# 1. Introduction

## 1.1 Motivation

Among the most critical and contentious contemporary international issues is the phenomenon of global warming which is expected to inflict significant damages on most societies. There appears to be substantial scientific support for the hypothesis that global warming is caused by the accumulation of carbon in the earth's atmosphere.[1] Since all carbon emissions inflict global damages by adding to the stock of atmospheric carbon, the problem of global warming can be characterized as one of managing a global stock externality. In response to this problem, the Kyoto protocol (United Nations (1997)) suggests a three-step procedure to regulate fresh flows into the carbon stock. The first step is to allocate endowments of emission rights among the participating nations. The second step involves each nation creating a mechanism for allocating its endowment among domestic interested parties. The third step envisages the creation of a mechanism for the international re-allocation of emission rights across interested parties.[2]

The objectives of this paper are twofold. First, we wish to propose a conceptual and institutional framework for the first step of the Kyoto procedure. This framework is formally expressed in a non-cooperative model of emission capping, i.e., the creation of endowments of emission rights. As our results apply to every solution of this model, they also apply to the solution that is optimal in terms of some normative criterion.

In addition to providing a basic model that formalizes a positive and normative theory of emission capping, we extend the basic model in natural ways to incorporate and analyze the strategic maneuvering entailed by it. Our basic model is constructed to satisfy four principles: domestic national sovereignty, non-cooperative voluntarism, transfer neutrality and optimality.

Domestic national sovereignty means that every nation is free to choose the mechanism for domestic regulation of emissions, a principle that is explicitly enshrined in the Kyoto protocol. This principle has two important implications. First, the endowments of emission

[1] We use "carbon" as shorthand for all greenhouse gases that contribute to global warming.

[2] See Hahn & Stavins (2000) for a description of the Kyoto protocol in general and the proposals related to the third step in particular.

rights must be awarded to nation states rather than to private parties within states, i.e., they must take the form of national emission caps. It follows that a model of the capping procedure must have nations as the relevant decision-makers. Secondly, the methods chosen by nations for the domestic allocation of their endowments of emission rights should have no effect on the determination of national emission caps.

Since emissions generate an international negative externality, a nation's decision regarding whether to accept the cap assigned to it will depend on its preference and the expected emissions of other nations. Non-cooperative voluntarism means that we can expect a nation to accept its assigned cap only if it cannot improve its position unilaterally. This rules out the possibility of implementing a non-Nash-equilibrium contract using incentives or sanctions not explicitly incorporated in the specified preferences.

The theory of emission capping represented by the model of Sections 2-4 is transfer neutral as it does not allow for wealth transfers. Therefore, our approach amounts to a pure theory of managing the emission flow externality that can serve as a neutral starting point for an evaluation of international wealth transfers. The justification of this principle in some specific contexts is deferred until Sections 5.3 and 5.4.

Optimality means that the chosen method of assigning caps should be optimal among the class of methods that satisfy the above three requirements. Our notion of optimality is strong, yet flexible enough to incorporate diverse normative and ideological positions.

### 1.2 Nature of modeling

A nation's decision regarding acceptance of an assigned emission cap will depend on the competing claims of the economic benefits accompanying emissions and the economic costs imposed by a global emission externality. We formalize this conflict by postulating that a nation consists of two groups: firms and consumers. This formalization is sharpened by using the principle of national sovereignty to aggregate the firms and the consumers in a given nation into an aggregate firm and an aggregate consumer respectively. A nation's benefit is represented by the firm's profit, which is related to its own emission, while the cost is represented by the damage suffered by the consumer from the international emission externality. This externality is formalized by postulating that the consumer in every country consumes the sum of emissions of all countries.

While the nation is the relevant decision-maker with respect to the choice of national emission cap, the actual emission level is chosen by the firm subject to the national cap.

Since it is the actual emissions that determine profits and damages, our model should endogenously account for the firms' responses to emission caps. As a firm's objective is to maximize profit, while the nation cares about emission-related damage as well, the preferences of the firm and the nation are not congruent. The potential difference between emission caps and actual emissions implies that a model determining both endogenously must have nations and firms as players. As we wish to focus on the state's decision-making with respect to the emission caps, we adopt a simple formulation of the firm's decision problem so as to identify the firm's chosen emission with the cap imposed on it. This halves the number of endogenous variables to be determined and also halves the number of players whose actions are immediately relevant for the determination of caps. Section 5.3 contains further discussion of this issue.

Apart from the profile of emissions, a nation's utility depends on its characteristics. A nation is characterized by a pair of parameters. One is a shift parameter for the firm's profit function, which we interpret as private capital that embodies technology. The other is a shift parameter for the consumer's damage function, which we interpret as social capital that determines a nation's vulnerability to environmental damage.

Our formal model is a two-stage game.[3] The Stage 2 subgames involve two nations that play a strategic form game selected out of a parametrized family of such games; our results can be readily extended to a model with more nations. The game played in Stage 2 determines the profile of national emission caps. In Stage 1, Nation/Firm 1 will choose the values of a subset of the above-mentioned parameters. The resulting profile of parameters, some of which are exogenously given and the remainder chosen in Stage 1, determines the Stage 2 subgame played by the two nations. In this subgame, each nation's action

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[3] The actual procedure used for emission capping is best characterized as multilateral bargaining. The problem with this procedure is that our models of multilateral bargaining are far from robust and seem inadequate when we attempt to model actual bargaining carried out in multiple dimensions. This lack of theoretical perspective renders the procedure opaque as there do not seem to be systematically applicable principles for determining the caps. The model presented in Sections 2-4 of this paper can be interpreted as a reduced-form representation of the current procedure for cap allocation or as a structural description of an alternative institution for optimal cap allocation that is described in Section 5.

Game-theoretic models of the possibility of sustaining international environmental agreements includes Barrett (1992), Barrett (1994) and Mäler (1989). While these works feature static abstractly specified games, our work features a dynamic game derived explicitly from a rich economic structure. This economic structure and the dynamic nature of our model allow us to analyze issues that cannot be framed adequately in a static framework.

space is the set of permitted emission cap levels and its payoff is generated by the utility function corresponding to the parameters chosen by Nation/Firm 1 in Stage 1. We study the subgame perfect equilibria (SPE) of the two-stage game.

### 1.3 Plan of paper

We formally set up our model in Section 2. In Section 3, we do some comparative statics exercises involving the Stage 2 Nash equilibrium. In Section 4, we analyze Nation/Firm 1's choice of parameters in Stage 1. In Section 5, we interpret the results of Sections 3 and 4 in a specific institutional and procedural setting. We suggest how our results can be interpreted as pertaining to an optimal scheme for determining emission caps. We also discuss (a) mechanisms for international re-allocation of emission rights as envisioned in the third step of the Kyoto process, (b) the relationship between emission capping and economic growth, and (c) the demand for the use of equity-based criteria in the assignment of emission rights endowments. Section 6 contains the analysis of a simple dynamical system suggested by our model. We conclude in Section 7 by summarizing our findings.

## 2. The model

### 2.1 Formalism

The data of our model is $\{N, \Theta, E, (u_i)_{i \in N}\}$, where $N = \{0, 1, 2\}$ is the set of players, $\Theta$ is a parameter space, $E \subset \Re_+$ is an action space, $u_0 : \Theta^2 \times \Re_+^2 \to \Re$ is Player 0's utility function and $u_i : \Theta \times \Re_+^2 \to \Re$ is Player $i$'s utility function for $i \in \{1, 2\}$.

The formal model is a three player two-stage game $\Gamma$. In Stage 1 of $\Gamma$, Player 0 selects a pair of parameters $\theta = (\theta_1, \theta_2) \in \Theta^2$; we interpret $\theta_i$ as Player $i$'s characteristic. In Stage 2, knowing $\theta$, Players 1 and 2 play the strategic form subgame $\Gamma(\theta)$. In $\Gamma(\theta)$, Players 1 and 2 choose actions $e_1(\theta) \in E$ and $e_2(\theta) \in E$ respectively. Given the chosen action profile $e(\theta) = (e_1(\theta), e_2(\theta))$, Player 0's utility is $u_0(\theta, e(\theta))$ and Player $i$'s utility is $u_i(\theta_i, e(\theta))$ for $i \in \{1, 2\}$.

In $\Gamma$, while Player 0's strategy is to pick $\theta \in \Theta^2$, Player $i$'s strategy, for $i \in \{1, 2\}$, is to select a function $e_i : \Theta^2 \to E$ that generates $i$'s action $e_i(\theta)$ in the subgame $\Gamma(\theta)$.

**Definition 2.1.1.** *$(\theta^*, e_1, e_2) \in \Theta^2 \times E^{\Theta^2} \times E^{\Theta^2}$ is an SPE of $\Gamma$ if*

*(a) for every $\theta \in \Theta^2$, $(e_1(\theta), e_2(\theta))$ is a Nash equilibrium of $\Gamma(\theta)$, and*

*(b)* $u_0(\theta^*, e_1(\theta^*), e_2(\theta^*)) \geq u_0(\theta, e_1(\theta), e_2(\theta))$ *for every* $\theta \in \Theta^2$.

*If* $e : \Theta^2 \to E^2$ *satisfies (a), then it is called an equilibrium plan for* $\Gamma$.

