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An Extention of the Model of Inequity Aversion by Fehr and Schmidt

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Abstract. The aim of this paper is to improve on the model by Fehr and Schmidt (1999) by developing a non-linear model (that leads to interior rather than corner solutions) and by taking into account that different levels of income imply different reactions of fair-minded people. We suggest to modify the inequity-aversion utility function proposed by Fehr and Schmidt by taking into account not only the difference between players' payoffs, but also their absolute value. This allows for a non-linear utility function where different stakes lead to different unique optimal interior solutions.

JEL classification: A13, C72, C91, D63, D64

Keywords: Fairness, Inequity-aversion

Introduction

The inequity-aversion model by Fehr and Schmidt (FS onwards) represents one of the most important theoretical contributions to fairness studies. Its relevance is due to its simplicity and to the consistency of theoretical solutions with experimental evidence in different games. However, too much emphasis has been assigned to the second positive feature. In fact, the FS model fails to explain two relevant experimental results: the significant percentage of interior solutions in some distributions games (i.e. ultimatum game, Güth et al., 1982; and dictator game, Forsythe et al., 1994) and the relevance of different monetary stakes in players' decisional process (i.e. Slonim and Roth, 1998; Munier and Zaharia, 2002).

The aim of this paper is to improve on the model by FS by developing a non-linear model (that leads to interior rather than corner solutions) and by taking into account that different levels of income imply different reactions of fair-minded people.

1. The model

The inequity-aversion model by FS, in a two-player game, states that:

$$U_i(x_i, x_j) = x_i - \alpha_i \max\{x_j - x_i, 0\} - \beta_i \max\{x_i - x_j, 0\}, \qquad i \neq j \tag{1}$$

where:

$x_{i,j}$ is the payoff of player i (or j)

α_i is a parameter of envy

β_i is a parameter of altruism

$0 \leq \beta_i < 1$ and $\alpha_i \geq \beta_i$ since the disutility that comes from a position of disadvantage is higher than the disutility that comes from a position of advantage;

$\partial U_i / \partial x_j \geq 0$ iff $x_i \geq x_j$ since the marginal utility of an increase in others' income is positive if and only if they have a lower level of income with respect to subject i.

A problem with this model is that, when applied, for example, to the *Ultimatum Game* and to the *Dictator Game*, it provides corner solutions depending on the value of β. In particular, when $\beta_i \in [0, 0.5)$, player i always maximizes her own payoff choosing not to transfer any sum to player j. On the other hand, when $\beta_i \in (0.5, 1)$, player i always maximizes her own payoff choosing to share equally the total amount of money with player j. Finally, when β_i is equal to 0.5, player i is indifferent between any distribution of the total payoff S where $x_i \in [S/2, S]$. In other words, this model is unable to clearly explain players' optimal interior choices, that are the most common results (for a survey, Fehr and Camerer, 2003).

The assumption of a linear utility function, which awards simplicity to the model, is the reason why interior solutions are not well defined. Kohler (2003) argues that 'an increasing degree of difference aversion resolves the shortcoming that only two "focal" equilibria exist'.[1] However, his model holds only when the initial endowment S is normalized to 1. Consequently, this does not allow any analysis concerning different levels of endowment, which is our main concern. Moreover, if we consider the actual value of the initial endowment, the (for example) quadratic difference becomes extremely high with high numbers and even an almost selfish Proposer will decide to transfer half of the sum.

To reach our goal, we suggest to modify the initial inequity-aversion utility function (1) by taking into account not only the difference between player i 's and player j's payoffs, but also their

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[1] Kohler, 2003, p.7.

absolute value.[2] This allows for a non-linear utility function where different stakes lead to different unique optimal interior solutions. Our utility function is:

$$V(x_i, x_j) = f(x_i) - \alpha_i f(x_j - x_i, x_i) - \beta_i f(x_i - x_j, x_j) \tag{2}$$

We assume that the second term of the previous function is increasing with respect to the difference and decreasing with respect to the value of x_i. At the same time, the third term is increasing with respect to the difference and decreasing with respect to the value of x_j.

