Who participates in tax amnesties? Self-selection of risk-averse taxpayers

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Abstract

In this paper we model taxpayer participation in an unanticipated tax amnesty which can be entered by paying a fixed amount. Taxpayers are characterized by a Constant Relative Risk Aversion (CRRA) utility function and differ in relative risk aversion coefficient and in income. With minor changes the same model also describes a FATOTA (Fixed Amount of Taxes or Tax Audit) system. Our results show that amnesties may fail as a self-selective device to fully separate large-scale from small-scale tax evaders and to extract resources from the former. Only taxpayers whose relative risk aversion falls within a given interval participate, while those whose evasion is too small or too large do not enter. The model is used to estimate relative risk aversion and tax evasion of participants in the 1991 and 1994 Italian income tax amnesties.

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1 Introduction

Attitudes toward risk play a key role in determining citizens reactions to taxation. Risk aversion, which is often assumed, helps explain why tax systems are viable, as it commands the willingness to pay risk premia, even when the expected penalties are small. On the other hand, differences in risk aversion may give rise to equity problems. Taxpayers who differ in their risk attitudes may differ in their degree of tax compliance. The actual tax burden thus varies without any equity justification. Efficiency problems are also likely to arise, such as the "excess burden" of tax evasion pointed out by Yitzachi (1987).

Taxation could, however, be designed with a view to exploiting differences in risk attitude. Some specific tax law provisions seem in fact to do so in promising to remove, at a given price, the uncertainty stemming from random tax audits, thus offering insurance in order to induce the self-selection of taxpayers. The FATOTA system (see Chu [1990]), for example, allows taxpayers to pay a Fixed Amount of Taxes (called FAT), rather than to report income as usual and run the risk of a control. By assuming that all taxpayers have the same preferences and are risk averse[1], but differ in income, Chu (1990) shows that the fixed tax will be chosen by those prone to the greatest evasion. The welfare improving characteristics of the FATOTA system resemble those of plea bargaining in criminal proceedings[2], which, at given conditions, induces the self-selection of the indicted. Unanticipated tax amnesties in which participants make fixed payments may be modelled in a similar way, as they remove the threat of (further) controls.

In this paper we present a tax amnesty model, based upon the expected utility approach, in which participants must pay a fixed amount. We depart from previous research

[1] Chu (1997) assumes in an example logarithmic utility (implying DARA, Decreasing Absolute Risk Aversion) which belongs to the family of utility functions we study here.

[2] On this topic see Grossman and Katz (1983).

conducted by Marchese and Privileggi (1997), which used a Constant Absolute Risk Aversion (CARA) utility function, and here assume that all taxpayers are characterized by a Constant Relative Risk Aversion (CRRA) utility function. Heterogeneity among taxpayers is assumed. Taxpayers thus differ in relative risk aversion coefficient and in income, which are treated as continuous unobservable variables.

The main result of this paper is that amnesties may fail as a self-selective device to fully separate large from small evaders and to extract resources from the former. If taxpayers display CRRA preferences, very rich taxpayers with very low risk aversion might not enter[3]; participants are only those whose evasion belongs to a given interval. This result accords with available empirical research, which provides case studies (see Fisher et al. [1989]) in which only small evaders participate in amnesties. It also accords with conjecture and examples put forth in the literature about plea bargaining (see, e.g., Grossman and Katz [1983]). However, the modelling of possible self-selection failures of FATOTA or tax amnesties has not been considered in previous literature (see for example Chu [1990], Franzoni [2000] and Marchese and Privileggi [1997]) and does not represent a trivial extension from the plea bargaining literature.

In Section 2 we model the reaction to an unanticipated amnesty by a partial evader who has previously optimally reported his income. With minor changes, the same model also describes the reaction to a FAT proposal. The non-trivial analytical difficulties arising with the CRRA utility function specification have been overcome by constructing and solving an approximated version of the model. In Section 3, amnesty participants are characterized with reference to their relative risk aversion and tax evasion. In Section 4, results of a deterministic estimation on data pertaining to the 1991 and 1994 Italian tax amnesties are

[3] With CARA preferences this cannot happen, and all taxpayers with a percentage of concealed income equal to or greater than a given threshold enter the amnesty (see Marchese and Privileggi [1997]).

reported. The conclusions focus on the efficiency and equity implications of tax amnesties.

2 Modelling the Taxpayer's Problem

The utility that taxpayers enjoy out of their income w is assumed to be of the standard CRRA form, with constant relative risk-aversion coefficient α:

$$u(w) = \frac{w^{1-\alpha} - 1}{1-\alpha}. \tag{1}$$

In the class (1) we also include the case $\alpha = 1$ by taking $u(w) = \lim_{\alpha \to 1} \left(w^{1-\alpha} - 1 \right) / (1-\alpha) = \ln w$. To focus upon the tax evasion problem, it is assumed that income is exogenous and non random, thus ignoring, on the one hand, the feed-back of taxation upon hours of work, savings, etc., and, on the other hand, background risks the agent may face.

A progressive tax system is considered, where the income tax is approximated by the following function:

$$t(y) = \gamma y^{\delta}, \tag{2}$$

where y denotes the *reported income* and $0 < \gamma < 1$, $\delta > 1$ are parameters such that $\gamma y^{\delta} < y$; *i.e.*, the reported income must always be higher than the amount of tax to be paid. This implies that $y < \gamma^{1/(1-\delta)}$ must hold.

As we rule out rewards to honest taxpayers by assumption, a taxpayer will report $y \leq w$, where $w > 0$ denotes the *true income*. For reasons of analytical tractability that will become clear in Section 3, we assume that the sanction to be paid in case of detection is proportional to the amount of concealed income:

$$\widehat{s}(w, y) = \sigma (w - y), \tag{3}$$

where $\sigma > 0$ is a penalty rate. Since we are considering a progressive tax system defined as in (2) joined with a sanction function $\widehat{s}(w, y)$ which is linear in concealed income, the selection of a suitable range of values for parameter σ becomes a critical issue that will be extensively discussed in Appendix B.

To ensure that the taxpayer can always bear the loss in case of detected evasion, it is assumed that:

$$\sigma\left(w-y\right) \leq w-\gamma y^{\delta}, \tag{4}$$

which implies that, for each given w, there is some[4] m_w such that $0 \leq m_w \leq y$, with $m_w > 0$ whenever $y < w$.

To summarize, letting

$$M_w = \min\left(w, \gamma^{1/(1-\delta)} - \varepsilon\right),$$

with $\varepsilon > 0$ arbitrarily small, then the feasible set of values for the reported income y is the closed interval $[m_w, M_w]$.

2.1 Rational Taxpayer's Behavior

We assume that, while filling in their income tax forms, taxpayers are unaware of a coming tax amnesty, and thus only concerned with standard income tax parameters.

A rational taxpayer who earned a true income $w > 0$ will choose to report the amount v that maximizes her expected utility

$$\mathbb{E}u(y) = \frac{(1-p)\left(w-\gamma y^{\delta}\right)^{1-\alpha} + p\left[w-\gamma y^{\delta} - \sigma\left(w-y\right)\right]^{1-\alpha} - 1}{1-\alpha} \tag{5}$$

with respect to the reported income y, where $0 < p < 1$ is the probability of detection. Note that, by considering $u(w) = \ln w$ when $\alpha = 1$, $\mathbb{E}u(y)$ is well defined for all $\alpha > 0$ and for all feasible y. Note also that $\mathbb{E}u(y)$ is strictly concave over $[m_w, M_w]$ for all $\alpha > 0$, hence, there exists a unique $m_w \leq v \leq M_w$ that maximizes the expected utility.

[4]Note that m_w cannot be explicitly calculated, since (4) cannot be solved with respect to y; moreover $m_w = 0$ if and only if $y = w$.

Since we are interested exclusively in the behavior of partial evaders, we shall assume that the optimal amount of reported income v lies in the interior of the feasible set, *i.e.,* $m_w < v < M_w$; we rule out full evaders and full compliers. Thus, utility maximization is completely described by F.O.C. in (5), which leads to

$$\frac{w - \gamma v^{\delta} - \sigma (w - v)}{w - \gamma v^{\delta}} = \left[\left(\frac{p}{1-p} \right) \left(\frac{\sigma}{\gamma \delta v^{\delta - 1}} - 1 \right) \right]^{1/\alpha}. \tag{6}$$

Note that the assumption of interiority of solution v implies the left hand of (6) to be strictly positive and less than one, which translates into

$$0 < \left(\frac{p}{1-p} \right) \left(\frac{\sigma}{\gamma \delta v^{\delta - 1}} - 1 \right) < 1. \tag{7}$$

A necessary condition for (7) clearly is

$$\sigma > \gamma \delta v^{\delta - 1}, \tag{8}$$

which envisages a sanction rate larger than the marginal tax rate. It will thus be assumed that (8) holds in the following.