Consider an SPE $(\theta^*, e_1, e_2)$. Given $\theta^*$, the Stage 2 choices are $(e_1(\theta^*), e_2(\theta^*))$. Suppose $\theta^0$ is the choice that Player 0 would make if the Stage 2 choices $(e_1(\theta^*), e_2(\theta^*))$ were given, i.e., setting $\theta = \theta^0$ maximizes $u_0(\theta, e_1(\theta^*), e_2(\theta^*))$. The difference between $\theta^*$ and $\theta^0$ represents *ex post* chagrin on the part of Player 0; Player 0 would prefer to have $\theta^0$ *ex post*, but is stuck with $\theta^*$. It also represents *ex ante* strategic manipulation by Player 0 because of his ability to choose the Stage 2 subgame *via* his Stage 1 choice. We focus on the nature of this strategic effect in our interpretation of the formalism.

### 2.2 Interpretations

Our specification and interpretation of $\Gamma$ will involve the functions $g : \Re^2_+ \to \Re_+$, $h : \Re^2_+ \to \Re_+$, $v : \Re_+ \to \Re_+$ and $\delta : \Re^2_+ \to \Re_+$.

Consider two nations, 1 and 2, with Nation $i$ populated by aggregate Firm $i$ and aggregate Consumer $i$. If Firm $i$ with private capital $t_i \in \Re_+$ employs variable input $v_i \in \Re_+$, then Firm $i$'s profit is $g(t_i, v_i)$ and its emission is $h(t_i, v_i)$; we also refer to $g(t_i, v_i)$ and $h(t_i, v_i)$ as Nation $i$'s profit and emission respectively. The total emission $h(t_1, v_1)+h(t_2, v_2)$ is consumed by Consumers 1 and 2, thereby causing damage $\delta(k_i, h(t_1, v_1) + h(t_2, v_2))$ to Consumer $i$ who has social capital $k_i$.

Private capital $t_i$ consists of all fixed inputs such as plant and machinery that embody the technology available to Firm $i$. Social capital $k_i$ consists of all assets that are used to mitigate the damage caused by emissions.[4]

Let $v(t)$ be the variable input chosen by a profit-maximizing firm with private capital $t$ in the absence of emission constraints.[5] *The following assumption is made, without explicit reference, throughout the rest of this paper.*

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[4] Social capital includes water management systems, meteorological facilities, etc. As it is difficult to distinguish between private and social capital by physical criteria, the economic criterion that can be employed to determine whether some asset is private is to check if it directly affects the profit of the aggregate firm.

[5] The precise way in which this function is generated is irrelevant for our purposes.

**Assumption 2.2.1.** *$g$, $h$ and $v$ satisfy the following conditions.*

*(a) For every $t \in \Re_+$, $g(t,.) : \Re_+ \to \Re_+$ is strictly increasing on $[0, v(t)]$.*

*(b) $h(t,.) : \Re_+ \to \Re_+$ is strictly increasing and continuous.*

*(c) There exists $\overline{e} \in \Re_{++}$ such that, for every $t \in \Re_+$, we have $h(t, v(t)) > \overline{e}$.*

Suppose a profit-maximizing firm with private capital $t$ faces the emission constraint $h(t,v) \leq e$ with $e \in \Re_+$, and $v_c(t,e)$ is its constrained choice of variable input. Therefore, $h(t, v_c(t,e)) \leq e$. If $e \in [0, \overline{e}]$, then (c) implies $h(t, v(t)) > \overline{e} \geq e \geq h(t, v_c(t,e))$, and (b) implies $v_c(t,e) < v(t)$. Suppose $h(t, v_c(t,e)) < e$. By the continuity of $h(t,.)$, there exists $\epsilon > 0$ such that $h(t, v_c(t,e) + \epsilon) < e$. By (a), we have $g(t, v_c(t,e) + \epsilon) > g(t, v_c(t,e))$, a contradiction. Therefore, if $e \in [0, \overline{e}]$, then $h(t, v_c(t,e)) = e$.

In both our interpretations of the game $\Gamma$, we shall set $E = [0, \overline{e}]$ and interpret $e_i \in E$ as Nation $i$'s emission cap. Consequently, $\overline{e}$ will represent the largest possible emission cap in our model. $\overline{e}$ will perform two roles in our model. First, by the above argument, it will guarantee that each nation's emission cap will be a binding constraint when the cap is chosen from $E$. Secondly, by bounding $E$, it will enable us to apply the existence results quoted in Section 5.1. The uniformity of this bound across all $t$ is merely a matter of convenience; with minor modifications, our purposes will be served even if $\overline{e}$ varies continuously with $t$.

If Nation $i$'s emission is capped at $e_i \in \Re_+$ and it imposes the constraint $h(t_i, v_i) \leq e_i$ on Firm $i$, then Firm $i$'s profit is

$$f(t_i, e_i) = g(t_i, v_c(t_i, e_i)) \tag{2.2.2}$$

While $g$ may be called the direct profit function, $f : \Re^2_+ \to \Re_+$ may be called the indirect profit function. If $f$ is differentiable, then $D_e f(t_i, e_i)$ is Firm $i$'s shadow value of emission rights.[6] If this shadow value increases (resp. decreases) with private capital, i.e., the constraint tightens (resp. slackens), then we say that technology is emission enhancing (resp. retarding).

If Nation $i$'s emission is capped at $e_i \in [0, \overline{e}]$ and it imposes the constraint $h(t_i, v_i) \leq e_i$ on Firm $i$, then Firm $i$'s emission is $h(t_i, v_c(t_i, e_i)) = e_i$. Consequently, if the profile of

[6] $D$ is the differential operator. Partial derivatives are denoted by $D$ with appropriate subscripts.

emission caps is $e = (e_1, e_2) \in [0, \overline{e}]^2$, then Consumer $i$ consumes the aggregate emission $h(t_1, v_c(t_1, e_1)) + h(t_2, v_c(t_2, e_2)) = e_1 + e_2 = e_+$ and suffers damage $\delta(k_i, e_+)$. $D_{e_+}\delta(k_i, .)$ is called Nation $i$'s vulnerability to damage. Apart from simplifying the damage expression, Assumption 2.2.1 eliminates the dependence of Nation $i$'s damage on $t_1$ and $t_2$. We now present the first interpretation of $\Gamma$.

**Interpretation I.** *In $\Gamma$, set $\Theta = \Re^2_+$ and $E = [0, \overline{e}]$. Given $\theta = (\theta_1, \theta_2) \in \Theta^2$, with $\theta_1 = (t_1, k_1)$ and $\theta_2 = (t_2, k_2)$, and $e = (e_1, e_2) \in E^2$, Player 0's utility is*

$$u_0(\theta, e) = f(t_1, e_1) - \delta(k_1, e_+) - t_1 - k_1 - t_2 - k_2$$

*and Player $i$'s utility, for $i \in \{1, 2\}$, is*

$$u_i(\theta_i, e) = f(t_i, e_i) - \delta(k_i, e_+) \tag{2.2.3}$$

In this interpretation, Players 0 and 1 are Nation 1's agents. Player 0 represents Nation 1 in the choice of parameters $\theta = ((t_1, k_1), (t_2, k_2))$ in Stage 1 of $\Gamma$. Given $\theta$, Nations 1 and 2 play $\Gamma(\theta)$ in the forms of Players 1 and 2 respectively. Nation $i$'s gross utility is given by the difference between profit $f(t_i, e_i)$ and damage $\delta(k_i, e_1 + e_2)$. Nation 1's investment $C(\theta) = t_1 + k_1 + t_2 + k_2$ in choosing parameters is reflected in Player 0's decision problem, but not in Player 1's decision problem since the cost $C(\theta)$ is already sunk when Nation 1 chooses an action in Stage 2. If some components of $\theta$ are exogenously given, then those components are dropped from the definition of $C(\theta)$. Nation 1's choice of $\theta$ represents the allocation of Nation 1's public investment in, domestic and foreign, private and social assets. We now present an alternative interpretation of $\Gamma$.

**Interpretation II.** *Everything is identical to Interpretation I, except that Player 0's utility is $u_0(\theta, e) = f(t_1, e_1) - t_1 - k_1 - t_2 - k_2$.*

In this interpretation, Players 0 is Firm 1, Player 1 is Nation 1 and Player 2 is Nation 2. Firm 1 chooses parameters $\theta = ((t_1, k_1), (t_2, k_2))$ in Stage 1 of $\Gamma$. Given these parameters, Nations 1 and 2 play $\Gamma(\theta)$. Thus, the decision-makers with respect to emission caps continue to be the nations. Firm 1's choice of $\theta$ represents the allocation of Nation 1's private investment in, domestic and foreign, private and social assets.

There are two motives for treating Nations/Firms 1 and 2 asymmetrically. First, it allows us to uncover in a simple way the strategic motives guiding the choice of parameters.