We consider the following form of the utility function presented in equation (2) to analyse the implications of the model:

$$U_i(x_i, x_j) = x_i - \alpha_i \max\left\{\frac{x_j - x_i}{\gamma_i x_i + 1}, 0\right\} - \beta_i \max\left\{\frac{x_i - x_j}{\sigma_i x_j + 1}, 0\right\}, \quad i \neq j. \tag{3}$$

Let's re-write equation (3) as follows:

$$U_i = \begin{cases} x_i - \alpha_i\left(\dfrac{x_j - x_i}{\gamma_i x_i + 1}\right) & x_j > x_i \quad (4) \\ x_i - \beta_i\left(\dfrac{x_i - x_j}{\sigma_i x_j + 1}\right) & x_i \geq x_j \quad (5) \end{cases}$$

In equation (4), we consider the utility of player i when her payoff is lower than player j's payoff. We assume that the level of x_i influences the disutility due to the payoffs' difference. In particular, the higher the level of x_i, the lower the discomfort due to the difference. However, we assume that subjects' concern for their income as a weight of the difference is not equal among the population. This is represented by the parameter $\gamma_i \in [0,1]$. When $\gamma_i = 1$, player i's income has the same relevance as the difference. When $\gamma_i = 0$, player i's income has no relevance and the model corresponds to the model by FS. Obviously, for a given value of α_i, x_i and x_j, the higher γ_i, the higher the level of the utility.

The lowest level of equation (4) is reached when $x_i = 0$. In this case:

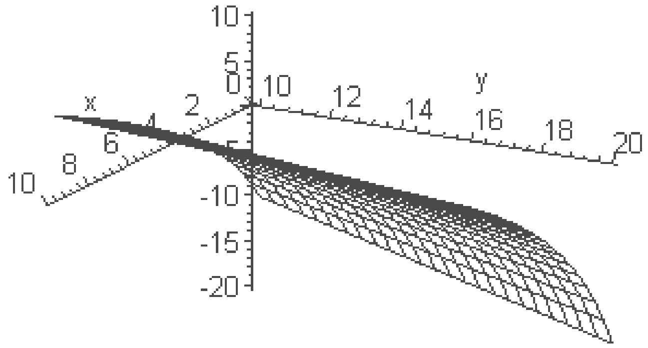
[2] It comes from common sense that the discomfort due to inequality decreases as the income of the worse-off player increases. Consider, for example, two subjects in two different scenarios. In the first case, player A has 10 euro and player B 0 euro. In the second case player A has 1000 euro and player B 990 euro. Player A will suffer more in the first situation.

$$U_i = -\alpha_i (x_j) \tag{6}$$

and the higher α_i the higher the disutility (see Figure 1).

Figure 1.

Envy function: an example



where:

$\alpha_i = 1$

$\gamma_i = 0.3$

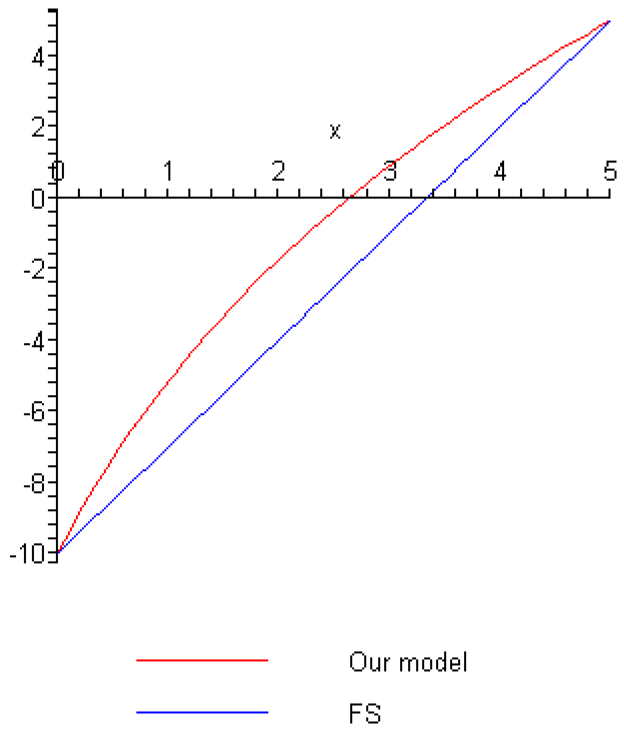
$x = x_i \in [0,10]$

$y = x_j \in [10,20]$

It is interesting to notice that the corner points of our function ($x_i = 0$, $x_i = x_j$) correspond to the corner points of FS's function (see Figure 2).[3]

[3] For simplicity, we consider a typical ultimatum or dictator scenario, where the sum of the payoffs is constant.

Figure 2.



where:

$\alpha_i = 1$

$\gamma_i = 0.3$

$x = x_i$

$x_i + x_j = 10$

In equation (5), we consider the utility of player *i* when her payoff is higher than player *j'*s payoff. As in FS, player *i'* s utility is negatively influenced by the payoffs difference. However, this difference has a decreasing 'weight' with respect to x_j.

