Economic Efficiency, Distributive Justice and Liability Rules

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Abstract

The main purpose of this paper is to show that the conflict between the considerations

involving economic efficiency and those of distributive justice, in the context of

assigning liability, is not as sharp as is generally believed to be the case. The condition of

negligence liability which characterizes efficiency in the context of liability rules has an

all-or-none character. Negligence liability requires that if one party is negligent and the

other is not then the liability for the entire accident loss must fall on the negligent party.

Thus within the framework of standard liability rules efficiency requirements preclude

any non-efficiency considerations in cases where one party is negligent and the other is

not. In this paper it is shown that a part of accident loss plays no part in providing

appropriate incentives to the parties for taking due care and can therefore be apportioned

on non-efficiency considerations. For a systematic analysis of efficiency requirements, a

notion more general than that of a liability rule, namely, that of a decomposed liability

rule is introduced. A complete characterization of efficient decomposed liability rules is

provided in the paper. One important implication of the characterization theorems of this

paper is that by decomposing accident loss in two parts, the scope for distributive

considerations can be significantly broadened without sacrificing economic efficiency.

Key Words: Tort Law, Liability Rules, Decomposed liability Rules, Efficient Rules,

Nash Equilibria, Negligence Liability, Distributive Justice

JEL Classification: K13

Considerations relating to the efficiency of liability rules have occupied an important place in the law and economics literature right from its inception. The pioneering contribution by Calabresi (1961) analyzed the effect of liability rules on parties' behaviour. In his seminal contribution Coase (1960) looked at liability rules from the point of view of their implications for social costs. The rule of negligence was analyzed by Posner (1972) from the perspective of economic efficiency. The first formal analysis of liability rules was done by Brown (1973). His main results demonstrated the efficiency of both the rule of negligence and the rule of strict liability with the defense of contributory negligence. Formal treatment of some of the most important results of the extensive literature on liability rules is contained in Landes and Posner (1987), Shavell (1987), and Miceli (1997). A complete characterization of efficient liability rules is contained in Jain and Singh (2002).

In the literature dealing with the question of efficiency of liability rules, the problem has generally been considered within the framework of accidents resulting from interaction of two risk-neutral parties, the victim and the injurer. The social goal is taken to be the minimization of total social costs, which are defined to be the sum of costs of care taken by the two parties and expected accident loss. The probability of accident and the amount of loss in case of occurrence of accident are assumed to depend on the levels of care taken by the two parties. A party is called nonnegligent if its care level is at least equal to the due care level; otherwise it is called negligent. A liability rule determines the proportions in which the two parties are to bear the loss in case of occurrence of accident as a function of whether and by what proportion the parties involved in the interaction are negligent. A liability rule is efficient if it invariably induces both parties to behave in ways which result in socially optimal outcomes, i.e., outcomes under which total social costs are minimized. The central result regarding the efficiency question that has emerged is that a liability rule is efficient if and only if it satisfies the condition of negligence liability. The condition of negligence liability requires that (i) if the victim is nonnegligent and the injurer is negligent then the entire loss, in case of occurrence of accident, must be borne by the injurer; and (ii) if the injurer is nonnegligent and the victim is negligent then the entire loss, in case of occurrence of accident, must be borne by the victim.

The condition of negligence liability, which has an all-or-none character, completely specifies the assignment of liability shares in cases where one party is negligent and the other is not. Consequently, it would seem that if the choice of a liability rule is to be from the set of efficient liability rules then the non-efficiency considerations, including distributive and restitutive considerations, cannot possibly play any role in assigning liability in cases where one party is negligent and the other is not. Other considerations at best can have a role only in situations when either both parties are negligent or both are nonnegligent.

If liabilities of the parties are to be specified as proportions of total accident loss, as is

done in tort law, then what has been said above about efficiency considerations precluding other considerations in cases where one party is negligent and the other nonnegligent is indeed correct. But if one is willing to go outside the traditional tort law framework then it turns out that the scope for non-efficiency considerations is much greater than is generally thought to be the case. In providing correct incentives to the parties, part of accident loss, equal to the optimal loss when both parties are taking the due care, suitably adjusted to take into account differing probabilities of accident with different care levels, seems to play no role and can therefore be apportioned between the two parties independently of their care levels. It is the apportionment of the accident loss over and above the adjusted optimal loss which turns out to be crucial from the point of view of providing correct incentives to the parties. An example may help illustrate the point.

Consider a two-party interaction in which the accident occurs with certainty; but the magnitude of accident loss depends on the care levels of the parties. Let the loss be 100 if neither party takes care, 98.5 if one party takes care and the other does not, and 97 if both parties take care. Let taking care by either party cost 1. As the rule of strict liability with the defense of contributory negligence is an efficient liability rule, it would induce in the context of the scenario of this example both the parties to take care and thus lead to the socially optimal outcome. It can easily be checked that the rule would lead to the socially optimal outcome of both parties taking care even if part of the loss equal to the optimal loss, which is 97 here, is assigned to the injurer regardless of the care levels of the two parties.

This example makes it clear that the efficiency requirement does not preclude altogether a role for distributive considerations even when one party is negligent and the other is not. In principle, part of the accident loss can be assigned between the parties purely on non-efficiency considerations without affecting the efficiency property. For a systematic treatment of this question the notion of a liability rule needs to be generalized so that all possible decompositions of accident loss could be considered to find out the precise constraints imposed by the efficiency requirement.

A liability rule apportions the accident loss between the parties as a function of whether and by what proportions the two parties are nonnegligent. Corresponding to any liability rule one can define a two-parameter family of rules in the following way: (i) A specified multiple (θ) of adjusted optimal loss is to be assigned between the two parties in fixed proportions $(\lambda, 1 - \lambda)$ (ii) The remainder of the loss is to be apportioned between the two parties as specified by the liability rule in question as a function of proportions of nonnegligence of the parties. This more general notion of a rule would be called a (λ, θ) -decomposed liability rule. For $\theta = 0$ the two notions coincide. We show that if $0 \le \theta \le 1$ then the decomposed liability rule is efficient if and only if the corresponding liability rule is efficient. If $\theta > 1$ then it turns out that no decomposed liability rule can be efficient.

