NON-LINEARITY IN THE CANADIAN AND US LABOUR MARKETS: UNIVARIATE AND MULTIVARIATE EVIDENCE FROM A BATTERY OF TESTS

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ABSTRACT

The non-linearity of macroeconomic processes is becoming an increasingly important issue both at theoretical and empirical level. This trend holds for labour market variables as well. Reallocation theory of unemployment relies on non-linearities. At the same time there is mounting empirical evidence of business cycles asymmetries. Thus the assumption of linearity /non-linearity becomes crucial for the corroboration of labour market theories. This paper turns on the microscope on the assumption of linearity and investigates the presence of asymmetries on aggregate and disaggregate labour market variables. The assumption of linearity is tested using five statistical tests for the US and Canadian unemployment rates, growth rates of the employment sectoral shares of construction, finance, manufacturing and trade sectors. An AR(*p*) model was used to remove any linear structure from the series. Evidence of non-linearity is found for the sectoral shares with all five statistical tests in the US case but not in the aggregate level. The results for Canada are not clear-cut. Evidence of unspecified non-linearity is found in the unemployment rate and in the sectoral shares. Overall important asymmetries are found in disaggregated labour market variables in the univariate setting. The linearity hypothesis was also examined in a multivariate framework. Evidence is provided that important asymmetries exist and a linear VAR cannot capture the dynamics of employment reallocation.

Keywords: Non-linearity, Sectoral Shares
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1. INTRODUCTION

Recent empirical work has focused on the non-linear characteristics of economic and financial time series. Although much evidence has been found favouring the presence of non-linearities in financial data, this has not always been the case for macroeconomic data. As Barnett and Seletis (2000) observe in a review paper, the analysis of macroeconomic time series has not led to particularly encouraging results, mainly due to small samples and high noise levels in the aggregate data. Nevertheless, non-linearity is a well-documented feature of the US unemployment rate. Time series models, such as Markov-switching, threshold autoregression (TAR) or smooth transition autoregression (STAR), have been considered for describing and forecasting the non-linearity of unemployment (see Koop and Potter, 1999, and van Dijk et al, 2002). Recently, non-linearity has become a crucial issue in the analysis of (un)employment fluctuations since intersectoral (and intrasectoral) reallocations would bring about (un)employment cycles through non-linear, asymmetric shocks (for a survey c.f. Gallipoli and Pelloni, 2001). However, some researchers have overlooked this key feature of the labour market and continued to focus their attention on linear representations. For instance, in a multivariate setting Campbell and Kuttner (1996) introduce a structural VAR (SVAR) model for aggregate employment and employment sectoral shares for the US. They disregard the potential non-linearity underlining the process of job reallocation and treat aggregate and sectoral shocks symmetrically. Pelloni and Polasek (1999; 2003) and Panagiotidis, Pelloni and Polasek (2003) have stressed how this overlooking of the non-linear structure of sectoral shocks could distort analysis and throw into doubt the reliability of empirical results. To correct this analytical shortcoming, they suggest a specific analytic structure. They propose a VAR-GARCH-M model as a potential framework, which could accommodate the intrinsic non-linearity of employment reallocations. None of the aforementioned studies develop detailed exploratory tests for the potential non-linearity of the univariate series. Similarly, before carrying out their specific experiments at a multivariate level, these studies do not test for non-linearity in general terms. They either exclude non-linearity or directly develop an a priori chosen form of non-linearity[1].

It is the purpose of this paper to fill this vacuum. Since the presence of non-linearity would put a priori constraints on testable theories, we deem it essential that researchers know whether certain macroeconomic series contain a linear or a non-linear structure. There are good reasons why nonlinearities should be investigated. If the presence of nonlinearities is empirically supported, then researchers should try to incorporate them into theoretical models and empirical analyses. If researchers do not take them into account: i) estimates will violate certain assumptions, ii) important dependencies will be left out of the linear model and iii) forecasts will behave poorly. We will focus on Canadian and US labour market time series which are relevant for the analysis of the macroeconomic effects of aggregate shocks vis-à-vis those of sectoral shocks. Of course, we do not expect to exhaust the testing potentialities of those markets, but simply hope to provide a set of standard tests that can help future work.

If linearity were to emerge as a characterising feature of the unemployment and employment sectoral shares series, we could safely reject reallocation shocks as a triggering force and avoid further expensive explorations of the data. Thus the linearity

[1] Pelloni and Polasek (1999, 2003) use the Bayes-factor for model selection. However their testing is limited to a specific range of models compatible with a standard linear VAR.

or not of unemployment is crucial for discriminating among hypotheses. For, instance, as pointed out above, it is still an unsolved macroeconomic puzzle whether unemployment fluctuations reflect aggregate or reallocation disturbances. The solution of this puzzle is probably connected with linearity or non-linearity of the aggregate unemployment process and of the relevant sectoral variables (e.g. see Davis and Haltiwanger 1999; Pelloni and Polasek, 1999 and 2003).

The outline of the paper is as follows. In Section 2, we provide some background information. The methodological issues are discussed in section 3, where the various tests are briefly described, along with the pre-whitening model. The data employed are presented in Section 4. The results of the pre-whitening model and of the univariate non-linearity tests for the Canadian and US labour markets are discussed in Section 5. Section 6 presents the outcomes of the multivariate case and Section 7 concludes.

2. BACKGROUND

A priori testing for non-linearity has normally been limited to the unemployment rate, with no attention given to disaggregate labour market variables. Brock and Sayers (1988) and Frank and Stengos (1988) follow similar approaches and use an autoregressive (AR) model and the BDS test statistic (Brock et al 1996, BDS hereafter) to test the linearity assumption for the unemployment rate in the US and Canada respectively. While Brock and Sayers (1988) find strong evidence for non-linearity in the US case, Frank and Stengos (1988) fail to reject the linearity assumption for the Canadian time series. Furthermore, Frank, Sayers and Stengos (1993) examine Canadian provincial unemployment data for evidence of significant non-linear structure. They follow the work by Nickell (1990) on unemployment persistence, which suggests that the reduced form unemployment equations can be modelled by linear autoregressions. Their findings do not bear out the presence of an important non-linear structure. They conclude that the suggestion that aggregation was responsible for the series "linearity" does not appear to be supported by the data. Recently, Panagiotidis and Pelloni (2003), using a battery of tests, found evidence in favour of non-linearity in the German growth rates of employment sectoral shares but could not corroborate non-linearity in the UK case.

In this paper we wish to extend the work of Brock and Sayers (1988), Frank and Stengos (1988), Frank, Sayers and Stengos (1993) and Panagiotidis and Pelloni (2003) in several dimensions. The first three of these articles only employ the BDS test statistic while we introduce a much larger battery of tests. Our testing analysis is not just aimed at measures of aggregate or regional unemployment, but also considers measures of sectoral employment. We are unaware of a similar exercise run for US and Canadian employment sectoral shares. Additionally, differently from all the above papers, we extend our analysis to the multivariate case.

