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A NON-COOPERATIVE THEORY OF QUANTITY-RATIONING INTERNATIONAL TRANSFRONTIER POLLUTION

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A non-cooperative theory of quantity-rationing international transfrontier pollution

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We study the problem caused by international transfrontier pollution. Our results are derived from the analysis of an incomplete information, non-cooperative game model of the determination of emissions in a quantity-rationing setting. We model the emission capping negotiations using the best response dynamic process and provide natural conditions under which the process has a unique and globally asymptotically stable stationary point. We then analyze the link between type profiles and the stationary points of the negotiation process to derive various comparative statics results and the type-contingent ordering of emission allocations. Finally, we study the investment strategies that nations can use prior to the negotiations in order to manipulate the equilibrium emission caps. The results have implications regarding the political economy of emission capping.

JEL classification: D74, H41, Q21, Q25, R11

Key words: Emission capping, non-cooperative game, negotiations, incomplete information, manipulation

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1. Introduction

1.1 The problem

Consider n nations whose firms emit some pollutant into a shared medium. The aggregate emission damages the interests of consumers in all these nations. The *status quo* situation is that each emitting firm chooses its production plan, and therefore emission, to maximize its own profit, subject to domestic regulatory constraints, without taking into account the externality suffered by the consumers. For standard specifications of national welfare that take consumer welfare into account, the resulting outcome is inefficient. Suppose these nations attempt to improve upon this situation by means of a protocol that caps each nation's emission to some country-specific level.[1]

Each nation consists of two classes of entities: firms that emit pollution and consumers who suffer the damage from the aggregate international emission. While the production and emission decisions are made by the firms, the emission caps under the protocol are negotiated by the national governments without the firms being at the table. As seems reasonable, we model the negotiating national governments as being concerned not only about the profits of domestic firms but also about the damage suffered by domestic consumers.

We assume that every nation has the sovereign right to choose the mechanism for allocating among the domestic firms the emission rights received by it in the form of the national emission cap. We do not model this choice and take it as exogenously given. In effect, domestic regulation is black-boxed and assumed to be effective, which allows us to focus on the issue of incentive compatibility in the emission capping process without worrying about subsequent domestic implementation issues. It also allows us to aggregate each nation's firms into a single national firm.[2] For the sake of convenience, we shall also aggregate each nation's consumers into a single national consumer.

Our model has three stages. In Stage 1, privately informed Nations choose investments to alter their exogenously given types. A nation's type consists of two parameters. The first is private capital, which consists of the fixed inputs that determine the nation's (equivalently, the national firm's) production technology. The second is adaptation capital, which determines the relationship between aggregate emission and the damage suffered by that nation (equivalently, by the national consumer). Adaptation capital consists of all assets that are used to mitigate the damage caused by emissions, e.g., water and forest management systems, meteorological facilities, knowledge of the ways to cope with the effects of pollution, research facilities that generate such technologies, etc. The distinction between the two kinds of capital is functional in nature. In Stage 1, each Nation may enhance these parameters *via* investment. In Stage 2, the national governments use the best response dynamic to negotiate emission caps: given a proposed profile of caps, the new profile of proposed caps is the profile of best responses to the given profile. As a cap cannot be imposed on a sovereign nation, the negotiations continue until a stationary profile is reached.[3] In Stage 3, a stationary point of the best response dynamic is implemented as the profile of caps and the national firm in each nation chooses a production and emission plan subject to that nation's emission cap.

We now flesh out the model described above, first with some remarks regarding its theoretical underpinnings and our motives for constructing such a model, and then some comments regarding its special features.

The caps negotiated in Stage 2 of the model are self-enforcing as, by definition, no country can unilaterally improve upon a stationary profile. Accordingly, the first objective of this paper is to provide a theory of self-enforcing emission cap determination. This involves showing the existence, uniqueness and appropriate stability properties of stationary profiles of caps, as is done in Section 2.3. Our substantive goal in this paper is to use this theory to answer the following broad question in a number of settings: given that nations differ in terms of technology and resources, what investment strategies might a nation (or its firm) pursue prior to capping so as to bring about an emission cap profile that is optimal from its point of view?

An issue that is central to a large portion of the comparable literature (see Section 1.2) is a nation's participation decision *vis-a-vis* the protocol. We shall formally endogenize each nation's participation decision with respect to the protocol by identifying "non-participation" with an infinite cap in equilibrium. While we formally allow this possibility, in equilibrium every nation will accept a finite emission cap that is lower than its *status quo* emission level, i.e., every nation will choose to "participate". As the caps are incentive compatible (by definition) and implementable by the national governments (by assumption), the equilibrium aggregate emission will be lower than the *status quo* level. Indeed, the equilibrium outcome will be a Pareto improvement over the *status quo* and this result does not rely on international transfers.

We assume that the emission rights implied by a cap are not tradeable internationally. It is natural to wonder why a model with non-tradeable emission rights should even be considered as voluntary trading after nations are endowed with rights should be Pareto improving. This improvement, however, is merely with respect to the *given* endowments. In our context, endowments of emission rights are not given exogenously but are determined endogenously *via* negotiations. Thus, *ex ante* welfare analysis, i.e., prior to the determination of emission rights endowments, requires the comparison of equilibrium outcomes resulting from various arrangements for determining emission rights endowments. In this paper, we explore the implications of an autarkic post-protocol regime.

Another implicit assumption underlying our approach is that standard tax-subsidy type remedies for dealing with externalities are inapplicable in the modelled environment because of the lack of appropriate institutions that can exercise supranational fiscal authority over sovereign nations.

As will become clear, our model applies directly to a pollutant that dissipates quickly and therefore does not have cumulative effects *via* its stock. However, it can also be used as a building block for a model to study the problem of restricting the future flow of a cumulative pollutant as, at a point of time, accounting and paying for past emissions is an analytically distinct problem from that of regulating future flows: while the former is the problem of apportioning a noxious cake of exogenously given size, the latter is the problem of regulating the marginal externalities generated by future emissions. However, unlike in the case of evanescent pollutants, cumulative pollutants will require our model to serve as a stage game in a dynamic game with the pollution stock serving as the state variable.

We now discuss the following features of our model: (a) the dynamic cap negotiation process, (b) the heterogeneity of nations, (c) the asymmetry of information among the nations, (d) the possibility of manipulating the capping process *via* the investment decisions, and (e) the decomposition of nations into entities with divergent attitudes to emissions.

Feature (a) is the technical fulcrum of this paper. For each profile of national types, the equilibrium profile of caps is the stationary point of the capping negotiations, which are formalized using the best response dynamic. This formalism has a number of useful technical properties that enable a general and straightforward attack on our substantive questions. We first characterize the set of stationary points of the dynamic process as the set of Nash equilibria of an artificial complete information game. Given this characterization, the existence of stationary points follows simply from an application of Nash's existence theorem. Additional natural assumptions regarding the primitives of the model imply that the artificial game is dominance-solvable. This guarantees the existence of a unique Nash equilibrium of the artificial game, and therefore, a unique stationary point of the capping negotiations. Moreover, the steady-state solution of the best response dynamical system corresponding to the unique stationary point is globally asymptotically stable:

starting from any proposed profile of caps, the negotiations will converge asymptotically to the unique stationary profile. Pulling together the various technical results, we know that, for every profile of types, there exists a unique equilibrium profile of caps, and that the resulting mapping from type profiles to cap profiles is simply characterized and sufficiently smooth under natural assumptions regarding the primitives of the model. This facilitates the comparative statics exercises that underly the substantive results. Moreover, as suggested by the correspondence principle, the assumptions that imply the stability properties of the dynamics also yield useful information for signing the comparative statics exercises.

With respect to feature (b), it is obvious that the potential participants in a capping regime may be heterogeneous. We model the heterogeneity by allowing national types to vary across nations and analyze its effects on Stage 2 outcomes (see Proposition 3.1.5 and Section 3.2). It is also instructive to account for the evolution of national types. We study this issue by interpreting Stage 1 investment as the means for adjusting a nation's type. This allows the analysis of an asymmetry among nations that is distinct from heterogeneity of types. Since the ability to invest is constrained by a nation's wherewithal, a second source of heterogeneity among nations is their differing ability to invest. In Section 4, we derive a number of results related to strategic investment choices. The political-economic implications and interpretations are discussed in Section 5.1.

Feature (c) is intended to capture the view that national negotiators have incomplete information, not only about environmental processes but also about the capabilities of other nations. Technological innovations that allow the use of new resources, or the more efficient use of traditional resources, boost the size of private capital. Examples, just in the energy industry, include the development of non-conventional energy sources and safer nuclear technology, new drilling techniques for deep-sea oil and gas exploration, more efficient use of coal for thermal energy and the improved capture of coal-bed methane. Equally, new farming techniques, better water management systems and improved meteorological forecasting will improve a nation's ability to deal with environmental degradation, i.e., they will increase a nation's adaptation capital. Given the value of patents for such innovations, especially under an emission-capping regime, such technologies are not symmetrically available to all nations. Moreover, the complex nature, secrecy and relatively small size of innovative technology make it unlikely to be inferred accurately from commonly available aggregative data. This asymmetry is important as bargainers at capping negotiations will take into account not only their legacy of well-known technology that is embodied in the commonly available data, but also their knowledge of emerging technologies at the margin.

Features (d) and (e) are significant innovations in the comparable literature. With respect to (d), strategic manipulation of protocols has not been studied. With respect to (e), other models treat the participating nations as unitary entities that choose emissions and the emission caps, in effect conflating the two variables and identifying the interests of the entities choosing these variables.[4]

1.2 The literature

We now describe the theoretical literature on the regulation of international pollution externalities paying particular attention to whether the model in question assumes: (α) the existence of robust institutions for enforcing international contracts, (β) that players are symmetric, and (γ) that players have complete information.

The cooperative game approach. While this literature employs a variety of solution concepts, the approach using the core (Chander and Tulkens 1992, 1995, 1997) has yielded the most striking general theoretical results. It characterizes a system of zero-sum transfers for many-player emission allocation games that implements efficient emission allocations and satisfies individual and group rationality axioms.

This approach assumes that the members of a coalition of nations can correlate decisions so as to maximize the sum of their utilities, thereby yielding the value of the coalition. Thus, it implicitly relies on robust institutions for enforcing contracts among coalitions of nations, as the assumed correlation of decisions cannot be guaranteed otherwise. While this methodology has no problem dealing with heterogeneous players, the generalization of cooperative solution concepts to incomplete information environments is very problematic.

Cooperative models implicitly make strong assumptions regarding the existence of appropriate international institutions and consequently generate large improvements from the *status quo*. Our non-cooperative model makes weak institutional assumptions, resulting in smaller improvement from the *status quo*. While the strength of the cooperative approach is its normative attractiveness, the strength of the non-cooperative approach is its descriptive plausibility.

The stable coalition approach. Studies such as Barrett (1994), Black et al. (1993) and Carraro and Siniscalco (1993) adopt a methodology initiated by d'Aspremont et al. (1983) to study the likely size of a "stable coalition".[5] As this methodology does not slot readily into either the cooperative or the non-cooperative game categories, we describe it in the following abstract fashion.

Consider a game in strategic form given by data Γ, with N as the set of players. Given Γ, we have the family of games $\{\Gamma_{N'} \mid N' \subset N\}$, where $\Gamma_{N'}$ is the game derived from Γ, with N' acting as a coalition while the players in $N - N'$ act as singletons. N' being a coalition means that the players in N' act as a unit and distribute payoffs within the coalition using some cooperative solution concept. Given a solution concept applicable to the games in $\{\Gamma_{N'} \mid N' \subset N\}$, let $E(N')$ denote an equilibrium profile of payoffs for the game $\Gamma_{N'}$. A coalition N' is said to be stable if: (a) $E(N')_i \geq E(N' - \{i\})_i$ for every $i \in N'$, and (b) $E(N')_i \geq E(N' \cup \{i\})_i$ for every $i \in N - N'$. (a) requires that no one in N' should want to defect unilaterally, while (b) requires that no one outside N' should want to join it unilaterally. Clearly, multiple equilibria create severe conceptual problems.

As shown in Tulkens (1998), this approach is not congruent with the cooperative approach as it does not yield a characteristic function. However, a stable set can be generated as the Nash equilibrium outcome of the following "metagame". Consider a treaty that determines the actions of all players who sign it. If $i \in N$ signs (denoted by action $a_i = 1$), i's actions in Γ are chosen by the treaty, but otherwise (denoted by action $a_i = 0$) i acts as a singleton. A profile of actions $a = (a_i)_{i \in N}$ generates a coalition $N(a) = \{i \in N \mid a_i = 1\}$ of players who have signed and a collection of singletons $N - N(a)$ who have not signed. Thus, a profile of actions a picks out the game $\Gamma_{N(a)}$. N' is a stable coalition if and only if there is a Nash equilibrium a of the metagame such that $N' = N(a)$.

