Transmission of Policy Shocks in a Monetary Asset-Pricing Model

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Abstract

In a framework of a two-country monetary asset-pricing model with production the effects of stochastic and structural fiscal and monetary policy shocks are investigated. The model is kept simple enough to allow the derivation of closed form solutions of the functional equation system for the equilibrium price functions. With money yielding liquidity services in the exchange process some correlation results are derived, especially for the impact of structural and stochastic policy shocks on stock prices, exchange rates etc. Furthermore it is investigated whether shares can provide protection against inflation resulting from monetary shocks.

1 Introduction

International asset markets have grown in size and importance during the last decades and there has been a expansion in the variety of traded assets. It is of increased interest in assessing the efficiency and examining the behavior of asset prices denominated in alternative currencies. In this paper we study the influence of government policy, i.e. fiscal and monetary policy, on such assets. The main questions posed concern the transmission effects of policy shocks, the consequences of government policy for exchange rates and the transmission of inflation to other countries.

For this purpose, we describe a two-country world with cash-in-advance constraints in which purchases of goods must be made with producer's currencies. We combine elements from research on general equilibrium asset pricing by Lucas (1978), and Lucas/Stokey (1987), with investment and production by Brock (1982), and Stockman/Svensson (1987), and asset pricing in monetary economics by Svensson (1985), and Danthine/Donaldson (1986). The setup delivers an integration of money with real general equilibrium theory, where the use of money is motivated by a Clower-type (1967) cash-in-advance constraint. The cash-in-advance constraint is in a sense similar to the direct transactions cost approach to rationalize that agents hold diversified portfolios. When transaction costs are expressed as a function of (real) money balances relative to consumption, then the cash-in-advance constraint can be considered as a limiting case according to Feenstra (1986). However, we do not follow Lucas (1978), Giovannini (1989) or Eckwert (1990) in simply assuming or securing that the cash-in-advance constraint (the liqidity constraint) is always binding, rather we precisely state conditions with respect to the monetary policy variable when this occurs. The simplicity of our model enables us to calculate explicit equilibrium price functions based on utility maximizing agents and profit maximizing firms.

In this paper we introduce a monetary cash-in-advance model with investment, international capital flows, and governments in both countries. We consider a monetary economy in discrete time with infinitely living consumers maximizing the expected utility of a consumption stream, where the expectation essentially is over realizations of output, monetary and fiscal policy shocks. The model is a variant of the monetary two-country asset pricing model of Lucas (1982) combined with the market opening structure of Svensson (1985b). Into that model the governments of both countries are integrated as decision units of monetary and fiscal policy measures (compare Schittko and Müller (1999) for a model of a closed economy). Furthermore we include an endogeneous production process modelled like in Stockman and Svensson (1987), retaining basically their notation. We employ a crucial simplification with respect to the information content of shocks, namely we assume them to be serially uncorrelated.

The paper is organized as follows. In section 2 we formulate our model. The decision problems of the representative consumers of both countries are described and we present the optimal investment decisions of the producers which is followed by the budget restrictions of the governments of both countries. In section 3 we describe the equilibrium of our model, beginning with the market equilibrium conditions, followed by the equilibrium price functions and the explicit equilibrium solution. Section 4 deals with the comparative statics of fiscal and monetary policy. In this section we investigate the effects of both stochastic and structural policy shocks. Finally we give some concluding remarks (section 5). At the end of the paper we include an appendix, which supplies one technical proof.

2 The Model

Our world economy consists of two countries, the home and foreign country. The two countries are completely specialized producing their own goods. These goods are aggregated to country-specific tradable goods, which can be demanded from the consumption and government sectors. Both goods are non-storable so that the production quantities have to be used up in the production period. The output of the domestic good in period t, x_t, depends on an endogenously determined domestic capital stock k_t and an exogenously given random disturbance term ε_t,

$$x_t = x(k_t, \varepsilon_t) \ . \tag{1}$$

The production function x is strictly concave and monotonically increasing in k, i.e. $x_k > 0$ and $x_{kk} < 0$. Output of the foreign good y_t is costlessly produced from a non-depreciating foreign capital stock. The foreign output variable y_t follows a stochastic process and is assumed to be given exogenously.

Only the foreign good can be used for domestic investment. Investment transforms foreign goods at time t into domestic capital in the following period k_{t+1}, so $y_t - k_{t+1}$ equals total world consumption of the foreign good in equilibrium at time t. Furthermore we assume that the domestic capital stock (in the form of foreign goods) is used up completely in producing the domestic good. We choose this formulation like in Stockman und Svensson (1987) because otherwise no explicit investment function can be derived.

We have two governments and two national currencies. The governments conduct a stochastic monetary policy which is described by

$$M_{t+1} = \omega_t M_t \quad , M_0 > 0 \tag{2}$$

and

$$N_{t+1} = \omega_t^* N_t \quad , N_0 > 0 \ . \tag{3}$$

M_t, respectively N_t, denote the home respectively the foreign quantity of money (=supply of money) at the beginning of time t and ω_t, respectively ω_t^*, denote the gross rates of monetary expansion for the corresponding currencies.

Let P_t and P_t^* denote the own-currency prices of domestic and foreign goods. The flexible exchange rate f_t gives the price of one unit of foreign currency in terms of domestic money. The relative price of the domestic good in terms of the foreign good is denoted by p_t and is calculated by $p_t = P_t/(f_t P_t^*)$. The purchasing power of domestic money in terms of foreign goods is $\pi_{Mt} = (f_t P_t^*)^{-1}$ and the purchasing power of foreign money in terms of foreign goods is equal to $\pi_{Nt} = 1/P_t^*$. All real prices p_t, π_{Mt} and π_{Nt} therefore are measured in units of the foreign good. The real price of the foreign consumption good in all transaction periods is therefore equal to one.

Both countries have equally sized constant populations, so that the consumption sector of each country can be modelled by a representative individual. These two consumers are risk-averse and identically with respect to their preferences, which can be represented by

$$E\left\{\sum_{\tau=t}^{\infty}\beta^{\tau-t}U(x_\tau^d, y_\tau^d)\right\} \quad , \; 0<\beta<1 \tag{4}$$

The variable x_τ^d denotes private consumption of home goods in period τ, y_τ^d consumption of foreign goods, $\beta \in (0,1)$ is a constant discount factor, and E stands for the expectations operator, conditional upon the available information at the arbitrary starting time t. We assume the time-invariant period utility function $U(\cdot,\cdot)$ is strictly monotone, increasing in both arguments, strictly concave, bounded and twice continuously differentiable. Furthermore U is additive separable, i.e. $U_{xy} = 0 = U_{yx}$, and the Inada conditions hold. Because of the identity of the representative consumers the utility function U and the discount rate β in both countries are identical.

Uncertainty in our model results from the specification of exogenously given stationary stochastic processes for the goods production in the foreign country y_t, for the random term of the endogenous domestic production ε_t, as for government consumption and the money supply process in both countries, g_t, g_t^*, ω_t and ω_t^*. The vector $s_t := (y_t, \varepsilon_t, \omega_t, \omega_t^*, g_t, g_t^*)$ denotes the state of the economy at the beginning of the transaction period t. We assume, that s_t is identically and independently distributed and denote the distribution function by F.

The output quantities x_t and y_t are produced by an aggregate firm in the corresponding country and are sold on the goods market. The shares of these firms are held privately and the total stock of home and foreign shares is normed to one. Let α_{1t} (α_{1t}^*) be the domestic (foreign) household's share of the domestic firm at the beginning of the transactions period t and α_{2t} (α_{2t}^*) his share of the foreign firm.

