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Abstract

The tremendous growth of markets for credit derivatives since the mid 1990's has raised questions regarding the role of these instruments in the banking industry which is heavily exposed to credit risk. However, while recent literature mainly focused on pricing and optimal decisions regarding volumes of credit derivatives the present paper centers the strategic role of these instruments in the competition between banking firms. We use a duopolistic version of the industrial organization approach to banking to find out that credit derivatives may influence banking competition. For this result to hold observability of the volume of credit derivatives held by banks is not necessary.

Keywords: bank, risk, duopoly, hedging.

JEL classification: D21, D40, D43, G21, L13

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The tremendous growth of markets for credit derivatives since the mid 1990's has raised questions regarding the role of these instruments in the banking industry which is heavily exposed to credit risk. However, while recent literature mainly focused on pricing and optimal decisions regarding volumes of credit derivatives the present paper centers the strategic role of these instruments in the competition between banking firms. We use a duopolistic version of the industrial organization approach to banking to find out that credit derivatives may influence banking competition. For this result to hold observability of the volume of credit derivatives held by banks is not necessary.

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1 Introduction

The increasing amount of credit default events together with the recent economic development as well as the Basel II project have shown that credit risk is not only the oldest but also the most significant kind of risk in banking business. Prior to the introduction of credit derivatives banks had mainly to rely on a buy–and–hold strategy and were only able to influence the structure of their portfolio by selecting and monitoring borrowers. Further, the selection of borrowers and thus diversification was more or less restricted due to a restricted scope of accessible borrowers and industry branches. This situation changed considerably with the introduction of credit securitization and credit derivatives. Both instruments put banks and corporations in a position to move from a mere buy–and–hold strategy to the active management of credit risk. The importance and impact of this recent development is emphasized by the exponential growth of trading volume of credit derivatives and the increasing use of securitization. Among credit derivatives credit default swaps, by far, possess the highest market share and liquidity. According to the Credit Derivatives Report 2002 of the British Bankers Association (British Bankers' Association (2002)), credit default swaps made up almost half of the market, while securitization carried a market share of 22%, leaving each of the remaining instruments with a portion of no more than 8%. Partly, this can be attributed to the increasing use of synthetic securitization in which a credit default swap is used to transfer credit risk to a special purpose vehicle instead of the underlying pool of debt itself (cf. Watzinger (2000)). A credit default swap is a bilateral financial contract usually based on a ISDA (International Swaps and Derivatives Association) master agreement that standardizes contractual details. The protection seller grants an insurance payment in case of default and the protection buyer pays a periodic fee in return. Under certain conditions (cf. O'Kane and McAdie (May 2001)), this fee equals the spread between the yield of the underlying and the risk free interest rate. It is distinguished between the typical physical settlement which is the delivery of the defaulted debt to the protection seller for the face value and cash settlement where the protection seller compensates the loss given default. As the difference between face value and recoverable value of a physical delivery coincides with the loss given default, we are able to model both types of settlement in the same way.

The ongoing globalization process and the tremendous growth of markets for credit derivatives since the mid 1990's have substantially increased the interdependencies of banking business. Due to this development banks should not only try to just optimize their credit portfolio with respect to measures like credit–value–at–risk (CVaR) and risk adjusted performance measures (RAPM) but also take into account a possible strategic impact of their business decisions on interest rates and hedging on the behavior of competitors. As regards market risk, the literature already suggests a motive to use derivative instruments not only for hedging but also for strategic reasons – see for example Allaz and Vila (1993) and Hughes and Kao (1997).[1] Hence, it is reasonable expecting credit derivatives to be used as strategic devices in the competition of banks too. In this context a business decision of bank A is said to have a strategic effect or impact, respectively, when it can be considered as a "strategic move" in the sense of Schelling (1960) who defined a strategic move as one that influences the choice of the competitor by affecting his expectations about how A will behave in future in a way that is beneficial for A. Schelling outlines four features that characterize a strategic move. Firstly, such a move is *sequential*, that is to say, A is able to move before the competitor makes his or her final move. Secondly, the move has to be *known* to the competitor before he moves. Thirdly, A's move must *affect incentives* of A so that it changes the future optimal behavior of A. Moreover, the move must affect what the competitor considers as the optimal behavior of A and it must influence the competitors behavior accordingly. This fourth feature is termed *rational expectations*. The strategic move is successful if it increases the profit of bank A.

Our paper will figure out situations in which these requirements are met and in this way presents a formal argument of when credit derivatives may be consid-

[1] In an oligopolistic setting of producing firms of the Cournot type Allaz and Vila (1993) show that even without a hedging motive firms may use forward markets to threaten harder competition with competitors. However, their key assumption is observability of the futures positions. Hughes and Kao (1997) prove that risk averse duopolists use forwards even without such an observability.

ered to be strategic devices in the competition of banks. That is, in view of the impressing evolution in the use of credit derivatives as a means to separate credit risk form market risk and to enable its active trading, our analysis provides first insights into the strategic role of credit derivatives in the context of an oligopolistic banking industry where banks face credit risk, and a market for credit derivatives, more precisely credit default swaps, exists. Furthermore we analyze how the timing of the decision process influences the strategic impact of an active portfolio management with credit derivatives. Banking practice suggests three kinds of decision processes. Assume any arbitrary fixed volume H to be tradable at the market for all available underlying, and notice, a bank's risk aversion is reflected in its (unhedged) net position in loans. Then the first possibility is production to influence hedging, because a bank is very likely to lower interest rates in order to obtain a substantial and lucrative credit volume that fits into its portfolio, given the bank is able to hedge the new credit at the market. As this will substantially increase the unhedged loan volume the bank needs to increase hedging activities in order to keep the net position in loans constant that reflects its risk aversion. The second possible order of the decision process is when hedging influences production – i.e. the managements decision on interest rates for loans and deposits. Under beneficial market conditions a bank may be inclined to higher its hedging volume or even to speculate increasingly via going long the credit derivatives market, that is to say, buying protection. In order to keep the net position constant, the loan volume will be increased via lower loan interest rates. These two possible scenarios in banking practice made us not only to investigate a possible strategic effect of hedging on production but also the other way round. Additionally, we take into account a third scenario in which the decision on hedging and the level of interest rates for loans and deposits is made simultaneously – even though it does obviously not meet the qualitative characteristics set out by Schelling (1960) and is less relevant in practice.

The plan of the paper is as follows: section 2 establishes the model. Section 3 successively investigates the strategic effects of hedging. Firstly within simultaneous decisions (section 3.1), then within an decision on interest rates that precedes the hedging decision(section 3.2), and finally in section 3.3 the other way round. For the first two cases we will not find any strategic effects, neither of hedging nor of production. In contrast, when the hedging decision precedes the decision on interest rates for loans and deposits we find an increasing hedge volume to higher the demanded interest rate for loans. Section 4 concludes and the appendix covers the formal proofs of the stated propositions.

2 A Model of Competing Banks

The model applied for the following analysis is based on suggestions by (Yanelle, 1985, p. 5) and (Yanelle, 1988, p. 4ff.) who argues that in the banking industry competition via prices – that is interest rates for deposits and loans – is much more likely than competition using volumes of deposits and loans. This is obvious since overall deposits and loans may be considered homogeneous and hence lenders care for a high commission for making funds available for financing investment projects and borrowers are interested in loans at low cost. Thus, competition in the banking industry takes place by simultaneously deciding on interest rates in the deposit and loan market.

In particular consider a one period setting with two identical banking firms (say A and B) which simultaneously decide on deposit and loan rates. We assume that banks behave non-cooperatively in the decision making process. Since both banks may have market power in the deposit market as well as in the loan market deposit supply and loan demand depend on the level of deposit and loan rates, respectively. Furthermore, the volume of deposits taken and the volume of loans issued by a particular bank is influenced by the competitor's behavior. That is, the effective deposit supply ($\hat{D}_i$) and the effective loan demand ($\hat{L}_i$) function for any bank $i = A, B$ may be written as follows:

$$\hat{D}_i(r_{D_i}, r_{D_j}) = \begin{cases} 0, & r_{D_i} < r_{D_j}; \\ \frac{1}{2}D(r_{D_i}), & r_{D_i} = r_{D_j}; \quad i,j = A,B; i \neq j \\ D(r_{D_i}), & r_{D_i} > r_{D_j}; \end{cases} \tag{1}$$

and

$$\hat{L}_i(r_{L_i}, r_{L_j}) = \begin{cases} 0, & r_{L_i} > r_{L_j}; \\ \frac{1}{2}L(r_{L_i}), & r_{L_i} = r_{L_j}; \quad i,j = A,B; i \neq j. \\ L(r_{L_i}), & r_{L_i} < r_{L_j} \end{cases} \tag{2}$$

The interpretation of these functions is obvious. If one considers the effective deposit supply function at first it should be noted that the effective deposit supply depends on both his own and the deposit rate of the competitor, i.e. $\hat{D}_i = \hat{D}_i(r_{D_i}, r_{D_j})$; $i, j = A, B$; $i \neq j$. The distinction of the cases regarding the relations between deposit rates states in which way actual volumes of deposits are determined contingent on the actual interest rate in the deposit market. That is, we assume that there is a unique deposit supply function in the market denoted $D(r_D)$ which is allocated among both banks depending on their deposit rate. In case of different deposit rates among banks the one with the higher interest rate serves the entire market while the one with the lower rate is not getting offered any deposits.

However, if deposit rates are the same, both banks will share the market equally. In this context the deposit supply function is assumed to to be twice continuously differentiable, strictly increasing and concave, i.e.

$$\frac{dD(r_D)}{dr_D} > 0 \text{ and } \frac{d^2D(r_D)}{(dr_D)^2} < 0.$$

Furthermore let $D(0) > 0$, because even with a zero deposit rate customers are inclined to hold a certain level of deposits for transactional reasons.[2] As a result the effective deposit supply is differentiable in sections, increasing and concave. Moreover, the deposit supply of one bank increases with its own and decreases with the competitor's deposit rate:

$$\frac{\partial \hat{D}_i(r_{D_i}, r_{D_j})}{\partial r_{D_i}} \geq 0 \text{ and } \frac{\partial \hat{D}_i(r_{D_i}, r_{D_j})}{(\partial r_{D_j})} \leq 0.$$

The interpretation of the effective loan demand function follows analogously if one considers $L(r_L)$ as the unique loan demand function in the market. Hence, the effective loan demand states how to allocate loan demand at the current loan rates among both banks. In this context loan demand $L(r_L)$ is assumed to be twice continuously differentiable, strictly decreasing and concave:

$$\frac{dL(r_{L_i})}{dr_{L_i}} < 0 \text{ and } \frac{d^2L(r_{L_i})}{(dr_{L_i})^2} < 0.$$

Moreover, since worldwide available funds are bounded, it is reasonable to assume that $L(0) < \infty$. Thus the effective loan demand function is differentiable in sections, decreasing and concave. Further a bank's loan demand decreases with its own loan rate and increases with the competitor's loan rate:

$$\frac{\partial \hat{L}_i(r_{L_i}, r_{L_j})}{\partial r_{L_i}} \leq 0 \text{ and } \frac{\partial \hat{L}_i(r_{L_i}, r_{L_j})}{\partial r_{L_j}} \geq 0.$$

Further assume taking deposits and issuing loans to cause no operating costs. Of course, this assumption seems not very reasonable since in reality operating costs play some role. One might also argue that these costs arise to an extend that is relevant for banks' decision making. However, abstracting from operating costs is much less restrictive than it might appear at a first glance. While an introduction of constant operating costs would sophisticate notation there were no additional or different insight from such a model. In addition we neglect any reserve requirements to be met by the banks. This is feasible since consideration of any certain share α of deposits to be held with the central bank at zero interest just complicates notation but again does not affect our results.