In other words, efficiency properties remain unaffected provided the quantum of loss that is assigned independently of care levels does not exceed adjusted optimal loss. Regardless of whether a decomposed liability rule corresponds to an efficient liability rule or an inefficient liability rule, if $\theta > 1$ then the decomposed liability rule would be inefficient. In view of these results it is clear that it is the amount of loss that is in excess of the adjusted optimal loss which constitutes the irreducible minimum which must be assigned to the negligent party to ensure efficiency, in cases where one party is negligent and the other is not. Thus the requirements imposed by efficiency considerations could be quite mild depending on the context.

The paper is divided in two sections. The first section sets out the framework of analysis and introduces the notion of a decomposed liability rule. The next section shows that, given $0 \le \theta \le 1$, a (λ, θ) -decomposed liability rule is efficient if and only if it satisfies the condition of negligence liability. It is also shown that for $\theta > 1$ every (λ, θ) -decomposed liability rule is inefficient.

1 Definitions and Assumptions

We consider accidents resulting from interaction of two parties, assumed to be strangers to each other, in which, to begin with, the entire loss falls on one party to be called the victim (plaintiff). The other party would be referred to as the injurer (defendant). We denote by $a \ge 0$ the index of the level of care taken by the victim; and by $b \ge 0$ the index of the level of care taken by the injurer.

Let

 $A = \{a \mid a \geq 0 \text{ is the index of some feasible level of care which can be taken by the victim}\}, and$

 $B = \{b \mid b \ge 0 \text{ is the index of some feasible level of care which can be taken by the injurer}\}.$

We assume:

$$0 \in A \land 0 \in B. \tag{A1}$$

We denote by c(a) the cost to the victim of care level a and by d(b) the cost to the injurer of care level b.

Let $C = \{c(a) \mid a \in A\}$, and $D = \{d(b) \mid b \in B\}$.

We assume:

$$c(0) = 0 \land d(0) = 0. \tag{A2}$$

We also assume that c and d are strictly increasing functions of a and b respectively.

(A3)

In view of (A2) and (A3) it follows that:

$$(\forall c \in C)(c > 0) \land (\forall d \in D)(d > 0).$$

A consequence of (A3) is that c and d themselves can be taken as indices of levels of care of the two parties.

Let Π denote the probability of occurrence of accident and $H \geq 0$ the loss in case of occurrence of accident. Both Π and H will be assumed to be functions of c and d; $\Pi = \Pi(c,d), H = H(c,d)$. Let $L = \Pi H$. L is thus the expected loss due to accident. We assume:

$$(\forall c, c' \in C)(\forall d, d' \in D)[[c > c' \to L(c, d) \le L(c', d)] \land [d > d' \to L(c, d) \le L(c, d')]]. \tag{A4}$$

In other words, it is assumed that a larger expenditure on care by either party, given the expenditure on care by the other party, results in lesser or equal expected accident loss.

Total social costs (TSC) are defined to be the sum of cost of care by the victim, cost of care by the injurer, and the expected loss due to accident; TSC = c + d + L(c, d). Let $M = \{(c', d') \in C \times D \mid c' + d' + L(c', d') \text{ is minimum of } \{c + d + L(c, d) \mid c \in C \land d \in D\}\}$. Thus M is the set of all costs of care configurations (c', d') which are total social cost minimizing. It will be assumed that:

$$C, D \text{ and } L \text{ are such that } M \text{ is nonempty.}$$
 (A5)

Let I denote the closed unit interval $[0,1]^1$. Given C, D, Π, H and $(c^*, d^*) \in M$, we define functions p and q as follows:

$$p: C \mapsto I \text{ by:}$$

$$p(c) = 1 \text{ if } c \ge c^*$$

$$= \frac{c}{c^*} \text{ if } c < c^*;$$

$$q: D \mapsto I \text{ by:}$$

$$q(d) = 1 \text{ if } d \ge d^*$$

$$= \frac{d}{d^*} \text{ if } d < d^*.$$

In case there is a legally binding due care level for the plaintiff, it would be taken to be identical with c^* figuring in the definition of function p; and in case there is a legally binding due care level for the defendant, it would be taken to be identical with d^* figuring in the definition of function q. Thus implicitly it is being assumed that the legally binding due care levels are always set appropriately from the point of view of minimizing total social costs.

¹In addition to denoting the set $\{x \mid 0 \le x \le 1\}$ by [0,1], we use the following standard notation to denote:

by [0,1) the set $\{x \mid 0 \le x < 1\}$,

by (0,1] the set $\{x \mid 0 < x \le 1\}$, and

by (0,1) the set $\{x \mid 0 < x < 1\}$.

p and q would be interpreted as proportions of nonnegligence of the victim and the injurer respectively. (1-p) and (1-q) consequently would denote the proportions of negligence of the victim and the injurer respectively.

Let $\lambda \in [0,1]$ and $\theta \geq 0$. Denote $H(c^*,d^*), \Pi(c^*,d^*)$ and $L(c^*,d^*)$ by H^*,Π^* and L^* respectively.

Define functions S and G as follows:

$$S(c,d) = H(c,d) - \theta H^* \frac{\Pi^*}{\Pi(c,d)} \quad \text{if } \Pi(c,d) \neq 0 \ \land \ H(c,d) > \theta H^* \frac{\Pi^*}{\Pi(c,d)}$$
$$= 0 \quad \text{otherwise.}$$

$$G(c,d) = \theta H^* \frac{\Pi^*}{\Pi(c,d)}$$
 if $S(c,d) > 0$
= $H(c,d)$ if $S(c,d) = 0$.

Let $R(c,d) = \Pi(c,d)S(c,d)$ and $F(c,d) = \Pi(c,d)G(c,d)$.

