosen good holder will accept money in exchange for

good, and assume this is the money holder's belief regarding the probability that money will be accepted.

It follows that the money holder believes money will be traded for good with probability $\theta(1-M)xP$.

This event provides $1-\varepsilon$ units of utility in current period consumption and βV_G^M units of utility from

future transactions. The alternative no trade event occurs with probability $1 - \theta(1-M)xP$, which

yields βV_M^M units of utility, associated with starting the next period as a money holder.

An equilibrium for the Fiat Money Economy is a state (P, M) such that $P = p$, so expectations

are rational, and $M \in [0, \overline{M}]$, so the fraction of money holders is consistent with the fraction of agents

initially provided the option to hold money. Following Kiyotaki and Wright (1993), Ljungqvist and

Sargent (2000) show that there are three values for the belief P consistent with the rational expectation

$P = p$. These are $P = 0$, $P = 1$, and $P = x$. The corresponding fractions of agents holding money in

equilibrium are $M = 0$, $M = \overline{M}$, and $M \in [0, \overline{M}]$. These results follow from the fact that $P < x$

implies $V_M^M < V_G^M$, and agents maximize by choosing $p = 0$ (never accept money), while $P > x$ implies

$V_M^M > V_G^M$ and agents maximize by choosing $p = 1$ (always accept money). Thus, we learn that the

circulation of fiat money can be supported if people have enough faith in it.

The introduction of money as a medium of exchange can improve welfare, but need not. Using

the welfare criterion $W = \overline{M}V_M^M + (1-\overline{M})V_G^M$ to examine well being in the pure monetary equilibrium

(where $P = 1$), Ljungqvist and Sargent (2000) show that the introduction of fiat money can increase

welfare as long as $x < 1/2$. That is, money enhances well-being, or is "essential," if barter is difficult

enough. Assuming $x < 1/2$, the quantity of money that maximizes societal welfare is

$\overline{M}^* = 1 - 2x/(2-2x)$, and maximum societal welfare is $W^* = [\theta(1-\varepsilon)x(1-x)]/[(1-\beta)(2-2x)^2]$.

5. The Store Economy

Here, rather than introducing money, we introduce a highly stylized store into the Barter Economy. A store mediates trade by accepting the one unit of good from an agent, providing the amount γ of some other good in exchange. The store's technology, which the store must purchase at a positive cost, allows the store to mediate exchange. This technology also allows the store to divide goods so γ need not equal one. The store must markup the goods it buys to cover its costs, and the store seeks profit. The fee $1-\gamma$ an agent must pay for trading through the store is the same for each good and is public information, so agents make their plans observing γ in the range

(8) $0 \leq \gamma < 1$.

The positive markup implies trading through the store is a second best option for agents, adopted if and only if the social interaction does not yield a barter transaction. We assume the store's technology allows the store to fully coordinate the trade of agents who do not barter. A rationale for this assumption is the following. When the $1-\theta$ agents are not matched, it is not because they cannot be matched, but because they do not happen to meet another agent. An advantage of a store is people know its location. If the $1-\theta$ agents not matched in the social interaction go to the store, the store can ensure each is matched with another. The store would also have the opportunity to coordinate trade for the $\theta\left[1-x^2\right]$ agents who are matched with another in the social interaction but do not experience a double coincidence of wants. Thus, there will be $1-\theta x^2$ agents who arrive at the store because they do not complete a barter transaction. Because production and matching each occur at random, the preferences of these $1-\theta x^2$ agents would be randomly distributed, just as all agents preferences were distributed before the social interaction. If these agents could have experienced a double coincidence of wants in the social interaction, but did not because the matching process is random rather than selective, then it is reasonable

14

to think that the store could, by paying a cost to implement a selective matching process, coordinate the trade of any measure of agents.

Assuming trade through the store also generates the transactions cost ε for agents, no agent would be willing to trade through the store if $\gamma < \varepsilon$. Trading through the store in this case would produce a net current period loss of utility, while holding over good to the next period would not. Thus, the store must set the trading rate so

(9) $\gamma \geq \varepsilon$.

Given the assumptions just described, the value function for an agent in the Store Economy is

(10) $V_G^S = \theta x^2 \left[1 - \varepsilon + \beta V_G^S \right] + \left[1 - \theta x^2 \right] \left[\gamma - \varepsilon + \beta V_G^S \right]$.

The first term on the right side of (10) is the value associated with the barter possibility, while the second term is the value associated with the opportunity to trade through the store. Solving for V_G^S , each agent expects the lifetime discounted expected utility

(11) $V_G^S = \dfrac{\theta x^2 \left[1 - \varepsilon \right] + \left[1 - \theta x^2 \right] \left[\gamma - \varepsilon \right]}{1 - \beta}$.

Comparing (11) and (5), we see that the introduction of the store increases the expected lifetime utility of the agent if and only if $\gamma > \varepsilon$.

The store captures as revenue the fraction $1 - \gamma$ of each unit of good delivered by an agent, and $1 - \theta x^2$ agents trade through the store. Therefore, measured in units of utility, the expected total revenue of the store is $\left[1 - \gamma \right] \left[1 - \theta x^2 \right]$.

The store operation cost is measured in units of utility, and can be paid by the store using any good. Because the form of the cost will matter in the monetary economy below, we consider two cases. In one case, the store's operating cost is the fixed cost

(12) $\bar{c} > 0$.

In the other case, the store's operating cost is the variable per unit cost

(13) $\hat{c} > 0$.

Using these cost assumptions, the store's profit level is $\pi = [1 - \gamma][1 - x^2] - \bar{c}$ in the fixed cost case and

$\pi = [1 - \gamma - \hat{c}][1 - x^2]$ in the variable cost case.   A store is viable if and only if it earns a non-negative

profit; i.e., $\pi \geq 0$.

The store is a profit maximizing entity. If a profit is earned it would be in the form of good that

could be distributed back to the owners of the firm, which would enhance the well being of the agent

owners. It is convenient to examine a contestable market equilibrium. When the mediation market is

contestable, the existing store must prevent other stores from capturing the market. To do this, it turns out

that the existing store must set its mark up so as to earn zero profit. If the existing store does not earn a

profit, then there is no need to model how profits are distributed, which simplifies the model. Given

conditions (1)-(4) and (12)-(13), a Contestable Market Equilibrium for the Store Economy is a value for

γ such that (i) the store is viable; i.e., $\pi \geq 0$, (ii) the store attracts customers; i.e., $\gamma \geq \varepsilon$, (iii) no other

store can enter and capture the mediation market from the existing store; i.e. there is no $\gamma' > \gamma$ yielding

store profit $\pi' \geq 0$, and (iv) the store's profit π is maximized under conditions (i)-(iii).

Theorem 1 (Existence of a Contestable Equilibrium for the Store Economy):   A Contestable Market

Equilibrium for the Store Economy exists if and only if

(16) $\bar{c} \leq [1 - \theta x^2][1 - \varepsilon]$ (fixed cost case)

(17) $\hat{c} \leq 1 - \varepsilon$ (variable cost case)

Proof: See the Appendix

In either the fixed or variable cost case, the store's desire to maximize its profit is superseded by

its need to retain control of the contestable market. The trading rate γ in the contestable market

equilibrium is as attractive as it can be for agents because it must be set so profit equals zero to prevent

entry. In the fixed cost case, the equilibrium trading rate is $\gamma = [[1 - \theta x^2] - \bar{c}]/[1 - \theta x^2] = 1 - \bar{c}/[1 - \theta x^2]$.