[2]In Germany so called Giro accounts can serve as an example.

An important assumption which distinguishes our model from other settings of the literature is the existence of an interbank market where both banks are able to lend or borrow any desired amount of funds at an exogenously given and riskless interest rate r.[3] This assumption is important since the interbank market separates the banks' decisions on deposit and loan rates.[4] In this regard let M_i ; $i = A, B$ denote bank i's difference between loan and deposit volume that is borrowed from or lent to the interbank market at an interest rate of r. Thus bank i's balance sheet constraint may be written as

$$M_i + \hat{L}_i(r_{L_i}, r_{L_j}) = \hat{D}_i(r_{D_i}, r_{D_j}) + K \; ; \; i, j = A, B \; ; \; i \neq j \tag{3}$$

where K denotes an exogenously given certain amount of equity capital which is the same for both banks.

Both banks are assumed to face the same credit risk, modelled by the random variable $\tilde{\theta} = \tilde{\delta} \cdot \tilde{\lambda} \in [0, 1]$.[5] Thus credit risk can be split up into the default event $\tilde{\delta} \in \{0, 1\}$ and loss given default (LGD) $\tilde{\lambda} = (\tilde{\theta} \mid \tilde{\delta} = 1) \in (0, 1]$. Hence, if $\tilde{\delta} = 1$ a bank i looses the share $\tilde{\lambda}$ of the end–of–period payment $(1 + r_{L_i})L_i$ from its debtors. The symmetry of credit risk in our framework appears due to the fact that $\tilde{\theta}$ and the respective distribution function $F(\tilde{\theta})$ are the same for both banks. This latter assumption seems reasonable since one might argue that in reality competing banks often focus major parts of their loan business on the same geographical regions – no matter whether we deal with locally or globally playing banks. And in this way, generally speaking, they lend to the same kind of borrowers and are thus exposed to the same kind of credit risk.

However, both banks can hedge credit risk by entering a market for credit default swaps (CDS).[6] These derivative financial instruments pay in the case of borrower default (i.e. $\tilde{\delta} = 1$) at the end of the period a random amount of $\tilde{\lambda} \cdot H$ to the seller of protection, where H is the face value. Due to the definition of the credit risk $\tilde{\theta}$ this is equivalent to CDS paying the stochastic amount $\tilde{\theta} \cdot H$. In return the seller of credit risk pays a deterministic premium $\bar{\theta} \cdot H$ to the buyer of credit risk. Additionally, we assume an unbiased market in which $E(\tilde{\theta}) = \bar{\theta}$ holds. The market for CDS is also

[3] To motivate the assumptions with respect to r consider this interest rate to be determined by the central bank.

[4] Yanelle (1985) and Yanelle (1988) do not consider an interbank market. Accordingly, the volumes of loans lent and deposits taken must be equivalent. At the end, this restriction is crucial for the main result of these papers by which there does not appear the Bertrand Paradox in the price competition of symmetric banks. However, we will show in the following that this assumption is not necessary to derive similar results.

[5] During the paper all random variables will be denoted by a " ˜ ".

[6] Credit default swaps are the most widely used kind of credit derivative (see e.g. Broll et al., 2003, p. 5) which is the reason why we focus the analysis on these instruments.

assumed to provide a perfect hedge, i.e. there is no basis risk left.[7]

If one denotes by H_i the volumes of CDS sold by bank $i = A, B$ it is obvious that buying proctection with a CDS grants the bank a stochastic claim $\tilde{\theta} H_i$ in exchange for the deterministic insurance premium $\bar{\theta} H_i$. The stochastic claim is paid in case of default, that is at the end of our 1-period setting. On the other hand the premium is paid at the settlement date in practice. To circumvent the need to discount the premium we decided to let the premium paid at the end of the period too. With the further assumption that H_i is not restricted in any way it is obvious that hedging contributes $(\tilde{\theta} - \bar{\theta})H_i$ to bank i's profit. As a result bank i's profit may be written as

$$\begin{aligned} \tilde{\Pi}_i &= \Big((1-\tilde{\theta})r_{L_i} - r\Big)\hat{L}_i(r_{L_i}, r_{L_j}) - \tilde{\theta}\hat{L}_i(r_{L_i}, r_{L_j}) + (\tilde{\theta} - \bar{\theta})H_i + \\ &\quad + (r - r_{D_i})\,\hat{D}_i(r_{D_i}, r_{D_j}) + rK. \end{aligned} \tag{4}$$

In (4) the balance sheet constraint (3) has been already considered.

Both banks are considered to behave risk averse with identical von Neumann-Morgenstern utility function $U(\cdot)$ having $U'(\cdot) > 0$ and $U''(\cdot) < 0$.[8] Thus, bank i maximizes its expected utility of profit with respect to deposit rates, loan rates and the hedging volume. In this regard it is assumed that decisions on deposit and loan rates are made simultaneously at the beginning of the period. That is, when making decisions a bank takes optimal deposit and loan rates of the competitor as given and maximizes the own expected utility in order to achieve a best response on this behavior. When doing so the respective bank conjectures that there is no impact of its own decisions on the competitor's behavior.

While the assumption regarding simultaneous decisions on deposit and loan rates seem quite reasonable, the timing of the hedging decision is not so obvious. In fact, banking practice shows that depending on the current market situation hedging decisions can be made before, at the same time of, or after decisions on interest rates, but every time before uncertainty regarding non-performing loans is resolved. That is, depending on the timing of the hedging decision there are three opportunities for the final structure of the game: at first, when decisions on hedging and interest rates are made simultaneously we are still in a one period setting where at the beginning of the period all relevant decisions are made and at the end of the period uncertainty is resolved and payouts are realized. However, while a game of this structure provides a good benchmark for comparisons with other settings, in reality it seems to be of minor interest.

[7] Of course, these assumptions are quite restrictive but provide a good starting point for a detailed formal analysis of the subject. Dropping these assumptions is thus left for further research.

[8] See for example Froot et al. (1993) for a detailed analysis of the explanation for risk averse behavior of firms in general and Froot and Stein (1998) and Pausch and Welzel (2002) for the application of similar arguments in the context of the banking sector in particular.

The second opportunity, where decisions on the optimal hedging volumes are made after decisions on interest rates but before uncertainty is resolved, is a preferred proceeding with large scale loans. There are situations in which a bank would like to issue additional loans which substantially increase the banks' overall loan portfolio. However, with risk aversion the additional loan would alter the size of the loan portfolio beyond the optimal level which forces the bank to adjust loan rates. In order to avoid these adjustments, the bank might issue the loan at the present loan rate and afterwards use hedging techniques to ensure the new situation being still optimal. Note, for this reasoning to hold the additional loan has to be sufficiently large. Small scale loans do influence the size of the overall loan portfolio only marginally. However, when hedging decisions are made at an intermediate date of the analyzed period of time, the final game is de facto a two stage one: at the first stage (beginning of the period) decisions on interest rates are made, at the second stage (intermediate date during the period) decisions on hedging are made. Again, at the end of the period uncertainty is resolved and payouts are realized.

As already outlined by the introduction, in the third possible setting, the banks firstly decide on optimal hedging levels and choose optimal deposit and loan rates thereafter. Hence, the final game here is again a two stage one. The structure is the same as in the previous case except that now decisions on interest rates are made at an intermediate date.

Finally assume that in any of the three mentioned cases above a bank takes the hedging volume of the respective competitor as given when deciding on its own optimal hedging level.

3 The Role of Hedging in a Duopolistic Banking Industry

The arguments of bankers which have been presented in the previous section suggest very interesting interpretations of the potential role of hedging for banks. That is to say, deciding on interest rates first – setting number two above – might force the bank to adjust hedging levels in a further step in order to guarantee optimality of the situation. Hence, in this case one might expect hedging to work as a opportunity for adjusting the bank's risk exposure in an optimal way. In contrast, when hedging takes place before decisions on deposit and loan rates – setting number three above – the bank seems to prepare for potential changes regarding the decisions on deposits an loan rates. That is, from the arguments of bank practitioners one might expect an influence of hedging on future interest rates in the latter case. Hence hedging may serve as a strategic device in the competition of banks in this situation. In the following we will analyze these conjectures in formal detail.

3.1 Simultaneous Decisions on Hedging and Interest Rates

As explained earlier, the setting where decisions on hedging and interest rates are made simultaneously is less relevant from a practical point of view than for a formal analysis. That is we will use this case as a benchmark for the arguments to follow in later sections and to become familiar with the basics of our model.

The optimization problems of the respective banks in the current situation are given as follows:

$$\max_{r_{D_i}, r_{L_i}, H_i} \mathrm{E}[U(\tilde{\Pi}_i)] \;;\; i,j = A, B \tag{5}$$

where $\tilde{\Pi}_i$ is given by equation (4) above.

The corresponding first order necessary conditions may then be written as:

$$-\hat{D}_i(r_{D_i}, r_{D_j}) + (r - r_{D_i})\frac{\partial \hat{D}_i(r_{D_i}, r_{D_j})}{\partial r_{D_i}} = 0 \tag{6}$$

$$\begin{aligned} &\mathrm{E}\left[U'(\tilde{\Pi}_i)\left(\hat{L}_i(r_{L_i}, r_{L_j}) + (r_{L_i} - r)\frac{\partial \hat{L}_i(r_{L_i}, r_{L_j})}{\partial r_{L_i}}\right)\right] - \\ &-\mathrm{E}\left[U'(\tilde{\Pi}_i)\tilde{\theta}\left(\hat{L}_i(r_{L_i}, r_{L_j}) + r_{L_i} + 1\right)\frac{\partial \hat{L}_i(r_{L_i}, r_{L_j})}{\partial r_{L_i}}\right] = 0 \end{aligned} \tag{7}$$

$$\mathrm{E}\left[U'(\tilde{\Pi}_i)\left(\tilde{\theta} - \bar{\theta}\right)\right] = 0 \tag{8}$$

$$i,j = A, B \;;\; i \neq j.$$

Note, from the first order condition for the optimal deposit rate (6) one can observe that credit risk does not matter in this regard. This, however, is obvious since on the one hand the deposit business is not exposed to any source of risk directly. On the other hand, from the assumptions regarding operating costs, there appears no effect which influences decisions on optimal deposit rates caused by possible changes of the optimal loan rates. Hence, one can restrict attention to decisions on loan rates and hedging.

It is easy to see that with simultaneous decisions there appears no strategic impact of hedging. That is, by definition a strategic move requires a sequential proceeding of the considered game which is not the case in the present situation. (cf. Schelling, 1960, p. 119ff.) Hence one can immediately start analyzing optimal decisions of the banks.