Secondly, there are interesting situations that can be represented accurately by our asymmetric game. Since the choice of parameters in Stage 1 involves a cost, one can interpret Nation/Firm 1 as being an affluent player with resources to invest and Nation/Firm 2 as being an impoverished player with no spare resources to invest. *The following assumption is made, without explicit reference, throughout the rest of this paper.*

**Assumption 2.2.4.** *Suppose $f$, defined by (2.2.2), satisfies the following properties.*

*(a) $f$ is continuous, strictly increasing and strictly concave.*

*(b) $f$ is twice continuously differentiable on $\Re^2_{++}$; for every $t \in \Re_+$, $f(t,.)$ is twice continuously differentiable on $\Re_{++}$.*

*(c) Either $D_{et}f > 0$ on $\Re^2_{++}$ or $D_{et}f < 0$ on $\Re^2_{++}$.*

*(d) For every $e \in \Re_{++}$, there exists $t' \in \Re_{++}$ such that $D_t f(t', e) = 1$.*

*(e) For every $t \in \Re_{++}$, $\lim_{x \downarrow 0} D_e f(t, x) = \infty$.*

*Suppose $\delta : \Re^2_+ \to \Re_+$ satisfies the following properties.*

*(f) $\delta$ is continuous, and for every $k \in \Re_{++}$, $\delta(k,.)$ is strictly increasing and strictly convex.*

*(g) $\delta$ is twice continuously differentiable on $\Re^2_{++}$; for every $k \in \Re_+$, $\delta(k,.)$ is twice continuously differentiable on $\Re_{++}$.*

*(h) $D_{ke_+}\delta < 0$ on $\Re^2_{++}$.*

*(i) For every $e_+ \in \Re_{++}$, $\delta(., e_+)$ is strictly decreasing and strictly convex, and there exists $k' \in \Re_{++}$ such that $D_k\delta(k', e_+) = -1$.*

(a), (b), (f) and (g) have straightforward interpretations. (c) requires that technology uniformly be either emission enhancing or emission retarding. This assumption is made only for convenience and sharpness of results. (h) means that greater social capital reduces a nation's vulnerability to damage. (d) and (i) rule out uninteresting corner solutions, while (e) rules out an equilibrium plan that awards a zero emission cap to some nation.

**Lemma 2.2.5.** *For $i \in \{1, 2\}$, define $u_i : \Theta \times \Re^2_+ \to \Re$ by (2.2.3).*

*(A) $u_i$ is continuous.*

*(B) $u_i$ is twice continuously differentiable on* $\mathrm{Int}\,(\Theta \times \Re^2_+)$.

*(C) For every $(\theta_i, e) \in \Theta \times \Re^2_+$, $u_i(\theta_i, ., e_j) : \Re_+ \to \Re$, where $i \neq j$, is strictly concave.*

*(D) For every $(\theta_i, e) \in \Theta \times \Re^2_{++}$ such that $\theta_i = (t_i, k_i)$, we have*

$$|D_{e_i e_i} u_i(\theta_i, e)| > |D_{e_+ e_+}\delta(k_i, e_+)|$$

*(E) For every $(\theta, e) \in \Theta^2 \times \Re^2_{++}$, we have $\det A(\theta, e) > 0$, where*

$$A(\theta, e) = \begin{pmatrix} D_{e_1 e_1} u_1(\theta_1, e) & D_{e_1 e_2} u_1(\theta_1, e) \\ D_{e_2 e_1} u_2(\theta_2, e) & D_{e_2 e_2} u_2(\theta_2, e) \end{pmatrix}$$

Assumption 2.2.4, in particular parts (e) and (f), yields the following fact.

**Lemma 2.2.6.** *If $e$ is an equilibrium plan for $\Gamma$, then $e(\theta) \gg 0$ for every $\theta \in \Theta^2$.*

**Definition 2.2.7.** *Given an equilibrium plan $e$ for $\Gamma$, $\theta = (\theta_1, \theta_2) \in \Theta^2$ is a critical type profile for $e$ if for every $i \in \{1, 2\}$,*

$$D_{e_i} u_i(\theta_i, e(\theta)) = 0 \tag{2.2.8}$$

Clearly, if $e$ is an equilibrium plan for $\Gamma$ and $e(\theta) \in (0, \overline{e})^2$, then $\theta$ is a critical type profile for $e$.

**Definition 2.2.9.** *An equilibrium plan $e$ for $\Gamma$ is said to be regular if for every $\theta \in \Theta^2$ and $i \in \{1, 2\}$, $e_i(\theta) = \overline{e}$ implies $D_{e_i} u_i(\theta_i, e(\theta)) > 0$.*

Let $e$ be a regular equilibrium plan for $\Gamma$. It follows that, if $D_{e_i} u_i(\theta_i, e(\theta)) = 0$ for some $i$ and $\theta \in \Theta^2$, then $e_i(\theta) < \overline{e}$. Thus, if $\theta$ is a critical type profile for $e$, then $e(\theta) \in (0, \overline{e})^2$.

## 3. Comparative statics for the Stage 2 subgames

### 3.1 The formulae

Suppose $e$ is a regular equilibrium plan for $\Gamma$ and $\theta$ is a critical type profile for $e$. It follows from Assumption 2.2.4, (2.2.8) and the implicit function theorem that $e$ is continuously differentiable on an open neighborhood of $\theta$ and

$$\begin{aligned} D_{\theta_1} e_1(\theta) &= \frac{-D_{e_1 \theta_1} u_1(\theta_1, e(\theta)) D_{e_2 e_2} u_2(\theta_2, e(\theta))}{\det A(\theta, e(\theta))} \\ D_{\theta_1} e_2(\theta) &= \frac{D_{e_1 \theta_1} u_1(\theta_1, e(\theta)) D_{e_2 e_1} u_2(\theta_2, e(\theta))}{\det A(\theta, e(\theta))} \\ D_{\theta_1} e_+(\theta) &= \frac{D_{e_1 \theta_1} u_1(\theta_1, e(\theta)) [D_{e_2 e_1} u_2(\theta_2, e(\theta)) - D_{e_2 e_2} u_2(\theta_2, e(\theta))]}{\det A(\theta, e(\theta))} \end{aligned} \tag{3.1.1}$$

Suppose $e$ is a regular equilibrium plan for $\Gamma$ and $\theta$ is a non-critical type profile for $e$. Let $e_1(\theta) < \overline{e} = e_2(\theta)$. (The other cases, $e_1(\theta) = \overline{e} > e_2(\theta)$ and $e_1(\theta) = \overline{e} = e_2(\theta)$, can be analyzed analogously.) In this case, (2.2.8) is replaced by

$$D_{e_1} u_1(\theta_1, e(\theta)) = 0 \qquad e_2(\theta) = \overline{e} \qquad D_{e_2} u_2(\theta_2, e(\theta)) = \beta(\theta) > 0 \tag{3.1.2}$$

$\beta(\theta)$ is Nation 2's shadow value of emission rights. Regularity of $e$ guarantees that $e_2(\theta) = \overline{e}$ if and only if $\beta(\theta) > 0$. It follows from (3.1.2) and the implicit function theorem that there exists an open neighborhood of $\theta$ on which $(e_1, \beta)$ is continuously differentiable and $\beta$ is strictly positive, which implies $e_2(\theta) = \overline{e}$ on this neighborhood. Consequently,

$$\begin{aligned} D_{\theta_1} e_1(\theta) &= \frac{-D_{e_1\theta_1} u_1(\theta_1, e_1(\theta), \overline{e})}{D_{e_1e_1} u_1(\theta_1, e_1(\theta), \overline{e})} \\ D_{\theta_1}\beta(\theta) &= D_{e_2e_1} u_2(\theta_2, e_1(\theta), \overline{e}) D_{\theta_1} e_1(\theta) \end{aligned} \tag{3.1.3}$$

Naturally, $D_{\theta_1} e_2(\theta) = 0$ and $D_{\theta_1} e_+(\theta) = D_{\theta_1} e_1(\theta)$. Differentiating (3.1.2) with respect to $\theta_2$, we have

$$D_{e_1e_1} u_1(\theta_1, e_1(\theta), \overline{e}) D_{\theta_2} e_1(\theta) = 0$$

and

$$D_{\theta_2}\beta(\theta) = D_{e_2\theta_2} u_2(\theta_2, e_1(\theta), \overline{e}) + D_{e_2e_1} u_2(\theta_2, e_1(\theta), \overline{e}) D_{\theta_2} e_1(\theta)$$

Since $D_{e_1e_1} u_1(\theta_1, e_1(\theta), \overline{e}) < 0$, we have

$$D_{\theta_2} e_1(\theta) = 0 \qquad \text{and} \qquad D_{\theta_2}\beta(\theta) = D_{e_2\theta_2} u_2(\theta_2, e_1(\theta), \overline{e}) \tag{3.1.4}$$

Naturally, $D_{\theta_2} e_2(\theta) = 0$ and $D_{\theta_2} e_+(\theta) = 0$.

### 3.2 Variations in private capital

Suppose $e$ is a regular equilibrium plan for $\Gamma$ and $\theta = (\theta_1, \theta_2)$ is a critical type profile for $e$, with $\theta_i = (t_i, k_i)$. Suppose $t_1$ varies and all the other parameters are fixed. Specializing (3.1.1), we have

$$\begin{aligned} D_{t_1} e_1(\theta) &= \frac{-D_{et} f(t_1, e_1(\theta)) D_{e_2e_2} u_2(\theta_2, e(\theta))}{\det A(\theta, e(\theta))} \\ D_{t_1} e_2(\theta) &= \frac{-D_{et} f(t_1, e_1(\theta)) D_{e_+e_+} \delta(k_2, e_+(\theta))}{\det A(\theta, e(\theta))} \\ D_{t_1} e_+(\theta) &= \frac{-D_{et} f(t_1, e_1(\theta)) D_{ee} f(t_2, e_2(\theta))}{\det A(\theta, e(\theta))} \end{aligned}$$

Application of Assumption 2.2.4 and Lemma 2.2.5 yields the following facts.