This rate worsens as the double coincidence likelihood increases because the store has fewer customers over which to spread the fixed cost. Using this trading rate to compare life in the Store Economy to that in the Barter Economy, we find $V_G^S - V_G^B = \left[\left[1 - \theta x^2\right]\left[1 - \varepsilon\right] - \bar{c}\right]/\left[1 - \beta\right]$, which implies a store will improve well-being when it can arise. In the variable cost case, the equilibrium trading rate $\gamma = 1 - \hat{c}$ is independent of the volume of trade through store. Using this rate, we find

$V_G^S - V_G^B = \left[\left[1 - \varepsilon\right] - \left[1 - \theta x^2\right]\hat{c}\right]/\left[1 - \beta\right]$. Again, for the variable cost case, we find the store will enhances well-being if it can arise.[14]

In both the fixed cost and variable cost cases, as the cost of store operation goes to zero, the equilibrium trading rate γ goes to one. With zero store operation costs, the store in a contestable market provides a perfectly efficient medium of exchange, and the expected lifetime utility of an agent is

$V_G^S = \left[1 - \varepsilon\right]/\left[1 - \beta\right]$.

6. A Money-Store Economy

The existence theorems for the Store Economy indicate there is reason to think a store would naturally arise in the Barter Economy. Individual agents have an incentive to introduce it, earning economic profit out of equilibrium until the competition from the assumed contestability reduces it to zero in equilibrium. This begs another question. If the store solves the double coincidence of wants problem in the Barter Economy, then is there any niche left for money? We will now show there is a remaining niche. By recognizing this niche, we explain why money might naturally arise as a second medium of exchange in the economy. The remaining niche is a speculative niche, for holding money as a medium of exchange potentially allows the money holder to avoid the store's markup.

Suppose, in addition to being able to trade good for good, an agent can also choose to accept an IOU from the store valued at γ units of desired good. Because the agent discounts future consumption, there is no reason to hold the IOU unless the store offers some additional incentive. The incentive we

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[14] Store viability implies $\hat{c} \leq 1 - \varepsilon$ and we know $0 < 1 - \theta x^2 < 1$. Together, these two conditions imply $1 - \varepsilon > \left[1 - \theta x^2\right]\hat{c}$, so $V_G^S - V_G^B = \left[\left[1 - \varepsilon\right] - \left[1 - \theta x^2\right]\hat{c}\right]/\left[1 - \beta\right]$ implies $V_G^S > V_G^B$.

explore is a store policy that allows the IOU to be traded, so the agent who redeems the IOU at the store need not be the agent who initially receives the IOU from the store. Transferable modern day store gift cards are an example of such an IOU. When the IOU is held from one period to the next, it becomes money held as a store of value. When the IOU is traded from one agent to another, it becomes money circulating as a medium of exchange.

As is typical in search theoretic models of money, assume production cannot occur as long as money is held. Let M denote the measure of agents holding money, so $1-M$ is the measure not holding money. Figure 1 presents the trading possibilities for an agent in the Money-Store Economy, depending upon whether the agent enters the given period holding money or holding good.

Figure 1: The Money-Store Economy
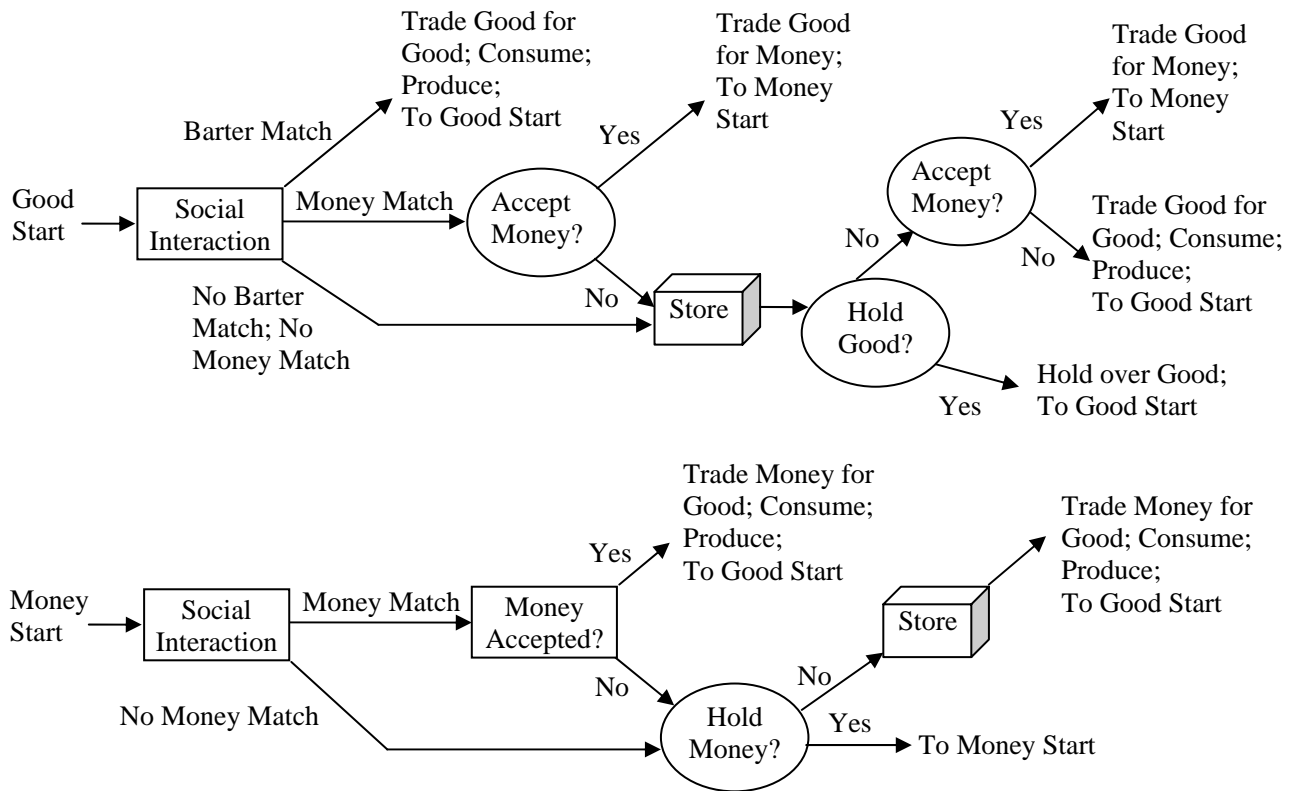
Let V_M and V_G denote the lifetime discounted expected utility of an agent who starts a period holding money and good, respectively. The value function for a money holder can be written as

18

(18) $\quad V_M = \theta[1-M]xP[1-\varepsilon+\beta V_G] + [1-\theta[1-M]xP]\underset{p}{Max}\{p\beta V_M + [1-p][\gamma-\varepsilon+\beta V_G]\}.$

The first term is associated with the possibility of trading money for good. The probability is $\theta[1-M]$ that an agent holding money is matched with an agent holding good, and x is the probability that the agent holding money wants the good held by the other. Thus, the probability of a "money match" is $\theta[1-M]x$. When a money match occurs, the money holder moves to a chance node, where the agent holding good decides whether or not to accept the money. We assume all agents, including the money holder, subjectively perceive other agents will accept money with probability $P \in [0,1]$. If the money is accepted, the money holder trades money for good, consumes, produces, and enters the next time period holding good. This transaction occurs with probability $\theta[1-M]xP$ and yields $1-\varepsilon+\beta V_G$ units of lifetime expected utility, explaining why $\theta[1-M]xP[1-\varepsilon+\beta V_G]$ appears in the value function.