This approach leans very heavily on assumptions (α), (β) and (γ). As in the cooperative approach, (α) is intrinsic to this methodology as it is required to model the collusive behavior of coalition N' in the game $\Gamma_{N'}$. Indeed, when the above structure is expanded to allow transfers, (α) is strengthened substantially by allowing various sets of players to "commit" to being "cooperative". While some examples with heterogeneous players have been studied, general results that allow player heterogeneity and incomplete information are not known.

The cost-effectiveness approach. This approach studies the implications of the postulate that nations will seek to implement pollution caps or abatement targets at minimum abatement cost (Breton et al. 2004, Kaitala et al. 1993, Petrosjan and Zaccour 2003). If a sufficiently rich set of such strategies is available, then the determination of a profile of strategies to implement a given profile of emission caps itself becomes a matter of strategic analysis.

While this approach can explain how a given profile of emission caps may be implemented, it cannot serve our agenda in this paper, which is to generate a theory of how the emission caps are arrived at in the first place. This is because abatement costs do not capture fully the costs and benefits from emissions. However, in the spirit of "perfection", the use of equilibrium cost minimizing strategies *after* emission rights are awarded should be anticipated and taken into account when the emission rights are allocated. This refinement does not create difficulties in the model analyzed in the main body of this paper as the range of implementation options modelled by us is very limited: the only way for a country to implement an emission cap in our model is to cut domestic emission.

Non-cooperative multi-period approach. A fourth set of studies considers multi-period games in which each nation chooses how much to emit in every period. The formal framework can be a repeated game, in which the stage-game in every period is identical, or a dynamic game such as a differential or stochastic game, in which the stage-game is parametrized by an endogenously-determined time-varying state variable. Although this literature does not model regulatory interventions *via* quantitative restrictions on emissions, it does allow for regulation *via* international fiscal incentives. Even in the absence of fiscal incentives, the existence of a future in a multi-period game can sustain *implicit* agreements in equilibrium, with penal codes (e.g., "grim trigger", "tit-for-tat", "carrot-and-stick") providing appropriate incentives. Two main issues are considered in this literature (e.g., Hoel 1993, van der Ploeg and de Zeeuw 1992, Cesar 1994, Dockner and van Long 1993, Martin et al. 1993): (1) the extent to which implicit cooperation can be induced in equilibrium, and (2) the extent to which international fiscal incentives can be used to influence the equilibrium.

The literature addressing (1) applies the so-called folk theorems of game theory; while an array of such theorems are known for repeated games for a variety of equilibrium concepts, there do not appear to be very general analogous results for dynamic games. With respect to (2), although fiscal incentives are a standard method of correcting distortions caused by externalities, the application of fiscal tools across nations requires various facilitating international institutions. While this approach does not require assumption (β), it does rely on assumption (γ).

1.3 Outline of the paper and results

In Section 2.1, we formally state our three stage model. As is standard, the model is solved backwards. Accordingly, Stages 3, 2 and 1 of the model are studied in Sections 2.2, 2.3 and 2.4 respectively. The equilibrium of the entire model is defined in Section 2.5.

In Section 3, we analyze the outcomes of Stage 2 and derive the comparative statics results that link the stationary points of the bargaining process to the underlying data of the model. We use these results in Section 4 to derive the properties of equilibrium behavior in Stage 1. We conclude in Section 5 with an interpretation of our results and suggestions for interesting extensions. The proofs are collected in the Appendix.

Other than the results related to the theory of emission caps determination analyzed in Section 2.3, we derive two classes of substantive results. First, we derive general comparative statics results (Proposition 3.1.5) that show the systematic and easily interpretable dependence of the emission caps (i.e., the stationary points of the negotiation process) on the underlying data consisting of national types. Secondly, we use the comparative statics results to show (Propositions 4.2.7, 4.2.8, 4.3.4, 4.3.6, 4.4.1) how equilibrium investment choices in Stage 1 are used to manipulate the equilibrium emission caps. As we show, the qualitative and quantitative properties of the manipulations that a nation implements depend systematically on factors such as the nature of technology, the initial endowment of that nation and whether the investment is made by the state or the emitting firm within it.

2. The model

2.1 Extensive form

Let N be the set of nations with $|N| = n$. Nation i's type space is $\Theta = \Re^2_+$ and the space of type profiles is Θ^n. $\lambda_i \in \mathcal{P}(\Theta)$ is the common knowledge distribution of Nation i's initial type $\bar{\theta}_i$. Let $\lambda_{-i} = \prod_{j \in N - \{i\}} \lambda_j$ be Nation i's belief about the initial types of the other nations.

Nation i consists of Firm i and Consumer i. Nation i's initial type is a pair $(\bar{t}_i, \bar{k}_i) = \bar{\theta}_i \in \Theta$, where $\bar{t}_i$ is Firm i's private capital and $\bar{k}_i$ is Consumer i's adaptation capital. As *per* the usual interpretation of "type", $\bar{\theta}_i$ is Nation (and Firm) i's private information.

The play of the game proceeds as follows. After Nature picks a profile of initial types $\bar{\theta} = (\bar{\theta}_j)_{j \in N} \in \Theta^n$, the game has three stages: (1) investment, (2) capping, and (3) production. While the sequencing of stages is important, the passage of time *per se* is ignored; consequently, there is no time-discounting.

In Stage 1, the nations simultaneously modify their initial types by investing: if Nation i invests $I_i \in \Theta$, then its type is modified from $\bar{\theta}_i$ to $\bar{\theta}_i + I_i$. The profile of investments $I = (I_j)_{j \in N} \in \Theta^n$ chosen in Stage 1 is common knowledge in Stage 2. In Section 4, we shall consider a number of variations of this stage, depending on whether Nation i or Firm i chooses I_i and bears the resulting cost.

In Stage 2, with Nation i knowing $\bar{\theta}_i$ and I, the nations negotiate to determine the emission caps to be allocated to the various nations. We model the negotiations formally using the best response dynamic procedure and postulate that the implemented profile of caps, $e \in \bar{\Re}^n_+$, is a stationary point of this dynamic procedure. In Section 2.3, we show that the set of stationary points of the dynamic process is identical to the set of Nash equilibria of a complete information game denoted $G(\bar{\theta}, I)$, where $\bar{\theta} = (\bar{\theta}_j)_{j \in N}$. Given this characterization, the existence of stationary points is easy to demonstrate using Nash's existence theorem (see Moulin 1986). Moreover, this characterization is exploited in Sections 3 and 4 to derive the substantive results of this paper. There are additional reasons for seeing the stationary points as compelling predictions of the negotiations. We will provide a simple condition to supplement our primitive assumptions that will ensure that $G(\bar{\theta}, I)$ is dominance-solvable. This will guarantee the existence of a unique Nash equilibrium of $G(\bar{\theta}, I)$, and therefore, a unique stationary point of the best response dynamic process. Consequently, the steady-state solution of the dynamical system corresponding to the unique stationary point is globally asymptotically stable: starting from *any* proposed profile of caps, the dynamic process will drive the proposed profiles of caps towards the unique stationary profile.

In Stage 3, Firm i knows $\bar{\theta}_i$, I and e, and chooses variable input v_i, which determines Firm (and Nation) i's profit and emission.The profile of emissions by all the nations determine the damage suffered by Consumer (and Nation) i. The choice of v_i is subject to the constraint that the resulting emission cannot exceed Nation i's cap e_i.

Suppose Nation i's modified type is $\theta_i = (t_i, k_i)$. If Firm i employs variable input v_i, then Firm (and Nation) i's profit is $g(t_i, v_i)$ and its emission is $h(t_i, v_i) \leq e_i$; g and h are to be interpreted as incorporating the effects of domestic regulations, e.g., emission taxes and subsidies for adoption of clean technologies, other than the quantity-rationing implied by the emission rights regime. The resulting total world emission $\sum_{j \in N} h(t_j, v_j)$ is consumed by the consumer of every nation and this causes damage $\delta(k_i, \sum_{j \in N} h(t_j, v_j))$ to Consumer (and Nation) i.

2.2 Stage 3: properties of optimal action and value functions

Suppose the profile of modified types is θ and the profile of emission caps is e. Given this data, we analyze Firm i's decision-making assuming that it maximizes its profit.

Assumption 2.2.1. *$g : \Re^2_+ \to \Re_+$ and $h : \Re^2_+ \to \Re_+$ are continuous functions such that, for every $t \in \Re_+$,*

(a) $g(t, .)$ has a unique maximum at $V(t)$ where $V : \Re_+ \to \Re_{++}$ is continuous,

(b) $g(t,.)$ is strictly increasing on $[0, V(t)]$ and strictly concave on $\Re_+$, and

(c) $h(t,0) = 0$ and $h(t,.)$ is strictly increasing and strictly convex.

(a) and (b) imply that profit increases with variable input until the unconstrained maximum is attained and use of the variable input faces diminishing returns. (c) implies that emission increases at an increasing rate with the variable input.

Consider a firm with modified private capital by $t \in \Re_+$ and emission cap $e \in \bar{\Re}_+$. The firm's problem is to choose variable input v to maximize profit $g(t,v)$ subject to the constraint $h(t,v) \leq e$. Since $V(t)$ is an unconstrained optimal choice for the firm and $h(t,.)$ is increasing, the firm's constraint can be written as $v \in \Gamma(t,e) \equiv \{v \in \Re_+ \mid h(t,v) \leq e\} \cap [0, V(t)]$.

Let $v : \Re_+ \times \bar{\Re}_+ \to \Re_+$ be such that, for every $(t,e) \in \Re_+ \times \bar{\Re}_+$, $v(t,e)$ solves the firm's problem; consequently, for every $(t,e) \in \Re_+ \times \bar{\Re}_+$, we have $h(t, v(t,e)) \leq e$. Define $f : \Re_+ \times \bar{\Re}_+ \to \Re$ by $f(t,e) = g(t, v(t,e))$. Consequently, Firm i's choice of variable input is $v(t_i, e_i)$, its profit is $f(t_i, e_i)$ and its emission is $h(t_i, v(t_i, e_i))$. The next result records some consequences of the above structure.

Proposition 2.2.2. *Given Assumption 2.2.1,*

(A) for every $(t,e) \in \Re_+ \times \bar{\Re}_+$ there exists a unique $v(t,e) \in \Gamma(t,e)$ such that $g(t, v(t,e)) \geq g(t,v)$ for every $v \in \Gamma(t,e)$, and

(B) $v : \Re_+ \times \bar{\Re}_+ \to \Re_+$ and $f : \Re_+ \times \bar{\Re}_+ \to \Re$ are continuous functions.

Moreover, for every $(t,e) \in \Re_+ \times \bar{\Re}_+$,

(C) $e \geq h(t, V(t))$ if and only if $v(t,e) = V(t)$, and

(D) $e \leq h(t, V(t))$ if and only if $h(t, v(t,e)) = e$.

Furthermore, for every $t \in \Re_+$,

(E) $f(t,.)$ is strictly increasing on $[0, h(t, V(t))]$,

(F) $f(t,.)$ is strictly concave on $[0, h(t, V(t))]$, and

(G) $f(t,e) = g(t, V(t))$ for $e \geq h(t, V(t))$.

$V(t)$ may be interpreted as the *status quo*, i.e., prior to emission capping, level of variable input use. Therefore, $g(t, V(t))$ and $h(t, V(t))$ are the *status quo* profit and emission. Thus, a cap e is a binding constraint on the firm if and only if $e \leq h(t, V(t))$. Given our interpretation of $V(t)$, the above results have straightforward interpretations.

2.3 Stage 2: existence, uniqueness and stability of stationary points

Suppose the profile of initial types is $\bar{\theta}$ and the profile of investments in Stage 1 is I, resulting in the profile of modified types $\theta = \bar{\theta} + I = (t_j, k_j)_{j \in N}$. Suppose the profile of *emission caps $e = (e_j)_{j\in N} \in \bar{\Re}^n_+$ is proposed publicly by an uninformed mediator, i.e., one who is not privy to the types of the various nations. The nations inform the mediator about their best response to the proposed profile. The best responses are used as the proposals for the next round and the procedure carries on iteratively. A stationary point of this procedure is the finally implemented profile of caps.*

To ease analysis, we enrich Assumption 2.2.1 as follows. *Henceforth, a function being $\mathcal{C}^p$ means it is p times continuously differentiable on the specified set.*

Assumption 2.3.1. *In addition to the requirements of Assumption 2.2.1, suppose that,*

(a) for every $t \in \Re_+$, $g(t,.)$ and $h(t,.)$ are $\mathcal{C}^1$ on $\Re_{++}$, and

(b) for every $k \in \Re_+$, $\delta(k,.)$ is continuous, $\mathcal{C}^1$ on $\Re_{++}$, strictly increasing and strictly convex.