The ownership of shares of the home firm on the one hand involves an obligation to buy investment goods for domestic goods production in the following period, and on the other hand the right to obtain part of the dividend payments. To hold shares α_{1t} means for the domestic consumer that in period t he has to supply investment goods $\alpha_{1t}k_{t+1}$ to the domestic firm. In equilibrium ($\alpha_{1t} + \alpha_{1t}^* = 1$) the consumption sector finances the total investment k_{t+1}, and the home firm can distribute the real revenue from sale $p_t x_t =: \delta_{1t}$ to the households corresponding to their shares as dividends $\alpha_{1t}\delta_{1t}$ and $\alpha_{1t}^*\delta_{1t}$ without having to build up the capital stock of the following period. Production of the foreign good is exogenously given, the total sales revenue $y_t =: \delta_{2t}$ in equilibrium is distributed to the consumers according to their shares.

The governments of both countries finance their government consumption by money creation and an endogenously determined lump-sum tax. Although only national consumers are taxed, risk-averse consumers have an incentive to diversify the risk of varying tax payments, so that trade with tax liabilities results. For this purpose the governments emit tax shares which are traded on the asset market like the shares of the firms. This assumption can be interpreted as a form of source-based taxation of revenues. The total stock of the tax shares is normed to one. Let α_{3t} (α^*_{3t}) be the share of the home (foreign) consumer of the domestic tax liabilities in period t and α_{4t} (α^*_{4t}) his share of the foreign tax liabilities. The proportionate tax payments of the households to the home government can be interpreted as dividends which the consumption sector receives for his share holdings in the form of tax liabilities at time t. The home consumer therefore gets a dividend from the home government in the amount of $\alpha_{3t}\delta_{3t} = -\alpha_{3t}T_t$. Equivalently T^*_t can be interpreted as a negative dividend payment δ_{4t} of the foreign government to the consumers.

Money holdings in our model are motivated by cash-in-advance constraints, which give rise to finance constraints for the consumers of both countries in buying goods. The domestic household in period t begins with money holdings m_t in home currency and n_t in foreign currency and share holdings α_t. The consumers observe the current state $s_t = (y_t, \varepsilon_t, \omega_t, \omega^*_t, g_t, g^*_t)$, i.e. they receive complete information on the output of goods in the foreign country and on the random term for the domestic production in the current period as well as on government consumptions and money growth in both countries. Additional to that they know the quantities of money supply M_t and N_t and they know the capital stock k_t which is build up by investment in the previous period $t-1$. Subsequently the goods markets open, where home and foreign goods are exchanged for money. Home goods have to be payed by home currency and foreign goods by foreign currency. The home household can aquire goods according to his real finance constraints

$$\pi_{Mt}m_t \geq p_t x^d_t \tag{5}$$

and

$$\pi_{Nt}n_t \geq y^d_t + \alpha_{1t}k_{t+1} \tag{6}$$

The purchase of foreign goods is not only for private consumption y^d_t, but also for investment, which the consumer according to his share holding α_{1t} has to supply to the domestic firm. After the close of the goods markets the asset markets open, where home and foreign shares can be traded against money and currencies can be exchanged at the exchange rate f_t. By that the households aquire the desired assets to start with at the beginning of next period. At the same time sales revenues are payed out according to the share holdings as dividends to the consumers and the governments collect the tax liabilities. The home consumer maximizes his objective function (4) subject to the cash-in-advance constraints (5) and (6) as well as his real budget constraint measured in foreign goods

$$\begin{aligned} p_t x^d_t + y^d_t + \alpha_{1t}k_{t+1} \quad &+ \quad \pi_{Mt}m_{t+1} + \pi_{Nt}n_{t+1} + q_t\alpha_{t+1} + \alpha_{3t}T_t + \alpha_{4t}T^*_t \\ &= \quad \pi_{Mt}m_t + \pi_{Nt}n_t + q_t\alpha_t + \alpha_{1t}p_t x_t + \alpha_{2t}y_t \ , \end{aligned} \tag{7}$$

where

$$q_t\alpha_t = \sum_{i=1}^{4} q_{it}\alpha_{it} \quad \text{und} \quad q_t\alpha_{t+1} = \sum_{i=1}^{4} q_{it}\alpha_{i,t+1}$$

holds. On the expenditure side we have the purchases of goods, the tax payments as well as the value of the end-of-period demands for money and securities. On the revenue side we have the dividends paid out by the firms, and the value of the initial holdings of shares and money at the beginning of the transactions period t. The foreign household solves a comparable problem, which is distinguished notationally by an *. Prices and dividends are formed on the same markets and will be identical to the above mentioned entities.

Because of the time specification we have in our model a precautionary motive to hold money. The money needed to buy goods has to be demanded in the previous period before the relevant state s_t to buy the goods is known. After receiving all relevant information economic agents have no possibility to restructure the money holdings in their portfolio for the purchase of goods. Because of the described market opening structure money has a liquidity advantage compared to the other assets. These liquidity services of money result, because shares can be transformed into money only after the close of the goods markets. Furthermore dividends can be used for the purchase of goods at the earliest in the next period.

The task is now to determine the equilibrium allocations and the real prices of the shares, goods and monies. Of all possible intertemporal equilibrium solutions we restrict ourselves to stationary ones, where the endogenously determined prices are independent of the regarded time instant.

Furthermore we assume that the agents have rational expectations with respect to market clearing prices.

Proceeding as usual in this context gives the first order conditions for the representative household in the home country, where non-primed variables refer to the current period and the primed variables to the future period:

$$U_x(x^d, y^d) = (\lambda + \mu)p \ , \tag{8}$$

$$U_y(x^d, y^d) = \lambda + \nu \ , \tag{9}$$

$$\beta \int (\lambda' + \mu')\pi'_M \ dF(s') = \lambda \pi_M \ , \tag{10}$$

$$\beta \int (\lambda' + \nu')\pi'_N \ dF(s') = \lambda \pi_N \ , \tag{11}$$

$$\beta \int \Big[\ \lambda'(q'_1 + p'x') - (\lambda' + \nu')k'' \ \Big] \ dF(s') = \lambda q_1 \ , \tag{12}$$

$$\beta \int \lambda'(q'_i + \delta'_i) \ dF(s') = \lambda q_i \ \ , \ i = 2, 3, 4 \ , \tag{13}$$

$$px^d + y^d + \alpha_1 k' + \pi_M m' + \pi_N n' + q\alpha' = \pi_M m + \pi_N n + (q + \delta)\alpha \ , \tag{14}$$

$$\begin{array}{rcl} px^d & = & \pi_M m \ , \ \mu \geq 0 \ , \\ px^d & < & \pi_M m \ , \ \mu = 0 \ , \end{array} \tag{15}$$

$$
\begin{aligned}
y^d + \alpha_1 k' &= \pi_N n \ , \ \nu \geq 0 \ , \\
y^d + \alpha_1 k' &< \pi_N n \ , \ \nu = 0 \ ,
\end{aligned} \tag{16}
$$

By solving the comparable maximization problem of the foreign consumer we get similar conditions with same prices, while shares and quantities are distinguished notionally by an *.

