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Information Gathering, Disclosure and Contracting in Competitive Markets

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Abstract

The paper studies the determinants of information gathering in insurance and credit markets. In our set-up, information may have either operational or strategic value, e.g. it may improve allocative decisions or allow agents to appropriate a larger share of gains from trade at the contracting stage. The timing of information gathering is endogenous and agents can gather information either before or after contracting. Access to precontractual information generates a negative contracting externality, which was first identified in Hirshleifer.s (1971) seminal contribution. In contrast with a well established conventional wisdom and a substantial literature, we prove that, if the operational value of information is positive and not "too small", private returns of information fall short of its social returns, and pre-contractual access to information leads to under-investment . On the contrary, agents over-invest in information gathering activities, when the operational value of the available signals is sufficiently low. Consistently with contractual arrangements observed in the real world, we also show that equilibrium contracts have also a very simple shape when private information can be voluntarily disclosed.

Keywords: private information, information gathering, value of information.

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1 Introduction

Agents trading in insurance and credit markets spend large amounts of resources to obtain better assessments of the likelihood of future events, such as the occurrence of health disease or a monetary loss, or the success of an entrepreneurial project. What determines individual decision to acquire and disclose information and the timing of information gathering activities ? How financial contracts should be designed in order to provide appropriate private incentives to gather information ? Can private returns of information exceed or fall short of its social returns? These are key issues for the functioning of financial markets where most of the information used by the agents is indeed acquired. As dramatic efficiency advances of information technologies are rising policy concerns in many sectors, including the medical and genetic industries[1], these issues have recently gained a prominent place also in the public debate. Understanding whether and under which conditions private and social returns of information may diverge is indeed the necessary first step of any welfare analysis investigating the effects of better access to information.

Following Jack Hirshleifer (1971), several contributions in the literature have showed that public diffusion of information before contracting reduces agents welfare by destroying trading opportunities (Green (1981), Marshall (1974), Wilson (1975) Schlee (2001), Morris-Shin (2002)). A substantial related literature has also argued that agents overinvest in information gathering activities whenever they can acquire private information before trading (Hirshleifer (1971),Shavell (1994), Reingaum (1989) Khalil-Kremer -Rochet (1998), Bergeman-Valimaki (2002)), among others). In another seminal contribution, Khalil and Kremer (1992) show that agents never gather socially wasteful information (e.g. information with negative social value) under optimal contracting. One of the main purpose of this paper is to argue that, in a richer contracting environment, opposite results often hold true : under mild conditions, agents under-invest in (private) information gathering activities if the operational value of information is positive and not "too small"; while they over-invest in these activities when information has negative social value.

We study a market where agents trade with intermediaries to obtain funds or to ensure themselves, and can acquire information on their own characteristics (types), or on the value of an asset they own. This information can be useful either for operational purposes, in order to improve the quality of allocative decisions, or for strategic reasons, to appropriate larger shares of gains from trade. Following a line of research opened by Khalil and Kremer, our purpose is to investigate the determinants of information gathering activities in an environment where their timing is endogenous. The key assumptions of our model are that agents can disclose information to their contractual counterpart either before or after contracting, while contracts can set payments contingent on the information disclosed either at the contracting stage or after contracting. Noteworthy, these assumptions are in line with real world contracts. For instance, insurance companies commonly offer health insurance policies which set health care reimbursement contingent on the disclosure of medical tests to be passed after contracting. While,

[1] In this past decade much attention has been centered on the Human Genome Project for promoting health and preventing disease. Human Genome knowledge can provide individuals with the opportunity for screening and identifying many genetic disorders and opens important issues concerning the insurability of health risks.

often, insurance contracts also prescribe or allow insurance premia to be revised after the disclosure of new information on clients' prospective health. Moreover, the availability and the price of credit for an entrepreneur opening a credit line generally depends also on the information concerning the quality of his project, which is disclosed after contracting as time goes by.

On the modelling side, allowing agents to gather information after contracting avoids imposing arbitrary restrictions of the contract space. Indeed, even in the first best-benchmark where asymmetric information issues are absent[2], information must in general be acquired after contracting for potential gains from trade to be fully exploited[3], .

The point of departure of our analysis is that, according to the key observation of Hirshleifer, private information acquired before contracting has a positive strategic value, independently from its operational value, as it allows to better assess the terms of trade which are offered in the market. To the extent that an agent's access to private information reduces expected gains of his potential contractual parties, however, it also creates a welfare reducing contractual externality. To illustrate, consider a test which permits to better identify the likelihood of a disease or a genetic disorder. Prior to testing, competing insurance companies are willing to offer him insurance at a "fair" unitary price, equal to the probability that the health problem is identified. A major problem arises, however, if individuals can acquire a (sufficiently accurate and cheap) health test *before* seeking insurance and keep private the result of the test : ex ante fair prices become unprofitable since only those who discover bad news, and hence are more likely to be unhealthy, will insure themselves. Thus, precontractual testing results in an adverse selection of the pool of insurance applicants, making unviable mutually beneficial trades. [4]. Private benefits from private testing may then lead private and social returns of information to diverge. As a by product, the fear of bad selection effects may induce insurance companies to ration the amount of insurance offered, or to impose test-contingent discriminatory pricing in order to reduce asymmetric information at the contracting stage.

A large literature exploring the effects of the negative contractual externalities created by private information gathering activities has proved in several context, and at an increasing level of generality, that these externalities may lead agents to overinvest in information. These results, however, rely on somewhat restrictive assumptions; indeed, they are proved in settings where agents can gather information only before contracting[5], or financial trades are unnecessary to fully exploit information [6]. By relaxing these assumptions, we show that a careful analysis of intermediaries' (principals) market behavior may change the conventional wisdom prevailing in the literature, and that the Pigouvian logic, according

[2] An appropriate comparison between social and private returns of information gathering activities, e.g., between a fully efficient and a second best setting, then requires the agents' actions space to be the same in the two cases.

[3] This is just an application of the general principle that trades must precede actions for markets to be efficient.

[4] In the extreme case where the agent is fully informed about his individual state an intermediary offering a contract to him will make losses with certainty.

[5] A very important exception in the literature is the article of Khalil and Kremer. They consider a setting where information has negative social vaue (e.g. no operational value) and agents' information gathering choices are of a 0-1 type - agents can decide to gather either a perfectly informative signal or no signal- and show that agents do not gather information in the second best).

[6] This assumption is common in the innovation literature on patent races where the overinvestment result is also common.

to which negative (resp. positive) externalities result in overinvestment (resp. under-investment) may turn out to be inappropriate for understanding the *external effects* of private information gathering. Specifically, in contrast with all the previous literature, we demonstrate that agents generally under-invest in information when the social value of information is not too small. Moreover, in contrast with the seminal work of Khalil and Kremer, we also show that agents generally over-invest in information gathering whenever information has a negative net social value, but is relatively inexpensive and not perfectly accurate.

The value of precontractual information, and hence agents' private incentives to gather information before contracting, are generally determined by the whole set of contracts they are offered; moreover, an agent who decides to gather precontractual information causes a non negligible reduction of the profit made by th principals he trade with. For these reasons, *rational* intermediaries do not take agents' actions as given, but design contracts aimed at protecting themselves against the contractual externalities created by the agents. This paper characterizes equilibrium contracts and information gathering strategies, and shows how in a competitive environment principals use different contractual instruments, e.g. payments schedule and information requirements, to deter agents from gathering precontractual information. Precisely, we show that in equilibrium intermediaries generally offer contracts which prescribe agents to gather socially suboptimal amount of information after contracting and ration transfers across agnts individual states, in order to make less advantageous private precontractual information. In a final section of the paper, we extend the model to consider the case where agents can gather information only after contracting but can opt-out from the contract they have signed after contracting.

In our setting agents's endowment as well the return of their investment in a production or a loss reduction technology are uncertain. Agents can gather private information by acquiring signals which allow to update their initial assessments on the likelihood of future states. Available signals are ordered according to their informativeness, and either earlier or more accurate information is costlier. Agents can also voluntarily disclose the information they have gathered; crucially, however, they cannot prove their ignorance to contractual parties at the contracting stage. For instance, an agent can disclose the result of a medical test to his insurer, but cannot prove that he has not undertaken any test, whenever this is the case. Contracts between principals and agents can set payments contingent either on publicly observable variables or on the information disclosed by agents.