The second term of value function (18) is associated with a money holder who is not able to trade money to another agent for good. This disappointing outcome occurs with probability $1-\theta[1-M]xP$, and the agent holding money proceeds to a choice node, where a decision must be made as to whether to continue holding money or not. This choice involves solving the problem

(19) $\quad \underset{p \in [0,1]}{\max}\{p\beta V_M + [1-p][\gamma-\varepsilon+\beta V_G]\},$

where p is the probability with which the agent chooses to continue to hold money. When the agent continues to hold money, consumption is forgone, no production occurs, and the next period is experienced as a money holder, which yields an expected lifetime utility of βV_M. Alternatively, when the agent decides against holding money, γ units of good are received from the store, consumption occurs, production occurs, and the next period is experienced as a good holder, which yields the expected lifetime utility of $\gamma-\varepsilon+\beta V_G$. When $\gamma-\varepsilon+\beta V_G < \beta V_M$, the pure money holding strategy $p=1$ is best. When $\gamma-\varepsilon+\beta V_G > \beta V_M$, the pure strategy of not holding money $p=0$ is best. When $\gamma-\varepsilon+\beta V_G = \beta V_M$, the agent is indifferent between holding money and not, so any mixed

19

strategy $p \in [0,1]$ is as good as any other such strategy.   Regardless of the solution of problem (19), the second term of the value function, $[1-\theta[1-M]xP]Max_p\{p\beta V_M, [1-p][\gamma-\varepsilon+\beta V_G]\}$, represents value an agent holding money expects when the agent is unable to trade money to another agent for good. [15]

The value function for a good holder can be written as

(20) $V_G = \theta[1-M]x^2[1-\varepsilon+\beta V_G] + [1-\theta[1-M]x^2]Max\{Max_p\{p\beta V_M + [1-p][\gamma-\varepsilon+\beta V_G]\}, \beta V_G\}$

The first term of this value function comes from the barter possibility. The probability of being matched with another good holder is $\theta[1-M]$, and the probability of double coincidence of wants is x^2. Therefore, the probability that a good holder will experience a barter trade is $\theta[1-M]x^2$. When barter occurs, each agent in the pair consumes, produces, and enters the next period starting with good. Thus, barter yields $1-\varepsilon+\beta V_G$ units of utility. Because this is at least as much utility as any other possibility, a barter trade is executed whenever possible. Consequently, the expected value associated with the barter possibility is $\theta[1-M]x^2[1-\varepsilon+\beta V_G]$.

The second term of value function (20) is associated with the possibility that the agent does not complete a barter transaction, which occurs with probability $1-\theta[1-M]x^2$. In this event, the agent must decide whether or not to hold over the good to the next period. Utility βV_G is obtained from holding over the good, and $Max_p\{p\beta V_M + [1-p][\gamma-\varepsilon+\beta V_G]\}$ is obtained from solving problem (19) otherwise. To understand why problem (19) is always faced when good is not held over, first note that one reason for not completing a barter trade is the agent is matched with another good holder, but no double coincidence of wants occurs. In this case, the agent holding good proceeds to the store and solves

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[15] In order for an agent to have ever decided to hold money $\beta V_M \geq \gamma - \varepsilon + \beta V_G$ would have to have held at some point in the past.   Thus, in order to have this problem to solve, we know that the agent will not choose $p=0$, and we could present the value function of the money holder as $V_M = \theta[1-M]xP[1-\varepsilon+\beta V_G] + [1-\theta[1-M]xP]\beta V_M$. We choose to present the more general value function for the money holder as we do in (18) so that the reader can see all of the possible trading options in the value function, even those that will not be used in equilibrium.

problem (19) to determine whether to accept money from the store or trade good for good. Alternatively, barter might not occur because the good holder is matched with a money holder. If the money holder does not want the good held by the good holder, then the good holder again proceeds to the store and solves problem (19). If the money holder wants the good holder's good, the good holder must decide whether to accept the money holder's money, or not. This again involves solving problem (19) for the alternative is trading the good to the store in exchange for good, as long as holding over good is not better.

At this point, it is useful to illustrate why the introduction of money offers potential benefit. The probability is $p\theta Mx$ that a good holder will trade away good for money, for this requires that the good holder be matched with a money holder, that the money holder wants the good holder's good, and that the good holder will accept the money. Because a good holder experiences barter with probability $\theta[1-M]x^2$, the probability that an agent holding good neither experiences a barter match nor trades after a money match is $1-\theta[1-M]x^2 - p\theta Mx$. In this case, the agent proceeds to the store. In the Store Economy, the probability that a good holder will arrive at the store is $1-\theta x^2$. Subtracting the former probability from the latter, we find the difference is $\theta Mx[p-x]$, which is positive and increasing in M when $p > x$. Thus, when the likelihood a good holder will accept money exceeds the likelihood of a single coincidence of wants, more money in circulation makes it less likely that good holders will have to go to the store and pay the store's markup. This is why the circulation of money has the potential to increase well being.

Note that we are maintaining the assumption that agents cannot divide good. The implication of this indivisibility assumption is that the money holder receives all of the trading surplus, $1-\gamma$, available when a money holder and good holder meet and trade. It is the possibility of obtaining this surplus, obtaining 1 unit of good through money trade in the future rather than γ units through store trade in the present, which motivates an agent to accept money. If goods were

divisible, it would be reasonable to expect the trading surplus to be split between the two traders.

In particular, rather than receiving 1 unit of good from the good holder, the money holder would

receive something less, though more than γ, so the good holder would also reap a present benefit from the

monetary trade. To allow for this divisibility, we would have to complicate the model with

bargaining assumptions, or a price mechanism, so the split of the trading surplus is determined

by the model. While we believe this extension offers the exciting possibility of modeling price

determination in the search theoretic framework in a more natural way than has been done

before, we have chosen not to pursue this here for it would blur the main result. However, what

the reader should understand is that, if we can find conditions where the issuance and holding of

money is incentive compatible in this context, where a producer gets nothing in the current

period from trading for good for money, but is entirely motivated by the potential surplus gained

in a future period, we should be also able to do so in the case where divisibility would allow

money trade to provide immediate rewards. That is, the indivisibility assumption implies we are

examining an extreme case, where we should have the most difficulty finding conditions under

which money can arise and circulate.

