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Sustained Output Growth Under Uncertainty: A Simple Model with Human Capital

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Abstract

In a model where agents use their labour/education choice to adjust their consumption profile over time, I show that the impact of uncertainty on growth depends, critically, on agents' attitudes towards risk, reflected by the coefficient of relative risk aversion. In this respect, the well known result from the literature on 'saving under uncertainty' can be extended into a broader context, whereby the intertemporal profile of consumption is determined via human capital accumulation rather than saving and physical capital investment.

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1 Introduction

During the late 1960s and early 1970s, a variety of theoretical analyses began exploring the impact of aggregate uncertainty on saving decisions (e.g., Levhari and Srinivasan, 1969; Sandmo, 1970; Mirman, 1971; Rothschild and Stiglitz, 1971). Under different settings, all these analyses seemed to reach a consensus on the importance of attitudes towards risk on determining the reaction of saving rates to higher degrees of future uncertainty. Specifically, the main conclusion derived from the aforementioned analyses is that, in response to higher degrees of uncertainty, saving rates increase (decrease) if the coefficient of relative risk aversion is above (below) unity. When the coefficient of relative risk aversion is equal to one (i.e., the case of logarithmic utility) saving is unresponsive to aggregate uncertainty.

More recently, the work of Romer (1986) revived an important idea (originally proposed by Frankel, 1962) within a context of a production economy with intertemporal consumer maximisation. He showed that if the investment activity that adds to the aggregate stock of capital can generate and spread additional knowledge, and if the relative importance of knowledge on productivity is sufficiently high, then the economy can reach an equilibrium with ever increasing levels of output (or, equivalently, a *sustainable* and *endogenously determined* growth rate of output). The upshot from Romer's analysis was that the factors normally impinging on saving rates (and, therefore, aggregate investment) can improve our understanding of the differences in growth rates and, to some extent, potential standards of living across economies.

Of course, it was not long before theorists made the apparent connection and understood that, as long as uncertainty is an important consideration behind saving motives and behaviour, higher degrees of uncertainty may have significant long-term implications in terms of output growth trends. In particular, the theoretical analyses by Smith (1996), de Hek (1999) and Jones *et al.* (2005) addressed the issue of the interaction between uncertainty and long-run output growth within the context of dynamic, general equilibrium models with endogenous mechanisms for productivity improvements and stochastic elements arising from the presence of technology (or productivity) shocks. Their results verify the importance of the coefficient of relative risk aversion as this was described within the various analytical frameworks of the literature on optimal savings under uncertainty – a literature to which I alluded earlier.[1]

The models constructed by Smith (1996), de Hek (1999) and Jones *et al.* (2005), despite being different in terms of their overall structure, share one common future: all types of capital accumulate through savings – that is, agents decide to sacrifice their current consumption and devote a certain fraction of their produced output with the purpose building up some capital stock that will facilitate future production and consumption. Nevertheless, pecuniary elements need not be the only ones to serve in the accumulation of capital. As the work of Uzawa (1968), Razin (1972) and Lucas (1988) suggested, another important aspect in the formation of capital (especially human capital), the accumulation of knowledge and, therefore, the driving force behind long-run growth involves the various human resources, like time or effort, that individuals devote with the purpose of improving their future productive capacity.

Naturally, in such scenarios the nature of the trade-offs between current and/or future benefits are slightly different from the standard consumption-saving choice. For example, we devote more time towards human capital accumulation in order to improve our future consumption possibilities, rather than working in order to achieve more current consumption. We may even choose to devote more time/effort towards both labour and (human capital) investment at the expense of our leisure. The question emerging is the following: to what extent do the aforementioned results on saving and growth under uncertainty survive within a framework in which the endogenous process behind sustainable growth resembles the one put forward by Uzawa (1968), Razin (1972) and Lucas (1988)? In this paper I construct a simple model in which I show that, indeed, the main implication of the papers comprising the literature on 'growth under uncertainty' is not just a mere extension of the conclusions reached from the literature on 'optimal saving under uncertainty'. My model shows that the basic premise of uncertainty promoting (impeding) trend growth whenever the coefficient of relative risk aversion is above (below) unity may still emerge in an environment where there is no actual saving involved and growth is driven through purposeful time/effort devoted towards human capital accumulation.