Within this setting, private precontractual information gathering is formally equivalent, from the viewpoint of an agent, to acquire an option which gives him the *opportunity* to choose an offer after some payoff relevant uncertainty is resolved. Crucially, however, the "price" an agent has to pay for this option is simply equal to the cost of precontractual information gathering, and is not paid to the intermediaries offering the "underlying" contracts, whose profitability is reduced by the option. For these reasons, whenever the cost of precontractual information is sufficiently low, access to private precontractual information by the agents reduces the set of mutually convenient trades that intermediaries can offer, and makes unviable first best trades. Competing intermediaries can then offer either contracts which prescribe precontractual information disclosure and make non negative profits on informed agents (on agents who disclose information before contracting), or, in alternative, contracts designed to attract only the uninformed (which meet appropriate incentive compatibility). As we show, relevant incentive compatibility conditions for the latter types of contracts are then jointly determined by all the contracts offered in the market : either the ones designed to attract uninformed agents or those requiring precontractual information disclosure. In particular, this is true since offering more favorable contracts to the informed agents makes precontractual information gathering more advantageous and hence more difficult to deter. By exploiting the properties of these constraints, we demonstrate that the set of contracts offered in equilibrium contains : (i) all "interim efficient" contracts which make zero profit on agents who gather and disclose information before contracting and (ii) the non negative profit contract which is preferred by the agent within the set of contracts that deter precontractual information, given that all "interim efficient" contracts are offered. In equilibrium, agents always accept the latter, i.e. all information is taken after contracting. However, the fear of inducing agents to gather precontractual information will lead intermediaries to ration transfers towards "bad" individual states in order to discourage precontractual information gathering. For instance, in an health insurance market insurers will generally reduce either promised reimbursements or corresponding premia, in order to deter the precontractual information. Lower reimbursement in bad health states, indeed, reduce the benefit that an agent obtains by gathering precontractual information and purchasing more insurance conditional on bad news. Thus, in equilibrium, competition by financial intermediaries plays a double role: it defines the market opportunities for uninformed agents, and, at the same time, it contributes to determine the value of precontractual information. Indeed, competitive pressures lead intermediaries to offer the *best possible deals* not only to the agents who do not gather precontractual information but also to the ones that gather and disclose it. Noteworthy, the latter effect of competition is welfare reducing as it makes incentive constraints more severe.

We also shows that minimizing second best losses leads intermediaries to condition equilibrium payoffs not only to the realization of the state of the world, as it would be the case in the first best, but also on the news gathered after contracting. In the real world, contracts of this type are in fact widely used either in insurance or in financial markets. Moreover, under mild conditions competitive contracts have also a simple shape, consistently with real world arrangements, but in contrast with the predictions of most second best models. In particular in insurance settings, equilibrium contracts offered by the insurers are standard insurance schemes imposing a positive deductible and a maximal repayment. While contracts offered by lenders to agents dealing with a funding problem are debt contracts with a very limited umber of covenants.

We then exploit the characterization of equilibrium allocations to investigate the determinants of information gathering decisions. We first show that if available signals are not perfectly informative, and have negative social (operational net) value but relatively low cost, agents overinvest in information gathering in equilibrium. Intermediaries require socially wasteful information to be gathered after contracting in order to discourage precontractual information gathering. Essentially, this result follows from the fact that expected gains from precontractual information are proportional to the magnitude of the transfers towards "bad" individual states that the agent can implement by trading. Thus, if the cost of information gathering is small, contracts offered to non informed agents must implement small transfers across individual states in order to deter precontractual information gathering. If information is sufficiently noisy and inexpensive, however, informed and non informed are similar types from an intermediary's viewpoint (e.g. face similar distributions of future states). For this reason, an agent who gathers and discloses information before contracting can obtain in the market an allocation close to his first best allocation (the one he would receive were access to precontractual information precluded), whatever news he reveals. Whenever this is the case, contracts prescribing to gather information with negative social value minimimize second best losses due to private access to information, and are offered in equilibrium; so that agents oveinvest in information gathering. The opposite result holds true when the operational value of information is positive and not "too small". In this case, as we show, precontractual access to private information leads agents to under-invest in information gathering. This is because a (slight) reduction in the informativeness of the signal that the agent is required to gather after contracting allows to relax incentive constraints, while having only negligible (second order) effects on the returns of information. Specifically, we prove that, this incentive effect arises because, under mild conditions, the less informative is the signal that an agent is required to gather after contracting the less advantageous is for him to acquire that signal before contracting. Intuitively, this is for the following reason. First, expected gains from precontractual information are proportional to the expected utility that the agent obtains when he discloses *good news* before contracting, since disclosing these news at the contracting stage allows to obtain more favorable terms of trades in the market. Second, as we show, the expected utility an agent can obtain in the market conditional on disclosing *good news (*e.g. realization of a signal indicating that favorable contingencies are more likely*)* is increasing in the quality of the signal delivering those news. Thus, reducing the quality of the signal that the agent is required to gather after contracting also decreases the expected consumption that informed agents observing good news before contracting can obtain by revealing their information to the market, and, hence, makes it easier to deter precontractual information.

Finally, in a section which contains some extensions to the basic model, we show that the incentive problem that principals face whether agents can gather precontractual information or can acquire information only after contracting, but can opt-out from the contract they have signed after contracting, are essentially the same; and, for this reason, all the results proved in one setting extend in a straightforward way to the other. These results allow to demonstrate our analysis also applies to insurance and credit settings where long terms contracts are often not enforceable.

2 Set-up

We consider an economy with a continuum of agents, and large number of competing intermediaries (principals). Intermediary compete by offering exclusive contracts to the agents, they can either fund their investments in a risky technology, or insure them against idiosyncratic shocks to their endowment. Each agent can gather private information on his distribution of individual states either *before* or *after* contracting.

Preferences and endowments Agents are ex ante identical and consume one physical good, x, in different individual states of the world. There exists a finite number, S, of individual states of the world which are identically and independently distributed across agents; p_s denotes the prior probability of state s, while w_s is the state s contingent endowment of the agent; throughout it will be convenient to assume $w_{s+1} \geq w_s$ for all s. Principals are risk-neutral and maximize their expected profit. Agents' preferences are assumed to be state independent, and are represented by the certainty utility function $u(x)$, which is strictly concave and twice differentiable.

Production Each agent can choose an action a belonging to the set $A \subseteq \Re$, representing an investment whose net return, $r(a, s)$, depends on s. The function $r(a, s)$ is assumed to be strictly concave in a, with $r_a(a, s) > 0$ for all s. We shall also assume that $r_a(a, s)$ is monotone in s; e.g.,the return function $r()$ may display either increasing or decreasing first differences. In the former case, returns and the endowments are independently distributed or positively correlated; and $r()$ can be suitably interpreted as a *production* technology. Indeed, while independence between $r()$ and w is often a natural assumption in the analysis of production decisions, positive correlation between these two variables may arise in the presence of non transferable resources such as health or human capital. For instance, the investment returns of an entrepreneur undertaking a project will depend positively on his own human capital endowment whenever human capital is not perfectly transferable and hence cannot be purchased. Differently, the assumption that $r_a(a, s)$ is decreasing in s is often appropriate to describe the returns of a *loss reduction* technology. For instance, medical treatments usually have larger returns when health losses are more consistent; while obviously being ineffective when the agent is healthy[7].