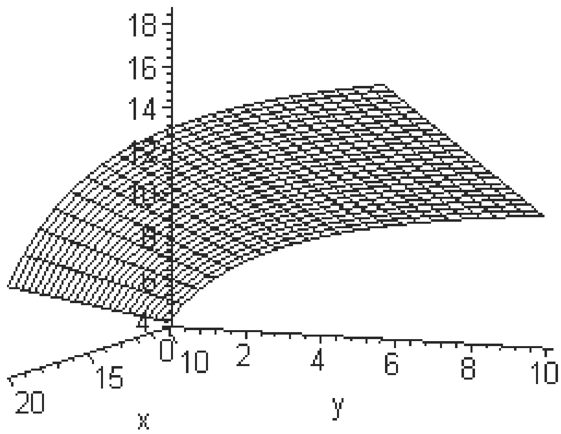
The lowest level of Figure 1 is reached when x_j is equal to 0. In this case:

$$U_i = (1 - \beta_i)x_i \tag{7}$$

This means that when $x_j = 0$, player *i'*s utility is a weighted function of x_i. The higher β_i, the lower the utility (see Figure 3).

Figure 3.

Altruism function: an example



where:

$\beta_i = 0.6$

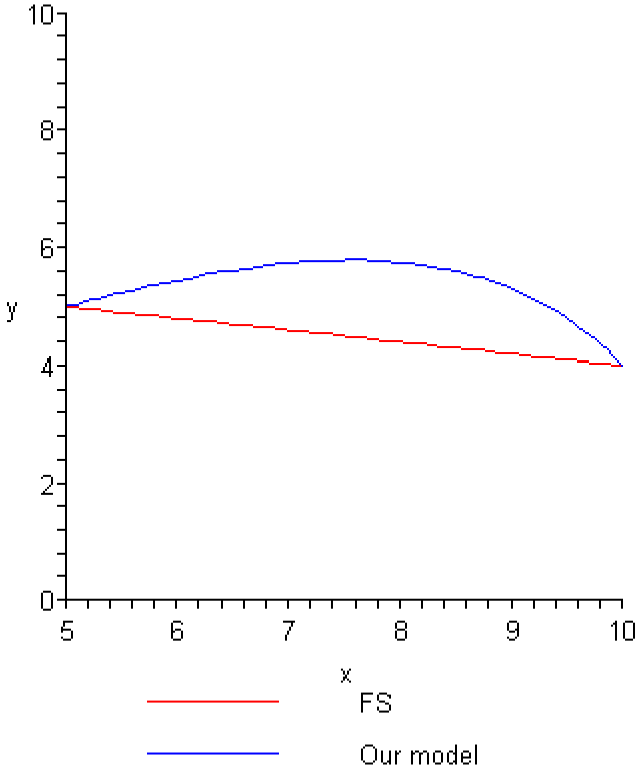
$\sigma_i = 0.3$

$x = x_i \in [10,20]$

$y = x_j \in [0,10]$

Also in this case, the corner points of our function ($x_i = x_j, x_j = 0$) correspond to the corner points of FS's function, whatever the level of σ_i (see Figure 4).

Figure 4



where:

$\beta_i = 0.6$

$\sigma_i = 0.3$

$x = x_i \in [10,20]$

$y = x_j \in [0,10]$

As for FS, $\alpha_i \geq \beta_i$ and $0 \leq \beta_i < 1$.

Consider now the first and the second derivatives of equation (4) with respect to x_i:

$$\frac{\partial U_i}{\partial x_i} = 1 + \frac{\alpha_i}{\gamma_i x_i + 1} + \frac{\alpha_i (x_j - x_i)\gamma_i}{(\gamma_i x_i + 1)^2} > 0 \qquad (8)$$

$$\frac{\partial^2 U_i}{\partial x_i^2} = -2\frac{\alpha_i \gamma_i}{(\gamma_i x_i + 1)^2} - \frac{2\alpha_i (x_j - x_i)\gamma_i^2}{(\gamma_i x_i + 1)^3} < 0 \qquad (9)$$

The marginal utility of x_i is positive and decreasing, while in FS it is positive and constant. Obviously, as in FS, the first derivative of i's utility function with respect to x_j is negative:

$$\frac{\partial U_i}{\partial x_j} = -\frac{\alpha_i}{\gamma_i x_i + 1} < 0 \tag{10}$$

Consider now the first derivative of equation (5) with respect to x_i and the first and second derivatives with respect to x_j:

$$\frac{\partial U_i}{\partial x_i} = 1 - \frac{\beta_i}{\sigma_i x_j + 1} > 0 \tag{11}$$

$$\frac{\partial U_i}{\partial x_j} = \frac{\beta_i}{\sigma_i x_j + 1} + \frac{\beta_i (x_i - x_j)\sigma_i}{(\sigma_i x_j + 1)^2} > 0 \tag{12}$$

$$\frac{\partial^2 U_i}{\partial x_j^2} = -2\frac{\beta_i \sigma_i}{(\sigma_i x_j + 1)^2} - \frac{2\beta_i (x_i - x_j)\sigma_i{}^2}{(\sigma_i x_j + 1)^3} < 0 \tag{13}$$

The implication is that both a higher value of x_i and a higher value of x_j lead to a higher value of i's utility function.