The smoothness assumptions are for made to obtain simple characterizations. (b) also assumes that damage increases at an increasing rate with respect to total emission. We note some consequences.

Proposition 2.3.2. *Given Assumption 2.3.1 and $t \in \Re_+$,*

(A) $D_v g(t, V(t)) = 0$,

(B) $D_e v(t,e) = 0$ and $D_v g(t, v(t,e)) = 0$ for $e > h(t, V(t))$,

(C) $v(t,.)$ is $\mathcal{C}^1$ on $(0, h(t, V(t)))$, and

(D) $g(t, v(t,.))$ is $\mathcal{C}^1$ on $(0, h(t, V(t))) \cup (h(t, V(t)), \infty)$.

Description of dynamic negotiation procedure. Suppose a profile of caps $e \in \bar{\Re}^n_+$ is proposed. Given e, we say that Nation i accepts e_i if there is no alternative cap for i that is preferred to e_i, assuming that the caps e_{-i} will be implemented by the other nations. Since caps cannot be imposed exogenously on sovereign nations, assuming that the other nations will implement e_{-i} amounts to assuming that every Nation $j \in N - \{i\}$ accepts e_j given e.

Let $\Lambda_i(I, e_{-i})$ be Nation i's belief about θ_{-i}, conditional on (a) private information θ_i, (b) public information I, and (c) the assumption that the other nations accept the caps e_{-i}. Thus, Nation i's conditional expected payoff from accepting e_i is $U_i(\theta_i, I, e)$, given by

$$f(t_i, e_i) - \int_{\Theta^{n-1}} \Lambda_i(I, e_{-i}, dx)\, \delta\left(k_i, h(t_i, v(t_i, e_i)) + \sum_{j\in N-\{i\}} h(t_j(x), v(t_j(x), e_j))\right) \tag{2.3.3}$$

The equal weights we have given to the national firm and the national consumer in specifying U_i are just a matter of notational convenience; specifying other weights will increase the number of model parameters but not affect the nature of our results. The set of Nation i's best responses to a proposal e is

$$\beta_i(e_{-i};\theta_i,I) = \bigcap_{x\in\bar{\Re}_+} \{b \in \bar{\Re}_+ \mid U_i(\theta_i,I,b,e_{-i}) \geq U_i(\theta_i,I,x,e_{-i})\}$$

We require that a proposal e be implemented only if it is accepted by all nations, i.e., $e_j \in \beta_j(e_{-j};\theta_j,I)$ for every $j \in N$. As there is no guarantee that an uninformed mediator will pick a proposal e that all nations will accept, we require that the mediator make fresh proposals until one is accepted by all nations. A new proposal is generated from the current one by using the best response dynamic:

$$e_j(\tau+1) = \beta_j(e_{-j}(\tau);\theta_j,I) \tag{2.3.4}$$

for $j \in N$ and $\tau \in \mathcal{N}$, where τ represents time.[6] We say that $e \in \bar{\Re}^n_+$ is a stationary point of (2.3.4), given $(\theta,I) \in \Theta^n \times \Theta^n$, if $e_j = \beta_j(e_{-j};\theta_j,I)$ for every $j \in N$.

Characterization of best response mapping. We begin our analysis of (2.3.4) by characterizing Nation j's best responses to a given proposal e. In principle, it seems possible that $h(t_j,v(t_j,b)) < b$ for some $b \in \beta_j(e_{-j};\theta_j,I)$, i.e., Nation j's actual emission given a best response cap might be less than the cap. This divergence can complicate the analysis of emission capping as the costs and benefits of capping are generated by the actual emissions that result from capping, rather than the caps *per se*. The following result eliminates this potential problem.

Proposition 2.3.5. *Given Assumption 2.3.1, $j \in N$, $(t_j,k_j) = \theta_j \in \Theta$ and $e \in \bar{\Re}^n_+$, if $b \in \beta_j(e_{-j};\theta_j,I)$, then $h(t_j,v(t_j,b)) = b < h(t_j,V(t_j))$.*

We immediately have the following consequences.

Corollary 2.3.6. *Nation i's belief that the other nations will accept the caps e_{-i} amounts to believing, almost surely, that $e_j \in \beta_j(e_{-j};\theta_j,I)$ for every $j \in N-\{i\}$. Thus, for almost every $(t_j,k_j)_{j\in N-\{i\}} \in \operatorname{supp}\Lambda_i(I,e_{-i})$ and for every $j \in N-\{i\}$, $h(t_j,v(t_j,e_j)) = e_j < h(t_j,V(t_j))$. This simplifies (2.3.3) to*

$$U_i(\theta_i,I,e) = f(t_i,e_i) - \delta\left(k_i, h(t_i,v(t_i,e_i)) + \sum_{j\in N-\{i\}} e_j\right) \equiv u_i(\theta_i,e) \tag{2.3.7}$$

Using Proposition 2.2.2,

$$u_i(\theta_i, e) = \begin{cases} f(t_i, e_i) - \delta\left(k_i, \sum_{j \in N} e_j\right), & \text{if } e_i \in [0, h(t_i, V(t_i))] \\ f(t_i, e_i) - \delta\left(k_i, h(t_i, V(t_i)) + \sum_{j \in N - \{i\}} e_j\right), & \text{if } e_i > h(t_i, V(t_i)) \end{cases} \tag{2.3.8}$$

Corollary 2.3.9. *(2.3.7) and Proposition 2.3.5 imply*

$$\beta_i(e_{-i}; \theta_i, I) = \bigcap_{x \in \bar{\Re}_+} \{b \in \bar{\Re}_+ \mid u_i(\theta_i, b, e_{-i}) \geq u_i(\theta_i, x, e_{-i})\} \subset [0, h(t_i, V(t_i))) \tag{2.3.10}$$

Combining (2.3.8), Proposition 2.2.2(F) and Assumption 2.3.1(b), $u_i(\theta_i, ., e_{-i})$ is strictly concave on $[0, h(t_i, V(t_i))]$. Therefore, Nation i's best response is unique, i.e., $\beta_i(.; \theta_i, I)$ is a function, which justifies formula (2.3.4). As u_i depends only on Nation i's modified type θ_i, and not directly on I, we may replace $\beta_i(e_{-i}; \theta_i, I)$ by $\beta_i(e_{-i}; \theta_i)$ in (2.3.4).

Corollary 2.3.11. *If e is a stationary point of (2.3.4), then $e_j < h(t_j, V(t_j))$ for every $j \in N$, i.e., in equilibrium, every nation accepts to cap its emission strictly below the level that the domestic firm would choose in the absence of capping.*

Thus, Proposition 2.3.5 has a number of useful implications. First, it reduces the number of variables involved in the analysis of an equilibrium by identifying equilibrium caps with equilibrium emissions. Secondly, the complicated expected damage term in (2.3.3) is simplified to the damage term in (2.3.7). This is a vital simplification as it ensures that the private values assumption is satisfied when equilibrium caps are identified. Thirdly, Nation i's payoff in the relevant range $[0, h(t_i, V(t_i))]$ is *not* monotonically increasing with respect to e_i, for a larger cap in this range induces greater emission by Firm i, thereby increasing Firm i's profit, but also increasing Consumer i's damage. Thus, by attaching an endogenously generated shadow value to emission rights in the form of damages, our model forces nations to trade-off profits against damages, thereby preventing them from pursuing arbitrarily large caps. Consequently, results in Section 4 asserting that a nation manipulates its type to increase its emission cap do not reflect a trivial desire to have an arbitrarily large amount of a free positive-valued option, but a desire to have a specific larger cap for strategic reasons.

Existence, uniqueness and stability of stationary points. Given $\theta = (\theta_j)_{j \in N} \in \Theta^n$, where $\theta_j = (t_j, k_j)$ is Nation j's modified type, define the non-cooperative game

$$G(\theta) = \{N, ([0, h(t_j, V(t_j))], u_j(\theta_j, .))_{j \in N}\} \tag{2.3.12}$$

where N is the set of players, $[0, h(t_j, V(t_j))]$ is player j's strategy space and $u_j(\theta_j, .) : \prod_{r \in N} [0, h(t_r, V(t_r))] \to \Re$ is player j's payoff function.

Proposition 2.3.13. *Given $\theta \in \Theta^n$, e^* is a stationary point of (2.3.4) if and only if e^* is a Nash equilibrium of $G(\theta)$.*

Needless to say, this result neither assumes nor implies that the nations are playing $G(\theta)$ in Stage 2. However, the characterization is very useful as it allows the application of many standard results; e.g., an application of Nash's existence theorem yields the following.

Proposition 2.3.14. *Given Assumption 2.3.1 and $\theta \in \Theta^n$,*

(A) there exists a stationary point of (2.3.4), and

(B) if e is a stationary point of (2.3.4), then $e \in \prod_{j \in N}[0, h(t_j, V(t_j)))$.

Theorem 3 in Moulin (1986) implies the following strong result.

Proposition 2.3.15. *If Assumption 2.3.1 is satisfied and*

$$-D_{ee} f(t_j, e_j) > (n-2) D_{e_+ e_+} \delta(k_i, e_+) \tag{2.3.16}$$

for every $j \in N$ and $e \in \prod_{j \in N}[0, h(t_j, V(t_j))]$, then $G(\theta)$ is dominance-solvable and has a unique Nash equilibrium that is globally asymptotically stable with respect to (2.3.4).

Combining this result with Proposition 2.3.13, we have

Corollary 2.3.17. *Given the assumptions of Proposition 2.3.15, (2.3.4) has a unique stationary point that is globally asymptotically stable.*

Thus, given appropriate assumptions, the postulated negotiation process converges to the same stationary point, independent of the initial proposed profile. For $n = 2$, Assumption 2.3.1 implies that (2.3.16) holds trivially, and therefore the conclusions of Corollary 2.3.17 hold automatically. (2.3.16) becomes easier to satisfy as (a) n decreases, (b) the curvature of f increases, and (c) the curvature of δ decreases.

Welfare properties of stationary points. Proposition 2.3.14(B) implies that $e \ll (h(t_j, V(t_j)))_{j \in N}$ for every stationary point e of (2.3.4). Compared to the *status quo*, every consumer is better off and every firm is worse off. From the perspective of the nations, the stationary point represents a Pareto improvement over the *status quo*. Indeed, the stationary profile is Pareto superior to every intermediate profile also.

Proposition 2.3.18. *Suppose Assumption 2.3.1 is satisfied. If e is a stationary point of (2.3.4) and $z \in \prod_{j \in N}[e_j, h(t_j, V(t_j))] - \{e\}$, then $u_i(\theta_i, z) < u_i(\theta_i, e)$ for every $i \in N$.*

2.4 Stage 1: definition of equilibrium investment choices

Suppose it is anticipated that the emission caps accepted in Stage 2 will be generated by a function $(\bar{\theta}, \theta) \mapsto e(\bar{\theta}, \theta) \in \bar{\Re}^n_+$, where $\bar{\theta} \in \Theta^n$ is the profile of initial types and θ is the profile of modified types. $I = \theta - \bar{\theta}$ is the profile of investments. Let investment I_i cost $C(I_i)$ to Nation i.

If the profile of initial types is $\bar{\theta}$ and the profile of Stage 1 investments is I, then Nation i's payoff is $u_i(\bar{\theta}_i + I_i, e(\bar{\theta}, \bar{\theta} + I)) - C(I_i)$, where the first term is given by (2.3.8); indeed, if $e(\bar{\theta}, \bar{\theta} + I)$ is a stationary point, then Corollary 2.3.11 implies that the first line of (2.3.8) applies. We assume that the profile of investments is generated by a Bayesian equilibrium $(\sigma_i)_{i \in N}$, where Nation i's strategy $\sigma_i : \Theta \to \Theta$ is such that, for every $\bar{\theta}_i \in \Theta$, $I_i = \sigma_i(\bar{\theta}_i)$ maximizes

$$\int_{\Theta^{n-1}} \lambda_{-i}(d\bar{\theta}_{-i})\, u_i(\bar{\theta}_i + I_i, e(\bar{\theta}_i, \bar{\theta}_{-i}, \bar{\theta}_i + I_i, \bar{\theta}_{-i} + \sigma_{-i}(\bar{\theta}_{-i}))) - C(I_i) \tag{2.4.1}$$

Rational behavior in this environment amounts to being a Bayesian decision-maker, i.e., every nation should maximize its subjective expected payoff conditional on its type and some belief regarding the decisions of the other nations, or equivalently as in (2.4.1), some belief about the types and strategies of the other nations. In addition to rationality, a Bayesian equilibrium requires the profile of beliefs and strategies to be consistent, i.e., the subjective belief of every nation should be justified by the actual strategies of the other nations. We shall be content to characterize Bayesian equilibrium strategies without attempting to prove the existence of such an equilibrium.