In other words,

$$R(c,d) = L(c,d) - \theta L^*$$
 if $L(c,d) > \theta L^*$
= 0 otherwise.

$$F(c,d) = \theta L^* \qquad \text{if } L(c,d) > \theta L^*$$

= $L(c,d) \qquad \text{if } L(c,d) \le \theta L^*.$

Thus,

$$R(c,d) = max\{L(c,d) - \theta L^*, 0\}$$

$$F(c,d) = \min\{L(c,d), \ \theta L^*\}.$$

G(c,d) will be referred to as the specified multiple of the adjusted optimal loss; and S(c,d) will be called the excess loss. Consistent with this nomenclature, F(c,d) will be referred to as the specified multiple of the expected optimal loss; and R(c,d) as the expected excess loss.

A liability rule is a rule which specifies the proportions in which the two parties are to bear the loss in case of occurrence of accident as a function of proportions of two parties' nonnegligence. Formally, a liability rule is a function f from I^2 to I^2 , $f: I^2 \mapsto I^2$, such that: f(p,q) = (x,y), where x + y = 1.

A (λ, θ) - decomposed liability rule is a rule which specifies the proportions in which the two parties are to bear the excess loss in case of occurrence of accident as a function of proportions of two parties' nonnegligence; and assigns specified multiple of adjusted optimal loss in the $(\lambda, 1 - \lambda)$ proportions to the victim and the injurer respectively. Formally, a (λ, θ) - decomposed liability rule is a function f from I^2 to I^2 , $f: I^2 \mapsto I^2$, such that: f(p,q) = (x,y) = [x(p,q),y(p,q)], where x + y = 1.

Let C, D, Π, H and $(c^*, d^*) \in M$ be given. If accident takes place and loss of H(c, d) materializes, then $xS(c, d) + \lambda G(c, d)$ will be borne by the victim and $yS(c, d) + (1 - \lambda)G(c, d)$ by the injurer. As, to begin with, in case of occurrence of accident, the entire loss falls upon the victim, $yS(c, d) + (1 - \lambda)G(c, d)$ represents the liability payment by

the injurer to the victim. The expected costs of the victim and the injurer, to be denoted by EC_1 and EC_2 respectively, therefore, are $[c + xR(c,d) + \lambda F(c,d)]$ and $[d + yR(c,d) + (1-\lambda)F(c,d)]$ respectively. Both parties are assumed to prefer smaller expected costs to larger expected costs and be indifferent between alternatives with equal expected costs.

Remark 1 The notion of a (λ, θ) - decomposed liability rule is a generalization of the notion of a liability rule. For $\theta = 0$, the definition of a (λ, θ) - decomposed liability rule reduces to that of a liability rule.

It should be noted that for every liability rule there corresponds a class of (λ, θ) -decomposed liability rules. The class corresponding to liability rule f is given by: $\{g \mid g \text{ is } a \ (\lambda, \theta) \text{ - decomposed liability rule, } \lambda \in [0, 1], \theta \geq 0, (\forall p, q \in [0, 1])[g(p, q) = f(p, q)]\}.$

Let f be a (λ, θ) - decomposed liability rule. An application of f consists of specification of C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5). f is defined to be efficient for a given application C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5) iff $(\forall (\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium $\to (\overline{c}, \overline{d}) \in M]$ and $(\exists (\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium]. f is defined to be efficient iff it is efficient for every possible application. In other words, a (λ, θ) - decomposed liability rule is efficient for given f given f and f an

Remark 2 It should be noted that if (A5) is not satisfied then no (λ, θ) - decomposed liability rule can be efficient.

To illustrate some of the above ideas we consider below several examples.

Example 1: Let $(\frac{1}{3}, 1)$ - decomposed liability rule f be defined by:

$$(\forall p \in [0,1])(\forall q \in [0,1])[f(p,q) = (\frac{1}{2}, \frac{1}{2})].$$

Consider an application of f such that:

$$C = D = \{0, 1\};$$

$$L(0,0) = 9, L(0,1) = L(1,0) = 7.5, L(1,1) = 6.$$

Here (1,1) is the unique TSC-minimizing configuration of costs of care.

$$Let(c^*, d^*) = (1, 1).$$

As $\theta L^* = 6$, we obtain:

$$R(0,0) = 3, R(0,1) = R(1,0) = 1.5, R(1,1) = 0$$

$$F(0,0) = F(0,1) = F(1,0) = F(1,1) = 6.$$

$$EC_1(0,0) = 3.5, EC_1(0,1) = 2.75, EC_1(1,0) = 3.75, EC_1(1,1) = 3;$$

²Throughout this paper we consider only pure-strategy Nash equilibria.

$$EC_2(0,0) = 5.5, EC_2(0,1) = 5.75, EC_2(1,0) = 4.75, EC_2(1,1) = 5.$$

Therefore (0,0) is the only Nash equilibrium. Thus f is inefficient.

Example 2: Let $(\frac{1}{2}, \frac{1}{2})$ - decomposed liability rule f be defined by:

$$(\forall p \in [0,1))(\forall q \in [0,1))[f(p,q) = (\frac{1}{2},\frac{1}{2})] \land (\forall p \in [0,1))[f(p,1) = (1,0)] \land (\forall q \in [0,1))[f(1,q) = (0,1)] \land [f(1,1) = (\frac{1}{2},\frac{1}{2}].$$

Consider an application of f such that:

$$C = D = \{0, 1\};$$

$$L(0,0) = 10, L(0,1) = L(1,0) = 8.5, L(1,1) = 7.$$

Let $(c^*, d^*) = (1, 1)$, which is the unique TSC-minimizing configuration of costs of care.

As $\theta L^* = 3.5$, we obtain:

$$R(0,0) = 6.5, R(0,1) = R(1,0) = 5, R(1,1) = 3.5$$

$$F(0,0) = F(0,1) = F(1,0) = F(1,1) = 3.5$$

We have:

$$EC_1(0,0) = 5$$
, $EC_1(0,1) = 6.75$, $EC_1(1,0) = 2.75$, $EC_1(1,1) = 4.5$

$$EC_2(0,0) = 5, EC_2(0,1) = 2.75, EC_2(1,0) = 6.75, EC_2(1,1) = 4.5$$

Here (1,1) is the only Nash equilibrium. Thus the rule is efficient for the application considered here.³

Example 3: Let $(\frac{1}{2}, 2)$ - decomposed liability rule f be defined by:

$$(\forall p \in [0,1))(\forall q \in [0,1))[f(p,q) = (\frac{1}{2},\frac{1}{2})] \land (\forall p \in [0,1))[f(p,1) = (1,0)] \land (\forall q \in [0,1))[f(1,q) = (0,1)] \land [f(1,1) = (\frac{1}{2},\frac{1}{2}].$$

Consider an application of f such that:

$$C = D = \{0, 1\};$$

$$L(0,0) = 10, L(0,1) = L(1,0) = 8.5, L(1,1) = 7.$$

Let $(c^*, d^*) = (1, 1)$, which is the unique TSC-minimizing configuration of costs of care.