Information gathering Each agent can gather information on the distribution of his individual states by choosing one signal from the family $E = \{\eta_l\}_{l \in 0}^{L}$. A signal η_l is a random variable with finite support $\mathcal{N} = \{\sigma_1, ..., \sigma_N\}$[8]. For each l, $p_l(\sigma_n | s)$ represents the probability of observing σ_n, conditional on s being the true state of the world, while $p_l(\sigma_n, s)$ is the joint probability of the two events. Signals are assumed to be ordered according to their Blackwell informativeness. Namely, for each pair η_{l+1} and η_l, the matrix P_l of conditional densities of η_l is equal to the matrix P_{l+1} of conditional densities of η_{l+1} pre-multiplied by a stochastic matrix B_l, i.e. $P_{l+1} = B_l P_l$. For convenience, η_0 will denote the completely uninformative signal such that $p_0(\sigma_n | s) = p_0(\sigma_{n'} | s)$ for all s, and for all pairs $(\sigma_n, \sigma_{n'})$.

We shall also assume that the marginal distribution of σ, i.e. the vector $g_l(\sigma_n) = \sum_s p_l(\sigma_n, s)$, is independent from η_l. Finally, we impose that the distribution of conditional densities satisfies the monotone likelihood ratio property. Formally, for each l,

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In the language of financial economics, a technology with increasing first difference corresponds to a speculative asset, while one with decreasing first differences corresponds to an asset offering hedging opportunities.

8 In the next sections we shall assume whenever convenient that the set of possible realization is finite but arbitrarily large. This is almost without loss of generality as conditional probabilities are not assumed to be different for all possible realizations of a signal. In other words, a subset of signals realizations can be interpreted as pure sunspots.

$$\frac{p_l(\sigma_n\,|s)}{p_l(\sigma_{n+1},s)} \geq \frac{p_l(\sigma_n\,|s+1)}{_l(\sigma_{n+1}\,|s+1)} \text{ for all } n \leq N-1 \text{ and } s \leq S-1.$$

In words, individual states and realizations are affiliated random variables, so that the likelihood of higher (better) realizations increases relatively more with better states.

Costs and returns of information gathering activities Information can be gathered at two different stages (the complete timing of agents' actions is specified below). Either earlier information or better information are more expensive : the cost $c_\tau(\eta_l)$ of the signal η_l is decreasing with respect to the gathering stage, τ, and increasing in the informativeness of the signal, l. Moreover, $c_\tau(\eta_0) = 0$ for all τ, e.g. the uninformative signal is costless. As it has been argued by Khalil and Kremer (1992), the assumption that earlier information is costlier is realistic in most economic applications. The extra cost of earlier information indeed, may represent an opportunity cost (which is positive when the interest rate is positive), or may be due to the fact that larger availability of time reduces the costs of information gathering, possibly because some uncertainty naturally disappears as time elapses.

Information The signal gathered by an agent , its realization, and the timing of information acquisition are his own private information. Signals and realizations are "hard" (trasmissible) information, and may be voluntarily disclosed by agents who acquires them to their principals. Crucially, however, an agent cannot prove that he has not gathered any information before a certain stage, whenever this is the case. Moreover, agents disclosing information cannot provide verifiable evidence of the stage in which their information has been gathered.

Information is assumed to be transmissible in order to simplify the analysis. Assuming away voluntary disclosure, would not change most qualitative results of the analysis (but would require the introduction of an additional set of incentive constraints). Finally agents' investment and consumption choices are verifiable and contractible.

Timing of actions Principals compete by offering exclusive contracts at the initial stage, $\tau = 0$; while agents choose which information , to gather and disclose, accept principals offers, invest and consume according to the following timing. Either before choosing a contract, at $\tau = \tau_1$, or after contracting at $\tau = \tau_2$ agents may gather information . At $\tau = \bar{\tau}$, with $\tau_1 < \bar{\tau} < \tau_2$, each agent can apply for one of the contracts offered, and can disclose the information previously acquired to his contractual counterpart; at the same stage, applications are accepted by principals. At τ_2, agents receive funds and invest (i.e. implement the action a); while at the final stage $\tau = \tau_3$, uncertainty is completely resolved, agents receive their endowment and investment returns, contractual payments are delivered, and consumption takes places.

Contracts Merely for simplicity, we shall impose some restrictions on the set of possible disclosure policies. Precisely, we assume without loss of generality that any contract prescribing the agent to acquire the signal η_l at stage τ also requires that, at the same same stage, (i) the agent provides verifiable evidence that he has gathered η_l, and (ii) discloses the realization of η_l.[9] .Noteworthy, the fact that information

[9]Considering a larger space of contracts, so as to allow principals discretion on how much information disclosure to

can be voluntarily disclosed limits, but do not eliminate at all the effects of informational asymmetries. Indeed, an agent who accept a contract prescribing not to gather information, neither before nor after contracting, can violate such a prescription, by gathering any signal either at τ_1 or at τ_2, without bearing any punishment. Similarly, an agent accepting a contract that prescribes to gather η_l with $l > 0$ after contracting, has still the opportunity to acquire η_l before contracting, since the timing of his information gathering activity is private information.

It is convenient to formally represent the action of not gathering any signal at a given stage as that of gathering the costless and completely uninformative signal η_0. Consistently, let an *information gathering plan*, $\boldsymbol{\eta} = (\eta^1, \eta^2)$, specify the signal that the agent gather at the stages $\tau = 1$ and $\tau = 2$. Since each agent can gather at most one signal, $\eta^{\tau} = \eta_0$ for each information gathering plan such that $\eta^{\tau'} \neq 0$. A contract, $b = (\eta, a, z)$, then specifies an information gathering plan, $\boldsymbol{\eta}$, an $N + 1$ dimensional vector of actions $\mathbf{a} = (...a(\sigma_n), ...)$, and an $(N + 1) \times S$ dimensional vector of transfers to the agent z=$(..., z(\sigma_n, s), ...)$. The interpretation of a and z is the following : $a(\sigma_n)$ represents the action prescribed by the contract conditional on the disclosure of σ_n, $z(\sigma_n, 0)$ is the amount of funds the agent receives at the investment stage conditional on σ_n, and $z(\sigma_n, s)$ is the final stage contingent transfer to the agent, which is conditional on both s and σ_n. Finally, we shall assume that intermediaries offer exclusive contracts. It will also be notationally convenient, and without loss of generality to assume that intermediaries fund entirely agents' investments, which is they issue contracts such that $a(\sigma_n) = z(0, \sigma_n)$.

Strategies Each principal can offer a finite menu of contracts[10]. A principal's strategy $f^p = B^P$ is simply the choice of a finite set containing $m \geq 0$ of contracts. An agent's strategy consists of two sequential actions, through which he chooses (i) a signal to be gathered before contracting (possibly the uninformative signal η_0) and (ii) a contract in the set of principals' offers. More formally, let $\mathcal{B}$ be the set of vectors of possible offers, and define a *history h observed by the agent* at the contracting stage as a set of of offers B, together with the signal η_l^1 that he has gathered before contracting and its realization σ_n. An agent strategy, $f^a = (f^{1a}$,$f^{2a})$, then is formally defined by a pair of maps f^{1a} and f^{2a}. The map $f^{1a} : \mathcal{B} \to E$, associates to each possible vector of contractual offers, $B \in \mathcal{B}$, a signal, η_l in E (possibly, with $\eta_l = \eta_0$) that the agent gathers before contracting. The map $f^{2a} : H \!\to \mathcal{B}$ associates a contract c contained in the set of possible vectors of offers B to each possible *history* $h \in H$ observed by the agent at the contracting stage τ_1.

In the following we shall focus attention on pure strategy symmetric equilibria where all principals and agents choose the same strategy; this is unrestrictive in our setting. We shall also assume that whenever M principals offer the same contract, each of them receives the same fraction of applications for that contract.

Payoffs Define $p_l(s \,|\sigma_n\,)$ the probability of observing s conditional on the signal η_l and the realization σ_n.

require, would not change the results of the analysis. Classical unraveling results show that in equilibrium all payoff relevant information will be disclosed whenever full disclosure is possible.

[10] This will allow to restrict attention tot symmetric equilibria.