**Proposition 3.2.1.** *Suppose $e$ is a regular equilibrium plan for $\Gamma$ and $\theta = (\theta_1, \theta_2)$ is a critical type profile for $e$, with $\theta_i = (t_i, k_i)$. If technology is emission enhancing (resp. emission retarding), then $D_{t_1} e_1(\theta) > 0$ (resp. $D_{t_1} e_1(\theta) < 0$), $D_{t_1} e_2(\theta) < 0$ (resp. $D_{t_1} e_2(\theta) > 0$) and $D_{t_1} e_+(\theta) > 0$ (resp. $D_{t_1} e_+(\theta) < 0$). It also follows that $|D_{t_1} e_1(\theta)| > |D_{t_1} e_2(\theta)|$.*

An analogous result derived from the analysis of (3.1.3) and (3.1.4) is

**Proposition 3.2.2.** *Suppose $e$ is a regular equilibrium plan for $\Gamma$ and $\theta = (\theta_1, \theta_2)$ is such that $e_1(\theta) < \overline{e} = e_2(\theta)$, with $\theta_i = (t_i, k_i)$.*

*(A) If technology is emission enhancing (resp. emission retarding), then $D_{t_1} e_1(\theta) > 0$ (resp. $D_{t_1} e_1(\theta) < 0$), $D_{t_1} e_2(\theta) = 0$, $D_{t_1} \beta(\theta) < 0$ (resp. $D_{t_1} \beta(\theta) > 0$) and $D_{t_1} e_+(\theta) > 0$ (resp. $D_{t_1} e_+(\theta) < 0$).*

*(B) If technology is emission enhancing (resp. emission retarding), then $D_{t_2} e_1(\theta) = D_{t_2} e_2(\theta) = D_{t_2} e_+(\theta) = 0$ and $D_{t_2} \beta(\theta) > 0$ (resp. $D_{t_2} \beta(\theta) < 0$).*

In (A), the private capital of the unconstrained nation is perturbed. The qualitative effect of a change in $t_1$ on $e_1$ is the same as in Proposition 3.2.1. As Nation 2's constraint is binding, the burden of adjustment is shifted to the shadow value of the constraint: if the constraint tightens (resp. loosens), then the shadow value increases (resp. decreases). In (B), the private capital of the constrained nation is perturbed. This has no effect on the allocation of caps and the burden of adjustment falls on the shadow value of the constraint. The following result records the effect on emission caps of international differences in private capital.

**Assumption 3.2.3.** *Suppose $e$ is a regular equilibrium plan for $\Gamma$ and $\theta = (\theta_1, \theta_2)$ is a critical type profile for $e$, with $\theta_1 = (t_1, k)$, $\theta_2 = (t_2, k)$ and $t_1 > t_2$.*

(2.2.8) and Assumption 3.2.3 imply that $D_e f(t_1, e_1(\theta)) = D_e f(t_2, e_2(\theta))$. Therefore,

$$\begin{aligned}\int_{e_1(\theta)}^{e_2(\theta)} dy\, D_{ee} f(t_2, y) &= D_e f(t_2, e_2(\theta)) - D_e f(t_2, e_1(\theta)) \\ &= D_e f(t_1, e_1(\theta)) - D_e f(t_2, e_1(\theta)) \\ &= \int_{t_2}^{t_1} dx\, D_{et} f(x, e_1(\theta))\end{aligned}$$

If $D_{et} f > 0$ (resp. $D_{et} f < 0$), then $\int_{t_2}^{t_1} dx\, D_{et} f(x, e_1(\theta)) > 0$ (resp. $\int_{t_2}^{t_1} dx\, D_{et} f(x, e_1(\theta)) < 0$), i.e., $\int_{e_1(\theta)}^{e_2(\theta)} dy\, D_{ee} f(t_2, y) > 0$ (resp. $\int_{e_1(\theta)}^{e_2(\theta)} dy\, D_{ee} f(t_2, y) < 0$). By the concavity of $f$, we have $e_1(\theta) > e_2(\theta)$ (resp. $e_1(\theta) < e_2(\theta)$). Consequently, we have

**Proposition 3.2.4.** *Given Assumption 3.2.3, if technology is emission enhancing (resp. retarding), then $e_1(\theta) > e_2(\theta)$ (resp. $e_1(\theta) < e_2(\theta)$).*

The stated relationships between private capital stock and emission caps hold when technology is either uniformly emission enhancing or uniformly emission retarding. Interesting intermediate cases, such as that of technology being emission enhancing for low levels of private capital and emission retarding for high levels of private capital, are not addressed by Proposition 3.2.4.

### 3.3 Variations in social capital

Suppose $e$ is a regular equilibrium plan for $\Gamma$ and $\theta = (\theta_1, \theta_2)$ is a critical type profile for $e$, with $\theta_i = (t_i, k_i)$. Suppose $k_1$ varies and all the other parameters are fixed. Specializing (3.1.1) and using the facts that

$$D_{k_1 e_1} u_1(\theta_1, e(\theta)) = -D_{k_1 e_+} \delta(k_1, e_+(\theta)) \quad \text{and} \quad D_{e_2 e_1} u_1(\theta_1, e(\theta)) = -D_{e_+ e_+} \delta(k_1, e_+(\theta))$$

we have

$$D_{k_1} e_1(\theta) = \frac{D_{k_1 e_+} \delta(k_1, e_+(\theta)) D_{e_2 e_2} u_2(\theta_2, e(\theta))}{\det A(\theta, e(\theta))}$$
$$D_{k_1} e_2(\theta) = \frac{D_{k_1 e_+} \delta(k_1, e_+(\theta)) D_{e_+ e_+} \delta(k_2, e_+(\theta))}{\det A(\theta, e(\theta))}$$
$$D_{k_1} e_+(\theta) = \frac{D_{k_1 e_+} \delta(k_1, e_+(\theta)) D_{ee} f(t_2, e_2(\theta))}{\det A(\theta, e(\theta))}$$

Application of Assumption 2.2.4 and Lemma 2.2.5 yields the following facts.

**Proposition 3.3.1.** *Suppose $e$ is a regular equilibrium plan for $\Gamma$ and $\theta = (\theta_1, \theta_2)$ is a critical type profile for $e$, with $\theta_i = (t_i, k_i)$. Then, $D_{k_1} e_1(\theta) > 0$, $D_{k_1} e_2(\theta) < 0$ and $D_{k_1} e_+(\theta) > 0$. It also follows that $D_{k_1} e_1(\theta) > |D_{k_1} e_2(\theta)|$.*

An analogous result derived from the analysis of (3.1.3) and (3.1.4) is

**Proposition 3.3.2.** *Suppose $e$ is a regular equilibrium plan for $\Gamma$ and $\theta = (\theta_1, \theta_2)$ is such that $e_1(\theta) < \overline{e} = e_2(\theta)$, with $\theta_i = (t_i, k_i)$.*

*(A) Then $D_{k_1} e_1(\theta) > 0$, $D_{k_1} e_2(\theta) = 0$, $D_{k_1} \beta(\theta) < 0$ and $D_{k_1} e_+(\theta) > 0$.*

*(B) Then $D_{k_2} e_1(\theta) = D_{k_2} e_2(\theta) = D_{k_2} e_+(\theta) = 0$ and $D_{k_2} \beta(\theta) > 0$.*

We now check how international differences in social capital affect the emission caps.

**Assumption 3.3.3.** *Suppose $e$ is a regular equilibrium plan for $\Gamma$ and $\theta = (\theta_1, \theta_2)$ is a critical type profile for $e$, with $\theta_1 = (t, k_1)$, $\theta_2 = (t, k_2)$ and $k_1 > k_2$.*

Assumptions 2.2.4 and 3.3.3 imply that, $0 < D_{e_+} \delta(k_1, e_+(\theta)) < D_{e_+} \delta(k_2, e_+(\theta))$. By (2.2.8), $D_e f(t, e_1(\theta)) < D_e f(t, e_2(\theta))$. Concavity of $f$ implies $e_1(\theta) > e_2(\theta)$.

**Proposition 3.3.4.** *Given Assumption 3.3.3,* $e_1(\theta) > e_2(\theta)$.

## 4. Strategic choice of investment in Stage 1

### 4.1 Set-up

Consider Stage 1 of $\Gamma$ in which Nation/Firm 1 chooses some of the parameters prior to the allocation of emission caps in Stage 2, while the remaining parameters are given exogenously. In Section 4.2 (resp. 4.3), $t_1$ and $k_1$ are endogenized using Interpretation I (resp. II). In Section 4.4, $t_2$ and $k_2$ are endogenized using Interpretations I and II. Linear additive investment costs permit this piecemeal approach with no loss of generality.

### 4.2 Nation 1 chooses domestic private and social capital

Consider an SPE $(\theta, e_1, e_2)$. We begin by normalizing Nation 1's private and social capital before Stage 1 to 0.[7] Nation 1 chooses private capital $t_1$ and social capital $k_1$ at cost $t_1 + k_1$. We assume that $\theta_2 = (t_2, k_2)$ is exogenously given, while $\theta_1 = (t^*, k^*)$, with $t^* > 0$ and $k^* > 0$, is chosen by Nation 1 in Stage 1. By definition, $(t^*, k^*)$ must maximize $u_1(t, k, e(t, k)) - t - k$. We simplify our formulae by suppressing $\theta_2$.