For this purpose rewrite equation (8) to arrive at

$$\mathrm{E}\left[U'(\tilde{\Pi}_i)\tilde{\theta}\right] = \mathrm{E}\left[U'(\tilde{\Pi}_i)\right]\bar{\theta} \;;\; i = A, B.$$

Using this expression to transform the first order necessary condition (7) yields

$$\begin{aligned} \mathrm{E}\left[U'(\tilde{\Pi}_i)\right]\left(\hat{L}_i(r_{L_i},r_{L_j})+(r_{L_i}-r)\frac{\partial \hat{L}_i(r_{L_i},r_{L_j})}{\partial r_{L_i}}\right)&+\\ +\mathrm{E}\left[U'(\tilde{\Pi}_i)\right]\bar{\theta}\left(\hat{L}_i(r_{L_i},r_{L_j})+r_{L_i}+1\right)\frac{\partial \hat{L}_i(r_{L_i},r_{L_j})}{\partial r_{L_i}} &= 0\\ i,j=A,B\ ;\ i\neq j& \end{aligned}$$

which can be rewritten to arrive at

$$(1-\bar{\theta})\hat{L}_i(r_{L_i},r_{L_j})+\left((1-\bar{\theta})r_{L_i}-r-\bar{\theta}\right)\frac{\partial \hat{L}_i(r_{L_i},r_{L_j})}{\partial r_{L_i}}=0. \tag{9}$$

The interesting fact regarding equation (9) is that it is equivalent to the first order necessary condition which would be derived in a situation without credit risk. That is, if there were a certain share $\bar{\theta}$ instead of a random share $\tilde{\theta}$ of non-performing loans at the end of the period it is easily verified that the resulting first order necessary condition for the optimal loan rate would exactly be equal to equation (9). Therefore it is immediately clear that optimal deposit and loan rates are exactly the same as in the case without credit risk. Remember, the first order necessary condition for the optimal deposit rate is not at all affected by credit risk.

Therefore we can go on deriving the optimal hedging decisions: from $\mathrm{E}\left[U'(\tilde{\Pi}_i)\tilde{\theta}\right]=\mathrm{E}\left[U'(\tilde{\Pi}_i)\right]\bar{\theta}$ it is easy to see that $\mathrm{Cov}(U'(\tilde{\Pi}_i),\tilde{\theta})=0$ must hold in equilibrium.[9] Therefore, from the profit function (4) it is obvious that this is true if and only if $H_i=(1+r_{L_i})\hat{L}_i(\cdot)$. That is, in the optimum both banks fully hedge their exposure to risk.

As a result, in case of simultaneous decisions on hedging and interest rates we find both banks to fully hedge their exposure to risk and to choose deposit and loan rates that are the same as in the case without credit risk. Hence, one can observe a separation of the decisions regarding risk management and production (i.e. interest rates). However, these results are well known from the literature having perfect competition or monopoly.[10] The reason for this result to appear is that simultaneous decision making prevents strategic affects.

Moreover, one can show that in the equilibrium both banks set identical deposit and loan rates, i.e. $r^*_{D_A}=r^*_{D_B}=r_D$ and $r^*_{L_A}=r^*_{L_B}=r_L$ where a "*" denotes optimal values.[11] As a result it follows that in the equilibrium it must hold true that

[9] This follows immediately from the well known covariance formula $\mathrm{E}[U'(\tilde{\Pi}_i)\tilde{\theta}]=\mathrm{E}[U'(\tilde{\Pi}_i)]\mathrm{E}[\tilde{\theta}]+\mathrm{Cov}[U'(\tilde{\Pi}_i),\tilde{\theta}]$ where $\mathrm{E}[\tilde{\theta}]=\bar{\theta}$.

[10] See for example Wahl and Broll (2000) for a presentation of a monopolistic situation.

[11] For a proof of this equilibrium strategy see the appendix.

$\hat{D}_A^* = \hat{D}_B^* = \frac{1}{2}D(r_D)$ and $\hat{L}_A^* = \hat{L}_B^* = \frac{1}{2}L(r_L)$ and hence $H_A^* = H_B^* = (r_L + 1)\frac{1}{2}L(r_L)$ which is a further concretization of the full hedging result derived above.

To summarize results, we can now state our

Proposition 1 *In a duopolistic banking industry with symmetric banks that simultaneously decide on hedging as well as deposit and loan rates both banks fully hedge exposures to risk and choose the same deposit and loan rates as in a situation without credit risk. Moreover, the equilibrium deposit and loan rates as well as the optimal hedging levels are the same for both banks.*

However, there are two further aspects regarding the symmetric equilibrium stated in Proposition 1 which have to be mentioned. For this purpose consider Bank A c.p. reducing deposit rate compared to equilibrium strategy. As a consequence deposit supply of A drops down to zero and deposit supply of B increases by $\frac{1}{2}D(r_D)$. As a result B's first order necessary condition for an optimal deposit rate (6) is no longer met and, from the reasoning in the proof of r_D being an equilibrium strategy, it is obvious that B has to cut its deposit rate too. Therefore one can observe deposit rates to be strategic complements in the sense of (Bulow et al., 1985, p. 501) – formally (cf. Bulow et al., 1985, p. 494):

$$\frac{\partial^2 \mathrm{E}\left[U(\tilde{\Pi}_i)\right]}{\partial r_{D_i} \partial r_{D_j}} > 0 \; ; \; i, j = A, B \; ; \; i \neq j. \tag{10}$$

An analogous argument may be presented for the loan business: consider c.p. Bank A increases the loan rate compared to the equilibrium strategy r_L. As a result, bank B's demand for loans increases by $\frac{1}{2}L(r_L)$ which forces bank B to increase its loan rate too. This is a direct result of the reasoning in the respective proof of r_L to be an equilibrium strategy. Hence loan rates are also strategic complements in the sense of (Bulow et al., 1985, p. 501), formally (cf. Bulow et al., 1985, p. 494)

$$\frac{\partial^2 \mathrm{E}\left[U(\tilde{\Pi}_i)\right]}{\partial r_{L_i} \partial r_{L_j}} > 0 \; ; \; i, j = A, B \; ; \; i \neq j. \tag{11}$$

With the observation that deposit and loan rates are strategic complements one has to care about stability of the equilibrium. Note, when in the above example B raises its deposit rate as a reaction to A's increase of the deposit rate stability of the equilibrium requires that B's reduction of the deposit rate (as a reaction on A's reduction) has not to be stronger than A's decrease of its deposit rate. Hence the stability condition can be written formally (cf. Vives, 1999, p. 51):

$$det \begin{pmatrix} \frac{\partial^2 \mathrm{E}\left[U(\tilde{\Pi}_i)\right]}{(\partial r_{D_i})^2} & \frac{\partial^2 \mathrm{E}\left[U(\tilde{\Pi}_i)\right]}{\partial r_{D_i} \partial r_{D_j}} \\ \frac{\partial^2 \mathrm{E}\left[U(\tilde{\Pi}_i)\right]}{\partial r_{D_j} \partial r_{D_i}} & \frac{\partial^2 \mathrm{E}\left[U(\tilde{\Pi}_i)\right]}{\partial r_{D_j})^2} \end{pmatrix} > 0 \; ; \; i, j = A, B \; ; \; i \neq j. \tag{12}$$

Analogously, in the above example B's reaction to rise its loan rate to answer A's behavior mustn't be as strong as A's increase of loan rate. Otherwise the equilibrium would not be stable, formally (cf. Vives, 1999, p. 51)

$$det \begin{pmatrix} \frac{\partial^2 \mathrm{E}\left[U(\tilde{\Pi}_i)\right]}{(\partial r_{L_i})^2} & \frac{\partial^2 \mathrm{E}\left[U(\tilde{\Pi}_i)\right]}{\partial r_{L_i} \partial r_{L_j}} \\ \frac{\partial^2 \mathrm{E}\left[U(\tilde{\Pi}_i)\right]}{\partial r_{L_j} \partial r_{L_i}} & \frac{\partial^2 \mathrm{E}\left[U(\tilde{\Pi}_i)\right]}{\partial r_{l_j})^2} \end{pmatrix} > 0 \ ; \ i,j = A,B \ ; \ i \neq j. \tag{13}$$

Therefore, in the following we assume that stability conditions (12) and (13) will be met in order to perform the desired analysis.

Another interesting aspect of our model is that although we consider a duopolistic model of symmetric banks producing homogeneous goods – that is deposits and loans – the well known Bertrand Paradox which is the usual consequence in similar situations does not appear. That is, both banks realize strictly positive profits in the equilibrium. This is obvious because our proof of (r_D, r_L) being an equilibrium shows that $r > r_D > 0$ and $r_L > r > 0$ is valid, see appendix. The interpretation of both inequalities is that in equilibrium prices of deposits and loans are larger than marginal costs in the respective cases. A similar result is well known from the papers of Yanelle (1985) and Yanelle (1988) which argue that in case of simultaneous price competition in input and output markets – this is, indeed, what happens in the banking industry – the Bertrand Paradox does not appear.

However, while in Yanelle (1985) and Yanelle (1988) the reason for this result is that the volume of deposits taken restricts the volume of issued loans, our model has to be analyzed closer to find out the reasons for the same result. Nevertheless, the reasoning is quite similar to the one of Yanelle if one considers the balance sheet constraint (3). Remember, in section 2 it was explained that the existence of the interbank market separates deposit and loan business in the sense that a bank may realize any desired volume of deposits and loans since any excess or shortage of funds may be balanced with using the interbank market. The problem, however, is that using the interbank market causes costs – that is the interbank rate r. Therefore, on the one hand, issuing loans of larger volume than deposits taken requires the bank to borrow additional funds in the interbank market which is more expensive than using deposits for issuing loans – remember, in the equilibrium $r > r_D$ holds. On the other hand, taking a larger volume of deposits than issued loans puts a bank in a situation of lending the excess funds in the interbank market. This, however, causes profits to decrease since in the equilibrium $r_L > r$ holds. Hence, our model creates a kind of implicit relation between deposit and loan business in a sense that deviating from $D(r_D) = L(r_L)$ causes costs for the respective bank. As a result, the budget constraint which has been explicitly assumed in Yanelle (1985) and Yanelle (1988) appears implicitly in our model from the costs which arise when volumes of deposits and loans differ.

3.2 Decisions on Interest Rates before Hedging

As explained in section 2, when banks first set deposit and loan rates and decide on hedging levels thereafter, the decision problem looks as follows:

stage 1 Both banks simultaneously decide on deposit and loan rates. In this way the respective bank takes decisions of competitor as given

stage 2 Given decisions on interest rates in stage 1 both banks determine optimal volumes of CDS.

In order to proceed the analysis in the present case we apply the concept of subgame perfect Nash equilibrium. In this way we solve the game backwardly: we first determine optimal hedging decisions taking decisions on interest rates of stage 1 as given. Thereafter we determine optimal deposit and loan rates where the result of stage 1 is taken into account.

Hence on stage 2 both banks solve the following optimization problem:

$$\max_{H_i} \mathrm{E}\left[U(\tilde{\Pi}_i)\right] \ ; \ i,j = A,B \ ; \ i \neq j \tag{14}$$

where $\tilde{\Pi}_i$ is again given by equation (4) above.

The corresponding first order necessary conditions are then:

$$\mathrm{E}\left[U'(\tilde{\Pi}_i)\left(\tilde{\theta} - \bar{\theta}\right)\right] = 0 \ ; \ i = A,B. \tag{15}$$

Now turning to stage 1 of the game it proves advantageous to replace H_i by $H_i(r_{D_i}, r_{L_i}, r_{D_j}, r_{L_j})$; $i,j = A,B$; $i \neq j$ to account for the opportunity of the optimal hedging level to depend on deposit and loan rates of both banks – i.e. there is the opportunity of the existence of strategic effects of interest rates on the hedging behavior of competitors. Remember, in section two there have been presented some arguments from practice which state that in the present case a certain decision on interest rates at stage 1 might affect decisions on hedging at stage two. We explicitly allow for such effects to appear with our notation. In this regard we further assume any bank to take decisions on interest rates of the competitor as given when making own decisions. This is reasonable because interest rates of both banks are chosen simultaneously. That is, neither bank can observe and react to the respective competitor's decisions on deposit and loan rates.

