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Pareto-improving intergenerational transfers

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Abstract

In the presence of endogenous growth intergenerational transfer from the young to the old reduce per capita income growth and harm future generations. On the other hand, competitive equilibria are inefficient if externalities sustain long-run growth. This paper shows that if individuals retire in the last period of their life, the inefficiency of the market economy can be removed by an investment subsidy without making the current or future generations worse off only if coupled with intergenerational transfers from the young to the old.

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1. Introduction

Standard endogenous growth economies are dynamically efficient in the sense that there is no other feasible growth path which provides at least as much consumption in every period and more in some periods [see Saint-Paul (1992) and King and Ferguson (1993)]. This result suggests that in endogenous growth settings intergenerational transfers from the young to the old can hardly be based on efficiency grounds since overinvestment cannot occur. By reducing aggregate investment, transfers from the young to the old lower the rate of per capita income growth and, henceforth, negatively affect the welfare of future generations.

There is another important result in the theory of endogenous growth stating that the aggregate technology in the one sector overlapping generations model with finite individual lifetimes must be non-convex to generate endogenous long-run per capita income growth [see Boldrin (1992) and Jones and Manuelli (1992)]. For this reason, the literature on growth with overlapping generations has widely confined attention to the case of non-convex technologies. In order to be consistent with the competitive equilibrium concept, the non-convexity is typically modeled by introducing some externality which emanates from aggregate investment in physical or human capital or the stock of technological knowledge on labour productivity. Since these effects are not priced, the market economy is Pareto-inefficient.

Clearly, the inefficiency of the market economy is a static one as is illustrated in Fig. 1. It contains the Pareto-frontier of an endogenous growth economy, where U is the welfare of the current generation and V is a discounted sum of the welfare of all future generations. The competitive equilibrium is given by point A which is characterized by static inefficiency and, henceforth, strictly inside the Pareto-frontier. The inefficiency of the market economy can be removed by means of a Pigouvian investment subsidy. However, as shown in this paper, if individuals retire when old, such a policy can exploit all possible efficiency gains without making the current generation worse off, if and only if the policy is accompanied by intergenerational transfers from the working population to the old. Removing the inefficiency without employing intergenerational transfers, only allocations in Fig. 1 can be reached that are located to the south-east of A, say, for instance, B, implying that the current generation is made worse off.

Here seems to lurk a paradox since intergenerational transfers from the young to the old as such are harmful for growth. However, the underlying intuition is very simple. The subsidy spurs private investment in productive resources. This trans-

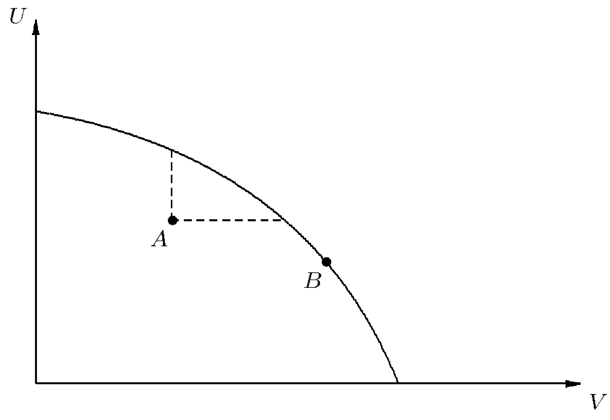

Fig. 1. Static inefficiency of the market economy

lates into higher labour productivity that benefits future working generations. If the financing of the investment subsidy does not rely on intergenerational transfers from the young to the old, the individuals who trigger the increase in productivity by investing more than they would do otherwise, cannot be compensated by the individuals who enjoy the productivity increase in the form of higher wages.

The result is first derived employing a simple two-period overlapping generations model in which a spillover from capital formation on labour productivity as emphasized by Arrow (1962) and Romer (1986) sustains long-run growth. In this framework no Pareto-improvement is possible without intergenerational transfers if individuals retire in their second period of life. Subsequently, it is shown that a Pareto-improvement can be achieved without relying on intergenerational transfers if the working life takes more than one period. However, not all efficiency gains can be exploited without intergenerational transfers if individuals retire when old. Thereafter, it is demonstrated that a case for intergenerational transfers can also be made in other models of endogenous growth such as the endogenous innovation and the human capital formation model.

2. The basic model

2.1. Individuals

The economy consists of overlapping generations of the Diamond (1965) type in which individuals live for two periods. When young, they inelastically supply one unit of labour in the labour market, consume, and save for old age. When old, they retire and consume the proceeds of their savings.

Preferences of a representative member of the generation born at time t, denoted as generation t, are given by $u_t = u(c_t^y, c_{t+1}^o)$, where c_t^y and c_{t+1}^o are the amounts of young- and old-age consumption, and u is a twice continuously differentiable utility function satisfying the standard monotonicity, concavity and Inada assumptions. Young- and old-age consumption are constrained by:

$$c_t^y = w_t - s_t - \tau_t, \tag{1}$$

$$c_{t+1}^o = (1 + i_{t+1})\, s_t - \theta_{t+1}, \tag{2}$$

where w_t and s_t are the wage rate and the amount of savings at time t, and i_{t+1} is the interest rate at time $t+1$. In both periods of life individuals are subject to taxation, where τ_t and θ_{t+1} are lump sum taxes that an individual born at time t has to pay in his first and second period of life. The first-order condition for maximum utility is:

$$-u_{1,t} + (1 + i_{t+1})\, u_{2,t} = 0, \tag{3}$$

where $u_{1,t}$ and $u_{2,t}$ are the partial derivatives of u at (c_t^y, c_{t+1}^o). Equation (3) implicitly defines savings as a function of first and second period disposable income and the interest rate:

$$s_t = s(w_t - \tau_t, -\theta_{t+1}, i_{t+1}). \tag{4}$$

Assuming that young- and old-age consumption are both normal goods, it follows $s_{1,t} > 0$ and $s_{2,t} < 0$. The impact of an increase in the interest rate on savings, in contrast, is generally ambiguous since it triggers both an income and a substitution effect which are of different sign under the normality assumption.