Thus, the purpose of this paper is to use five statistical tests for non-linearity to examine the linearity assumption for unemployment rates, and the growth rates of US and Canadian aggregate employment and sectoral shares. We wish to encompass the works of Brock and Sayers (1988) for the US and of Frank and Stengos (1988) and Frank, Sayers and Stengos (1993) for Canada within a unique common framework while simultaneously expanding the testing procedure. Our aims are: i) to expand the univariate methodology employed by both Brock and Sayers (1988) and Frank and Stengos (1988)) by introducing a larger array of tests; ii) to apply such a methodology to both Canadian and US data sets so as to bring the evidence about these two countries under a common methodological umbrella; iii) to broaden the focus of the analysis by investigating, alongside aggregate (un)employment data, the series of employment sectoral shares, iv) to introduce non-linear testing in the multivariate setting for the sets of variables under analysis.

The outcomes of our analysis can provide useful knowledge for further study of cyclical fluctuations. For instance, the presence of non-linear dynamics (in the examined time series) is crucial for the separation of the macroeconomic effects of aggregate disturbances from those of sectoral shocks.

Our work can also show whether there are sectors characterised by relatively more complex behaviour and, thus, if asymmetries are more important for some sectors than others. Finally, by addressing the issue of transition from a univariate to a multivariate non-linear testing methodology, we wish to stress the mutually important informational content of these two settings.

3. METHODOLOGY

Given the nature of this paper, we will avoid imposing directly a specific non-linear data generating process. This choice is dictated by three different reasons: the number of non-linear data generators is infinite and, as a result, it is extremely difficult and dangerous to impose an a priori structure; given our methodology, we do not have to make any heroic assumptions; and, more importantly, we do need to reject linearity before we proceed with imposing a non-linear structure of some form. As Potter (1999) points out: "*Successful nonlinear time series modelling would improve forecasts and produce a richer notion of business cycle dynamics than linear time series models allow. For this to happen two conditions are necessary. First, economic time series must contain nonlinearities. Second, we need reliable statistical methods to summarize and understand these nonlinearities suitable for time series of the typical macroeconomic length*".

Many statistical tests for non-linear dependence have been proposed in the recent literature. We make use of five different tests for detecting non-linear serial dependence. This strategy permits us both to obtain a deeper insight into the properties of the time series and to minimise the possibility of drawing the wrong conclusion. If our battery of tests displays a "consensus" in favour of a specific result, we could interpret this "consensus" as corroboration of that outcome.

The five tests that we use are: McLeod and Li (1983), Engle (1982), Brock et al (1996) (BDS), Tsay (1986), and Hinich and Patterson (1995) and Hinich (1996) (bicovariance test). All these tests share the same principle: once any linear serial dependence is removed from the data, any remaining serial dependence must be due to non-linearities in the data generating mechanism. Other well known nonlinearity tests including the bispectrum based tests (Hinich 1982, Hinich and Rothman 1998) are not included since they do not employ the same pre-whitening methodology (for more discussion on non-linearity tests see Barnett et al 1997).

The linear serial dependence is removed from the data through a pre-whitening model as follows: we fit an AR(p) model[2] to the sample data for values from p = 0 to p =

[2] See also Brock and Sayers (1988), Frank and Stengos (1988), Frank, Sayers and Stengos (1993), Patterson and Ashley (2000) and Panagiotidis and Pelloni (2003).

10. The optimal lag length is chosen to minimise the Schwartz criterion (SC). The SC, unlike some alternatives, such as the AIC (Akaike Information Criterion), is known to be consistent for AR(p) order determination under the null hypothesis of a linear generating mechanism (see Judge, et al. 1985, p.246 and Patterson and Ashley, 2000). AR models are quite popular in capturing the linear characteristics of unemployment (e.g. Blanchard and Summers 1986, Alogoskoufis and Manning 1988, and Nickell 1990). The residuals $\{e_t\}$ of the AR(p), which are serially uncorrelated by construction[3], are then tested for non-linear independence using each of the procedures.

All the procedures operate under the null hypothesis that the whitened series under consideration is *i.i.d.* The McLeod and Li test is for ARCH effects and looks at the autocorrelation function of the squares of the prewhitened data and tests whether corr (e_t^2, e_{t-k}^2) is non-zero for some k. The test suggested by Engle (1982) is an LM test, which should have power against GARCH alternatives. The BDS test is a nonparametric test for serial independence based on the correlation integral of the scalar series $\{e_t\}$ (for more on the BDS test see Brock, Hsieh and LeBaron, 1991). The Tsay (1986) test looks explicitly for quadratic serial dependence in the data and has proven to be powerful against a TAR (Threshold Autoregressive) process. The Bicovariance test assumes that $\{e_t\}$ is a realisation from a third-order stationary stochastic process and tests for serial independence using the sample bicovariances of the data. It can be considered a generalisation of the Box-Pierce portmanteau statistic[4].

[3] See the Breusch-Godfrey results in Table 5

[4] Rejections secured by the bicovariance statistic could call into question results indicating ARCH/GARCH structure since the bicovariance test might be detecting significant non-zero bicovariances that should not be present of the process is a martingale difference process that underpins ARCH/GARCH models. Finally, one further option for testing for the presence of ARCH/GARCH using the Hinich bicovariance

The reader is referred to the technical appendix and to the detailed discussion of these tests in Barnett et al (1997) and Patterson and Ashley (2000). Both these papers, in line with the results of other studies (e.g., Brock, Hsieh and LeBaron, 1991), reckon the BDS to be the most powerful of the tests for a non-specified form of non-linearity as an alternative. The other tests could detect specific forms of non-linearity; MacLeod and Li and Engle test the presence of volatility clustering and the Tsay for threshold effects. The combination of the BDS and the other tests would allow us to draw more precise conclusions on the presence and nature of non-linearity in the series. Thus, for example, if the BDS and Engle tests reject linearity while the others do not, we could not bear out the presence of GARCH effects.

4. DATA

The data employed in this exercise are the US and Canadian unemployment rates, aggregate employment and the employment sectoral shares published by the Bureau of Labor Statistics (http://www.bls.gov) and Statistics Canada (http://www.statcan.ca). These are seasonally adjusted monthly observations from 1983:01 to 2000:12 and the sectors under consideration are construction, finance, manufacturing and trade (Figures 1 and 2): for the US the following series were obtained: EES 00000001 Aggregate Employment, EES 20000001 Construction, EES 30000001 Manufacturing, EES 60000001 Retail Trade + EES 51000001 Wholesale Trade, EES 70000001 Finance, Insurance and real estate and LNS14000000 Un Rate, and for Canada: D980745 Un Rate,

(and covariance) test is to use the testing procedure outlined in Brooks and Hinich (1998) and Brooks, Hinich and Molyneux (2000). We are grateful to an anonymous referee for pointing this to us.

D980595 Aggregate Employment, L95660 Manufacturing, L95661 Construction, L95664 Trade, L95665 Finance, Insurance and Real Estate.

We follow Wallis (1987) and employ the logistic transformation for the unemployment rates (this transformation has to be preferred to the logarithmic one; for a discussion and applications see Wallis, 1987):

$$\log\left\{\frac{y_t}{(1-y_t)}\right\},\ 0 \le y_t \le 1$$

We use the growth rate of the logistic transformation owing to as a unit root.

The growth of the sectoral shares is defined as

$x_t = \log u_t^i - \log u_{t-1}^i$ where u_t^i is the share of sector *i* at time *t*.

Table 3 presents the summary statistics of the series and Table 4 the unit root tests (ADF, PP and Breitung's nonparametric unit root test).