As a result the following relations hold:

$$\frac{dH_i(r_{D_i}, r_{L_i}, r_{D_j}, r_{L_j})}{dr_{D_i}} = \frac{\partial H_i(r_{D_i}, r_{L_i}, r_{D_j}, r_{L_j})}{\partial r_{D_i}}$$

$$\frac{dH_i(r_{D_i}, r_{L_i}, r_{D_j}, r_{L_j})}{dr_{L_i}} = \frac{\partial H_i(r_{D_i}, r_{L_i}, r_{D_j}, r_{L_j})}{\partial r_{D_i}}$$

$$i,j = A,B \ ; \ i \neq j$$

Hence the optimization problem for stage 1 of the game may be written as

$$\max_{r_{D_i}, r_{L_i}} \mathrm{E}\left[U(\tilde{\Pi}_i)\right] \ ; \ i = A, B \tag{16}$$

with corresponding first order necessary conditions

$$\begin{aligned} \mathrm{E}\left[U'(\tilde{\Pi}_i)\left(-\hat{D}_i(r_{D_i}, r_{D_j}) + (r_{D_i} - r)\frac{\partial \hat{D}_i(r_{D_i}, r_{D_j})}{\partial r_{D_i}}\right)\right] + & \\ +\mathrm{E}\left[U'(\tilde{\Pi}_i)(\tilde{\theta} - \bar{\theta})\right] \frac{dH_i(r_{D_i}, r_{L_i}, r_{D_j}, r_{L_j})}{dr_{D_i}} &= 0 \qquad (17) \\ \mathrm{E}\left[U'(\tilde{\Pi}_i)\left(\hat{L}_i(r_{L_i}, r_{L_j}) + (r_{L_i} - r)\frac{\partial \hat{L}_i(r_{L_i}, r_{L_j})}{\partial r_{L_i}}\right)\right] - & \\ -\mathrm{E}\left[U'(\tilde{\Pi}_i)\tilde{\theta}\left(\hat{L}_i(r_{L_i}, r_{L_j}) + (r_{L_i} + 1)\frac{\partial \hat{L}_i(r_{L_i}, r_{L_j})}{\partial r_{L_i}}\right)\right] + & \\ +\mathrm{E}\left[U'(\tilde{\Pi}_i)\left(\tilde{\theta} - \bar{\theta}\right)\right] \frac{dH_i(r_{D_i}, r_{L_i}, r_{D_j}, r_{L_j})}{dr_{L_i}} &= 0 \qquad (18) \\ i, j = A, B \ ; \ i \neq j & \end{aligned}$$

The interesting fact about equations (17) and (18) is that they only differ from the respective first order necessary conditions (6) and (7) of simultaneous decisions on hedging and interest rates in line two of (17) and line three of (18). That is, since lines two and three in equations (17) and (18), respectively, represent the strategic effect of interest rates on hedging it is immediately clear that differences in optimal interest rates compared to the case of section 3.1 can appear if and only if these strategic effects are not zero.

Therefore, in order to determine optimal deposit and loan rates as well as optimal hedging decisions it is necessary to analyze the strategic effect in more detail. But having a closer look at equations (17) and (18) and taking into account first order conditions for the optimal hedging decision (15) it is easy to see that the second line of (17) and the third line of (18) diminish.[12] Note further, when strategic effects drop

[12] That is, for this reasoning to hold we apply the envelope theorem: from (15) we know that in the equilibrium $\mathrm{E}[U'(\tilde{\Pi}_i)(\tilde{\theta}-\bar{\theta})] = 0$ and hence the sign of $\frac{H_i(r_{D_i}, r_{D_j}, r_{L_i}, r_{L_j})}{dr_{D_i}}$ and $\frac{H_i(r_{D_i}, r_{D_j}, r_{L_i}, r_{L_j})}{dr_{L_i}}$ are not longer relevant for the optimal decisions on interest rates.

out first order necessary conditions for the optimal deposit and loan rates become

$$
\begin{aligned}
-\hat{D}_i(r_{D_i}, r_{D_j}) + (r - r_{D_i})\frac{\partial \hat{D}_i(r_{D_i}, r_{D_j})}{\partial r_{D_i}} &= 0 \quad \text{and} \\
\mathrm{E}\left[U'(\tilde{\Pi}_i)\left(\hat{L}_i(r_{L_i}, r_{L_j}) + (r_{L_i} - r)\frac{\partial \hat{L}_i(r_{L_i}, r_{L_j})}{\partial r_{L_i}}\right)\right] - & \\
-\mathrm{E}\left[U'(\tilde{\Pi}_i)\tilde{\theta}\left(\hat{L}_i(r_{L_i}, r_{L_j}) + r_{L_i} + 1\right)\frac{\partial \hat{L}_i(r_{L_i}, r_{L_j})}{\partial r_{L_i}}\right] &= 0 \\
i, j = A, B \; ; \; i \neq j. &
\end{aligned}
$$

Comparing these equations with (6) and (7) in 3.1 shows that first order necessary conditions are equivalent in both cases. Furthermore, by the same reasoning as in 3.1 it is obvious that from (15) $\mathrm{Cov}[U'(\tilde{\Pi}_i), \tilde{\theta}] = 0 \; ; \; i = A, B$ follows immediately. That is, in the equilibrium there is no interrelation between credit risk and a bank's profit. And, again, from the argument of section 3.1 it is well known that this appears if and only if both banks fully hedge their exposure $(1+r_{L_i})L_i(r_{L_i}, r_{L_j}) - H_i = 0 \; ; \; i = A, B$.

Moreover, rewriting (15) to arrive at

$$
\mathrm{E}\left[U'(\tilde{\Pi}_i)\tilde{\theta}\right] = \mathrm{E}\left[U'(\tilde{\Pi}_i)\right]\bar{\theta} \; ; \; i = A, B
$$

and using this equation to rewrite (18) yields

$$
(1 - \bar{\theta})\left(\hat{L}(r_{L_i}, r_{L_j}) + r_{L_i}\frac{d\hat{L}(r_{L_i}, r_{L_j})}{dr_{L_i}}\right) - (r + \bar{\theta})\frac{d\hat{L}(r_{L_i}, r_{L_j})}{dr_{L_i}} = 0.
$$

This latter equation, however, is equivalent to (9) which is the first order necessary condition for the optimal loan rate when the optimal hedging strategy is applied.

As a result the optimal loan rate is the same as the one of the riskless case which is obvious since in the current situation it was argued that a full hedge is optimal. Thus banks are no longer exposed to credit risk and choose loan rates accordingly. Since, in addition, the first order necessary condition for the optimal deposit rate does not change compared to the situation in section 3.1 it is obvious that optimal deposit rates remain the same, too.

Moreover, due to perfect symmetry of the model in equilibrium both banks set the same deposit rates as well as the same loan rates. That is, the proof of the symmetric Nash Equilibrium in pure strategies in Proposition 1 which is presented in the appendix may be applied in an analogous way for the situation in the current section. Hence in equilibrium $r^*_{D_A} = r^*_{D_B} \equiv r_D$ and $r^*_{L_A} = r^*_{L_B} \equiv r_L$ holds. And, of course, arguments regarding the uniqueness and the stability of the Nash equilibrium applied in section 3.1 still apply in the present context. Therefore, it is clear, too, that both banks realize strictly positive profits and the Bertrand Paradox does not arise.

At the end it is clear that the equilibrium on the hedging stage is symmetric, too: since both banks choose the same deposit and loan rates the levels of deposits taken and loans issued are the same, too. Because of the optimal hedging strategy being to fully hedge exposure to risk both banks choose same amount of CDS in the optimum: $H_A = H_B = H = (1 + r_L)L(r_L)$. Hence, we can summarize results in

Proposition 2 *When decisions on interest rates are made before decisions on hedging a unique symmetric Nash equilibrium will appear in which both banks fully hedge exposures to risk and optimal deposit and loan rates are the same as in a situation without credit risk. A strategic effect does not appear in this situation. Neither do interest rates influence hedging nor the other way round.*

The interpretation of this result is straightforward: even when decisions on hedging are made after decisions on deposit and loan rates the results do not change compared to the situation in section 3.1. In particular we observe the appearance of the strong separation property which is well known from the literature.(cf. Wahl and Broll, 2000, for example) Therefore, both banks separate decisions on hedging and interest rate. This separation, however is the reason why there does not appear a strategic effect of interest rates although the game is a sequential one. Note, as soon as decisions on interest rates are made there exists a unique dominant hedging strategy which maximizes expected utility of a bank. This dominant strategy is to fully hedge exposure to risk since doing so ensures that on the one hand profits are maximized. On the other hand neither bank can credibly deviate from this strategy. Note, since the banks compete in prices (i.e. loan rates in this situation) a bank may only realize an advantage compared to the competitor when the own loan rate is lower. However, when one bank deviates from the strategy to fully hedge its exposure to risk this bank's profit is no longer riskless. This, in turn, forces a risk averse bank to choose loan rate such that a sufficient compensation for bearing risk is achieved. Therefore, the deviating bank has to rise its loan rate and fails in the competition of banks. Hence, the only optimal hedging strategy in the present situation is to fully hedge exposures to risk and thus there is no chance to use hedging as a strategic device.

3.3 Hedging before Decisions on Interest Rates

Regarding the arguments of practitioners which have been mentioned in section 2 one may expect a situation where hedging takes place before decisions on deposit and loan rates the most interesting one. The following analysis will provide arguments on whether this impression is correct.

With the assumptions of section 2 the structure of the game is as follows:

stage 1 Both banks may enter the market for CDS and simultaneously decide on optimal levels of derivatives. The level of CDS of the respective competitor is taken as given which is a direct consequence of the simultaneity of decisions.

stage 2 Both banks simultaneously set deposit and loan rates taking optimal levels of CDS from stage 1 as given. Of course due to simultaneity of decisions on interest rates neither bank expects the respective competitor to react on own decisions.

Just like in the previous section we apply the concept of subgame perfect Nash-equilibrium and backward induction in order to analyze the current situation. That is, we first determine optimal interest rates in stage 2 given decisions on hedging and afterwards determine optimal levels of CDS in stage 1 considering optimal interest rates from stage two.

Therefore, given hedging decision, both banks solve the following optimization problem on stage 2:

$$\max_{r_{D_i}, r_{L_i}} \mathrm{E}\left[U(\tilde{\Pi}_i)\right] \; ; \; i = A, B \tag{19}$$

where $\tilde{\Pi}_i$ is given by equation (4) above.

The corresponding first order necessary conditions are:

$$-\hat{D}_i(r_{D_i}, r_{D_j}) + (r - r_{D_i})\frac{\partial \hat{D}_i(r_{D_i}, r_{D_j})}{\partial r_{D_i}} = 0 \tag{20}$$

$$\begin{aligned} &\mathrm{E}\left[U'(\tilde{\Pi}_i)\left(\hat{L}_i(r_{L_i}, r_{L_j}) + (r_{L_i} - r)\,\frac{\partial \hat{L}_i(r_{L_i}, r_{L_j})}{\partial r_{L_i}}\right)\right] - \\ &-\mathrm{E}\left[U'(\tilde{\Pi}_i)\tilde{\theta}\left(\hat{L}_i(r_{L_i}, r_{L_j}) + (r_{L_i} + 1)\frac{\partial \hat{L}_i(r_{L_i}, r_{L_j})}{\partial r_{L_i}}\right)\right] = 0 \end{aligned} \tag{21}$$

$$i, j = A, B \; ; \; i \neq j.$$

A comparison of (20) and (21) with the corresponding first order necessary conditions (6) and (7) in the case of simultaneous decisions on interest rates and hedging shows that, in general, first order necessary conditions for the optimal deposit and loan rates are equivalent. Therefore, different results may appear if and only if the first order necessary condition for the optimal hedging level differs from (8) in section 3.1.