The choice of joining an exponential function to determine the amount of taxes due, $t(y) = \gamma y^{\delta}$ in (2), to an affine function to calculate the sanction in case of detection, $s(y) = \sigma (w - y)$ in (3), does not allow for an explicit solution v of the maximization problem. Our goal, however, is that of estimating the true income w for a given (optimally) reported income v, and for this it is enough to have existence and uniqueness of an interior solution for the maximization problem of rational taxpayers, characterized by (6).

2.2 Participation in an Unexpected Amnesty

Suppose that after taxpayers have reported their optimal income v, but before audits begin, the tax administration offers the taxpayers the possibility of paying some fixed amount x in order to avoid any applicable sanction with certainty. Ignoring, for the sake of simplicity, inter-temporal discounting, and assuming that no other relevant variables (e.g., true income, penalty rate, etc.) have changed in the meantime, the taxpayer will accept the offer if she is at least indifferent between paying the certain amount x or maintaining her status as partial evader. Thus, in order to participate in the amnesty, the following condition must be met:

$$\frac{\left(w-\gamma v^{\delta}-x\right)^{1-\alpha}}{1-\alpha} \geq \frac{(1-p)\left(w-\gamma v^{\delta}\right)^{1-\alpha}+p\left[w-\gamma v^{\delta}-\sigma\left(w-v\right)\right]^{1-\alpha}}{1-\alpha}, \tag{9}$$

where the additive constants $-\left(1-\alpha\right)^{-1}$ have already been dropped from both sides. Obviously, a necessary condition for (9) to hold is that the extra payment x must be lower than the sanction:

$$0<x<\sigma\left(w-v\right). \tag{10}$$

From the point of view of the tax administration, the reported income v is available (observable) information, and the fixed amount x is assumed to be a parameter exogenously provided by some government decision maker[5], while the true income w and the individual constant relative risk-aversion coefficient α are unobservable variables. By considering jointly the optimal behavior of taxpayers as expressed in (6) and the threshold condition (9), we are led to the following system:

$$\left\{\begin{array}{l} \left[w-\gamma v^{\delta}-\sigma\left(w-v\right)\right]\left(w-\gamma v^{\delta}\right)^{-1}=\left\{p\left(1-p\right)^{-1}\left[\sigma\left(\gamma\delta v^{\delta-1}\right)^{-1}-1\right]\right\}^{1/\alpha} \\ \left(1-\alpha\right)^{-1}\left(w-\gamma v^{\delta}-x\right)^{1-\alpha} \geq \left(1-\alpha\right)^{-1}\left\{\left(1-p\right)\left(w-\gamma v^{\delta}\right)^{1-\alpha}\right. \\ \qquad\qquad\qquad\qquad \left.+p\left[w-\gamma v^{\delta}-\sigma\left(w-v\right)\right]^{1-\alpha}\right\} \end{array}\right. \tag{11}$$

where the reported income v and the amnesty payment x are given and α and w are the unknowns. All pairs (α, w) solving system (11), characterize the subset of taxpayers who previously (optimally) reported an income v and now participate in the amnesty for the fixed amount x, in terms of their relative risk-aversion α and true earned income w.

[5]We do not address in this paper the symmetric problem of the government, which chooses tax and amnesty parameters in order to maximize some objective function.

2.3 An Alternative Interpretation of the Model: Fixed Amount of Taxes

Model (11) can also be referred to an alternative scenario, in which there are no tax amnesties, but, rather, the government offers a FAT (Fixed Amount of Taxes) as a substitute for ordinary taxation. It is assumed in this case that the government holds some *a priori* belief[6] about a taxpayer's optimally reported income v. The first equation of system (11) then links v to the taxpayer's true income w according to (6). The FAT offer includes the tax calculated according to government beliefs, and amounts to $\gamma v^{\delta} + x$. All pairs (α, w) solving system (11), characterize the subset of taxpayers whose (believed) reported income is v and who accept the FAT, in terms of their relative risk-aversion α and true earned income w, which are unknown variables. The FAT scenario is more general than the amnesty one, as taxpayers facing a FAT offer based on *a priori* government beliefs have nothing to gain from altering their actual tax report, which is requested only if they refuse the FAT. Note that the FAT can be offered immediately as an alternative to ordinary taxation, while the amnesty must be unexpected to work according to the system (11).

In practice, implementation of the FAT approach may be somehow difficult for the government, as *a priori* information about taxpayers' (believed) reported income v may be poor. To overcome this difficulty, FAT offers are often designed for income classes rather

[6] In the standard approach, the taxpayer's true income is the realization w of a random variable $\widehat{w}$, known only by himself and not by the government, while value $v(w)$ of reported income is the outcome of the taxpayer's optimal strategy. The probability distribution of r.v. $\widehat{w}$ conditional to some other information β (such as profession, branch of activity, properties owned, etc.) is common knowledge among taxpayers and the government itself, which, by solving the same optimization problem of the taxpayers, may calculate the induced conditional probability distribution of r.v. $\widehat{v}|\beta = v(\widehat{w})|\beta$ as well. Hence, in this scenario, the value for v used in model (11) could be some proxy of r.v. $\widehat{v}|\beta$, such as, for instance, the conditional expected value $\mathbb{E}[\widehat{v}|\beta]$.

than for pointwise income values. Tax amnesties, instead, may hinge upon actual tax reports to provide the government with the information needed to determine personalized entrance payments, as the examples quoted in Section 4 show. Each scenario thus has advantages and disadvantages[7].

3 Approximating Solutions for the Model

While we have not been able to find explicit solutions of (11), in what follows we shall study a slight simplification of (11) that provides a sufficiently clear portrait of the solution set. Thanks to the choice made in Section 2 of a sanction function that is linear in the true income w, we can transform the left hand side of the first equation in (11) as follows:

$$\begin{aligned}\left[w-\gamma v^{\delta}-\sigma\left(w-v\right)\right]\left(w-\gamma v^{\delta}\right)^{-1} &= 1-\sigma\left(w-v\right)\left(w-\gamma v^{\delta}\right)^{-1}\\ &= 1-\sigma(w-\gamma v^{\delta}+\gamma v^{\delta}-v)\left(w-\gamma v^{\delta}\right)^{-1}\\ &= 1-\sigma+\sigma\left(v-\gamma v^{\delta}\right)\left(w-\gamma v^{\delta}\right)^{-1}\end{aligned}$$

With this position, substituting into the first equation yields

$$w-\gamma v^{\delta}=\frac{\sigma\left(v-\gamma v^{\delta}\right)}{\sigma-1+r^{1/\alpha}}, \tag{12}$$

where

$$r=\left(\frac{p}{1-p}\right)\left(\frac{\sigma}{\gamma\delta v^{\delta-1}}-1\right). \tag{13}$$

[7] The Italian government is still, in fact, oscillating between these approaches. Since 1999, a reference reported income (and implicitly a kind of FAT tax) was introduced for small business and self-employment income. The amount of the reference incomes v are determined on the basis of physical and economic indicators, according to the so called *Studi di Settore* (economic branch studies). The method for calculating reference incomes has been agreed upon by the tax administration and the small business and self-employment representative organizations.

On the other hand a new tax amnesty, based upon the disclosure of resort to moonlighters by firms and professionals, is presently (summer 2001) under parliamentary examination.

For a given reported income v, F.O.C. expressed in the first equation of (11), describing the maximizing behavior of taxpayers, provides through (12) a representation for the true income w as a function of the relative risk aversion α:

$$w = \sigma \left(v - \gamma v^{\delta}\right) \left(\sigma - 1 + r^{1/\alpha}\right)^{-1} + \gamma v^{\delta}. \tag{14}$$

Therefore, we shall henceforth focus exclusively on the variable α to study solutions of (11).