[1] In the same paper, de Hek (1999) analyses a second model with human capital. However, he restricts his attention to logarithmic utility.

The rest of the paper is organised as follows: Section 2 presents the basic model. In section 3 I define and derive the dynamic equilibrium and in Section 4 I show the impact of uncertainty on economic growth. Section 5 discusses and concludes.

2 The Model

The framework belongs to the 'representative agent' class of models. Imagine an artificial economy populated by a single producer/consumer of a perishable commodity. At the beginning of each period, the representative agent is endowed with a unit of time or effort and she produces y_t units of the good according to

$$y_t = \omega_t (1 - \chi_t), \tag{1}$$

where $1 - \chi_t$ indicates the amount of time allocated to output production and ω_t is the agent's productivity in transforming her labour into consumable output.

Productivity growth is the driving force behind a sustainable growth rate of output. I shall assume that productivity has two components – an exogenous, stochastic component and an endogenous element allowing the agent to devote resources as to improve her future productivity. This idea can be captured by assuming that ω_t takes the form

$$\omega_t = \mathrm{A}_t h_t^{\psi}, \ \psi \in (0,1]. \tag{2}$$

In equation (2), A_t captures the stochastic component of productivity. I assume that it grows exogenously according to

$$\mathrm{A}_t = a_t \mathrm{A}_{t-1}, \tag{3}$$

where $\{a_t\}_{t=1}^{\infty}$ is a sequence of positively valued and bounded, independently and identically distributed random variables with constant mean, denoted by $\overline{a}$, and constant variance. The endogenous component of productivity is captured by the variable h_t, in equation (2), which grows according to

$$h_{t+1} = \mathrm{H}\chi_t^{\eta} h_t, \ \eta \in (0,1]. \tag{4}$$

Given the specification in (4), h_t can be narrowly defined as human capital but may also be defined in a broader sense if χ_t is thought as including all types of activities that the individual undertakes as to improve her efficiency in producing output (e.g., R&D, training etc.).[2]

The representative agent's preferences are described by a lifetime expected utility function of the CRRA form. That is,

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\varrho} - 1}{1-\varrho} \right), \ \varrho > 0, \beta \in (0,1), \tag{5}$$

where c_t denotes consumption, E_0 is the conditional expectations operator, β is a discount factor and ϱ is the coefficient of relative risk aversion.

The agent can consume only out of her available income/output. Therefore, her per-period budget constraint has the simple form

$$c_t \leq \omega_t (1 - \chi_t). \tag{6}$$

Having described the underlying characteristics of the artificial economy, my next step is the solution of the model. This is done in the following Section.

3 Equilibrium

In this section, I provide the analytical derivation the model's equilibrium which is characterised by the following

Definition. *Given the initial value* $h_0 > 0$*, the dynamic, general equilibrium is a sequence of quantities* $\left\{c_t, y_t, \chi_t, \mathrm{A}_t, a_{t+1}, h_{t+1}\right\}_{t=0}^{\infty}$ *such that:*

(i) *Given* $\left\{\mathrm{A}_t, a_{t+1}\right\}_{t=0}^{\infty}$*, the quantities* $\left\{c_t, \chi_t, h_{t+1}\right\}_{t=0}^{\infty}$ *solve the representative agent's optimisation problem.*

(ii) χ_t *is stationary.*

(iii) *The goods market clears every period, i.e.,* $c_t = y_t \ \ \forall t \geq 0$.

(iv) *The transversality condition holds.*

[2] Sustainable output growth is feasible because productivity can sustain its growth rate in the long-run. To see this, use (2)-(4) to write productivity as $\omega_t = a_t (\mathrm{H}\chi_{t-1}^{\eta})^{\phi} \omega_{t-1}$. The presence of the variable χ_{t-1} justifies the labeling of the growth model as 'endogenous'.