Suppose $e = (e_1, e_2)$ is a regular equilibrium plan for $\Gamma$ and $\theta = (\theta_1, \theta_2)$ is a critical type profile for $e$. We assume that $\theta_2 = (t_2, k_2)$ is exogenously given, while $\theta_1 = (t^*, k^*)$, with $t^* > 0$ and $k^* > 0$, is chosen by Nation 1 in Stage 1. Given these assumptions, the equilibrium caps $e(\theta) = (e_1(\theta), e_2(\theta))$ will be characterized by (2.2.8). Regularity of $e$ implies that $e(\theta)$ will be in the interior of $E^2$.

Given $(t^*, k^*)$, the emission caps $e(t^*, k^*) = (e_1(t^*, k^*), e_2(t^*, k^*))$ are characterized by (2.2.8). Therefore, $(t^*, k^*)$ must maximize $u_1(t, k, e(t, k)) - t - k$. Using (2.2.8), the first order conditions characterizing $(t^*, k^*)$ reduce to

$$D_t f(t^*, e_1(t^*, k^*)) - D_{e_+}\delta(k^*, e_+(t^*, k^*))D_t e_2(t^*, k^*) = 1 \tag{4.2.1}$$

$$-D_k\delta(k^*, e_+(t^*, k^*)) - D_{e_+}\delta(k^*, e_+(t^*, k^*))D_k e_2(t^*, k^*) = 1 \tag{4.2.2}$$

We first analyze (4.2.2). $-D_k\delta(k^*, e_+(t^*, k^*)) > 0$ is the marginal direct benefit to Nation 1 from investment in social capital. Since $-D_{e_+}\delta(k^*, e_+(t^*, k^*)) < 0$ and $D_k e_2(t^*, k^*) < 0$

[7] This change of origin has no effect on the qualitative results developed below. It merely enables us to disregard corner solutions.

(by Proposition 3.3.1), $-D_{e_+}\delta(k^*, e_+(t^*, k^*))D_k e_2(t^*, k^*) > 0$ is the marginal strategic benefit to Nation 1 from such investment *via* the manipulation of Nation 2's emission cap.

Let $k^0$ be Nation 1's choice of social capital given the emission caps $e(t^*, k^*)$. $k^0$ is the social capital Nation 1 would like to have *ex post*, i.e., after the strategic choice $k^*$ has lead to the award of caps $e(t^*, k^*)$. $k^* - k^0$ is a measure of Nation 1's *ex post* chagrin; $k^* > k^0$ indicates strategic overinvestment in social capital, while $k^* < k^0$ indicates strategic underinvestment. It follows from Assumption 2.2.4 that there exists $k^0 > 0$ that maximizes $f(t^*, e_1(t^*, k^*)) - \delta(k, e_+(t^*, k^*)) - t^* - k$. $k^0$ is characterized by the equation

$$D_k\delta(k^0, e_+(t^*, k^*)) = -1 \tag{4.2.3}$$

Since $D_{e_+}\delta > 0$ and $D_k e_2(t^*, k^*) < 0$, it follows from (4.2.2) and (4.2.3) that

$$\begin{aligned} D_k\delta(k^0, e_+(t^*, k^*)) &= D_k\delta(k^*, e_+(t^*, k^*)) + D_{e_+}\delta(k^*, e_+(t^*, k^*))D_k e_2(t^*, k^*) \\ &< D_k\delta(k^*, e_+(t^*, k^*)) \end{aligned}$$

which implies $k^0 < k^*$ as $D_{kk}\delta > 0$ by Assumption 2.2.4. Therefore, by Proposition 3.3.1, $e_1(., k^*) > e_1(., k^0)$, $e_2(., k^*) < e_2(., k^0)$ and $e_+(., k^*) > e_+(., k^0)$.

**Proposition 4.2.4.** *Nation 1 strategically overinvests in social capital. This manipulation results in a higher emission cap for Nation 1, a lower emission cap for Nation 2 and a higher aggregate emission cap.*

We now analyze (4.2.1). Let $t^0$ be Nation 1's choice of technology given the emission caps $e(t^*, k^*)$. By analogy with the situation considered above, $t^* - t^0$ is a measure of Nation 1's *ex post* chagrin; $t^* > t^0$ indicates strategic overinvestment in private capital, while $t^* < t^0$ indicates strategic underinvestment. By Assumption 2.2.4, there exists $t^0 > 0$ that maximizes $f(t, e_1(t^*, k^*)) - \delta(k^*, e_+(t^*, k^*)) - t - k^*$. $t^0$ is characterized by the equation

$$D_t f(t^0, e_1(t^*, k^*)) = 1 \tag{4.2.5}$$

It follows from (4.2.1) and (4.2.5) that

$$D_t f(t^0, e_1(t^*, k^*)) = D_t f(t^*, e_1(t^*, k^*)) - D_{e_+}\delta(k^*, e_+(t^*, k^*))D_t e_2(t^*, k^*)$$

If $D_{et}f > 0$, then $D_t e_2(t^*, k^*) < 0$ by Proposition 3.2.1. Since $D_{e_+}\delta > 0$, this implies $D_t f(t^0, e_1(t^*, k^*)) > D_t f(t^*, e_1(t^*, k^*))$. Since $f$ is concave, we have $t^0 < t^*$. By a symmetric argument, if $D_{et}f < 0$, then $t^0 > t^*$.

**Proposition 4.2.6.** *Nation 1 strategically overinvests (resp. underinvests) in private capital if technology is emission enhancing (resp. retarding). In both cases, the manipulation results in a higher emission cap for Nation 1, a lower emission cap for Nation 2 and a higher aggregate emission cap.*

$G(t,k) = f(t, e_1(t,k)) - t - k$ is Firm 1's net profit from investing $t$ and $k$ in private and social capital respectively. We examine Firm 1's incentive to invest in private capital at the parameter level $(t^*, k^*)$ chosen by Nation 1. If technology is emission enhancing. It follows from (2.2.8), Proposition 3.2.1 and (4.2.1) that

$$\begin{aligned} D_t G(t^*, k^*) &= D_t f(t^*, e_1(t^*, k^*)) + D_e f(t^*, e_1(t^*, k^*)) D_t e_1(t^*, k^*) - 1 \\ &= D_t f(t^*, e_1(t^*, k^*)) + D_{e_+} \delta(k^*, e_+(t^*, k^*)) D_t e_1(t^*, k^*) - 1 \\ &> D_t f(t^*, e_1(t^*, k^*)) - D_{e_+} \delta(k^*, e_+(t^*, k^*)) D_t e_2(t^*, k^*) - 1 \\ &= 0 \end{aligned} \tag{4.2.7}$$

Thus, Firm 1 would locally prefer a higher level of private capital than the $t^*$ chosen by Nation 1. If technology is emission retarding, then the inequality in (4.2.7) is reversed, implying that Firm 1 would locally prefer a lower level of private capital than the $t^*$ chosen by Nation 1.

One can also ask how the firm would like to perturb social capital, starting from $k^*$. Let $\Delta(t,k) = \delta(k, e_+(t,k))$. Using (2.2.8) and (4.2.2), we have

$$\begin{aligned} D_k G(t^*, k^*) &= D_e f(t^*, e_1(t^*, k^*)) D_k e_1(t^*, k^*) - 1 \\ &= D_{e_+} \delta(k^*, e_+(t^*, k^*)) D_k e_1(t^*, k^*) - 1 \\ &= D_k \delta(k^*, e_+(t^*, k^*)) + D_{e_+} \delta(k^*, e_+(t^*, k^*)) D_k e_+(t^*, k^*) \\ &= D_k \Delta(t^*, k^*) \end{aligned} \tag{4.2.8}$$

Thus, if damage $\Delta$ is increasing (resp. decreasing) in $k$ at $(t^*, k^*)$, then Firm 1 would locally prefer a higher (resp. lower) level of social capital.

The above results are local in nature. They answer the question: starting from $(t^*, k^*)$, in what direction would the firm like to make "small" perturbations in private and social capital? Comparisons of the globally optimal choices made by the nation and the firm require global information that we do not have.

### 4.3 Firm 1 chooses domestic private and social capital

Suppose Firm 1 chooses $t^{**} > 0$ and $k^{**} > 0$ as domestic private and social capital respectively. Given $(t^{**}, k^{**})$, the emission caps $e(t^{**}, k^{**}) = (e_1(t^{**}, k^{**}), e_2(t^{**}, k^{**}))$ are determined in Stage 2 by (2.2.8). Therefore, $(t^{**}, k^{**})$ must maximize $f(t, e_1(t, k)) - t - k$. This implies

$$D_t f(t^{**}, e_1(t^{**}, k^{**})) + D_e f(t^{**}, e_1(t^{**}, k^{**})) D_t e_1(t^{**}, k^{**}) = 1 \qquad (4.3.1)$$

$$D_e f(t^{**}, e_1(t^{**}, k^{**})) D_k e_1(t^{**}, k^{**}) = 1 \qquad (4.3.2)$$

We begin by analyzing (4.3.2). In (4.3.2), the left-hand-side is the marginal strategic benefit of investment in social capital to Firm 1, *via* the manipulation of Nation 1's emission cap. In the absence of this strategic effect, Firm 1 will choose social capital $k^{00} = 0$. Thus, the possibility of manipulating its own cap induces strategic *overinvestment* in social capital by Firm 1. Therefore, by Proposition 3.3.1, $e_1(., k^{**}) > e_1(., k^{00})$, $e_2(., k^{**}) < e_2(., k^{00})$ and $e_+(., k^{**}) > e_+(., k^{00})$.