In order to find out whether this is true we first have to conduct some modifications: note first that due to the sequential nature of the present game there may occur strategic effects of the hedging decision on optimal deposit and loan rates. Therefore we write in the following

$$r_{D_i} = r_{D_i}(H_i, H_j) \text{ and } r_{L_i} = r_{L_i}(H_i, H_j) \; ; \; i, j = A, B \; ; \; i \neq j$$

to account for this opportunity. Furthermore, since decisions on hedging volumes are made simultaneously every bank conjectures that the respective competitor does not change its decision regarding hedging as a reaction on own decisions. That is $\frac{dH_i}{dH_j} = 0$; $i, j = A, B$; $i \neq j$. Consequently, the following relations hold:

$$\frac{dr_{D_i}(H_i, H_j)}{dH_i} = \frac{\partial r_{D_i}(H_i, H_j)}{\partial H_i} \quad \text{and} \quad \frac{dr_{L_i}(H_i, H_j)}{dH_i} = \frac{\partial r_{L_i}(H_i, H_j)}{\partial H_i} \tag{22}$$

$$\frac{dr_{D_i}(H_i, H_j)}{dH_j} = \frac{\partial r_{D_i}(H_i, H_j)}{\partial H_j} \quad \text{and} \quad \frac{dr_{L_i}(H_i, H_j)}{dH_j} = \frac{\partial r_{L_i}(H_i, H_j)}{\partial H_j} \tag{23}$$

$$i, j = A, B \ ; \ i \neq j.$$

Hence, one can state the banks' optimization problems on stage 1 as

$$\max_{H_i} \mathrm{E}\left[U(\tilde{\Pi}_i)\right] \ ; \ i = A, B \tag{24}$$

with corresponding first order necessary conditions

$$\begin{aligned}
&\left(-\hat{D}_i(r_{D_i}, r_{D_j}) + (r - r_{D_i})\frac{\partial \hat{D}_i(r_{D_i}, r_{D_j})}{\partial r_{D_i}}\right)\frac{dr_{D_i}(H_i, H_j)}{dH_i} + \\
&+\mathrm{E}\left[U'(\tilde{\Pi}_i)\left(\hat{L}_i(r_{L_i}, r_{L_j}) + (r_{L_i} - r)\frac{\partial \hat{L}_i(r_{L_i}, r_{L_j})}{\partial r_{L_i}}\right)\right]\frac{dr_{L_i}(H_i, H_j)}{dH_i} - \\
&-\mathrm{E}\left[U'(\tilde{\Pi}_i)\tilde{\theta}\left(\hat{L}(r_{L_i}, r_{L_j}) + (r_{L_i} + 1)\frac{\partial \hat{L}_i(r_{L_i}, r_{L_j})}{\partial r_{L_i}}\right)\right]\frac{dr_{L_i}(H_i, H_j)}{dH_i} + \\
&(r - r_{D_i})\frac{\partial \hat{D}_i(r_{D_i}, r_{D_j})}{\partial r_{D_j}}\frac{dr_{D_j}(H_i, H_j)}{dH_i} + \\
&+\mathrm{E}\left[U'(\tilde{\Pi}_i)\left((1 - \tilde{\theta})r_{L_i} - r - \tilde{\theta}\right)\frac{\partial \hat{L}_i(r_{L_i}, r_{L_j})}{\partial r_{L_j}}\right]\frac{dr_{L_j}(H_i, H_j)}{dH_i} + \\
&+\mathrm{E}\left[U'(\tilde{\Pi}_i)\left(\tilde{\theta} - \bar{\theta}\right)\right] = 0 \\
&i, j = A, B \ ; \ i \neq j.
\end{aligned}$$

By applying the envelope theorem[13] yields

$$\begin{aligned} (r-r_{D_i})\frac{\partial \hat{D}_i(r_{D_i},r_{D_j})}{\partial r_{D_j}}\frac{dr_{D_j}(H_i,H_j)}{dH_i}&+\\ +\mathrm{E}\left[U'(\tilde{\Pi}_i)\left((1-\tilde{\theta})r_{L_i}-r-\tilde{\theta}\right)\frac{\partial \hat{L}_i(r_{L_i},r_{L_j})}{\partial r_{L_j}}\right]\frac{dr_{L_j}(H_i,H_j)}{dH_i}&+\\ +\mathrm{E}\left[U'(\tilde{\Pi}_i)\left(\tilde{\theta}-\bar{\theta}\right)\right] &= 0 \quad (25)\\ i,j=A,B\ ;\ i\neq j.& \end{aligned}$$

After having derived first order necessary conditions we are interested in the optimal levels of deposit and loan rates as well as the optimal hedging volume. In this regard we proceed like in section 3.2. We first determine optimal hedging decision and analyze optimal deposit and loan rates afterwards. Unfortunately, from the condition for the optimal hedging level (25) it is not obvious what the optimal hedging volume looks like. The reason is that (25) covers terms which include the strategic effects $\frac{dr_{D_j}(H_i,H_j)}{dH_i}$ and $\frac{dr_{L_j}(H_i,H_j)}{dH_i}$ that do not diminish by simply applying the envelope theorem. Therefore, before one can make statements about a bank's assets and liabilities management – that is deposit and loan rates as well as hedging levels – the strategic effects have to be analyzed in more detail.

3.3.1 The Strategic Role of Hedging

As suggested earlier strategic effects may be seen from the signs of expressions:

$$\frac{dr_{D_i}(H_i,H_j)}{dH_i}\ ;\ \frac{dr_{D_j}(H_i,H_j)}{dH_i}\ ;\ \frac{dr_{L_i}(H_i,H_j)}{dH_i}\ ;\ \frac{dr_{L_j}(H_i,H_j)}{dH_i}\ ;\ i,j=A,B\ ;\ i\neq j.$$

That is, strategic effects exist if one can show that a bank's decision regarding the optimal volume of hedging affects deposit and loan rates of both banks. And in this way hedging plays an important (strategic) role in the competition between banks. Hence, we have to determine the signs of the expressions above.

Regarding the deposit business at first we can state and prove

[13] Note, the first term in the first line of this latter equation is equivalent to the first order necessary conditions (20) for the optimal deposit rate which have to be zero in the optimum. Analogously, the first terms in lines two and three are equivalent to the first order necessary conditions (21) for the optimal loan rates which have to be zero in the optimum, too.

Proposition 3 *In the deposit business there is no strategic effect of hedging, i.e.*

$$\frac{dr_{D_i}(H_i, H_j)}{dH_i} = 0 \quad (26)$$

$$\frac{dr_{D_j}(H_i, H_j)}{dH_i} = 0 \quad (27)$$

$$i, j = A, B \ ; \ i \neq j.$$

Proof: See the Appendix.

The interpretation of this result is obvious, since credit risk does not influence the deposit business. Note, this has been already explained in detail earlier, and because there does not appear an interrelation between deposit and loan business via operating costs hedging is not able to affect optimal decisions on deposit rates.

Turning now to the loan business we can state and prove

Proposition 4 *In the loan business there appears a strategic effect of hedging. In particular, as long as the banks' profit function exhibits constant or decreasing absolute risk aversion in the Arrow-Pratt sense hedging positively affects loan rates of both banks and hence dilutes the intensity of competition, i.e.*

$$\frac{dr_{L_i}(H_i, H_j)}{dH_i} > 0 \quad (28)$$

$$\frac{dr_{L_j}(H_i, H_j)}{dH_i} > 0 \quad (29)$$

$$i, j = A, B \ ; \ i \neq j.$$

Proof: See the Appendix.

There are some interesting observations which can be derived from the proof of proposition 4: in the proof it was clarified that the crucial factor for the strategic effect to appear is the sign of the term $\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{L_i} \partial H_i}$. This term characterizes the effect of hedging on the expected marginal utility of a bank's profit with respect to the loan rate. If this effect is positive – which is true in the present setting – a higher hedging volume increases expected marginal utility in the loan business. If one further remembers that the optimal loan rate can be determined using the expected marginal utility [14] it is immediately clear that due to the effect of hedging on the expected

[14]Note, the first order necessary conditions (21) state that in equilibrium the expected marginal utility of a bank with respect to the loan rate has to be zero.

marginal utility the optimal loan rate changes. In other words, choosing the hedging volume before interest rates sends a signal to a bank's competitor which supplies him with information on the bank's future loan rate. As a rational competitor takes this information into account when choosing the optimal loan rate it is obvious that hedging works as a strategic move in the competition among banks in the current setting. In fact, this result is in line with the theoretical literature on the strategic role of hedging.(Allaz and Vila, 1993, cf.)

A closer look at the expression for $\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{L_i} \partial H_i}$ allows an additional very important interpretation. That is, the appearance of the strategic effect crucially depends on our assumption of risk aversion ($U''(\cdot) < 0$). In case of risk neutrality, i.e. $U''(\cdot) = 0$, it turns out that $\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{L_i} \partial H_i} = 0$ and the strategic effect of hedging disappears. This leads to a very important conclusion: as long as a bank knows that the respective competitor is risk averse and, in turn, the competitor knows that the considered bank is risk averse, too, there appears a strategic effect of hedging. In this context it is not necessary that a bank can observe the hedging activities of the competitor. The common information that both banks are risk averse will suffice for hedging to act as a strategic move in the competition among banks. It is the risk aversion which makes hedging beneficial in our model. Hence, risk aversion provides the foundation for the strategic effect. Thus, our result confirms the results of Hughes and Kao (1997) who argue that it is not the observability of the hedging activity per se which is necessary for a strategic effect of hedging to appear. Rather it is the hedging motive, i.e. the fact that hedging is beneficial for the firms, which is important in the competition among firms.

3.3.2 Assets and Liabilities Management

Knowing the strategic effects of hedging allows to analyze optimal decisions on hedging as well as optimal decisions on deposit and loan rates. We can thus state our

Proposition 5 *In case of constant or decreasing absolute risk aversion in the sense of Arrow and Pratt the unique equilibrium hedging volume H of both banks is larger than the bank's exposures to credit risk, i.e. there appears an over hedge in the equilibrium, i.e. $H > (1 + r_L)L(r_L)$. Consequently, the unique optimal loan rate r_L of both banks increases compared to the situations in 3.1 and 3.2. The unique optimal deposit rate r_D, however, does not change.*

Proof: See the Appendix.

The interpretation of this result is straightforward. Firstly consider optimal decisions on interest rates to note that in the present context hedging does not completely eliminate risk. Due to the over hedge the banks are still exposed to risk and hence the loan rate increasing effect of risk aversion still works. This is a direct consequence of the strategic effect of hedging in this context.

Furthermore, due to the strategic effect a bank can force its respective competitor to increase the loan rate. The reason is that loan rates are strategic complements in the sense of Bulow et al. (1985). Hence, the over hedge of a bank leaves this bank exposed to risk which, in turn, requires to increase the loan rate compared to a situation with a full hedge. However, due to loan rates being strategic complements the competing bank has to increase the loan rate too. Otherwise there were no optimal situation.[15] Thus, with hedging a bank can gain an advantage at the expense of the competitor. This strategic effect of hedging works towards an increase of the hedging volume.