5. EMPIRICAL RESULTS

As a first step we estimate the pre-whitening model. The order of the AR process is chosen to minimise the SC (Table 5). The results vary from an AR(0) (regress on a constant) to a maximum AR(6) for US construction. An AR(1) was the preferred specification for the US unemployment rate (as opposed to an AR(2) in Brock and Sayers, 1988) and an AR(0) for the Canadian one (as opposed to the AR(2) in Frank and Stengos, 1988).

Our next step is to use the residuals of the AR(*p*) regression to compute the five test statistics for non-linearity. The results are summarised in Table 1 and presented in

Tables 6 and 7. Under "asymptotic theory", the values are those based on the large sample distributions of the relevant test statistics. For the "Bootstrap" results, 1000 new samples were independently drawn from the empirical distribution of the pre-whitened data. Each new sample is used to calculate a value for the test statistic under the null hypothesis of serial independence. The fraction of the 1000 test statistics that exceed the sample value of the test statistic from the original data is then reported as the significance level at which the null hypothesis can be rejected. The outcomes from both approaches are reported for a given sample size (for a discussion on the sample size, the asymptotic theory and the bootstrap, see Patterson and Ashley 2000).

Most of the tests provide evidence against the hypothesis that the US unemployment rate is characterised by non-linearities. This suggests that the naïve AR(1) specification is capable of capturing the dynamics of the series. On the other hand, the Tsay tests question the above conclusion at the 10% significance level. However, there is evidence across the spectrum of tests that non-linear dynamics are present in the case of aggregate employment. This result is further endorsed by the outcome of the sectoral shares where a strong rejection of the linearity hypothesis emerges across the battery of tests. Construction is a noticeable exception, in the sense that linearity is rejected but ARCH (see McLeod-Li test) and TAR (see Tsay test) effects could be excluded from the infinite set of non-linear specifications.

TABLE 1: SUMMARY OF RESULTS

	ARCH	GARCH	TAR	General Linearity Test	
US	McLeod-Li	Engle	Tsay	Bicovariance	BDS
Unemployment Rate	v	v	v	v	v
Aggregate Employment	x	x	v	x	x
Construction	v	x	v	v	x
Finance	x	x	x	x	x
Manufacturing	v	x	x	x	x
Trade	x	x	x	x	x
Canada					
Unemployment Rate	x	x	x	x	x
Aggregate Employment	v	v	x	v	v
Construction	v	x	x	x	x
Finance	x	x	v	v	x
Manufacturing	v	v	v	x	x
Trade	x	x	x	x	x

Note: v denotes acceptance of the null of *iid* and x rejection at the 5% level of significance.

The results for Canada display a different picture. First, there is a consensus in favour of non-linearity for the unemployment rate. In the case of aggregate employment an interesting result emerges; all tests support the linearity hypothesis except the Tsay test. As a result we could argue that the data generating process might be captured by a TAR process. GARCH effects seem to drive the Finance sector, since only the McLeod-Li and Engle tests reject the linearity hypothesis. For the remaining sectors, evidence against linearity is given by the BDS test statistic. Although we cannot argue in favour of a specific non-linear alternative, we could exclude GARCH and TAR type of structures in the case of manufacturing. The latter can be explained by the power of some tests against specific non-linear structures (the Engle and McLeod-Li tests against (G)ARCH processes and the Tsay test against TAR processes), whereas the BDS is defined against unspecified alternative non-linear structures.

6. MULTIVARIATE ANALYSIS

In this section, we examine how the evidence on asymmetries of the previous part is affected by extending the analysis to a multivariate model. To explore this issue, we estimate a bivariate unrestricted VAR model, similar to the one presented in Campbell and Kuttner (1996). The latter does not use a dispersion proxy as a summary measure of reallocation, but models the relationship between aggregate and sectoral employment explicitly using dynamic time series models. We choose this multivariate structure because of its importance and popularity in the recent sectoral shifts literature. Though our testing is not aimed at corroborating sectoral shifts, it is of great consequence for empirical analyses of the macroeconomic impact of job reallocations. In fact, such a testing procedure may inform us about the admissibility of certain models.

Our benchmark model is a two-dimensional VAR of the Campbell and Kuttner (1996) type:

$$y_t = A_1 y_{t-1} + \ldots + A_p y_{t-p} + Bx_t + u_t$$

where $y_{at} = \begin{bmatrix} y_{1t} \\ y_{3t} \end{bmatrix}$, y_{1t} is the growth rate of total employment and y_{3t} is the growth rate of the manufacturing employment share. Furthermore, we investigate the behaviour of a different specification where aggregate employment is replaced by the unemployment rate; $y_{bt} = \begin{bmatrix} y_{2t} \\ y_{3t} \end{bmatrix}$, y_{2t} is the growth rate of the unemployment rate and u_t's are the corresponding residuals. The obtained results for the US are:

$$\begin{bmatrix} y_{1t} \\ y_{3t} \end{bmatrix} = \begin{bmatrix} 0.149** & 0.101* \\ 0.336*** & 0.103 \end{bmatrix} \begin{bmatrix} y_{1t-1} \\ y_{3t-1} \end{bmatrix} + \begin{bmatrix} 0.395*** & 0.133** \\ -0.0001 & 0.217*** \end{bmatrix} \begin{bmatrix} y_{1t-2} \\ y_{3t-2} \end{bmatrix} + \begin{bmatrix} 0.001*** \\ -0.001*** \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{3t} \end{bmatrix} \quad (1.1)$$

Adj R-squared = 0.281 and 0.157

$$\begin{bmatrix} y_{2t} \\ y_{3t} \end{bmatrix} = \begin{bmatrix} 0.183*** & -0.244 \\ -0.012** & 0.040 \end{bmatrix} \begin{bmatrix} y_{2t-1} \\ y_{3t-1} \end{bmatrix} + \begin{bmatrix} -0.039 & 2.753*** \\ -0.0004 & 0.286*** \end{bmatrix} \begin{bmatrix} y_{2t-2} \\ y_{3t-2} \end{bmatrix} + \begin{bmatrix} -0.011*** \\ -0.001*** \end{bmatrix} + \begin{bmatrix} u_{2t} \\ u_{3t} \end{bmatrix} \quad (1.2)$$

Adj R-squared = 0.05 and 0.109

While for Canada we obtain:

$$\begin{bmatrix} y_{1t} \\ y_{3t} \end{bmatrix} = \begin{bmatrix} 0.083 & 0.068** \\ 0.701*** & 0.121* \end{bmatrix} \begin{bmatrix} y_{1t-1} \\ y_{3t-1} \end{bmatrix} + \begin{bmatrix} 0.338*** & 0.062** \\ 0.228 & 0.032 \end{bmatrix} \begin{bmatrix} y_{1t-2} \\ y_{3t-2} \end{bmatrix} + \begin{bmatrix} 0.001*** \\ -0.001*** \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{3t} \end{bmatrix} \quad (2.1)$$

Adj R-squared = 0.22 and 0.11

$$\begin{bmatrix} y_{2t} \\ y_{3t} \end{bmatrix} = \begin{bmatrix} 0.042 & -0.535* \\ -0.042*** & 0.072 \end{bmatrix} \begin{bmatrix} y_{2t-1} \\ y_{3t-1} \end{bmatrix} + \begin{bmatrix} -0.079 & -0.774** \\ -0.037** & 0.072 \end{bmatrix} \begin{bmatrix} y_{2t-2} \\ y_{3t-2} \end{bmatrix} + \begin{bmatrix} -0.003** \\ -0.001** \end{bmatrix} + \begin{bmatrix} u_{2t} \\ u_{3t} \end{bmatrix} \quad (2.2)$$

Adj R-squared = 0.05 and 0.09

Significance at the 1%, 5% and 10% is denoted by ***, **, * respectively.