For any strategy profile such that agents gather the signal η_l^τ at stage τ[11], invests $a(\sigma_n)$ conditional on observing σ_n and receives the transfer $z(s,\sigma_n)$ contingent on s and σ_n, the agent expected utility is :

$$\sum_{n\in N}\sum_{s\in S} p_l(\sigma_n,s)U(x(\sigma_n,s)) = \sum_{n\in N} g(\sigma_n)\sum_{s\in S} p_l(s\,|\sigma_n)U(x(\sigma_n,s))$$

where, $x(s,\sigma_n) = w_s - c_\tau(\eta_l) + r(a(\sigma_n),s) + z(\sigma_n,s))$.

Given any strategy profile where agents do not gather precontractual information, an intermediary who signs a contract prescribing to gather η_l after contracting obtains the per capita profit :

$$\Pi(b,l) = -\left[z(0,\sigma_n) + \sum_{n\in N, s\in S} g(\sigma_n)p_l(s\,|\sigma_n)z(s,\sigma_n)\right]$$

Finally, in any strategy profile where agents gather η_l before contracting and accept the contract $b = (\boldsymbol{\eta},\mathbf{a},\mathbf{z})$ if and only if he observes the subset $\hat{\Theta}$ of possible realizations, a principal offering b obtains $\Pi(b,l,n)$

$$\Pi(b,l) = -\left[z(0,\sigma_n) + \sum_{\sigma_n\in\hat{\Theta}, s\in S} g(\sigma_n)p_l(s\,|\sigma_n)z(s,\sigma_n)\right]$$

A *competitive equilibrium* is a Bayesian perfect equilibrium of the game just described.

In the next sections, we shall characterize agents and principals equilibrium strategies and equilibrium contracts. Preliminarily, we describe the properties of first best allocations.

3 The value of information in the first best

Information has positive operational value, to the extent that it allows to take investment decisions better suited to the circumstances. In order to precisely define the operational value of information, let $a_l^*(\sigma_n) = \arg\max \sum_{s\in S} p_l(s\,|\sigma_n)r(a(\sigma_n),s)$ with

$$r(\eta_l) = \sum_{n\in N} g_l(\sigma_n)\sum_{s\in S} p_l(s\,|\sigma_n)r(a^*(\sigma_n),s).$$

The operational *gross* value of η_l can then be formally defined as the difference $\Delta r(\eta_l) = r(\eta_l) - r(\eta_0)$. While its net value is $\Delta r(\eta_l) - c_\tau(\eta_l)$.

By the Blackwell Sufficiency Theorem, the gross operational value of information is increasing in l (in the informativeness of the signal), as less noisy induce the agent to invest more (respectively, less) in states where their technology is more (respectively, less) productive. It is also well known in the literature that if signals and states are affiliated random variable (i.e. satisfy the monotone likelihood ratio property),

[11] As we explained before, the signal gathered by the agent at stage τ is not necessarily the one prescribed by the contract.

$\Delta r_a(s) = r_a(a,s) - r_a(a,s+1)$ is increasing (resp.decreasing) in s, while the optimal action $a^*(\sigma_n)$ is increasing (resp. decreasing) in σ_n , (see Athey (1997) and Jewitt (1987)). Intuitively, under increasing differences, larger values of σ indicates that the expected productivity of r is larger than initially assessed , and this leads the agent to invest more. The opposite holds true under decreasing differences.

The next result, which is proved in Persico (2000) and will be used in the next sections, indicates that information has larger operational value when optimal actions are more "risk sensitive", which is when the schedule of actions, $a^{FB}(s)$ that maximize the return function $r(a,s)$in each state is steeper.

Proposition 1 *For any pair of return functions $\hat{r}(a,s)$ and $\tilde{r}(a,s)$ such that $sign(\Delta\hat{r}_a(s)) = sign(\Delta\tilde{r}_a(s))$ and $|\Delta\hat{r}(a,s)| > |\Delta\tilde{r}(a,s)|$, one has $\hat{r}(\eta_l) > \tilde{r}(\eta_l)$.*

Intuitively, the larger are the differences $\left|a^{FB}(s+1) - a^{FB}(s)\right|$, the larger is the increase in the expected return that the agent obtains by gathering a signal which allows to take a better suited action (while information has zero operational value if the optimal action is state independent : $a^{FB}(s+1) - a^{FB}(s) = 0$).

As it was first explained by Hirshleifer (1971), a signal may also have strategic value independently from its operational value, since it may increase agents' expected gains from trading. The information that the signal conveys, indeed, provides the agent the opportunity to accept a contract only when he observe news indicating that the probability of receiving a positive transfer from the contract is sufficiently large. In general, however, whether in equilibrium precontractual information has positive strategic value for the agent depends on the set of contracts offered in the market. Moreover, if information gathering activities are publicly observable, in order to discourage precontractual information gathering activities, principals can offer different contracts to informed and non informed agents, and to agents who have observed different news before contracting. Next proposition summarizes the main properties of the first best competitive equilibrium and states that under symmetric information principals precontractual information has never a positive strategic value, and hence is never acquired at equilibrium.

Proposition 2 *If information gathering activities are publicly verifiable, the set of competitive equilibria coincides with that of first best allocations preferred by the agents. Moreover, in a competitive equilibrium (i) intermediaries make zero profit and offer different contracts to agents gathering information before and after contracting, respectively; (ii) $a^*_\eta(\sigma_n)$ maximizes the operational value of information; and (iii) agents do not gather any signal before contracting.*

The formal proof of the proposition is omitted for brevity as it relies on standard undercutting arguments; we now detail informally the arguments of the proof. Under symmetric information, intermediaries' competitive (undercutting) behavior leads the agents to appropriate all the potential gains from trades so that properties (i) and (ii) hold in equilibrium. Moreover there exists no equilibrium where : (I) the set of offers includes a subset $B = \{b_{l1}, ..., b_{lN}\}$, of contracts such that the contract b_{ln} is offered to agents who gather η_l and disclose both η_l and σ_n before contracting; (II) a subset of agents gathers the signal, η_l, with $l > 0$, before contracting and sign b_{ln} when observing σ_n. Two different arguments can alternatively be used to explain this result. The first is based on the fact that precontractual information is costlier. Let denote $z_{b_{lm}}$ the vector of state contingent payment of the contract b_{ln}. consider now the contract $\hat{b}$, which prescribes to gather η_l after contracting and pays the vector $z_{b_n} - \varepsilon$ conditional on observing σ_n. Since $\hat{b}$ allows to save the extra cost of precontractual information, for ε sufficiently small, agents prefer not to gather precontractual information and to accept $\hat{b}$, , if this contract is added to the set B of offers. Moreover $\hat{b}$ makes a profit larger than b_n, for all n, hence any intermediary will find it profitable to offer it. [12].The alternative argument that one can use for proving Proposition 2 is based on the insight that precontractual information destroys trading opportunities. If the first best optimal allocations preferred by the agents is not the autarky allocation, which is typically the case whenever agents are risk averse or need funding in order to invest, gathering information before contracting prevents efficient transfers across individual states. Under competition, indeed, each of the contract offered to agents who have observed different news (realizations of a signal) must satisfy a zero profit condition, conditional on those news, thus preventing the equalization of marginal rates of substitution across states.[13] Differently, a contract offered to an agent who gather information after contracting must only meet the ex ante non negative profit non negative profit constraint, And, hence can implement transfers across agents observing different news. In the absence of asymmetric information competition will lead to efficiency and hence equilibrium contracts will deter precontractual information gathering.

4 Private information and incentive compatibility

Differently than in the first best, under private information an agent who acquires the private signal η_l before contracting can apply either for contracts which prescribe to not gather information at any stage or for those prescribing to gather η_l after contracting. This gives rise to a contractual externality which is key in determining of intermediaries' market behavior. To understand its nature, let consider first the simple case where signals have no operational value and agents do not produce. In the absence of asymmetric information, risk-averse agents obtain in equilibrium the state independent a contract b implementing first best allocation $x(\sigma_n, s) = \bar{x} = \sum p_s w_s$, for all s, , and do not gather information in any stage. Consider now the same environment but assume that agents can gather private information before contracting. An agent who gather η_l and observes σ_n before contracting will accept b if and only if $U(\bar{x} - c_1(\eta_l)) > \sum\limits_{s \in S,} p_l(s\,|\sigma_n) U(w_s - c_1(\eta_l))$. Therefore, agents will find it rational to acquire precontractual information if and only if

[12] A version of this argument is developped by Khalil-Kremer (1991) to prove that in a principal agent model wih asymmetric inofrmatin agents never gather precontractual information.