On the other hand, any hedging volume that differs from a full hedge leaves the bank exposed to risk which causes its expected utility to decrease due to risk aversion. The validity of this argument is evident when one has a closer look at the first order necessary condition for the optimal hedging levels in sections 3.1 and 3.2 (8) and (15), respectively, which are[16]

$$\mathrm{E}\left[U'(\tilde{\Pi}_i)\left(\tilde{\theta}-\bar{\theta}\right)\right]=0\ ;\ i=A,B.$$

Hence, an under hedge can not be optimal. On top of this we are also able to exclude the possibility of a hedging volume that runs to infinity, because for any over-hedge we perceive a trade off between the strategic effect that promotes the over-hedge and the decreasing expected utility. Overall this yields a finite over-hedge. More formally, by using the results derived so far the first order necessary conditions for the optimal hedging levels in the current situation (25) can be rewritten to yield

$$\mathrm{E}\left[U'(\tilde{\Pi}_i)\left((1-\tilde{\theta})r_L-r-\tilde{\theta}\right)\tfrac{1}{2}\frac{dL(r_L)}{dr_L}\right]\frac{dr_{L_j}(H_i,H_j)}{dH_i}+\mathrm{E}\left[U'(\tilde{\Pi}_i)\left(\tilde{\theta}-\bar{\theta}\right)\right]=0.$$

Note, the second term on the left hand side of this equation is equivalent to the first order necessary conditions for the optimal hedging levels in sections 3.1 and 3.2 (8) and (15) and, thus, represents the risk effect. The first term on the left hand side of the above equation represents the strategic effect of hedging on r_{L_j}.[17] In this regard we already know that the strategic effect is positive and hence leads to an increase of the hedging level in any situation. The risk effect of hedging, however,

[15] Note, in section 3.1 this argument has been presented in more detail.

[16] Note, in the situations analyzed in sections 3.1 and 3.2 there did not appear a strategic effect of hedging. Hence the optimal hedging levels have been the full hedge in both situations which is obvious since this strategy minimizes the disutility of bearing risk on both situations. The resulting effect – call it risk effect of hedging – thus works towards a full hedge of the bank's exposure.

[17] This was derived in the proof of proposition 5 in the appendix.

always acts towards a full hedge. That is, as long as the bank's hedging volume is less than the exposure to risk (under hedge) both effects work in the same direction and make the bank increase the hedging volume. Therefore, a under hedge is not optimal in the current setting. When a bank, however, chooses a hedging volume beyond the level of the full hedge there appears a trade-off between both effects: the strategic effect still works towards increasing the hedging level, while the risk effect demands to decrease the hedging volume. As a result, there must be a certain hedging level where both effects balance out. This optimal hedging level then is necessarily achieved with an over hedge.

4 Conclusion

Due to the growing importance and international liquidity of credit derivatives and because of the increasing interdependencies of competitors in the banking industry our paper investigated possible strategic effects of management decisions concerning interest rates for loans and deposits as well as the hedging volume on the behavior of competitors. We considered three scenarios of the decision process that can be observed in banking practice. From intuition it seems clear that with simultaneous decisions strategic effects can not occur. Our model is able to confirm this impression formally. In case of a sequential process where the decision on interest rates precedes the hedging decision matters are not that clear. Nonetheless, our model rejects both a strategic effect of interest rates and of hedging. In contrast, when the hedging decision is made at first there is a strategic effect of hedging on the loan rates of the competitor. More precisely, when one bank increases its hedging this will not only cause its own loan rate to rise but also the competitors loan rate to keep equilibrium and optimality. Moreover, and unlike the other scenarios, this case leads to an over-hedge of exposure.

We are confident that our paper will not only help to promote the scientific discussion in this area but also sensitize management in banking industry for this issue. The usefulness our paper for practice can be further improved by extending the set of credit derivatives considered within our theoretical framework to cover credit default swaps with basis risk and credit default digitals. Future research should also try to relax our assumption of a risk neutral market.

Appendix

Proof of the Symmetric Equilibrium in Proposition 1

Consider the first order necessary conditions (6) and (7) and note that an equilibrium strategy $[(r^*_{D_A}, r^*_{D_B}), (r^*_{L_A}, r^*_{L_B})]$ simultaneously solves systems (6) and (7). In order to prove the existence of a Nash-Equilibrium in pure strategies where both banks set identical deposit and loan rates we show that a one-sided deviation from strategies $r^*_{D_A} = r^*_{D_B} \equiv r_D$ and $r^*_{L_A} = r^*_{L_B} \equiv r_L$ puts the deviating bank worse off. Hence there is no incentive to choose another strategy than the one claimed to be an equilibrium.

In this way consider first the deposit business, i.e. equation (6): with strategy r_D equations (6) can be rewritten as follows:

$$-\tfrac{1}{2}D(r_D) + (r - r_D)\tfrac{1}{2}\frac{dD(r_D)}{dr_D} = 0. \tag{30}$$

It is easy to see that for this equation to hold $r > r_D > 0$ must be true since $D(r_D) > 0$ and $\frac{dD(r_D)}{dr_D} > 0$. Therefore taking deposits adds strictly positive amount to banks' profits and a situation like the Bertrand Paradox – which is the usual result in symmetric models of oligopolies with price competition – does not appear.[18]

We will now argue that it is not beneficial to one of the banks to deviate from the equilibrium strategy r_D:

Let for example c.p. $r_{D_A} < r_D$. Then - from definition of deposit supply function - Bank A's deposit supply drops to zero and as a result profits from taking deposits drop out. Therefore the profit of bank A decreases due to the one-sided reduction of the deposit rate.

Let now c.p. $r_{D_A} > r_D$. Then - due to the definition of deposit supply and $\frac{dD(r_{D_i})}{dr_{D_i}} > 0$ it must be true that

$$\hat{D}_A(r_{D_A}, r_D) = D(r_{D_A}) > D(r_D) > \tfrac{1}{2}D(r_D) = \hat{D}_A(r_D, r_D).$$

Furthermore, due to $\frac{d^2D(r_{D_i})}{(dr_{D_i})^2} < 0$ following relation holds

$$\frac{dD(r_{D_A})}{dr_{D_A}} < \frac{dD(r_D)}{dr_D}.$$

[18] For the appearance of the Bertrand Paradox our model would require $r_D = r$. The interpretation of this condition is straightforward: r represents the revenue and r_D the cost per unit of deposits taken since a bank has to pay r_D per unit to depositors and receives r per unit of deposit from lending in the interbank market. Hence the relation for the Bertrand Paradox stated above says that in this case price equals marginal costs.

Hence, in order for (6) to be satisfied $r_{D_A} < r_D$ must hold which contradicts assumption $r_{D_A} > r_D$, and $r_{D_A} > r_D$ cannot be an equilibrium.

Since this reasoning analogously holds for bank B an one-sided deviation from r_D is obviously not reasonable for any of the two banks. Consequently r_D must be an equilibrium in the deposit market.

Consider now the banks' loan business, i.e. equations (7): with strategy r_L equations (7) can be rewritten:

$$\mathrm{E}\left[U'(\tilde{\Pi}_i)\left((1-\tilde{\theta})\left(\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}}+r_L\right)-(r+\tilde{\theta})\right)\right]=0. \tag{31}$$

From $r > 0$, $\tilde{\theta} \in [0,1]$, and $U'(\cdot) > 0$ it is immediately clear, that equation (31) holds if and only if $\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + r_L > 0$ – otherwise the expected marginal utility would be strictly negative.

Using the well known covariance formula $\mathrm{E}(XY) = \mathrm{E}(X)\mathrm{E}(Y) + \mathrm{Cov}(X,Y)$ and rearranging terms yields

$$\begin{aligned} & \mathrm{E}[U'(\tilde{\Pi}_i)]\underbrace{\left(\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}}+r_L-r\right)}_{a}- \\ -\left(\mathrm{E}(U'(\tilde{\Pi}_i))\mathrm{E}(\tilde{\theta})+\mathrm{Cov}(U'(\tilde{\Pi}_i),\tilde{\theta})\right) & \underbrace{\left(\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}}+r_L+1\right)}_{b} = 0. \end{aligned}$$

Since $U'(\cdot) > 0$ and $\mathrm{Cov}(U'(\tilde{\Pi}_i),\tilde{\theta}) > 0$, where the sign of the covariance immediately follows from (4) from which one can observe $\tilde{\Pi}_i$ to shrink and, due to $U''(\tilde{\Pi}_i) < 0$, $U'(\tilde{\Pi}_i)$ to increase when $\tilde{\theta}$ rises, the latter equation holds if and only if terms a and b have the same sign. Due to $\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + r_L > 0$ – see arguments above – this is true if an only if

$$\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + r_L > r.$$

After rearranging terms this relation may be rewritten as

$$r_L - r > -\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}}.$$

Since $\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} < 0$ it follows that

$$r_L > r > 0$$

in equilibrium.

As a result, in equilibrium the loan business adds a strictly positive amount to the banks' profits and, again, the Bertrand Paradox does not appear.[19]

Just like in the deposit business we will now argue that it ist not beneficial for at least one of the banks to deviate from the strategy r_L.

Let for example c.p. $r_{L_A} > r_L$: then from definition of loan demand Bank A's level of loans drops to zero. However, since loans add a positive amount to bank A's profits this behavior reduces profits and is thus not beneficial.

Consider now c.p. $r_{L_A} < r_L$: in this case bank A's loan demand increases and the following relation holds:

$$\hat{L}_A(r_{L_A}, r_L) = L(r_{L_A}) > L(r_L) > \tfrac{1}{2}L(r_L) = \hat{L}_A(r_L, r_L)$$

Furthermore, due to $\frac{d^2L(r_{L_i})}{(dr_{L_i})^2} < 0$ it follows that $\frac{dL(r_{L_A})}{dr_{L_A}} > \frac{dL(r_L)}{dr_L}$ and hence

$$\frac{L(r_{L_A})}{\frac{dL(r_{L_A})}{dr_{L_A}}} < \frac{L(r_L)}{\frac{dL(r_L)}{dr_L}}.$$

Thus, in order to (7) be satisfied, $r_{L_A} > r_L$ must hold which contradicts $r_{L_A} < r_L$. As a consequence $r_{L_A} < r_L$ cannot be an equilibrium.

The same reasoning holds for bank B deviating form r_L in an analogous way. Therefore, r_L must be an equilibrium.

Consider now equation (8) which determines optimal hedging levels of both banks. In section 3.1.1 it was argued that for any bank this condition implies to fully hedge the exposure to credit risk. With this strategy it was shown in section 3.1.1 that equation (7) reduces to equation (9). However, this latter equation is just a special case of (7). Therefore, the arguments regarding equilibrium strategies in the loan business presented earlier even hold for (9). Hence both banks set the same loan rates (r_L)in the equilibrium.

Moreover, since this implies that both banks issue the same volume of loans in the equilibrium ($\frac{1}{2}L(r_L)$) it is obvious that both banks realize the same equilibrium hedging levels. That is, in the equilibrium it must be true that $H_A^* = H_B^* \equiv H = (1 + r_L)\frac{1}{2}L(r_L)$. $\square$

[19]If the Bertrand Paradox would appear, it would be true that $r_L = r$ which would have to be interpreted as "price equals marginal costs" in the equilibrium.