Denoting the Lagrange multipliers associated with (6) and (4) by λ_t and ξ_t respectively, the first order conditions for the agent's maximisation problem are

$$c_t^{-\varrho} = \lambda_t , \tag{7}$$

$$\lambda_t \mathrm{A}_t h_t^{\psi} = \xi_t \eta \mathrm{H} \chi_t^{\eta-1} h_t , \tag{8}$$

$$\xi_t = \beta E_t(\mathrm{H}\chi_{t+1}^{\eta}) + \beta E_t[\lambda_{t+1}\psi \mathrm{A}_{t+1} h_{t+1}^{\psi-1}(1-\chi_{t+1})] . \tag{9}$$

The condition in (7) equates the marginal utility of consumption with the shadow value of foregone wealth. The condition in (8) equates the marginal benefits from devoting more time/effort towards output production and the accumulation of capital. The marginal benefit from increased labour effort corresponds to the utility benefit of higher current consumption. Given the condition in (9), the marginal benefit for devoting more time to human capital accumulation is associated with the discounted expected utility value of future consumption (resulting from the corresponding increase of future output) and with the further evolution of the capital stock in the future.

Let us embark on the solution of the model by using (1), (2) and (4) in (8) to derive

$$\frac{\lambda_t y_t}{1-\chi_t} = \frac{\eta \xi_t h_{t+1}}{\chi_t} . \tag{10}$$

Multiplying both sides of (9) by h_{t+1} and taking account of (1), (2) and (4) yields

$$\xi_t h_{t+1} = \beta E_t(\xi_{t+1} h_{t+2}) + \beta\psi E_t(\lambda_{t+1} y_{t+1}) . \tag{11}$$

Next, define $\xi_t h_{t+1} \equiv J_t$ and substitute it together with (7) and the equilibrium condition $c_t = y_t$ in (9) to derive

$$J_t = \beta E_t(J_{t+1}) + \beta\psi E_t(y_{t+1}^{1-\varrho}) . \tag{12}$$

We can use (12) to substitute recursively for T times and apply the law of iterated expectations. Eventually, this procedure yields

$$J_t = \beta^T E_t(J_{t+T}) + \beta\psi E_t(y_{t+1}^{1-\varrho}) + \beta^2\psi E_t(y_{t+2}^{1-\varrho}) + \beta^3\psi E_t(y_{t+3}^{1-\varrho}) + ... + \beta^T\psi E_t(y_{t+T}^{1-\varrho}) . \tag{13}$$

The next step involves the derivation of a solution for (13). Taking account of (1) and (2) we get

$$\frac{y_{t+T}}{y_{t+T-1}} = \frac{\mathrm{A}_{t+T}}{\mathrm{A}_{t+T-1}} \left(\frac{h_{t+T}}{h_{t+T-1}} \right)^{\psi} \frac{1-\chi_{t+T}}{1-\chi_{t+T-1}} . \tag{14}$$

At this point we can make a guess concerning the form of time allocation decisions in equilibrium. Specifically, we may conjecture that $\chi_t = \overline{\chi} \forall t$.[3] Consequently, taking account of (3) and (4), we can rewrite (14) as

$$y_{t+T} = va_{t+T}\, y_{t+T-1}\,, \tag{15}$$

where $v = (\mathrm{H}\overline{\chi}^{\eta})^{\psi}$. Substituting recursively in (15) yields

$$y_{t+T} = y_0 v^{t+T} \prod_{i=1}^{t+T} a_i \,. \tag{16}$$

Recall that the realisations of the shock for all periods $i = 0,1...t$ are part of the agent's information set available at period t. We can take account of this when substituting (16) in (13). Doing so, and factorising with $\beta\psi y_0^{1-\varrho}(v^{t+1})^{1-\varrho}\left(\prod_{i=1}^{t} a_i\right)^{1-\varrho} E_t(a_{t+1}^{1-\varrho})$, yields