We turn now to the optimization problem of the domestic firm. The value of the domestic firm is the value of the outstanding stock of asset 1, i.e. q_1. Owners of the firm, the shareholders, make the investment decision to maximize the value of the firm. According to (12) we get with forward iteration

$$
q_{1t} = \frac{1}{\lambda_t} E \left\{ \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \left[\lambda_\tau p_\tau x(k_\tau, \varepsilon_\tau) - (\lambda_\tau + \nu_\tau) k_{\tau+1} \right] \right\} \ . \tag{17}
$$

We assume that firms act als price takers, so each firm treats $(\lambda_\tau, \nu_\tau, p_\tau)$ as given for all $\tau = t+1, t+2, \ldots$ (compare Stockman and Svensson (1987)). k_{t+1} denotes the capital stock in period $t+1$, which is determined by the purchase of investment goods on the goods market at time t.

At the opening of the asset market in t, where the share prices are determined, k_{t+1} is already determined. Therefore the value of the firm is maximized by the optimal choice of the next possible investment k_{t+2}. Choosing the investment to maximize the value of a share, then the following equation holds:

$$
\beta \int \lambda'' p'' x_{k''}(k'', \varepsilon'') \ dF(s'') = U_y(x^{d\prime}, y^{d\prime}) \ . \tag{18}
$$

The right side of (18) shows the marginal utility of the foreign consumption good. The variable λ stands for the marginal utility of wealth (non-money wealth) and x_k denotes the marginal productivity of capital. The left side describes the discounted expected utility of future higher dividend payments. (18) therefore says that in a producer optimum the marginal utility of foreign goods consumption has to be equal to the loss in utility stemming from the purchase of one additional unit of goods for investment purposes. This diminished utility is measured by the expected higher dividends, which are implied by the increased capital input.

Al last we turn to the the governments of both countries, which conduct a monetary policy according to (2) and (3). Government consumption extends excluxively to home produced goods. To finance the stochastic expenditures g_t the home government uses newly created money $(\omega_t - 1)M_t$ and an endogenously determined real lum-sum tax T_t. The government's budget constraint measured in foreign goods therefore is

$$
p_t g_t = T_t + \pi_{Mt}(\omega_t - 1) M_t \ . \tag{19}
$$

In analogy the foreign government has to fulfill the following restriction:

$$
g_t^* = T_t^* + \pi_{Nt}(\omega_t^* - 1) N_t \ . \tag{20}
$$

3 The equilibrium

We begin this section with the market clearing conditions, which are then inserted into the first order conditions. The individual decisions are compatible with balanced goods, money and share markets, if we have:

$$x^d + x^{d*} + g = x \ , \tag{21}$$

$$y^d + y^{d*} + k' + g^* = y \ , \tag{22}$$

$$m' + m^{*\prime} = M' = \omega M \ , \tag{23}$$

$$n' + n^{*\prime} = N' = \omega^* N \ , \tag{24}$$

$$\alpha' + \alpha^{*\prime} = \ (1, 1, 1, 1) \ . \tag{25}$$

Definition 1 *A stationary rational expectations equilibrium for our model consists of a set of functions V, λ, μ, ν, m', n', α', x^d, y^d, $m^{*\prime}$, $n^{*\prime}$, $\alpha^{*\prime}$, x^{d*}, y^{d*}, an investment function k', tax functions T und T^* as well as price functions q, π_M, π_N and p such that (8)–(16), the corresponding functions for the foreign land, (18)–(20) and (21)–(25) are fulfilled for all s. All these functions are dependent on (k, s).*

Let us define $\bar{m} := \pi_M M$ and $\bar{n} := \pi_N N$; $\bar{m}$ and $\bar{n}$ describe the home and foreign real quantities of money measured in foreign goods. Furthermore we define $u_x(x) := U_x(x/2, y)$ and $u_y(y) := U_y(x, y/2)$. This abreviating notation is possible as the utility function U is additiv separable by assumption. U_x therefore is only dependent on x. At the same time U_y depends only on y.

Let us denote the investment function by $K(k, s) := k'(k, s)$. The investment function describes the formation of the domestic capital stock in the subsequent period in dependence on (k, s).

If we insert these definitions and the market clearing conditions (21)–(25) into the first order conditions then we end up with the following equation system (26)–(37), which describes a perfectly pooled stationary equilibrium in the sense of Lucas (1982):

$$x^d = \frac{x(k,\varepsilon) - g}{2} = x^{d*} \ , \ y^d = \frac{y - K(k,s) - g^*}{2} = y^{d*} \ , \tag{26}$$

$$m' = m^{*\prime} = \frac{\omega M}{2} \ , \ n' = n^{*\prime} = \frac{\omega^* N}{2} \ , \tag{27}$$

$$\alpha' = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) = \alpha^{*\prime} \ , \tag{28}$$

$$u_x(x(k,\varepsilon) - g) = [\lambda(k,s) + \mu(k,s)]p(k,s) \ , \tag{29}$$

$$u_y(y - K(k,s) - g^*) = \lambda(k,s) + \nu(k,s) \ , \tag{30}$$

$$\frac{\beta}{\omega} \int [\lambda(K(k,s),s') + \mu(K(k,s),s')]\bar{m}(K(k,s),s') \ dF(s') = \lambda(k,s)\bar{m}(k,s) \ , \tag{31}$$

$$\frac{\beta}{\omega^*}\int[\lambda(K(k,s),s')+\nu(K(k,s),s')]\bar{n}(K(k,s),s')\ dF(s')=\lambda(k,s)\bar{n}(k,s)\ , \tag{32}$$

$$\begin{aligned}\lambda(k,s)q_1(k,s) &= \beta\int\Big\{\ \lambda(K,s')[q_1(K,s')+p(K,s')x(K,\varepsilon')] \\ &\qquad -[\lambda(K,s')+\nu(K,s')]*K(K,s')\ \Big\}\ dF(s')\ ,\end{aligned} \tag{33}$$

$$\lambda(k,s)q_i(k,s)=\beta\int\lambda(K,s')\ [\ q_i(K,s')+\delta_i(K,s')\ ]\ dF(s')\ ,\ i=2,3,4\ , \tag{34}$$

$$\begin{aligned}p(k,s)[x(k,\varepsilon)-g] &= \bar{m}(k,s)\ ,\ \mu(k,s)\geq 0\ , \\ p(k,s)[x(k,\varepsilon)-g] &< \bar{m}(k,s)\ ,\ \mu(k,s)=0\ ,\end{aligned} \tag{35}$$

$$\begin{aligned}y-g^* &= \bar{n}(k,s)\ ,\ \nu(k,s)\geq 0\ , \\ y-g^* &< \bar{n}(k,s)\ ,\ \nu(k,s)=0\ ,\end{aligned} \tag{36}$$

$$\beta\int\lambda(K,s')p(K,s')x_k(K,\varepsilon')\ dF(s')=u_y(y-K-g^*)\ . \tag{37}$$

With risk-averse consumers having identical preferences this pooled equilibrium is unique. The identical utility functions imply that the individuals each demand half of the quantity supplied which is left after government consumption and investment demand are taken into account. Furthermore money holdings of each consumer of both currencies are each half of the respective money supply. As already mentioned risk-averse agents have an incentive to diversify the risk of varying tax liabilities in both countries by trade with tax liability shares. The identical consumers hold each half of the tax liability shares of both countries in their portfolio to minimize the risk to be taxed. Furthermore in equilibrium each individual holds half of the shares of the firms (which are normed to one) to minimize the risk of the endogenously determined dividend payments in the home country and to minimize the risk of the stochastic dividend payments in the foreign country. With the interpretation as source-based taxation it is straightforward that the identical consumers hold half of the tax liabilities in each country since they get the same revenues due to their firm share holdings. These properties of our pooled equilbrium are described by the equations (26)–(28). The optimal decisions of both consumers on the goods and financial markets are identical.