We now turn to the store's problem. For the store to exist, condition (9) must hold, and if $\gamma \geq \varepsilon$

then no good holder will ever hold over good from one period to the next, for nothing is gained. This

implies all agents who arrive at the store holding good will either trade good for good or trade good for

money. Let α denote the fraction that trade good for good, so $1-\alpha$ trade good for money. Let δ

denote the fraction of those arriving at the store with money that trade money for good, so that

$1-\delta$ choose to hold money. It follows that the store experiences the following transactions. There are

$\alpha[1-M][1-\theta[1-M]x^2 - p\theta Mx]$ agents who deliver good to the store in exchange for good. There are

$[1-\alpha][1-\theta[1-M]x^2 - p\theta Mx]$ agents who deliver good to the store in exchange for money. There are

$\delta M \left[1 - \theta [1 - M] xP\right]$ agents who deliver money to the store in exchange for good. It follows that the revenue of the store is

(21) $R = [1 - \gamma] \alpha [1 - M] \left[1 - \theta [1 - M] x^2 - p\theta Mx\right] + [1 - \alpha] \left[1 - \theta [1 - M] x^2 - p\theta Mx\right] - \gamma \delta M \left[1 - \theta [1 - M] xP\right]$.

To keep the model as simple as possible, we assume the store sets its trading rate γ with the expectation that a stationary equilibrium will prevail, where the new money it issues will equal to old money it redeems, so

(22) $[1 - \alpha] \left[1 - \theta [1 - M] x^2 - p\theta Mx\right] = \delta M \left[1 - \theta [1 - M] xP\right]$.

Condition (22) and revenue definition (21) together imply the revenue function for the store reduces to

(23) $R = [1 - \gamma] [1 - M] \left[1 - \theta [1 - M] x^2 - p\theta Mx\right]$.

Examining condition (23), we see that, as in the store economy, the store's revenue depends upon the markup $1 - \gamma$ and the number of agents holding good who end up trading through the store, which is

$[1 - M] \left[1 - \theta [1 - M] x^2 - p\theta Mx\right]$. The store's equilibrium profit level is

$\pi = [1 - \gamma] [1 - M] \left[1 - \theta [1 - M] x^2 - p\theta Mx\right] - \bar{c}$ for the fixed cost case and

$\pi = [1 - \gamma - \hat{c}] [1 - M] \left[1 - \theta [1 - M] x^2 - p\theta Mx\right]$ for the variable cost case. We can now define an equilibrium for the Money-Store Economy:

Definition (Equilibrium for Money-Store Economy): Given conditions (1)-(4) and (12)-(13), an equilibrium for the Money-Store Economy is a quadruple (p , P , γ , M) such that (i) agents maximize their lifetime expected utility; i.e., the value functions (18) and (20) are satisfied, (ii) each agent holds money if and only if it is optimal; i.e., p solves problem (19), (iii) agents have rational expectations; i.e., $P = p$, (iv) the store is viable; i.e., $\pi \geq 0$, (v) the existing store prevents viable entry; i.e., there is no $\gamma' > \gamma$ yielding store profit $\pi' \geq 0$, (vi) the store attracts customers; i.e., $\gamma \geq \varepsilon$, (vii) the store maximizes profit π , given conditions (iv)-(vi), (viii) there is a determinant amount of money in circulation; i.e., $0 < M < 1$, (ix) both the store and money mediate exchange; i.e., $\gamma - \varepsilon + \beta V_G = \beta V_M$.

Conditions (i)-(vii) are conditions on agent and store behavior. Condition (viii) indicates we are looking for an equilibrium where money circulates, and condition (ix) indicates we are looking for an equilibrium where money does not dominate the store as a medium of exchange, nor vice versa.

Lemma 1 (Excess Demand Function for Money): Let $\beta V_M - \left[\gamma - \varepsilon + \beta V_G\right]$ denote the excess demand for money. If $0 \le M < 1$, and $\varepsilon \le \gamma < 1$, then

$$(24) \quad \beta V_M - \left[\gamma - \varepsilon + \beta V_G\right] \begin{pmatrix} > \\ = \\ < \end{pmatrix} 0 \Leftrightarrow P - x \begin{pmatrix} > \\ = \\ < \end{pmatrix} \frac{\gamma - \varepsilon}{\beta \theta x [1 - M][1 - \gamma]}$$

Proof: See Appendix

Lemma 2 (Equilibrium Quantity of Money): In any equilibrium where $P \ne x$ and $\gamma \ne 1$,

$$(25) \quad M = \frac{[1 - \gamma]\beta \theta x [P - x] - [\gamma - \varepsilon]}{[1 - \gamma]\beta \theta x [P - x]} = 1 - \frac{\gamma - \varepsilon}{[1 - \gamma]\beta \theta x [P - x]}$$

Proof: From Lemma 1, the equilibrium condition $\gamma + \beta V_G = \beta V_M$ implies $P - x = [\gamma - \varepsilon]/[\beta \theta x [1 - M][1 - \gamma]]$. Solving for M then yields condition (25).

Lemma 3 (Equilibrium Trading Rate Restriction): In an equilibrium where $0 < M < 1$, the trading rate γ must satisfy

$$(26) \quad \varepsilon < \gamma < \frac{\beta \theta x [P - x] + \varepsilon}{1 + \beta \theta x [P - x]}$$

Proof: Using the solution (25) to replace the quantity of money in the equilibrium condition $0 < M < 1$, solving for γ directly yields condition (26).

Using Lemma 1, we can obtain a restriction on the willingness of agents to accept money that must hold if a monetary equilibrium is to exist.

Lemma 4 (Restriction on willingness to accept money): In an equilibrium where $0 < M < 1$, the probability of accepting money must satisfy

$$(27) \quad P > x + \frac{\gamma - \varepsilon}{[1 - \gamma]\beta \theta x}.$$

Proof: If $P \le x + [\gamma - \varepsilon]/[1 - \gamma]\beta \theta x$, then condition (25) indicates $M \le 0$, which violates the money circulation condition $0 < M < 1$.

Lemma 4 indicates agents must have enough confidence in money in order for it to circulate in

equilibrium. When this confidence exists, and money circulates, condition (25) allows us to understand

how the store's choice for γ influences the quantity of money in circulation. As the store's markup

increases so that the trading rate γ decreases toward the transactions cost ε, store trade becomes less

attractive and money becomes the dominant medium of exchange; i.e., $M \to 1$. Alternatively, as the

markup decreases and the trading γ increases and approaches $[\beta\theta x[P-x]+\varepsilon]/[1+\beta\theta x[P-x]]$, store

trade becomes more attractive and money is driven out of the economy; i.e., $M \to 0$. The condition

$[\beta\theta x[P-x]+\varepsilon]/[1+\beta\theta x[P-x]]$ is increasing in P and β. Thus, when there is more confidence in

money, or when agents discount the future less, a higher trading rate γ is necessary to drive money out of

the economy.

Remember that, in the Kiyotaki and Wright Fiat Money Economy, $P > x$ had to hold for money

to circulate. Condition (27) indicates, in our Money-Store Economy, agents must generally have even

more confidence in money for it to circulate. This is because monetary trade does not just compete with

the barter opportunity, but it must also compete with the store. The store is no competition when its

markup is so high that $\gamma = \varepsilon$. However, as the store's trading rate increases above this threshold level,

agents must have increasing confidence in money for it to circulate while competing with the store.

