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Equation**

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Abstract: This paper seeks to demonstrate that a backward looking specification of the IS curve using UK data can encompass the forward looking model recently discussed by Kara and Nelson (2004). By relaxing the restriction that the interest rate and the inflation rate enter the IS curve with coefficients of equal magnitude but opposite sign, we obtain IS curve estimates which are empirically plausible and which encompass the rival specification.

Keywords: IS curve, forward looking, real interest rate.

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I. Introduction

Recent years have seen the growing popularity of a class of small macroeconomic models consisting of an IS curve, a Phillips curve and a Taylor rule. For example, King (2000) provides a theoretical survey of this literature while Henry and Pagan (2004) concentrate on empirical applications. What becomes clear on reading this literature is that these models have been interpreted in very different ways. At one extreme they are generated by the assumption of forward-looking optimising agents operating in more-or-less frictionless markets while at the other they are simply shorthand representations of the old-fashioned Keynesian IS-LM structure. A good example of the former approach is the paper by McCallum (2001) while Rudebusch and Svensson (1999) provide a typical example of the latter.

A potentially useful application of the new IS-LM model is in assessing the effects of structural change. If the policy regime in operation does not change then it is well known that models generated by different underlying structures may be indistinguishable. However, once policy regime shifts occur then the Lucas critique demonstrates that we should expect to see changes in empirical relationships which are not based on structural parameters. This has led a number of authors to seek evidence of such changes in the new IS-LM model. Estrella and Fuhrer (2002 and 2003) have argued that this evidence is not supportive of the optimising interpretation of the new IS-LM model in that these models are (a) inconsistent with the dynamic relationships between the variables of the model which we observe and (b) exhibit at least as much instability as essentially backward looking models. This argument is disputed by Kara and Nelson (2004) who argue that optimising models are empirically stable whereas backward looking models produce insignificant estimates for the most important parameters such as the response of output to the real interest rate. Direct comparisons are difficult here since Estrella and Fuhrer use US data whereas Kara and Nelson present evidence based on UK and Australian data.

In this paper we re-examine the evidence of Kara and Nelson (KN) concerning one particular feature of the new IS-LM model. Our aim is to assess the impact of structural change on estimates of the IS equation. However, the conclusion is quite different from that of KN who find a stable relationship for the optimising form of the equation. The argument here is that the superior performance of the optimising equation in their model results from the invalid restriction that it is the *real* interest rate which is of sole importance in the backward looking version of the IS curve. If the restriction that the coefficients on the nominal interest rate and the expected rate of inflation are not equal and opposite in sign is not imposed then the backward looking equation outperforms the forward looking version. Moreover, estimates of the backward looking equation can explain the results of the forward looking model thus ‘encompassing’ the rival model.

II. The forward looking IS curve

The forward looking IS curve estimated by KN takes the form:

$$y_t = b_0 + E_t y_{t+1} - \sigma (i_t - E_t \pi_{t+1}) + s_g (g - E_t g_{t+1}) + v_t \quad (1)$$

where y is (log) real GDP, i is the nominal interest rate, π is the inflation rate and g is the ratio of real government purchases to real GDP. v is a random error and the parameter σ can be interpreted as the reciprocal of the coefficient of relative risk aversion. This is contrasted with a backward looking IS curve which takes the form:

$$y_t^d = a_0 + \sum_{i=1}^j \alpha_i y_{t-i}^d + \sum_{i=1}^k \beta_i (i_{t-i} - \pi_{t-i}) + u_t \quad (2)$$

where y^d is the deviation of output from trend (generated by a variety of different techniques) and u is a random error.

Since the purpose of this paper is to provide an alternative interpretation of the results present by KN it is useful to demonstrate that their results can be replicated. The data used here consist of quarterly real GDP at 2003 prices taken from the ONS database, the three month Treasury Bill yield as a measure of the short term interest rate and $\pi_t = 100 \times (\log p_t - \log p_{t-4})$ as a measure of the inflation rate. The empirical equivalent of equation (1) takes the form:

$$y_t = b_0 + E_t y_{t+1} + b_1 (i_t - E_t \pi_{t+1}) + b_2 D7304_t + s_g (g - E_t g_{t+1}) + v_t \quad (3)$$

where $D7304_t$ is an intercept dummy variable which is equal to 1 from 1973.4 onwards. This equation is estimated using lagged values of the model variables as instruments for the forward looking expectations. The results are given in Table 1 alongside those of KN.