The equilibrium equations (29)–(37) are directly derived from the necessary conditions of home and foreign households.

Equation (29) tells us, that the marginal utility of domestic good consumption u_x is equal to marginal utility of wealth plus the multiplier μ. A binding cash-in-advance constraint ($\mu \geq 0$ in (35)), i.e. where the money holding of home currency is completely used up in buying home goods, prevents that in equilibrium there is an equivalence between the marginal utility of consumption and the marginal utility of wealth. Money possesses a liquidity advantage compared with dividend paying shares. These liquidity services of home money are measured by the Lagrangean multiplier μ. If we have a non-binding finance constraint ($\mu = 0$ in (35)), then marginal utility of wealth λ is equal to the marginal utility of consumption u_x. In the purse of the agents home money is left, which could be used for further consumption. Because of that, shares have no payment disadvantage. Equation (30) can be interpreted analogously.

The asset-pricing equation (34) shows that the discounted expected utility of future dividend payments in the consmumer optimum has to balance the loss in current utility which is caused by the purchase of one additional share of the foreign firm, the home or the foreign tax liability. Finally equation (33) for the price of a share of the home firm can be interpreted similarly to (34), but the expected utility of future dividend payments has to be reduced, because of the expected future obligations to invest, which come hand in hand with the purchase of one additional unit of home shares.

Now we use the equilibrium conditions to discuss the explicit solution in more detail. First of all starting with (37) the following lemma can be proofed.

Lemma 1 *The investment function $K(k,s)$ depends only on (y, g^*), i.e. $K(k,s) = K(y, g^*)$. Furthermore it holds: $K_y \in (0,1)$, $K_{g^*} \in (-1, 0)$ and $K_y = -K_{g^*}$.*

Proof: Compare Stockman and Svensson (1987).

Lemma 1 states that an increase of the foreign output y stimulates investment ($K_y > 0$), but the additional output is not totally used building up the future domestic capital stock ($K_y < 1$). Part of the increased y agents want to consume directly. The reaction to increased foreign government expenditures g^* is, that consumers react in case of an unchanged supply of goods with a reduction in consumption and investment demand ($-1 < K_{g^*} < 0$).

There exists a critical value $\tilde{\omega}^*$ of the growth rate of foreign money with the property, that a higher gross rate of growth than $\tilde{\omega}^*$ implies the equivalence of the real quantity of foreign money $\bar{n}$ and the goods supply net of the foreign goverment expenditures, $(y - g^*)$; whereas a lower growth of money involves a non-binding liquidity constraint. To calculate this $\tilde{\omega}^*$ we define

$$A^* := \beta \int u_y(y' - K(y', g^{*\prime}) - g^{*\prime}) \bar{n}(y', \omega^{*\prime}, g^{*\prime}) \, dF(s')$$

as the discounted expected marginal utility of the real quantity of foreign money and insert (30) into (32) to get

$$\tilde{\omega}^*(y, g^*) = \frac{A^*}{u_y(y - K(y, g^*) - g^*)(y - g^*)} \tag{38}$$

There also exists a critical value $\tilde{\omega}$ for the gross rate of growth of the home quantity of money, where the liquidity constraint is just fulfilled with equality. We define the function

$$A(y, g^*) := \beta \int \frac{u_x(x(K(y, g^*), \varepsilon') - g')}{p(K(y, g^*), s')} \, \bar{m}(K(y, g^*), s') \, dF(s')$$

as the discounted expected marginal utility of the real quantity of home money. Inserting (29) into (31) gives then

$$\tilde{\omega}(k, y, \varepsilon, g, g^*) = \frac{A(y, g^*)}{[x(k, \varepsilon) - g]u_x(x(k, \varepsilon) - g)} \tag{39}$$

If the home government increases the quantity of money with a lower rate than $\tilde{\omega}$, the liquidity constraint does not bind strictly. With a home rate of monetary expansion of $\omega \geq \tilde{\omega}$ the cash-in-advance constraint, however, will be fulfilled with equality.

With that the explicit solutions for the model can be calculated as in the following transparent form. The marginal utility of wealth is given by

$$\lambda(y, \omega^*, g^*) = \begin{cases} u_y(y - K(y, g^*) - g^*) & , \quad \omega^* < \tilde{\omega}^* \\ A^* / [\omega^*(y - g^*)] & , \quad \omega^* \geq \tilde{\omega}^* \end{cases} . \tag{40}$$

The real quantity of foreign in equilibrium can be described by

$$\bar{n}(y, \omega^*, g^*) = \begin{cases} A^* / [\omega^* u_y(y - K(y, g^*) - g^*)] & , \quad \omega^* < \tilde{\omega}^* \\ y - g^* & , \quad \omega^* \geq \tilde{\omega}^* \end{cases} , \tag{41}$$

and the liquidity services of foreign money are

$$\nu(y, \omega^*, g^*) = \begin{cases} 0 & , \quad \omega^* < \tilde{\omega}^* \\ u_y(y - K(y, g^*) - g^*) - \dfrac{A^*}{\omega^*(y - g^*)} & , \quad \omega^* \geq \tilde{\omega}^* \end{cases} . \tag{42}$$

For the real quantity of home money we have

$$\bar{m}(y, \omega, \omega^*, g^*) = \frac{A(y, g^*)}{\omega \lambda(y, \omega^*, g^*)} = \begin{cases} \dfrac{A(y, g^*)}{\omega u_y(y - K - g^*)} & , \ \omega^* < \tilde{\omega}^* \\ \\ \dfrac{A(y, g^*)\omega^*(y - g^*)}{\omega A^*} & , \ \omega^* \geq \tilde{\omega}^* \end{cases} . \tag{43}$$

For the real price of the home consumption good we obtain

$$p(k, s) = \begin{cases} u_x(x(k, \varepsilon) - g) / \lambda(y, \omega^*, g^*) & , \quad \omega < \tilde{\omega} \\ \bar{m}(y, \omega, \omega^*, g^*) / [x(k, \varepsilon) - g] & , \quad \omega \geq \tilde{\omega} \end{cases} , \tag{44}$$

and the liquidity services of holding home money are

$$\mu(k, s) = \begin{cases} 0 & , \quad \omega < \tilde{\omega} \\ \dfrac{u_x(x(k, \varepsilon) - g)}{p(k, s)} - \lambda(y, \omega^*, g^*) & , \quad \omega \geq \tilde{\omega} \end{cases} . \tag{45}$$

The purchasing power of home money measured in foreign goods can be calculated as $\pi_M = \bar{m}/M$,

$$\pi_M(M, y, \omega, \omega^*, g^*) = \begin{cases} A(y, g^*) / [\omega u_y(y - K - g^*)M] & , \quad \omega^* < \tilde{\omega}^* \\ A(y, g^*)\omega^*(y - g^*) / [\omega A^* M] & , \quad \omega^* \geq \tilde{\omega}^* \end{cases} , \tag{46}$$

and the purchasing power of the foreign money is equal to

$$\pi_N(N, y, \omega^*, g^*) = \frac{\bar{n}}{N} = \frac{1}{P^*}\begin{cases} A^* / [\omega^* u_y(y - K - g^*)N] & , \quad \omega^* < \tilde{\omega}^* \\ (y - g^*)/N & , \quad \omega^* \geq \tilde{\omega}^* \end{cases} . \tag{47}$$