The corresponding residuals, u_{1t}, from the total employment equation, u_{2t}, from the unemployment rate equation, and u_{3t}, from the manufacturing equation, were used to calculate the battery of tests (see Tables 8 and 9, summary of the results in Table 2).

For the US, evidence emerges in favour of linearity only for u_{2t} (the residuals from the unemployment rate equation). In all other series there is strong evidence against linearity. This result may have interesting implications for sectoral shifts analysis. First, the results of equation (1.1) suggest that, besides theoretical considerations, the linear specification chosen by Campbell and Kuttner (1996) for the US has to be rejected. Second, we can neither support nor rule out the presence of GARCH effects in equation 1.1, thus leaving open the issue of volatility clustering for sectoral shocks (conflicting results from McLeod-Li and Engle test). Third, the residuals in (1.1) and (1.2) are not shocks, but a mixture of shocks. Thus in (1.1) either the non-linear shocks dominate or all the shocks are linear, while in (1.2) clearly we have a mixture of linear and non-linear shocks. In the latter case, linearity is accepted for the aggregate employment residuals, while it is rejected for the sectoral component. Although, given the character of our analysis, it would be improper to draw further inferences, we could argue, ex-ante, that the theoretical non-linear nature of idiosyncratic shocks is compatible with the data. Thus, non-linearity seems to be an exploitable feature for building models aimed at discriminating between the macroeconomic effects of aggregate and sectoral shocks.

TABLE 2: SUMMARY OF RESULTS

	US	ARCH	GARCH	TAR	General Linearity Test	
		McLeod-Li	Engle	Tsay	Bicovariance	BDS
	Aggregate Employment	x	x	x	x	x
equation 1.1		x	x	x	x	x
	Manufacturing	v	x	x	x	x
		v	x	x	x	x
	Unemployment rate	v	v	x	v	v
equation 1.2		v	v	x	v	v
	Manufacturing	x	x	x	x	x
		x	x	x	x	x
	Canada					
	Aggregate Employment	v	v	v	v	v
equation 2.1		v	v	v	v	v
	Manufacturing	v	v	v	v	x
		v	v	v	v	x
	Unemployment rate	v	v	x	v	x
equation 2.2		v	v	x	v	x
	Manufacturing	v	v	v	v	v
		v	v	v	v	v

For Canada, linearity is supported for u_{1t} (equation 2.1) throughout by our five-test procedure (see tables 8 and 9). This support was not corroborated in the case of u_{3t}, as all the tests, except the BDS, accept linearity (equation 2.1). Although linearity seems to be the rule, not the exception, the BDS casts doubts of whether a Cambell and Kuttner (1996) type of linear VAR could be employed in the case of Canada. Furthermore, the rejection of linearity is even stronger when aggregate employment is substituted for aggregate unemployment. The residuals of the unemployment series are clearly non-linear (maybe threshold?) though not driven by a GARCH processes (McLeod-Li and Engle accept linearity for u_{2t} but Tsay and BDS reject it). Overall, though, the evidence seems not to be as strong as for the US.

7. CONLUSIONS

The purpose of our work was to investigate the presence of non-linear serial dependence in the US and the Canadian labour markets. A robust methodology was employed to investigate the existence of asymmetries in aggregate and disaggregate labour market variables in the US and Canada. Asymmetries were examined both in a univariate and a multivariate setting. Five tests for non-linearity were employed and the statistics were estimated using both the asymptotic theory and the bootstrap. We have shown that important asymmetries exist in the sectoral labour market variables. For the US univariate series, our evidence bears out linearity for aggregate unemployment, whereas suggesting non-linearity for aggregate employment. At sectoral level the tests strongly reject the hypothesis of linearity. Thus the non-linear characteristics of US unemployment (Brock and Sayers 1988) are not confirmed by our study but important non-linearities are uncovered at the sectoral level.

The conclusion for Canada is almost reversed: evidence against linearity for unemployment coexists with almost unanimous evidence (with the exception of the Tsay test) in support of linearity for aggregate employment. At sectoral level, GARCH and TAR type of effects were not detected in manufacturing but the BDS test statistic rejected the linearity assumption in all cases. These mixed results are not in line with either Frank and Stengos (1988) or Frank, Sayers and Stengos (1993), which failed to detect a significant non-linear structure in the Canadian labour market.

Finally, we considered the multivariate case, where specifications (dynamic time series models) already used in the sectoral shifts literature were employed to examine the power of linear structures. Evidence of significant non-linearities rules out the use of linear VAR models for both countries and endorses the potential importance of asymmetries in testing the impact of employment reallocations.

Additionally, Clements and Krolzig (1998) show that AR models have a competitive forecasting performance from nonlinear (Markov Switching and threshold autoregressive) time series. As a result, the estimated AR models can be used for forecasting. Structural breaks and outliers might be responsible for these results (see Koop and Potter, 2000), although the results of this exercise are not altered when some values (max and min) where omitted. The evidence for non-linearity seems to suggest that asymmetric behaviour is present at a disaggregate level and a fundamental asymmetry exists between the expansion and contraction phases of the sectoral shares.

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Figure 1: US Unemployment Rate and Sectoral Shares Growth Rates