As $\theta L^* = 14$, it follows that:

$$(\forall (c,d) \in C \times D)[R(c,d) = (0,0) \land F(c,d) = L(c,d)].$$

$$EC_1(0,0) = 5$$
, $EC_1(0,1) = 4.25$, $EC_1(1,0) = 5.25$, $EC_1(1,1) = 4.5$

$$EC_2(0,0) = 5$$
, $EC_2(0,1) = 5.25$, $EC_2(1,0) = 4.25$, $EC_2(1,1) = 4.5$

Consequently, (0,0) is the only Nash equilibrium; and thus the rule is inefficient.

Example 4: Let f be a(0,1) - decomposed liability rule defined by:

$$(\forall p \in [0,1])(\forall q \in [0,1))[f(p,q) = (0,1)] \land (\forall p \in [0,1])[f(p,1) = (1,0)].$$

Consider:

 $^{3(\}frac{1}{2},\frac{1}{2})$ - decomposed liability rule of Example 2 is efficient for every C,D,Π,H and $(c^*,d^*)\in M$ satisfying (A1) - (A5). Efficiency of this decomposed liability rule for every application follows from Theorem 1.

$$C = D = \{0, 1, 2\};$$

$$L(0,0) = 10, L(0,1) = 6, L(0,2) = 5, L(1,0) = 5.5, L(1,1) = 1.5, L(1,2) = .5, L(2,0) = 5, L(2,1) = 1, L(2,2) = 0.$$

Here
$$M = \{(1,1), (1,2)\}$$
. Let $(c^*, d^*) = (1,1)$. Hence, $L^* = 1.5$

Therefore,

$$R(0,0) = 8.5, R(0,1) = 4.5, R(0,2) = 3.5, R(1,0) = 4, R(1,1) = 0, R(1,2) = 0, R(2,0) = 3.5, R(2,1) = 0, R(2,2) = 0;$$
 and

$$F(0,0) = F(0,1) = F(0,2) = F(1,0) = F(1,1) = F(2,0) = 1.5, F(1,2) = .5, F(2,1) = 1, F(2,2) = 0.$$

We have:

$$EC_1(0,0) = 0, EC_1(0,1) = 4.5, EC_1(0,2) = 3.5, EC_1(1,0) = 1, EC_1(1,1) = 1,$$

$$EC_1(1,2) = 1, EC_1(2,0) = 2, EC_1(2,1) = 2, EC_1(2,2) = 2;$$

$$EC_2(0,0) = 10, EC_2(0,1) = 2.5, EC_2(0,2) = 3.5, EC_2(1,0) = 5.5, EC_2(1,1) = 2.5,$$

$$EC_2(1,2) = 2.5, EC_2(2,0) = 5, EC_2(2,1) = 2, EC_2(2,2) = 2.$$

Here there are two Nash equilibria, namely, (1, 1) and (1, 2).

The rule is efficient for the application considered here.⁴

Example 5: Let (1,1) - decomposed liability rule f be defined by:

$$(\forall p \in [0,1))(\forall q \in [0,1])[f(p,q) = (1,0)] \land (\forall q \in [0,1])[f(1,q) = (0,1)].$$

Let:

$$C = \{0, 1, 2\}, D = \{0, 1\};$$

$$L(0,0) = 10, L(0,1) = 6, L(1,0) = 5, L(1,1) = 1, L(2,0) = 4, L(2,1) = 0.$$

Here $M = \{(1,1), (2,1)\}.$

$$Let(c^*, d^*) = (2, 1).$$

Therefore,

$$L^* = 0$$
, and $(\forall (c, d) \in C \times D)[R(c, d) = L(c, d) \land F(c, d) = 0]$; and

$$EC_1(0,0) = 10, EC_1(0,1) = 6, EC_1(1,0) = 6, EC_1(1,1) = 2, EC_1(2,0) = 2, EC_1(2,1) = 2;$$

$$EC_2(0,0) = 0, EC_2(0,1) = 1, EC_2(1,0) = 0, EC_2(1,1) = 1, EC_2(2,0) = 4, EC_2(2,1) = 1.$$

Here (2,1) is the only Nash equilibrium; and f is efficient for the given application.⁵

2 Characterization of Efficient Decomposed Liability Rules

Condition of Negligence Liability (NL): A (λ, θ) - decomposed liability rule f satisfies the condition of negligence liability iff $[[\forall p \in [0, 1)][f(p, 1) = (1, 0)] \land [\forall q \in [0, 1)][f(1, q) = (1, 0)]$

⁴The rule is efficient for every application.

⁵This rule is also efficient for every application.

(0,1)]].

In other words, a decomposed liability rule satisfies the condition of negligence liability iff its structure is such that (i) whenever the injurer is nonnegligent and the victim is negligent, the entire excess loss in case of accident is borne by the victim, and (ii) whenever the victim is nonnegligent and the injurer is negligent, the entire excess loss in case of an accident is borne by the injurer.

Lemma 1 Let $0 \le \theta \le 1$. If a (λ, θ) - decomposed liability rule satisfies condition NL then for any arbitrary choice of C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5), (c^*, d^*) is a Nash equilibrium.

Proof: Let f be a (λ, θ) - decomposed liability rule satisfying NL, where $0 \le \theta \le 1$. Take any C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5).