Furthermore the explicit solutions for the four securities prices follow with (40)–(47) directly from (33) and (34) as

$$q_{1t}(y, \omega^*, g^*) = \frac{1}{\lambda_t} E\Big\{ \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \Big[\lambda_\tau p_\tau x_\tau - (\lambda_\tau + \nu_\tau) k_{\tau+1} \Big] \Big\} =: \frac{B_1(y, g^*)}{\lambda_t} , \tag{48}$$

$$q_2(y, \omega^*, g^*) = \frac{\beta}{\lambda(1-\beta)} \int \lambda' y' \, dF(s') =: \frac{B_2}{\lambda_t} , \tag{49}$$

$$q_{3t}(y, \omega^*, g^*) = \frac{1}{\lambda_t} E\Big\{ \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \Big[A \frac{\omega_\tau - 1}{\omega_\tau} - \lambda_\tau g_\tau p_\tau \Big] \Big\} =: \frac{B_3(y, g^*)}{\lambda_t} , \tag{50}$$

and

$$q_4(y, \omega^*, g^*) = \frac{\beta}{\lambda(1-\beta)} \int \Big[A^{*\prime} \, \frac{\omega^{*\prime} - 1}{\omega^{*\prime}} - \lambda' g^{*\prime} \Big] \, dF(s') =: \frac{B_4}{\lambda_t} . \tag{51}$$

Finally the spot market exchange rate is given by

$$f(M, N, y, \omega, \omega^*, g^*) = \frac{M \omega A^*}{N \omega^* A(y, g^*)} , \tag{52}$$

which is completely independent of the binding or non-binding liquidity constraints in the home and foreign country.

4 Monetary and fiscal policy shocks

In this section we investigate the impact of monetary and fiscal policy measures and the transmission effects on the equilibrium price functions. Changes in government consumption and changes in money growth can be caused by stochastic or by structural policy changes (compare Eckwert (1990)). A structural policy change means a permanent change of government consumption, respectively a permanent change of the growth rate of money by a non-stochastic amount. Stochastic shocks, which are restricted to the current transactions period, can be understood as stochastic changes of the exogenously given random variables.[1]

We restrict ourselves to the investigation of serially uncorrelated shocks. First we study how the equilibrium price functions depend on government consumption, given a certain realization of goods supply and of monetary expansion. Secondly we examine the dependence of the equilibrium solution on the money growth rates, given unchanged outputs and government consumption levels. From models of closed economies one knows, that with a positive or negative correlation of the shocks the equilibrium price

[1]For the distinction of temporary and permanent shocks also compare Karayalcin (1999).

functions can react completely different, which inherits to open economies. Even for closed economies to study correlated shocks one has to rely on numerical methods (compare Schittko (1992)).

As expectations are formed rationally, agents deriving optimal consumption and savings plans take into account the possibility of stochastic policy changes and of supply shocks. Temporary stochastic changes amount to anticipated changes of government consumption and of the money and goods supply processes. The resulting effects depend essentially on the fact, whether the financial constraints for one of the goods or for both goods will be binding or non-binding.

Stochastic policy changes

Changes in foreign government consumption g^* imply a shifting of the bounds $\tilde{\omega}$ and $\tilde{\omega}^*$ (see (38) and (39)); additional to that $\tilde{\omega}$ will be affected by the level of home government expenditures. An investigation of the effects of home and foreign fiscal policy on the goods and share prices therefore is only possible under certain restrictions.

To that end we define

$$\bar{x} := x(k,\varepsilon) - g \ , \ \bar{x}' := x(K(y,g^*),\varepsilon') - g' \ , \ \bar{y} := y - K(y,g^*) - g^* \ . \qquad (53)$$

The Arrow-Pratt-coefficient of relative risk aversion is given by $r_x = -\,[x u_{xx}(x)]/u_x(x)$.

Now we make the following technical assumption, to guarantee that the intrinsically variable bounds between binding and non-binding cash-in-advance constraints are independent of the realization of the state variables:

Assumption 1 *We assume:*

$$\begin{aligned} r_{\bar{x}} &= -\frac{\bar{x}u_{xx}(\bar{x})}{u_x(\bar{x})} = 1 \ , \\ r_{\bar{x}'} &= -\frac{\bar{x}'u_{xx}(\bar{x}')}{u_x(\bar{x}')} = 1 \quad \text{and} \\ r_{\bar{y}} &= -\frac{\bar{y}u_{yy}(\bar{y})}{u_y(\bar{y})} = \frac{1}{1-K_y(y,g^*)} + K(y,g^*)\frac{u_{yy}(\bar{y})}{u_y(\bar{y})} \ , \end{aligned}$$

where $\bar{x}$, $\bar{x}'$ and $\bar{y}$ are given by (53).

Our analysis below is made under Assumption 1, which, if seen for the whole aggregated economy, is rather natural as mentioned by Arrow (1971), who argues that the 'relative risk aversion must hover around 1'. Because of this assumption we have fixed liquidity bounds in the home and foreign country.

Lemma 2 *Suppose Assumption 1 is fulfilled. Then $\tilde{\omega}(k,y,\varepsilon,g,g^*)$ and $\tilde{\omega}^*(y,g^*)$ are constant functions.*

Proof: See Appendix

In the sequel of section 4 we suppose Assumption 1 to be fulfilled. By the resulting constant liquidity bounds $\tilde{\omega}$ and $\tilde{\omega}^*$ the equilibrium price functions under fiscal policy shocks remain in their original liquidity region.[2]

First we study the impact of an expansive fiscal policy in the foreign country on the goods and share prices. As in comparative statics the monetary expansion is treated as given, the foreign government finances the increased government expenditures exclusively by tax increases. The equilibrium price of home goods measured in foreign goods p is given by (44). Especially we look at the impact of an expansionary foreign expenditure policy on the marginal utility of the foreign consumption good $u_y(y - K(y, g^*) - g^*)$. According to Lemma 1 an increase of g^* leads to a reduction of consumption and of purchases of investment goods, i.e. $0 > K_{g^*} > -1$. The remaining quantity of goods for consumption purposes $(y - K(y, g^*) - g^*)$ diminishes, so that because of the strict monotonicity and concavity of the utility function U the marginal utility u_y is increasing. With this the relative price of the home consumption good is in any case negatively correlated with the stochastic foreign government consumption, independent of the state of binding of both cash-in-advance constraints. The reason for this lies in the reduction of the supply of foreign goods for the private sector. The unchanged quantity of home goods, measured in foreign goods, therefore has a lower value. For the equilibrium nominal price of the foreign good we have with (47) and $P^* = \pi_N^{-1}$ that increased government expenditures g^* lead to an increase of P^*, i.e. they are inflationary, because of the higher marginal utility u_y, respectively because of the smaller denominator $(y - g^*)$. However the nominal price of the home good $P = pP^*f$ is unaffected by a foreign fiscal policy change.

What is the effect of a short-term foreign fiscal policy shock on the real prices of the four assets? To arrive at an answer we look at the impact of g^* on the marginal utility of (non-monetary) wealth in (40). An increase of g^* increases the marginal utility of wealth (of securities) λ, as the crowding out of private consumption leads to inflation and therefore to an increase in the esteem of non-monetary wealth.

As the real price of a share of the foreign firm can be calculated with (49) as $q_2(y, \omega^*, g^*) = B_2/\lambda(y, \omega^*, g^*)$ and the value of a foreign tax liability share with (51) as $q_4(y, \omega^*, g^*) = B_4/\lambda(y, \omega^*, g^*)$, we can infer negative price effects for these two securities following an increase of foreign government consumption.