**Proposition 4.3.3.** *Proposition 4.2.4 holds, with "Nation 1" replaced by "Firm 1".*

In (4.3.1), $D_t f(t^{**}, e_1(t^{**}, k^{**}))$ is the marginal direct benefit to Firm 1 of investment in private capital, while $D_e f(t^{**}, e_1(t^{**}, k^{**})) D_t e_1(t^{**}, k^{**})$ is the marginal strategic benefit to Firm 1, *via* the manipulation of Nation 1's emission cap.

Let $t^{00}$ be Firm 1's choice of private capital given the emissions $e(t^{**}, k^{**})$. By Assumption 2.2.4, there exists $t^{00} > 0$ that maximizes $f(t, e_1(t^{**}, k^{**})) - \delta(k^{**}, e_+(t^{**}, k^{**})) - t - k^{**}$. $t^{00}$ is characterized by the equation

$$D_t f(t^{00}, e_1(t^{**}, k^{**})) = 1 \qquad (4.3.4)$$

It follows from (4.3.1) and (4.3.4) that

$$D_t f(t^{00}, e_1(t^{**}, k^{**})) = D_t f(t^{**}, e_1(t^{**}, k^{**})) - D_{e_+} \delta(k^{**}, e_+(t^{**}, k^{**})) D_t e_2(t^{**}, k^{**})$$

Suppose $D_{et} f > 0$. By Proposition 3.2.1, we have $D_t e_2(t^{**}, k^{**}) < 0$. It follows that $D_t f(t^{00}, e_1(t^{**}, k^{**})) > D_t f(t^{**}, e_1(t^{**}, k^{**}))$. As $f$ is concave, we have $t^{00} < t^{**}$. If $D_{et} f < 0$, we have $t^{00} > t^{**}$ by an analogous argument.

**Proposition 4.3.5.** *Proposition 4.2.6 holds, with "Nation 1" replaced by "Firm 1".*

$H(t,k) = f(t, e_1(t,k)) - \delta(k, e_+(t,k)) - t - k$ is Nation 1's net welfare from investing $t$ and $k$ in private and social capital respectively. We examine Nation 1's incentive to invest in private and social capital at the parameter level $(t^{**}, k^{**})$ chosen by Firm 1. As one would expect, the results are the exact opposite of the results at the end Section 4.2. If technology is emission enhancing, then it follows from (4.3.1) that

$$D_t H(t^{**}, k^{**}) = -D_{e_+}\delta(k^{**}, e_+(t^{**}, k^{**}))D_t e_+(t^{**}, k^{**}) < 0$$

Thus, Nation 1 would locally prefer a lower level of private capital than the $t^{**}$ chosen by Firm 1. If technology is emission retarding, then $D_t H(t^{**}, k^{**}) > 0$, which means that Nation 1 would locally prefer a higher level of private capital than $t^{**}$.

As for social capital, we have $D_k H(t^{**}, k^{**}) = -D_k \Delta(t^{**}, k^{**})$. Consequently, if damage $\Delta$ is increasing (resp. decreasing) in $k$ at $(t^{**}, k^{**})$, then Firm 1 would locally prefer a lower (resp. higher) level of social capital.

### 4.4 Nation/Firm 1 chooses foreign private and social capital

In this section we consider the two-stage game in which Nation 1 can invest in private and social capital in Nation 2, keeping all other parameters fixed; the fixed parameters are suppressed from our expressions. This amounts to the following economic question: if Nation 1 transfers resources to Nation 2 that can be tied to specific purposes, what will be the nature of the restrictions on the use of this foreign aid?

We begin by normalizing Nation 2's initial private and social capital to 0. While Nation 1's type remains fixed, let $t^*$ and $k^*$ be Nation 1's investment in private and social capital respectively in Nation 2.

Given the Stage 1 choices $t^*$ and $k^*$, the emission caps $e(t^*, k^*)$ will be determined in Stage 2 by (2.2.8). Therefore, $(t^*, k^*)$ must maximize $u \circ e(t,k) - t - k$ subject to the constraints: $t \geq 0$ and $k \geq 0$. As $(t^*, k^*)$ solves this problem, there exist $\lambda$, $\mu \in \Re_+$ such that $\lambda t^* = 0$, $\mu k^* = 0$, and

$$D_{e_1} u_1(e(t^*, k^*))D_t e_1(t^*, k^*) + D_{e_2} u_1(e(t^*, k^*))D_t e_2(t^*, k^*) = 1 - \lambda \qquad (4.4.1)$$

$$D_{e_1} u_1(e(t^*, k^*))D_k e_1(t^*, k^*) + D_{e_2} u_1(e(t^*, k^*))D_k e_2(t^*, k^*) = 1 - \mu \qquad (4.4.2)$$

If $k^* > 0$, then $\mu = 0$. Applying (2.2.8), (4.4.2) reduces to

$$D_{e_+}\delta(e_+(t^*, k^*))D_k e_2(t^*, k^*) = -1 \tag{4.4.3}$$

By Assumptions 2.2.4, $D_{e_+}\delta(e_+(t^*, k^*)) > 0$. Moreover, $D_k e_2(t^*, k^*) > 0$ by Proposition 3.3.1. It follows that (4.4.3) cannot hold. Thus, $k^* = 0$. The reason for this result is that an increase in $k_2$ decreases $e_1$ and increases $e_+$, thereby hurting Firm 1's profit and causing greater social damage to Nation 1. The same argument also rules out the possibility of Firm 1 investing in Nation 2's social capital.

**Proposition 4.4.4.** *Neither Nation 1, nor Firm 1, will invest in Nation 2's social capital.*

If $t^* > 0$, then (4.4.1) implies $\lambda = 0$. Applying (2.2.8), (4.4.1) reduces to

$$D_{e_+}\delta(e_+(t^*, k_2))D_t e_2(t^*, k_2) = -1 \tag{4.4.5}$$

If technology is emission enhancing, then $D_t e_2(t^*, k_2) > 0$, which violates (4.4.5). Thus, $t^* = 0$. The reason for this result is that, when technology is emission enhancing, an increase in $t_2$ decreases $e_1$ and increases $e_+$, thereby hurting Firm 1's profit and causing greater social damage to Nation 1. The same argument also rules out the possibility of Firm 1 investing in Nation 2's private capital when technology is emission enhancing.

**Proposition 4.4.6.** *Neither Nation 1, nor Firm 1, will invest in Nation 2's private capital if technology is emission enhancing.*

If technology is emission retarding, then it is possible that Nation 1 and Firm 1 will choose Nation 2's private capital as $t^* > 0$ and $t^{**} > 0$ respectively. $t^*$ is characterized by (4.4.5) while $t^{**}$ is characterized by

$$D_e f(t_1, e_1(t^{**}))De_1(t^{**}) = 1 \tag{4.4.7}$$

Clearly, in the absence of a strategic effect, neither Nation 1, nor Firm 1, would invest in Nation 2's private capital. Thus, $t^* > 0$ and $t^{**} > 0$ represent strategic overinvestment in Nation 2's private capital. The result of such overinvestment is a higher $e_1$, a lower $e_2$ and a lower $e_+$. Thus, Firm 1 benefits strategically from $t^{**} > 0$ by raising $e_1$, and thereby, its own profit. Nation 1 benefits from $t^* > 0$ by raising $e_1$, thereby raising Firm 1's profit, and by lowering $e_+$, thereby lowering its own damages. Thus, Nation 1 has a stronger marginal incentive to overinvest in Nation 2's private capital, which should translate into a greater degree of overinvestment by Nation 1, relative to Firm 1. This can be shown formally as follows. $H(t) = f(e_1(t)) - \delta(e_+(t)) - t$ is Nation 1's net welfare from investing $t$ in Nation 2's private capital; we have suppressed $t_1$ and $k_1$ as they are assumed to be fixed and $k_2 = 0$ by Proposition 4.4.4. Using (2.2.8), (4.4.7) and Proposition 3.2.1, we have

$$DH(t^{**}) = -D_{e_+}\delta(e_+(t^{**}))De_2(t^{**}) - 1 = -D_e f(e_1(t^{**}))De_+(t^{**}) > 0$$

Thus, at the investment level chosen by Firm 1, Nation 1 would prefer to invest even more.

**Proposition 4.4.8.** *If technology is emission retarding, Nation 1 and Firm 1 may choose to invest $t^* > 0$ and $t^{**} > 0$ respectively in Nation 2's private capital. If positive, such choices represent strategic overinvestment, with Nation 1 having a greater marginal incentive to overinvest.*

## 5. Extensions

### 5.1 Existence and optimality of equilibrium plans

An SPE for our model requires that the Stage 2 emission caps profiles be generated by an equilibrium plan $e : \Theta^2 \to E^2$. There are two natural questions to ask in this regard. First, are there relatively weak assumptions about the data of our model that ensure the existence of an equilibrium plan? Secondly, given these conditions and some notion of optimality, is there an optimal equilibrium plan? Given such a plan, all the results of Sections 2-4 can be interpreted as applying to that plan.