Theorem 2 (Non-Existence of a Monetary Equilibrium for Fixed Cost Case): When store operation cost is
the fixed cost $\bar{c}$ and expectations are rational (i.e. $P = p$), no equilibrium exists with $\varepsilon \le \gamma < 1$ for the
Monetary Economy.

Proof: See Appendix

What we learn from Theorem 2 is, if a store of the fixed cost type can exist, it will drive money

out of the economy. Increasing the trading rate reduces the store's markup, which makes the store more

attractive. This increases the number of traders through the store and reduces the number of money

holders. Profits increase because the fixed cost is spread over more traders, and the store becomes

increasingly efficient. Eventually, the attractiveness of the store drives money out of the economy, so a

monetary equilibrium cannot exist.

Theorem 3 (Existence of a Monetary Equilibrium for Variable Cost Case): When the variable store cost $\hat{c}$ satisfies

(28) $$\frac{1-\varepsilon}{1+\beta\theta x[1-x]} < \hat{c} < 1-\varepsilon,$$

then a set of equilibria exists for the Monetary Economy with a store trading rate

(29) $\gamma = 1 - \hat{c}$.

The set of equilibria contains an infinite number of elements, parameterized by the equilibrium expectation P^*, where and P^* satisfies

(30) $$\overline{P} \equiv x + \frac{1-\hat{c}-\varepsilon}{\beta\theta x\hat{c}} < P^* \leq 1,$$

and the equilibrium quantity of money M^* satisfies

(31) $$M^* = 1 - \frac{1-\varepsilon-\hat{c}}{\beta\theta x\hat{c}[P^*-x]}.$$

Proof: See Appendix

From conditions (29) and (31), we see that, as the per unit store operation cost $\hat{c}$ increases to $1-\varepsilon$, the equilibrium trading rate γ decreases to ε and economy's money supply M increases to 1. That is, money becomes a more dominant medium of exchange as the cost of operating the store increases. As the store operation cost $\hat{c}$ decreases to $[1-\varepsilon]/[1+\beta\theta x[1-x]]$, the expectation P^* must increase to 1 for a monetary equilibrium to exist, the trading rate γ increases to $[\beta\theta x[1-x]+\varepsilon]/[1+\beta\theta x[1-x]]$ and the money in circulation M decreases to zero.

The threshold level for money confidence is $\overline{P} = x + [1-\varepsilon-\hat{c}]/[\hat{c}\beta\theta x]$ in condition (30), so there are multiple monetary equilibria associated with expectations in the range $P \in (\overline{P},1]$. As the store operation cost $\hat{c}$ increases, the expectation P^* can be lower with money still circulating, and the low money confidence bound $\overline{P} = x$ is reached as $\hat{c}$ reaches $1-\varepsilon$.   As x increases, agents must have more confidence in money for it to exist, and there is some value for x large enough that no store can

exist and support the circulation of money, because barter becomes so effective.   Condition (30)

indicates that if people discount the future enough (β small enough), if the store operations cost is low

enough ($\hat{c}$ small enough), if a match in the social interaction is too unlikely (θ small enough), or a single

coincidence of wants too unlikely (x small enough), then a monetary equilibrium cannot be supported

even if agents are entirely confident that money will be accepted (i.e., $P = 1$).

In a monetary equilibrium, when an agent arrives at the store with money, it is equivalent to

arriving with good because in either case the store offers γ units of good in exchange. In equilibrium, all

such agents are indifferent between holding the money into the next good and trading with store and

consuming this period. Because of this indifference in equilibrium, our model does not distinguish how

much money is being redeemed at the store, how much money the store is issuing, nor how long a

particular agent holds money. Imposing the indifference assumption on our equilibrium has allowed us to

keep our model simple, so as to keep the focus on how this framework can be used to endogenize the

introduction of private money. Extending the model by adding enough structure to remove this

indifference assumption, while moving beyond the scope of this paper, would be interesting because

offers potential for examining the velocity of money, and other issues.

Theorem 4 (Stability of Monetary Equilibrium---Variable Cost Case):   Let M^{*} denote the equilibrium
quantity of money, where $\beta V_M = \gamma - \varepsilon + \beta V_G$. For all $M \in [0, M^{*})$, $\beta V_M > \gamma - \varepsilon + \beta V_G$, and for all
$M \in (M^{*}, 1)$, $\beta V_M < \gamma - \varepsilon + \beta V_G$.

Proof: See Appendix

Theorem 4 is intuitive. When there is no money in circulation, or very little, accepting money

offers the largest advantage it can offer, for the probability of being matched with a good holder who will

accept the money in exchange is as high as it can be. Assumption (28) ensures each agent views holding

money as being better than holding good in this situation, so good holders exchange good for money at

any opportunity, and the quantity of money increases. As the quantity of M increases toward its

equilibrium value, there are fewer good holders for a money holder to be matched with, so the expected

value associated with holding money decreases to that associated with not holding money.   Assumption

(28) ensures that there can also be so much money that its expected value is less than that associated with holding good. In this high money quantity situation, money holders will exchange good for money at any opportunity, and the quantity of money decreases toward the equilibrium quantity.

Theorem 4 also indicates that a store equilibrium, as defined in section 4, cannot persist when the store operation cost satisfies condition (28) and there is enough confidence in money to satisfy condition (30). To see this, assume $\gamma = 1 - \widehat{c}$, so that the existing store in the store economy equilibrium is earning zero profits, and all agents not making barter transactions are trading through the store. Theorem 4 indicates every agent would prefer to go to a store with trading rate $\gamma = 1 - \widehat{c}$ that also issues money, for higher expected value could be obtained by accepting money from such a store. Such a store would not only rightly perceive that it could capture the mediation market because of the added expected value offered to agents, but it would also rightly perceive that pure profit would be earned on each new unit of money issued in the disequilibrium as money is initially introduced. Thus, to prevent the entry of such a store in the contestable market, the existing store would be motivated to create the money and earn the profit in disequilibrium by issuing it.

7. Welfare

To examine welfare, we use the criterion $W = MV_M + (1 - M)V_G$. In equilibrium, when $\beta V_M = \gamma - \varepsilon + \beta V_G$, the value functions (18) and (20) reduce to $V_M = [[1 - M]\theta x P[1 - \gamma]]/[1 - \beta]$ and $V_G = [[1 - M]\theta x^2[1 - \gamma] + [\gamma - \varepsilon]]/[1 - \beta]$, so we can write the welfare function as

$$(32) \quad W = \frac{M[1 - M]\theta x P[1 - \gamma]}{1 - \beta} + \frac{[1 - M][[1 - M]\theta x^2[1 - \gamma] + [\gamma - \varepsilon]]}{1 - \beta}.$$

Note that, if a monetary economy cannot form so $M = 0$, the welfare function reduces to $W = V_G$, where the welfare level V_G in condition (32) is the same as that for the Store Economy in condition (11). Because money holding precludes production and reduces the likelihood of

barter, we see in condition (32) that the portion of welfare associated with good holding is monotonically decreasing in M. The portion of condition (32) associated with money holding indicates that more money contributes to welfare when the quantity of money is low, but detracts from welfare when the quantity of money is high. An increase in the money level increases the fraction of agents who reap the reward of trading money for good, but more money reduces production and reduces the probability of a money match. The former effect dominates when the money level is low, but the latter effect dominates when the money level is high.