To find out the impacts of fiscal policy changes in the foreign country on home securities' prices requires a more elaborate analysis. The real price of a share of the home firm is given in (48) as $q_1(y, \omega^*, g^*) = B_1(y, g^*)/\lambda(y, \omega^*, g^*)$. We already know that an increase of g^* leads to a reduction of $1/\lambda$. Since one can calculate $B_{1g^*}(y, g^*) > 0$ the direction of the effect of a stochastic increase in foreign government consumption on the real share price of the home firm therefore cannot be determined exactly; the effect depends on whether λ or B_1 is stimulated more strongly.

Equation (50) gives the value of a share of domestic tax liabilities as $q_3(y, \omega^*, g^*) =$

[2]Without Assumption 1 we would have to consider complicated special cases, where prices in the vicinity of the bounds $\tilde{\omega}$ or $\tilde{\omega}^*$ change the liquidity region, if we have a short-run policy change.

$B_3(y, g^*)/\lambda(y, \omega^*, g^*)$. By differentiating with respect to g^* we get $B_{3g^*}(y, g^*) < 0$, so B_3 is, exactly as $1/\lambda$, a monotone declining function of g^*. Foreign fiscal policy therefore is correlated negatively with q_3.

Let us now consider the effects of a temporary change of home government expenditures. An expansionary home fiscal policy stimulates the relative price of the home consumption good $p(k, s)$ in (44). The reason is, that an increase of g reduces the supply of home goods for the private sector. With an unchanged quantity of foreign goods the real price of the home good measured in foreign goods increases. Numerically the positive price effect with a binding home liquidity constraint ($\omega \geq \tilde{\omega}$) results from the reduction of the quantity of goods for consumption purposes ($x - g$). The diminished supply of consumption goods with a non-binding cash-in-advance constraint ($\omega < \tilde{\omega}$) increases u_x, because of the monotonicity and concavity of the utility function U. By the same argument we conclude, that a stochastic increase of g at home is inflationary, i.e. the nominal goods price P increases.

From (48)–(51) we can see, that anticipated fiscal policy shocks at home do not affect the real (and nominal) equilibrium prices of shares. Only temporary changes of the foreign government expenditures therefore have an effect on the share prices. This is due to the asymmetric modelling of the production decisions. Finally neither an anticipated change of home government expenditures nor an anticipated change of foreign government expenditures affects the equilibrium exchange rate f.

Now we turn to the transmission effects of temporary nominal shocks, which can be understood as increases of the home and foreign quantity of money (compare Svensson (1985b)). Here we can do without Assumption 1, as the functions for the liquidity bounds $\tilde{\omega}$ and $\tilde{\omega}^*$ are independent of both money growth rates ω and ω^*. The analysis of an expansive monetary policy however has to be restricted to infinitesimal small increases of the monetary growth rates, to prevent that the price functions change over from the region of non-binding finance constraints ($\omega < \tilde{\omega}$ and $\omega^* < \tilde{\omega}^*$) to regions where the finance constraints are fulfilled with equality ($\omega \geq \tilde{\omega}$ and $\omega^* \geq \tilde{\omega}^*$).[3] Let us note, that a stochastic change of the gross rates of growth ω_t and ω_t^* influence the quantities of money supply only in the following period, as the monetary policy of both governments was modelled according to $M_{t+1} = \omega_t M_t$ and $N_{t+1} = \omega_t^* N_t$. By that economic agents already know at the beginning of the current transactions period t the level of the supplied quantities of money in $t + 1$ and they know that with a higher money supply there is an increase in the rate of inflation in $t + 1$. How does this knowledge change the equilibrium price functions for goods and shares in the current period?

The nominal exchange rate f (see (52)) increases with a stochastic expansionary monetary policy of the home government. We can see, that the nominal home goods price

$$P(M, k, s) = pP^*f = \begin{cases} M\omega u_x(x(k, \varepsilon) - g)\,/A(y, g^*) & , \quad \omega < \tilde{\omega} \\ M\,/[x(k, \varepsilon) - g] & , \quad \omega \geq \tilde{\omega} \end{cases} \qquad (54)$$

[3] To investigate the effects of a smaller supply of money in the home country the case $\omega = \tilde{\omega}(k, y, \varepsilon, g, g^*)$ has to be excluded and for the foreign country the case $\omega^* = \tilde{\omega}^*(y, g^*)$ must be ruled out, because on these bounds even a small lowering of the money growth rates cause such a transition.

is only stimulated by an increase of the home growth rate of money ω, if we have a non-binding cash-in-advance constraint in the home country ($\omega < \tilde{\omega}$). In this case consumers do not spend their total holdings of home money for purchases of goods. But as they know, that the home money supply will increase in the following period and therefore the non-spent money will have a lower value, they try to obtain more home goods in the current period. This increased demand has its inflationary effects already in the current transactions period. However with a binding finance constraint we have no increased goods demand, as the agents have already spent their total home money holdings. Inflation can be seen not before the following period.

With a finance constraint fulfilled with equality ($\omega \geq \tilde{\omega}$) the relative price p falls with a stochastically increased ω. However the real price of home goods in case of a non-binding liquidity constraint following a monetary shock in the home country remains unchanged. As a matter of fact $p = P/fP^*$ should also fall in this region because of the increased nominal exchange rate. But the also higher nominal price P counterbalances the negative price effect of the exchange rate.

A temporary increase of the home money growth rate ω has positive effects on the nominal home share prices

$$Q_1(M, y, \omega, g^*) = \frac{q_1}{\pi_M} = \frac{M\omega B_1(y, g^*)}{A(y, g^*)} \tag{55}$$

and

$$Q_3(M, y, \omega, g^*) = \frac{q_3}{\pi_M} = \frac{M\omega B_3(y, g^*)}{A(y, g^*)} \ . \tag{56}$$

The agents expecting higher home goods prices in the following period try to conserve the value of home money beyond the current period, where the monetary expansion takes plase, by increased share purchases, because the currently purchased shares in t can at first be transferred into money on the asset market in period $t+1$ and be used to buy goods earliest in $t+2$. Share holdings therefore protect against inflation.

The real share prices q_1, q_2, q_3 and q_4, as well as the nominal foreign share prices Q_2 and Q_4, are unaffected by a stochastic monetary policy change of the home country.

What are the effects of a foreign monetary shock? A lowering of the nominal exchange rate f (compare (52)) can be caused by an expansionary monetary policy of the foreign government. The nominal foreign share prices, i.e. the price of a share of a foreign firm,

$$Q_2(N, \omega^*) = \frac{q_2}{\pi_N} = \frac{N\omega^* B_2}{A^*} \ , \tag{57}$$

and the price of a foreign tax liability share

$$Q_4(N, \omega^*) = \frac{q_4}{\pi_N} = \frac{N\omega^* B_4}{A^*} \ , \tag{58}$$

increase following a stochastic foreign money growth. The agents try by an increased demand for foreign shares (from which finally the price increase results) to transfer the purchasing power of their foreign money holdings into period $t+2$, to protect themselves from the expected inflation in $t+1$.

With an expansive foreign monetary policy the marginal utility of wealth λ falls in case we have a binding liquidity constraint. In this case the agents already have spent their total foreign money holdings on the goods markets. Because of the higher quantity of money they expect a price increase in the following period and they want to increase their goods demand. But the shares can be transformed into money at first on the asset markets, their marginal utility falls, and the liquidity services of holding foreign money ν (compare (42)) are valued higher. Because of the lower λ the real share prices q_1, q_2, q_3 and q_4 increase with a cash-in-advance constraint fulfilled with equality.