Suppose (c^*, d^*) is not a Nash equilibrium. This implies:

$$(\exists c' \in C)[c' + x[p(c'), q(d^*)]R(c', d^*) + \lambda F(c', d^*) < c^* + x[p(c^*), q(d^*)]R(c^*, d^*) + \lambda F(c^*, d^*)] \lor (\exists d' \in D)[d' + y[p(c^*), q(d')]R(c^*, d') + (1 - \lambda)F(c^*, d') < d^* + y[p(c^*), q(d^*)]R(c^*, d^*) + (1 - \lambda)F(c^*, d^*)].$$

(1.1)

First suppose
$$(\exists c' \in C)[c'+x[p(c'),q(d^*)]R(c',d^*) + \lambda F(c',d^*) < c^*+x[p(c^*),q(d^*)]R(c^*,d^*) + \lambda F(c^*,d^*)]$$
 holds. (1.2)

We first consider the case $c' < c^*$.

$$c' < c^* \to x[p(c'), q(d^*)] = 1$$
, by condition NL. (1.3)

As $(c^*, d^*) \in M$ it follows that:

$$c' < c^* \ \rightarrow \ L(c',d^*) > L^*$$

$$\to L(c', d^*) - \theta L^* > 0$$
, as $0 < \theta < 1$

$$\to R(c', d^*) = L(c', d^*) - \theta L^* \wedge F(c', d^*) = \theta L^*. \tag{1.4}$$

In view of (1.3) and (1.4), (1.2) implies:

$$c' + L(c', d^*) - \theta L^* + \lambda \theta L^* < c^* + x^1 (1 - \theta) L^* + \lambda \theta L^*, \text{ where } x^1 \text{ denotes } x(1, 1).$$

$$(1.5) \rightarrow c' + d^* + L(c', d^*) < c^* + d^* + \theta L^* + x^1 (1 - \theta) L^*$$

$$\rightarrow c' + d^* + L(c', d^*) < c^* + d^* + L^*, \text{ as } 0 \le x^1 \le 1.$$

This is a contradiction as $(c^*, d^*) \in M$, and therefore $TSC(c', d^*)$ cannot be less than $TSC(c^*, d^*)$.

This contradiction establishes that
$$c' < c^* \rightarrow (1.2)$$
 cannot hold. (1.6)

Next consider the case when $c' > c^* \wedge L(c', d^*) > \theta L^*$.

If
$$c' > c^* \wedge L(c', d^*) > \theta L^*$$
 then (1.2) implies:

$$c' + x^{1}[L(c', d^{*}) - \theta L^{*}] + \lambda \theta L^{*} < c^{*} + x^{1}(1 - \theta)L^{*} + \lambda \theta L^{*}$$

$$\rightarrow c' + x^1 L(c', d^*) < c^* + x^1 L^*$$

$$\rightarrow (1-x^1)c' + x^1[c' + d^* + L(c', d^*)] < (1-x^1)c^* + x^1[c^* + d^* + L^*]$$

$$\rightarrow (1-x^1)c' < (1-x^1)c^*, \text{ as } x^1 \ge 0 \text{ and } TSC(c', d^*) \ge TSC(c^*, d^*).$$
 (1.7)

If
$$(1 - x^1) > 0$$
 then $[(1.7) \rightarrow c' < c^*]$. (1.8)

If
$$(1-x^1) = 0$$
 then $[(1.7) \rightarrow 0 < 0]$. (1.9)

 $c' < c^*$ contradicts the hypothesis that $c' > c^*$ and 0 < 0 is a contradiction. Therefore we conclude that:

$$c' > c^* \wedge L(c', d^*) > \theta L^* \rightarrow (1.2) \text{ cannot hold.}$$
 (1.10)

Finally consider the case when $c' > c^* \land L(c', d^*) \le \theta L^*$.

If $c' > c^* \wedge L(c', d^*) \leq \theta L^*$ then (1.2) implies:

$$c' + \lambda L(c', d^*) < c^* + x^1 (1 - \theta) L^* + \lambda \theta L^*. \tag{1.11}$$

$$(1.11) \rightarrow (1 - \lambda)c' + \lambda[c' + d^* + L(c', d^*)] < (1 - \lambda)c^* + \lambda[c^* + d^* + L^*]$$

$$-\lambda L^* + x^1(1-\theta)L^* + \lambda \theta L^*$$

$$\rightarrow (1 - \lambda)c' < (1 - \lambda)c^* - \lambda L^* + x^1(1 - \theta)L^* + \lambda \theta L^*,$$

as $TSC(c', d^*) \ge TSC(c^*, d^*)$ and $\lambda \ge 0$

$$\rightarrow (1 - \lambda)c' < (1 - \lambda)c^* - \lambda L^* + (1 - \theta)L^* + \lambda \theta L^*, \text{ as } 0 \le x^1 \le 1$$

$$\to (1 - \lambda)c' < (1 - \lambda)c^* + (1 - \theta)(1 - \lambda)L^* \tag{1.12}$$

If
$$(1 - \lambda) = 0$$
 then $[(1.12) \rightarrow 0 < 0$, a contradiction]. (1.13)

If
$$(1 - \lambda) > 0$$
 then $[(1.12) \rightarrow c' < c^* + (1 - \theta)L^*].$ (1.14)

But we have: $L^* - L(c', d^*) \le c' - c^*$, as $(c^*, d^*) \in M$;

and

 $L(c',d^*) < \theta L^*$, by hypothesis.

Consequently, $L^* \leq c' - c^* + \theta L^*$

$$\to c^* + (1 - \theta)L^* \le c', \tag{1.15}$$

contradicting $c' < c^* + (1 - \theta)L^*$.

In view of (1.13) - (1.15), it follows that:

$$c' > c^* \wedge L(c', d^*) \le \theta L^* \rightarrow (1.2) \text{ cannot hold.}$$
 (1.16)

$$(1.6)$$
, (1.10) and (1.16) establish that (1.2) cannot hold. (1.17)

By an analogous proof one can show that:

$$(\exists d' \in D)[d' + y[p(c^*), q(d')]R(c^*, d') + (1 - \lambda)F(c^*, d') < d^* + y[p(c^*), q(d^*)]R(c^*, d^*) + (1 - \lambda)F(c^*, d^*)] \text{ cannot hold.}$$

$$(1.18)$$

(1.17) and (1.18) establish the proposition.