2.2. Firms

Firms hire the available labour force which equals the size of the young generation, given by N_t at time t, and the aggregate capital stock, K_t, to produce the homogeneous output $Y_t = F(K_t, A_t N_t)$, where the technology F exhibits constant returns to scale. The index A_t measures labour productivity at time t which each single firm takes as given. Normalizing the size of each generation to one, marginal product pricing leads to the following prices for capital and labour:

$$r_t = f'(k_t), \tag{5}$$

$$w_t = A_t\,[f(k_t) - k_t\, f'(k_t)], \tag{6}$$

where $k_t \equiv K_t/A_t$ and $f(k_t) \equiv F(K_t/A_t, 1)$. To endogenize A_t, a positive spillover from cumulated aggregate investment on labour productivity as suggested by Arrow (1962) and Romer (1986) is considered. A very simple form of the Arrow-Romer model consistent with endogenous long-run growth is the following:[1]

$$A_t = \frac{1}{a} K_t, \tag{7}$$

where a is a positive technological parameter. Substituting (7) into (5) and (6) yields:

$$r_t = r \equiv f'(a), \tag{8}$$

$$w_t = \omega\, K_t, \quad \text{with } \omega \equiv [f(a) - af'(a)]/a. \tag{9}$$

Thus, the price for capital is constant over time and the wage rate is proportional to the capital stock. Note that the factor of proportionality ω represents the external return on capital caused by the spillover from cumulated investment on labour productivity. Because of the positive externality of investments on labour productivity, the interest rate differs from the social marginal return on capital which is given by $dY_t/dK_t = r + \omega$.

2.3. Competitive equilibrium without government activity

If the government does not play any active economic role ($\tau_t = \theta_{t+1} = 0$), the competitive equilibrium is determined by the savings function (4), the factor price conditions (8) and (9), a product market equilibrium condition requiring that aggregate investment and aggregate savings are equalized:

$$K_{t+1} = s_t, \tag{10}$$

and a no arbitrage condition, implying that the interest rate equals the private

[1] In fact, the linear specification of the Arrow-Romer model is now standard in the endogenous growth literature. See, e.g., Saint-Paul (1992), Grossman and Yanagawa (1993), and King and Ferguson (1993).

rate of return on capital:

$$i_{t+1} = r_{t+1}. \tag{11}$$

2.4. A subsidy to private savings

Consider now a subsidy to private savings paid by the government in order to internalize the external effect of cumulated investment on labour productivity. The government announces at time t that it will pay an amount σ in addition to the private rate of return on savings at time $t+1$. The no arbitrage condition then becomes:

$$i_{t+1} = r_{t+1} + \sigma. \tag{12}$$

2.5. Subsidy financing without intergenerational transfers

If the subsidy is financed without intergenerational transfers, one can distinguish two polar cases: Either the old at time $t+1$ pay a tax and its revenues are used to pay the savings subsidy in the same period, or the young at time t pay a tax whose revenues are invested in the capital market and distributed in the form of the savings subsidy in the next period to the then old.

If the subsidy is financed by a tax imposed on the old at time $t+1$, the government budget constraint reads:

$$\theta_{t+1} = \sigma\, s_t, \tag{13}$$

and the competitive equilibrium is implicitly defined by equations (4), (8), (9), (10), (12), and (13).

If, on the other hand, the subsidy is financed by a tax imposed on the young at time t, the latter is given by the present discounted value of the savings subsidy:

$$\tau_t = \frac{\sigma\, s_t}{1 + r_{t+1}}, \tag{14}$$

where it has been considered that the government as an investor in the capital market yields the market return r_{t+1} at time $t+1$. Because of the public investment, the product market equilibrium condition becomes:

$$K_{t+1} = s_t + \tau_t, \tag{15}$$

so that the competitive equilibrium is determined by (4), (8), (9), (12), (14), and (15). Generally, a mix of the two cases considered above is also conceivable to finance the subsidy without intergenerational transfers. However, the argument is clearest in the two polar cases.

2.6. Employing intergenerational transfers

Consider now the case where the government may employ intergenerational transfers. If only part of the subsidy is financed by the generation which receives the subsidy and another part by the succeeding young, the government budget constraint becomes:

$$\theta_{t+1} + \tau_{t+1} = \sigma\, s_t, \tag{16}$$

where τ_{t+1} represents the intergenerational transfer from the young to the old distributed as a savings subsidy. The product market equilibrium condition is again given by (10). Thus, the competitive equilibrium is determined by (4), (8), (9), (10), (12), and (16).

3. Welfare analysis

Consider as a benchmark the competitive equilibrium without government intervention. A straightforward implication of the positive spillover from cumulated aggregate investment on labour productivity is that the laissez faire economy is Pareto-inefficient. In fact, a Pareto-improvement could be reached, if the young saved more.

Proposition 1. The competitive equilibrium without government intervention is Pareto-inefficient. A Pareto-improvement is possible if the young save more.

The intuition behind this result which has been formally proved by King and Ferguson (1993, pp. 95-96) is very simple. In the competitive equilibrium without government intervention individuals are willing to trade one unit of young-age for $1+r$ units of old-age consumption. If one unit of young-age consumption is added to the capital stock, however, $1+r+\omega$ units of consumption can be achieved in the next period as the additional unit of savings not only creates a private return of r but also an external return of ω. Thus, future consumption can be increased by more than it is necessary to compensate each generation for forgoing consumption when young.

In what follows it will be studied how the inefficiency of the competitive equilibrium can be removed by a savings subsidy. The analysis starts with financing schemes that do not rely on intergenerational transfers. The following lemma reveals how these schemes affect the wage rate at time $t+1$.

Lemma 1. $dw_{t+1}/d\sigma > 0$ under both financing schemes without intergenerational transfers.

Proof: See the Appendix.

The intuition behind this result is straightforward. On the aggregate level the savings subsidy policy encourages investment which translates into increased labour productivity growth. This result is obtained even though the effect of an increase in the subsidy rate σ on savings as such is ambiguous since it triggers both an income and a substitution effect of opposite sign. However, since the increase in the subsidy rate is accompanied by an increase in lump-sum taxes, the income effect of a higher savings subsidy is outweighed by the income effect of higher lump-sum taxes so that it is the substitution effect of a higher savings subsidy which drives the result.

If the scheme is based on taxes that individuals born at time t pay when old, their indirect lifetime utility may be written as:

$$v_t(\sigma) = u\left[w_t - s_t, (1+r+\sigma)\, s_t - \frac{\sigma}{\omega}\, w_{t+1}\right],$$

where equations (1), (2) and (8) have been considered and where use has been made of the fact that $\theta_{t+1} = (\sigma/\omega) w_{t+1}$, which follows by substituting (9) and (10) into (13). Now differentiate $v_t(\sigma)$ while considering the Envelope Theorem and taking into account that w_t is already predetermined at time t. After replacing

s_t by w_{t+1}/ω which follows by substituting (9) into (10), one then finds that:

$$v_t'(\sigma) = -u_{2,t}\,\frac{\sigma}{\omega}\,\frac{dw_{t+1}}{d\sigma}.$$

From Lemma 1 it is known that $dw_{t+1}/d\sigma > 0$ so that $v_t'(\sigma) < 0$. Hence, a savings subsidy financed with a tax levied on the same generation definitely reduces that generation's lifetime utility and, therefore, is not Pareto-improving.