2. Application to Dictator and Ultimatum Games

Consider a ***Dictator Game*** where the total sum to be divided between the Dictator and the Receiver is equal to *k*. In this game where the only active player is the Dictator, the utility function to be considered is her utility function. Since we are not analysing a mini Dictator Game (see for example, Abbink, Sadrieh and Zamir, 2004) but a traditional Dictator Game, we consider only the part of the utility function where the Dictator has a payoff that is equal or higher than the payoff of the Receiver. Consequently, the sum that the Dictator transfers to the Receiver is equal to $s \in [0, \frac{k}{2}]$. We can rewrite equation (5) as follows:

$$U_i = k - s - \beta_i \left(\frac{k - 2s}{\sigma_i s + 1} \right) \tag{14}$$

In equilibrium:

$$s^* = \begin{cases} k/2 & \text{if } \beta_i \geq \frac{1}{2}\left(1 + \frac{1}{2}\sigma_i k\right) \\ \dfrac{-1 + \sqrt{\beta_i \sigma_i k + 2\beta_i}}{\sigma_i} & \text{if } \dfrac{1}{2 + \sigma_i k} < \beta_i < \frac{1}{2}\left(1 + \frac{1}{2}\sigma_i k\right) \\ 0 & \text{if } \beta_i \leq \dfrac{1}{2 + \sigma_i k} \end{cases}$$

In FS the value of β_i determines the optimal transfer to the Receiver. In this case, β_i, σ_i and *k* determine the optimal value of *s*. However, this modified version of their model allows unique optimal interior solutions, given the value of the parameters.

As an example, consider the case where $k = 10$, $\sigma_i = 0.1$:

$$s^* = \begin{cases} 5 & \text{if } \beta_i \geq 0.75 \\ \dfrac{-1+\sqrt{3\beta_i}}{0.1} & \text{if } 0.33 < \beta_i < 0.75 \\ 0 & \text{if } \beta_i \leq 0.33 \end{cases}$$

Let's now consider a ***Generalized Dictator Game*** where the Dictators decides to transfer a sum (s) to the Receiver who receives *ms,* with $m \geq 1$. Now $s \in [0, \frac{k}{m+1}]$ and:

$$U_i = k - s - \beta_i \left(\frac{k - s - ms}{\sigma_i ms + 1} \right) \tag{15}$$

In equilibrium:

$$s^* = \begin{cases} k/(m+1) & \text{if } \beta_i \geq l \\ \dfrac{-1+\sqrt{\beta_i m + \beta_i \sigma_i km + \beta_i}}{\sigma_i m} & \text{if } h < \beta_i < l \\ 0 & \text{if } \beta_i \leq h \end{cases}$$

where:

$$h = \frac{1}{m+1+\sigma_i km}$$

$$l = \frac{m+1+\sigma_i km}{(m+1)^2}$$

Finally, consider now an ***Ultimatum Game***, where:

- player i is the Proposer;
- player j is the Responder;
- s is the offer of the Proposer;
- k is the sum to be divided.

Since the Proposer will never offer to the Responder more than a half of the total sum *k*, the utility function of player *j* is:

$$U_j(s) = s - \alpha_j \left\{ \frac{k-2s}{\gamma_i s + 1} \right\}, \qquad i \neq j. \tag{16}$$

The Responder will accept any sum that provides a positive value of equation (16), since the utility of her rejection of the Proposer's offer provides a level of utility equal to 0. The Responder will reject any offer:

$$s < s'(\alpha_j, k) \equiv \frac{1}{2} \frac{-1 - 2\alpha_i + \sqrt{1 + 4\alpha_i + 4\alpha_i^2 + 4\alpha_i \gamma_i k}}{\gamma_i} \tag{17}$$

The value of *s*' depends positively both on the level of α_j and on the value of *k.*