In order to discuss formally the notion of optimality, suppose there is an international welfare function $w : \Theta^2 \times E^2 \to \Re$ that embodies various ideological and ethical positions. An example of the former is a "green" welfare function that lexicographically prefers aggregate emissions to be low. An example of the latter is an "egalitarian" welfare function that prefers an equal distribution of emission rights.

Given $w$, we say that an equilibrium plan $e$ is optimal if $w(\theta, e(\theta)) \geq w(\theta, \overline{e}(\theta))$ for every equilibrium plan $\overline{e}$ and type profile $\theta \in \Theta^2$. This is a strong notion of optimality as it requires an equilibrium plan that is superior to every other plan for every possible profile of types. It can be shown that an optimal equilibrium plan exists under very general conditions.[8]

### 5.2 The institutional framework

The model presented in Sections 2-4 operates in an institutional vacuum. For instance, there is no procedural account of how an equilibrium plan is chosen and how it becomes common knowledge. In addition, there can be informational asymmetries among the nations; in the case of our model, there could be incomplete information regarding the stocks of private and social capital. Keeping these issues in mind, we outline an institutional setting in which our abstract results can be interpreted.

**Step 1.** Nations negotiate to create an international mediator and the mediator's welfare function $w : \Theta^2 \times E^2 \to \Re$. This mediator serves as the common agent of the participating nations and is invested with the following powers: (a) to verify Nation $i$'s type $\theta_i \in \Theta$, and (b) to use the join of this information, $\theta = (\theta_1, \theta_2) \in \Theta^2$, to recommend emission cap $e_i(\theta) \in E$ to Nation $i$.

While emission caps are currently determined as a product of bargaining among the principals, our proposed procedure limits the role of bargaining to the determination of $w$. When negotiating the principles that shape $w$, the nations may not know each others characteristics, nor will they know the equilibrium plan that is ultimately to generate the caps. So, the nations are forced to negotiate behind a veil of ignorance. This disjunction between the process of formulating $w$ and its consequences in terms of allocation of caps is appropriate if $w$ is to reflect the ideological and ethical stances of the negotiating nations.

**Step 2.** Knowing the data $\{N, \Theta, E, (u_i)_{i \in N}\}$ and $w$, the mediator announces an optimal equilibrium plan $e$.

Since the principals are sovereign nations, we postulate that the assignment of caps for each profile of types is such that each nation will voluntarily choose to abide by its assigned cap, given the caps assigned to the other nations.[9] Consequently, the recommendations

[8] Theorems 4.1 and 4.2 in Shah (1999) provide very general sufficient conditions.

[9] Even prior to the issue of accepting prescribed caps, one must deal with the problem of inducing nations to participate in the emission capping regime for they might be tempted to stay out of the regime and free-ride on the emission cuts effected by participating nations. This can be prevented by the usual clause preventing the treaty from coming into force until everyone ratifies it. We assume that every nation prefers to join the agreement rather than maintain the *status quo*.

have to be generated by an equilibrium plan $e$. In case there are a number of equilibrium plans, conflict among the principals regarding the choice of implemented plan is resolved by picking the optimal plan with respect to $w$. Note that an optimal equilibrium plan is invariant with respect to the profile of national characteristics, regardless of whether the characteristics are determined exogenously or endogenously. If national characteristics are not exogenously given, then the procedure has to account for their endogenous determination.

**Step 3.** Knowing the data $\{N, \Theta, E, (u_i)_{i \in N}\}$, $w$ and $e$, the Nations/Firms choose their characteristics.

If characteristics are given exogenously, then Step 3 is skipped. In either case, the mediator can verify the characteristics.

**Step 4.** Given the profile of characteristics, the mediator uses $e$ to publicly announce caps. Since $e$ is an equilibrium plan, each nation will choose to abide by the announced cap.

### 5.3 International re-allocations

Our formulation of the firm's decision problem in Sections 2-4 does not allow for the international re-allocation of emission rights. Keeping in mind the possibility of internationally trading emission rights under the Kyoto protocol, an obvious extension of our model would be to allow firms to use their national emission caps domestically or as a resource for international trade.

We first consider the effects of such trade *after* endowments have been created as in our model. Since each nation's emission cap is a binding constraint on the domestic firm, the firm's shadow value of emission rights is positive. If Firm 1's shadow value of emission rights is greater than Firm 2's shadow value, then both benefit if Firm 2 "sells" some emission rights to Firm 1 at a "price" between the two shadow values. The "terms of trade" will depend on the trading mechanism used. As total emission is unchanged, so is the profile of damages. Therefore, a Pareto improving re-allocation of emission rights for the firms amounts to a Pareto improvement for the nations. Note that the trade between Firms 1 and 2 implies Nation 1 pays Nation 2 for emission rights, a resource that derives its value from the scarcity decreed by the capping regime.

Emission capping followed by trade means that the regulation of emissions is accompanied by a 'wealth effect' in the form of a transfer of resources that would not have occurred in the absence of a capping regime. The model presented in Sections 2-4 represents a pure theory of endowment determination that is free of this 'wealth effect'. Our pure theory is intended to serve as a benchmark against which a sensible and transparent discussion of international resource transfers can be conducted. A transfer neutral model also allows for the design of the best mechanism for making the transfers should such transfers be acceptable to all concerned parties; it is far from obvious that the distortion of emission caps is the best mechanism for implementing general resource transfers.

In spite of the above arguments, suppose we wish to dispense with a pure theory of endowment determination and allow the incorporation of trading into our framework. Naturally, the solution of the extended model will hinge on the chosen mechanism for trade. The choice of a trading mechanism is a normative issue that can be studied formally using the theory of mechanism design (see Mas-Colell, et al. (1995)). It is desirable, for ethical as well as practical reasons, that mechanisms be evaluated in terms of the properties of their outcomes (e.g., Pareto efficiency, equity, fairness) across a rich domain of endowments. Given the prescriptive nature of these requirements, it is entirely appropriate to choose the mechanism prior to the determination of endowments.

The model of Sections 2-4 can be extended to formally incorporate a given trading mechanism as follows. First, expand the interpretation of a firm's profit to include not only the profit from production, but also the profit from trading in emission rights. The profit derived by Firm/Nation $i$ from a given endowment of emission rights will now reflect the choice made by Firm $i$ with respect to the use of emission rights for domestic production or international trade. As a result, the mappings from emission caps to profits and damages are not exogenously given data, as in the model of Section 2-4, but are determined endogenously by the equilibria of the game generated by the given trading mechanism.

This formal extension reduces the power of our results. First, since the equilibrium re-allocations will be a function of the characteristics of all firms, a given nation's profit and damage will no longer be functions exclusively of its own characteristics but functions of the entire profile of characteristics. While this "non-private-values" feature is easily incorporated into the abstract formalism discussed in Sections 5.1 and 5.2, the results of Sections 3 and 4 have to carry a significantly heavier load of assumptions. Secondly, not only are the equations complicated by the additional terms, these new terms are difficult to sign since they reflect myriad 'general equilibrium' effects.

### 5.4 Other criteria for determining emission caps

As noted in Section 1, the Kyoto protocol attempts to solve a stock externality problem by regulating flows into that stock. We have modeled the creation of flow endowments without addressing the critical issue of how responsibility for the historically given stock of carbon should be fixed, nor do we deal with the problem of calculating the appropriate compensation for the stock externality.

It can be argued that the nations that are largely responsible for the historically given stock should compensate the other nations, perhaps through a distortion of the allocation of flow rights. While there is a strong ethical case for such compensation, it is far from obvious that compensation should take the particular form of distorting the caps on emission flows. An alternative remedy for righting the historical wrong is a one-time lump-sum transfer. Once the slate is wiped clean through such compensation, there is no reason to worry about the origins of the stock externality in the determination of caps on emission flows. This separation allows us to concentrate exclusively on the flow externality problem.

A second issue that is raised in the context of emission capping is that a nation's emission cap should reflect some notion of "equity" and "fairness", perhaps extending to a *per capita* emission entitlement. While these moral positions can be debated, we comment only on their economic implications and implementability.

In the short-run, i.e., with fixed capital stocks, the principal reason for the advocacy of such rights by nations with low *per capita* emissions is the desire to exploit the caps *via* trade. Slightly adapting the argument made in Section 5.3, it is obvious that the award of equity-linked emission entitlements is tantamount to a transfer of other resources between nations in the guise of trade in emission rights. Even if the ethical desirability of such transfers is conceded, their implementability is doubtful. For instance, would the potential 'donor' nations (i.e., the buyers) want to make the required transfers?[10] Besides, even if

[10] The paucity of such transfers in the past indicates otherwise. When the question is posed squarely as one of resource transfer, there is no reason to be optimistic that potential donors who have been tightfisted in the past will turn generous if transfers are disguised as emission entitlements.

donor nations wish to make the transfers, it is not obvious that they should be made *via* emission cap distortions rather than other mechanisms, say by lump-sum transfers.

Alternatively, if emission rights are not internationally tradeable, then the only way to exploit the rights is to expand domestic production upto the point where the cap becomes binding. Consequently, the international profile of emissions will replicate the international profile of caps. However, as equity-based caps are not necessarily a Nash equilibrium, some nation will reject this arrangement. While ethical, non-equilibrium requirements such as equity are enforceable within a nation if the prevailing "social contract" permits, these requirements are difficult to impose on sovereign nations.