The effect of the money level on welfare is given by the derivative

$$(33) \qquad \frac{dW}{dM} = \frac{\theta x[1-\gamma]}{1-\beta}[P - 2x - 2[P-x]M] - \frac{\gamma - \varepsilon}{1-\beta}.$$

Evaluating at $M = 0$, with $\gamma = 1 - \hat{c}$, we find that the introduction of money to the Store Economy can increase welfare only if

$$(34) \qquad P > \tilde{P} = 2x + \frac{1 - \hat{c} - \varepsilon}{\theta x \hat{c}}.$$

When $P = 1$, as in the monetary equilibrium for the Kiyotaki and Wright Fiat Money Economy, and $\hat{c} = 1 - \varepsilon$ is imposed to rule out the existence of the store, then condition (34) reduces to $x < 1/2$, which is the condition Ljungqvist and Sargent (2000) show must hold in order for fiat money to be welfare improving in the Fiat Money Economy.

For the Store-Money Economy, when money endogenously arises and finds its equilibrium value, the equilibrium quantity of money will be welfare maximizing only by chance. The optimal quantity of money, found by setting the derivative (33) equal to zero, can be written as

$$(35) \quad \hat{M} = 1 - \frac{\beta[1 - \hat{c} - \varepsilon]}{2\beta\theta x \hat{c}[P - x]} - \frac{P}{2[P - x]}$$

Comparing this optimal quantity of money to the equilibrium quantity of money M^* given by (31), we find

$$(36) \quad M^* \begin{pmatrix} > \\ = \\ < \end{pmatrix} \hat{M} \Leftrightarrow P^* \begin{pmatrix} > \\ = \\ < \end{pmatrix} \hat{P} = \frac{[1-\hat{c}-\varepsilon][2-\beta]}{\beta\theta x \hat{c}}$$

That is, we find that, when an interior optimum exists for the quantity of money, there exists a unique confidence level $\hat{P}$ that will bring forth the optimal quantity. Full confidence in fiat money (i.e., $P=1$), or any confidence level $P > \hat{P}$, brings forth more money than is optimal, while $P < \hat{P}$ brings forth too little. The following theorem delineates how the introduction of money affects welfare, in general.

Theorem 5 (Welfare): For any monetary equilibrium, when $x < [1-\beta]/[1+[1-\beta]]$, the effect of an increase in the quantity of money affects welfare depends upon the store's operation cost:

Case 1: $\dfrac{1-\varepsilon}{1+\beta\theta x[1-x]} < \hat{c} < \dfrac{[2-\beta][1-\varepsilon]}{[2-\beta]+\beta\theta x} \Leftrightarrow \dfrac{\partial W}{\partial M^*} > 0$ for all $P^* \in [\overline{P},1]$

Case 2: $\dfrac{[2-\beta][1-\varepsilon]}{[2-\beta]+\beta\theta x} < \hat{c} < \dfrac{[1-\beta][1-\varepsilon]}{[1-\beta]+\beta\theta x^2} \Leftrightarrow \dfrac{\partial W}{\partial M^*} > 0$ for all $P^* \in [\overline{P},\hat{P})$,

$\dfrac{\partial W}{\partial M^*} < 0$ for all $P^* \in (\overline{P},\hat{P}]$

Case 3: $\dfrac{[1-\beta][1-\varepsilon]}{[1-\beta]+\beta\theta x^2} < \hat{c} < 1-\varepsilon \Leftrightarrow \dfrac{\partial W}{\partial M^*} < 0$ for all $P^* \in [\overline{P},1]$.

However, when $x \geq [1-\beta]/[1+[1-\beta]]$, an increase in the quantity of money only reduces welfare in that $\dfrac{[1-\varepsilon]}{1+\beta\theta x[1-x]} < \hat{c} < 1-\varepsilon \Leftrightarrow \dfrac{\partial W}{\partial M^*} < 0$ for all $P^* \in [\overline{P},1]$.

Proof: See Appendix

Theorem 5 indicates that, in order for the introduction of money to be welfare enhancing, agents must discount the future enough and barter cannot be too easy. When barter is difficult enough or when agents discount the future enough, the level of the store's operation cost

determines whether or not more money enhances welfare. When the store has a low operation cost (i.e., Case 1), more money only increases welfare, and more money is present in the economy as the confidence agents have in money increases from $\overline{P}$ to 1. When the store operation cost is high (i.e., Case 3), more confidence in money only decreases welfare. The intermediate case (Case 2), where there is an interior optimum for the quantity of money, occurs when the store cost is in an intermediate range. In this case, regarding the impact on welfare, agents can have either too much confidence in money (i.e., $P^* > \hat{P}$) or too little (i.e., $P^* < \hat{P}$).

8. Discussion

We have presented a model in which fiat money naturally arises from the competition among profit seeking stores for a contestable mediation market. The equilibrium quantity of fiat money is determined by the microfoundation of the model, not by government.   As in the traditional search theoretic models of fiat money, the confidence agents have in money is important for determining whether the circulation of fiat money can be supported. However, in our model, for a given confidence level P, the equilibrium quantity of money M^* is stable, meaning there are market forces that will drive the quantity of money back to M^* if the quantity is not at M^*.

Our model's one to one mapping from money confidence to the quantity of money in circulation has implications for monetary policy. For example, suppose government introduces a form of fiat money before our store recognizes it can profit by introducing money. Assume agents view government money as equivalent to that introduced by the store in our model. Our model indicates government could introduce a quantity of money up to the equilibrium amount of money M^* for out model, and in doing so would crowd out the ability of the store to introduce

money. However, government could not get agents to voluntarily accept an amount of money above M^* because agents would prefer to trade their good to the store in exchange for good.   In general, our model suggests that recognizing competing mediums of exchange is important for evaluating the scope of money's impact.

Our model also has fiscal policy implications. Specifically, taxes on trade can impact the quantity of money in circulation in our model and, consequently, the level of welfare.   Only by coincidence will the level of confidence in money P^* support an equilibrium quantity of money M^* that maximizes welfare. We can examine the implementation of a transaction tax by noting in our model this would effectively increase the transactions cost ε.   Condition (31) indicates this would increase the equilibrium quantity of money, providing an increase in welfare if the original equilibrium quantity of money was too low.   From the discussion in the previous paragraph, we know a government attempt to increase the money supply would not accomplish this same improvement in welfare.   This suggests further exploration of models with competing mediums of exchange may enhance the understanding of how welfare may be affected by the interaction of monetary and fiscal policies.

Banking can readily be introduced to our model because our model has characteristics similar to that of He, Huang, and Wright (2005). In fact, we may reinterpret our model as having a banking sector. The store's issuance of IOUs can be interpreted as the agent making a deposit in a bank after selling good to the store. Money is created when the bank credits the account of the agent for this deposit. Our assumptions here imply the bank never makes loans, but holds 100 percent of deposits as reserves, and only facilitates transactions by delivering the purchasing power when our money holder makes a trade for good. However, many agents holding money will not experience the single coincidence of wants necessary to make a trade, so some bank

reserves will go unused. By altering the model we have presented here so bank loans can made from excess reserves, the impact of fractional reserve banking could be examined. Following He, Huang, and Wright (2005), we could also extend our model by introducing the assumption that money is subject to theft, but not bank deposits, which would make it possible to distinguish coin and currency circulation from demand deposits.