Dividing the second inequality in (11) by $\left(w - \gamma v^{\delta}\right)^{1-\alpha} > 0$, we have

$$\frac{1}{1-\alpha} \left(1 - \frac{x}{w - \gamma v^{\delta}}\right)^{1-\alpha} \geq \frac{1}{1-\alpha} \left[1 - p + p \left(\frac{w - \gamma v^{\delta} - \sigma (w - v)}{w - \gamma v^{\delta}}\right)^{1-\alpha}\right],$$

and by substituting the left hand side as in (12) and the right hand side as in the first equation of (11), system (11) boils down to a single inequality where the unknown is the sole variable α:

$$\frac{1}{1-\alpha} \left(A - Br^{1/\alpha}\right)^{1-\alpha} \geq \frac{1}{1-\alpha} \left[1 - p + p \left(r^{1/\alpha}\right)^{1-\alpha}\right]$$

where A and B are constants defined by

$$A = 1 - \frac{(\sigma - 1)x}{\sigma \left(v - \gamma v^{\delta}\right)} \tag{15}$$

$$B = \frac{x}{\sigma \left(v - \gamma v^{\delta}\right)}. \tag{16}$$

Since, by (13) and (7), $0 < r < 1$, the right hand side is well defined. In order for the left hand side to make sense as well, we need $A - Br^{1/\alpha} > 0$ for all $\alpha > 0$, which is equivalent to $A - B > 0$. A direct application of (15) and (16) to this inequality, leads to the following assumption that will hold throughout the entire paper.

A. 1 $0 < x < v - \gamma v^{\delta}$.

Assumption A.1 also ensures full liability of detected evaders when tax evasion is small, *i.e.*, v is very close to w. Note that, by (15), (16) and A.1, both $0 < B < A < 1$ and $0 < A - B < A - Br < 1$ also hold.

Under Assumption A.1 we can define a function f on $\mathbb{R}_{++}$ by

$$f(\alpha) = \left(A - Br^{1/\alpha}\right)^{1-\alpha} - p\left(r^{1/\alpha}\right)^{1-\alpha} - (1-p). \tag{17}$$

Note that $f(\alpha)$ is well defined for all $\alpha > 0$ and is C^∞. Moreover, $f(1) = 0$. With this notation at hand, system (11) proves equivalent to

$$\begin{cases} f(\alpha) \geq 0 & \text{if } 0 < \alpha \leq 1 \\ f(\alpha) \leq 0 & \text{if } \alpha \geq 1. \end{cases} \tag{18}$$

3.1 The Approximated Model

Function f defined in (17) does not permit a direct mathematical approach to characterizing solutions of (18). Hence, we shall characterize solutions of a slightly simplified system and under some further conditions. Specifically, we shall use a suitable lower bound $l < f$ for $0 < \alpha < 1$, while for $\alpha \geq 1$ we will be able to characterize solutions only for a subclass of models. However, we shall see that our technique covers the most meaningful cases.

Let

$$\phi(\alpha) = \left(A - Br^{1/\alpha}\right)^{1-\alpha}$$

$$\varphi(\alpha) = -p\left(r^{1/\alpha}\right)^{1-\alpha}.$$

Clearly $f = \phi + \varphi - (1-p)$. Now define

$$\psi(\alpha) = 1 - \ln(A - Br)(\alpha - 1)$$

and

$$l(\alpha) = \begin{cases} \psi(\alpha) + \varphi(\alpha) - (1-p) & \text{for } 0 < \alpha < 1 \\ f(\alpha) & \text{for } \alpha \geq 1. \end{cases}$$

We shall characterize solutions of the system

$$\begin{cases} l(\alpha) \geq 0 & \text{if } 0 < \alpha \leq 1 \\ l(\alpha) \leq 0 & \text{if } \alpha \geq 1. \end{cases} \tag{19}$$

Lemma 1 *Under A.1, $\psi(\alpha) < \phi(\alpha)$ for all $0 < \alpha < 1$.*

Proof. Since $r < 1$ and A.1 implies $B < A$,

$$(A - Br)^{1-\alpha} < \left(A - Br^{1/\alpha}\right)^{1-\alpha} = \phi(\alpha) \tag{20}$$

for all $0 < \alpha < 1$. Since $(A - Br)^{1-\alpha}$ is strictly convex for all $\alpha > 0$, by the superdifferentiability property the following is true:

$$\psi(\alpha) = 1 - \ln(A - Br)(\alpha - 1) < (A - Br)^{1-\alpha}$$

for all $0 < \alpha < 1$, which, coupled with (20), proves the assertion. ■

The proof of the lemma above explains our construction of function l. Since function ϕ is neither convex nor concave over $(0, 1)$, we replace it with the convex lower bound $(A - Br)^{1-\alpha}$, then we further lower it by taking its first order approximation centered on $\alpha = 1$. Thus, l turns out to be a lower bound for f on the interval $(0, 1)$, with an improved (linearized) shape for component ϕ in f; while, by construction, $l = f$ for all $\alpha \geq 1$. Therefore, solutions to (19) are a subset of solutions to (18); in particular, some points in the "left-side" solution set of (18), a subset of interval $(0, 1)$, are lost through our approximation. Note that, by construction, $l'(1) = f'(1) = -\ln(A - Br) + p \ln r$.

3.2 The Main Result

The following result completely describes the solution set of our simplified model, system (19). First we further restrict the admissible range of the parameters and the reported income.

A. 2 *Parameters $p, \gamma, \delta, \sigma$ and reported income v altogether satisfy*

i)

$$r = \left(\frac{p}{1-p}\right)\left(\frac{\sigma}{\gamma\delta v^{\delta-1}} - 1\right) \leq e^{-2};$$

ii)

$$1 - r < \sigma \leq \frac{1}{2}\left(1+\sqrt{5}\right).$$

Both (i) and (ii) are technical restrictions. The proof of Proposition 1 is based on condition (i), which has been chosen because $r \leq e^{-2}$ is met by all available data considered in Section 4. The idea behind the proof of Proposition 1, however, can be applied through a symmetrical argument to obtain analogous results for the case $r > e^{-2}$, as will become clear in the sequel. The left inequality in (ii) is necessary to let both conditions (21) and (23) in the next Proposition 1 to be well defined. Note that, thanks to the same inequality, condition (22) is also well defined whenever (i) is satisfied, as $\sigma > 1 - r$ implies $\sigma > 1 - e^{-2}$ provided that (i) is true. The right inequality in (ii) is a sufficient condition for full liability assumption (4) to be satisfied[8], obtained by means of (14). Hence, since $1 - e^{-2} \simeq 0.865$ and $(1/2)\left(1+\sqrt{5}\right) \simeq 1.618$, Assumption A.2 narrows the admissible values for the penalty rate σ to a subset of the interval $(0.865, 1.618]$.

Proposition 1 *Suppose A.1 and A.2 hold. Then the solution set $S \subseteq \mathbb{R}_{++}$ of system (19) has the following properties.*

i) *If condition (i) of A.2 holds with equality, then S is a nonempty interval*[9]*: $S = [\underline{\alpha}, \overline{\alpha}]$,*

[8]It should be remarked, however, that the right inequality in (ii) is only a sufficient condition for (4). Actually it could be relaxed a little, since from both (4) and (14) the necessary and sufficient condition for (4) turns out to be $\sigma \leq 1+ (1/2)\left(\sqrt{4 + r^{2/\alpha}} - r^{1/\alpha}\right)$ and so, as $r < 1$, less restrictive than the right inequality in (ii). Nonetheless, since the last condition does depend on α, such an assumption would exogenously impose some lower bound on α itself, thus further complicating the subsequent analysis.

In general, progressive taxation is problematic to model as, on the one hand, the sanction must be high enough to sustain increasing marginal tax rates, while, on the other, it should not be prohibitive, as full liability is assumed.

[9]To be precise $S = (0, \overline{\alpha}]$ whenever $\underline{\alpha} = 0$, since $\mathbb{E}u(\cdot)$ is not defined for $\alpha = 0$.

with $0 \leq \underline{\alpha} < 1 < \overline{\alpha} < +\infty$, *if and only if*

$$x < \frac{\sigma\left(v - \gamma v^{\delta}\right)}{\sigma - (1-r)}\left(1 - r^{p}\right). \tag{21}$$

ii) *If condition (i) of A.2 holds with strict inequality, a sufficient condition for* S *to be non-empty and of the form* $S = [\underline{\alpha}, \overline{\alpha}]$ *with* $0 \leq \underline{\alpha} < 1 < \overline{\alpha} < +\infty$, *is the following:*

$$x \leq \frac{\sigma\left(v - \gamma v^{\delta}\right)}{\sigma - (1-e^{-2})}\left\{1 - \left[1 + p\left(\ln r\right)\left(\frac{\ln r}{2} + 1\right)\right]^{2/(2+\ln r)}\right\}. \tag{22}$$

iii) *If*

$$x > \frac{\sigma\left(v - \gamma v^{\delta}\right)}{\sigma - (1-r)}\left[1 - \exp\left(\frac{4e^{-2}p}{r \ln r}\right)\right], \tag{23}$$

then S *is empty.*

The proof of Proposition 1 will be accomplished through several steps. First we need a preliminary lemma.