Proof of Proposition 3

In order to determine the sign of $\frac{dr_{D_i}(H_i,H_j)}{dH_i}$ and $\frac{dr_{D_j}(H_i,H_j)}{dH_i}$ we apply the implicit function theorem and the Cramer rule to arrive at

$$\frac{dr_{D_i}(H_i,H_j)}{dH_i} = \frac{\det\begin{pmatrix} -\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{D_i}\partial H_i} & \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{D_i}\partial r_{D_j}} \\ -\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_j)]}{\partial r_{D_j}\partial H_i} & \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_j)]}{(\partial r_{D_j})^2} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{(\partial r_{D_i})^2} & \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{D_i}\partial r_{D_j}} \\ \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_j)]}{\partial r_{D_i}\partial r_{D_j}} & \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_j)]}{(\partial r_{D_j})^2} \end{pmatrix}} = \frac{\det J_{D_i}}{\det J_D} \tag{32}$$

$$\frac{dr_{D_j}(H_i,H_j)}{dH_i} = \frac{\det\begin{pmatrix} \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{(\partial r_{D_i})^2} & -\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{D_i}\partial H_i} \\ \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_j)]}{\partial r_{D_j}\partial r_{D_i}} & -\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_j)]}{\partial r_{D_j}\partial H_i} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{(\partial r_{D_i})^2} & \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{D_i}\partial r_{D_j}} \\ \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_j)]}{\partial r_{D_i}\partial r_{D_j}} & \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_j)]}{(\partial r_{D_j})^2} \end{pmatrix}} = \frac{\det J_{D_j}}{\det J_D} \tag{33}$$

$$i,j = A,B\ ;\ i \neq j$$

Since our aim is to determine the sign of expressions (32) and (33) we have to find out the signs of the single components of the respective expressions. Furthermore, as we consider a banks' behavior in the equilibrium of the game we calculate all expressions for the equilibrium values of deposit and loan rates. In this regard the arguments from the proof of the symmetric equilibrium above can be applied analogously. Hence we can use the unique equilibrium strategy r_D in our subsequent reasoning.

From the arguments of section 3.1 concerning deposit rates beeing strategic complements one can observe (see equation (10)):

$$\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{D_i}\partial r_{D_j}} > 0\ ;\ i,j = A,B\ \ i \neq j$$

Furthermore from differentiating (20) with respect to r_{D_i} one arrives at

$$\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{(\partial r_{D_i})^2} = -2\frac{\partial \hat{D}_i(r_{D_i},r_{D_j})}{\partial r_{D_i}} + (r - r_{D_i})\frac{\partial^2 \hat{D}_i(r_{D_i},r_{D_j})}{(\partial r_{D_i})^2} < 0\ ;\ i,j = A,B\ ;\ i \neq j$$

where the inequality follows from the arguments of the proof of r_D to be the equilibrium strategy and hence in the equilibrium it must be true that

$$\begin{aligned}
r - r_{D_i} = r - r_D &> 0, \\
\frac{\partial \hat{D}_i(r_{D_i}, r_{D_j})}{\partial r_{D_i}} = \frac{dD(r_D)}{dr_D} &> 0, \\
\frac{\partial^2 \hat{D}_i(r_{D_i}, r_{D_j})}{(\partial r_{D_i})^2} = \frac{d^2 D(r_D)}{(dr_D)^2} &< 0 \\
i, j = A, B \; ; \; i \neq j.
\end{aligned}$$

Moreover, differentiating (20) with respect to H_i and H_j yields

$$\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{D_i} \partial H_i} = 0$$

and

$$\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{D_i} \partial H_i} = 0$$

respectively.

At least, again from section 3.1, the equilibrium is stable if equation (12) holds, that is

$$\det J_D > 0.$$

As a result $\det J_{D_i} = \det J_{D_j} = 0$ and $\det J_D > 0$ and hence in the equilibrium it must be true that

$$\frac{dr_{D_i}(H_i, H_j)}{dH_i} = 0 \tag{34}$$

$$\frac{dr_{D_j}(H_i, H_j)}{dH_i} = 0 \tag{35}$$

$$i, j = A, B \; ; \; i \neq j.$$

□

Proof of Proposition 4

Analogous to the former case we determine the sign of $\frac{dr_{L_i}(H_i,H_j)}{dH_i}$ and $\frac{dr_{L_j}(H_i,H_j)}{dH_i}$ by applying the implicit function theorem and the Cramer rule:

$$\frac{dr_{L_i}(H_i,H_j)}{dH_i} = \frac{\det\begin{pmatrix} -\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{L_i}\partial H_i} & \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{L_i}\partial r_{L_j}} \\ -\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_j)]}{\partial r_{L_j}\partial H_i} & \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_j)]}{(\partial r_{L_j})^2} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{(\partial r_{L_i})^2} & \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{L_i}\partial r_{L_j}} \\ \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_j)]}{\partial r_{L_i}\partial r_{L_j}} & \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_j)]}{(\partial r_{L_j})^2} \end{pmatrix}} = \frac{\det J_{L_i}}{\det J_L} \tag{36}$$

$$\frac{dr_{L_j}(H_i,H_j)}{dH_i} = \frac{\det\begin{pmatrix} \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{(\partial r_{L_i})^2} & -\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{L_i}\partial H_i} \\ \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_j)]}{\partial r_{L_j}\partial r_{L_i}} & -\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_j)]}{\partial r_{L_j}\partial H_i} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{(\partial r_{L_i})^2} & \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{L_i}\partial r_{L_j}} \\ \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_j)]}{\partial r_{L_i}\partial r_{L_j}} & \frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_j)]}{(\partial r_{L_j})^2} \end{pmatrix}} = \frac{\det J_{L_j}}{\det J_L} \tag{37}$$

$$i,j = A,B\ ;\ i \neq j$$

In general, the proceeding of the proof is the same like the one in the proof of proposition 3. That is, we analyze the effect of hedging on the equilibrium loan rate r_L which is the same for both banks.[20]

From the arguments of section 3.1 loan rates are known to be strategic complements, i.e.

$$\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{L_i}\partial r_{L_j}} > 0\ ;\ i,j = A,B\ ;\ i \neq j.$$

Furthermore it must be true that in equilibrium

$$\begin{aligned}\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{(\partial r_{L_i})^2} &= \tfrac{1}{2}\mathrm{E}\left[U''(\tilde{\Pi}_i)\left((1-\tilde{\theta})L(r_L) + \left((1-\tilde{\theta})r_L - r - \tilde{\theta}\right)\frac{dL(r_L)}{dr_L}\right)^2\right] + \\ &+ 2\mathrm{E}\left[U'(\tilde{\Pi}_i)(1-\tilde{\theta})\right]\frac{dL(r_L)}{dr_L} + \\ &+ \mathrm{E}\left[U'(\tilde{\Pi}_i)\left((1-\tilde{\theta})r_L - r - \tilde{\theta}\right)\right]\frac{d^2L(r_L)}{(dr_L)^2} < 0 \\ &i,j = A,B\ ;\ i \neq j\end{aligned}$$

[20] Again it is easily verified that the arguments of the proof of the symmetric equilibrium of proposition 1 are still valid in the current situation. Hence both banks choose the same loan rates in the equilibrium.

where the inequality follows from risk aversion ($U''(\cdot) > 0$) and from the fact that quadratic terms are non-negative ($(\cdot)^2 \geq 0$). Moreover, from the proof of proposition 5 we know $\mathrm{E}\left[U'(\tilde{\Pi}_i)(1-\tilde{\theta})\right] > 0$ and $\mathrm{E}\left[U'(\tilde{\Pi}_i)\left((1-\tilde{\theta})r_L - r - \tilde{\theta}\right)\right] > 0$ and by assumption in the equilibrium $\frac{dL(r_L)}{dr_L}, \frac{d^2L(r_L)}{(dr_L)^2} < 0$ holds.

Now, differentiating (21) with respect to H_i and H_j yields

$$\begin{aligned}
\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{L_i}\partial H_i} &= \mathrm{E}\left[U''(\tilde{\Pi}_i)\left((1-\tilde{\theta})\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + \left((1-\tilde{\theta})r_L - r - \tilde{\theta}\right)\right)\left(\tilde{\theta}-\bar{\theta}\right)\right] \\
\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{L_i}\partial H_j} &= 0 \\
& \quad i,j = A,B \ ; \ i \neq j
\end{aligned}$$

where equation (21) was divided by $\frac{dL(r_L)}{dr_L}$ before differentiating.

In order to determine the sign of the first equation rearrange terms to arrive at

$$\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{L_i}\partial H_i} = \mathrm{E}\left[U''(\tilde{\Pi}_i)\left(\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + (r_L - r) - \tilde{\theta}\left(\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + (r_L+1)\right)\right)\left(\tilde{\theta}-\bar{\theta}\right)\right]. \tag{38}$$

Consider now $(\tilde{\theta}-\bar{\theta})$: adding $\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + (r_L - r) - \frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + (r_L - r)$ and multiplying $\frac{\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}}+(r_L+1)}{\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}}+(r_L+1)}$ yields

$$\tilde{\theta}-\bar{\theta} = \frac{1}{\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + (r_L+1)}\left(\bar{c}-\tilde{c}\right)$$

where

$$\begin{aligned}
\bar{c} &= \frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + (r_L - r) - \bar{\theta}\left(\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + (r_L+1)\right) \\
& \text{and} \\
\tilde{c} &= \frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + (r_L - r) - \tilde{\theta}\left(\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + (r_L+1)\right)
\end{aligned}$$

Hence (38) can be rewritten as follows

$$\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{L_i}\partial H_i} = \frac{1}{\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + (r_L+1)}\left(\mathrm{E}[U''(\tilde{\Pi}_i)\tilde{c}]\bar{c} - \mathrm{E}[U''(\tilde{\Pi}_i)\tilde{c}^2]\right).$$

In this latter equation it is obvious that $\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}}+(r_L+1) > 0$ – see the arguments in the proof of the symmetric equilibrium from proposition 1 above. Furthermore,$\mathrm{E}[U''(\tilde{\Pi}_i)\tilde{c}^2] < 0$ since $U''(\cdot) < 0$ and $\tilde{c}^2 > 0$.

Therefore, in order to determine the sign of (38) the sign of $\mathrm{E}[U''(\tilde{\Pi}_i)\tilde{c}]\bar{c}$ has to be figured out: applying the covariance formula to the first order necessary conditions (21) yields

$$\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + (r_L - r) - \bar{\theta}\left(\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + (r_L + 1)\right) = \\ = \left(\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + (r_L + 1)\right)\frac{\mathrm{Cov}(U'(\tilde{\Pi}_i), \tilde{\theta})}{\mathrm{E}[U'(\tilde{\Pi}_i)]}.$$

Since the first line of this latter equation is equivalent to $\bar{c}$, $\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + (r_L + 1) > 0$ and $\mathrm{E}[U'(\cdot)] > 0$ by assumption the sign of $\bar{c}$ only depends on the sign of $\mathrm{Cov}(U'(\cdot), \tilde{\theta})$. In this regard inspection of the profit function (4) shows

$$\mathrm{Cov}(U'(\tilde{\Pi}_i), \tilde{\theta}) \overset{>}{\underset{<}{=}} 0 \;\Leftrightarrow\; (r_L + 1)L(r_L) - H_i \overset{>}{\underset{<}{=}} 0$$

and as a result

$$\bar{c} \overset{>}{\underset{<}{=}} 0 \;\Leftrightarrow\; (r_L + 1)L(r_L) - H_i \overset{>}{\underset{<}{=}} 0.$$

Unfortunately without additional assumption regarding the utility function no further results can be derived. However, it is acknowledged that increasing absolute risk aversion in the Arrow-Pratt sense (IARA) is not very reasonable for real life situations. Therefore, in the following we restrict attention to constant (CARA) and decreasing (DARA) absolute risk aversion in the sense of Arrow and Pratt.