$$\begin{aligned} J_t = \beta^T E_t(J_{t+T}) + \beta\psi y_0^{1-\varrho}(v^{t+1})^{1-\varrho}\left(\prod_{i=1}^{t} a_i\right)^{1-\varrho} E_t(a_{t+1}^{1-\varrho})\big[1+...\\ ...+\beta v^{1-\varrho}E_t(a_{t+2}^{1-\varrho}) + \left(\beta v^{1-\varrho}\right)^2 E_t(a_{t+3}^{1-\varrho})E_t(a_{t+2}^{1-\varrho}) + ...\\ ...+\left(\beta v^{1-\varrho}\right)^3 E_t(a_{t+4}^{1-\varrho})E_t(a_{t+3}^{1-\varrho})E_t(a_{t+2}^{1-\varrho}) + ...\\ ...+\left(\beta v^{1-\varrho}\right)^{t+T-1} E_t(a_{t+T}^{1-\varrho})E_t(a_{t+T-1}^{1-\varrho})\cdots E_t(a_{t+2}^{1-\varrho})\big]. \end{aligned} \tag{17}$$

Given that shocks are iid and generate constant mean and variance, we can define

$$E_t(a_{t+s}^{1-\varrho}) = \Theta \forall s \geq 1\,. \tag{18}$$

Substituting (18) in (17) and considering the limiting case of $T \to \infty$ yields

$$\begin{aligned} J_t = \lim_{T\to\infty}\beta^T E_t(J_{t+T}) + \beta\psi y_0^{1-\varrho}(v^{t+1})^{1-\varrho}\left(\prod_{i=1}^{t} a_i\right)^{1-\varrho} E_t(a_{t+1}^{1-\varrho})[1+...\\ ...+\beta v^{1-\varrho}\Theta + (\beta v^{1-\varrho}\Theta)^2 + (\beta v^{1-\varrho}\Theta)^3 + (\beta v^{1-\varrho}\Theta)^4 ...\ \big]. \end{aligned} \tag{19}$$

Equation (15) implies that

$$\left(\frac{y_{t+s}}{y_{t+s-1}}\right)^{1-\varrho} = \left(va_{t+s}\right)^{1-\varrho}.$$

Taking expectations on both sides and using (18), we derive

[3] This will be true because the equilibrium solution for χ_t will depend on the current expectation made about the future value for a_{t+1}. The equilibrium value for χ_t will be time invariant, given that this random variable generates constant mean and variance.

$$E_t\left[\left(\frac{y_{t+s}}{y_{t+s-1}}\right)^{1-\varrho}\right] = v^{1-\varrho}E_t(a_{t+s}^{1-\varrho}) = \Theta v^{1-\varrho} \equiv \tilde{g}. \tag{20}$$

Furthermore, we can use (16) to derive

$$y_0^{1-\varrho}(v^{t+1})^{1-\varrho}\left(\prod_{i=1}^{t} a_i\right)^{1-\varrho} E_t(a_{t+1}^{1-\varrho}) = E_t\left[y_0^{1-\varrho}(v^{t+1})^{1-\varrho}\left(\prod_{i=1}^{t+1} a_i\right)^{1-\varrho}\right] = E_t(y_{t+1}^{1-\varrho}). \tag{21}$$

Substituting (20) and (21) in (19) yields

$$J_t = \lim_{T\to\infty}\beta^T E_t(J_{t+T}) + \beta\psi E_t(y_{t+1}^{1-\varrho})[1+\beta\tilde{g}+(\beta\tilde{g})^2+(\beta\tilde{g})^3+...]. \tag{22}$$

Assuming $\beta\tilde{g}<1$ and imposing the transversality condition $\lim_{T\to\infty}\beta^T J_{t+T}=0$, we can get a solution for (22) as [4]

$$J_t \equiv \xi_t b_{t+1} = \frac{\beta\psi E_t(y_{t+1}^{1-\varrho})}{1-\beta\tilde{g}}. \tag{23}$$

Substituting (23) and (7) in (10), using $\chi_t=\overline{\chi}$, multiplying both sides by $\overline{\chi}/y_t^{1-\varrho}$ and taking account of (20) yields

$$\frac{\overline{\chi}}{1-\overline{\chi}} = \eta\psi\beta\frac{E_t\left[(y_{t+1}/y_t)^{1-\varrho}\right]}{1-\beta\tilde{g}} = \frac{\eta\psi\beta\tilde{g}}{1-\beta\tilde{g}}, \tag{24}$$

which solving for $\overline{\chi}$ leads us to

$$\overline{\chi} = \frac{\eta\psi\beta\tilde{g}}{1-\beta\tilde{g}(1-\eta\psi)}. \tag{25}$$