Lemma 2 *Under A.1, function* $\phi(\alpha) = \left(A - Br^{1/\alpha}\right)^{1-\alpha}$ *is strictly convex for* $\alpha \geq 1$, *while function* $\varphi(\alpha) = -p\left(r^{1/\alpha}\right)^{1-\alpha}$ *is strictly concave for* $0 < \alpha \leq -(1/2)\ln r$ *and is strictly convex for* $\alpha \geq -(1/2)\ln r$.

Proof. A tedious direct computation of the second derivatives of both ϕ and φ gives the result. ■

Proof of Proposition 1. *Part (i).* This is a very peculiar (and fortunate) circumstance, as, in most cases, available data meet condition (i) of A.2 with strict inequality. We consider this case in detail mainly for expository reasons, because it helps in clarifying the main idea behind the whole proof.

Since equality in condition (i) of A.2 is equivalent to $-(1/2)\ln r = 1$, by Lemma 2 function $l(\alpha) = \psi(\alpha) + \varphi(\alpha) - (1-p)$ turns out to be strictly concave over $(0,1)$ and strictly convex over $[1, +\infty)$. This is true since l is the sum of a constant, and functions

ψ and φ, which are linear and strictly concave respectively over $(0,1)$, and both strictly convex over $[1,+\infty)$. In other words, l has a unique flex-point at $\alpha = 1$. Moreover, since $l(1) = 0$, S is non-empty and has the form $S = [\underline{\alpha}, \overline{\alpha}]$ if and only if its derivative is strictly negative at $\alpha = 1$, that is,

$$l'(1) = -\ln(A - Br) + p \ln r < 0$$

which, after some algebra, is the same as condition (21), which makes sense thanks to (ii) of A.2. Note also that $\overline{\alpha} < +\infty$ since $l(\alpha) \to +\infty$ as $\alpha \to +\infty$.

Part (ii). Strict inequality in condition (i) of A.2 is equivalent to

$$-\frac{\ln r}{2} > 1. \tag{24}$$

To simplify notation, let $c = -(1/2)\ln r > 1$. In this case we extend the argument above by constructing a function $h(\alpha)$ that is as similar to l as possible but is better shaped than l over the interval $(1,c)$ where the required convexity property of l cannot be verified directly. Define

$$\chi(\alpha) = 1 + \frac{\phi(c) - 1}{c - 1}(\alpha - 1)$$

and

$$h(\alpha) = \begin{cases} l(\alpha) & \text{for } 0 < \alpha \le 1 \text{ and } \alpha \ge c \\ \chi(\alpha) + \varphi(\alpha) - (1-p) & \text{for } 1 < \alpha < c. \end{cases}$$

As in the construction of function l, where we replaced the badly shaped function ϕ with a linear one, ψ, over $(0,1)$, function h constitutes an improvement of function l again by linearizing ϕ, which, by Lemma 2, is convex for all $\alpha \ge 1$. As a result, h turns out to be strictly concave over $(1,c)$, being the sum of a constant, a linear and a concave function.

Function $h(\alpha)$ turns out to be the same as $l(\alpha)$ outside the interval $(1,c)$, where the same argument of Part (i) applies. Specifically, h is strictly concave over $(0,1)$ and $l(\alpha) \ge 0$ has a non-empty interval $[\underline{\alpha}, 1)$ as the solution as long as $l'_{-}(1) = l'(1) < 0$; while h is strictly convex over $(c, +\infty)$. Inside interval $(1, c)$, we have seen that $h(\alpha) = \chi(\alpha) + \varphi(\alpha) - (1-p)$ is strictly concave. Moreover, since h is obtained by replacing the strictly convex function ϕ with the segment joining two points of its graph, $h(\alpha) > l(\alpha)$ holds true for all $\alpha \in (1, c)$, while $h(1) = l(1)$ and $h(c) = l(c)$. Note that h is not differentiable at points $\alpha = 1$ and $\alpha = c$, where it is only left and right-differentiable, while $l'(1)$ exists.

Hence, $h'_+(1) \leq 0 \Longrightarrow h(\alpha) < 0 \Longrightarrow l(\alpha) < 0$ for all $\alpha \in (1, c]$. Furthermore, $l(c) < 0$ plus its convexity over $(c, +\infty)$ implies $l(\alpha) \leq 0$ for all $\alpha \in [c, \overline{\alpha}]$, where $c < \overline{\alpha} < +\infty$. To conclude, $h'_+(1) \leq 0 \Longrightarrow l(\alpha) \leq 0$ for all $\alpha \in (1, \overline{\alpha}]$, while, on the other side, $h'_+(1) \leq 0 \Longrightarrow l'_+(1) = l'(1) < 0$, thus also establishing the non-emptiness of the interval $[\underline{\alpha}, 1)$. A direct computation shows that condition $h'_+(1) \leq 0$ is equivalent to condition (22), and the proof is complete. Note that again, under our construction, we obtain a function h with a unique flex point $\alpha = c$.

Part (iii). By construction, $\psi'(\alpha) = \phi'(1) = -\ln(A - Br)$ over $(0, 1]$. By Lemma 2,

$$\phi'(\alpha) \geq \phi'(1) = -\ln(A - Br) \qquad \forall \alpha \geq 1 \quad \text{and}$$

$$\varphi'(\alpha) \geq \varphi'\left(-\frac{\ln r}{2}\right) = \frac{4e^{-2}p}{r \ln r} \qquad \forall \alpha > 0.$$

Therefore,

$$l'(\alpha) \geq \phi'(1) + \varphi'\left(-\frac{\ln r}{2}\right) = -\ln(A - Br) + \frac{4e^{-2}p}{r \ln r}$$

and $l'(\alpha) > 0$ if $-\ln(A - Br) + (4e^{-2}p)(r \ln r)^{-1} > 0$, which is equivalent to (23); hence, (23) $\Longrightarrow l'(\alpha) > 0$ for all $\alpha > 0$. Since $l(1) = 0$, $l' > 0$ means that l "crosses" level zero increasingly at $\alpha = 1$, and system (19) has empty solution set, as was to be shown. ■

Clearly Proposition 1 is only theoretically meaningful: it provides the intrinsic shape of the solution set of a model that approximates (18). It basically states that all participants in the amnesty, if any, have a relative risk aversion coefficient belonging to some interval which includes 1. From a different perspective, it states that taxpayers with a relative risk aversion coefficient below some lower bound $\underline{\alpha} < 1$ (that is, those who, by (14), concealed a large amount $w - v$) and taxpayers with a relative risk aversion coefficient above some upper bound $\overline{\alpha} > 1$ (that is, those who, again by (14), concealed only a small income amount) do not enter the amnesty. In order to find values of both extrema $\underline{\alpha}$ and $\overline{\alpha}$, one must rely on numerics, as the following sections show.

3.3 Robustness of our Result: an Example

The proof showed how condition (i) of A.2 forces the unique flex-point of function φ to lie to the right of $\alpha = 1$. Clearly, it is possible to reproduce a similar technique for the case $r > e^{-2}$. Since, in view of most of the possible applications of the model, condition (i) of A.2 is always satisfied, we did not pursue this analysis.

One shortcoming of our proof is that it does not work for "small intervals": when $\overline{\alpha}$ approaches 1 from the right, the sufficient (but not necessary) condition (22) fails, since $h'(1) \leq 0$ does not hold anymore. On the other hand, condition (23) is also only sufficient, and not necessary, to have an empty set as the solution. We illustrate these facts in the following example, where it will be shown that for values of the fixed amount x between the two thresholds (22) and (23) a solution may or may not exist, and, if there is any, it could be an interval not containing 1. This reinforces the argument that our conditions in Proposition 1, even if they are only sufficient conditions, are calibrated well enough to capture most situations.

Also, in view of Section 4 which follows, let us study an example with the following values of parameters: $\gamma = 0.002$, $\delta = \sigma = 1.28$, $p = 0.01$ and $v = 20,000,000$. Assumption A.2 is clearly satisfied since $r \simeq 0.0355 < 0.135 \simeq e^{-2}$ and $\sigma = 1.28$ lies between $1 - e^{-2} \simeq 0.865$ and $(1/2)\left(1 + \sqrt{5}\right) \simeq 1.618$. In particular, condition (i) of A.2 holds with strict inequality and thus parts (ii) and (iii) of Proposition 1 will be relevant. Any fixed payment x such that $0 < x < v - \gamma v^{\delta} \simeq 15,570,564$ satisfies A.1 and will be a good candidate for checking conditions (22) and (23) of Proposition 1.