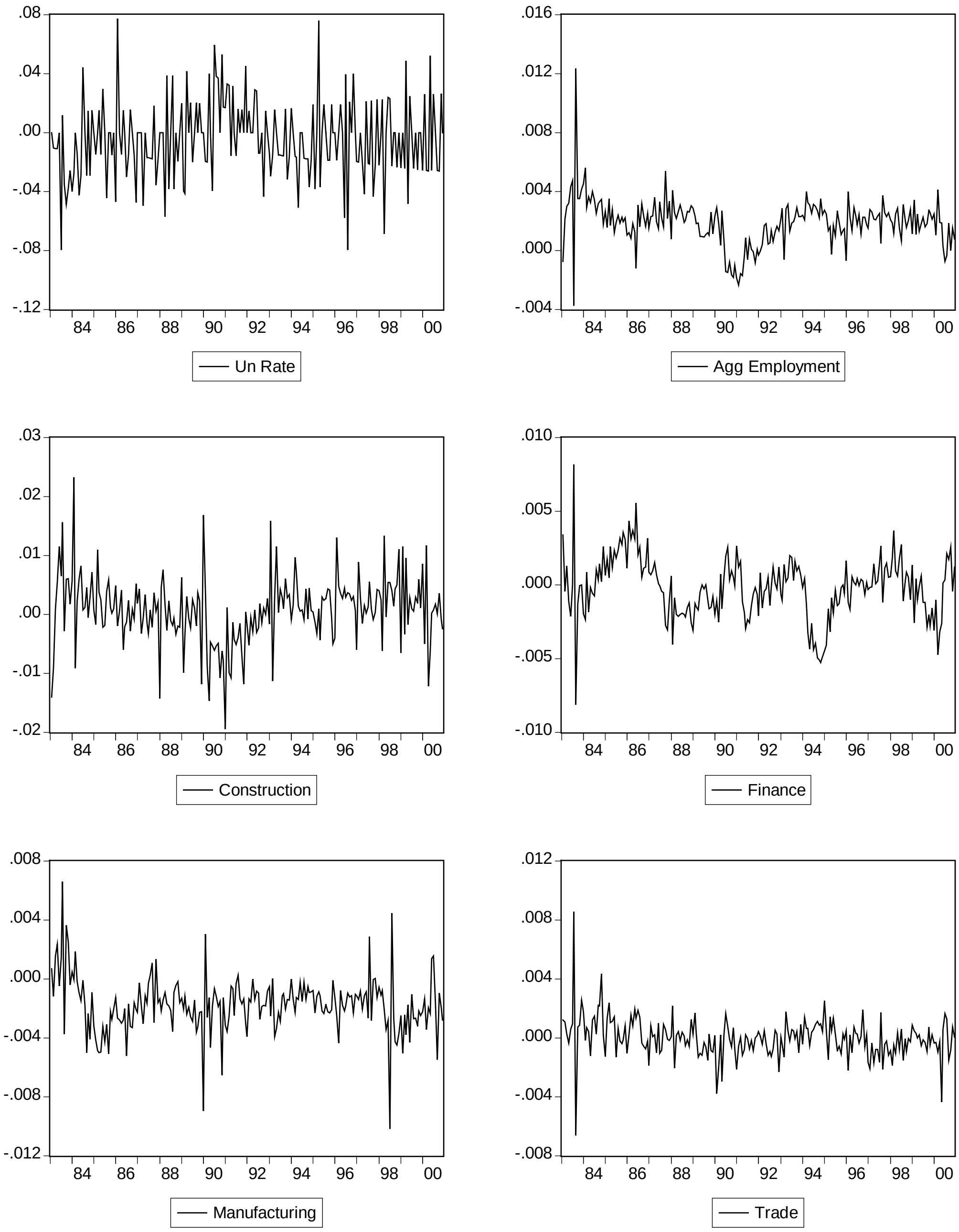

Figure 2: Canadian Unemployment Rate and Sectoral Shares Growth Rates

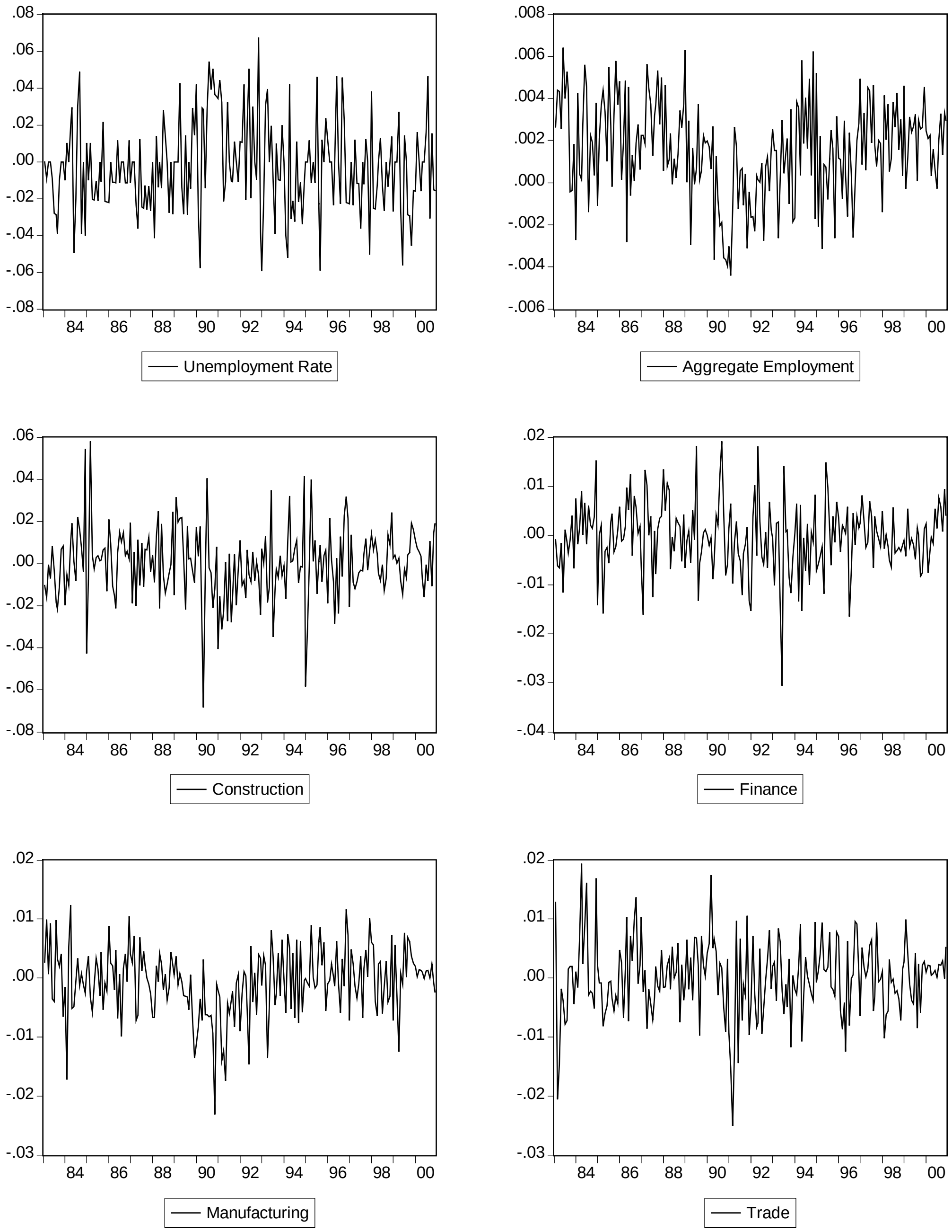

TABLE 3: SUMMARY STATISTICS

	US						Canada					
	UN RATE	AG EMPL	CONSTRUCTION	FINANCE	MANUF	TRADE	UN_RATE	AG EMPL	CONSTRUCTION	FINANCE	MANUF	TRADE
Mean	-0.005	0.002	0.001	0.000	-0.002	0.000	-0.003	0.002	0.000	0.000	-0.001	0.000
Std. Dev.	0.026	0.002	0.006	0.002	0.002	0.001	0.024	0.002	0.017	0.007	0.006	0.006
Skewness	0.146	0.707	-0.010	-0.092	0.057	0.552	0.307	-0.273	-0.126	-0.290	-0.580	-0.171
Kurtosis	3.489	11.449	4.814	4.512	7.297	12.591	2.921	2.662	5.110	4.564	4.081	4.246
J-B	2.898	657.369	29.497	20.777	165.496	835.047	3.442	3.686	40.445	24.914	22.509	14.947
Probability	0.235	0.000	0.000	0.000	0.000	0.000	0.179	0.158	0.000	0.000	0.000	0.001
Obs	215	215	215	215	215	215	215	215	215	215	215	215