2.5 Equilibrium

We now pull together the discussion of Sections 2.1 to 2.4 in the following definition.

Definition 2.5.1. *$\{(\sigma_i)_{i \in N}, e, v\}$ is an equilibrium if it satisfies the following properties.*

(a) $v : \Re_+ \times \bar{\Re}_+ \to \Re_+$ is such that, for every $(t, e) \in \Re_+ \times \bar{\Re}_+$, $v = v(t, e)$ maximizes $g(t, v)$ subject to the constraint $h(t, v) \le e$.

(b) $e : \Theta^n \times \Theta^n \to \Re^n_+$ is such that, for every $j \in N$ and every $(\bar{\theta}, \theta) \in \Theta^n \times \Theta^n$, $e_j = e_j(\bar{\theta}, \theta)$ maximizes $u_j(\theta_j, e_{-j}(\bar{\theta}, \theta), e_j)$.

(c) for every $i \in N$, $\sigma_i : \Theta \to \Theta$ is such that $\sigma_i(\bar{\theta}_i)$ maximizes (2.4.1) for every $\bar{\theta}_i \in \Theta$.

3. Analysis of Stage 2 outcomes

3.1 Comparative statics

In this section we analyze the properties of the mapping $e : \Theta^2 \times \Theta^2 \to \Re^2_+$ that describes stationary cap profiles. We simplify notation in this section by suppressing the profile of initial types $\bar{\theta}$; thus, $e(\bar{\theta}, \theta)$ is abbreviated to $e(\theta)$. Moreover, using Proposition 2.3.5, $e(\theta)$ may be referred to as the profile of emissions (identical to the profile of emission caps) when the profile of modified types is θ. Naturally, variations in the modified type θ_i reflect one-for-one the variations of the investments I_i. This analysis requires assumptions regarding the effects of type variations on emission, profit and damage.

Assumption 3.1.1. *In addition to Assumption 2.3.1 and (2.3.16), assume that*

(a) for every $v \in \Re_+$, $g(., v)$ is strictly increasing and $h(., v)$ is strictly decreasing,

(b) for every $e \in \Re_+$, $f(., e)$ is strictly concave,

(c) g and h are $\mathcal{C}^3$ and δ is $\mathcal{C}^2$ on $\Re^2_{++}$.

(d) $D_{ke_+}\delta < 0$, and

(e) for every $e_+ \in \Re_+$, $\delta(., e_+)$ is strictly decreasing and strictly convex.

(a) implies that profit is increasing and emission is decreasing with respect to private capital. (b) implies that private capital faces diminishing returns. (d) implies that greater adaptation capital reduces a nation's vulnerability to damage. (e) means that a nation's damage decreases with adaptation capital but this beneficial effect is subject to diminishing returns. We note the following consequences of these assumptions.

Proposition 3.1.2. *Given Assumption 3.1.1,*

(A) v and f are $\mathcal{C}^2$ on $\{(t, e) \in \Re^2_{++} \mid e < h(t, V(t))\}$,

(B) given $e, t, t' \in \Re_+$, if $t < t'$, $e \leq h(t, V(t))$ and $e \leq h(t', V(t'))$, then $f(t, e) < f(t', e)$, and

(C) u_i is $\mathcal{C}^2$ at $(\theta_i, e) \in \Re^4_{++}$ such that $e_i < h(t_i(\theta_i), V \circ t_i(\theta_i))$.

Let $j \in N$ and $\theta \in \Theta^N$. By Definition 2.5.1(b), $e_j = e_j(\theta)$ maximizes $u_j(\theta_j, e_j, e_{-j}(\theta))$. Proposition 2.3.5 implies $e_j(\theta) < h(t_j(\theta), V \circ t_j(\theta))$. Therefore, Proposition 3.1.2(C) implies that $u_j(\theta_j, ., e_{-j}(\theta))$ is $\mathcal{C}^2$ at $e_j(\theta)$. Thus, we have the first order condition for every $j \in N$:

$$D_{e_j} u_j(\theta_j, e(\theta)) = 0 \tag{3.1.3}$$

$D_{e_j} u_j(\theta_j, e(\theta))$ is the shadow value of emission rights to Nation j of type θ_j when the emission cap profile is $e(\theta)$. Thus, (3.1.3) means that, in equilibrium, the shadow value of emission rights for every nation is equal to the cost of acquiring the marginal right, which is zero. Analogously, $D_e f(t_j, e_j)$ is the shadow value of emission rights to Firm j with private capital t_j when Nation j's emission cap is e_j. It follows from Proposition 2.2.2 that the firm's shadow value of emission rights is positive in equilibrium. Thus, *ceteris paribus*, the national firm will prefer a larger cap than the equilibrium one negotiated by the national government. This divergence of interests will explain differences between the socialist and capitalist cases considered in Sections 4.2 and 4.3.

Combining Propositions 2.2.2(F) and 3.1.2(A) with Assumptions 3.1.1(c) and 2.3.1(b) implies the second order condition

$$D_{e_j e_j} u_j(\theta_j, e(\theta)) < 0 \tag{3.1.4}$$

for every $j \in N$. The effects on $e(\theta)$ of varying either component of θ_1 are as follows; with appropriate notational adjustments, the same result holds for all nations.

Proposition 3.1.5. *Given Assumption 3.1.1, and interpreting x as either t_1 or k_1,*

(A) Sign $D_x e_j = -$Sign $D_{e_1 x} u_1$ for $j \in N - \{1\}$,

(B) Sign $D_x e_1 =$ Sign $D_{e_1 x} u_1$, and

(C) Sign $D_x \sum_{j \in N} e_j =$ Sign $D_{e_1 x} u_1$.

Evidently, the signs of all the variational formulae depend on how the type variations affect the shadow values of emission rights for the nation whose type is being perturbed.

3.2 Applications

We set $N = \{1, 2\}$ in the following applications. As should be clear from the proofs and Proposition 3.1.5, these results apply to any pair of nations in N. The following definition classifies technology as locally clean (resp. dirty) if the firm's shadow value of emission rights decreases (resp. increases) with increases in private capital.

Definition 3.2.1. *Technology f is dirty (resp. clean) at (t', e') if $D_{te} f(t', e') > 0$ (resp. $D_{te} f(t', e') < 0$).*

Corollary 3.2.2. *Let $N = \{1, 2\}$ and $\theta = ((t_1, k_1), (t_2, k_2))$. Given $e(\theta) = (e_1(\theta), e_2(\theta))$, let $e_-(\theta) = e_1(\theta) - e_2(\theta)$.*

(A) If $D_{te} f(t_1, e_1(\theta)) < 0$, then $D_{t_1} e_1(\theta) < 0$, $D_{t_1} e_2(\theta) > 0$, $D_{t_1} e_+(\theta) < 0$ and $D_{t_1} e_-(\theta) < 0$.

(B) If $D_{te} f(t_1, e_1(\theta)) > 0$, then $D_{t_1} e_1(\theta) > 0$, $D_{t_1} e_2(\theta) < 0$, $D_{t_1} e_+(\theta) > 0$ and $D_{t_1} e_-(\theta) > 0$.

(C) $D_{k_1} e_1(\theta) > 0$, $D_{k_1} e_2(\theta) < 0$ and $D_{k_1} e_+(\theta) > 0$.

(A) (resp. (B)) means that the growth of private capital in a clean (resp. dirty) nation implies lower (resp. higher) domestic emission, higher (resp. lower) foreign emission, lower (resp. higher) aggregate emission, and assuming $e_-(\theta) > 0$, convergence (resp. divergence) of national emissions. (C) means that the growth of adaptation capital implies higher domestic emission, lower foreign emission and higher aggregate emission. We next derive the ordering of emission caps implied by the ordering of adaptation capital.

Corollary 3.2.3. *If $N = \{1, 2\}$, $\theta = ((t, k_1), (t, k_2))$ and $k_1 > k_2$, then $e_1(\theta) > e_2(\theta)$.*

Ceteris paribus, nations with more adaptation capital have larger emissions. In the case of private capital, the analogous result is more complicated as the nature of technology affects the directions in which the emissions change as private capital varies.

Corollary 3.2.4. *If $N = \{1, 2\}$, and*

(a) $\theta = ((t_1, k), (t_2, k))$ and $t_1 > t_2$,

(b) $D_{te}f(t_1, e_1(\theta)) > 0$ and $D_{te}f(t_2, e_2(\theta)) > 0$ (resp. $D_{te}f(t_1, e_1(\theta)) < 0$ and $D_{te}f(t_2, e_2(\theta)) < 0$), and

(c) for every e', $D_{te}f(., e')$ is decreasing,

then $e_1(\theta) > e_2(\theta)$ (resp. $e_1(\theta) < e_2(\theta)$).

If (a) both nations have the same adaptation capital stock, (b) both nations have clean (resp. dirty) technology, and (c) technology becomes cleaner as private capital grows, then the nation with the greater private capital stock has lower (resp. higher) emission.

4. Analysis of Stage 1 choices

4.1 Set-up

In this section we analyze Nation 1's choices in Stage 1. In Section 4.2, we consider the "socialist" case: investment in Nation 1's private and adaptation capital is chosen by the government. In Section 4.3, we consider the "capitalist" case: Firm 1 chooses investment in Nation 1's private and adaptation capital. In Section 4.4, we consider the "global" case when Nation 1 or Firm 1 can choose foreign adaptation and private capital. It is also possible to consider the mixed economy case when the government chooses Nation 1's domestic adaptation capital while Firm 1 chooses domestic private capital. Since these decision-makers have different objectives, their joint decisions have to conform to some notion of equilibrium. The joint decisions can be modelled using Nash equilibrium as the solution concept. However, we do not present the details here as the results qualitatively mimic those of Sections 4.2 and 4.3, and the analysis does not throw up any new phenomena.

Let $\{(\sigma_i)_{i\in N}, e, v\}$ satisfy Definition 2.5.1. We studied v in Section 2.2, and e in Sections 2.3 and 3. Finally, we study $(\sigma_i)_{i\in N}$ to derive the motives guiding the investments in Stage 1. It suffices to study the decision-making of just one nation, say Nation 1, as the same arguments apply to every nation. For notational and expositional simplicity, we study a special case of our model.

Assumption 4.1.1. *The special model is specified by the following assumptions:*

(a) $N = \{1, 2\}$,

(b) $\bar{\theta}_1 \equiv 0 \in \Re^2$, *and*

(c) $C(x, y) = x + y$.

By Proposition 3.1.5(A), the directional effect of Nation 1's Stage 1 choice on the equilibrium Stage 2 outcomes is identical across the other nations. Given that the results in this section rely only on this directional effect, we may replace $N - \{1\}$ with a representative Nation 2 without loss of generality, as we do in Assumption (a). Assumption (b) is a change-of-origin so that we can ignore non-negativity constraints that would result from positive initial levels of private and adaptation capital and an inability to choose smaller levels by disinvesting. *As* $\bar{\theta}_1 \equiv 0$*, we lighten notation by eliminating it from our expressions.* Assumption (c) allows investment to move without friction between private and adaptation capital. Our piecemeal analysis of investment decisions is without loss of generality because the cost of investments is linear and additive. This allows us to focus on the strategic role of these investments.

Assumption 4.1.2. $\{(\sigma_i)_{i\in N}, e, v\}$ *is an equilibrium of the special model such that*

(a) $\sigma_1(0) = (t_1, k_1) \gg 0$, *and*

(b) $e(\bar{\theta}_2, I_1, \bar{\theta}_2 + I_2) \in \Re^2_{++}$ *for* $(\bar{\theta}_2, I_1, \bar{\theta}_2 + I_2) \in \Theta \times \Theta^2$.

4.2 The socialist case

If Nation 1's investment is $I_1 = (t_1, k_1)$ and Nation 2's type is $\bar{\theta}_2$, then Nation 1's payoff is $u_1(I_1, e(\bar{\theta}_2, I_1, \bar{\theta}_2 + \sigma_2(\bar{\theta}_2))) - C(I_1)$. Consequently, Nation 1's Stage 1 problem is: choose $I_1 = (t_1, k_1) \in \Theta$ to maximize

$$\int_{\Theta} \lambda_2(d\bar{\theta}_2)\, u_1(I_1, e(\bar{\theta}_2, I_1, \bar{\theta}_2 + \sigma_2(\bar{\theta}_2))) - C(I_1) \qquad (4.2.1)$$

Let $e : \Theta^2 \times \Theta^2 \to \Re^2_+$ be such that, for every $(\bar{\theta}, \theta) \in \Theta^2 \times \Theta^2$, $e(\bar{\theta}, \theta)$ is accepted by both nations. Given e, let I_1 solve (4.2.1). Let $I_1' \in \Theta$ maximize

$$\int_\Theta \lambda_2(d\bar{\theta}_2)\, u_1(I_1', e(\bar{\theta}_2, I_1, \bar{\theta}_2 + \sigma_2(\bar{\theta}_2))) - C(I_1') \tag{4.2.2}$$

Clearly, Nation i prefers θ_1' to θ_1 *ex post*. The difference between θ_1' and θ_1 represents Nation 1's ability to manipulate the choice of caps in Stage 2 by affecting the choice of the Stage 2 continuation game *via* its Stage 1 choice of investment.