The upper bound for x in condition (22) turns out to be $\overline{x} \simeq 1,558,294$, therefore any fixed payment that satisfies $x \leq 1,558,294$ produces a nonempty interval $[\underline{\alpha}, \overline{\alpha}]$ of relative risk aversion coefficients characterizing participants in the amnesty. For example, with $x = 800,000$, the interval has $\underline{\alpha} \simeq 0.2$ and $\overline{\alpha} \simeq 4.85$ as its extremes, as shown in figure 1 (a), where the function h discussed in part (ii) of the proof of Proposition 1 is plotted. These two values, through (14), correspond to a minimum true income $w \simeq 29,900,965$ (corresponding to $\overline{\alpha} \simeq 4.85$) and a maximum true income $w \simeq 75,609,144$ (corresponding to $\underline{\alpha} \simeq 0.2$), which imply an evasion (in terms of share of concealed income) of around 33% and around 74% respectively.

The lower bound for x expressed by condition (23) is $\underline{x} \simeq 2,820,247$, hence any fixed payment that satisfies $x > 2,820,247$ produces an empty set of participants in the amnesty. This means that function l is strictly increasing and crosses the x axis at the unique point $\alpha = 1$, as is shown in figure 1 (b) for $x = 3,000,000$.

For any value between $\overline{x} \simeq 1,558,294$ and $\underline{x} \simeq 2,820,247$ Proposition 1 cannot be applied and, in principle, nothing can be said. The sufficient condition (22), fails to detect the existence of a nonempty interval of values of relative risk aversion coefficients, even if such an interval exists for some values of parameter $x \in [\overline{x}, \underline{x}]$. On the other hand, also sufficient condition (23) fails to establish the emptiness of the solution set for other values of the parameter $x \in [\overline{x}, \underline{x}]$.

To illustrate the first case, take, for example, $x = 1,900,000$. Even if (22) is not satisfied, figure 2 (a) shows that all taxpayers with risk aversion coefficients in the interval $[0.87, 1.75]$ will enter the amnesty[10]. The plot clearly shows why (22) fails: the approxima-

[10]The two extrema of the interval $[0.87, 1.75]$, through (14), correspond in this case to a minimum

tion of function l (dot line) through function h (solid line) used in part (ii) of the proof of Proposition 1, which differs from l only in the interval $(1, -(1/2)\ln r) \simeq (1, 1.669)$, turns out to be too rough as the interval $[\underline{\alpha}, \overline{\alpha}]$ shrinks. In particular, for $x = 1,900,000$, the right derivative $h'_+(1)$ turns out to be positive while $l'(1)$ is negative.

A similar situation is shown in figure 2 (b), where only function l is plotted for $x = 2,115,000$. This is a peculiar circumstance since, even if $l'(1)$ itself turns out to be positive, a nonempty interval of risk aversion coefficients characterizing taxpayers who enter the amnesty exists: the interval[11] $[1.11, 1.26]$. Note that, unlike the solution sets characterized by condition (22), this interval does not contain the value $\alpha = 1$.

Figure 2 (c) shows a situation where condition (23) is not satisfied but the solution set is empty for $x = 2,125,000$. Here function l crosses the x axis increasingly at $\alpha = 1$, but $l'(\alpha)$ is negative for some $\alpha > 1$.

The last three counterexamples considered support the robustness of our Proposition 1 by showing that outside the conditions used in the proposition anything goes, that is, conditions (22) and (23), while being only sufficient, seem to be "nearly necessary". Moreover, these counterexamples show that Proposition 1 proves to be useless only in special cases where the possible interval of participants is extremely tiny, a circumstance that does not seem very appealing while planning a tax amnesty program. It is not coincidental that all estimates reported in the next section fall comfortably within the range provided by condition (22).

A last remark regards the error on the lower bound $\underline{\alpha}$ introduced by considering the

true income $w \simeq 50,945,526$ (corresponding to $\overline{\alpha} \simeq 1.75$) and a maximum true income $w \simeq 70,519,592$ (corresponding to $\underline{\alpha} \simeq 0.87$), which imply an evasion of around 61% and around 72% respectively.

[11]The two extrema of the interval $[1.11, 1.26]$, through (14), correspond in this case to a minimum true income $w \simeq 61,258,459$ (corresponding to $\overline{\alpha} \simeq 1.26$) and a maximum true income $w \simeq 64,929,017$ (corresponding to $\underline{\alpha} \simeq 1.11$), which imply an evasion of around 67% and around 69% respectively.

approximated model (19) in place of (18). Again numerics show that such an error is small compared to the size of the whole interval $[\underline{\alpha}, \overline{\alpha}]$, in part because it affects only the side on the left of $\alpha = 1$.

4 Estimation on Data Pertaining to the 1991 and 1994 Italian Tax Amnesties

Italian Law no. 413/1991 introduced a general tax amnesty regarding basic Italian taxes[12]. This amnesty was considered a success in terms of participation and revenue. Participation was highly concentrated among taxpayers with self-employment and business income (where it encompassed 40% of those who had filed a tax report), while it was scanty among wage earners. While this amnesty followed another major one granted ten years before, it was nevertheless not easy for taxpayers to anticipate its timing and characteristics in order to suitably modify their tax reports. Thus, treating the amnesty as unanticipated seemed an acceptable starting point, to be checked *ex post* by comparing estimation results with those available from other sources.

With reference to the income tax, for taxpayers not yet audited, par. 38 of the amnesty law provides rules for calculating the extra payment necessary to enter, *i.e.*, for calculating variable x in our model. They are summarized in Table 1. Requested x payment is an increasing function of the income tax already paid by the participant and thus of her reported income v. This schedule seems to be dictated by the aim of extracting each taxpayer's willingness to pay: i.e., Italian legislators exploited the information conveyed by income reports about potential "amnesty demand".

Payment x was due for each year for which the amnesty was entered, from 1985 to

[12] More details are provided in Marchese and Privileggi (1997).

1990. To calculate the income tax amount due from each taxpayer, parameters of the tax function (2) have been estimated with reference to the actual income brackets, for each year for which the amnesty could be entered: details are reported in Appendix A. Amount x due was calculated on the basis of the estimated tax due, according to the rules described in Table 1. In view of the discussion in Appendix B, the sanction parameter σ in (3) has been set equal to the tax parameter δ in (2). The audit rate[13] in Italy in the relevant years was around 1%; this is thus the value used for parameter p.

Following the same procedure carried out in the example of Section 3.3, for every amount x requested, we calculated upper and lower threshold levels of relative risk aversion, $\overline{\alpha}$ and $\underline{\alpha}$, and percentage evasion, $\underline{e}$ and $\overline{e}$, needed for participation in the amnesty. Values of parameters $p, \gamma, \sigma = \delta$ and all reported incomes v, are such that condition (i) of Assumption A.2 holds with strict inequality. Moreover, it is important to remark that all values of parameter x turned out to be well below the bounds given by condition (22) in Proposition 1.

Here we report numeric solutions of model (19) for some representative taxpayers. Table 2 reports the results for a taxpayer endowed with an average net reported income, and who enjoys average deductibles (case a); Table 3 refers to a taxpayer endowed with average net reported business income[14] (case b); Table 4 to a taxpayer endowed with average net reported self-employment income[15] (case c). All the taxpayers considered paid a tax which belongs to the first bracket of Table 1.

[13] For data about controls, see Ministero delle Finanze, Ufficio di Statistica, *Accertamenti effettuati ai fini delle imposte dirette*, Roma, various issues.

[14] The average value is calculated without taking into account taxpayers who report income equal to zero, and refers to entrepreneurs in the ordinary tax regime (thus excluding cases of forfeit).

[15] The average value is calculated without taking into account the taxpayers who report an income equal to zero, and refers to all types of self-employment.

The estimated relative risk aversion interval ranges from a high of around $\alpha = 4.5$ to virtually zero (as some lower threshold values are likely to be strictly positive only because the solution of system (18) is approximated by the solution of system [19]) and is thus well inside the range $1 - 10$ usually considered[16]. The upper risk aversion threshold is not far from that calculated in Marchese and Privileggi (1997), which ranged from 6.3 to 4.6[17].

The lower threshold percentage evasion in the examples examined always lies between 34 and 35% of the true income (to be compared with 33 and 34% calculated in Marchese and Privileggi [1997]). The upper threshold evasion is 73/75%, thus suggesting that only quasi-full evaders were left out of the amnesty.