The same is true also of the second financing scheme without intergenerational transfers. If generation t pays a tax when young whose revenues are invested in the capital market and redistributed to them when they are old, their indirect lifetime utility is given by:

$$v_t(\sigma) = u\left[w_t - s_t - \frac{1}{1+r+\sigma}\,\frac{\sigma}{\omega}\,w_{t+1}, (1+r+\sigma)\,s_t\right],$$

where again (1), (2), and (8) have been considered and where τ_t has been replaced employing equations (8), (9), (14), and (15). Differentiating with respect to σ while again employing the Envelope Theorem, replacing s_t by $(1+r/1+r+\sigma)(w_{t+1}/\omega)$ which follows by substituting (8), (9) and (14) into (15), and then considering (3) yields after some manipulations:

$$v_t'(\sigma) = -u_{2,t}\,\frac{\sigma}{\omega}\,\frac{dw_{t+1}}{d\sigma}.$$

Thus, also under the second financing scheme without intergenerational transfers the savings subsidy cannot be Pareto-improving since generation t again suffers a utility loss. The underlying economic mechanism is the same in both cases. Marginally increasing the savings subsidy leads to additional savings of generation t. The increase in savings causes a higher tax burden which exerts a negative first-order effect on lifetime utility of generation t. Furthermore, higher savings trigger a positive labour productivity effect which benefits generation $t+1$ in the form of higher wages. Since intergenerational transfers are excluded by assumption, generation t cannot be compensated by generation $t+1$ and, consequently, is worse off.

In order to complete the argument, it will now be shown that a Pareto-improvement can be obtained if transfers from the young to the old are employed. First, it will be computed which share of the subsidy financing burden can be devoted to individuals born at time t without making them worse off, i.e. it will be asked which share can be financed without intergenerational transfers. The indirect lifetime utility function of generation t in case of the third financing scheme reads:

$$v_t(\sigma) = u[w_t - s_t, (1 + r + \sigma)\, s_t - \theta_{t+1}].$$

Differentiating with respect to σ while again employing the Envelope Theorem, one finds that a marginal increase in the savings subsidy leaves lifetime utility of generation t unchanged if:

$$s_t = \frac{d\theta_{t+1}}{d\sigma}. \tag{17}$$

If generation t pays a tax when old that is consistent with (17) and generation $t + 1$ pays the remainder in order to meet the government budget constraint as defined by (16), this again causes an increase in the wage rate at time $t + 1$, as the following lemma states.

Lemma 2. $dw_{t+1}/d\sigma > 0$ under the financing scheme with intergenerational transfers.

Proof: See the Appendix.

Considering this result it can easily be shown that generation $t + 1$ is made better off. Substituting (9) and (10) into (16), one gets:

$$\frac{\sigma}{\omega}\, w_{t+1} = \theta_{t+1} + \tau_{t+1}.$$

Differentiating with respect to σ and rearranging terms yields:

$$\frac{d\theta_{t+1}}{d\sigma} = \frac{1}{\omega}\, w_{t+1} + \frac{\sigma}{\omega}\, \frac{dw_{t+1}}{d\sigma} - \frac{d\tau_{t+1}}{d\sigma}.$$

Substituting this expression into (17) and considering that savings are again determined by $s_t = w_{t+1}/\omega$, one obtains:

$$\frac{d\tau_{t+1}}{d\sigma} = \frac{\sigma}{\omega}\, \frac{dw_{t+1}}{d\sigma}. \tag{18}$$

Now assume, for a moment, that the savings subsidy is only paid at time $t{+}1$. Then generation $t + 1$ is made better off by a marginal increase in σ if its disposable income when young, given by $w_{t+1} - \tau_{t+1}$, increases. Differentiating the latter expression with respect to σ, then substituting for $d\tau_{t+1}/d\sigma$ by employing (18), and finally considering Lemma 2, it follows that this indeed is the case as long as $\sigma < \omega$. In fact, generation $t+1$ is made best off if the savings subsidy equals the external return on capital, i.e. if $\sigma = \omega$. In this case generation t fully internalizes the social return on private savings and pushes labour productivity of generation $t+1$ to its socially optimal level.

What about generation $t+2$? If the savings subsidy is only paid at time $t+1$, first period disposable income of generation $t+2$ equals the wage rate w_{t+2}. Combining updates of equations (4), (8), (9) and (10), w_{t+2} may be written as:

$$w_{t+2} = \omega\, s(w_{t+1} - \tau_{t+1}, 0, r).$$

Since $w_{t+1} - \tau_{t+1}$ is increasing in σ for all $\sigma < \omega$ and since young-age consumption is a normal good, it follows that $dw_{t+2}/d\sigma > 0$ for all $\sigma < \omega$. This implies that also generation $t+2$ is made better off. By induction it then follows that paying a savings subsidy at rate $\sigma \leq \omega$, which is partly financed by intergenerational transfers from the young to the old, does not harm generation t and makes all subsequent generations better off. Of course, the savings subsidy should not only be paid at time $t+1$ but also in all subsequent periods.

The discussion so far has demonstrated that in the two-period model with retirement intergenerational transfers from the young to the old are necessary in order to achieve a Pareto-improvement. Only then those generations which induce a higher labour productivity of subsequent generations by saving more than they would choose to do otherwise can be sufficiently compensated. The following proposition summarizes this result.

Proposition 2. In the two-period model with retirement a Pareto-improvement can be decentralized as a competitive equilibrium by subsidizing private savings at rate $\sigma \leq \omega$ if and only if the subsidy is coupled with intergenerational transfers from the young to the old.

4. Length of working life and retirement

The results derived so far rest on the assumption that the working life of each generation is so short that the generation targeted by the subsidy does not benefit from the productivity gain spurred by the subsidy. The present section qualifies this result in two respects. First, a two-period overlapping generations economy is considered in which individuals work in both periods of life. It is demonstrated that in this economy a first best allocation which is Pareto-superior to the initial laissez faire equilibrium can be achieved without employing intergenerational transfers.[2] Second, a three-period overlapping generations economy is considered in which individuals work only in the first two periods of life. In this economy a Pareto-improvement is possible without intergenerational transfers. However, coupling a savings subsidy policy with intergenerational transfers allows the policy maker to achieve further efficiency gains relative to the scenario in which no such transfers are allowed without reducing the welfare of any generation.