[Table One here]

Table 1 indicates a close match between our parameter estimates and those of KN. The signs and magnitudes of the parameters are very similar in all cases. Our standard errors appear to be somewhat higher, leading to the lower values of the t-ratios reported. However, if we adopt a 5% significance value for statistical tests, then the higher values of the standard errors we obtain makes no difference as to which parameter estimates are considered significant or not significant. Our overall judgement is therefore that our results match those of KN closely. Similarly, in Table 2 we present estimates of the backward looking model (2) estimated by KN along with their parameter estimates and again, we find that there is a high level of agreement between the two sets of estimates.

[Table Two here]

The estimates presented in Tables 1 and 2 both impose the restriction that it is the *real* interest rate which is important in the IS curve. This implies *ceteris paribus* that an

increase in expected inflation leads to an increase in current aggregate demand, since it produces a lower real interest rate when the nominal rate is held fixed, and there is no independent effect of inflation on the demand side of the economy. This assumption is so widespread in modern macroeconomics that it is hardly ever questioned. However, there is also a long tradition in empirical macroeconomics of allowing for a separate effect of inflation other than the real interest rate effect. For example, Davidson *et al* (1978) find a negative equilibrium effect of inflation on consumption expenditures which they interpret as reflecting the effect of changing prices on real balances. More recently Fair (2002) has found evidence that the restriction of equal and opposite signs on the interest rate and expected inflation is consistently rejected for a variety of macroeconomic relationships estimated using data for a range of different economies. Finally, Goodhart and Hoffman (2005) have also relaxed this restriction and found that it significantly improves the empirical fit of their estimates of the new IS curve for a variety of different economies.

Given the weight of evidence that the restriction imposed by inclusion of the real interest rate alone may be invalid, we therefore estimate the backward looking IS curve given in equation (4).

$$y d_t = \beta_0 + \beta_1 y d_{t-1} + \beta_2 y d_{t-2} + \beta_3 i_t + \beta_4 \pi_t + u_t \quad (4)$$

The lag length for deviations of output from trend is set at 2 since this appears to be sufficiently general to capture the autoregressive structure of the data while the interest rate and inflation are included as contemporaneous variables only. We did experiment with the inclusion of lags of these variables but this did not change the results significantly. Estimates of equation (4) are reported in Table 3.

[Table Three here]

The results shown in Table 3 indicate that the real interest rate restriction receives little support. For the whole sample period, the coefficients on the nominal interest

rate and inflation are both negative. While we cannot reject the null that the signs are equal and opposite when the output variable is the deviation from the Hodrick-Prescott trend, this is simply because the standard errors for both coefficients are so large that neither is significantly different from zero. When we use either deviations from either a linear or a quadratic trend, the interest rate coefficient becomes statistically significant with the correct negative sign and we can reject the null that the inflation coefficient is equal but opposite in sign. However, we are reluctant to read too much into the significant negative coefficient for these cases since there is highly likely that this method of separating trend from cycle may fail to deal adequately with the presence of a unit root in the series and therefore the apparent significance of the interest rate may be spurious.

In the lower panel of Table 3 we present the results of estimating equation (4) for the period after 1979.2. These confirm the presence of a structural break in the model and mark a noticeable improvement over the full sample estimates. However, these estimates are still not consistent with the real interest rate restriction. Again, we consider the results for the deviation of output from the Hodrick-Prescott trend the most reliable. These indicate a positive but insignificant nominal interest rate coefficient while inflation is negative and significant. The results for alternative trend-cycle decompositions are similar and in all cases we reject the null that the coefficients on the nominal interest rate and inflation are equal and opposite in sign.

In summary therefore, our results do not indicate any empirical support for the restriction which would enable us to write the IS curve in terms of the real interest rate rather than including separate nominal interest rate and inflation terms. Indeed, we find little evidence of a nominal interest rate effect on deviations of output from trend with the most important effect appearing to be a negative effect of inflation after the 1979 change in monetary regime.