[13] It is worth to note that under symmetric information the result that precontractual information gathering is not an equilibrium outcome holds in a very large class of economies. Indeed, it remains valid also in settings where agents and firms are heterogenous and it may be efficient to acquire information before choosing the appropriate partners in production activities. It suffices that before gathering the information necessary to choose the partner, agents can sign a financial contract, whose payoffs are appropriately made contingent on the information obtained Under perfect competition among intermediaries such a contract will indeed be offered in equilibrium.

$$\sum_{n\in N,} g(\sigma_n)\max\left\{U(\bar{x}-c_1(\eta_l)),\sum_{s\in S,} p_l(s\,|\sigma_n)U(w_s-c_1(\eta_l))\right\} > U(\bar{x}) \text{ for some } l>0$$

For $c_1(\eta_l))=0$ and η_l perfectly informative, this inequality is satisfied since $\sum_{s\in S,} p_l(s\,|\sigma_n)w_s > \bar{x}$ for all n sufficiently large.

In words, agents who can obtain perfect information for free will accept b only when they discover that their endowment is low and they can receive a sure positive net payment form the contract. According to the same logic, agents who are offered a full insurance contract prefer to acquire private information for $c_1(\eta_l))$ sufficiently small and η_l sufficiently informative, and accept b only after observing sufficiently low values of the realizations σ_n. The counterpart of this fact is that, under private access to information, a principal offering the full insurance contract b would attract a pool of clients of worse quality (having higher probability of facing a bad states) than under public information. In other words, access to private information reduces the expected profit they can make on the contracts they offer.

More generally, agents' incentives to gather private information before contracting depend on the features of the whole set of contracts they can apply for. Sequentially rational principals, however, recognize the effects of contractual externalities created by agents' access to precontractual information and can appropriately design contracts (choose their set of offers) in order to *protect* themselves against these effects. By the revelation principle, the analysis of intermediaries' *market* behavior under asymmetric information can then be simplified by assuming that only contracts satisfying appropriate incentive compatibility conditions, which induce the agents to follow contractual prescriptions, are offered in equilibrium. In the rest of this section, we derive these conditions.

An agent, who gathers η_l before contracting (in the interim stage $\tau=1$) and observes σ_n, discovers some characteristics about his true type which are summarized by the bidimensional vector (l,n) with $l\geq 1$. The set of possible *interim types of the agent* is then $T=\{...(n,l),...\}$, with $n=1,...,N$ and $l=1,...,N$. To any finite set of exclusive contracts B offered in the market it corresponds a set of allocations, $X(n,l\,|B\,)$ that an agent with interim type (n,l) can obtain in the market either by disclosing the information he has gathered at the contracting stage, or by pretending to be uninformed at that stage, respectively.

Formally,

$$X(n,l\,|B\,)=\left\{\begin{array}{c}(x(\sigma_n)):\exists b, \text{ such that } x(s,\sigma_n)\ =w_s-c_\tau(\eta_l)+r(s,a(\sigma_n))+z(s,\sigma_n)\\ \text{and } \eta=(\eta_l,\eta_0) \text{ or } \eta=(\eta_0,\eta_l)\end{array}\right\}$$

The expected utility that an interim type (n,l) can obtain by trading is thus equal to :

$$V(X(n,l\,|B\,))=\sum_s \max_{\mathbf{x}\in X(n,l|B\,)} p_l(s\,|\sigma_n)U(x(\sigma_n,s))$$

.

Thus, a contract, b that prescribes not to gather information before contracting and implements the allocation x=(..., $x(\sigma_0, s)$,...)attracts only non informed agents, if and only :

$$(1) \qquad \sum_{s,n} p_0(s,\sigma_n)U(x(\sigma_n,s)) \geq \sum_n g(\sigma_n)V(X(n,l\,|B\,)) \text{ for all } l$$

Similarly, a contract implementing x= (..., $x(\sigma_n, s)$, ...), that prescribes to gather $\eta_{\hat{l}}$ with $\hat{l} > 0$ after contracting, attracts non informed agents only, if :

$$(2) \qquad \sum_{s,n} p_{\hat{l}}(s,\sigma_n)U(x(\sigma_n,s)) \geq \sum_n g(\sigma_n)V(X(n,\hat{l}\,|B\,)$$

Noteworthy, the incentive constraints defined by (1) and (2) differ in that (1) must hold for all l while (2) only for $l = \hat{l}$ Indeed, (1) takes into account that an agent who accepts a contract prescribing not to gather information at any stage may have chosen any possible signal before contracting; while (2) reflects the fact that an agent who must disclose η_l after contracting may possibly have gathered this signal, but not anyone else, before contracting.

Let $IC_0(B))$ and $IC_{\eta_l}(B))$ be the set of allocations satisfying (1) and (2), respectively, when the set of contracts B is offered in the market. The following lemma will be useful in the characterization of the competitive equilibrium. It states that the larger is the set of contracts offered in the market, the smaller are $IC_0(B))$ and $IC_{\eta_l}(B))$.

Lemma 3 *For any pair B' and B'', with $B' \subset B''$, $IC_{\eta_l}(B')) \subseteq IC_{\eta_l}(B''))$).*

Intuitively, when a larger set of offers is available to informed agents, the ex ante option value of private information increases and this makes incentive constraints more severe.

5 Competitive equilibria

Let $x_i(n,l) = (\ldots.., x_i(n,s,l), \ldots) \in \Re^S$ be the (interim efficient) allocation most preferred by an agent with *interim type* (n,l) in the set of allocations satisfying the (interim) non negative profit constraint :

$$\sum_{n \in N, s \in S} p_l(s\,|\sigma_n\,)(x(\sigma_n,s) - w_s + r(a_l^*(\sigma_n),s) - c_1(\eta_l)) = 0,$$

with $a_l^*(\sigma_n) = \arg\max \sum_{n \in N, s \in S} p_l(s\,|\sigma_n\,) r(a_l(\sigma_n),s)$; and define $X^I = \{\ldots, \mathbf{x}_i(n,l) \ldots\}$.

Since in our setting preferences are strictly convex and state independent, the interim efficient allocation is such that consumption is constant across individual states, which is $x_i(n,,s,l) = x_i(n,l)$. Thus, for notational simplicity, in the following we shall omit the reference to the individual state s.

Let Π_l be the set of allocations satisfying the ex ante non negative profit constraint :

$$\sum_{n\in N,s\in S} g(\sigma_n)p_l(s\,|\sigma_n)(x(n,l)-w_s+r(a_l^*(\sigma_n),s)-c_2(\eta_l))=0,$$

on agents gathering and disclosing η_l after contracting.

Finally define $b_i(n,l)$ the contract supporting the interim efficient allocation $x_i(n,l)$, with $B^I = \{...,b_i(n,l),...\}$.

The next proposition shows that there exists a competitive equilibrium where the whole set of interim efficient zero profit contracts in B^I is offered in the market, together with the contract b^e. This contract prescribes not to gather information before contracting, sets the efficient action $a_l^e(\sigma_n) = a_l^*(\sigma_n)$, given the information acquired after contracting, and maximizes the agent expected utility subject to the (ex ante) non negative profit and to the incentive constraints, where constraints take into account the fact that all interim efficient allocations are offered. In this equilibrium, all agents accept b^e. Finally, all competitive equilibria in which principals do not offer contracts making negative profit out of equilibrium are payoff equivalent.