4.1. Two periods, no retirement

In this section a savings subsidy is considered which is financed by a tax imposed on the old. This implies that the subsidy is not accompanied by intergenerational transfers. If individuals work in both periods of their life, savings of generation t become:

$$s_t = s[w_t, w_{t+1} - \theta_{t+1}, r + \sigma],$$

where θ_{t+1} meets the constraint $\theta_{t+1} = \sigma\, s_t$. Since now at each point in time two generations belong to the labour force, the total size of the latter is given by 2 if the size of each generation is again normalized to one. Therefore, the interest rate becomes $r = f'(a/2)$, the external return on capital becomes $\omega = [f(a/2) - (a/2)\, f'(a/2)]/(a/2)$, and the wage rate at time t becomes $w_t = (\omega/2)\, K_t$. Without retirement generation t can enjoy some of the gains from higher productivity triggered by the subsidy since its second period labour income, w_{t+1}, increases. It turns out that even though generation t cannot fully appropriate the productivity gains from the subsidy as also the next generation enjoys a higher wage (since both generations are equal in size, each generation gets half of the external return ω), the productivity gain which benefits generation t is sufficiently large to compensate for the burden of subsidy financing. In fact, as the next proposition states, no intergenerational transfers are needed to achieve a Pareto-improvement by a savings subsidy. Moreover, if the utility function satisfies some further re-

[2] In Fig. 1 of Section 1 a first best allocation which is Pareto-superior to the initial laissez faire equilibrium is characterized by a point on the part of the Pareto-frontier which is located to the north-east of point A.

strictions, it is possible to exploit all efficiency gains in a Pareto-improving way without making use of intergenerational transfers.

Proposition 3. In the two-period model without retirement

i) a Pareto-improvement can be decentralized as a competitive equilibrium by subsidizing private savings at rate $\sigma \leq \omega/2$. Such intervention does not require intergenerational transfers from the young to the old.

ii) a first best allocation which is Pareto-superior to the initial laissez faire equilibrium can be decentralized by subsidizing private savings at rate $\sigma = \omega$. Such intervention does not require intergenerational transfers if the cross derivative of the utility function is non-negative and private savings are weakly concave with respect to the subsidy rate.

Proof: See the Appendix.

As long as the subsidy rate σ is smaller than $\omega/2$, a further increase in σ benefits generation t as well as all future generations. However, the subsidy rate is at the socially optimal level only if it equals the external return on capital, i.e. only if $\sigma = \omega$. Yet, once the subsidy rate equals $\omega/2$, any further increase in σ marginally decreases the welfare of generation t. In order to achieve a first best allocation which is Pareto-superior to the initial laissez faire equilibrium without employing intergenerational transfers, one has to take care that generation t is at least as well off for $\sigma = \omega$ as for $\sigma = 0$. It turns out that this is the case if an increase in old-age consumption does not reduce marginal utility of young-age consumption and the encouraging effect of the subsidy on savings grows weaker at high levels of subsidization. Both conditions limit the extent to which savings increase due to the subsidy policy and, henceforth, dampen the distortionary effect of the subsidy on the welfare of generation t.

4.2. Three periods, retirement

Consider next an economy in which individuals live for three periods. In their first two periods of life, i.e. when young or middle-aged, they work and in their third period of life, i.e. when old, they are retired. Utility of a member of generation t is now given by $u_t = u(c_t^y, c_{t+1}^m, c_{t+2}^o)$, where:

$$c_t^y = w_t - s_t^y,$$
$$c_{t+1}^m = w_{t+1} + (1 + r + \sigma_{t+1}^m)\, s_t^y - s_{t+1}^m - \theta_{t+1}^m,$$
$$c_{t+2}^o = (1 + r + \sigma_{t+2}^o)\, s_{t+1}^m - \theta_{t+2}^o,$$

define young-, middle- and old-age consumption which are again assumed to be normal. On his young-age savings, s_t^y, the individual receives a subsidy at the rate σ_{t+1}^m when middle-aged and on his middle-age savings, s_{t+1}^m, he receives a subsidy at the rate σ_{t+2}^o when old. Without intergenerational transfers the individual himself bears the full financial burden of the savings subsidy, given by θ_{t+1}^m when middle-aged and θ_{t+2}^o when old. Distinguishing between both subsidy rates at different points in time and subsidy rates for the middle-aged and the old admittedly inflates the notation. However, it facilitates separating the effects of savings subsidy policies on different generations at certain points in time.

The first-order conditions for maximum utility now read:

$$-\, u_{1,t} + (1 + r + \sigma_{t+1}^m)\, u_{2,t} = 0, \tag{19}$$
$$-\, u_{2,t} + (1 + r + \sigma_{t+2}^o)\, u_{3,t} = 0. \tag{20}$$

These two equations implicitly define young- and middle-age savings as functions of young-, middle- and old-age disposable income and the returns on young- and middle-age savings:

$$s_t^y = s^y[w_t, w_{t+1} - \theta_{t+1}^m, -\theta_{t+2}^o, r + \sigma_{t+1}^m, r + \sigma_{t+2}^o], \tag{21}$$
$$s_{t+1}^m = s^m[w_t, w_{t+1} - \theta_{t+1}^m, -\theta_{t+2}^o, r + \sigma_{t+1}^m, r + \sigma_{t+2}^o]. \tag{22}$$

The government budget constraints at time $t+1$ are given by:

$$\theta_{t+1}^m = \sigma_{t+1}^m\, s_t^y, \tag{23}$$
$$\theta_{t+1}^o = \sigma_{t+1}^o\, s_t^m, \tag{24}$$

and the product market equilibrium condition reads:

$$s_t^y + s_t^m = K_{t+1},$$

as aggregate savings at time t are now given by young-age savings of generation t and middle-age savings of generation $t-1$. Considering that $K_{t+1} = 2\, w_{t+1}/\omega$ if the young and the middle-aged work, the product market equilibrium condition becomes:

$$w_{t+1} = \frac{\omega}{2}\,(s_t^y + s_t^m), \tag{25}$$

which implicitly defines the sequence of equilibrium wage rates for a given policy of savings subsidization.

Considering the results of Section 4.1, it is obvious that introducing a savings subsidy on young-age savings of generation t, i.e. augmenting the return on generation t's young-age savings, s_t^y, by an amount $\sigma_{t+1}^m \leq \omega/2$, leads to a Pareto-improvement. Generation t is still working at time $t+1$ and sufficiently benefits from the increase in the wage rate, all future generations do also benefit from higher labour productivity, and, finally, generation $t-1$ is not affected by the savings subsidy as it neither participates in its financing nor in the resulting productivity gains.

However, given the positive externality from cumulated investment on labour productivity, also generation $t-1$'s middle-age savings, s_t^m, is inefficiently low. Thus, a policy of savings subsidization should also affect savings of the current middle-aged in order to exploit all possible efficiency gains. Yet, introducing a subsidy on middle-age savings of generation $t-1$, given by σ_{t+1}^o, will not benefit them in the form of higher wages as they will be retired at time $t+1$. In fact, an argument for intergenerational transfers similar to the one set forth in Section 3 can be made in the three-period model if there is retirement. As the second part of the next proposition states, introducing a savings subsidy policy that exploits all possible efficiency gains without making any generation worse off is possible if and only if there are intergenerational transfers from the working generation to the old.