Before I proceed to the derivation of the growth rate, there is the issue of identifying the characteristics of the solution for $\overline{\chi}$. A useful initial step comes in the form of

Lemma 1. *Define* $P(\overline{\chi})=\beta\tilde{g}=\beta\Theta\mathrm{H}^{\psi(1-\varrho)}\overline{\chi}^{\eta\psi(1-\varrho)}$ *and* $Q(\overline{\chi})=\overline{\chi}/[\overline{\chi}(1-\eta\psi)+\eta\psi]$. *Then* $P(0)=0$ *for* $\varrho\in(0,1)$, $P(0)\to\infty$ *for* $\varrho>1$, $P(1)<1$, $Q(0)=0$ *and* $Q(1)=1$. *Furthermore,* $P'(\overline{\chi})>0$ *and* $P''(\overline{\chi})<0$ *if* $\varrho\in(0,1)$ *and* $P'(\overline{\chi})<0$ *and* $P''(\overline{\chi})>0$ *if* $\varrho>1$. *Also* $Q'(\overline{\chi})>0$ *and* $Q''(\overline{\chi})<0$.

[4] Some slightly tedious algebra can verify the solution in (23) after direct substitution back in (12).

Proof. After appropriate substitution, it is obvious that $Q(0) = 0$, $Q(1) = 1$, $P(0) = 0$ and $P(1) < 1$ because $\beta\tilde{g} < 1$ must hold by assumption. Differentiating, we get $Q'(\overline{\chi}) = \eta\psi / [\overline{\chi}(1-\eta\psi) + \eta\psi]^2 > 0$ and $Q''(\overline{\chi}) = -2\eta\psi(1-\eta\psi) / [\overline{\chi}(1-\eta\psi) + \eta\psi]^3 < 0$, as well as $P'(\overline{\chi}) = \eta\psi(1-\varrho)\beta\Theta\mathrm{H}^{\psi(1-\varrho)}\overline{\chi}^{\eta\psi(1-\varrho)-1}$ and $P''(\overline{\chi}) = [\eta\psi(1-\varrho)-1]\eta\psi(1-\varrho)\beta\Theta\mathrm{H}^{\psi(1-\varrho)}\overline{\chi}^{\eta\psi(1-\varrho)-2}$. Obviously, for $\varrho \in (0,1)$ we have $P'(\overline{\chi}) > 0$ and $P''(\overline{\chi}) < 0$, while for $\varrho > 1$ we have $P'(\overline{\chi}) < 0$ and $P''(\overline{\chi}) > 0$. ■

Given the above, the characterisation of the solution for $\overline{\chi}$ comes in the form of

Lemma 2. *As long as the optimal time allocation decisions are characterised by an interior solution for* $\overline{\chi}$*, then this solution is unique.*

Proof. Rearrange equation (25) to get $\beta\tilde{g} = \beta\Theta\mathrm{H}^{\psi(1-\varrho)}\overline{\chi}^{\eta\psi(1-\varrho)} = \overline{\chi} / [\overline{\chi}(1-\eta\psi) + \eta\psi]$ or, equivalently, $P(\overline{\chi}) = Q(\overline{\chi})$. According to the result in Lemma 1, both functions are continuous. Additionally, notice that $P'(0) \to \infty$ for $\varrho \in (0,1)$. Thus, $P'(0) > Q'(0) = 1/\eta\psi$ for $\varrho \in (0,1)$. Consequently, taking account of Lemma 1, we can conclude that $\forall \varrho > 0$ there exists some $\overline{\chi} \in (0,1)$ such that $P(\chi) > Q(\chi)$ for $\chi \in (0, \overline{\chi})$ and $P(\chi) < Q(\chi)$ for $\chi \in (\overline{\chi}, 1]$. ■

The only situation where the solution for $\overline{\chi}$ is explicit, is when $\eta = \psi = 1$. In that case, we can use (25) to get $\overline{\chi} = \beta\tilde{g} = \beta\Theta v^{1-\varrho} = \beta\Theta(\mathrm{H}\overline{\chi})^{1-\varrho}$ which, solving for $\overline{\chi}$, yields

$$\overline{\chi} = \mathrm{H}^{(1-\varrho)/\varrho}(\beta\Theta)^{1/\varrho}. \tag{26}$$

Nonetheless, despite the fact that, in most cases, we get the solution for $\overline{\chi}$ implicitly, its response to uncertainty (which also determines the impact of uncertainty on output growth) is clear-cut. This is an issue to which I turn in the subsequent Section.