The utility function of the Proposer is:

$$U_i = k - s - \beta_i \left(\frac{k-2s}{\sigma_i s + 1} \right) \tag{18}$$

In equilibrium, the Proposer who knows the type (α_j) of the Responder, will offer:

$$s^* = \begin{cases} k/2 & \text{if } \beta_i \geq \frac{1}{2}\left(1 + \frac{1}{2}\sigma_i k\right) \\ \dfrac{-1 + \sqrt{\beta_i \sigma_i k + 2\beta}}{\sigma_i} & \text{if } q < \beta_i < \frac{1}{2}\left(1 + \frac{1}{2}\sigma_i k\right) \\ s'(\alpha_j, k) & \text{if } \beta_i \leq q \end{cases}$$

where:

$$q = \frac{1}{2} \frac{\left(\gamma_j^2 + 4\alpha_j \gamma_j^2 - \gamma_j^2 r - 2\gamma_j \sigma_i + 4\alpha_j^2 \gamma_j^2\right)}{\sigma_i^2 (2 + \gamma_j k)} + $$
$$- \frac{1}{2} \frac{\left(2\alpha_j \gamma_j^2 r - 4\alpha_j \gamma_j \sigma_i + 2\alpha_j \gamma_j^2 \sigma_i k + 2\gamma_j \sigma_i r + 2\sigma_i^2\right)}{\sigma_i^2 (2 + \gamma_j k)}$$

with:

$$r = \sqrt{1 + 4\alpha_i + 4\alpha_i^2 + 4\alpha_i \gamma_i k}$$

For instance, if $k = 10$, $\alpha_j = 1$, $\gamma_j = \sigma_i = 0.1$:

$$s^* = \begin{cases} 5 & \text{if } \beta_i \geq 0.75 \\ \dfrac{-1 + \sqrt{3\beta_i}}{0.1} & \text{if } 0.565 < \beta_i < 0.75 \\ 3.02 & \text{if } \beta_i \leq 0.565 \end{cases}$$

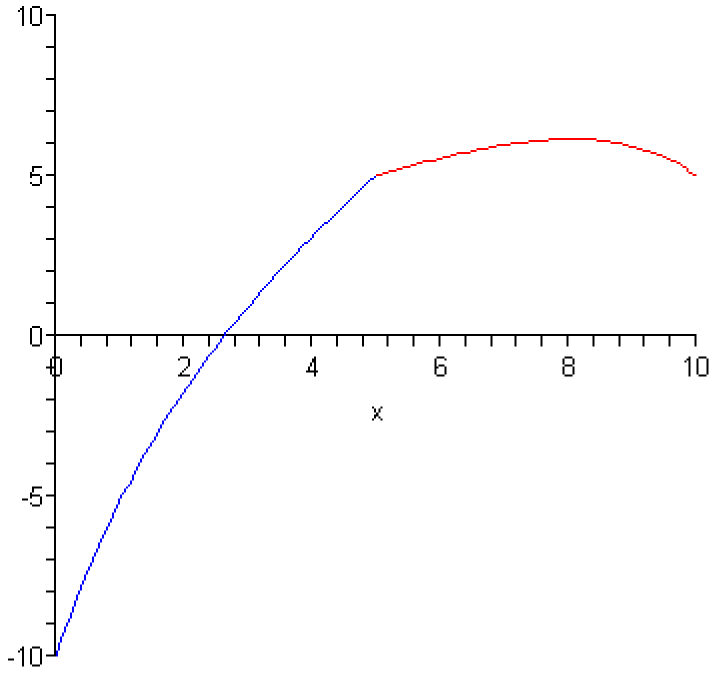
Our modified version of the model by FS is consistent with the empirical evidence. The results obtained by Slonim and Roth (1998) and by Munier and Zaharia (2002) show that the higher the sum to be divided (*k*), the lower in percentage the minimum accepted transfer and, at the same time, the optimal transfer (*s*) to the Responder. We provide an example to show that this is the case in this modified version of the model by FS, but not in their original model.

When $k = 10$, $\gamma = 0.1$ and $\sigma = 0.2$ **:**

$$U_i = \begin{cases} x_i - \alpha_i \left(\dfrac{10 - 2x_i}{0.1x_i + 1} \right) & x_i < 5 \\ x_i - \beta_i \left(\dfrac{2x_i - 10}{0.2(10 - x_i) + 1} \right) & x_i \geq 5 \end{cases}$$

For $\alpha_i = 1, \beta_i = 0.5$, the function is depicted in Figure 5.