TABLE 4: UNIT ROOT TESTS

		US				Canada			
		Levels		First Differences		Levels		First Differences	
		t-statistic	p-value*	t-statistic	p-value*	t-statistic	p-value*	t-statistic	p-value*
Unemployment Rate	ADF	-1.993769	0.2895	-17.3396	0.0000	-0.91394	0.7825	-13.4267	0.0000
	PP	-1.854422	0.3535	-17.2277	0.0000	-1.16889	0.688	-13.7176	0.0000
	Breitung	0.05961	0.4000	0.0013	0.0000	0.02351	0.2667	0.00138	0.0000
Aggregate Employment	ADF	-0.656457	0.8538	-3.58577	0.0068	-0.51049	0.8854	-4.80668	0.0001
	PP	-0.347737	0.9142	-13.155	0.0000	-1.10515	0.714	-13.7094	0.0000
	Breitung	0.09501	0.9000	0.00584	0.0000	0.087	0.9000	0.00449	0.0000
Construction	ADF	-1.441373	0.5614	-4.26764	0.0007	-1.15747	0.6928	-14.2723	0.0000
	PP	-0.871631	0.7958	-15.133	0.0000	-1.18091	0.6829	-14.2678	0.0000
	Breitung	0.01462	0.1100	0.00327	0.0000	0.04737	0.5000	0.00082	0.0000
Finance	ADF	-0.975216	0.7621	-4.63097	0.0002	-0.82917	0.8084	-14.1809	0.0000
	PP	-0.653247	0.8546	-9.97072	0.0000	-0.86948	0.7964	-14.1825	0.0000
	Breitung	0.06595	0.7000	0.0052	0.0000	0.05693	0.6000	0.00075	0.0000
Manufacturing	ADF	-0.318197	0.9187	-6.68422	0.0000	-0.87618	0.7944	-12.4923	0.0000
	PP	-0.194812	0.9357	-14.3321	0.0000	-0.91689	0.7815	-13.1766	0.0000
	Breitung	0.09832	1.0000	0.00154	0.0000	0.07717	0.9000	0.00328	0.0000
Trade	ADF	-0.507229	0.8860	-14.6069	0.0000	-1.93341	0.3165	-12.8368	0.0000
	PP	-0.897863	0.7876	-14.966	0.0000	-2.53839	0.1079	-13.2916	0.0000
	Breitung	0.04323	0.4000	0.0011	0.0000	0.01832	0.1000	0.00031	0.0000

TABLE 5: THE ORDER OF THE AR(*P*) PRE-WHITENING MODEL

LAG	US UNEMPLOYMENT RATE AR(1)	AGGREGATE EMPLOYMENT AR(4)	CONSTRUCTION AR(6)	FINANCE AR(2)	MANUFACTURING AR(2)	TRADE AR(1)	CANADA UNEMPLOYMENT RATE AR(0)	AGGREGATE EMPLOYMENT AR(3)	CONSTRUCTION AR(0)	FINANCE AR(0)	MANUFACTURING AR(0)	TRADE AR(0)
	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient
1	-0.172	-0.04	-0.109	0.32	0.079	-0.001		0.042				
	(2.56)	(0.588)	(1.57)	(5.07)	(1.212)	(0.01)		(0.624)				
2		0.304	0.08	0.395	0.293			0.358				
		(4.77)	(1.23)	(6.29)	(4.473)			(5.665)				
3		0.349	0.208					0.181				
		(5.44)	(3.15)					(2.664)				
4		0.176	0.247									
		(2.619)	(3.69)									
5			0.085									
			(1.26)									
6			0.143									
			(2.19)									
Adj R2	0.025	0.361	0.164	0.389	0.09	0.00	0.00	0.19	0.00	0.00	0.00	0.00
SC	-4.428	-10.386	-7.546	-9.944	-9.739	-10.333	-4.566	-9.437	-5.31	-7.076	-7.505	-7.251
BG	0.98	0.07	0.08	0.183	0.03	0.44	0.24	0.61	0.79	0.71	0.03	0.06

Note: In Table 1, probability refers to the J-B stat. In (•) the *t*-ratio, adj is the Adjusted R^2 and SC the Schwartz Criterion.
ADF is the Augmented Dickey-Fuller test statistic, PP is the Phillips-Perron test statistic, * MacKinnon (1996) one-sided p-values.
Breitung is the Breitung (2002) nonparametric test for unit root where the null hypothesis is that there is a unit root. Breitung CV: 5% 0.01003, 10% 0.01433. The *p*-values reported in this case are simulated. BG is the *p*-value of the Breush-Godfrey test for serial correlation.

TABLE 6: TESTS FOR NON-LINEAR SERIAL DEPENDENCE: US AND CANADA UN. RATE AND SECTORAL SHARE GROWTH RATES.

US	**UNEMPLOYMENT RATE**		**AGGREGATE EMPLOYMENT**		**CONSTRUCTION**		**FINANCE**		**MANUFACTURING**		**TRADE**	
	BOOTSTRAP	ASYMPTOTIC	BOOTSTRAP	ASYMPTOTIC	BOOTSTRAP	ASYMPTOTIC	BOOTSTRAP	ASYMPTOTIC	BOOTSTRAP	ASYMPTOTIC	BOOTSTRAP	ASYMPTOTIC
MCLEOD-LI TEST												
UP TO LAG 20	0.960	0.976	0.006	0.000	0.134	0.149	0.044	0.001	0.057	0.045	0.054	0.008
UP TO LAG 24	0.983	0.992	0.008	0.000	0.179	0.223	0.062	0.003	0.091	0.102	0.075	0.033
BICOVARIANCE TEST												
UP TO LAG 8	0.630	0.736	0.000	0.000	0.551	0.730	0.004	0.000	0.010	0.000	0.078	0.028
ENGLE TEST												
UP TO LAG 1	0.407	0.425	0.003	0.000	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000
UP TO LAG 2	0.664	0.660	0.006	0.000	0.014	0.002	0.000	0.000	0.000	0.000	0.002	0.000
UP TO LAG 3	0.690	0.694	0.018	0.000	0.011	0.002	0.000	0.000	0.000	0.000	0.002	0.000
UP TO LAG 4	0.812	0.827	0.025	0.001	0.016	0.004	0.000	0.000	0.001	0.000	0.007	0.000
TSAY TEST	0.095	0.114	0.197	0.212	0.205	0.211	0.000	0.000	0.017	0.006	0.008	0.001

CANADA	**UNEMPLOYMENT RATE**		**AGGREGATE EMPLOYMENT**		**CONSTRUCTION**		**FINANCE**		**MANUFACTURING**		**TRADE**	
	BOOTSTRAP	ASYMPTOTIC	BOOTSTRAP	ASYMPTOTIC	BOOTSTRAP	ASYMPTOTIC	BOOTSTRAP	ASYMPTOTIC	BOOTSTRAP	ASYMPTOTIC	BOOTSTRAP	ASYMPTOTIC
MCLEOD-LI TEST												
UP TO LAG 20	0.003	0.002	0.890	0.902	0.129	0.168	0.016	0.009	0.159	0.206	0.016	0.005
UP TO LAG 24	0.013	0.007	0.754	0.791	0.205	0.311	0.021	0.014	0.164	0.197	0.025	0.015
BICOVARIANCE TEST												
UP TO LAG 8	0.001	0.000	0.150	0.160	0.025	0.003	0.116	0.108	0.000	0.000	0.015	0.005
ENGLE TEST												
UP TO LAG 1	0.004	0.007	0.572	0.577	0.014	0.006	0.005	0.001	0.052	0.070	0.000	0.000
UP TO LAG 2	0.000	0.000	0.320	0.342	0.011	0.003	0.007	0.001	0.111	0.137	0.002	0.001
UP TO LAG 3	0.000	0.000	0.500	0.523	0.006	0.003	0.013	0.003	0.134	0.184	0.002	0.000
UP TO LAG 4	0.001	0.001	0.671	0.678	0.011	0.002	0.010	0.003	0.093	0.130	0.004	0.000
TSAY TEST	0.000	0.001	0.014	0.019	0.000	0.000	0.224	0.239	0.640	0.660	0.005	0.008

Note: Only *p*-values are reported, under the null hypothesis that the time series is a serially *i.i.d.* process. The logistic transformation is adopted for the unemployment rate and the Sectoral share growth rate is defined as ($\log(U_T)-\log(U_{T-1})$) where U is the Sectoral share ($0<U<1$).