Proposition 4. In the three-period model with retirement

i) a Pareto-improvement can be decentralized as a competitive equilibrium by subsidizing young-age savings at rate $\sigma^m \leq \omega/2$. Such intervention does not require intergenerational transfers from the working generation to the old.

ii) a first best allocation which is Pareto-superior to the initial laissez faire equilibrium can be decentralized as a competitive equilibrium by subsidizing young- and middle-age savings at rate $\sigma^m = \sigma^o = \omega$ if and only if the subsidy is coupled with intergenerational transfers from the working generation to the old.

Proof: See the Appendix.

There is an interesting parallel between Proposition 4 and a result recently provided by Jappelli and Pagano (1999). Employing an Arrow-Romer endogenous growth model in which individuals live for three periods and only earn income in the second period of life, these authors have demonstrated that imposing a borrowing constraint on the current young may lead to a Pareto-improvement. The borrowing constraint prevents the current young from borrowing as much as they would like which leads to an increase in aggregate savings and growth. If the utility loss the current young derive from the distortion in its intertemporal consumption path due to the borrowing constraint is more than compensated by the gain they receive from enjoying higher wages when middle-aged, a Pareto-improvement obtains. As in the present model, all depends on whether the current generation can sufficiently appropriate the productivity increase generated by their additional savings respectively their lower indebtedness. The results of this section suggest that while imposing a borrowing constraint may lead to a Pareto-improvement, it is insufficient to exploit all possible efficiency gains as it only reduces borrowing of the current young but does not increase savings of the current middle-aged.

5. Alternative Models of Endogenous Growth

The analysis so far has focused on a simple Arrow-Romer growth model in which a spillover from capital formation on labour productivity supports long-run growth. In this section it is demonstrated that a case for intergenerational transfers from the young to the old can also be made in other endogenous growth models that rely on externalities, namely the endogenous innovation and the human capital formation model.

5.1. Endogenous Innovation

The result of Sections 3 and 4 most directly apply to the endogenous innovation model with increasing product varieties emphasized by Romer (1987). There, final output is expressed as a function of labour and differentiated intermediate inputs. More precisely, final output at time t is determined by:

$$Y_t = N_t^{\alpha} \sum_{i=1}^{n_t} x_{i,t}^{1-\alpha}.$$

where n_t is the number of differentiated intermediate varieties known at time t, $x_{i,t}$ is the amount of the i-th intermediate good, and N_t again is the size of the working population. Producing one unit of a known intermediate good requires one unit of final output. A new type of intermediate variety can be developed by devoting $\tilde{a}$ units of final output to research. The research sector is fully competitive so that the price for a blueprint needed to produce an intermediate good is $\tilde{a}$ units of the final good. Blueprints are patented so that each intermediate variety is exclusively produced by one firm. The return on a blueprint is given by the monopoly rent that an intermediate goods producing firm earns when selling its output to the final good sector. It can be shown [see, e.g., Grossman and Yanagawa (1993, pp. 12-13) for more detail] that the equilibrium allocation is characterized by a fixed amount of each type of intermediate good. Denote this amount by $\bar{x}$ and normalize the size of the working generation to one. Final output at time t then becomes:

$$Y_t = n_t \, \bar{x}^{1-\alpha},$$

and the wage rate reads:

$$w_t = \tilde{\omega} \, n_t, \qquad \text{with } \tilde{\omega} \equiv \alpha \, \bar{x}^{1-\alpha}.$$

If the working generation at time t devotes $\tilde{a}$ units of final output to research, the number of varieties known in the next period increases by one and creates a return of $\bar{x}^{1-\alpha}$ units of final output. The share $\tilde{\omega}$, however, accrues as an external return to the working generation at time $t+1$. Therefore, the argument set forth in Sections 3 and 4 directly applies to the endogenous innovation model.

5.2. Human Capital Formation

Another growth model that figures prominently within the endogenous growth literature is that of endogenous human capital formation emphasized by Lucas (1988). Human capital formation models generate endogenous long-run growth provided that human capital per worker can increase over time without bound. Within an infinite horizon continuous time framework Lucas meets this requirement by assuming that the evolution of human capital is given by $\dot{h} = \delta \, \lambda \, h$, where h is the stock of human capital per worker, λ is the fraction of non-leisure time devoted to human capital formation, and δ is a technological parameter. In an overlapping generations framework with finite lifetimes an ever increasing stock of human capital requires that human capital accumulated by the current generation is somehow linked to the stock of human capital of preceding generations. To capture the underlying mechanism of the Lucas growth model, it is natural to assume that human capital evolves according to [see, e.g., Azariadis and Drazen (1990) for a similar assumption]:[3]

$$h_t = \delta\,\lambda_t\,h_{t-1}, \tag{26}$$

where h_{t-1} is the stock of human capital acquired by a member of generation $t-1$. In the laissez faire economy there is no mechanism that signals to young individuals the effect of their human capital investment decision on the human capital endowment of the next generation. This constitutes a positive externality between successive generations and the competitive allocation will be characterized by inefficiently low human capital investment. The allocation can be improved by an education subsidy. As some of the returns on education accrue to the next generation, again a case for intergenerational transfers from the young to the old may arise.

In contrast to the Arrow-Romer and the endogenous innovation model, however, the market mechanism may provide a channel through which the current generation has access to the returns on additional education. Since an increase in education augments the future stock of human capital, it will also increase the future interest rate if there is some complementary relationship between human and physical capital in production. This will benefit the old and may provide a compensation for additional human capital investments in the past. This, in turn, may weaken the case for combining a subsidy policy with intergenerational transfers as will be demonstrated in what follows.

Consider the two-period model with retirement in which the government engineers an education subsidy but does not employ intergenerational transfers. Young- and old-age consumption are then defined by:

$$c_t^y = (1-\lambda_t)\,h_t\,w_t + \sigma\,h_t - s_t - \tau_t,$$
$$c_{t+1}^o = (1+r_{t+1})\,s_t,$$

[3] A more direct translation of the Lucas model into a discrete time framework would be to assume that the increment of human capital is determined by $h_t - h_{t-1} = \delta\,\lambda_t\,h_{t-1}$. This, however, would necessitate considering the possibility of a corner solution with respect to time devoted to education. This, in turn, would complicate the analysis in a rather irrelevant respect.

where $1 - \lambda_t$ is non-leisure time devoted to work and σ now denotes a subsidy on education. The subsidy is financed by the tax τ_t imposed on the young generation which is targeted by the subsidy. Therefore, the government budget reads: $\tau_t = \sigma\, h_t$. It is straightforward to show that the time devoted to education chosen by a member of generation t is determined by $\lambda_t = 1/2 + \sigma/2w_t$ so that $d\lambda_t/d\sigma > 0$. Aggregate production is given by $Y_t = F[K_t, (1-\lambda_t)\, h_t]$, where the labour force again has been normalized to one, and the interest and wage rate read $r_t = F_{1,t}$ and $w_t = F_{2,t}$. The indirect utility function of generation t may be written as:

$$v_t(\sigma) = u[(1-\lambda_t)\, h_t\, w_t + \sigma\, h_t - s_t - \tau_t, (1+r_{t+1})\, s_t].$$