Figure 5



where:

$x = 1 - s$

$y = U_i$

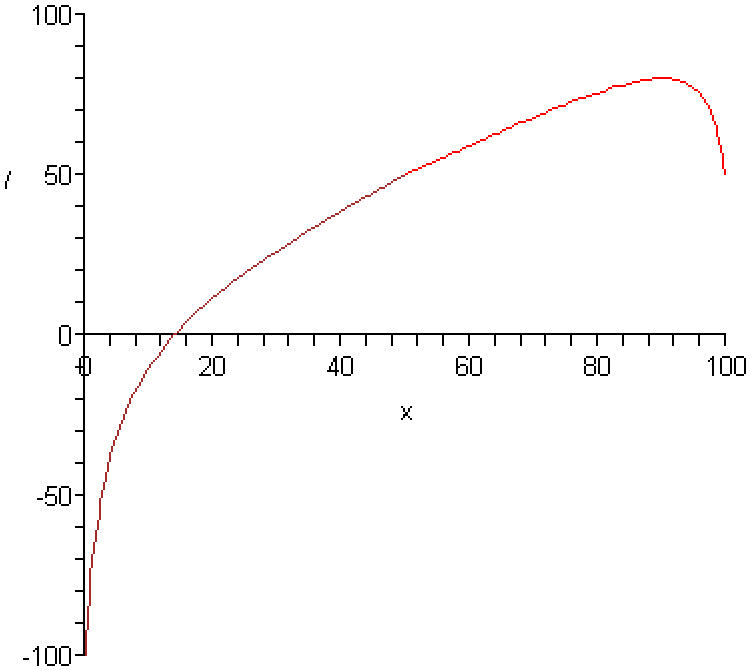
If player *i* is a Proposer, the value of (1 – *s*) that maximizes her utility function is 7.9. If she is a Responder, the minimum accepted transfer (*s*) is 3.03.

When $k = 100$, , $\gamma = 0.1$ and $\sigma = 0.2$ **:**

$$U_i = \begin{cases} x_i - \alpha_i\left(\dfrac{100 - 2x_i}{\gamma_i x_i + 1}\right) & x_i < 50 \\ x_i - \beta_i\left(\dfrac{2x_i - 100}{\sigma_i(100 - x_i) + 1}\right) & x_i \geq 50 \end{cases}$$

For $\alpha_i = 1$ and $\beta_i = 0.5$, the function is depicted in Figure 6.

Figure 6



where:

$x = 1 - s$

$y = U_i$

If player *i* is a Proposer, the value of (1 – *s*) that maximizes her utility function is 88.4. If she is a Responder, the minimum accepted transfer (*s*) is 20.

If we compare the results, we can notice that when *k* is higher, a Proposer maximizes her utility by keeping a higher percentage of the total sum (88.4% against 79%) and a Responder's utility goes to 0 with a lower percentage of the total sum (20% against 30%).[4]

[4] When $k = 10$, σ_i=0.2, $\alpha_i = 1$, $\beta_i = 0.9$, the value of 1-*s* that maximizes the Proposer's utility function is now about 5.5. When $k = 10$, σ_i=0.2, $\alpha_i = 1$, $\beta_i = 0.2$, the value of 1-*s* that maximizes the Proposer's utility function is 10. When $k = 10$, γ_i=0.1, $\alpha_i = 2$, $\beta_i = 0.5$, the value of *s* that makes the Responder's utility function equal to 0 is about 3.7. This suggests that a higher level of altruism implies a lower level of 1-*s* to maximize the Proposer's utility. On the other hand, a higher level of envy implies that a higher level of *s* is required for the Responder to have a positive utility.

3. Application to Public Good Games

In this section we analyse how fair-minded people behave in a Pubic Good Game. There are $n \geq 2$ players who have to decide how to invest their initial endowment y. They can either keep the whole sum or invest (g_i) part of it (or the total sum) in a public good whose return is $a \in \left(\frac{1}{n},1\right)$. Player i's monetary payoff is:

$$x_i\left(g_1,\ldots,g_n\right) = y - g_i + a\sum_{j=1}^{n} g_j \tag{19}$$

Since $a < 1$, any contribution implies a loss of (1-a). Consequently, the selfish *Homo Oeconomicus* always find it profitable not to contribute.