To roughly assess the results, the calculated percentage evasion thresholds can be compared with available evasion estimates from other sources[18]. For the average Italian taxpayer during the relevant time period, the minimum evasion estimate is 19.9%, the maximum 36%. For self-employment and business income, the corresponding values are 42.9% - 58.1%. For income from wages or pensions, evasion is relatively quite low (8.1% - 16.1%). Our results show, for the average taxpayer, a lower-threshold percentage evasion (needed to make the amnesty worth entering) which is close to the top evasion values calculated by other studies. This fact is roughly consistent with the idea that the amnesty was not designed to be appealing for the average taxpayer. For those endowed with the average business or self-employment income instead, the mean value of our estimated thresholds is at most 10% higher than the mean of the evasion estimates available from other studies. The interval we found is thus consistent with an amnesty design aimed to be appealing for mean/high evaders in business or self-employment.

[16] Epstein and Zin (1990) quote a tradition concerning relative risk aversion, which should not be greater than $\alpha = 10$.

[17] Remember that when CARA is assumed there is no lower risk aversion threshold.

[18] For surveys, see Bernasconi (1995) and Monacelli (1996).

Law 656/1994 granted a further amnesty (*concordato di massa*) reserved for entrepreneurs and the self-employed. Rules regulating this amnesty provided an entrance payment based upon relative gross revenue and profitability revealed by the tax reports[19]. Estimation results based on model (19) are shown in Table 5.

They largely confirm results reached for the 1991 tax amnesty. Calculated lower threshold evasion and upper threshold risk aversion parallel previous findings in Marchese and Privileggi (1997) with a somewhat larger downward correction for risk aversion and upward correction for evasion than for the 1991 tax amnesty.

An interesting reference point for assessing risk aversion estimates is represented by results reported in Guiso and Paiella (2001). To elicit risk attitudes they exploited household reactions to a hypothetical lottery offered to the sample of the Italian population involved in the periodical Bank of Italy household survey of 1995. According to their estimates, relative risk aversion range from 0.2 to 36.3, with a right-skewed distribution, a median value of 4.8 and a mean of 5.38. Our estimates are thus coherent with an amnesty design aimed at being selective and appealing for median-low risk averse agents.

5 Conclusions

Optimal taxation literature has pointed out (see, e.g., Brito et al. [1995]) that differences in risk aversion may signal other relevant taxpayers characteristics, such as income or productive ability. This observation has been mainly exploited to develop models of random taxation. Suitably designed random taxes may in fact help in overcoming problems of asymmetrical information between government and taxpayers. Moreover, differences in

[19] See Marchese and Privileggi (1997) for details. The amnesy was not available for those who had already benefited from the 1991 one according to par. 38. For many taxpayers, the 1994 amnesty was cheaper than the previous one.

risk aversion imply that the least risk averse citizens are those most willing to play lotteries. Random taxation may increase efficiency by offering lotteries which are cheaper to implement[20] than the "tax evasion lottery", which implies running tax controls (see Pestieau et al. (1998).

Actual tax systems, however, do not explicitly seem to resort to the introduction of forms of gambling, perhaps because of the traditional uncertain statute of this criterion on moral and also religious ground. Moreover, if one assumes that taxpayers are risk averse and that a benevolent government is at most risk neutral, randomization seems to be a costly way of inducing taxpayer self selection, whenever it increases the overall amount of risk with reference to the *status quo ante.* Taxpayer self-selection systems that resort to insurance offers, such as FATOTA or unexpected tax amnesties, seem thus a more natural and efficient way to pursue the same goals.

Some nice efficiency and welfare improving characteristics of the latter instruments have been clarified in the literature. Both FATOTA and tax amnesties, however, may imply equity problems, as they introduce some kind of discrimination. Specifically, taxpayers with the same true income may pay different total amounts according to their attitudes toward risk. Moreover, if one introduces time discounting and taxpayer anticipation, amnesties may increase tax evasion by prospective participants, with effects upon total tax revenue that could turn from positive into negative. In this paper we have added a further *caveat* for the use of these instruments, by demonstrating that self-selection may fail when a CRRA specification of the taxpayer utility function is considered. Our result is in line with findings in the literature about plea-bargaining: Grossman and Katz (1983) have noted that self-selection of the guilty may be problematic when the indicted differ in risk-aversion[21].

[20]For instance, by offering the possibility of opting for a high tax and giving to those who accept it a ticket for a lottery that provides for a given expected rebate.

[21]Other related results pertain to the so called cut-off rule, according to which taxpayers who report a

Extension of their observations to either FATOTA or amnesty models, however, is not straightforward, as in their model committing the crime is a discrete choice (not explicitly studied), and each defendant has an exogenously given degree of risk aversion. When taxes are considered instead, the amount of the law breach (tax evasion), is a continuous variable, while risk attitudes (which arguably vary depending on income) contribute to motivating both the amount of the law breach and the willingness to accept a settlement proposal. Self-selection of those guilty of large-scale tax evasion through FATOTA or tax amnesties is thus quite likely to work, as happens in the examples in Chu (1990), Franzoni (2000), and Marchese and Privileggi (1997). However, the present paper shows that the case of an imperfect self-selection cannot be ruled out in general. With a CRRA specification, participation in amnesties or FATOTA programs at any rate occurs in a predictable pattern, and leaves out evaders below and above a given risk aversion interval.

With reference to empirical results, we find that the expected utility approach works reasonably well to describe the behavior of participants in Italian tax amnesties, thus reinforcing the findings of a former paper by Marchese and Privileggi (1997). The model studied in Marchese and Privileggi (1997) was constructed by means of an exponential utility function (with constant absolute risk aversion), and was capable of describing only the marginal (lower threshold tax evasion) participants; very little emerged about those (arguably the majority of participants) who received a strictly positive benefit from participating in the amnesty program. The new version presented in this paper provides more information on the characteristics and the evasion extent of taxpayers entering the amnesty. While one may argue that some quasi-full evaders did not participate, the coverage of the Italian amnesties within the target social groups seems large, thus providing a different

threshold income amount should not be audited. In this case, allowing for differences in risk-aversion may undermine the working of the mechanism as well.

picture from the case studied in Fisher et al. (1989) where the amnesty considered was appealing mainly to small evaders. On the other hand, empirical estimates of lower evasion thresholds based on the model presented in Section 2 are quite close to those of the previous study by Marchese and Privileggi (1997), which relied on a different utility function specification, thus further enhancing the appeal of the whole approach.

Note, finally, that the model discussed in Section 3 lends itself also to a converse view of the approach pursued in this paper, where we focussed exclusively on the problem of unravelling the information about taxpayers characteristics conveyed by amnesty participation. Since the solution set of participants in a FAT or amnesty offer is represented by an interval $[\underline{\alpha}, \overline{\alpha}]$ on the real line, a different model can be constructed aimed at finding a value for parameter x that maximizes a given social or government utility function. To solve this problem one must figure out the number of participants in the program when x is offered, which depends on both the length $\overline{\alpha} - \underline{\alpha}$ of the interval and the distribution of taxpayers over the interval itself. Hence, while the problem tackled in this work was to determine values for the unknown variable α, such a model would require the distribution of taxpayer risk aversion α to be given. This alternative approach, which could provide further insight into the equity and efficiency implications of tax amnesties, is left for future research.

6 Appendix

A The Tax Function

The progressive income tax due, as a function of the net taxable income, can be represented by as many linear segments as the number of income brackets; the higher the bracket, the higher the positive slope of the segment. O.L.S. estimation technique (with variables in logarithms) was used to interpolate each tax schedule in order to obtain a shape like in (2). As the income tax was often modified by the government in the period for which the amnesties described in this paper were available, there were six tax schedules to consider, as reported in Table 6.

B Constructing the Linear Sanction Function

In order to be consistent with the progressivity implied by the tax function (2), the sanction to be applied in case of detection should exhibit some degree of progressivity as well. The natural choice would be the same function on $\mathbb{R}^2$ as in Marchese and Privileggi (1997), that is

$$s(w, y) = g\gamma \left(w^\delta - y^\delta \right), \tag{25}$$

where $g > 1$ is a penalty rate. Here, parameter g replaces the more widely used term $1 + s$; that is, as in standard tax evasion models, the detected evader has to pay both the due tax and the penalty s times the evaded tax.

On the other hand, we have seen in Section 3 that a crucial step in simplifying system (11) requires a sanction function that is linear in the true income w; therefore we assumed the form $\widehat{s}(w, y) = \sigma(w - y)$ as in (3). In this appendix, we provide some arguments for determining a range for values of the coefficient σ so that $\widehat{s}(w, y)$ in (3) does not greatly differ from $s(w, y)$ in (25). To do this, let us discuss more thoroughly some restrictions required by (25).