Let $(t^*, k^*) \gg (0, 0)$ solve (4.2.1). Simplify notation by denoting $e(\bar{\theta}_2, t^*, k^*, \bar{\theta}_2 + \sigma_2(\bar{\theta}_2))$ by $e^*(\bar{\theta}_2)$. Using (3.1.3), (t^*, k^*) is characterized by

$$\int_\Theta \lambda_2(d\bar{\theta}_2)\, \left[D_t f(t^*, e_1^*(\bar{\theta}_2)) - D_{e_+}\delta(k^*, e_+^*(\bar{\theta}_2)) D_t e_2^*(\bar{\theta}_2)\right] = 1 \tag{4.2.3}$$

$$\int_\Theta \lambda_2(d\bar{\theta}_2)\, \left[D_k \delta(k^*, e_+^*(\bar{\theta}_2)) + D_{e_+}\delta(k^*, e_+^*(\bar{\theta}_2)) D_k e_2^*(\bar{\theta}_2)\right] = -1 \tag{4.2.4}$$

First consider the choice of adaptation capital. Combining Assumption 2.2.1, Proposition 2.2.3(A) and Corollary 3.2.2(C), the benefits of investment in adaptation capital are: (a) an increase in domestic profit caused by higher domestic emission, (b) lower domestic vulnerability to damage, and (c) a decrease in domestic damage on account of lower foreign emission. The costs are: (d) an increase in domestic damage caused by higher domestic emission, and (e) the opportunity cost of investment. In equilibrium, each nation's emission (cap) is chosen to balance benefit (a) and cost (d) at the margin. Thus, (4.2.4) ensures that marginal benefits (b) and (c) are balanced by the marginal cost (e). (b) is the direct benefit of investment in adaptation capital, while (c) is the indirect or strategic benefit.

Let (t^0, k^0) be Nation 1's choice of private and adaptation capital given the type contingent emissions $\bar{\theta}_2 \mapsto e^*(\bar{\theta}_2)$, i.e., $(t_1, k_1) = (t^0, k^0)$ maximizes (4.2.2). Then, (t^0, k^0) satisfies

$$\int_\Theta \lambda_2(d\bar{\theta}_2)\, D_t f(t^0, e_1^*(\bar{\theta}_2)) = 1 \tag{4.2.5}$$

$$\int_\Theta \lambda_2(d\bar{\theta}_2)\, D_k \delta(k^0, e_+^*(\bar{\theta}_2)) = -1 \tag{4.2.6}$$

$k^* > k^0$ (resp. $k^* < k^0$) is interpreted as strategic overinvestment (resp. underinvestment) by Nation 1 in domestic adaptation capital. (4.2.4), (4.2.6), Assumption 2.3.1(b) and Corollary 3.2.2(C) imply

$$\begin{aligned}\int_\Theta \lambda_2(d\bar{\theta}_2)\, D_k \delta(k^0, e_+^*(\bar{\theta}_2)) &= \int_\Theta \lambda_2(d\bar{\theta}_2)\, \left[D_k \delta(k^*, e_+^*(\bar{\theta}_2)) + D_{e_+}\delta(k^*, e_+^*(\bar{\theta}_2)) D_k e_2^*(\bar{\theta}_2)\right] \\ &< \int_\Theta \lambda_2(d\bar{\theta}_2)\, D_k \delta(k^*, e_+^*(\bar{\theta}_2))\end{aligned}$$

i.e., $\int_\Theta \lambda_2(d\bar\theta_2) \int_{k^*}^{k_0} dx\, D_{kk}\delta(x, e_+^*(\bar\theta_2)) < 0$. Assumption 3.1.1(e) implies $k^0 < k^*$.

Proposition 4.2.7. *Nation 1 overinvests in domestic adaptation capital, thereby strategically raising e_1, lowering e_2 and raising e_+.*

Now consider the choice of private capital. $t^* > t^0$ (resp. $t^* < t^0$) is interpreted as strategic overinvestment (resp. underinvestment) by Nation 1 in domestic private capital.

Proposition 4.2.8. *If $D_{te}f(t^*, e_1^*(\bar\theta_2)) < 0$ (resp. $D_{te}f(t^*, e_1^*(\bar\theta_2)) > 0$) for every $\bar\theta_2 \in \operatorname{supp}\lambda_2$, then Nation 1 underinvests (resp. overinvests) in domestic private capital, thereby strategically raising e_1, lowering e_2 and raising e_+.*

Proof. Suppose $D_{te}f(t^*, e_1^*(\bar\theta_2)) < 0$ for every $\bar\theta_2 \in \operatorname{supp}\lambda_2$. Then, (4.2.3), (4.2.5), Assumption 2.3.1(b) and Corollary 3.2.2(A) imply

$$\begin{aligned}\int_\Theta \lambda_2(d\bar\theta_2)\, D_t f(t^0, e_1^*(\bar\theta_2)) &= \int_\Theta \lambda_2(d\bar\theta_2)\, \left[D_t f(t^*, e_1^*(\bar\theta_2)) - D_{e_+}\delta(k^*, e_+^*(\bar\theta_2)) D_t e_2^*(\bar\theta_2)\right] \\ &< \int_\Theta \lambda_2(d\bar\theta_2)\, D_t f(t^*, e_1^*(\bar\theta_2))\end{aligned}$$

i.e., $\int_\Theta \lambda_2(d\bar\theta_2) \int_{t^*}^{t^0} dx\, D_{tt} f(x, e_1^*(\bar\theta_2)) < 0$. Assumption 3.1.1(b) implies $t^0 > t^*$. The other case is analogous. ■

Since, in equilibrium, e_1 is a best response to e_2 in terms of Nation 1's preference, we have the following observation.

Remark 4.2.9. *Nation 1's strategic manipulations (see (4.2.3) and (4.2.4)) in Propositions 4.2.7 and 4.2.8 work by manipulating downwards Nation 2's emission.*

4.3 The capitalist case

Suppose Firm 1 chooses investment $I_1 = (t_1, k_1)$. If Firm 1's investment is (t_1, k_1) and Nation 2's type is $\bar\theta_2$, then Firm 1's payoff is $f(t_1, e_1(\bar\theta_2, t_1, k_1, \bar\theta_2 + \sigma_2(\bar\theta_2))) - t_1 - k_1$. Assuming Nation 1 and Firm 1 have identical information, Firm 1's Stage 1 problem is: choose $(t_1, k_1) \in \Theta$ to maximize

$$\int_\Theta \lambda_2(d\bar\theta_2)\, f(t_1, e_1(\bar\theta_2, t_1, k_1, \bar\theta_2 + \sigma_2(\bar\theta_2))) - t_1 - k_1 \qquad (4.3.1)$$

Let $(t_1, k_1) = (t^{**}, k^{**}) \gg (0, 0)$ maximize (4.3.1). We simplify notation by writing $e(\bar\theta_2, t^{**}, k^{**}, \bar\theta_2 + \sigma_2(\bar\theta_2))$ as $e^{**}(\bar\theta_2)$. (t^{**}, k^{**}) is characterized by

$$\int_\Theta \lambda_2(d\bar\theta_2)\, [D_t f(t^{**}, e_1^{**}(\bar\theta_2)) + D_e f(t^{**}, e_1^{**}(\bar\theta_2)) D_t e_1^{**}(\bar\theta_2)] = 1 \qquad (4.3.2)$$

$$\int_\Theta \lambda_2(d\bar\theta_2)\, D_e f(t^{**}, e_1^{**}(\bar\theta_2)) D_k e_1^{**}(\bar\theta_2) = 1 \qquad (4.3.3)$$

Let (t^{00}, k^{00}) be Firm 1's choice of private and adaptation capital given the type contingent emissions $\bar{\theta}_2 \mapsto e^{**}(\bar{\theta}_2)$. $k^{**} > k^{00}$ (resp. $k^{**} < k^{00}$) indicates overinvestment (resp. underinvestment) in adaptation capital. Similarly, $t^{**} > t^{00}$ (resp. $t^{**} < t^{00}$) indicates overinvestment (resp. underinvestment) in private capital. Clearly, $k^{00} = 0$. Therefore, $k^{**} > 0$ represents overinvestment by Firm 1 in adaptation capital. Copying the proofs of Propositions 4.2.7 and 4.2.8, we have

Proposition 4.3.4. *Propositions 4.2.7 and 4.2.8 hold with "Firm 1" replacing "Nation 1".*

Nation 1's equilibrium emission is a best response to foreign emission, arrived at by balancing domestic profit considerations against domestic damage considerations. As Firm 1 does not take domestic damage into account, Nation 1's equilibrium emission is sub-optimal from Firm 1's perspective. Moreover, unlike Nation 1, Firm 1 is indifferent to foreign emission. This leads to the following observation, which may be contrasted with Remark 4.2.9.

Remark 4.3.5. *Firm 1's strategic manipulations (see (4.3.2) and (4.3.3)) in Proposition 4.3.4 work by manipulating upwards Nation 1's emission.*

We now ask: in what directions would Nation 1 like to perturb Firm 1's choice (t^{**}, k^{**})? Let $G(t_1, k_1) = \int_\Theta \lambda_2(d\bar{\theta}_2)\, u_1(t_1, k_1, e(\bar{\theta}_2, t_1, k_1, \bar{\theta}_2 + \sigma_2(\bar{\theta}_2))) - t_1 - k_1$. (4.3.2) implies that $D_t G(t^{**}, k^{**}) = -\int_\Theta \lambda_2(d\bar{\theta}_2)\, D_{e_+}\delta(k^{**}, e_+^{**}(\bar{\theta}_2)) D_t e_+^{**}(\bar{\theta}_2)$. Therefore, Assumption 2.3.1(b) and Corollary 3.2.2 imply

Proposition 4.3.6. *If $D_{te}f(t^{**}, e_1^{**}(\bar{\theta}_2)) < 0$ (resp. $D_{te}f(t^{**}, e_1^{**}(\bar{\theta}_2)) > 0$) for every $\bar{\theta}_2 \in \operatorname{supp} \lambda_2$, then Nation 1 prefers a higher (resp. lower) level of private capital than Firm 1.*

(4.3.3) implies that Nation 1's incentive for marginal domestic adaptation investment is $D_k G(t^{**}, k^{**}) = -\int_\Theta \lambda_2(d\bar{\theta}_2)\, [D_k\delta(k^{**}, e_+^{**}(\bar{\theta}_2)) + D_{e_+}\delta(k^{**}, e_+^{**}(\bar{\theta}_2)) D_k e_+^{**}(\bar{\theta}_2)]$. Assumptions 2.3.1(b) and 3.1.1(e), and Corollary 3.2.2(C), imply that this marginal incentive cannot be signed unambiguously as an increase in Nation 1's adaptation capital has two opposing effects on Nation 1's damage. On the one hand, it directly decreases domestic damage (the direct effect), but on the other hand, it increases domestic damage by inducing higher total emission (the indirect effect).

Remark 4.3.7. *If the direct effect is larger (resp. smaller) than the indirect effect, then Nation 1 prefers a higher (resp. lower) level of adaptation capital than Firm 1.*

4.4 The global case

We now consider a third variation on the model described in Section 2. Now suppose Nation 1 can invest in Nation 2's private and adaptation capital. By Corollary 3.2.2(C), an increase in Nation 2's adaptation capital hurts Firm 1 by reducing its emission, and therefore its profit, while it hurts Consumer 1 by increasing the total emission. Therefore, we have

Proposition 4.4.1. *Neither Nation 1, nor Firm 1, will invest in Nation 2's adaptation capital.*

The situation with respect to private capital is somewhat more complicated. Suppose Nation 1 invests $\alpha = \alpha^* > 0$ in Nation 2's private capital to maximize

$$\int_\Theta \lambda_2(d\bar{\theta}_2)\, u_1(\theta_1, e(\bar{\theta}_2, \theta_1, \bar{\theta}_2 + \sigma_2(\bar{\theta}_2) + (\alpha, 0))) - \alpha$$

α^* is characterized by

$$\int_\Theta \lambda_2(d\bar{\theta}_2)\ \left[D_e f(t_1, e_1^*(\bar{\theta}_2)) D_{t_2} e_1^*(\bar{\theta}_2) - D_{e_+}\delta(k_1, e_+^*(\bar{\theta}_2)) D_{t_2} e_+^*(\bar{\theta}_2)\right] = 1 \qquad (4.4.2)$$

where $e^*(\bar{\theta}_2) = e(\bar{\theta}_2, \theta_1, \bar{\theta}_2 + \sigma_2(\bar{\theta}_2) + (\alpha^*, 0))$ for $\bar{\theta}_2 \in \Theta$. If $\bar{\theta}_2$ is such that $D_{te} f(t_2(\bar{\theta}_2 + \sigma_2(\bar{\theta}_2)) + \alpha^*, e_2^*(\bar{\theta}_2)) > 0$ (resp. $D_{te} f(t_2(\bar{\theta}_2 + \sigma_2(\bar{\theta}_2)) + \alpha^*, e_2^*(\bar{\theta}_2)) < 0$), then Corollary 3.2.2 implies $D_{t_2} e_1^*(\bar{\theta}_2) < 0$ and $D_{t_2} e_+^*(\bar{\theta}_2) > 0$ (resp. $D_{t_2} e_1^*(\bar{\theta}_2) > 0$ and $D_{t_2} e_+^*(\bar{\theta}_2) < 0$).