Lemma 2 Let $0 \le \theta \le 1$. If a (λ, θ) - decomposed liability rule satisfies condition NL then for every possible choice of C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5):

$$(\forall (\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d}) \text{ is a Nash equilibrium } \rightarrow (\overline{c}, \overline{d}) \in M].$$

Proof: Let $0 \le \theta \le 1$. Let f be a (λ, θ) - decomposed liability rule satisfying NL. Take any C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5).

Let $(\overline{c}, \overline{d})$ be a Nash equilibrium. $(\overline{c}, \overline{d})$ being a Nash equilibrium implies:

$$(\forall c \in C)[\overline{c} + x[p(\overline{c}), q(\overline{d})]R(\overline{c}, \overline{d}) + \lambda F(\overline{c}, \overline{d}) \le c + x[p(c), q(\overline{d})]R(c, \overline{d}) + \lambda F(c, \overline{d})]$$
 and

$$(\forall d \in D)[\overline{d} + y[p(\overline{c}), q(\overline{d})]R(\overline{c}, \overline{d}) + (1 - \lambda)F(\overline{c}, \overline{d}) \leq d + y[p(\overline{c}), q(d)]R(\overline{c}, d) + (1 - \lambda)F(\overline{c}, d)]$$

(2.2)

$$(2.1) \rightarrow [\overline{c} + x[p(\overline{c}), q(\overline{d})]R(\overline{c}, \overline{d}) + \lambda F(\overline{c}, \overline{d}) \leq c^* + x[p(c^*), q(\overline{d})]R(c^*, \overline{d}) + \lambda F(c^*, \overline{d})]$$

$$(2.3)$$

$$(2.2) \rightarrow [\overline{d} + y[p(\overline{c}), q(\overline{d})]R(\overline{c}, \overline{d}) + (1 - \lambda)F(\overline{c}, \overline{d}) \leq d^* + y[p(\overline{c}), q(d^*)]R(\overline{c}, d^*) + (1 - \lambda)F(\overline{c}, d^*)]$$

$$(2.4)$$

Adding inequalities (2.3) and (2.4) we obtain:

$$\overline{c} + \overline{d} + R(\overline{c}, \overline{d}) + F(\overline{c}, \overline{d}) \le c^* + d^* + x[p(c^*), q(\overline{d})]R(c^*, \overline{d}) + y[p(\overline{c}), q(d^*)]R(\overline{c}, d^*) + \lambda F(c^*, \overline{d}) + (1 - \lambda)F(\overline{c}, d^*). \tag{2.5}$$

First we consider the case when $\overline{c} < c^* \wedge \overline{d} < d^*$.

$$\overline{c} < c^* \ \land \ \overline{d} < d^* \ \rightarrow \ R(\overline{c}, \overline{d}) = L(\overline{c}, \overline{d}) - \theta L^* \ \land \ F(\overline{c}, \overline{d}) = F(c^*, \overline{d}) = F(\overline{c}, d^*) = \theta L^*.$$

Therefore (2.5) reduces to:

$$\overline{c} + \overline{d} + L(\overline{c}, \overline{d}) - \theta L^* + \theta L^* \le c^* + d^* + \lambda \theta L^* + (1 - \lambda)\theta L^*,$$

as
$$x[p(c^*),q(\overline{d})]=0 \ \wedge \ y[p(\overline{c}),q(d^*)]=0$$
 by NL

$$\to \ \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \le c^* + d^* + \theta L^*$$

$$\to \ \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \le c^* + d^* + L^*.$$

As TSC is minimized at (c^*, d^*) , it follows that it must be the case that:

$$\overline{c} + \overline{d} + L(\overline{c}, \overline{d}) = c^* + d^* + L^*$$

This establishes that:

$$(\overline{c}, \overline{d})$$
 is a Nash equilibrium and $\overline{c} < c^* \land \overline{d} < d^* \rightarrow (\overline{c}, \overline{d}) \in M.$ (2.6)

Next we consider the case when $\bar{c} < c^* \land \bar{d} \ge d^*$.

If $\overline{c} < c^* \wedge \overline{d} \ge d^*$, in view of NL, (2.5) reduces to:

$$\overline{c} + \overline{d} + R(\overline{c}, \overline{d}) + F(\overline{c}, \overline{d}) \le c^* + d^* + x^1 R(c^*, \overline{d}) + \lambda F(c^*, \overline{d}) + (1 - \lambda) F(\overline{c}, d^*). \tag{2.7}$$

Now, by the definitions of functions R and F we have:

$$(\forall (c,d) \in C \times D)[R(c,d) + F(c,d) = L(c,d)].$$

Therefore (2.7) implies:

$$\overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \le c^* + d^* + x^1 R(c^*, \overline{d}) + \lambda F(c^*, \overline{d}) + (1 - \lambda)\theta L^*,$$

as
$$\overline{c} < c^* \rightarrow F(\overline{c}, d^*) = \theta L^*$$

$$\to \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \le c^* + d^* + x^1 R(c^*, \overline{d}) + \lambda \theta L^* + (1 - \lambda) \theta L^*,$$

as $F(c^*, \overline{d}) \leq \theta L^*$ by the definition of function F

$$\to \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \le c^* + d^* + R(c^*, \overline{d}) + \theta L^*$$
(2.8)

Now,
$$R(c^*, \overline{d}) + \theta L^* = L(c^*, \overline{d}), \quad \text{if } L(c^*, \overline{d}) > \theta L^*$$

= $\theta L^*, \quad \text{if } L(c^*, \overline{d}) \leq \theta L^*.$

As $\overline{d} \geq d^*$, we have $L(c^*, \overline{d}) \leq L^*$.

Also, $\theta L^* \leq L^*$, as $0 \leq \theta \leq 1$.

Therefore $R(c^*, \overline{d}) + \theta L^* \leq L^*$.

Consequently, (2.8) $\rightarrow \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \le c^* + d^* + L^*$.