Proposition 4 *In all competitive equilibria, agents do not gather precontractual information and choose the vector $a_l^e = a_l^*$ of state contingent actions. Moreover, there exists a competitive equilibrium in which agents gather, after contracting, the signal η_l^e, and consume the allocation $\mathbf{x}^e$ solving :*

$$\max_{\mathbf{x},\,\boldsymbol{\eta}_l}\sum_{s,n} p_l(s,\sigma_n)U(x(\sigma_n,s)) \qquad s.t.\;\; \mathbf{x}^e\in IC_{\eta_l}(\{b^e\}\cup\; B^i)\cap\Pi_{\eta_l}. \tag{3}$$

In this equilibrium a non empty subset of principals offer the set B^i of contracts together with the contract $b^e \equiv (z^e,\eta^e,\,a_l^e)$ such that : $\eta^e = (\eta_0^1,\eta_l^e)$, $a_l^e = a_l^*$, and $z^e(\sigma_n,s) = x^e(s,\sigma_n) - w_s + c_2(\eta_l) - r(a_l^e(\sigma_n,s)$. Finally, agents obtain the same expected utility in each equilibrium where none of the contracts offered make negative profits if accepted.

The set B^I contains all the *best deals* that principals can offer to the agents who acquire and disclose precontractual information. Similarly, b^e is the best contract that principals can offer to non informed agents, since this contract maximizes agents' expected utility under non negative profit constraint and the incentive constraint, when the set B^I of contracts is also offered. Thus, no principal can profitably deviate once b^e and all interim efficient contracts in B^I are offered, neither by attracting informed nor non informed agents. Essentially, interim efficient contracts define the set of competitive threats of being cream-skimmed for the intermediaries who offer contracts to non informed agents. These threats determine the fraction of potential gains from trading that agents and principals can exploit in equilibrium.

Uniqueness also results from competition : competitive pressures lead principals to offer either to informed or to uninformed agents the contracts they prefer in the set satisfying the non negative profit and the incentive constraints. Competitive equilibria must be robust to the introduction of interim efficient contracts (incentive compatible). This is because in equilibrium all contracts prescribing precontractual information, which make positive profits and are accepted by some agent if offered, must be offered by some competing principal.

The equilibrium feature that some contracts are offered but not accepted by any agent is a by-product of Bertrand-like undercutting behavior in several standard settings, and needs not be taken literally. In our set-up, this result rests upon the specific contracting assumptions we imposed in order to simplify the description of the game. It is possible to show that if one allows both principals and agents to make contractual proposals, the equilibrium allocation remains the same described above; but it can be supported by intermediaries' strategies such that no contract which is not accepted is offered in equilibrium.In addition, focusing attention on equilibria in which contracts making negative profit are not offered seems realistic and can be formally justified by a trembling hand argument.

Finally, Corollary 3 and Proposition 4 imply that competitive equilibria are not second best. In particular, let $L(l') = l$ for $\eta_l > \eta_0$, $L(l') = L$ for $\eta_l = \eta_0$, and $\Delta c(\eta_{l'}) = c_1(\eta_{l'}) - c_2(\eta_l)$, the set of second-best efficient allocations maximize the agent's expected utility under the incentive constraint

$$\max_{l' \in L(l')} \sum_{n \in N,} g(\sigma_n) \max \left\{ \sum_{s \in S} p_{l'}(s \,|\sigma_n) U(x(\sigma_n, s) - \Delta c(\eta_{l'})), U(w_s) \right\} - \sum_{s \in S,} p_l(s, \sigma_n) U(x(\sigma_n, s)) \leq 0 \ .$$

and the feasibility constraint :

$$\sum_{n \in N, s \in S} g(\sigma_n) p_l(s \,|\sigma_n)(x(n, l) - w_s + r(a_l^*(\sigma_n), s) - c_2(\eta_l)) = 0,$$

Since,

$$\max \left\{ \sum_{s \in S} p_{l'}(s \,|\sigma_n) U(x(\sigma_n, s) - \Delta c(\eta_{l'})), \sum_{s \in S} p_{l'}(s \,|\sigma_n) U(w_s) \right\} <$$
$$\max \left\{ \sum_{s \in S} p_{l'}(s \,|\sigma_n) U(x(\sigma_n, s) - \Delta c(\eta_{l'})), U(x_i(n, l')) \right\}$$

by risk aversion, competition makes incentive constraints more stringent.

Throughout, we shall focus on equilibrium contract. The reader can easily verify that all the characterization results stated in the following sections hold also in the second best.

6 Equilibrium payments' schemes and information gathering choices

Throughout this section we shall investigate the properties of the equilibrium payments' scheme received by the agent and his information gathering choices.

Equilibrium payments' scheme By proposition (4), the incentive constraint can be written as :

$$\max_{l' \in L(l')} \sum_{n \in N,} g(\sigma_n) \max \left\{ \sum_{s \in S} p_{l'}(s \,| \sigma_n) U(x(\sigma_n, s) - \Delta c(\eta_{l'})), U(x_i(n, l')) \right\} +$$

$$- \sum_{s \in S,} p_l(s, \sigma_n) U(x(\sigma_n, s)) \leq 0$$

where $L(l') = l$ for $\eta_l > \eta_0$, $L(l') = L$ for $\eta_l = \eta_0$. and $\Delta c(\eta_{l'}) = c_1(\eta_{l'}) - c_2(\eta_l)$[14].

The left-hand-side of this inequality represents the agent's option value of acquiring precontractual information. For simplicity, we shall assume in the following that there exists an unique signal $\eta_{l'} = \eta_{l^o}$ maximizing this value. Let $x^i(l^o)$, be the allocation such that an agent agent gathering η_{l^o} receives his interim efficient consumption $x_i(n, l^o)$ contingent on each possible σ_n.

To begin understanding the properties of incentive compatible allocations, note that, since $\Delta c(\eta_l) > 0$ for all l, any allocation in a sufficiently small neighborhood of $x^i(l^o)$, is incentive compatible; while only allocations belonging to a small neighborhood of $x^i(l^o)$ satisfy the incentive constraint for small values of $\Delta c(\eta_l)$.

Consider now the first best allocation, x^{FB}, which is constant across states and signals realizations because of agents' risk aversion. As one can readily verify, the option value of precontractual information is negative in x^{FB}, if and only if the extra cost, of precontractual information $\Delta c(\eta_l)$ is larger than a threshold value $\overline{\Delta c}(\eta_l) > 0$ for all l. In the following, we shall study the more interesting class of situations where $\Delta c(\eta_l) < \overline{\Delta c}(\eta_l)$. In this case, satisfying incentive conditions requires agents to consume more in states in which their produced and non produced wealth is relatively larger, or equivalently, transfers across states to be rationed (smaller than in the first best). To explain why, it is useful to consider the option value of the signal η_{l^o} as an average reflecting gains and losses that an agent gathering η_{l^o} before contracting makes, after observing good and bad news, respectively. Precisely, in our environment good news corresponds to realizations of η_{l^o} belonging to the subset Θ_1 such that the (expected) produced and non produced wealth conditional on σ_n, equal to $x_i(n, l^o)$, is larger than x^{FB} for all σ_n in Θ_1. While bad news corresponds to realizations of η_{l^o} belonging to the subset Θ_2 such that $x_i(n, l^o) < x^{FB}$ for all σ_n in Θ_2. Consider now a situation where principals offer interim efficient contracts to informed agents and the a contract implementing the allocation $x = (..., x(\sigma_n, s), ...)$ and prescribes to gather η_l after contracting. Conditional on gathering η_{l^o} and observing $\sigma_n \in \Theta_1$, the agent prefers to disclose at the contracting stage his information and apply for the interim efficient contract which guarantees either non negative expected transfers or allocative efficiency conditional on σ_n . Thus, his *utility gain conditional* on good news is equal to :

$$G(l^o) = \sum_{\sigma_n \in \Theta_1} g(\sigma_n)[U(x_i(n, l^o) - \sum_{s \in S} p_{l^o}(s \,| \sigma_n) U(x(\sigma_n, s)))] > 0$$

[14]Note that $\Delta c(\eta_{l'}) = c_1(\eta_{l'}) - c_2(\eta_l)$ is equal to $c_1(\eta_l) - c_2(\eta_l)$ for $l > 0$ while it is equal to $c_1(\eta_{l'})$ for $l = 0$.