III. Model comparisons and encompassing

So far our analysis has simply acted to provide an extra model for consideration. In this section we seek to demonstrate that this model is not only plausible in terms of its parameter estimates but that it effectively encompasses rival models. In this context the term encompassing is used to mean that the model of this paper can explain the results generated by other rival models while simultaneously demonstrating both a superior statistical performance and economic plausibility. The results so far have indicated a structural break post 1979 so we will concentrate on the period after this date. The data period is extended to the full sample available and the IS curve is complemented by a backward looking Phillips curve to determine inflation and a Taylor rule for the determination of the interest rate. Estimates of these three equations are given in Table 4:

[Table Four here]

The estimates in Table 4 are reasonably plausible in terms of a backward looking model. The IS curve has not changed noticeably with the addition of an extra three years of data. The Phillips curve has reasonable properties with a significant output gap (lagged one period) and a long-run effect of lagged inflation which is close to one. Finally, the Taylor rule exhibits significant effects of both inflation and the output gap but also a high degree of inertia. The ‘long-run’ effects of inflation and the output gap on the interest rate are 0.84 and 1.85 respectively. Although the fact that the long-run effect of inflation is less than unity might result in instability in models where the real interest rate is the relevant variable in the IS curve, this is not the case if the inflation rate enters the IS curve with an independent negative effect (cf. Fair (2000) for more discussion of this issue).

Having established that it is possible to estimate a plausible backward looking model, we now address the question of whether this encompasses the forward looking model

given by (1). To do this we solve the model presented in Table Four 10,000 times using randomly drawn values for $\hat{u}_{i,t} : i = 1, 2, 3; t = 1, \dots, T$. We then construct artificial data sets based on the model's solution which allow us to estimate the forward looking equation (1). Our aim is to show that results similar to those of KN can be generated even if the backward looking model is used as the data generation process. Figure One shows the distribution of the parameter estimate for the real interest rate in the forward looking equation obtained by the procedure described. The average value obtained is -0.0697 and the 95% percentile value is -0.003. On this basis we conclude that estimation of the forward looking model using data generated by model in Table Four produces an apparently significant negative effect of the real interest rate in a forward looking IS curve. Therefore, our model is capable of explaining the most important qualitative feature of the forward looking model.

[Figure One here]

Conclusions

This paper argues that the apparently favourable results for the forward looking IS curve in KN's comparison with the backward looking case is the result of the imposition of the untested assumption in the latter model that the nominal interest rate and the inflation rate enter with equal and opposite signs. If this assumption is relaxed then the inflation rate can be seen to have a short-run negative effect on the output gap which runs directly counter to the orthodox real interest rate model. Moreover, when a backward looking IS curve with a negative inflation effect is embedded within a small macroeconomic model, it can be shown to generate data which lead to significant negative real interest rate effect in a forward looking equation. Thus the fact that such an effect is found in estimates of forward looking models does not imply support for this approach, it simply suggests that the backward looking models with which they are being compared are insufficiently general.

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TABLE 1

	Kara and Nelson Estimates		This paper	
	1957.1-2002.4	1979.2-2002.4	1957.1-2002.4	1979.2-2002.4
b_0	-0.0057 (4.36)	0.0002 (0.29)	-0.0078 (4.89)	-0.0007 (0.35)
b_1	-0.0844 (3.92)	-0.1319 (5.17)	-0.0609 (2.15)	-0.1374 (2.77)
b_2	0.0029 (1.77)	-	0.0034 (1.76)	-
s_g	0.22*	0.22*	0.22*	0.22*

* indicates imposed coefficient

** sample period is 1958.4 to 2002.4 to allow for lags.

Instruments used were π_{t-1} , π_{t-2} , π_{t-3} , y_{t-1} , y_{t-2} , y_{t-3} , i_{t-1} , i_{t-2} , i_{t-3} , g_{t-1} , g_{t-2} , g_{t-3} plus dummy variable and constant.

TABLE 2

	Lags of y^d	Sum of coefficients*	Lags of $(i - \pi)$	Sum of coefficients*
Kara and Nelson Estimates				
1957.1-2002.4	1-4	0.717 (13.71)	1-4	0.003 (0.21)
1979.2-2002.4	1-4	0.819 (15.40)	1-4	0.029 (0.95)
This paper				
1957.1-2002.4	1-4	0.727 (13.49)	1-4	-0.001 (0.06)
1979.2-2002.4	1-4	0.852 (15.88)	1-4	0.037 (0.19)

* Numbers in parentheses are t-statistics for the hypothesis that the sum of the coefficients is equal to zero.