### 5.5 Growth and emissions

A dynamic variant of the above static equity argument is that less developed countries (LDCs), whose technology and product mix tend to be primitive and "dirty", need more liberal caps in order to grow. After all, the industrial growth of developed countries (DCs) was not constrained by emission caps when they were in their phase of "dirty" growth. Therefore, the argument goes, LDCs should not be burdened unduly with emission constraints. Even if one accepts the implicit 'iron law' that a nation must transit through a phase of "dirty" growth to reach the phase of clean growth, it is not obvious that the solution to the growth problem of LDCs is to award them emission rights in perpetuity. In the context of our model, the obvious response to this dynamic problem is to regularly revise caps to take into account the growth trajectories of the LDCs. Consequently, emission rights will be akin to leasehold rights rather than freehold rights.

Although our model is not a growth model, we can use the results of Section 3 to conjecture the effects of autonomous growth on emissions. Suppose, as postulated by the implicit 'iron law', technology is emission enhancing for low levels of private capital and emission retarding for high levels of private capital. Let Nation 1 be a DC with a stock of private capital sufficiently large for its technology to be emission retarding, and let Nation 2 be an LDC with a private capital stock sufficiently small for its technology to be emission enhancing. We identify a country's growth with increases in its private capital stock. Given that every nation will exploit its entire emission rights endowment, Proposition 3.2.1 implies that Nation 1's growth will decrease its own emission, increase Nation 2's emission and decrease total emission. On the other hand, Nation 2's growth will increase its own emissions, decrease Nation 2's emission and increase total emission. Therefore, autonomous growth, regardless of where it occurs, causes convergence of caps. Note that a thoroughgoing "green" will favor growth in the DC rather than the LDC as the former lowers total emission while the latter raises it! However, these conclusions would be valid only if the growth process was autonomous. It is more sensible to view growth as being endogenously determined by the investment decisions of nations/firms. While the results of Section 4 can be invoked in this context, we need to be careful as these results allow only one nation to choose investments.

Proposition 4.2.6 implies that Nation 1 will underinvest in domestic private capital. Indeed, if its private capital stock is sufficiently large, it will choose zero investment. Moreover, as Nation 2's technology is emission enhancing, Nation 1 will not invest in Nation 2's private capital. Since the developed countries control most investible resources, the emission capping regime seems to hurt growth prospects everywhere. However, Nation 1 can loosen its own emission cap and continue to grow by overinvesting in domestic social capital. This increases Nation 1's cap and increases the incentive for investment in domestic private capital. Thus, as far as a DC is concerned, overinvestment in social capital is the price for continued domestic economic growth under the capping regime we have postulated.

On the other hand, Propositions 4.2.6 and 4.4.8 imply that Nation 2 has an incentive to overinvest in domestic and foreign private capital as well as in domestic social capital. All three types of investment translate into a higher cap for Nation 2 and a lower cap for Nation 1. The problem, of course, is not that capping will restrict an LDC's incentive to invest and grow (as suggested by the 'iron law'), but that an LDC will simply not have the resources to make the desired investments. This, however, is the traditional problem faced by LDCs and not a new one created by emission capping.

## 6. Dynamics and stability

Suppose the mediator postulated in Section 5.2 uses an adaptive 'best response' algorithm to approximate the equilibrium mediation plan rather than compute the relevant fixed point. We check whether such a scheme can succeed.

### 6.1 The dynamical system

Let $e : \Theta^2 \to E^2$ be an equilibrium plan. Suppose the true type profile is $\theta \in \Theta^2$. Given $\theta$, if we start with a profile of caps $(e_1^0, e_2^0)$ and construct a sequence of caps $(e_1^n, e_2^n)$ where $e_1^n$ is a best response by Nation 1 to $e_2^{n-1}$ and $e_2^n$ is a best response by Nation 2 to $e_1^{n-1}$, then does the constructed sequence converge asymptotically to the equilibrium profile of caps $(e_1(\theta), e_2(\theta))$? Such a dynamic process can represent both, a process of negotiations and a process by which a mediator who is ignorant of the underlying parameters can arrive at an equilibrium profile of caps.

Since both players' types remain fixed, we drop them from our expressions. We postulate the continuous time "best response" dynamical system determining $e(\tau) = (e_1(\tau), e_2(\tau))$ for $\tau \in \Re_+$:

$$\begin{aligned} De_1(\tau) &= \beta_1 \circ e_2(\tau) - e_1(\tau) \\ De_2(\tau) &= \beta_2 \circ e_1(\tau) - e_2(\tau) \end{aligned} \tag{6.1.1}$$

where $\beta_i$ is the best response mapping of Nation $i$ and $e^* = (e_1^*, e_2^*)$ is the static equilibrium.

### 6.2 The stability argument

We wish to establish that the steady state solution $\tau \mapsto e^*$ is a locally asymptotically stable solution of (6.1.1). For this, it is sufficient that the solution of the following linear dynamical system derived from (6.1.1) be asymptotically stable:

$$\begin{pmatrix} De_1(\tau) \\ De_2(\tau) \end{pmatrix} = \begin{pmatrix} -1 & D\beta_1(e_2^*) \\ D\beta_2(e_1^*) & -1 \end{pmatrix} \begin{pmatrix} e_1(\tau) \\ e_2(\tau) \end{pmatrix} + \begin{pmatrix} e_1^* - D\beta_1(e_2^*)e_2^* \\ e_2^* - D\beta_2(e_1^*)e_1^* \end{pmatrix} \tag{6.2.1}$$

For the solution of (6.2.1) to be asymptotically stable, it is sufficient that all the roots of

$$B = \begin{pmatrix} -1 & D\beta_1(e_2^*) \\ D\beta_2(e_1^*) & -1 \end{pmatrix}$$

have negative real parts. By Hadamard's theorem, this is guaranteed if $B$ is a dominant diagonal matrix. Calculation of $D\beta_1(e_2^*)$ and $D\beta_2(e_1^*)$, and application of Lemma 2.2.5, implies that $B$ has a dominant diagonal.

## 7. Conclusions

The distribution of emission endowments will depend on a complicated interaction of the technological and environmental damage characteristics of the various nations. We have modeled and analyzed the situation with the aims of (a) providing a theoretical and practical means for organizing the discussion, (b) predicting the resulting pattern of emission caps, and (c) predicting the strategic behavior that can be expected to emerge in a dynamic extension of the basic model. Our broad results are as follows.

Other things being equal, nations with relatively less social capital will have smaller emission caps. One should caution that a nation's social capital devoted to environmental damage prevention and reduction should not be interpreted in absolute terms but in relation to that nation's natural vulnerability to such damage. The prediction of the pattern of caps on account of differences in private capital is ambiguous: if technology is emission enhancing (resp. retarding), then greater private capital translates into a higher (resp. lower) cap.

The effects of an exogenous increase of one nation's social capital is to increase that nation's cap, reduce the other nation's cap and raise the aggregate cap. If technology is emission enhancing (resp. retarding), then the effect of an exogenous increase of one nation's private capital is to increase (resp. decrease) that nation's cap, reduce (resp. raise) the other nation's cap and raise (resp. reduce) the aggregate cap.

If we model the levels of private and social capital as being chosen prior to the allocation of caps, then this endogenous choice will be affected by three factors: the cost of investment, the direct effect on profit and environmental damage, and the indirect benefit derived from manipulating emission caps. The last of these factors can be called the strategic effect.

In our model, a nation and the domestic firm will strategically overinvest in domestic social capital. Given the levels of private and social capital chosen by the nation, the firm will locally prefer a higher (resp. lower) level of social capital if the equilibrium damage suffered by the nation is increasing (resp. decreasing) in $k$. Neither the nation, nor the firm, will invest in foreign social capital as such investment reduces the domestic emission cap and raises the aggregate emission cap, thereby hurting domestic profit as well as causing greater domestic damage.

Suppose technology is emission enhancing. Then a nation and the domestic firm will overinvest in domestic private capital; they will not invest in foreign private capital as that reduces the domestic emission cap and raises the aggregate emission cap, thereby hurting domestic profit as well as causing greater domestic damage. Given the levels of private and social capital chosen by the nation, the firm locally prefers to increase the level of domestic private capital.

Suppose technology is emission retarding. Then a nation and the domestic firm will underinvest in domestic private capital and overinvest in foreign private capital. Given the levels of private and social capital chosen by the nation, the firm locally prefers to invest more in domestic private capital and less in foreign private capital.

In general, all the strategic maneuvers by the nation are designed to increase the domestic emission cap and reduce the foreign emission cap, while the strategic maneuvers by the domestic firm are designed simply to increase the domestic emission cap. Since caps are determined by an equilibrium plan, the domestic cap is a best response to the foreign cap. This means that, at an equilibrium allocation of caps, first-order perturbations in the domestic cap do not affect a nation's utility. Thus, local strategic maneuvers by a nation effectively work through a manipulation of the foreign cap.

Although our model does not make cap maximization an explicit objective of the players, they behave in a manner consistent with such an objective. It should be pointed out that this desire to increase the cap arises purely out of considerations such as profit and economic damage, and is completely independent of considerations of the market value of the caps in a trading regime that might follow the determination of endowments.

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