On the contrary, an agent who gathers η_{l^o}, and observes bad news, ends up by choosing the same contract that an uninformed agent would choose, as this allows him to obtain a positive transfer. Hence, conditional on bad news (on $\sigma_n \in \Theta_2$), his investment in information yields an *utility loss*, since earlier information is costly and yields no benefit. Precisely, this loss is equal to

$$L(l^o) = \sum_{\sigma_n \in \Theta_2,} g_l(\sigma_n)[\sum_{s \in S} p_{l^o}(s\,|\sigma_n)(U(x(\sigma_n, s) - (U(x(\sigma_n, s) - \Delta c(\eta_{l^o})] < 0.$$

As next proposition will show, whenever $G(l^o) + L(l^o) > 0$ in the first best, then the equilibrium expected consumption conditional on good news must be larger than in the first best (e.g., transfers from good to bad states must be reduced), in order to satisfy incentive conditions. Intuitively, this is true because $G(l^o)$ is increasing in the difference between the expected produced and non produced wealth that the agent receives conditional on good news, $\sum_{\sigma_n \in \Theta_1} g(\sigma_n) x_i(n, l^o)$, and $\sum_{\sigma_n \in \Theta_1} g(\sigma_n) \sum_{s \in S} p_{l^o}(s\,|\sigma_n) x(\sigma_n, s)$. Hence, reducing consumption conditional on good news reduces the gains from precontractual information.

More precisely, the equilibrium payment scheme of the agent is designed by competing intermediaries to deter precontractual information, by imposing the minimal welfare cos to the agent at the same time. In particular, the next proposition proves that the minimization of second best losses will lead principals either to ration agents' transfers towards the contingencies where bad news are observed or to offer consumption schedules which depend on the news that the agent acquires after contracting. Remarkably the equilibrium contract has also very simple properties, as it induces a completely flat consumption for all values of s and σ_n such that the agent receives a positive transfers in equilibrium, and at most two different values of contingent consumptions conditional on receiving a negative transfer. In addition, if the extra cost of earlier information is not too large or risk aversion is not "too decreasing with wealth", agents obtain a completely flat consumption schedule also for all "contingencies" (σ_n, s) in which their transfer is negative.

Denote $\xi(x) = \partial(u''(x)/u'(x))\partial x$ the first derivative of the agents' Arrow-Pratt risk aversion index.

Proposition 5 *whenever $\eta_l > \eta_0$ in equilibrium, the equilibrium allocation x^e is such $x((\sigma_n, s) = \bar{x}$ for all (σ_n, s) such that $z(\sigma_n, s) > 0$. Moreover,, the agent's consumption schedule satisfies the following properties (i) $x(\sigma_n, s) \geq x(\sigma_{n'}, s')$ whenever $s \geq s'$; (ii) for all (σ_n, η_l) such that $z(\sigma_n, s) < 0$, there exist at most two different values $\tilde{x}$ and $\underline{x}$ with $\bar{x} > \tilde{x} \geq \underline{x}$ and an individual state $\tilde{s}$ such that $x(\sigma_n, s) = \tilde{x}$ for all $s \geq \tilde{s}$ and $x(\sigma_n, s) = \underline{x}$ for all $s < \tilde{s}$;(iii) finally, $\tilde{x} = \underline{x}$ whenever $\xi(x) > -k$ with $0 < k < \bar{k}$, for some positive $\bar{k}$, or $\Delta c(\eta_l)$ sufficiently small.*

Providing a sure consumption $\bar{x}$ for all $\sigma_n \in \Theta_1$ lessens second best losses and improves incentives at the same time. In particular, a sure consumption reduces the expected utility gains agents obtain by accepting, after having gathered information and observed a particular realization, the corresponding interim efficient contract. Indeed, by risk aversion, lower consumption variability conditional on good news increases the utility that risk averse agents obtains within the contract .

A similar logic explains why $x(\sigma_n, s) = \underline{x}$ for all $\sigma_n \in \Theta_2$, whenever Δc is sufficiently small or $\xi(x)$ is not too negative.If $\xi(x)$ is sufficiently large, however, increasing the variability of agents' consumption conditional on $\sigma_n' \in \Theta_2$ may increase the loss that an agent bears when he gathers precontractual information, and observes bad news. Such a loss is indeed proportional to the agent's variability of consumption conditional on $\sigma_n \in \Theta_2$ whenever risk aversionis decreasing with wealth, and gets larger for larger values of the prudence index.

Finally a interesting feature of the equilibrium contracts characterized in the previous proposition is that they seem largely consistent with real world contracts. For instance, in an insurance context, an equilibrium contract can be easily interpreted as one with fixed deductible and maximal reimbursement.

Equilibrium information gathering choices

We now investigate the determinants of information acquisition choices, and compare private and social incentives to gather information. We shall show that in equilibrium agents always acquire information if the cost of *precontractual* information gathering is not too large for at least one of the available signals. Importantly, this remains true even if some or all signals have a negative net operational value (though this value cannot be too small), provided that a perfectly informative signal does not exist, or it is sufficiently costly. In such a case, information gathering costs paid by the agents in equilibrium results in a pure waste from a social (first best) point of view.

Proposition 6 *Assume that the most informative signal η_L is either sufficiently costly or not perfectly informative. In equilibrium, information gathering choices satisfy the following properties : (i) there exists a real positive number $\overline{c_2}$ such that if $c_2(\eta_l)$ $c_2(\eta_l) < \overline{c_2}$, the agent gathers η_l^e, with $l \geq 1$ in equilibrium; (ii) Assume $\Delta c(\eta_{l'})$ sufficiently small for all l. There exists a a positive and decreasing function $k_l(\Delta c(\eta_{l'}))$, such that if $r(\eta_l) - c_2(\eta_l) > -k_l(\Delta c(\eta_{l'}))$, for all l, the agent gathers η_l^e, with $l \geq 1$ in equilibrium; (iii) no information is gathered in equilibrium if $r(\eta_l) - c_2(\eta_l) < 0$ for all $l > 0$, and there exists δ sufficiently small such that, for each σ_n, $p_l(s/\sigma_n) \geq 1 - \delta$ for some s.*

Notably, these findings contrasts with the seminal contribution Khalil-Kremer (1991). who introduced the issue of the optimal timing of information gathering within a principal-agent problem where only a perfectly informative costly signal is available to the agents and information is not trasmissible. Within this setting, these authors proved that socially wasteful information is never gathered in equilibrium. Proposition 6 implies that this result does not generalize to the case where agents can choose between signals of different quality, or a perfectly informative signal is simply not available, and agents can voluntarily disclose the information they gather.[15]. The main insight behind this result is that acquiring

[15] For the sake of brevity, we only formally prove that that these properties holds in the competitive equilibrium, but exactly the same argument developed in proposition 6 can be applied to show that the same results extend to second best allocations where a single principal can choose the number of contracts offered.Moreover, one can easily check that the results in proposition 6 remain true if agents cannot disclose the information they gather.

and disclosing after contracting information with negative net operational value, while reducing agents expected consumption, makes it easier to deter agents from gathering precontractual information, i.e. it relaxes the incentive constraint. Information disclosure allows to design more powerful incentive schemes, by offering the agent a payment schedule contingent not only on the realization of individual states but also on the news that an agent would observe should he decide to gather precontractual information. Indeed, a contract that prescribes not to gather information neither before nor after trading can only ration transfers across different individual states in order to satisfy incentive constraints (to deter precontractual information gathering). Differently, in order to obtain the same objective, a contract which imposes to gather information and disclose information after contracting can also use this information to implement transfers across *interim types,* e.g. across agents disclosing different news.

As we showed in Proposition 5, making agents' payments dependent on the news they disclose after contracting allows to reduce second best losses. Proposition 6 demonstrates that this reduction may well overtake the loss of consumption that an agent bears when he gathers a signal with negative operational value. In particular, this is always the case when the (extra) cost of precontractual information is sufficiently small. In this case, indeed, a contract that prescribes not to gather precontractual information must necessarily implement very small transfers across states in order to satisfy incentive compatibility. Agents accepting such a contract will thus obtain an utility close to that associated to their autarky allocation. On the other hand, a contract prescribing to gather η_l after contracting yields to the agent an utility larger than that he obtains by gathering the same signal before contracting and accepting the interim effcient contract $b_i(l,n)$ whenever observing σ_n. If η_l is not perfectly informative and has a negative but not too small operational value , agents will then acquire information in equilibrium since the expected utility risk averse agents obtain by consuming $x_i(l,n)$ for each possible σ_n is larger than that they can achieve by consuming their autarky allocation .