Structural policy changes

Under a structural policy change we understand a permanent shift of government consumption or of the monetary rate of expansion by a non-stochastic amount (compare Eckwert (1990) and Schittko and Müller (1999)). More precisely, a structural increase of fiscal policy in the home country means a transition from g to $g+\delta$ for a small non-stochastic $\delta > 0$. Such long-run policy changes imply additional to the money and fiscal policy disturbances in the current period changed expectation values for the money supply and government expenditure processes in the subsequent periods.

To ensure constant liquidity bounds in the following investigation we need an additional assumption.

Assumption 2 *We suppose that:*

$$\begin{aligned} r_{\bar{x}''} &= -\frac{\bar{x}'' u_{xx}(\bar{x}'')}{u_x(\bar{x}'')} = 1 \quad \text{and} \\ r_{\bar{y}'} &= -\frac{\bar{y}' u_{yy}(\bar{y}')}{u_y(\bar{y}')} = \frac{1}{1-K_{y'}(y',g^{*\prime})} + K(y',g^{*\prime})\frac{u_{yy}(\bar{y}')}{u_y(\bar{y}')} \ , \end{aligned}$$

holds, where $\bar{x}'' := x(K(y',g^{*\prime}),\varepsilon'') - g''$ *and* $\bar{y}' := y' - K(y',g^{*\prime}) - g^{*\prime}$.

Under this assumption we get fixed bounds between binding and non-binding finance constraints in the following period.

Lemma 3 *Suppose Assumption 2 is fulfilled. Then* $\tilde{\omega}(K,y',\varepsilon',g',g^{*\prime})$ *and* $\tilde{\omega}^*(y',g^{*\prime})$ *are constant functions.*

PROOF:
The proof is analogous to the proof of Lemma 1, shifting the time index by one period.
□

Compared with a short-run expansive fiscal policy of the foreign government from the knowledge of increased government expenditures in the following period we get no further effects on the equilibrium goods prices p, P and P^* since $A_{g^{*\prime}}(y, g^*) = 0$ and $A^*_{g^{*\prime}} = 0$. A structural expansive fiscal policy abroad reduces the real price of the home consumption good p and stimulates the nominal price of the foreign good P^* to the same extent as a stochastic increase in government consumption. The marginal utility of wealth λ increases with a structural expansive fiscal policy abroad exactly in the same way as for a temporary increase in government expenditures, as $g^{*\prime}$ excerts no influence on A^*.

The direction of the effect of a permanent increase of foreign government expenditures on the value of a share of the home firm q_1 cannot be determined exactly as it was the case with a stochastic fiscal policy disturbance. The value of a share of the foreign firm is given by $q_2 = B_2/\lambda$. A temporary increase of government consumption increases λ and by that lowers q_2. From B_2 in (49) it follows, that an increased $g^{*\prime}$ increases the value of B_2, as $g^{*\prime}$ is positively correlated with λ', the marginal utility of wealth in the following period, i.e. $B_{2g^{*\prime}} > 0$. Therefore a structural expansive fiscal policy of the foreign government weakens the short-run negative price effect, respectively, under certain circumstances, it can even stimulate q_2.

By differentiation of B_3 and B_4 (see (50) and (51)) with respect to $g^{*\prime}$ one can infer that the temporary negative price effects on the tax liability shares are therefore strengthened by a long-run increase of foreign government expenditures since $B_{3g^{*\prime}} < 0$ and $B_{4g^{*\prime}} < 0$.

As the influence of $g^{*\prime}$ on B_1 cannot be determined unambigously, we cannot derive a definite result concerning the direction of the effect of a structural expansionary fiscal policy abroad on Q_1. Because of $B_{2g^{*\prime}} > 0$ the nominal price of a share of the foreign firm Q_2 increases following a permanent increase of foreign government expenditures. However the nominal price of the domestic tax liability share Q_3 due to $B_{3g^*} < 0$ and $B_{3g^{*\prime}} < 0$ will be reduced more strongly compared with a temporary fiscal policy shock. Finally we have negative price effects on Q_4 following a structural increase of foreign government expenditure, because $B_{4g^{*\prime}} < 0$.

Comparing the results of a stochastic and a structural expansionary fiscal policy abroad, we can state, that with a long-run increase of foreign government expenditures also the nominal prices of the foreign shares are affected. A higher government consumption is exclusively financed by higher tax claims and not by a monetary expansion. Therefore the price of shares of the firms increases and the price of a tax liability shares falls, because agents increasingly demand shares and demand fewer tax shares if a long-run tax increase is given. This effect could not be observed in the current period despite the tax increases, for the share holdings of the current transactions period are predetermined and cannot be changed after receiving all informations. By the changed nominal prices there are effects on the real foreign share prices. The increased attractivity of the share of the firm weakens the negative influence of a temporary increase of government expenditures on q_2 at least and intensivies the effect on q_4.

Let us now regard the influence of an expansionary home structural fiscal policy. We know, that a temporary increase of g does not influence the equilibrium share prices. Following a structural fiscal policy change at home the real and nominal prices of the foreign shares remain unchanged, because g' has no influence on B_2 and B_4. A long-run increase of expenditures of the home government only affects the prices of home shares; while Q_1 and q_1 are increasing, Q_3 and q_3 are falling.

At last we will study the price effects of structural monetary policy. The analysis can be conducted independently of Assumptions 1 and 2 because the monetary growth rates in the current and subsequent period have no influence on the liquidity bounds $\tilde{\omega}$, $\tilde{\omega}^*$, $\tilde{\omega}'$ and $\tilde{\omega}^{*\prime}$. However, we have to restrict our analysis again to infinitesimally small increases of monetary growth rates, to prevent a transition from a non-binding to a binding cash-in-advance constraint.

If we differentiate the function $A(y, g^*)$ with respect to ω', the gross rate of growth of the home quantity of money in the subsequent period, we obtain

$$A_{\omega'}(y, g^*) = -\beta \int\limits_{\omega' < \tilde{\omega}'} A(y', g^{*\prime}) \frac{1}{(\omega')^2} \, dF(s') \quad < 0 \ . \tag{59}$$

The expected marginal utility of the home quantity of money falls following a permanently increased supply of money at home. In a comparable model Svensson (1985a) arrives at the same conclusion. However, the expected marginal utility of the real quantity of money in the foreign country A^* will not be changed by monetary disturbances in the home country.

The nominal exchange rate f (see (52)) therefore increases because of the falling $A(y, g^*)$ more strongly following a structural expansionary monetary policy of the home country compared to the temporary increase of ω. By the same reasoning the nominal home goods price P with a non-binding liquidity constraint ($\omega < \tilde{\omega}$) is more strongly stimulated by a long-run monetary expansion at home if we compare this with a short-run expansion. If the finance constraint for the purchase of home goods is fulfilled with equality ($\omega \geq \tilde{\omega}$) P is unaffected not only by a stochastic but also by a structural monetary policy change. The real price of the home consumption good measured in foreign goods p falls more strongly compared to the case of a stochastic increase of ω. However, in the case of a non-binding liquidity constraint ($\omega < \tilde{\omega}$) structural monetary disturbances at home have no effect on the real price of home goods, because the changes of ω and ω' have no effect on λ. Likewise, neither ω nor ω' have an influence on the nominal price of the foreign consumption good P^*.

Therefore we conclude, that a structural expansionary monetary policy of the home government strengthens the goods price effects of a temporary monetary growth, because of the knowledge of an increasend inflation beyond the subsequent period consumers intensify their endeavour to get consumption goods in the current period at more favorable prices.