As we did in Section 2 through condition (4), we assume that cheating taxpayers caught by the authorities are always able to pay the sanction; in terms of (25) this implies that

$$g\gamma \left(w^\delta - y^\delta \right) \leq w - \gamma y^\delta, \tag{26}$$

which, in turn, forces the reported income y not to be smaller than a certain amount depending on parameters and the true income:

$$y \geq \left[\frac{g\gamma w^{\delta} - w}{(g-1)\,\gamma}\right]^{(1/\delta)}. \tag{27}$$

Moreover, the left hand in (27) is defined only if $g\gamma w^{\delta} - w > 0$, which yields a lower bound also for the true income w to be considered:

$$w > (g\gamma)^{1/(1-\delta)}. \tag{28}$$

Consider the first order Taylor expansion of (25) on the identity line, that is, on points $(w^*, y^*) \in \mathbb{R}^2$ such that $w^* = y^*$, which, in turn, implies $s(w^*, y^*) = 0$:

$$s(w,y) = g\gamma\delta a^{\delta-1}(w-y) + o\left(\|(w,y)-(a,a)\|^2\right),$$

where $a \in \mathbb{R}$ satisfies (28), that is $a > (g\gamma)^{1/(1-\delta)}$. Hence, function

$$L(w,y) = g\gamma\delta a^{\delta-1}(w-y)$$

is the linear approximation of $s(w,y)$ around some point $(a,a) \in \mathbb{R}^2$. By denoting

$$\sigma = g\gamma\delta a^{\delta-1}, \tag{29}$$

we get a linear approximation of the progressive sanction $s(w,y)$ in (25) of the same form as in (3). Now we need to find suitable values for the critical point a, which translate in suitable values for σ through (29). This will be achieved thanks to conditions (27) and (28), which will provide an upper and a lower bound respectively for the critical point a.

On one side we can always calculate a lower bound for a by using the infimum value[22]

$$\underline{a} = (g\gamma)^{1/(1-\delta)}. \tag{30}$$

[22] By using such a point as the critical value for our approximation, we actually assume the relative sanction $\widehat{s}$ to be parametrized on the lowest income bracket. In our model, this value represents the borderline "poorest full complier" who earns the infimum (not even feasible) income $w = (g\gamma)^{1/(1-\delta)}$ and is compelled to be honest by the model itself through condition (26). The resulting approximation $\widehat{s}$ would be biased in favor of richer evaders who would face a relatively lower sanction in case of detection.

On the other side, an upper bound is impossible to compute directly, since the true income w is unknown to the authorities, and condition (27) cannot be solved in terms of maximum true income w given some reported income y. To be more specific, condition (27) establishes the minimum reported income y that satisfies solvency condition (26); by reading condition (27) in the opposite direction, one might pessimistically assume y to be the minimum feasible reported income, that is,

$$y = \left[\frac{g\gamma w^{\delta} - w}{(g-1)\gamma} \right]^{(1/\delta)}, \tag{31}$$

and guess that the true income is the w in the right hand side of (31), for the given y. The income w obtained this way is the maximum true income[23] compatible with reported income y, that satisfy (26), and could be a good candidate for an upper bound $\overline{a}$ of our critical point a.

Unfortunately, (31) does not allow for calculation of w as a function of y. Hence we shall rely on some upper bound of such true income w, obtained by linearizing from below the strictly convex function $g\gamma w^{\delta} - w$. Since we are considering values $w > (g\gamma)^{1/(1-\delta)}$, a useful approximation turns out to be on the point $w^* = (g\gamma)^{1/(1-\delta)}$, where $g\gamma w^{*\delta} - w^* = 0$. Hence, if we let

$$y = \left\{ \frac{(\delta - 1)\left[w - (g\gamma)^{1/(1-\delta)}\right]}{(g-1)\gamma} \right\}^{(1/\delta)} < \left[\frac{g\gamma w^{\delta} - w}{(g-1)\gamma} \right]^{(1/\delta)}, \quad \text{for all } w > (g\gamma)^{1/(1-\delta)},$$

and, solve the equality on the left side for w as a function of y, we get an upper bound

$$\overline{a} = \frac{g-1}{\delta - 1}\gamma y^{\delta} + (g\gamma)^{1/(1-\delta)} \tag{32}$$

[23] Such an upper bound for the true income w would represent some fictitious "rich" taxpayer who cheated the most by reporting y as in (31). As opposed the previous case, the resulting approximation $\widehat{s}$ would punish the poorer income brackets by considering a relative (linear) sanction parametrized with respect to the richest.

for the true income which is larger than any true income w satisfying (27) for the given reported income y.

By calculating σ for both values $\underline{a}$ as in (30) and $\overline{a}$ as in (32), we get the lower and the upper bounds

$$\underline{\sigma} = \delta \tag{33}$$

$$\overline{\sigma} = g\gamma\delta \left[\frac{g-1}{\delta-1} \gamma y^{\delta} + (g\gamma)^{1/(1-\delta)} \right]^{\delta-1}, \tag{34}$$

where (increasing) dependency of the upper bound $\overline{\sigma}$ on the reported income y reflects its "progressivity" with respect to higher reported incomes.

The lower approximation (33) looks very terse, since it does not even depend on parameter g. Moreover, for values of parameters used in the estimates of Section 4, it satisfies both conditions (i) and (ii) in Assumption A.2, and numerics show that it behaves very well for the true incomes w that were the targets of the amnesties there considered. This justifies the adoption of the sanction function $\widehat{s}(w, y) = \delta\,(w - y)$ there. In general, however, some good theoretical compromise could be achieved by taking an average of (33) and (34), that is, by letting $\sigma = \lambda\underline{\sigma} + (1-\lambda)\,\overline{\sigma}$, with $0 < \lambda < 1$. Clearly, by doing so, one must keep an eye on the right inequality in condition (ii) of A.2, which can easily be violated.

It is interesting to remark, finally, that the choice of the minimum value $\underline{\sigma} = \delta$ implies the largest participation in the amnesty, as comparative static analysis shows. To see this, consider function f defined in (17) and rewrite it as a function of both α and σ:

$$f(\alpha, \sigma) = \left\{ A\,(\sigma) - B\,(\sigma) \left[r\,(\sigma) \right]^{1/\alpha} \right\}^{1-\alpha} - p \left\{ \left[r\,(\sigma) \right]^{1/\alpha} \right\}^{1-\alpha} - (1-p),$$

where, following definitions (13), (15) and(16),

$$r\,(\sigma) = \left(\frac{p}{1-p} \right) \left(\frac{\sigma}{\gamma\delta v^{\delta-1}} - 1 \right)$$

$$A\,(\sigma) = 1 + \frac{x}{v - \gamma v^{\delta}} \left(\frac{1}{\sigma} - 1 \right)$$

$$B(\sigma) = \frac{x}{\sigma\left(v - \gamma v^{\delta}\right)}.$$

Straightforward calculations show that

$$\begin{cases} \frac{\partial f}{\partial \sigma} < 0 & \text{if } 0 < \alpha < 1 \\ \frac{\partial f}{\partial \sigma} > 0 & \text{if } \alpha > 1, \end{cases} \tag{35}$$

for all feasible values of other parameters.

Now suppose that either condition (21) or (22) of Proposition 1 holds. Since, as we have seen in Section 3, the solution set of system (11) is equivalent to the solution set of system (18), which is,

$$\begin{cases} f(\alpha, \sigma) \geq 0 & \text{if } 0 < \alpha \leq 1 \\ f(\alpha, \sigma) \leq 0 & \text{if } \alpha \geq 1, \end{cases}$$

inequalities (35) mean that the graph of $f(\alpha)$ is uniformly lower if $0 < \alpha \leq 1$ and uniformly higher if $\alpha \geq 1$ for higher values of the sanction σ, which imply a "smaller" solution set $[\underline{\alpha}, \overline{\alpha}]$, as can easily be understood from figure[24] 1 (a). In other words, this heuristic argument shows that the effects of increasing the sanction is a narrowing of the interval of relative risk aversion which characterizes amnesty participants, which must be the largest for the minimum sanction $\underline{\sigma} = \delta$, all other parameters remaining fixed.

[24] In figure 1 (a) $h(\alpha)$ is actually plotted, which is an approximation of function $f(\alpha)$. Recall, however, that h is a lower bound for f if $0 < \alpha \leq 1$ and an upper bound for f if $\alpha \geq 1$; hence, inequalities (35) imply the same effects on the graph of h as on the graph of f.