It follows that, if $D_{te} f(t_2(\bar{\theta}_2 + \sigma_2(\bar{\theta}_2)) + \alpha^*, e_2^*(\bar{\theta}_2)) > 0$ for every $\bar{\theta}_2 \in \operatorname{supp} \lambda_2$, then Propositions 2.2.2(E) and 3.2.2 and Assumption 2.3.1(b) imply that (4.4.2) cannot hold. Thus, we have the following results.

Proposition 4.4.3. *(A) If Nation 1 believes with certainty that Nation 2's technology after investment will be dirty, then Nation 1 will not invest in Nation 2's private capital.*

(B) Nation 1 (resp. Firm 1) invests in Nation 2's private capital if and only if it believes with sufficiently high probability that Nation 2's technology after investment will be clean.

(C) If the belief about Nation 2's type is such that Firm 1 will choose to invest in

Nation 2's private capital, then Nation 1 has an even stronger incentive to make such an investment.

(A) follows from the fact that, given the hypothesis regarding Nation 2's technology, an increase in Nation 2's private capital hurts Firm 1 by reducing its emission and hurts Consumer 1 by raising total emission. (C) follows as Nation 1 stands to gain from the fall in total emission in addition to the rise in Firm 1's emission.

5. Conclusions

5.1 Summary of results

We summarize the main results obtained for the model described in Section 2.1. The results pertaining to Stages 2 and 3 of the model are largely technical and stated carefully in Sections 2.3 and 2.2 respectively. The substantive implication of these results is that our model of the negotiation process is theoretically tractable and possesses very useful properties: for every profile of types, there is a unique profile of caps that is the unique, globally asymptotically stable, stationary point of the negotiation process. The assumptions on the primitives of the model that imply such strong properties also facilitate the comparative statics results obtained in Section 3.1. Moreover, for every profile of types, the equilibrium (stationary) profile of caps represents a Pareto improvement over the *status quo*.

We describe and interpret here in somewhat greater detail the results pertaining to Stage 1 of the model. All nations will overinvest in domestic adaptation capital (Propositions 4.2.7). Nations with clean (resp. dirty) technologies will underinvest (resp. overinvest) in domestic private capital (Proposition 4.2.8). The effects of these manipulations are to raise domestic emission, lower foreign emissions and raise the total emission. The results regarding the domestic investment choices by firms' are qualitatively similar (Proposition 4.3.4), but there are significant differences. For instance, the variables targeted for manipulation by a nation are the foreign caps (Remark 4.2.9), while the variable targeted by a firm is the domestic cap (Remark 4.3.5). Nor is the extent of manipulation the same. With respect to domestic private capital, a nation with clean (resp. dirty) technology underinvests (resp. overinvests) less severely than its firm (Proposition 4.3.6), while the comparison is ambiguous in the case of adaptation capital (Remark 4.3.7).

Neither nations, nor their firms, will invest in foreign adaptation capital (Proposition 4.4.1). Neither a nation with clean technology, nor its firm, will invest in the private capital of a nation with dirty technology (Proposition 4.4.3). However, private investment across nations with clean technology is possible (Proposition 4.4.3) as this has the effect of raising domestic emission while lowering foreign and total emission.

Our model yields results regarding the ordering of emission caps. *Ceteris paribus*, nations with greater adaptation capital have larger caps in equilibrium (Corollary 3.2.3). The result in terms of private capital depends on the assumption that larger private capital implies cleaner technology. Given this assumption, *ceteris paribus*, the emission caps of nations with dirty (resp. clean) technology are positively (resp. negatively) related to the size of their private capital (Corollary 3.2.4). Obviously, care needs to be exercised when applying this result to actual nations. Comparisons based on casual empiricism are dubious as *ceteris paribus* rarely holds when comparing actual nations. However, these results do identify the principles governing the ordering of caps.

Now alter the above scenario by making type modifications exogenous. Suppose we identify a nation's growth with the growth of its private capital. Also suppose that (a) adaptation capital of both nations is fixed, (b) private capital grows in both nations for reasons exogenous to our model, and (c) nations with clean technology (the "North") emit more than nations with dirty technology (the "South"). Then, Corollary 3.2.2 suggests that Southern emission rises and Northern emission falls, i.e., their emissions converge. If technology becomes cleaner as private capital grows, then growth will automatically make Southern technology cleaner over time.

Our results also have some political-economic implications.

First, in our model, a nation's equilibrium emission cap is such that it is an active constraint on the domestic firm. Growth of a nation's adaptation capital is a means for relaxing this constraint. Thus, Green concern, i.e., increasing national welfare by reducing damage to the consumer, is not the only motive for investment in domestic adaptation capital; such investment is also a strategic way of gaining head-room for higher equilibrium emission by the domestic firm, resulting in higher profit and national welfare. The strategic motive is particularly strong for nations with clean technology (i.e., nations of the North) as investment in its private capital makes an already active emission constraint still tighter; thus, over-investment in adaptation capital is a strategic way of loosening this constraint or counteracting the effects of private capital growth.

Secondly, the growth of Southern private capital raises total emission, while the growth of Northern private capital lowers total emission. If a "Green" is someone who wishes to minimize total emission, then a "Green" will favor private investment (i.e., growth) in the

North and not in the South. This points to a conflict between the objectives of (Northern and Southern) green lobbies and the growth ambitions of the South.

Thirdly, investment (especially in adaptation capital) is a tool for manipulation in our model. Consequently, affluent nations with the wherewithal for substantial investments can manipulate the emission caps in ways that poorer nations cannot do.

5.2 Variations on the theme of this paper

First, we may consider the complement of the above model with tradeable emission permits instead of autarky. In this case, the first two stages of the model remain unchanged, while the third stage is modelled as a competitive equilibrium. A complicating aspect of the resulting model is that, unlike in the autarkic model, the Stage 3 emission of every nation depends on the entire profile of types *via* the equilibrium price of emission rights. An analysis of this variation remains as future work.

The second extension is to embed our model in a multi-period one. Clearly, national types will evolve over time on account of technical change and economic growth. Moreover, knowledge of the interaction between the economic process and the environment (modelled by g, h and δ) will evolve over time. The only certainty in this process is that emission rights will have to be re-allocated periodically. Therefore, learning about the relevant random processes is important for every nation so that it can position itself better for future renegotiations. A successful model in this context will have to feature forward-looking behavior and sophisticated belief formation by the negotiators. Such concerns have motivated a literature (e.g., Kelly and Kolstad 1999, Kolstad 1996, Ulph and Maddison 1997, Ulph and Ulph 1997) on learning in this context. However, a comprehensive general model is awaited.

Appendix

Proof of Proposition 2.2.2. (A) Fix $(t, e) \in \Re_+ \times \bar{\Re}_+$. As $h(t, 0) = 0 \leq e$, we have $\Gamma(t, e) \neq \emptyset$. As h is continuous, $\{v \in \Re_+ \mid h(t, v) \leq e\}$ is closed in $\Re_+$. As $V(t) \in \Re_+$, $[0, V(t)]$ is compact. Therefore, $\Gamma(t, e)$ is compact. As g is continuous, Weierstrass' theorem implies the existence of $v \in \Gamma(t, e)$ such that $g(t, v) = \sup g(t, \Gamma(t, e))$.

We now show that there is a unique solution. Suppose $v, v' \in \Gamma(t, e)$ such that $g(t, v) = g(t, v') = \sup g(t, \Gamma(t, e))$ and $v \neq v'$. Let $\lambda \in (0, 1)$. As $v, v' \in \Gamma(t, e)$, we have $h(t, v) \leq e$, $h(t, v') \leq e$ and $v, v' \in [0, V(t)]$. It follows that $\lambda v + (1 - \lambda)v' \in [0, V(t)]$, and as $h(t, .)$ is strictly convex, we have $h(t, \lambda v + (1 - \lambda)v') < \lambda h(t, v) + (1 - \lambda)h(t, v') \leq e$. Thus, $\lambda v + (1 - \lambda)v' \in \Gamma(t, e)$, and as $g(t, .)$ is strictly concave, we have $g(t, \lambda v + (1 - \lambda)v') > \lambda g(t, v) + (1 - \lambda)g(t, v') = \sup g(t, \Gamma(t, e))$, a contradiction.

(B) Define mappings Γ_1, Γ_2 and Γ, each from $\Re_+ \times \bar{\Re}_+$ to $\Re_+$, by $\Gamma_1(t, e) = \{v \in \Re_+ \mid h(t, v) \leq e\}$, $\Gamma_2(t, e) = [0, V(t)]$ and $\Gamma(t, e) = \Gamma_1(t, e) \cap \Gamma_2(t, e)$ respectively. Suppose Γ is continuous and has nonempty and compact values. Using this fact, (A) and the continuity of g, the result follows from the Theorem of the Maximum (Berge 1963, Theorem VI.3). We now confirm the hypothesized properties of Γ.

Assumptions 2.2.1 implies that $0 \in \Gamma(t, e)$ for every $(t, e) \in \Re_+ \times \bar{\Re}_+$. Thus, Γ has nonempty values. As h is continuous, $\operatorname{Gr} \Gamma_1$ is closed in $\Re_+ \times \bar{\Re}_+ \times \Re_+$. It follows immediately that Γ_1 has closed values. As $V(t) \in \Re_{++}$, Γ_2 has compact values. Thus, Γ has compact values.

If Γ_2 is upper hemicontinuous, then so is Γ (Berge 1963, Theorem VI.1.7). We show that Γ_2 is upper hemicontinuous. Consider $(t, e) \in \Re_+ \times \bar{\Re}_+$ and $\epsilon > 0$. As V is continuous, there exists an open neighborhood of t in $\Re_+$, say U, such that $V(U) \subset [0, V(t) + \epsilon)$. It follows that $[0, V(t')] \subset [0, V(t) + \epsilon)$ for every $t' \in U$. Thus, $\Gamma_2(U \times \bar{\Re}_+) = \cup_{t' \in U}[0, V(t')] \subset [0, V(t) + \epsilon)$. Suppose E is open in $\Re_+$ and $\Gamma_2(t, e) = [0, V(t)] \subset E$. It follows that there exists $\epsilon > 0$ such that $[0, V(t) + \epsilon)) \subset E$. By the above argument, there exists an open neighborhood of t in $\Re_+$, say U, such that $\Gamma_2(U \times \bar{\Re}_+) \subset [0, V(t) + \epsilon) \subset E$, as required.

We now show that Γ is lower hemicontinuous. Consider $(t, e) \in \Re_+ \times \bar{\Re}_+$ and E open in $\Re_+$ such that $\Gamma(t, e) \cap E \neq \emptyset$.

Suppose $e = 0$. Then $\Gamma(t, e) = \{0\} \subset E$. As Γ is upper hemicontinuous, there exists an open neighborhood of (t, e) in $\Re_+ \times \bar{\Re}_+$, say U, such that $\Gamma(U) \subset E$. Thus, $\Gamma(t', e') \cap E \neq \emptyset$ for every $(t', e') \in U$.

Suppose $e > 0$. Define mappings $\hat{\Gamma}_1$, $\hat{\Gamma}_2$ and $\hat{\Gamma}$, each from $\Re_+ \times \bar{\Re}_+$ to $\Re_+$, by $\hat{\Gamma}_1(t, e) = \{v \in \Re_+ \mid h(t, v) < e\}$, $\hat{\Gamma}_2(t, e) = [0, V(t))$ and $\hat{\Gamma}(t, e) = \hat{\Gamma}_1(t, e) \cap \hat{\Gamma}_2(t, e)$

respectively. As $\Gamma(t,e) \cap E \neq \emptyset$, there exists $v \in E$ such that $v \in [0, V(t)]$ and $h(t,v) \leq e$. If $v = 0$, then $h(t,v) = 0 < e$ and $v = 0 \in [0, V(t))$. Thus, $v \in \hat{\Gamma}(t,e) \cap E$. If $v > 0$, then $\emptyset \neq [0,v) \subset \hat{\Gamma}(t,e)$ as $h(t,.)$ is strictly increasing. As E is open in $\Re_+$ and $v \in E$, $\emptyset \neq [0,v) \cap E \subset \hat{\Gamma}(t,e) \cap E$. We conclude that $\hat{\Gamma}(t,e) \cap E \neq \emptyset$.