This establishes that:

$$(\overline{c}, \overline{d})$$
 is a Nash equilibrium $\wedge \overline{c} < c^* \wedge \overline{d} \ge d^* \to (\overline{c}, \overline{d}) \in M.$ (2.9)

By an analogous argument it follows that:

$$(\overline{c}, \overline{d})$$
 is a Nash equilibrium $\wedge \overline{c} \ge c^* \wedge \overline{d} < d^* \to (\overline{c}, \overline{d}) \in M.$ (2.10)

Finally consider the case when $\bar{c} \geq c^* \wedge \bar{d} \geq d^*$.

If $\overline{c} \geq c^* \wedge \overline{d} \geq d^*$, (2.5) reduces to:

$$\overline{c} + \overline{d} + R(\overline{c}, \overline{d}) + F(\overline{c}, \overline{d}) \le c^* + d^* + x^1 R(c^*, \overline{d}) + y^1 R(\overline{c}, d^*) + \lambda F(c^*, \overline{d}) + (1 - \lambda) F(\overline{c}, d^*),$$
 where $y^1 = y(1, 1)$.

$$\to \bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \le c^* + d^* + x^1 R(c^*, \bar{d}) + y^1 R(\bar{c}, d^*) + \theta L^*, \tag{2.11}$$

as $R(\overline{c}, \overline{d}) + F(\overline{c}, \overline{d}) = L(\overline{c}, \overline{d}), F(c^*, \overline{d}) \le \theta L^*$ and $F(\overline{c}, d^*) \le \theta L^*$.

Now, as
$$R(c^*, \overline{d}) = L(c^*, \overline{d}) - \theta L^*$$
, if $L(c^*, \overline{d}) - \theta L^* > 0$
= 0, if $L(c^*, \overline{d}) - \theta L^* < 0$,

and

$$L(c^*, \overline{d}) \le L^*,$$

it follows that:

$$R(c^*, \overline{d}) \le (1 - \theta)L^*. \tag{2.12}$$

Similarly,
$$R(\bar{c}, d^*) \le (1 - \theta)L^*$$
. (2.13)

In view of (2.12) and (2.13), it follows from (2.11) that:

$$\overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \le c^* + d^* + L^*,$$

which establishes that:

$$(\overline{c}, \overline{d})$$
 is a Nash equilibrium and $\overline{c} \ge c^* \wedge \overline{d} \ge d^* \rightarrow (\overline{c}, \overline{d}) \in M.$ (2.14)

(2.6), (2.9), (2.10) and (2.14) establish the proposition.

Lemma 3 Let $0 \le \theta \le 1$. If a (λ, θ) - decomposed liability rule is efficient for every possible choice of C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5), then it satisfies condition NL.

Proof: Let $0 \le \theta \le 1$. Suppose (λ, θ) - decomposed liability rule f violates NL. Then:

$$[\exists p \in [0,1)][f(p,1) \neq (1,0)] \lor [\exists q \in [0,1)][f(1,q) \neq (0,1)].$$

Suppose $[\exists q \in [0,1)][f(1,q) \neq (0,1)]$ holds.

Suppose for $q \in [0, 1)$ we have:

$$f(1,q) = (x_q, y_q), y_q \neq 1.$$

Let t be a positive number. As $y_q \in [0, 1)$, we have $y_q t < t$. Choose a positive number r such that $y_q t < r < t$.

As
$$q \neq 1, (1-q) \neq 0$$
. Let $d_0 = \frac{r}{1-q}$

Let
$$0 < \epsilon, 0 < c_0$$
 and $0 < \delta < r - y_q t$.

Now let C, D and L be specified as follows:

$$C = \{0, c_0, c_0 + \delta\}, D = \{0, qd_0, d_0, d_0 + \delta\},\$$

$$L(0,0) = t + qd_0 + c_0 + \epsilon + \delta, L(0,qd_0) = t + c_0 + \epsilon + \delta, L(0,d_0) = c_0 + \epsilon + \delta, L(0,d_0 + \delta) = c_0 + \epsilon + \frac{1}{2}\delta,$$

$$L(c_0,0) = t + qd_0 + \delta, L(c_0,qd_0) = t + \delta, L(c_0,d_0) = \delta, L(c_0,d_0 + \delta) = \frac{1}{2}\delta,$$

$$L(c_0 + \delta, 0) = t + qd_0 + \frac{1}{2}\delta, L(c_0 + \delta, qd_0) = t + \frac{1}{2}\delta, L(c_0 + \delta, d_0) = \frac{1}{2}\delta, L(c_0 + \delta, d_0 + \delta) = 0.$$

As $\epsilon > 0$ and $t > r = (1 - q)d_0$, it follows that (c_0, d_0) is the unique total social cost minimizing configuration.

Let
$$(c_0, d_0) = (c^*, d^*).$$

It should be noted that the above specification of C, D, L and $(c^*, d^*) \in M$ is consistent with (A1)- (A5).

Furthermore, the specification of L is done in such a way that no inconsistency would arise even if q = 0.

Now, expected costs of the injurer at $(c_0, qd_0) = EC_2(c_0, qd_0)$

$$= qd_0 + y_q R(c_0, qd_0) + (1 - \lambda)F(c_0, qd_0)$$

$$= qd_0 + y_q[L(c_0, qd_0) - \theta L^*] + (1 - \lambda)\theta L^*$$

$$= qd_0 + y_q(t+\delta) - y_q\theta\delta + (1-\lambda)\theta\delta$$

$$EC_2(c_0,d_0)$$

$$= d_0 + y[p(c_0), q(d_0)]R(c_0, d_0) + (1 - \lambda)F(c_0, d_0)$$

$$= d_0 + y^1(1-\theta)\delta + (1-\lambda)\theta\delta$$

$$EC_2(c_0, d_0) - EC_2(c_0, qd_0)$$

$$= d_0 + y^1(1-\theta)\delta + (1-\lambda)\theta\delta - qd_0 - y_a(t+\delta) + y_a\theta\delta - (1-\lambda)\theta\delta$$

$$= (r - y_q t) + y^1 (1 - \theta) \delta - y_q \delta (1 - \theta)$$

$$> (r-y_qt) + y^1(1-\theta)\delta - \delta$$

$$= (r - y_q t - \delta) + y^1 (1 - \theta) \delta > 0.$$

This establishes that (c_0, d_0) is not a Nash Equilibrium. Consequently f is not an efficient (λ, θ) - decomposed liability rule. The proof is completed by noting that, in case $[\exists p \in [0, 1)][f(p, 1) \neq (1, 0)]$ holds, an analogous argument shows that it is not the case that f is an efficient (λ, θ) - decomposed liability rule.