However, an interesting feature of our model is that fair-minded people may decide to cooperate. Without loss of generality we consider player 1 as the reference player, and we compare her utility when she decides not to contribute (g_1=0) to her utility when she contributes (g_1>0).

$$U\left(g_1 = 0\right) = y - a\sum_{j=2}^{n} g_j - \frac{\beta_1}{n-1}\sum_{j=2}^{n}\frac{g_j}{\sigma_1\left(y - g_j + a\sum_{j=2}^{n} g_j\right)+1} \tag{20}$$

$$U\left(g_1 > 0\right) = y - \left(1-a\right)g_1 + a\sum_{j=2}^{n} g_j - \frac{\beta_1}{n-1}\sum_{j=k+1}^{n}\frac{g_j - g_1}{\sigma_1\left(y - g_j + ag_1 + a\sum_{j=2}^{n} g_j\right)+1} +$$
$$- \frac{\alpha_1}{n-1}\sum_{j=2}^{k}\frac{g_1 - g_j}{\gamma_1\left(y - g_1 + ag_1 + a\sum_{j=2}^{n} g_j\right)+1} \tag{21}$$

If we compare (20) with (21), we obtain respectively the loss (22) and the gain (23) due to contribution:

$$\left(1-a\right)g_1 + \frac{\alpha_1}{n-1}\sum_{j=2}^{k}\frac{g_1 - g_j}{\gamma_1\left(y - g_1 + ag_1 + a\sum_{j=2}^{n} g_j\right)+1} \tag{22}$$

$$\frac{\beta_1}{n-1}\left(\sum_{j=2}^{n}\frac{g_j}{\sigma_1\left(y-g_j+a\sum_{j=2}^{n}g_j\right)+1}\right)-\frac{\beta_1}{n-1}\left(\sum_{j=k+1}^{n}\frac{g_j-g_1}{\sigma_1\left(y-g_j+ag_1+a\sum_{j=2}^{n}g_j\right)+1}\right) \tag{23}$$

When (23) is greater than (22), player 1 will contribute.

Consider now the effect of α_1, β_1 and *a* on the loss and the gain. When α_1 increases, the loss increases and it becomes less profitable for player 1 to cooperate. The opposite happens when β_1 increases: the gain increases and it becomes more profitable for player 1 to cooperate. The effect of an increase of *a* is less obvious. While the loss (equation 22) decreases, the gain (equation 23) increases. However, the final effect on contribution is positive. This means that the decrease of the loss is higher than the decrease of the gain.

Consider now a situation where the subjects can punish players who do not contribute to the provision of the public good. The payoff of player *i* is:

$$x_i = y - g_i + ag_i + a\sum_{j=2}^{n} g_j - z\sum_{j=1}^{n} p_{ji} - c\sum_{j=1}^{n} p_{ij} \tag{24}$$

where:
c = cost of each unit of punishment
z = yield of each unit of punishment
p_{ji} = units of punishment given by player j to player i
p_{ij} = units of punishment given by player i to player j

Without loss of generality, we analyse a 3-player game in order to avoid complex computations. Player 1 is a 'cooperative enforcer'[5] (she cooperates and she punishes a non-cooperative player), player 2 does not contribute to the provision of the public good and player 3 is a cooperative subject (she cooperates, but she does not punish non cooperators). We assume that $g_1 = g_3$ and $g_2 = 0$.

The payoff of player 1 when she decides to punish player 2 is:

[5] Fehr and Schmidt, 1999, p.19.

$$x_1 = y - g_1 + 2ag_1 - cp_{12} \tag{25}$$

and her utility is:

$$U_1 = x_1 - \frac{\alpha_1}{2}\left(\frac{g_1 + cp_{12} - zp_{12}}{\gamma_1 x_1 + 1} + \frac{cp_{12}}{\gamma_1 x_1 + 1}\right) \tag{26}$$

on the other hand, the payoff of player 1 when she decides not to punish player 2 is:

$$x_1 = y - g_1 + 2ag_1 \tag{27}$$

and her utility is:

$$U_1 = x_1 - \frac{\alpha_1}{2}\left(\frac{g_1}{\gamma_1 x_1 + 1}\right) \tag{28}$$

it is profitable for player 1 to punish player 2 when (26) is higher than (28). This means that:

$$cp_{12} < -\frac{\alpha_1}{2}\left(\frac{2cp_{12} - zp_{12}}{\gamma_1(y - g_1 + 2ag_1 - cp_{12})}\right) \tag{29}$$

and:

$$0 < p_{12} < \frac{1}{2}\frac{2c\gamma_1(y - g_1 + 2ag_1) + \alpha_1(2c - z)}{c^2\gamma_1} \tag{30}$$

under the following constraints:

$$p_{12} \leq \frac{y - g_1(2a - 1)}{c} \tag{31}$$

$$p_{12} \leq \frac{y + 2ag_1}{z} \tag{32}$$

$$p_{12} \le \frac{g_1}{z-c} \tag{33}$$

The first two constraints mean that the player 1 cannot pay for punishment more than her income and punishment cannot provide a negative income to player 2. The third constraint means that player 1's payoff after punishment cannot be higher than player's 2 payoff.