4 Output Growth Under Uncertainty

Prior to illustrating how uncertainty impinges on economic growth, I shall utilise a result that will facilitate us on understanding the mechanism involved behind the response of growth to higher degrees of aggregate uncertainty. This result is given as

Theorem 1. *Let x be a random variable and $f(x)$ a continuous function. A mean-preserving spread in the distribution of x increases (decreases) $Mean[f(x)]$ if the function $f(x)$ is strictly convex (strictly concave). If $f(x)$ is linear then a mean-preserving spread in the distribution of x does not affect $Mean[f(x)]$.*

Proof. This is a well known result that can be proven through a variety of approaches. One formal proof appears in Rothschild and Stiglitz (1970) among others. ■

A corollary derived from Theorem 1 takes the form of

Lemma 3. *Denote a mean-preserving spread in the distribution of $\{a_t\}_{t=1}^{\infty}$ by σ. Then $\Theta \equiv \theta(\sigma)$ such that $\theta'(\cdot) < 0$ iff $\varrho \in (0,1)$ and $\theta'(\cdot) > 0$ iff $\varrho > 1$. If $\varrho = 1$ then $\Theta = 1$.*

Proof. Revisit equation (18) and observe that the function $a_{t+s}^{1-\varrho}$ is strictly concave for $\varrho \in (0,1)$, equal to unity for $\varrho = 1$ and strictly convex for $\varrho > 1$. Consequently, application of Theorem 1 leads to the conclusion that a mean-preserving spread in the distribution of $a_i \forall i \geq 1$ either reduces, increases or leaves Θ unaffected depending on whether $\varrho \in (0,1)$, $\varrho > 1$ or $\varrho = 1$ respectively. ■

Now, I am ready to derive the main result from my analysis. This can be illustrated through

Theorem 2. *An increase in uncertainty, measured by a mean-preserving spread in the distribution of $a_i \forall i \geq 1$, leads to a decrease (increase) in output growth, γ, iff $\varrho \in (0,1)$ ($\varrho > 1$). If $\varrho = 1$ an increase in uncertainty does not have any effect on long-run growth.*

Proof. The growth rate is defined as y_{t+1} / y_t. Given (1), (3) and (4), then

$$\frac{y_{t+1}}{y_t} = a_{t+1} \frac{1-\chi_{t+1}}{1-\chi_t} \mathrm{H}^{\psi} \chi_t^{\eta\psi}.$$

In Lemma 2, I established that $\chi_t = \overline{\chi} \forall t$. Consequently, the average rate of output growth is

$$\gamma \equiv Mean\left(\frac{y_{t+1}}{y_t}\right) = \overline{a}\mathrm{H}^{\psi} \overline{\chi}^{\eta\psi}, \tag{27}$$

which means that, since output growth is monotonically increasing in the time devoted to human capital accumulation, the qualitative effects of uncertainty on growth will correspond to the qualitative effects of uncertainty on $\overline{\chi}$.

Let us begin with the case in which $\varrho \in (0,1)$. From Lemma 3, we know that a mean-preserving spread will reduce Θ, thus causing a reduction in the value of the function $P(\chi)$. Given that the function $Q(\chi)$ remains unaffected, it is true that the difference $Q(\chi) - P(\chi)$ increases. Now, suppose that, following the mean-preserving spread, the equilibrium value for χ increases to $\hat{\chi} > \overline{\chi}$. According to Lemma 2, $P(\chi) > Q(\chi)$ for $\chi \in (0, \hat{\chi})$ therefore $P(\overline{\chi}) > Q(\overline{\chi})$ given that $\hat{\chi} > \overline{\chi}$. However, taking account that $\overline{\chi}$ was the original equilibrium at which $P(\overline{\chi}) = Q(\overline{\chi})$, this analysis indicates that the difference $Q(\chi) - P(\chi)$ has actually fallen. Of course, this is a contradiction. As a result, we conclude that following a mean-preserving spread, the new equilibrium value should satisfy $\hat{\chi} < \overline{\chi}$. Hence, when $\varrho \in (0,1)$, uncertainty inhibits output growth.