Differentiating with respect to σ while considering the Envelope Theorem and the definition of h_t yields:

$$v_t'(\sigma) = u_{1,t}\left(-\sigma\, h_t\, \frac{1}{\lambda_t}\frac{d\lambda_t}{d\sigma} + \frac{1}{1+r_{t+1}}\frac{dr_{t+1}}{d\sigma}\, s_t\right). \tag{27}$$

The first term in brackets is negative. It is similar to the one studied in Sections 3 and 4. The education subsidy encourages the young to invest more in human capital than they would choose to do in a laissez faire economy. The additional investment leads to a higher tax burden born by the young whereas the benefits accrue to future generations. So, this effect again makes generation t worse off and suggests employing intergenerational transfers from the young to the old. However, the second term in brackets is positive if an increase in the stock of human capital augments the future return on physical capital. In fact, if there is a sufficiently strong positive impact of an increase in the stock of human capital on the return on physical capital, the current young may be sufficiently compensated for their additional investments in education as they receive a higher return on their savings when old. Consequently, in the human capital formation model the question of whether a generation can reap the benefits of its investment occurring in earlier stages of its own life is not only a matter of the length of the working life but also a matter of how physical and human capital are combined in production.

In order to make this point more precise, consider both the case of perfect substitution in production and the case of a Cobb Douglas economy. If human and physical capital are perfect substitutes, the technology can be written as $F[K, (1-\lambda)h] = \alpha\, K + \beta\,(1-\lambda)\, h$, where α and β are positive technological parameters. It then follows that $r_{t+1} = \alpha$ and, henceforth, $dr_{t+1}/d\sigma = 0$. This implies that in case of perfect substitution in production there is no compensation for additional education in terms of a higher return on savings. Then, the same argument for intergenerational transfers that has been made in the Arrow-Romer and the endogenous innovation model applies to the human capital formation model.

In the Cobb Douglas case the technology may be written as $F[K,(1-\lambda)\,h] = K^{\alpha}[1-\lambda)\,h]^{1-\alpha}$. Assume furthermore that also the utility function is Cobb Douglas so that savings of generation t becomes $s_t = \gamma\, y_t$, where y_t is lifetime income of generation t and $\gamma \in (0,1)$ is a preference parameter. Then, as is shown in the Appendix, an increase in σ at time t (and only at time t) affects utility of generation t as follows:

$$v_t'(\sigma) = u_{1,t}\left(-\frac{\sigma}{\lambda_t} + (1-\alpha)\,\frac{r_{t+1}}{1+r_{t+1}}\,\gamma\, w_t\right) h_t\,\frac{d\lambda_t}{d\sigma}, \tag{28}$$

which is positive for $\sigma \to 0$. This implies that in case of a Cobb Douglas economy at least a partial internalization of the externality from education can be achieved in a Pareto-improving way without employing intergenerational transfers.

6. Conclusion

This paper has based intergenerational transfers from the working population to the retired on the observation that in the presence of endogenous growth the current generation cannot fully reap the benefits generated by investments occurring in earlier stages of its own life. If an externality from cumulated investment on labour productivity sustains long-run growth and the working life of individuals is rather short, a Pareto-improvement can be achieved only if coupled with intergenerational transfers from the young to the old. In contrast, if the working life is sufficiently long, a Pareto-improvement is possible without intergenerational transfers as the current generation itself enjoys the productivity gains of the investment in the form of higher future wages. However, if individuals retire when old, a full exploitation of all possible efficiency gains still requires intergenerational transfers in order to make no generation worse off than in a laissez faire economy.

A case for intergenerational transfers can also be made if endogenous innovation or human capital formation generates long-run growth. In fact, the results derived in the model with a capital externality directly apply to the endogenous innovation model. In the human capital formation model, in contrast, the current generation may have some access to the future benefits of additional investment in education if an increase in the aggregate stock of human capital positively affects the return on physical capital and private savings. Yet, as long as this effect is limited, a subsidy on education should be combined with intergenerational transfers from the young to the old in order to achieve a Pareto-improvement.

The results derived in this paper have important implications in the context of already existing intergenerational transfer schemes, namely pay-as-you-go public pensions. In the presence of an investment subsidy policy a public pension scheme may have its merits as it compensates the old for their growth enhancing investments undertaken in the past. In a recent paper Belan, Michel and Pestieau (1998) have analyzed the option of a reform of a pay-as-you-go pension scheme in an endogenous growth economy. They have shown that a pay-as-you-go scheme can be abolished in a Pareto-improving way if, for a finite number of periods, the revenues of the pension system are distributed in the form of an investment subsidy. The results of the present paper clearly demonstrate that in the presence of a pay-as-you-go pension scheme a subsidy is Pareto-improving only because the old participate in the productivity gains via an increase in pension benefits as the latter depend on labour earnings of the young. In fact, an already existing pay-as-you-go scheme constitutes a formidable basis to engineer an investment subsidy.

Appendix

Proof of Lemma 1

Implicitly differentiating (4), it follows:

$$s_{1,t} = \frac{1}{D_t}\,[u_{11,t} - (1+i_{t+1})\,u_{12,t}], \tag{A.1}$$

$$s_{2,t} = \frac{1}{D_t}\,[u_{12,t} - (1+i_{t+1})\,u_{22,t}], \tag{A.2}$$

$$s_{3,t} = \frac{1}{D_t}\,[s_t\,u_{12,t} - u_{2,t} - (1+i_{t+1})\,s_t\,u_{22,t}], \tag{A.3}$$

where $D_t = u_{11,t} - 2\,(1+i_{t+1})\,u_{12,t} + (1+i_{t+1})^2\,u_{22,t}$ is the second derivative of the left hand side of (3) with respect to s_t and, henceforth, negative. If the financing scheme is defined by (13), one has $w_{t+1} = \omega\, s(w_t, -\theta_{t+1}, r+\sigma)$, which follows by considering (4), (8), (9), (10), and (11). Differentiating with respect to σ, yields:

$$\frac{dw_{t+1}}{d\sigma} = -\omega\left(s_{2,t}\,\frac{d\theta_{t+1}}{d\sigma} - s_{3,t}\right). \tag{A.4}$$

Differentiating (13) with respect to σ and replacing s_t by w_{t+1}/ω, then substituting the result into (A.4), and, finally, rearranging terms, one gets:

$$\frac{dw_{t+1}}{d\sigma} = -\frac{\omega\,(s_{2,t}\,s_t - s_{3,t})}{1+\sigma\,s_{2,t}}.$$

Considering equations (A.1) to (A.3), this expression becomes after some manipulations:

$$\frac{dw_{t+1}}{d\sigma} = -\frac{\omega\,u_{2,t}}{D_t\,[s_{1,t} - (1+r)\,s_{2,t}]}.$$

Since $s_{1,t} > 0$, $s_{2,t} < 0$, and $D_t < 0$, it follows $dw_{t+1}/d\sigma > 0$.