If $\hat{\Gamma}$ is lower hemicontinuous at (t,e), then there exists an open neighborhood of (t,e) in $\Re_+ \times \bar{\Re}_+$, say U, such that $\emptyset \neq \hat{\Gamma}(t',e') \cap E \subset \Gamma(t',e') \cap E$ for every $(t',e') \in U$. Thus, Γ is lower hemicontinuous at (t,e), as required.

It remains to show that $\hat{\Gamma}$ is lower hemicontinuous. Suppose E is open in $\Re_+$. As h and V are continuous, $\mathrm{Gr}\,\hat{\Gamma}_1 = \{(t,e,v) \in \Re_+ \times \bar{\Re}_+ \times \Re_+ \mid h(t,v) - e < 0\}$ and $\mathrm{Gr}\,\hat{\Gamma}_2 = \{(t,e,v) \in \Re_+ \times \bar{\Re}_+ \times \Re_+ \mid v - V(t) < 0\}$ are open in $\Re_+ \times \bar{\Re}_+ \times \Re_+$. Consequently, $\mathrm{Gr}\,\hat{\Gamma} = \mathrm{Gr}\,\hat{\Gamma}_1 \cap \mathrm{Gr}\,\hat{\Gamma}_2$ is open in $\Re_+ \times \bar{\Re}_+ \times \Re_+$. Note that

$$\{(t,e) \in \Re_+ \times \bar{\Re}_+ \mid \hat{\Gamma}(t,e) \cap E \neq \emptyset\} = \pi(\mathrm{Gr}\,\hat{\Gamma} \cap \Re_+ \times \bar{\Re}_+ \times E)$$

where $\pi : \Re_+ \times \bar{\Re}_+ \times \Re_+ \to \Re_+ \times \bar{\Re}_+$ is the projection mapping $\pi(t,e,v) = (t,e)$. As π is an open mapping and $\mathrm{Gr}\,\hat{\Gamma}$ and $\Re_+ \times \bar{\Re}_+ \times E$ are open in $\Re_+ \times \bar{\Re}_+ \times \Re_+$, we have $\{(t,e) \in \Re_+ \times \bar{\Re}_+ \mid \hat{\Gamma}(t,e) \cap E \neq \emptyset\}$ open in $\Re_+ \times \bar{\Re}_+$. Thus, $\hat{\Gamma}$ is lower hemicontinuous.

(C) and (D) follow immediately from Assumption 2.2.1.

(E) Let $e, e' \in [0, h(t, V(t))]$ and $e < e'$. (D) implies $h(t, v(t,e)) = e < e' = h(t, v(t,e'))$. Assumption 2.2.1 implies $v(t,e) < v(t,e') \leq V(t)$, and therefore, $f(t,e) = g(t, v(t,e)) < g(t, v(t,e')) = f(t,e')$.

(F) Let $e, e' \in [0, h(t, V(t))]$ and $\lambda \in (0,1)$, with $e \neq e'$. Let $v(t,e) = v$ and $v(t,e') = v'$. By definition, $h(t,v) \leq e$ and $h(t,v') \leq e'$. Assumption 2.2.1 implies $h(t, \lambda v + (1-\lambda)v') \leq \lambda h(t,v) + (1-\lambda)h(t,v') \leq \lambda e + (1-\lambda)e'$. Therefore, by Assumption 2.2.1,

$$f(t, \lambda e + (1-\lambda)e') \geq g(t, \lambda v + (1-\lambda)v') > \lambda g(t,v) + (1-\lambda)g(t,v') = \lambda f(t,e) + (1-\lambda)f(t,e')$$

(G) Using (C), if $e \geq h(t, V(t))$, then $f(t,e) = g(t, v(t,e)) = g(t, V(t))$. ■

Proof of Proposition 2.3.2. (A) follows immediately from the definition of $V(t)$.

(B) Consider $e > h(t, V(t))$. By Proposition 2.2.2(C), $v(t,e') = V(t)$ for every $e' > h(t, V(t))$. Therefore, $D_e v(t,e) = 0$, and using (A), $D_v g(t, v(t,e)) = D_v g(t, V(t)) = 0$.

(C) By Proposition 2.2.2(D), if $e \in (0, h(t, V(t)))$, then $h(t, v(t,e)) = e$. In this case, we can combine Assumptions 2.2.1(c) and 2.3.1(a), and use the implicit function theorem (Lang 1993, XIV, Theorem 2.1) to conclude that $v(t,.)$ is $\mathcal{C}^1$ on $(0, h(t, V(t)))$.

(D) Combining (B), (C) and Assumption 2.3.1(a), it follows that $f(t,.) = g(t, v(t,.))$ is $\mathcal{C}^1$ on $(0, h(t, V(t))) \cup (h(t, V(t)), \infty)$. ■

Proof of Proposition 2.3.5. If $e_i \geq h(t_i, V(t_i))$, then Proposition 2.2.2(C) implies $v(t_i, e_i) = V(t_i)$. It follows that $U_i(\theta_i, I, e) = U_i(\theta_i, I, h(t_i, V(t_i)), e_{-i})$ is independent of e_i for $e_i \geq h(t_i, V(t_i))$.

Consider $e_i \in (0, h(t_i, V(t_i)))$. Proposition 2.2.2(D) implies that $h(t_i, v(t_i, e_i)) = e_i$ and $U_i(\theta_i, I, e)$ is given by (2.3.3). Proposition 2.3.2(C) and Assumption 2.3.1(a) yield $D_{e_i} v(t_i, e_i) = 1/D_{v_i} h(t_i, v(t_i, e_i))$. Using Proposition 2.3.2(D), the derivative of the first term of (2.3.3) with respect to e_i is

$$D_{v_i} g(t_i, v(t_i, e_i)) D_{e_i} v(t_i, e_i) = \frac{D_{v_i} g(t_i, v(t_i, e_i))}{D_{v_i} h(t_i, v(t_i, e_i))} \tag{A.1}$$

As v is continuous, $\lim_{e_i \uparrow h(t_i, V(t_i))} v(t_i, e_i) = v(t_i, h(t_i, V(t_i))) = V(t_i)$ by Proposition 2.2.2(B). Using (A.1), Assumption 2.3.1(a) and Proposition 2.3.2(A), we have

$$\lim_{e_i \uparrow h(t_i, V(t_i))} D_{v_i} g(t_i, v(t_i, e_i)) D_{e_i} v(t_i, e_i) = \frac{D_{v_i} g(t_i, V(t_i))}{D_{v_i} h(t_i, V(t_i))} = 0$$

The derivative of the second term of (2.3.3) with respect to e_i, evaluated at $h(t_i, V(t_i))$, is

$$-\int_{\Theta^{n-1}} \Lambda_i(I, e_{-i}, dx)\, D_{e_+} \delta \left(k_i, h(t_i, V(t_i)) + \sum_{j \in N - \{i\}} h(t_j(x), v(t_j(x), e_j)) \right) < 0$$

Therefore, $e_i = h(t_i, V(t_i))$ cannot maximize (2.3.3). Since Nation i's expected payoff is invariant with respect to e_i for $e_i \geq h(t_i, V(t_i))$, we must have $b < h(t_i, V(t_i))$ for every $b \in \beta_i(e_{-i}; \theta_i, I)$. It follows from Proposition 2.2.2(D) that $h(t_i, v(t_i, b)) = b$ for every $b \in \beta_i(e_{-i}; \theta_i, I)$. ■

Proof of Proposition 2.3.13. Let e^* be a stationary point of (2.3.4). Consider $j \in N$. Using Corollary 2.3.9 and Proposition 2.3.5, we have $0 \leq e_j^* = \beta_j(e_{-j}^*; \theta_j) < h(t_j, V(t_j))$. (2.3.10) implies $u_j(\theta_j, e^*) \geq u_j(\theta_j, b, e_{-j}^*)$ for every $b \in \bar{\Re}_+$. Therefore, $u_j(\theta_j, e^*) \geq u_j(\theta_j, b, e_{-j}^*)$ for every $b \in [0, h(t_j, V(t_j))]$. Consequently, e^* is a Nash equilibrium of $G(\theta)$.

Conversely, suppose e^* is a Nash equilibrium of $G(\theta)$. Then, for every $j \in N$, $e_j^* \in [0, h(t_j, V(t_j))]$ and $u_j(\theta_j, e^*) \geq u_j(\theta_j, b, e_{-j}^*)$ for every $b \in [0, h(t_j, V(t_j))]$. As $u_j(\theta_j, ., e_{-j}^*)$ is strictly concave on $[0, h(t_j, V(t_j))]$, $u_j(\theta_j, e^*) > u_j(\theta_j, b, e_{-j}^*)$ for every $b \in [0, h(t_j, V(t_j))] - \{e_j^*\}$. (2.3.10) implies $\beta_j(e_{-j}^*; \theta_j) \subset [0, h(t_j, V(t_j))]$. Thus, if

$b' \in \bar{\Re}_+ - [0, h(t_j, V(t_j))]$, then there exists $b \in [0, h(t_j, V(t_j))]$, such that $u_j(\theta_j, b, e^*_{-j}) > u_j(\theta_j, b', e^*_{-j})$. Consequently, $u_j(\theta_j, e^*) > u_j(\theta_j, b', e^*_{-j})$. Therefore, $e^*_j = \beta_j(e^*_{-j}; \theta_j)$ for every $j \in N$. Thus, e^* is a stationary point of (2.3.4). ■

Proof of Proposition 2.3.14. Consider the game $G(\theta)$. By Assumption 2.2.1(a), each player's strategy set is a nonempty, compact and convex subset of $\Re$. Proposition 2.2.2(B) and Assumption 2.3.1(b) imply that each player's payoff function is continuous. Proposition 2.2.2(F) and Assumption 2.3.1(b) imply that each player's payoff is concave in his own strategy. Thus, by Nash's existence theorem, $G(\theta)$ has a Nash equilibrium, which is a stationary point of (2.3.4) by Proposition 2.3.13. ■

Proof of Proposition 2.3.18. Fix $i \in N$ and a stationary point e of (2.3.4). Let $R_j = [e_j, h(t_j, V(t_j))]$ for $j \in N$ and let $R = \prod_{j \in N} R_j$. Corollary 2.3.11 ensures that $\operatorname{Int} R \neq \emptyset$. Given $x \in R - \{e\}$, define $e^j = (e_1, \dots, e_j)$ and $x^j = (x_j, \dots, x_n)$ for $j = 1, \dots, n$; formally define $(e^0, x^1) = x$ and $(e^n, x^{n+1}) = e$.

Consider $x \in R$ such that $x \neq (h(t_j, V(t_j)))_{j \in N}$ and $x \neq e$. Assumption 2.3.1(b) implies $D_{e_k} u_i(\theta_i, x) = -D_{e_+} \delta(k_i, x_+) < 0$ for every $k \in N - \{i\}$. Corollary 2.3.9 and Assumption 2.3.1(b) imply that

$$
\begin{aligned}
D_{e_i} u_i(\theta_i, x) &= D_{e_i} u_i(\theta_i, e) + \sum_{k=1}^{n} \left[D_{e_i} u_i(\theta_i, e^{k-1}, x^k) - D_{e_i} u_i(\theta_i, e^k, x^{k+1}) \right] \\
&= D_{e_i} u_i(\theta_i, e) + \sum_{k=1}^{n} \int_{e_k}^{x_k} dy\, D_{e_i e_k} u_i(\theta_i, e^{k-1}, y, x^{k+1}) \\
&< D_{e_i} u_i(\theta_i, e) \\
&= 0
\end{aligned}
$$

Given $z \in R - \{e\}$, it follows that

$$
\begin{aligned}
u_i(\theta_i, z) - u_i(\theta_i, e) &= \sum_{k=1}^{n} \left[u_i(\theta_i, e^{k-1}, z^k) - u_i(\theta_i, e^k, z^{k+1}) \right] \\
&= \sum_{k=1}^{n} \int_{e_k}^{z_k} dy\, D_{e_k} u_i(\theta_i, e^{k-1}, y, z^{k+1}) \\
&< 0
\end{aligned}
$$

as required. ■

Proof of Proposition 3.1.2. (A) Consider $(t, e) \in \Re^2_{++}$ such that $e < h(t, V(t))$. It follows from Proposition 2.2.2(D) that $v = v(t, e)$ maximizes $g(t, v)$ subject to the constraint

$h(t,v)=e$. Thus, there exists $\lambda(t,e)\in\Re$ such that $D_v g(t,v(t,e))+\lambda(t,e)D_v h(t,v(t,e))=0$ and $h(t,v(t,e))-e=0$. Assumption 3.1.1(c) implies that the mapping

$$(t,e,v,\lambda)\mapsto(D_v g(t,v)+\lambda D_v h(t,v),h(t,v)-e)$$

is twice continuously differentiable. As Assumption 2.2.1(c) implies that the matrix

$$\begin{pmatrix} D_{vv}g(t,v(t,e))+\lambda(t,e)D_{vv}h(t,v(t,e)) & D_v h(t,v(t,e)) \\ D_v h(t,v(t,e)) & 0 \end{pmatrix}$$

is non-singular, the implicit function theorem (Lang 1993, XIV, Theorem 2.1) guarantees the existence of $\mathcal{C}^2$ functions $(t',e')\mapsto v(t',e')$ and $(t',e')\mapsto\lambda(t',e')$ for some open neighborhood of (t,e). It follows immediately that f is $\mathcal{C}^2$ on $\{(t,e)\in\Re^2_{++}\mid e<h(t,V(t))\}$.