Next, I shall consider the case where $\varrho > 1$. In that case, the formal proof can be derived by means of simple implicit differentiation. Define $Z(\overline{\chi}, \Theta) \equiv P(\overline{\chi}) - Q(\overline{\chi}) = 0$. Of course, $d\overline{\chi} / d\Theta = -[Z_{\Theta}(\cdot) / Z_{\overline{\chi}}(\cdot)]$. Using the definitions from Lemma 1 and differentiating yields, $Z_{\Theta}(\cdot) = \beta \mathrm{H}^{\psi(1-\varrho)} \overline{\chi}^{\eta\psi(1-\varrho)} > 0$, $Z_{\overline{\chi}}(\cdot) = \eta\psi(1-\varrho)\beta\Theta\mathrm{H}^{\psi(1-\varrho)} \overline{\chi}^{\eta\psi(1-\varrho)-1} - \{\eta\psi / [\overline{\chi}(1-\eta\psi) + \eta\psi]^2\} < 0$. Thus, $d\overline{\chi} / d\Theta > 0$. Combining with Lemma 3, we draw the conclusion that for $\varrho > 1$, uncertainty enhances output growth.

Finally, given Lemma 3, it is obvious that uncertainty does not bear any effect on output growth when $\varrho = 1$. Obviously, the optimal time allocation decisions are not affected by the degree of uncertainty when utility is logarithmic. ■