TABLE 7: **BDS TEST** STATISTICS FOR THE US AND CANADIAN UNEMPLOYMENT RATES AND SECTORAL SHARE GROWTH RATES

US																		
Bootstrap	**UN RATE**			**AGG EMPLOYMENT**			**CONSTRUCTION**			**FINANCE**			**MANUFACTURING**			**TRADE**		
Dimension	EPS=0.5	EPS=1	EPS=2	EPS=0.5	EPS=1	EPS=2	EPS=0.5	EPS=1	EPS=2	EPS=0.5	EPS=1	EPS=2	EPS=0.5	EPS=1	EPS=2	EPS=0.5	EPS=1	EPS=2
2	0.991	0.081	0.258	0.000	0.000	0.000	0.000	0.000	0.000	0.013	0.005	0.002	0.046	0.002	0.001	0.027	0.024	0.027
3	0.907	0.113	0.273	0.000	0.000	0.000	0.000	0.000	0.001	0.032	0.004	0.000	0.020	0.000	0.000	0.004	0.002	0.017
4	0.827	0.155	0.166	0.005	0.000	0.000	0.000	0.000	0.002	0.040	0.001	0.000	0.022	0.000	0.000	0.007	0.003	0.005
Asymptotic																		
2	0.999	0.073	0.259	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.002	0.000	0.030	0.000	0.000	0.010	0.014	0.004
3	0.975	0.111	0.293	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.002	0.000	0.007	0.000	0.000	0.000	0.001	0.003
4	0.945	0.152	0.153	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.003	0.000	0.000	0.000	0.000	0.001

CANADA																		
Bootstrap	**Un Rate**			**Agg Employment**			**Construction**			**Finance**			**Manufacturing**			**Trade**		
Dimension	EPS=0.5	EPS=1	EPS=2	EPS=0.5	EPS=1	EPS=2	EPS=0.5	EPS=1	EPS=2	EPS=0.5	EPS=1	EPS=2	EPS=0.5	EPS=1	EPS=2	EPS=0.5	EPS=1	EPS=2
2	0.000	0.000	0.001	0.841	0.826	0.713	0.035	0.008	0.002	0.014	0.011	0.002	0.000	0.001	0.010	0.000	0.005	0.003
3	0.000	0.000	0.000	0.623	0.505	0.362	0.000	0.000	0.000	0.027	0.010	0.002	0.000	0.000	0.002	0.000	0.011	0.003
4	0.000	0.000	0.000	0.488	0.318	0.262	0.000	0.000	0.000	0.004	0.004	0.000	0.000	0.000	0.001	0.000	0.021	0.002
Asymptotic																		
2	0.000	0.000	0.001	0.928	0.844	0.747	0.021	0.005	0.000	0.004	0.004	0.000	0.000	0.000	0.002	0.000	0.002	0.000
3	0.000	0.000	0.000	0.730	0.519	0.351	0.000	0.000	0.000	0.009	0.005	0.001	0.000	0.000	0.000	0.000	0.005	0.000
4	0.000	0.000	0.000	0.461	0.300	0.233	0.000	0.000	0.000	0.008	0.003	0.002	0.000	0.000	0.000	0.000	0.011	0.000

TABLE 8: TESTS FOR NON-LINEAR SERIAL DEPENDENCE: THE MULTIVARIATE CASE

US								
Equations	**1.1**				**1.2**			
	U1t		**U3t**		**U2t**		**U3t**	
	BOOTSTRAP	**ASYMPTOTIC**	**BOOTSTRAP**	**ASYMPTOTIC**	**BOOTSTRAP**	**ASYMPTOTIC**	**BOOTSTRAP**	**ASYMPTOTIC**
MCLEOD-LI TEST								
USING UP TO LAG 20	0.044	0.001	0.086	0.083	0.717	0.764	0.038	0.019
USING UP TO LAG 24	0.054	0.004	0.128	0.164	0.814	0.860	0.065	0.042
BICOVARIANCE TEST								
UP TO LAG 8	0.000	0.000	0.058	0.014	0.058	0.047	0.006	0.000
ENGLE TEST								
USING UP TO LAG 1	0.000	0.000	0.000	0.000	0.432	0.387	0.000	0.000
USING UP TO LAG 2	0.000	0.000	0.000	0.000	0.732	0.687	0.000	0.000
USING UP TO LAG 3	0.000	0.000	0.000	0.000	0.810	0.786	0.001	0.000
USING UP TO LAG 4	0.000	0.000	0.000	0.000	0.872	0.865	0.001	0.000
TSAY TEST	0.006	0.000	0.006	0.001	0.012	0.013	0.005	0.001
CANADA	**2.1**				**2.2**			
	U1t		**U3t**		**U2t**		**U3t**	
MCLEOD-LI TEST								
USING UP TO LAG 20	0.890	0.902	0.711	0.774	0.354	0.375	0.718	0.782
USING UP TO LAG 24	0.724	0.762	0.475	0.554	0.514	0.555	0.558	0.625
BICOVARIANCE TEST								
UP TO LAG 8	0.155	0.171	0.730	0.833	0.111	0.108	0.335	0.429
ENGLE TEST								
USING UP TO LAG 1	0.735	0.733	0.457	0.474	0.010	0.007	0.648	0.643
USING UP TO LAG 2	0.509	0.542	0.099	0.133	0.030	0.023	0.165	0.190
USING UP TO LAG 3	0.722	0.736	0.079	0.108	0.056	0.057	0.067	0.099
USING UP TO LAG 4	0.857	0.865	0.136	0.187	0.103	0.105	0.137	0.176
TSAY TEST	0.253	0.254	0.735	0.751	0.011	0.010	0.432	0.484

Note: Only *p*-values are reported, under the null hypothesis that the time series is a serially *i.i.d.* process.). U_{1t}, U_{2t} and U_{3t} are the residuals form the VAR analysis (equations 1.1, 1.2 , 2.1 and 2.2).