In general, the basic framework we have introduced here is amenable to a number of extensions that would appear to be useful for further developing the search theoretic approach to modeling money. By relaxing the assumption that agents cannot divide goods, a model with determinate prices might be obtained. By explicitly introducing banks, and allowing loans to be made with fractional reserves, money and banking might fruitfully be examined. Because this model contains a centralized market which we can think of as being operated by a Walrasian auctioneer, along with a decentralized market where money is a medium of exchange, this framework offers the potential for unifying the search theoretic approach with the Walrasian approach. Pursuing this unification will likely yield additional understanding about how monetary and fiscal policy interact to impact the economy as they differentially influence competing mediums of exchange.

Finally, there are technical extensions that could enhance the understanding obtained from this model. First, while the model would be less parsimonious, it would be interesting if the model could be given a complete microfoundation, so that both money and its competing intermediary form arise from the same microfoundation. Second, we might learn more about the development of a monetary economy if the model could be specified so that its disequilibrium dynamics can be traced, so we can watch the economy evolve from the point where money is initially introduced.

Appendix: Proofs

Proof of Theorem 1: Because profits are continuously decreasing in γ, any level for γ that generates a positive profit level cannot be sustained, for a viable competing store can be created with $\gamma' > \gamma$, so all agents will prefer the competing store. Thus, to prevent the successful entry of a competing store, the existing store must adjust γ to eliminate profit. In the fixed cost case, the store earns zero profit when $[1-\gamma][1-\theta x^2]-\bar{c}=0$, or when $\gamma = [[1-\theta x^2]-\bar{c}]/[1-\theta x^2]=1-\bar{c}/[1-\theta x^2]$. Given this value for γ, the store attracts customers if and only if $\gamma \geq \varepsilon$, which implies $\bar{c} \leq [1-\theta x^2][1-\varepsilon]$. In the variable cost case, the store earns zero profit when $[1-\gamma-\hat{c}][1-\theta x^2]=0$, or when $\gamma = 1-\hat{c}$. Given this value for γ, the store attracts customers if and only if $\gamma \geq \varepsilon$, which implies $\hat{c} \leq 1-\varepsilon$.

Proof of Lemma 1: Let p^* denote the optimal value for p in problem (19). If $\gamma \geq \varepsilon$, then the condition $Max\{Max_p\{p\beta V_M +[1-p][\gamma-\varepsilon+\beta V_G]\}, \beta V_G\}$ in value function (20) becomes $p^*V_M +[1-p^*][\gamma-\varepsilon+\beta V_G]$, for no agent will choose to hold good from one period to the next. Using the value functions (18) and (20) to construct the difference $D = \beta V_M -(\gamma-\varepsilon+\beta V_G)$, we can write $D = \beta\theta x[1-M](P-x)\{(1-\varepsilon+\beta V_G)-[p*\beta V_M +(1-p*)(\gamma-\varepsilon+\beta V_G)]\}-(\gamma-\varepsilon)$. Letting A denote the quantity $\beta\theta x(1-M)(P-x)$, the last condition can be rewritten as $D = A[1-\gamma-p^*D]-[\gamma-\varepsilon]$, which implies $[1+p^*A]D = A[1-\gamma]-[\gamma-\varepsilon]$. Because $0 < \beta\theta x[1-M]<1$ and $-1<P-x<1$, we know $-1 < A < 1$. Knowing $A>-1$ and $0\leq p^*\leq 1$, we know $[1+p^*A]>0$. Thus, $[1+p^*A]D = A[1-\gamma]-[\gamma-\varepsilon]$ implies the sign of D depends upon the sign of $A[1-\gamma]-[\gamma-\varepsilon]$. Condition (24) follows directly.

Proof of Theorem 2: By Lemma 2, the quantity of money in equilibrium must satisfy condition (25). Using this quantity of money, the profit function for the store in the fixed cost case becomes

$$\pi = \frac{\gamma-\varepsilon}{\beta\theta x[P-x]}\left[[1-p\theta x]+\frac{[\gamma-\varepsilon][p-x]}{\beta[1-\gamma][P-x]}\right]-\bar{c}$$

From Lemma 2, we know $P-x>0$ must hold for an equilibrium to exist with $\varepsilon \leq \gamma < 1$. Under rational expectations $p-x>0$ must also hold. It then follows that the store's profit is strictly increasing in the trading rate γ. Any store would therefore be motivated to increase γ to a level such that $\gamma > [\beta\theta x[P-x]+\varepsilon]/[1+\beta\theta x[P-x]]$, violating condition (26) of Lemma 3. The contestable market will not prevent this increase in the trading rate, but rather encourage it. This is sufficient to complete the proof.

Proof of Theorem 3: Together, the dynamic program conditions (18) and (20) and the equilibrium condition $\gamma-\varepsilon+\beta V_G = \beta V_M$ imply that the quantity of money in equilibrium must satisfy condition (25). Using this quantity of money, the profit function for the store in the variable cost case becomes

$$\pi = \left[1 - \gamma - \hat{c}\right]\left[\frac{\gamma - \varepsilon}{\beta\theta x\left[1 - \gamma\right]\left[P - x\right]}\left[\left[1 - p\theta x\right] + \frac{\left[\gamma - \varepsilon\right]\left[p - x\right]}{\beta\left[1 - \gamma\right]\left[P - x\right]}\right]\right]$$

Profit per store customer, $1 - \gamma - \hat{c}$, is decreasing in γ. Because $P > x$ and $P = p$ by the rational expectations assumption, the number of store customers,

$$\left[\frac{\gamma - \varepsilon}{\beta\theta x\left[1 - \gamma\right]\left[P - x\right]}\left[\left[1 - p\theta x\right] + \frac{\left[\gamma - \varepsilon\right]\left[p - x\right]}{\beta\left[1 - \gamma\right]\left[P - x\right]}\right]\right] \text{, is increasing in trading rate } \gamma. \text{ By setting } \gamma = \varepsilon, \text{ the}$$

store obtains a high profit per unit, but has no customers. As γ increases from ε to $1 - \hat{c}$, the number of customers continuously increases, while the profit per unit decreases continuously to zero. Therefore, a profit maximum exists in this domain for γ. Contestability implies the trading rate must be set at $\gamma = 1 - \hat{c}$ to eliminate the profit that would entice competing stores to enter. With $\gamma = 1 - \hat{c}$, condition (25) implies the money in circulation is the amount presented as condition (31). Because $\gamma - \varepsilon + \beta V_G = \beta V_M$, agents are indifferent between holding money and trading through the store, so agents are indifferent about the value of p. Thus, with $\gamma = 1 - \hat{c}$, any value for the expectation P that supports the circulation of a positive quantity of money can qualify as an equilibrium expectation. The restriction (28) ensures that there exists at least one such value for P. Condition (30) gives the range of values for P that support a money supply M such that $0 < M < 1$, and condition (31) gives the equilibrium money supply. Because all equilibrium conditions are satisfied for values of P satisfying condition (30), the proof is complete.