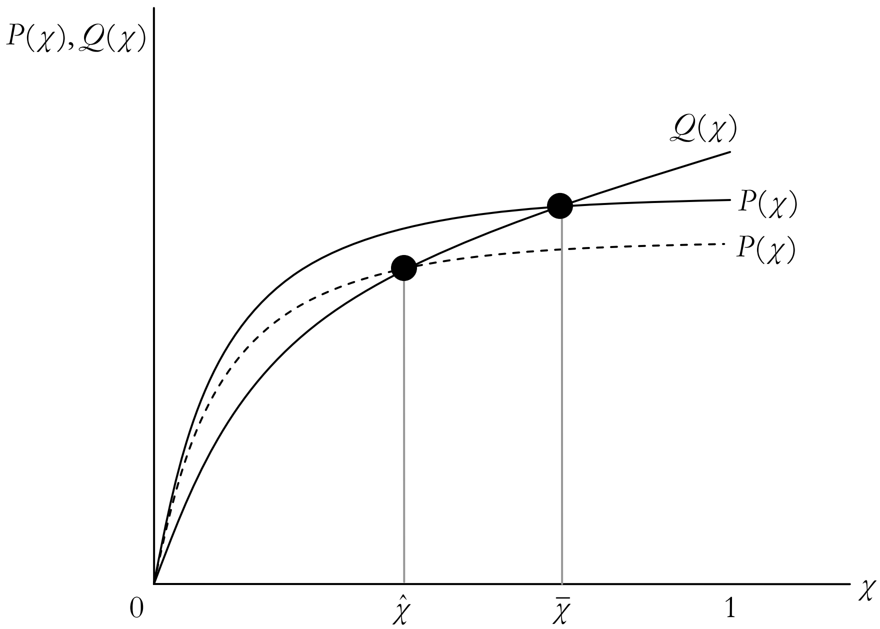

Figure 1. An increase in uncertainty when $\varrho \in (0,1)$

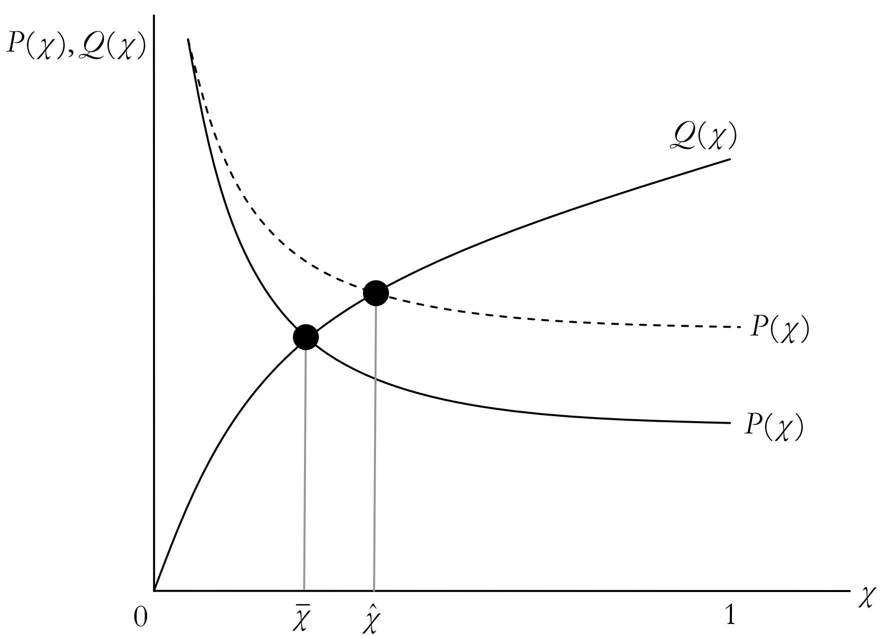

Figure 2. An increase in uncertainty when $\varrho > 1$

Once more, we can get an explicit solution for the growth rate in the scenario where both η and ψ are equal to unity. Substituting (26) in (27) and using Lemma 3 we derive

$$\gamma \equiv Mean\left(\frac{y_{t+1}}{y_t}\right) = \overline{a}[\beta \mathrm{H}\theta(\sigma)]^{1/\varrho}. \tag{28}$$

Obviously, the impact of uncertainty on growth is determined by the effect of σ (i.e., a mean-preserving spread) on γ. Clearly, $\partial\gamma / \partial\sigma = (1/\varrho)\overline{a}(\beta\mathrm{H})^{1/\varrho}[\theta(\sigma)]^{(1/\varrho)-1}\theta'(\sigma)$ which is negative if $\varrho \in (0,1)$, positive if $\varrho > 1$ and equal to zero if ϱ is equal to one.

5 Discussion

In this paper I have recovered a well known result – that is, the outcome whereby the preference parameter indicating relative risk aversion is crucial in determining the impact of uncertainty on output growth – within a context of an economy where the trade-off between labour and education is crucial for the evolution of human capital (and therefore sustainable growth). To complete the analysis, there is a need to provide sufficient intuition on why uncertainty impinges on the optimal human capital investment decisions, in the first place, and on why such investments may either be enhanced or inhibited by uncertainty, hence determining the impact that the latter bears on economic growth.

In general, the expectation of higher future productivity has two conflicting effects on the equilibrium allocation of time/effort between different activities. On the one hand, it induces agents to provide more effort towards learning activities, permanently, at the expense of labour, as they try to reap the relatively higher expected future benefits by accumulating human capital – i.e., the substitution effect. On the other hand, the expectation of enhanced future productivity raises lifetime income, thus generating an incentive for increasing the pattern of consumption in all periods, including the current one – i.e., the income effect. This can be achieved by a permanent increase in labour effort brought about at the expense of learning activities.

When $\varrho \in (0,1)$ $(\varrho > 1)$ the substitution (income) effect dominates. Furthermore, the concavity (convexity) of Θ in (18) indicates that the rise (fall) in $\overline{\chi}$ as a result of an expected increase in a_{t+1} is less pronounced than the fall (rise) in $\overline{\chi}$ resulting from an expected decrease in a_{t+1} of equal magnitude. As a result, greater uncertainty (measured by a mean-preserving spread in the distribution of a_{t+1}) reduces (increases) the benefits from accumulating human capital and ultimately impedes (enhances) output growth, following a fall (rise) in $\overline{\chi}$. Obviously, as long as $\varrho = 1$ income and substitution effects cancel each other out, therefore uncertainty has no effect on the growth rate of output.

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