TABLE 9:**BDS TEST** STATISTICS FOR THE MULTIVARIATE CASE

US	Eq 1.1						Eq. 1.2					
Bootstrap	U_{1t}			U_{3t}			U_{2t}			U_{3t}		
Dimension	EPS=0.5	EPS=1	EPS=2	EPS=0.5	EPS=1	EPS=2	EPS=0.5	EPS=1	EPS=2	EPS=0.5	EPS=1	EPS=2
2	0.003	0.001	0.001	0.152	0.000	0.002	0.483	0.155	0.215	0.001	0.000	0.001
3	0.001	0.001	0.000	0.109	0.000	0.000	0.442	0.297	0.226	0.000	0.000	0.000
4	0.000	0.000	0.000	0.038	0.000	0.000	0.310	0.396	0.162	0.002	0.000	0.000
Asymptotic												
2	0.000	0.000	0.000	0.133	0.001	0.000	0.532	0.120	0.172	0.000	0.000	0.000
3	0.000	0.000	0.000	0.088	0.000	0.000	0.475	0.300	0.216	0.000	0.000	0.000
4	0.000	0.000	0.000	0.016	0.000	0.000	0.253	0.442	0.141	0.000	0.000	0.000

Canada	Eq. 2.1						2.2					
Bootstrap	U_{1t}			U_{3t}			U_{2t}			U_{3t}		
Dimension	EPS=0.5	EPS=1	EPS=2	EPS=0.5	EPS=1	EPS=2	EPS=0.5	EPS=1	EPS=2	EPS=0.5	EPS=1	EPS=2
2	0.752	0.748	0.430	0.058	0.090	0.202	0.001	0.000	0.034	0.190	0.247	0.401
3	0.800	0.558	0.214	0.005	0.008	0.037	0.000	0.000	0.032	0.082	0.101	0.128
4	0.799	0.314	0.131	0.002	0.003	0.033	0.000	0.000	0.033	0.045	0.088	0.134
Asymptotic												
2	0.842	0.800	0.437	0.016	0.070	0.209	0.000	0.000	0.013	0.162	0.269	0.458
3	0.918	0.584	0.200	0.000	0.005	0.022	0.000	0.000	0.016	0.046	0.090	0.121
4	0.951	0.315	0.092	0.000	0.000	0.017	0.000	0.000	0.017	0.007	0.071	0.134

Note: Only *p*-values are reported, under the null hypothesis that the time series is a serially *i.i.d.* process.). U_{1t}, U_{2t} and U_{3t} are the residuals form the VAR analysis

The estimations in our exercise are carried out using GAUSS 3.2, Eviews 4.1 and the Nonlinear Toolkit 4.6 by Patterson and Ashley (2000).

TECHNICAL APPENDIX

BDS TEST FOR RANDOMNESS

A powerful test used for independence (and, under certain circumstances, for non-linear dependencies) was developed by Brock, Dechert, and Scheinkman (1996) and is based on the correlation integral. The BDS statistic tests the null hypothesis that the elements of a time series are independently and identically distributed (IID). For a time series which is IID, the distribution of the statistic:

$$W_{m,T}(\varepsilon) = \frac{\sqrt{T}\left\{C_{m,T} - C_{1,T}(\varepsilon)^m\right\}}{\sqrt{\sigma^2_{m,T}(\varepsilon)}}$$

is asymptotically $N(0,1)$.W_m(e) is known as the BDS statistic. C_m(e) denotes the fraction of m-tuples in the series, which are within a distance of each other and $\sigma_m(\varepsilon)$ is an estimate of the standard deviation under the null hypothesis of *IID*. The test statistic is asymptotically standard normal under the null of whiteness. The null is rejected if the test statistic is absolutely large, (say greater than 1.96). If the null hypothesis of *IID* cannot be accepted this implies that the residuals contain some kind of hidden structure, which might be non-linear (or even be chaotic).

MCLEOD AND LI TEST

The McLeod and Li test (McLeod and Li, 1983) can be used as a portmanteau test of non-linearity. To test for non-linear effects in time series data McLeod and Li have proposed the statistic:

$$Q(m) = \frac{n(n+2)}{n-k}\sum_{k=1}^{m} r_a^2(k)$$

where

$$r_a^2(k) = \frac{\sum_{t=k+1}^{n} e_t^2 e_{t-k}^2}{\sum_{t=1}^{n} e_t^2} \qquad k = 0,1,\ldots, n\text{-}1$$

are the autocorrelations of the squared residuals, e_t^2, obtained from fitting a model to the data. If the series e_t is independently and identically distributed (*IID*) then the asymptotic distribution of *Q(m)* is χ^2 with m degrees of freedom.

ENGLE LM TEST

This test was suggested by Engle (1982) to detect ARCH disturbances. Bollerslev (1986) suggests that it should also have power against GARCH alternatives. Since it is a Lagrange Multiplier test, the test statistic itself is based on the R^2 of an auxiliary regression, which in this case can be defined as:

$$e_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_k e_{t-i}^2 + v_t$$

Under the null hypothesis of a linear generating mechanism for e_t, NR^2 for this regression is asymptotically $\chi^2(p)$.

HINICH BICOVARIANCE TEST

This test assumes that $\{e_t\}$ is a realisation from a third-order stationary stochastic process and tests for serial independence using the sample bicovariances of the data. The (r,s) sample bicovariance is defined as :

$$C_3(r,s) = (N-s)^{-1} \sum_{t=1}^{N-s} e_t e_{t+r} e_{t+s} \qquad 0 \le r \le s$$

The sample bicovariances are thus a generalisation of a skewness parameter. The $C_3(r,s)$ are all zero for zero mean, serially *i.i.d.* data. One would expect non-zero values for the $C_3(r,s)$ from data in which e_t depends on lagged crossproducts, such as $e_{t-i}e_{t-j}$ and higher order terms.

Let $G(r,s) = (N\text{-}s)^{1/2} C_3(r,s)$ and define X_3 as

$$X_3 = \sum_{s=2}^{l} \sum_{r=1}^{s-1} [G(r,s)]^2$$

Under the null hypothesis that $\{e_t\}$ is a serially *i.i.d.* process, Hinich and Patterson (1995) show that X_3 is asymptotically distributed $\chi^2(l[l-1]/2)$ for $l<N^{1/2}$

Based on their simulations, they recommend using $l = N^4$. Under the assumption that $E(x_t^{12})$ exists, the X_3 statistic detects non-zero third order correlations. It can be considered a generalisation of the Box-Pierce portmanteau statistic.

TSAY TEST

The Tsay (1986) test is a generalisation of the Keenan (1985) test. It explicitly looks for quadratic serial dependence in the data.

Let $K=k(k-1)/2$ column vectors $V_1,\ldots,V_k$ contain all of the possible cross-products of the form $e_{t-i}e_{t-j}$, where $i \in [1,k]$ and $j \in [i,k]$. Thus, $v_{t,1} = e_{t-1}^2$, $v_{t,2}=e_{t-1}e_{t-2}$, $v_{t-3}=e_{t-1}e_{t-3}$, $v_{t,k+1}=e_{t-2}e_{t-3}$, $v_{t,k+2}=e_{t-2}e_{t-4}$,.., $v_{t,k} = e_{t-k}^2$ .And let $\hat{v}_{t,j}$ denote the projection of $v_{t,i}$ on the subspace orthogonal $e_{t-1},\ldots,e_{t-k}$, (i.e. the residuals from a regression of $v_{t,j}$ on $e_{t-1},\ldots,e_{t-k}$.)

The parameters $\gamma_1,\ldots,\gamma_k$ are then estimated by applying OLS to the regression equation

$$e_t = \gamma_0 + \sum_{i=1}^{K} \gamma_i \hat{v}_{t,i} + \eta_t$$

Note that the *jth* regressor in this equation is $\hat{v}_{t,j}$, the period t fitting error from a regression of $v_{t,j}$ on $e_{t-1},\ldots,e_{t-k}$. For p exceeding K, this projection is unnecessary for the dependent variable $\{e_t\}$ if it is pre-whitened using an AR(p) model. The Tsay test statistic then is just the usual F statistic for testing the null hypothesis that $\gamma_1,\ldots,\gamma_k$ are all zero.