Consider at first CARA, i.e. $-\frac{U''(\tilde{\Pi}_i)}{U'(\tilde{\Pi}_i)} = const. \; \forall \; \tilde{\Pi}_i$. Assuming CARA one can write

$$\mathrm{E}\left[U''(\tilde{\Pi}_i)\tilde{c}\right] = \frac{U''(\tilde{\Pi}_i)}{U'(\tilde{\Pi}_i)}\mathrm{E}\left[U'(\tilde{\Pi}_i)\tilde{c}\right] = 0$$

where the second equality follows from the fact that for the first order necessary conditions (21) in the equilibrium it must be true that $\mathrm{E}[U'(\cdot)\tilde{c}] = 0$.

Hence if utility function exhibits CARA it must be unambiguously true that

$$\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{L_i} \partial H_i} > 0.$$

In case of decreasing absolute risk aversion in the sense of Arrow-Pratt (DARA) the following relation must hold (cf. Ross, 1981, p. 623)

$$\frac{d}{d\tilde{\Pi}_i}\left(-\frac{U''(\tilde{\Pi}_i)}{U'(\tilde{\Pi}_i)}\right) = -\frac{U'''(\tilde{\Pi}_i)}{U'(\tilde{\Pi}_i)}\left(\frac{U'''(\tilde{\Pi}_i)}{U''(\tilde{\Pi}_i)} - \frac{U''(\tilde{\Pi}_i)}{U'(\tilde{\Pi}_i)}\right) < 0$$

$$\Leftrightarrow \quad U'''(\tilde{\Pi}_i) > \frac{(U''(\tilde{\Pi}_i))^2}{U'(\tilde{\Pi}_i)}$$

The following arguments are adopted from a proof of (Wong, 1997, p. 208f.):

First define a function $N(\tilde{\theta}) = \frac{U''(\tilde{\Pi}_i)}{U'(\tilde{\Pi}_i)}$ and then differentiate $N(\tilde{\theta})$ with respect to $\tilde{\theta}$ to determine behavior of $N(\tilde{\theta})$ when $\tilde{\theta}$ changes:

$$N'(\tilde{\theta}) = -\frac{(r_L+1)L(r_L) - H_i}{U'(\tilde{\Pi}_i)}\left[U'''(\tilde{\Pi}_i) - \frac{(U''(\tilde{\Pi}_i))^2}{U'(\tilde{\Pi}_i)}\right]$$

Due to DARA the following relationships hold:

$$N'(\tilde{\theta}) \overset{<}{\underset{>}{=}} 0 \;\Leftrightarrow\; (r_L+1)L(r_L) - H_i \overset{>}{\underset{<}{=}} 0$$

Now define $\hat{\theta}$ to be the realization of $\tilde{\theta}$ for which the following relation holds

$$c(\hat{\theta}) = \frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + (r_L - r) - \hat{\theta}\left(\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + (r_L+1)\right) = 0.$$

Thus for any $\tilde{\theta} > \hat{\theta}$ it follows that $c(\tilde{\theta}) < c(\hat{\theta})$ and $c(\tilde{\theta}) < 0$. Hence it must be true that

$$N(\tilde{\theta})c(\tilde{\theta}) \overset{>}{\underset{<}{=}} N(\hat{\theta})c(\tilde{\theta}) \;\Leftrightarrow\; (r_L+1)L(r_L) - H_i \overset{>}{\underset{<}{=}} 0.$$

Similarly for any $\tilde{\theta} < \hat{\theta}$ it follows that $c(\tilde{\theta}) > c(\hat{\theta})$ and $c(\tilde{\theta}) > 0$. As a result the following relations hold:

$$N(\tilde{\theta})c(\tilde{\theta}) \overset{>}{\underset{<}{=}} N(\hat{\theta})c(\tilde{\theta}) \;\Leftrightarrow\; (r_L+1)L(r_L) - H_i \overset{>}{\underset{<}{=}} 0.$$

From these arguments one can observe equivalent relations to hold regardless whether $\tilde{\theta} > \hat{\theta}$ or $\tilde{\theta} < \hat{\theta}$ Therefore multiplying both sides of the above relations by $U'(\tilde{\Pi}_i) > 0$ and taking expectations with respect to $\tilde{\theta}$ yields

$$\mathrm{E}\left[U''(\tilde{\Pi}_i)\tilde{c}\right] \;\begin{matrix} > \\ = \\ < \end{matrix}\; N(\hat{\theta})\mathrm{E}\left[U'(\tilde{\Pi}_i)\tilde{c}\right] = 0 \;\Leftrightarrow\; (r_L+1)L(r_L) - H_i \;\begin{matrix} > \\ = \\ < \end{matrix}\; 0$$

where the equality follows from the first order necessary conditions (21) which state that $\mathrm{E}[U'(\tilde{\Pi})\tilde{c}] = 0$.

As a result it is obvious that $\bar{c}$ and $\mathrm{E}[U'(\tilde{\Pi}_i)]$ have the same sign in any case and hence

$$\bar{c}\mathrm{E}\left[U'(\tilde{\Pi}_i)\tilde{c}\right] \geq 0.$$

Thus as long as utility function exhibits CARA or DARA it must be true that

$$\frac{\partial^2 \mathrm{E}[U(\tilde{\Pi}_i)]}{\partial r_{L_i} \partial H_i} > 0.$$

As a consequence one derives that $\det J_{L_i} > 0$ and $\det J_{L_j} > 0$. Furthermore from arguments regarding stability of the equilibrium it is already known that $\det J_L > 0$ must hold true. As a result the strategic effect of hedging on the optimal loan rate may be represented as

$$\frac{dr_{L_i}(H_i, H_j)}{dH_i} > 0 \tag{39}$$

$$\frac{dr_{L_j}(H_i, H_j)}{dH_i} > 0 \tag{40}$$

$$i, j = A, B \;;\; i \neq j$$

□

Proof of Proposition 5

To proceed the proof we first determine optimal hedging levels and second derive optimal deposit and loan rates.

For determining optimal hedging levels consider the corresponding first order necessary conditions (25). The third line of (25) can be transformed applying the covariance formula and using $\mathrm{E}(\tilde{\theta}) = \bar{\theta}$ to yield

$$\mathrm{E}\left[U'(\tilde{\Pi}_i)\left(\tilde{\theta} - \bar{\theta}\right)\right] = \mathrm{Cov}[U'(\tilde{\Pi}_i), \tilde{\theta}].$$

The sign of the first line of (25) only depends on the sign of $\frac{dr_{D_j}(H_i,H_j)}{dH_i}$. The reason is that due to the assumptions on the deposit supply function $\frac{\partial \hat{D}_i(r_{D_i},r_{D_j})}{\partial r_{D_j}} < 0$ and from the first order necessary conditions (20) it follows that

$$r - r_{D_i} = \frac{\hat{D}_i(r_{D_i}, r_{D_j})}{\frac{\partial \hat{D}_i(r_{D_i},r_{D_j})}{\partial r_{D_i}}} \geq 0.$$

Furthermore the sign of the second line of (25) only depends on the sign of $\frac{dr_{L_j}(H_i,H_j)}{dH_i}$ since due to assumptions on the loan demand function $\frac{\partial \hat{L}_i(r_{L_i},r_{L_j})}{\partial r_{L_j}} > 0$ and from the first order necessary conditions (21) it follows that

$$\mathrm{E}\left[U'(\tilde{\Pi}_i)\left((1-\tilde{\theta})r_{L_i} - r - \tilde{\theta}\right)\right] = -\frac{\mathrm{E}[U'(\tilde{\Pi}_i)(1-\tilde{\theta})]\hat{L}_i(r_{L_i}, r_{L_j})}{\frac{\partial \hat{L}_i(r_{L_i},r_{L_j})}{\partial r_{L_i}}} \geq 0.$$

In this regard one has to note that it must be true that $\mathrm{E}[U'(\tilde{\Pi}_i)(1-\tilde{\theta})] \geq 0$ since both random variables – while negatively correlated – are non-negative (marginal utility is strictly positive). Hence the expected value of the product of both random variables must be non-negative, too.

Moreover, in the proofs of propositions 3 and 4 it was shown that in the equilibrium the following relations hold:

$$\frac{dr_{D_j}(H_i, H_j)}{dH_i} = 0$$

and

$$\frac{dr_{L_j}(H_i, H_j)}{dH_i} > 0.$$

Therefore from the first order necessary conditions (25) it follows that in equilibrium

$$\mathrm{Cov}\left(U'(\tilde{\Pi}_i), \tilde{\theta}\right) = -\mathrm{E}\left[U'(\tilde{\Pi}_i)\left((1-\tilde{\theta})r_L - r - \tilde{\theta}\right)\tfrac{1}{2}\frac{dL(r_L)}{dr_L}\right]\frac{dr_{L_j}(H_i, H_j)}{dH_i} < 0. \tag{41}$$

[21] Due to risk aversion and examination of profit function (4) reveals that this is the case if and only if

$$(r_L + 1)L(r_L) - H_i < 0. \tag{42}$$

That is it is optimal for both banks to over hedge their risky positions. Furthermore, since both banks choose the same deposit and loan rates in the equilibrium the

[21] In this latter equation we have used the fact, that just like in the situations of section 3.1 and 3.2 both banks set the same deposit and loan rates in the equilibrium. That is, the arguments presented in the proof of the symmetric equilibrium of proposition 1 can be applied in the current situation as well.

optimal hedging levels have to be the same as well for both banks, i.e. $H_A^* = H_B^* \equiv H$.

Using this result one can now determine optimal deposit and loan rates:

Since hedging as well as risk does not affect the deposit side of the banks' businesses the optimal deposit rate remains the same compared to the cases of sections 3.1 and 3.2 and hence is the same like in a situation without credit risk.

However loan rates do change as follows: from the first order necessary conditions (41) in the equilibrium the following relation must hold:

$$\mathrm{E}\left[U'(\tilde{\Pi}_i)\tilde{\theta}\right] = \mathrm{E}\left[U'(\tilde{\Pi}_i)\right]\bar{\theta} - \mathrm{E}\left[U'(\tilde{\Pi}_i)\left((1-\tilde{\theta})r_L - r - \tilde{\theta}\right)\tfrac{1}{2}\frac{dL(r_L)}{dr_L}\right]\frac{dr_{D_j}(H_i,H_j)}{dH_i}.$$

Applying this equation to rewrite the first order necessary conditions for the optimal loan rate (21) yields

$$(1-\bar{\theta})\left(\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + r_L\right) - (r+\bar{\theta}) =$$

$$= -\left(\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + r_L + 1\right)\mathrm{E}\left[U'(\tilde{\Pi}_i)\left((1-\tilde{\theta})r_L - r - \tilde{\theta}\right)\tfrac{1}{2}\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}}\right]\frac{dr_{D_j}(H_i,H_j)}{dH_i} > 0$$

where the inequality follows due to $\frac{L(r_L)}{\frac{dL(r_L)}{dr_L}} + r_L + 1 > 0$.[22] Furthermore, it was shown earlier in this section that $\mathrm{E}\left[U'(\tilde{\Pi}_i)\left((1-\tilde{\theta})r_L - r - \tilde{\theta}\right)\tfrac{1}{2}L(r_L)\right]\frac{dr_{D_j}(H_i,H_j)}{dH_i} > 0$ and by assumption $\frac{dL(r_L)}{r_L} < 0$.

As a result the optimal loan rate is still higher than the one of the riskless case – i.e. $r_L > r_L^c$. $\square$

[22]See the proof of the symmetric equilibrium in proposition 1 above.

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