Proof of Theorem 4: Let $Z = \left[1 - \hat{c} - \varepsilon\right]/\left[\beta\theta x\left[1 - M\right]\hat{c}\right]$. Because $\hat{c} < 1 - \varepsilon$ by assumption (28), Z is monotonically increasing in M for all $M \in [0,1)$. Because $\gamma = 1 - \hat{c}$ and $\beta V_M = \gamma - \varepsilon + \beta V_G$ in equilibrium, condition (24) implies $P - x = Z^*$, where $Z^* = \left[1 - \hat{c} - \varepsilon\right]/\left[\beta\theta x\left[1 - M^*\right]\hat{c}\right]$. Because Z is increasing in M, it follows that $P < x + Z$, for all $M \in (M^*, 1)$, and condition (24) implies $\beta V_M < \gamma - \varepsilon + \beta V_G$. Because Z is increasing in M, it also follows that $P > x + Z$, for all $M \in [0, M^*)$, and condition (24) implies $\beta V_M > \gamma - \varepsilon + \beta V_G$.

Proof of Theorem 5: From (31), we know the equilibrium quantity of money M^* is strictly increases from zero to a quantity we will call M^*_{max} as P increases over the interval $P \in [\bar{P}, 1]$. The second derivative of the welfare function (32) is $\partial^2 W / \partial M^2 = -\left[2\theta x\hat{c}\left[P - x\right]\right]/\left[1 - \beta\right]$, which is strictly negative for all $M \in [0,1)$ when $P > x$. Since $P > x$ in any monetary equilibrium, this negative second derivative implies we have three possible cases. As P increases over the interval $P \in [\bar{P}, 1]$, so that M^* increases over the interval $M^* \in [0, M^*_{max}]$, welfare either strictly increases (Case 1), strictly increases to a peak and then strictly decreases (Case 2), or strictly decreases (Case 3).

From (36), we know Case 1 occurs when $\hat{P} > 1$ because $P < \hat{P}$ must hold for all $P \in [\bar{P}, 1]$, which implies $M^* < \hat{M}$ for all $M^* \in [0, M^*_{max}]$. Using the definition of $\hat{P}$ in (36), $\hat{P} > 1$ implies $\hat{c} < \left[\left[2 - \beta\right]\left[1 - \varepsilon\right]\right]/\left[\left[2 - \beta\right] + \beta\theta x\right]$. In order for M^* to be an equilibrium value,

(28) implies $\hat{c} > [1-\varepsilon]/[1+\beta\theta x[1-x]]$. These last two inequalities hold simultaneously if and only if $x < [1-\beta]/[1+[1-\beta]]$. Thus, when $x < [1-\beta]/[1+[1-\beta]]$, Case 1 holds if and only if $[1-\varepsilon]/[1+\beta\theta x[1-x]] < \hat{c} < [[2-\beta][1-\varepsilon]]/[[2-\beta]+\beta\theta x]$.

From (36), we know Case 2 occurs when $\overline{P} < \hat{P} < 1$. This is because $P < \hat{P}$ must hold for all $P \in [\overline{P}, \hat{P})$, which implies $M^* < \hat{M}$ for all $M^* \in [0, \hat{M})$, but then $P > \hat{P}$ must hold for all $P \in (\hat{P}, 1]$, which implies $M^* > \hat{M}$ for all $M^* \in (\hat{M}, M^*_{max}]$. Using the definitions of $\overline{P}$ and $\hat{P}$ in (30) and (36), $\hat{P} > \overline{P}$ implies $\hat{c} < [[1-\beta][1-\varepsilon]]/[[1-\beta]+\beta\theta x^2]$. Using the definition of $\hat{P}$ in (36), $\hat{P} < 1$ implies $\hat{c} > [[2-\beta][1-\varepsilon]]/[[2-\beta]+\beta\theta x]$. $\hat{c} > [1-\varepsilon]/[1+\beta\theta x[1-x]]$. These last two inequalities hold simultaneously if and only if $x < [1-\beta]/[1+[1-\beta]]$. Thus, when $x < [1-\beta]/[1+[1-\beta]]$, Case 2 holds if and only if $[[2-\beta][1-\varepsilon]]/[[2-\beta]+\beta\theta x] < \hat{c} < [[1-\beta][1-\varepsilon]]/[[1-\beta]+\beta\theta x^2]$.

From (36), we know Case 3 occurs when $\hat{P} < \overline{P}$ because $P > \hat{P}$ must hold for all $P \in [\overline{P}, 1]$, which implies $M^* > \hat{M}$ for all $M^* \in [0, M^*_{max}]$. Using the definitions of $\overline{P}$ and $\hat{P}$ in (30) and (36), $\hat{P} < \overline{P}$ implies $\hat{c} > [[1-\beta][1-\varepsilon]]/[[1-\beta]+\beta\theta x^2]$. The quantity $[[1-\beta][1-\varepsilon]]/[[1-\beta]+\beta\theta x^2]$ is strictly less than $[1-\varepsilon]$ for all the admissible parameter values $0 < \beta < 1$, $0 < x < 1$, $0 < \varepsilon < 1$, and $0 < \theta \le 1$. Thus, when $x < [1-\beta]/[1+[1-\beta]]$, Case 3 holds if and only if $[[1-\beta][1-\varepsilon]]/[[1-\beta]+\beta\theta x^2] < \hat{c} < 1-\varepsilon$.

When $x = [1-\beta]/[1+[1-\beta]]$, $[1-\varepsilon]/[1+\beta\theta x[1-x]] = [[1-\beta][1-\varepsilon]]/[[1-\beta]+\beta\theta x^2] = [[2-\beta][1-\varepsilon]]/[[2-\beta]+\beta\theta x]$, so it is not possible to find a value for $\hat{c}$ such that satisfies either Case 1 or Case 2. However, we may still find a value for $\hat{c}$ such that $[[1-\beta][1-\varepsilon]]/[[1-\beta]+\beta\theta x^2] < \hat{c} < 1-\varepsilon$, so Case 3 may still hold when $x = [1-\beta]/[1+[1-\beta]]$. When $x > [1-\beta]/[1+[1-\beta]]$, $[[2-\beta][1-\varepsilon]]/[[2-\beta]+\beta\theta x] < [1-\varepsilon]/[1+\beta\theta x[1-x]]$ and $[[1-\beta][1-\varepsilon]]/[[1-\beta]+\beta\theta x^2] < [[2-\beta][1-\varepsilon]]/[[2-\beta]+\beta\theta x]$. Thus, it is not possible to find a value for $\hat{c}$ such that satisfies either Case 1 or Case 2. However, we may still find a value for $\hat{c}$ such that $[[1-\beta][1-\varepsilon]]/[[1-\beta]+\beta\theta x^2] < \hat{c} < 1-\varepsilon$, so Case 3 may still hold when $x > [1-\beta]/[1+[1-\beta]]$. Thus, we can conclude that when $x \ge [1-\beta]/[1+[1-\beta]]$, only Case 3 may hold, which is sufficient to complete the proof.

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