Theorem 1 Let $0 \le \theta \le 1$. A (λ, θ) - decomposed liability rule is efficient for every possible choice of C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5) iff it satisfies the condition of negligence liability.

Proof: Let $0 \le \theta \le 1$. If a (λ, θ) - decomposed liability rule satisfies the condition of negligence liability then by Propositions 1 and 2 it is efficient for every possible choice of C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5). Proposition 3 establishes that if a (λ, θ) - decomposed liability rule is efficient for every possible choice of C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5), then it satisfies the condition of negligence liability.

Theorem 2 Let f be a (λ, θ) - decomposed liability rule. If $\theta > 1$ then f is not efficient.

Proof: Let f be a (λ, θ) - decomposed liability rule and $\theta > 1$.

Let $0 < \epsilon < \delta$. There exists a positive integer n such that:

$$\frac{\epsilon}{n\delta} < \theta - 1.$$

We consider the following specification of C, D and L.

$$C = D = \{0, \frac{\epsilon}{3}\}.$$

$$L(0,0) = n\delta + \epsilon, L(\frac{\epsilon}{3},0) = L(0,\frac{\epsilon}{3}) = n\delta + \frac{\epsilon}{2}, L(\frac{\epsilon}{3},\frac{\epsilon}{3}) = n\delta.$$

 $(\frac{\epsilon}{3}, \frac{\epsilon}{3})$ is the unique total social cost minimizing configuration.

Let
$$(c^*, d^*) = (\frac{\epsilon}{3}, \frac{\epsilon}{3}).$$

Now,
$$n\delta + \epsilon - \theta L^* = n\delta + \epsilon - \theta n\delta = n\delta \left[1 + \frac{\epsilon}{n\delta} - \theta\right] < 0.$$

Therefore,
$$R(0,0) = R(\frac{\epsilon}{3},0) = R(0,\frac{\epsilon}{3}) = R(\frac{\epsilon}{3},\frac{\epsilon}{3}) = 0.$$

And therefore, $(\forall (c,d) \in C \times D)[F(c,d) = L(c,d)].$

We have:
$$\lambda \leq \frac{1}{2} \vee 1 - \lambda \leq \frac{1}{2}$$

First consider the case when $\lambda \leq \frac{1}{2}$

The expected costs of the victim at $(\frac{\epsilon}{3}, \frac{\epsilon}{3}) = EC_1(\frac{\epsilon}{3}, \frac{\epsilon}{3}) = \frac{\epsilon}{3} + \lambda n\delta$.

$$EC_1(0, \frac{\epsilon}{3}) = 0 + \lambda(n\delta + \frac{\epsilon}{2}).$$

$$EC_1(\frac{\epsilon}{3}, \frac{\epsilon}{3}) - EC_1(0, \frac{\epsilon}{3})$$

$$= \frac{\epsilon}{3} + \lambda n\delta - \lambda n\delta - \lambda \frac{\epsilon}{2}$$

$$=\epsilon(\frac{1}{3}-\frac{\lambda}{2})$$

$$\geq \epsilon(\frac{1}{3} - \frac{1}{4})$$
, as $\lambda \leq \frac{1}{2}$

> 0.

Thus if $\lambda \leq \frac{1}{2}$ then the unique total social cost minimizing configuration $(\frac{\epsilon}{3}, \frac{\epsilon}{3})$ is not a Nash equilibrium.

If $1 - \lambda \leq \frac{1}{2}$ then an analogous argument shows that $(\frac{\epsilon}{3}, \frac{\epsilon}{3})$ is not a Nash equilibrium as the expected costs of the injurer at $(\frac{\epsilon}{3}, \frac{\epsilon}{3})$ are greater than at $(\frac{\epsilon}{3}, 0)$.

This establishes that f is not efficient.

References

Barnes, David W. and Stout, Lynn A. (1992), *The Economic Analysis of Tort Law*, St. Paul, West Publishing Company.

Brown, John Prather (1973), 'Toward an Economic Theory of Liability', 2 *Journal of Legal Studies*, 323-350.

Calabresi, Guido (1961), 'Some Thoughts on Risk Distribution and the Law of Torts', 70 Yale Law Journal, 499-553.

Cooter, Robert D. (1985), 'Unity in Torts, Contracts and Property: The Model of Precaution', 73 California Law Review, 1-51.

Cooter, Robert D. (1991), 'Economic Theories of Legal Liability', 5 Journal of Economic Perspectives, 11-30.

Cooter, Robert D. and Ulen, Thomas S. (1999), Law and Economics, 3rd ed., New York, Addison-Wesley.

Jain, Satish K. and Singh, Ram (2002), 'Efficient Liability Rules: Complete Characterization', 75 Journal of Economics (Zeitschrift für Nationalökonomie), 105-124.

Landes, William M. and Posner, Richard A. (1987), *The Economic Structure of Tort Law*, Cambridge (MA), Harvard University Press.

Levmore, Saul (1994), Foundations of Tort Law, Oxford, Oxford University Press.

Miceli, Thomas J. (1997), Economics of the Law: Torts, Contracts, Property, Litigation, Oxford, Oxford University Press.

Polinsky, Mitchell A. (1989), An Introduction to Law and Economics, 2nd ed., Boston, Little, Brown and Company.

Posner, Richard A. (1972), 'A Theory of Negligence', 1 Journal of Legal Studies, 28-96.

Posner, Richard A. (1992), *Economic Analysis of Law*, 4th ed., Boston, Little, Brown and Company.

Shavell, Steven (1980), 'Strict Liability versus Negligence', 9 Journal of Legal Studies, 1-25.

Shavell, Steven (1987), Economic Analysis of Accident Law, Cambridge (MA), Harvard University Press.

^{*} Complete list of working papers is available at the CDE website : http://www.cdedse.org/worklist.pdf