Consider equation (30) again. The value of punishment assigned by player 1 to player 2 (p_{12}) increases as the cost of punishment (c) decreases, the rent of punishment (z) decreases and envy of player 1 (α_1) increases.

4. Application to Gift Exchange Games

This game describes the principal-agent relation in an incomplete contracts contest. In the first stage, the Employer (the principal) offers a wage $w \in [\underline{w}, \overline{w}]$ (where $\underline{w} \geq 0$) to the Worker (the agent). The Worker decides whether to accept or not. If she rejects, both players receive nothing. In the second stage, the Worker decides her effort $e \in [\underline{e}, \overline{e}]$ (where $\underline{e} > 0$). The payoff of the Employer and of the Worker are respectively x_E and x_W:

$$x_E = ve - w \tag{34}$$

$$x_W = w - c(e) \tag{35}$$

where:

w = wage proposed by the Employer

v = marginal value of effort for the Employer

e = effort chosen by the Worker

$c(e)$ = effort cost for the Worker

We assume a positive income for both players when $e = \underline{e}$. This means that:

$$w \leq v\underline{e} \qquad \text{or} \qquad v \geq \frac{w}{\underline{e}}$$

and

$$\underline{w} \geq c(e)$$

Consider now a situation where $x_E > x_W$ at any feasible effort level. A fair-minded worker will always choose $e = \underline{e}$ when this effort level provides a positive utility. When $U_W \leq 0$ at any level of e, the Worker will never accept to work for the Employer.

When $x_W > x_E$ at $\underline{e}$, the Worker may decide to increase her effort to reduce the difference between her payoff and the payoff of the Employer. In this case, the utility of the Worker is:

$$U_w = w - c(e) - \beta_i \frac{(2w - c(e) - ve)}{\sigma_i (ve - w) + 1} \tag{36}$$

If we assume $c(e) = e$:

$$U_w = w - e - \beta_i \frac{(2w - e - ve)}{\sigma_i (ve - w) + 1} \tag{37}$$

and the optimal level of effort for the Worker is:

$$e^* = \frac{\sigma_i w - 1 + \sqrt{\beta_i (1 + \sigma_i vw + v - \sigma_i w)}}{\sigma_i v} \tag{38}$$

When β_i and w increase e^* increases as well. This result is consistent with the experimental evidence. In Fehr et al.(1993), a higher wage (w) corresponds to a higher effort (e^*).

5. Application to Trust Games

It is a two-stage game with two participants (the Investor and the Trustee). In the first stage the Investor can invest the whole initial endowment *S* or a part of it by sending any amount *y* (between 0 and *S*) to the Trustee. The experimenter triples the amount sent to the Trustee such that she receives 3*y*. In the second stage the Trustee can send part of the investment (any amount between 0 and 3*y*) back to the Investor.

The Utility of the Trustee is:

$$U_T = s + 3y - z - \beta_T \left(\frac{s + 3y - z - s + y - z}{\sigma_T (s - y + z) + 1} \right)$$

or:

$$U_T = s + 3y - z - \beta_T \left(\frac{4y - 2z}{\sigma_T (s - y + z) + 1} \right) \tag{39}$$

where:

y = sum sent to the Trustee by the Investor

z = sum sent back to the Investor by the Trustee

s = each player's initial endowment

Starting from (39), the optimal value of z is:

$$z^* = \frac{\sigma_T (y - s) - 1 + \sqrt{2\beta_T (\sigma_T s + \sigma_T y + 1)}}{\sigma_T} \tag{40}$$

This result means that the sum the Trustee sends back to the Investor depends positively on the sum received by the Investor and on the Trustee's degree of altruism. It can explain the results obtained by Berg et al. (1995): when y is equal to 0.5*S*, z is a bit less than y.[6] Moreover, z increases as y increases, as a sort of positive reciprocity between the Investor and the Trustee.

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[6] When $s = 10$, $y = 5$, $\sigma_T = 0.1$, $\beta_T = 0.8$, $z^* = 5$.

Summary and conclusions

Fehr and Schmidt assume that fair-minded people exist and they provide a model that explains extreme behavior. In this paper we provide a non-linear utility function of fair-minded people to explain interior solutions. In particular, we assume that the disutility due to unfair distribution of outcomes is influenced not only by the difference between the payoffs but also by the absolute value of the payoff of each player. This hypothesis looks plausible and allows to explain the behavior of players involved in Ultimatum Games. In addition, the model is consistent with the empirical evidence also in other games (i.e. Public Good Game, Dictator Game and Gift Exchange Game, Trust Game).

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