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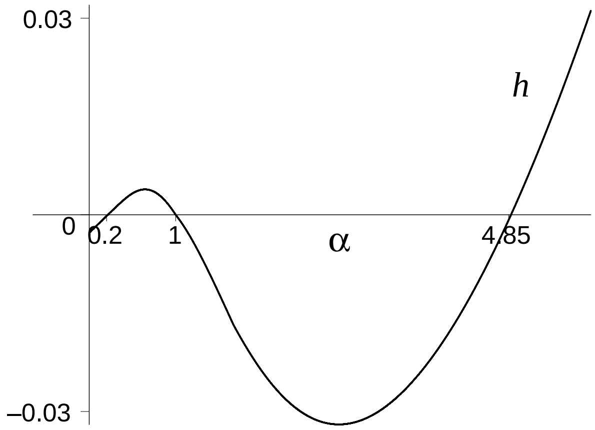
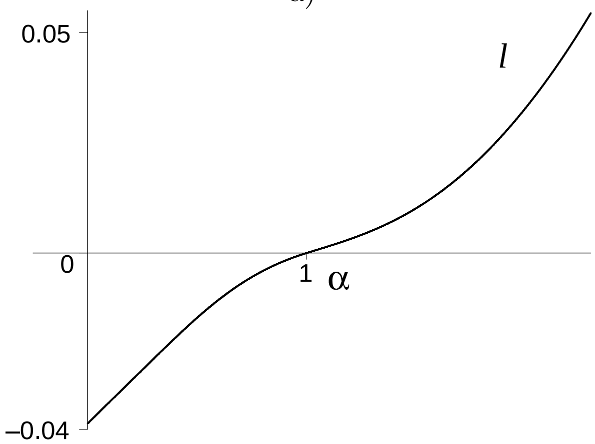
C Figures

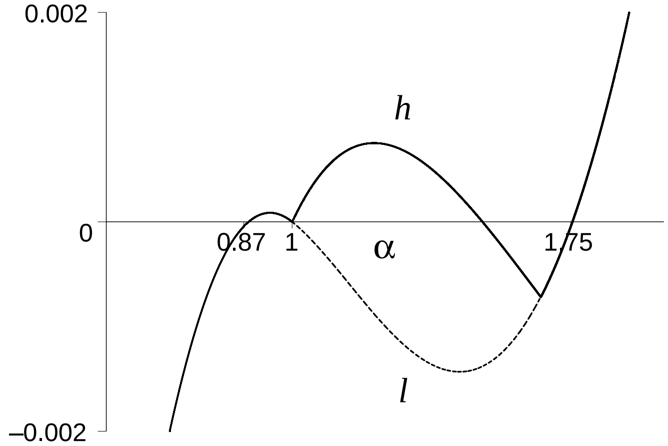


a)

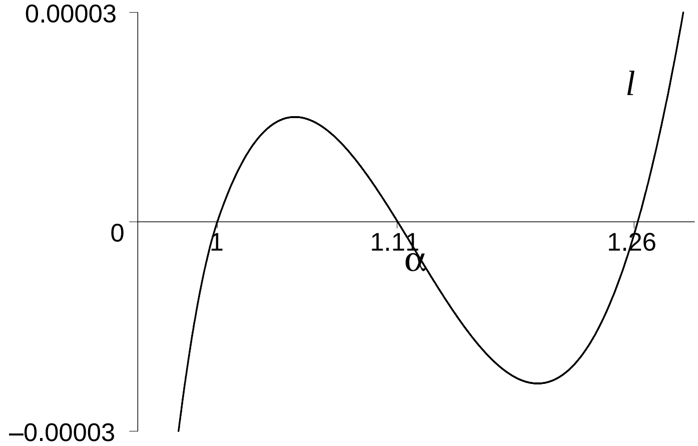


b)

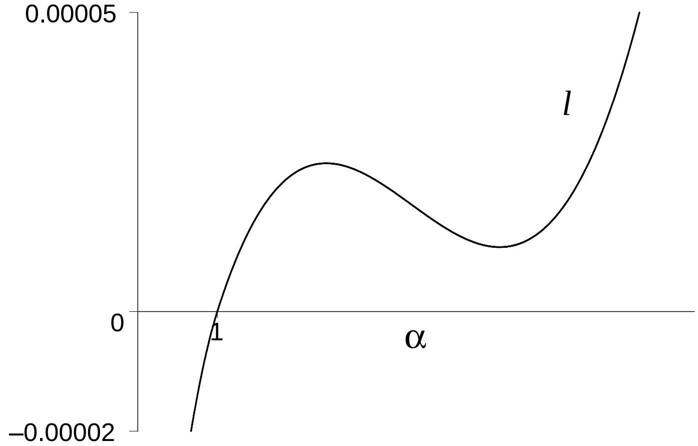
Figure 1: illustration of Proposition 1: a) plot of the function h for $x = 800,000$; b) plot of the function l for $x = 3,000,000$.



a)



b)



c)

Figure 2: failure of Proposition 1: a) plot of functions l (dot line) and h (solid line) for $x = 1,900,000$; b) plot of the function l for $x = 2,115,000$; c) plot of the function l for $x = 2,125,000$.

D Tables

Brackets of paid tax	**Extra payment due**
$0-10$	20% of the paid tax (with a minimum of 0.1)
$10-40$	18% of the paid tax
>40	15% of the paid tax

Table 1: payment x in the 1991 tax amnesty (millions of Italian Lire).

Year	v	$\overline{\alpha}$	$\underline{e}$	$\underline{\alpha}$	$\overline{e}$
1985	13.36	4.45	35%	0.21	75%
1986	14.18	4.44	34%	0.13	74%
1987	15.37	4.44	34%	0.15	74%
1988	16.74	4.44	34%	0.18	73%
1989	18.27	4.48	34%	0.08	75%
1990	19.45	4.48	34%	0.08	75%

Table 2: estimation results, 1991 tax amnesty, case (a) (v = reported income in millions of Italian lire; $\overline{\alpha}$ = upper threshold risk aversion; $\underline{e}$ = lower threshold percentage evasion; $\underline{\alpha}$ = lower threshold risk aversion; $\overline{e}$ = upper threshold percentage evasion).

Year	v	$\overline{\alpha}$	$\underline{e}$	$\underline{\alpha}$	$\overline{e}$
1985	12.80	4.45	35%	0.20	74%
1986	13.20	4.46	34%	0.11	74%
1987	14.00	4.45	34%	0.13	74%
1988	15.50	4.44	34%	0.16	74%
1989	17.92	4.48	34%	0.07	75%
1990	18.20	4.49	34%	0.06	75%

Table 3: estimation results, 1991 tax amnesty, case (b) (v = reported income in millions of Italian lire; $\overline{\alpha}$ = upper threshold risk aversion; $\underline{e}$ = lower threshold percentage evasion; $\underline{\alpha}$ = lower threshold risk aversion; $\overline{e}$ = upper threshold percentage evasion).

Year	v	$\overline{\alpha}$	$\underline{e}$	$\underline{\alpha}$	$\overline{e}$
1985	16.10	4.43	35%	0.25	74%
1986	17.60	4.43	34%	0.19	73%
1987	19.60	4.42	35%	0.22	73%
1988	23.10	4.40	35%	0.26	73%
1989	25.84	4.45	35%	0.17	74%
1990	28.05	4.45	35%	0.18	74%

Table 4: estimation results, 1991 tax amnesty, case (c) (v = reported income in millions of Italian lire; $\overline{\alpha}$ = upper threshold risk aversion; $\underline{e}$ = lower threshold percentage evasion; $\underline{\alpha}$ = lower threshold risk aversion; $\overline{e}$ = upper threshold percentage evasion).

Year	v	$\overline{\alpha}$	$\underline{e}$	$\underline{\alpha}$	$\overline{e}$
1987	19.32	4.26	36%	0.24	73%
1988	20.17	4.18	36%	0.26	73%
1989	22.98	4.04	37%	0.22	74%
1990	24.33	4.07	37%	0.21	74%
1991	25.31	4.08	37%	0.28	74%
1992	30.32	5.64	29%	0.10	73%

Table 5: estimation results for a taxpayer endowed with median business income from manufacturing industry, 1994 tax amnesty (v = reported income in millions of Italian lire; $\overline{\alpha}$ = upper threshold risk aversion; $\underline{e}$ = lower threshold percentage evasion; $\underline{\alpha}$ = lower threshold risk aversion; $\overline{e}$ = upper threshold percentage evasion).

Year	γ	δ
1985	0.002249	1.274515
1986 – 1988	0.001441	1.292886
1989	0.001537	1.282436
1988	0.001508	1.282429
1989	0.001739	1.279154
1990	0.001615	1.285728

Table 6: parameters of the tax function $t(y) = \gamma y^{\delta}$ for years 1985-1992 (OLS estimation on logarithms of data in Italian Lire).