Now let the financing scheme be defined by (14). Then, proceeding in the same way as above, one gets $w_{t+1} = \omega\, s(w_t - \tau_t, 0, r+\sigma) + \omega\,\tau_t$, where now (15) has been employed instead of (10). Differentiating with respect to σ yields:

$$\frac{dw_{t+1}}{d\sigma} = \omega\left((1-s_{1,t})\,\frac{d\tau_t}{d\sigma} + s_{3,t}\right). \tag{A.5}$$

Differentiating (14) with respect to σ, one gets:

$$s_t + \sigma \frac{ds_t}{d\sigma} = (1+r)\frac{d\tau_t}{d\sigma}. \tag{A.6}$$

Furthermore, from (9) and (15) it follows $s_t + \tau_t = w_{t+1}/\omega$ so that:

$$\frac{ds_t}{d\sigma} = \frac{1}{\omega}\frac{dw_{t+1}}{d\sigma} - \frac{d\tau_t}{d\sigma}. \tag{A.7}$$

Substituting (A.7) into (A.6), solving for $d\tau_t/d\sigma$, and substituting the result into (A.5), yields :

$$\frac{dw_{t+1}}{d\sigma} = \frac{\omega\,[(1-s_{1,t})\,s_t + (1+r+\sigma)\,s_{3,t}]}{1+r+\sigma\,s_{1,t}}.$$

Finally, considering (A.1) and (A.3), straightforward manipulation leads to:

$$\frac{dw_{t+1}}{d\sigma} = -\frac{\omega\,(1+r+\sigma)\,u_{2,t}}{D_t\,[1+r+\sigma\,s_{1,t}]},$$

implying $dw_{t+1}/d\sigma > 0$. *Q.E.D.*

Proof of Lemma 2

If the financing scheme is defined by (16), it follows $w_{t+1} = \omega\, s[w_t, -\theta_{t+1}, r + \sigma]$, where θ_{t+1} must be consistent with (17). Differentiating with respect to σ, one obtains:

$$\frac{dw_{t+1}}{d\sigma} = -\omega\left(s_{2,t}\frac{d\theta_{t+1}}{d\sigma} - s_{3,t}\right).$$

Substituting for $d\theta_{t+1}/d\sigma$ using (17) yields:

$$\frac{dw_{t+1}}{d\sigma} = -\omega\,(s_{2,t}\,s_t - s_{3,t}).$$

Finally, considering (A.2) and (A.3), straightforward algebra provides:

$$\frac{dw_{t+1}}{d\sigma} = -\frac{\omega\,u_{2,t}}{D_t}$$

which is positive. *Q.E.D.*

Proof of Proposition 3

i) Employing the same procedure as in the proof of Lemma 1, one finds:

$$\frac{dw_{t+1}}{d\sigma} = -\frac{\omega\, u_{2,t}}{D_t\,[2\,s_{1,t} - (2(1+r)+\omega)\,s_{2,t}]} > 0,$$

so that the subsidy again triggers a positive effect on labour productivity. Indirect utility of generation t in the case of the two-period model without retirement reads:

$$v_t(\sigma) = u\left[w_t - s_t, w_{t+1} + (1+r+\sigma)\,s_t - \frac{2\sigma}{\omega}\,w_{t+1}\right].$$

Differentiating with respect to σ, one gets after some manipulations:

$$v_t'(\sigma) = u_{2,t}\left(1 - \frac{2\sigma}{\omega}\right)\frac{dw_{t+1}}{d\sigma},$$

which is non-negative for $\sigma \leq \omega/2$.

ii) It will first be demonstrated that $v_t(\omega) \geq v_t(0)$ if $u_{12,t} \geq 0$ and $d^2 s_t/d\sigma^2 \leq 0$. Subsequently, it will be shown that a first best allocation obtains if $\sigma = \omega$ at each time t. Observe that v_t takes on a maximum if $\sigma = \omega/2$ since $v_t' > 0$ (< 0) if $\sigma < \omega/2$ $(> \omega/2)$. Thus, $v_t(\omega) \geq v_t(0)$ if the absolute slope of v_t for $\sigma < \omega/2$ is at least as steep as for $\sigma > \omega/2$. Since $v_t'[(\omega/2) - x] = x\left(u_{2,t}\frac{ds_t}{d\sigma}\right)|_{\sigma=(\omega/2)-x}$, it is straightforward to show that $v_t'[(\omega/2) - x] \geq -v_t'[(\omega/2) + x]$ for all $x \in [0, \omega/2]$ if $u_{12,t} \geq 0$ and $d^2 s_t/d\sigma^2 \leq 0$. It remains to show that a first best allocation obtains if σ equals ω at each time t. A first best allocation solves the problem

$$\max_{\{c_t^y, c_{t+1}^o, K_{t+1}\}} \sum_{t=0}^{\infty} \mu_t\, u(c_t^y, c_{t+1}^o),$$

subject to:

$$c_t^y + c_t^o = [1 + f(a/2)/(a/2)]\,K_t - K_{t+1},$$

$$K_0 > 0,\ c_0^o > 0,$$

for some sequence of positive weights $\{\mu_t\}_{t=0}^{\infty}$, where it has been considered that $Y_t = F(K_t, 2\,K_t/a) = K_t\, f(a/2)/(a/2)$ if both the young and the old work. The solution to this problem is implicitly defined by the following Euler equations:

$$\mu_{t+1}\,[1 + f(a/2)/(a/2)]\,u_{1,t+1} - \mu_t\,u_{1,t} = 0, \tag{A.8}$$

$$-\,\mu_{t+1}\,u_{1,t+1} + \mu_t\,u_{2,t} = 0, \tag{A.9}$$

and a transversality condition of the form:

$$\lim_{t\to\infty} \mu_t\,u_{1,t}\,K_t = 0.$$

Dividing (A.9) by (A.8), one gets:

$$u_{1,t} = [1 + f(a/2)/(a/2)]\,u_{2,t}, \quad t = 0, 1, 2, \ldots \tag{A.10}$$

Furthermore, the recursion in (A.8) implies:

$$\mu_t = \frac{1}{[1 + f(a/2)/(a/2)]^t}\,\frac{u_{1,0}}{u_{1,t}}\,\mu_0, \tag{A.11}$$

so that the transversality condition becomes:

$$\lim_{t\to\infty} \frac{K_t}{[1 + f(a/2)/(a/2)]^t} = 0, \tag{A.12}$$

as μ_0 and $u_{1,0}$ do not depend on t. In a decentralized economy with a savings subsidy equal to $\sigma = \omega$ at each time $t = 0, 1, 2, \ldots$ individual consumption plans satisfy:

$$u_{1,t} = (1 + r + \omega)\,u_{2,t}, \quad t = 0, 1, 2, \ldots$$

Since $r + \omega = f(a/2)/(a/2)$, the competitive allocation satisfies (A.10). To show that it satisfies (A.12), observe that $s_t \le w_t$ since only the young save. As $s_t = K_{t+1}$ and $w_t = (\omega/2)\,K_t$, this leads to $K_{t+1}/K_t \le (\omega/2)$ for all t. This, in turn, implies $K_t \le (\omega/2)^t\,K_0$ so that:

$$\frac{K_t}{(1 + r + \omega)^t} \le \left(\frac{\omega}{2\,(1 + r + \omega)}\right)^t K_0 \le \left(\frac{\omega}{1 + r + \omega}\right)^t K_0.$$

Since $K_t/(1 + r + \omega)^t \ge 0$ and $0 < \omega/(1 + r + \omega) < 1$, one finds:

$$\lim_{t\to\infty} \frac{K_t}{(1 + r + \omega)^t} = 0.$$

Considering that $r + \omega = f(a/2)/(a/2)$, it follows that the competitive allocation satisfies (A.12). The proposition now follows from the observation that the weights μ_t can be chosen so that they satisfy (A.11) for consumption levels that obtain in a competitive equilibrium with $\sigma = \omega$. *Q.E.D.*

Proof of Proposition 4

Part (i) is obvious from the analysis of Section 4.1. To prove part (ii) it will first be shown that a subsidy on middle-age savings of generation $t-1$, σ^o_{t+1}, in fact increases the wage rate w_{t+1}. Implicitly differentiating (25) while considering (21), a one period lag of (22) and the fact that $\theta^o_{t+1} = \sigma^o_{t+1}\,(2\,w_{t+1}/\omega - s^y_t)$, which follows by combining (24) and (25), one finds:

$$\frac{dw_{t+1}}{d\sigma^o_{t+1}} = -\frac{\omega\,[s^m_{3,t}\,s^m_t - s^m_{5,t}]}{(1+\sigma^o_{t+1}\,s^m_{3,t})\,(2-\omega\,s^y_{2,t})}.$$

Expressions of the partial derivatives of young- and middle-age savings can be found by applying the Implicit Function Theorem to (19) and (20) and to a one period lag of these equations. Doing this, tedious but straightforward algebra leads to:

$$\frac{dw_{t+1}}{d\sigma^o_{t+1}} = -\frac{\omega\,d_{t-1}\,u_{3,t-1}}{\Delta_{t-1}\,[s^m_{2,t} - \;(1+r)\,s^m_{3,t}]\,(2-\omega\,s^y_{2,t})}$$

where Δ_{t-1} is the determinant of the Jacobian of the system of equations defining the first-order conditions for maximum utility of generation $t-1$ and d_{t-1} is the first element of this Jacobian. The second-order conditions for maximum utility imply $\Delta_{t-1} > 0$ and $d_{t-1} < 0$. Furthermore, since consumption in all three periods is normal, $s^y_{2,t} < 0$, $s^m_{2,t} > 0$, and $s^m_{3,t} < 0$ hold true. Thus, $dw_{t+1}/d\sigma^o_{t+1} > 0$.

Indirect lifetime utility of generation $t-1$ may be written as:

$$v_{t-1}(\sigma^o_{t+1}) =$$
$$u[w_{t-1} - s^y_{t-1}, w_t + (1+r+\sigma^m_t)\,s^y_{t-1} - s^m_t - \theta^m_t, (1+r+\sigma^o_{t+1})\,s^m_t - \theta^o_{t+1}]$$

Differentiation yields under consideration of the Envelope Theorem and $\theta^o_{t+1} = \sigma^o_{t+1}\,(2\,w_{t+1}/\omega - s^y_t)$:

$$v'_{t-1}(\sigma^o_{t+1}) = -u_{3,t-1}\,\sigma^o_{t+1}\left(\frac{2}{\omega} - s^y_{2,t}\right)\frac{dw_{t+1}}{d\sigma^o_{t+1}}.$$

Since $dw_{t+1}/d\sigma^o_{t+1} > 0$ and $s^y_{2,t} < 0$, it follows $v'_{t-1} < 0$. Thus, without intergenerational transfers a further Pareto-improvement does not obtain by subsidizing savings of the middle-aged. However, a further Pareto-improvement is possible by coupling a subsidy on middle-aged savings with intergenerational transfers from the working population to the old. Moreover, all possible efficiency gains can be exploited by setting $\sigma^m = \sigma^o = \omega$ at each time t. The proof of the first claim is similar to the one of Proposition 2 and the proof of the second claim is similar to the last part of the proof of Proposition 3. Therefore, the proofs of these two claims can be dispensed with. *Q.E.D.*

Derivation of equation (28)

In the Cobb Douglas case r_{t+1} is given by:

$$r_{t+1} = \alpha\, K_{t+1}^{\alpha-1} H_{t+1}^{1-\alpha},$$

with $H \equiv (1-\lambda)\, h$. Considering (26) and the fact that $s_t = \gamma\,(1-\lambda_t)\, h_t\, w_t$, r_{t+1} becomes:

$$r_{t+1} = \alpha\, [\gamma\,(1-\lambda_t)\,\lambda_t\,\delta\, h_{t-1}\, w_t]^{\alpha-1}[(1-\lambda_{t+1})\,\lambda_{t+1}\,\delta^2\,\lambda_t\, h_{t-1}]^{1-\alpha}.$$

Differentiation with respect to σ leads to:

$$\begin{aligned}\frac{dr_{t+1}}{d\sigma} =& \alpha\,(1-\alpha)\,\delta\, h_{t-1}\,[-K_{t+1}^{\alpha-2} H_{t+1}^{1-\alpha}\,\gamma\,(1-2\lambda_t)\, w_t \\ &+ K_{t+1}^{\alpha-1} H_{t+1}^{-\alpha}\,(1-\lambda_{t+1})\,\lambda_{t+1}\,\delta]\,\frac{d\lambda_t}{d\sigma}.\end{aligned}$$

Considering (26), the product market equilibrium condition $s_t = K_{t+1}$ and the expression for r_{t+1}, straightforward manipulation leads to:

$$\frac{dr_{t+1}}{d\sigma} = (1-\alpha)\, r_{t+1}\,\gamma\, w_t\, h_t\,\frac{1}{s_t}\,\frac{d\lambda_t}{d\sigma}.$$

Substituting this expression into (27) yields (28).

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