We know that a temporary increase of ω does not change the real share prices. But differentiating the function $B_1(y, g^*)$ with respect to ω' we obtain

$$B_{1\omega'}(y, g^*) = -\, \beta \int\limits_{\omega' \geq \tilde{\omega}'} \frac{A(y', g^{*\prime}) x(K(y, g^*), \varepsilon')}{\bar{x}'(\omega')^2} \, dF(s') \quad < 0 \ , \tag{60}$$

i.e. q_1 falls following a long-run monetary expansion at home. If we differentiate the function $B_3(y, g^*)$ with respect to ω' we get

$$B_{3\omega'}(y, g^*) = \beta \int \frac{A(y', g^{*\prime})}{(\omega')^2}\, dF(s') + \beta \int\limits_{\omega' \geq \tilde{\omega}'} \frac{A(y', g^{*\prime}) g'}{\bar{x}'(\omega')^2}\, dF(s') \quad > 0 \ . \qquad (61)$$

A long-run increase of money supply at home leads to an increase in the value of the home tax liability share $q_3 = B_3/\lambda$. The positive price effect on q_3 and the negative effect on q_1 are due to the increased attractivity of the tax share following a structural expansionary monetary policy at home. A permanent increase in the supply of money at home at unchanged government expenditures leads to a reduction of current and future tax claims, and economic agents demand more tax shares with in absolute value smaller negative dividend. With a temporary change, restricted to the current period, this price effect, despite the tax reduction, cannot be observed, because of the predetermined share holdings. One should note, that a structural expansionary fiscal policy at home amounts to a tax increase at home, thereby giving opposite price effects for the home shares, i.e. q_1 increases and q_3 falls. However the real prices of foreign shares, q_2 and q_4, are unaffected by a long-run monetary growth in the home country, because neither ω nor ω' influence λ, B_2 or B_4.

5 Concluding remarks

In this paper we investigated the interactions between fiscal policy, monetary policy and financial markets in the framework of a international stochastic competitive equilibrium model, in which money is an asset providing liquidity services, and where agents' demand for money depends on the stream of expected liquidity services.

We have studied an asset pricing model addressing the effects of fiscal and monetary policy like in the traditional Mundell-Fleming model, but here in an explicit dynamic model. Thereby we have restricted the analysis to the case of i.i.d. processes. This restriction was essential to the derivation of simple explicit formulae for the prices. The model should be regarded as a bench-mark model for the discussed policy effects. We are fully aware of the recent advances in our ability to numerically simulate dynamic economic models.

Financial markets are seen to be responsive to fiscal and monetary policy changes. Summarizing we can say, that an expansive stochastic fiscal policy leads to increased inflation in the respective country. The real price of the home goods measured in foreign goods p is negatively correlated with foreign government expenditures and positively correlated with government expenditures of the home country. The nominal price of a share of the home firm is stimulated by an increase in foreign government consumption, whereas the price of the domestic tax liability share is reduced. Share holdings can be used to protect against inflation. Comparing the results of a stochastic and a structural expansionary fiscal policy abroad, we can state, that with a long-run increase of foreign government expenditures also the nominal prices of the foreign shares are affected. A higher government consumption is exclusively financed by higher tax claims and not by a monetary expansion. Therefore the price of shares of the firms increases and the price of a tax liability shares falls, because agents increasingly demand shares and demand fewer tax shares if a long-run tax increase is given. And a structural expansionary monetary policy of the home government strengthens the goods price effects of a temporary monetary growth, because of the knowledge of an increasend inflation beyond the subsequent period consumers intensify their endeavour to get consumption goods in the current period at more favorable prices.

Obviously nominal and real interest rates are influenced by the discussed policy measures. Furthermore the real rates of return on money and shares are affected by the policy disturbances. We leave the discussion of these effects and others for a future occasion. On analytical grounds we had to use very restrictive symmetry assumptions for both countries, using the the pooled equilibrium concept. Hopefully the results do not depend too much on this restricted specification.

Appendix

Proof of Lemma 2

According to (39) we have $\tilde{\omega}(k,y,\varepsilon,g,g^*) = A(y,g^*)/[\bar{x}u_x(\bar{x})]$. $r_{\bar{x}'} = 1$ from Assumption 1 implies $A_{g^*} = 0 = A_y$ since sign $A_{g^*}(y,g^*) = -\text{sign}[1-r_{\bar{x}'}]$ and sign $A_y(y,g^*) = \text{sign}[1-r_{\bar{x}'}]$. Therefore differentiating of $\tilde{\omega}$ with respect to y and g^* gives

$$\begin{aligned}
\tilde{\omega}_y(k,y,\varepsilon,g,g^*) &= \frac{A_y(y,g^*)}{\bar{x}u_x(\bar{x})} = 0 \ , \\
\tilde{\omega}_{g^*}(k,y,\varepsilon,g,g^*) &= \frac{A_{g^*}(y,g^*)}{\bar{x}u_x(\bar{x})} = 0 \ .
\end{aligned}$$

Differentiating with respect to g, k and ε leads to

$$\begin{aligned}
\tilde{\omega}_g(k,y,\varepsilon,g,g^*) &= \frac{A(y,g^*)}{(\bar{x}u_x(\bar{x}))^2}\ u_x(\bar{x})\Big[1-r_{\bar{x}}\Big] \quad = 0 \ , \\
\tilde{\omega}_k(k,y,\varepsilon,g,g^*) &= -\frac{A(y,g^*)}{(\bar{x}u_x(\bar{x}))^2}\ x_k(k,\varepsilon)u_x(\bar{x})\Big[1-r_{\bar{x}}\Big] \quad = 0 \ , \\
\tilde{\omega}_\varepsilon(k,y,\varepsilon,g,g^*) &= -\frac{A(y,g^*)}{(\bar{x}u_x(\bar{x}))^2}\ x_\varepsilon(k,\varepsilon)u_x(\bar{x})\Big[1-r_{\bar{x}}\Big] \quad = 0 \ .
\end{aligned}$$

Therefore $\tilde{\omega}$ is a constant function, independent of the exogenous variables of the model.

Now we look at $\tilde{\omega}^*$ from (38), $\tilde{\omega}^*(y,g^*) = A^*/[u_y(\bar{y})(y-g^*)]$. Then we get

$$\begin{aligned}
\tilde{\omega}^*_y(y,g^*) = -\frac{A^*}{[u_y(\bar{y})(y-g^*)]^2}\ u_y(\bar{y})\Big[& r_{\bar{y}}(K_y(y,g^*)-1) \\
& +K(y,g^*)\frac{u_{yy}(\bar{y})}{u_y(\bar{y})}(1-K_y(y,g^*))+1\Big] \ ,
\end{aligned}$$

and with $r_{\bar{y}} = \frac{1}{1-K_y(y,g^*)} + K(y,g^*)\frac{u_{yy}(\bar{y})}{u_y(\bar{y})}$ it follows from Assumption 1 $\tilde{\omega}^*_y(y,g^*) = 0$.

Proceeding analogously leads to

$$\begin{aligned}
\tilde{\omega}^*_{g^*}(y,g^*) = -\frac{A^*}{[u_y(\bar{y})(y-g^*)]^2}\ u_y(\bar{y})\Big[& r_{\bar{y}}(K_{g^*}(y,g^*)+1) \\
& +K(y,g^*)\frac{u_{yy}(\bar{y})}{u_y(\bar{y})}(-K_{g^*}(y,g^*)-1)-1\Big] \ .
\end{aligned}$$

With $r_{\bar{y}}$ from Assumption 4 and $K_{g^*} = -K_y$ it follows $\tilde{\omega}^*_{g^*}(y,g^*) = 0$. Therefore under Assumption 1 $\tilde{\omega}^*$ is also a constant function. □

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