(B) Suppose $e,t,t'\in\Re_+$, $t<t'$, $e\le h(t,V(t))$ and $e\le h(t',V(t'))$. Proposition 2.2.2(D) and Assumption 3.1.1(a) imply $h(t,v(t,e))=e=h(t',v(t',e))=h(t+t'-t,v(t',e))<h(t,v(t',e))$. Assumption 2.2.1(c) implies $v(t,e)<v(t',e)$ and the definition of v implies $v(t',e)\le V(t')$. Therefore, Assumptions 3.1.1(a) and 2.2.1(b) imply $f(t,e)=g(t,v(t,e))<g(t',v(t,e))<g(t',v(t',e))=f(t',e)$.

(C) This follows from Proposition 3.1.2(A) and Assumption 3.1.1(c). ■

Proof of Proposition 3.1.5. We simplify notation *via* the following conventions. As θ is given, we shall suppress it in all expressions. By convention: $f(t_j,.)\equiv f_j$ and $\delta(k_j,.)\equiv\delta_j$. We start with two preliminary steps, (α) and (β), before proving (A) and (B).

(α) Differentiating (3.1.3) with respect to x yields the equation $AD_x e=b$, where A and b are the $n\times n$ and $n\times 1$ matrices given by

$$A=\begin{bmatrix} D_{e_1e_1}u_1 & \dots & D_{e_1e_n}u_1 \\ \vdots & \ddots & \vdots \\ D_{e_ne_1}u_n & \dots & D_{e_ne_n}u_n \end{bmatrix} \qquad \text{and} \qquad b=\begin{bmatrix} -D_{e_1x}u_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Let A_j be the $(n-1)\times(n-1)$ matrix derived from A by eliminating the first row and the j-th column. Using Cramer's rule, we have

$$D_x e_j=\frac{(-1)^j D_{e_1x}u_1\det A_j}{\det A} \qquad (A.2)$$

(2.3.8) implies that, for every $i\in N$,

$$D_{e_ie_j}u_i=\begin{cases} D_{e_ie_i}f_i-D_{e_+e_+}\delta_i, & \text{if } j=i \\ -D_{e_+e_+}\delta_i, & \text{if } j\in N-\{i\} \end{cases}$$

We first evaluate $\det A_j$ for $j \in N - \{1\}$. Subtracting the first column of A_j from every other column yields $\det A_j = \det B_j$, where

$$B_j = \begin{bmatrix} -D_{e_+e_+}\delta_2 & D_{e_2e_2}f_2 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -D_{e_+e_+}\delta_{j-1} & 0 & \dots & D_{e_{j-1}e_{j-1}}f_{j-1} & 0 & \dots & 0 \\ -D_{e_+e_+}\delta_j & 0 & \dots & 0 & 0 & \dots & 0 \\ -D_{e_+e_+}\delta_{j+1} & 0 & \dots & 0 & D_{e_{j+1}e_{j+1}}f_{j+1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -D_{e_+e_+}\delta_n & 0 & \dots & 0 & 0 & \dots & D_{e_ne_n}f_n \end{bmatrix}$$

By a sequence of $j-2$ adjacent column interchanges, we can transform B_j into

$$C_j = \begin{bmatrix} D_{e_2e_2}f_2 & \dots & 0 & -D_{e_+e_+}\delta_2 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & D_{e_{j-1}e_{j-1}}f_{j-1} & -D_{e_+e_+}\delta_{j-1} & 0 & \dots & 0 \\ 0 & \dots & 0 & -D_{e_+e_+}\delta_j & 0 & \dots & 0 \\ 0 & \dots & 0 & -D_{e_+e_+}\delta_{j+1} & D_{e_{j+1}e_{j+1}}f_{j+1} & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & -D_{e_+e_+}\delta_n & 0 & \dots & D_{e_ne_n}f_n \end{bmatrix}$$

As interchanging adjacent columns changes the sign of a determinant, we have

$$\det A_j = \det B_j = (-1)^{j-2}\det C_j = (-1)^{j-1}\frac{D_{e_+e_+}\delta_j}{D_{e_1e_1}f_1 D_{e_je_j}f_j}\prod_{k=1}^{n} D_{e_ke_k}f_k$$

Thus,

$$D_x e_j = \left[\frac{(-1)^{2j-1}\prod_{k=1}^{n} D_{e_ke_k}f_k}{\det A}\right]\frac{D_{e_1x}u_1 D_{e_+e_+}\delta_j}{D_{e_1e_1}f_1 D_{e_je_j}f_j} \tag{A.3}$$

(β) Although the following arguments are standard, we provide them for the sake of completeness.

By the fundamental theorem of algebra (Markushevich 1965, Theorem 17.7), A has n roots. As A is real, its characteristic polynomial has real coefficients. Therefore, the conjugate of every complex root of A with multiplicity m is also a root of A with multiplicity m. Therefore, the product of all complex roots is positive. As A is similar to its Jordan canonical form (Gantmacher 1990, Section VI.6.3), $\det A$ equals the product of its roots. (2.3.16) implies that A is a row dominant diagonal matrix. Corollary 2.3.9 implies that A has a negative diagonal.

Suppose λ is a root of A with a non-negative real part. Let $|c|$ denote the modulus of a complex number c. As $a_{ii} < 0$ for every $i \in N$, $|a_{ii} - \lambda| \geq |a_{ii}| > \sum_{j \in N-\{i\}} |a_{ij}|$ for every $i \in N$. This implies $H = A - \lambda I$ is an $n \times n$ complex matrix with a row dominant diagonal. H is singular as λ is a root of A. Thus, there exists a complex n-tuple $x \neq 0$ such that $Hx = 0$, which implies $h_{ii}x_i + \sum_{j \in N-\{i\}} h_{ij}x_j = 0$ for every $i \in N$. The triangle inequality implies

$$|h_{ii}||x_i| = |h_{ii}x_i| = \left| \sum_{j \in N-\{i\}} h_{ij}x_j \right| \leq \sum_{j \in N-\{i\}} |h_{ij}x_j| = \sum_{j \in N-\{i\}} |h_{ij}||x_j|$$

Let $k \in N$ be such that $|x_k| \geq |x_j|$ for every $j \in N$. It follows that $|h_{kk}||x_k| \leq \sum_{j \in N-\{k\}} |h_{kj}||x_k| = |x_k| \sum_{j \in N-\{k\}} |h_{kj}|$. As $x \neq 0$, $|x_k| > 0$. Therefore, $|h_{kk}| \leq \sum_{j \neq k} |h_{kj}|$, a contradiction. Thus, all the roots of A have negative real parts.

It follows that $\det A > 0$ if and only if n is even. By copying this argument, we have $\det A_1 < 0$ if and only if n is even.

(A) Proposition 2.2.2(F) implies $D_{e_k e_k} f_k < 0$ for every $k \in N$. This and (β) imply

$$\frac{(-1)^{2j-1} \prod_{k=1}^{n} D_{e_k e_k} f_k}{\det A} < 0$$

for every n and $j \in N - \{1\}$. The result follows from ($A.3$) and Assumption 2.3.1(b).

(B) follows from ($A.2$) and (β).

(C) It follows from ($A.2$) that

$$D_x \sum_{j \in N} e_j = \frac{D_{e_1 x} u_1}{\det A} \sum_{j \in N} (-1)^j \det A_j = \frac{D_{e_1 x} u_1}{\det A} \det K$$

where

$$K = \begin{bmatrix} -1 & \dots & -1 \\ D_{e_2 e_1} u_2 & \dots & D_{e_2 e_n} u_2 \\ \vdots & \ddots & \vdots \\ D_{e_n e_1} u_n & \dots & D_{e_n e_n} u_n \end{bmatrix}$$

Subtracting the first column of K from every other column yields $\det K = \det L$, where

$$L = \begin{bmatrix} -1 & 0 & \dots & 0 \\ -D_{e_+ e_+} \delta_2 & D_{e_2 e_2} f_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -D_{e_+ e_+} \delta_n & 0 & \dots & D_{e_n e_n} f_n \end{bmatrix}$$

Thus, $D_x \sum_{j \in N} e_j = D_{e_1 x} u_1 \det L / \det A = -D_{e_1 x} u_1 \prod_{k=2}^{n} D_{e_k e_k} f_k / \det A$. Note that $\prod_{k=2}^{n} D_{e_k e_k} f_k < 0$ if and only if n is even. The result follows from (β). ■

Proof of Corollary 3.2.3. Let $k_1 > k_2$. Assumptions 2.3.1(b) and 3.1.1(d) imply $0 < D_{e_+} \delta(k_1, e_+(\theta)) < D_{e_+} \delta(k_2, e_+(\theta))$. This and (3.1.3) implies $\int_{e_2(\theta)}^{e_1(\theta)} dx\, D_{ee} f(t,x) = D_e f(t, e_1(\theta)) - D_e f(t, e_2(\theta)) < 0$. Proposition 2.2.2(F) implies $e_1(\theta) > e_2(\theta)$. ■

Proof of Corollary 3.2.4. Suppose $t_1 > t_2$, $D_{te} f(t_1, e_1(\theta)) > 0$ and $D_{te} f(t_2, e_2(\theta)) > 0$. Assumption (c) implies $D_{te} f(x, e_1(\theta)) > D_{te} f(t_1, e_1(\theta)) > 0$ for every $x \in [t_2, t_1]$. (3.1.3) implies $D_e f(t_1, e_1(\theta)) = D_e f(t_2, e_2(\theta))$. Therefore,

$$\begin{aligned}
\int_{e_1(\theta)}^{e_2(\theta)} dy\, D_{ee} f(t_2, y) &= D_e f(t_2, e_2(\theta)) - D_e f(t_2, e_1(\theta)) \\
&= D_e f(t_1, e_1(\theta)) - D_e f(t_2, e_1(\theta)) \\
&= \int_{t_2}^{t_1} dx\, D_{te} f(x, e_1(\theta)) \\
&> 0
\end{aligned}$$

Proposition 2.2.2(F) implies $e_1(\theta) > e_2(\theta)$. The other case follows analogously. ■

Notes

1. Our aim is to work out the consequences of the regime modelled in this paper, not to rationalize any actual environmental protocol. While the number of actual protocols is very large, most are merely exhortative. Improving on this situation requires us to understand, *ex ante*, the consequences of different strategies for constructing a protocol. In this paper, we analyze one version of the quantity-rationing strategy.

2. We should warn the reader that the "national firm" is not state-controlled. It is a standard profit-maximizing firm. A regulatory scheme determines the allocation of emission rights implied by an emission cap. This allocation determines each firm's constrained production set. By aggregating the constrained production sets, we may derive the national firm's emission-cap-constrained production set.

3. We identify "reached" with asymptotic convergence.

4. As a result, in parts of the literature (e.g., the cooperative game approach), the *status quo* emission choices are modelled as a Nash equilibrium of a non-cooperative game played among the nations. This is not a compelling description of the *status quo* as the emission decision is made by emitters rather than by the state. It is doubtful that, in the absence of an explicit capping protocol, domestic regulators attempt, much less succeed in, the alignment of domestic emitters' incentives with those of the state so as to force them to take into account the international emission externality. At best, *status quo* regulation is designed to force emitters to take into account domestic localized externalities, rather than international ones.

5. Apart from d'Aspremont et al. (1983), this approach has a formal affinity to the literature on stable cartels (Donsimoni et al. 1986), and an informal affinity with the literature on coalition formation (e.g., Aumann and Dreze 1983, Bernheim et al. 1987, Hart and Kurz 1983, Ray and Vohra 1997).

6. We are implicitly assuming in (2.3.4) that the best response mapping is single-valued, i.e., a function. This property will be derived below in Corollary 2.3.9.

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