TABLE 3

	<i>Deviations from Hodrick-Prescott trend</i>	<i>Deviations from linear trend</i>	<i>Deviations from quadratic trend</i>
	<i>1958.2 – 2002.4</i>	<i>1958.2 – 2002.4</i>	<i>1958.2 – 2002.4</i>
$\hat{\beta}_1$	0.725 (9.62)	0.834 (11.17)	0.831 (11.12)
$\hat{\beta}_2$	0.087 (1.14)	0.142 (1.84)	0.138 (1.81)
$\hat{\beta}_3$	-0.020 (0.63)	-0.078 (2.31)	-0.079 (2.32)
$\hat{\beta}_4$	-0.020 (0.98)	-0.023 (0.92)	-0.022 (0.92)
	$R^2 = 0.61$ $DW = 1.89$ $\hat{\sigma} = 0.0094$ $t = 1.71$	$R^2 = 0.91$ $DW = 1.89$ $\hat{\sigma} = 0.0099$ $t = 4.00$	$R^2 = 0.91$ $DW = 1.89$ $\hat{\sigma} = 0.0099$ $t = 4.09$
	<i>1979.2-2002.4</i>	<i>1979.2-2002.4</i>	<i>1979.2-2002.4</i>
$\hat{\beta}_1$	0.705 (7.16)	0.813 (8.47)	0.807 (8.42)
$\hat{\beta}_2$	0.249 (2.46)	0.179 (1.83)	0.181 (1.86)
$\hat{\beta}_3$	0.044 (0.97)	0.051 (1.06)	0.048 (0.98)
$\hat{\beta}_4$	-0.099 (2.41)	-0.153 (3.51)	-0.153 (3.51)
	$R^2 = 0.77$ $DW = 1.38$ $\hat{\sigma} = 0.0068$ $t = 2.40$	$R^2 = 0.94$ $DW = 1.40$ $\hat{\sigma} = 0.0072$ $t = 4.29$	$R^2 = 0.94$ $DW = 1.40$ $\hat{\sigma} = 0.0072$ $t = 4.48$

Equations in this table were estimated using instrumental variables with lags of the interest and inflation rate acting as instruments for their contemporaneous values. The t statistics listed below each equation are tests for $H_0 : \beta_3 = -\beta_4$ and are distributed as t_{n-k} under the null where n is the number of observations and k is the number of parameters.

TABLE FOUR: Estimates of full model 1979.2 – 2006.1

$$y_t = 0.0011 + 0.7121 y_{t-1} + 0.2379 y_{t-2} + 0.0398 i_t - 0.0945 \pi_t + \hat{u}_{1,t}$$

(0.60) (7.71) (2.51) (1.10) (2.64)

$$R^2 = 0.77 \quad \hat{\sigma} = 0.0064 \quad DW = 1.39$$

$$\pi_t = 0.0022 + 1.1107 \pi_{t-1} - 0.1453 \pi_{t-2} - 0.2330 \pi_{t-3} + 0.2118 \pi_{t-4} + 0.4264 y_{t-1} + \hat{u}_{2,t}$$

(1.60) (12.22) (1.06) (1.69) (2.39) (5.49)

$$R^2 = 0.96 \quad \hat{\sigma} = 0.0080 \quad DW = 2.03 \quad \sum \gamma_i = 0.9442$$

$$i_t = 0.0046 + 0.1023 \pi_t + 0.2264 y_t + 0.8777 i_{t-1} + \hat{u}_{3,t}$$

(2.20) (2.83) (3.96) (23.26)

$$R^2 = 0.96 \quad \hat{\sigma} = 0.0076 \quad DW = 1.77$$

The IS curve is estimated by instrumental variables using the same instruments used for the estimates presented in Table Three. The Phillips curve and the Taylor rule are estimated by Ordinary Least Squares. Figures in parentheses are t-ratios $\sum \gamma_i = 0.9442$ is the unconstrained sum of lagged inflation coefficients in the Phillips curve.

FIGURE ONE: Estimates of real interest rate coefficient in forward looking equation