A particularly striking example of a situation where acquiring socially wasteful information turns out to be welfare increasing is that where all available signals have either a negligible informational content, or a slightly negative net value , while $c_l + \Delta c(\eta_l)$ is sufficiently small for all l. In this case, indeed, for all $l > 0$ there exists an incentive compatible contract prescribing the agent to gather η_l after contracting, which makes non negative profit and allows the agent to obtain an expected utility *close* to that he would obtain in a competitive first best environment. This is simply all zero profit interim efficient allocations gets close to the first best allocation preferred by the agent when the informational content of the signals becomes negligible. On the other hand, any contract which induces the agent not to gather information must implement an allocation close to autarky when $\Delta c(\eta_l)$ becomes negligible. Thuhe feasible contract maximizing agents' expected utility clearly prescribes socially wasteful information to be gathered in equilibrium. Differently, whenever all available signals are very informative, it becomes approximately equivalent, in terms of the agent's incentives, to condition payments on signals' realizations or on individual states. For this reason, the acquisition of information with negative net value is never imposed in equilibrium.

Finally, the result that socially useless information may be gathered in equilibrium is consistent with what we often observe, for instance, in real health insurance markets Contractual forms which are widespread in these markets indeed often require agents to periodically undertake , after contracting, medical tests with negative net operational value and are simply aimed to obtain better information on the agent's prospective health. Moreover, these contracts contain clauses which impose to revise insurance premium according to the results of the tests. Our results shows that these apparently puzzling features of real contracts can be naturally explained in terms of the advantages of discouraging precontractual information acquisition.

The next corollary immediately follows from propositions 1 and 6.

Corollary 7 *If $(r_a(a,s) - r_a(a,s'))/c(\eta_1) + \Delta c(\eta_1)$ is sufficiently small for all s and s', η_1 is not perfectly informative and $c(\eta_1) + \Delta c$ is also sufficiently small, agents overinvest in information gathering in equilibrium.*

In the following we shall investigates the determinants of information gathering decisions and compare social and private incentives to gather information in the case where information has a positive net operational (social) value. We begin by proving a result that will play a crucial role in the rest of the section. In particular, we will show that under mild assumptions, the expected wealth of the agent conditional on observing a realization larger or equal than any threshold level $\sigma_{\bar{n}}$, with $\bar{n} > 1$, is larger (resp. smaller) under more (resp. less) informative signals.

Let $I(\sigma_n, s) = w_s + r(a_l^*(\sigma_n), s)$, and define

$$EI(\boldsymbol{\sigma}_n \geq \sigma_{\bar{n}} \,|\eta_l\,) = \frac{1}{P(\bar{n})} \sum_{s,n \geq \overline{n}} p_l(s_m, \sigma_n)(I(a(\sigma_n), s))$$

with $P(\bar{n}) = \sum_{s,n \geq \overline{n}} p_l(s_m, \sigma_n)$, the agent's expected wealth conditional on gathering η_l and observing a realization larger or equal than $\boldsymbol{\sigma}_{\bar{n}}$.

Lemma 8 *$EI(\boldsymbol{\sigma}_n \geq \sigma_{\bar{n}} \,|\eta_{l+1}\,) > EI(\boldsymbol{\sigma}_n \geq \sigma_{\bar{n}} \,|\eta_l\,)$ for all l and for all $\bar{n} > 1$. Moreover, there exists a strictly positive vector $\Phi \in \Re^S$ such that $EI(\boldsymbol{\sigma}_n \geq \bar{\sigma}_n \,|\eta_{l+1}\,) - I(\boldsymbol{\sigma}_n \geq \bar{\sigma}_n \,|\eta_l\,) > \sum_s \Phi_s[I_s - I_{s-1}]$ for all s.*

To clarify the economic intuition behind this result, consider the binary case where each signal has only two possible realizations, σ_1 and σ_2, and there exist only two individual states, s_1 and s_2, for each agent. Knowing that signals and states are positively correlated an agent observing the realization σ_2 can update his beliefs, so that by the Bayes rule $p_l(\sigma_2 \,|s_2\,) > p_2$. Moreover, the more informative is the signal the agent has gathered, the more confident he will be that the true state of nature is s_2 after having observed σ_2. Hence, Blackwell sufficiency, together with the assumptions of affiliation of individual states and signals, imply $p_{l+1}(s_2 \,|\sigma_2\,) > p_l(s_2 \,|\sigma_2\,) > p_2$.As a consequence, the agent expected wealth conditional on $\sigma = \sigma_2$ increases with l. Lemma 8 proves that this result generalizes to the case of many states and many signals' realizations. The lemma also states that the increase in expected wealth becomes more important when the slope of the state contingent wealth schedule $w_s + r(a_l^*(\sigma_n), s)$ gets larger.

By taking advantage of the previous lemma, we shall now compare the set of incentive compatible allocations corresponding to different information gathering choices and subsequently characterize equilibrium information gathering choices.For simplicity we shall assume that the numer of signals is sufficiently large, so that for any η_l there exists $\eta_{l'}$ less informative than η_l, such that $|p_{l'}(s\,|\sigma_n) - p_l(s\,|\sigma_n)| < \delta$, and $|c_2(\eta_{l'}) - c_2(\eta_l)| < \varepsilon$, with δ and ε sufficiently small, Specifically, next proposition proves that incentive compatibility constraints associated to more informative signals are more stringent, provided that $w_s + r(a_l^*(\sigma_n), s)$ is sufficiently increasing in s. Intuitively, this is because, by the previous lemma, a more informative signal gathered before contracting allows the agent to obtain in the market a larger consumption than a less informative one conditional on discovering good news. Since the expected net gains from precontractual information are proportional to the expected consumption that the agent can obtain in the market under good news, acquiring information before contracting is more tempting for an agent who is offered a contract prescribing η_l, than for one who is prescribed to gather the less informatve signal η_{l-k}.And hence ensuring incentive compatibility when agents are required to disclose more informative news also requires to ration more severely their trades.

Proposition 9 *If each signal has only two possible realizations, or* $I(\sigma_n, s) - I(\sigma_n, s-1)/(|u''(x)/u'(x)|) > D$, *with* D *sufficiently large for all possible* x *and* s. *For all* $l > 0$, *and for any* $x_l(\sigma_n, s) \in IC_{\eta_l}$, *there exists* $\hat{x}_{l-1}(\sigma_n, s) \in IC_{\eta_{l-1}}$ *such that* $E_{\eta_l} u(x_l(\sigma_n, s)) < E_{\eta_{l-1}} u(\hat{x}_{l-1}(\sigma_n, s))$ *and* $E_{\eta_l} x_l(\sigma_n, s) = E_{\eta_l - 1}(\hat{x}_{l-1}(\sigma_n, s))$.

The reason why the statement of the proposition requires $I_s - I_{s-1}$ to be sufficiently large relatively to the agents' Arrow-Pratt risk aversion index is the following. One cannot exclude a priori that gathering a more informative signal before contracting increases either the expected value or the riskiness of the distribution of consumption that the agent can obtain in the market conditional on observing a realization better than any given σ_n before contracting. When this happens, the former effect makes the option value of more informative signals larger , and hence incentive constraints associated to these signals more stringent, while the latter effect goes in the opposite direction. By the previous lemma, $I_s - I_{s-1}$ sufficiently large for all s with respect to the risk aversion index ensures that the forme effect prevails.

Let η^{FB} be the signal gathered by agents in the first best regime where information gathering activities are fully observable and contractible. Next proposition provides sufficient conditions for agents to underinvest in information gathering in equilibrium. Precisely, it shows that in the competitive equilibrium agents gather less information than in the first best, whenever a non empty subset of signals has positive net value, provided $w_s + r(a_l^*(\sigma_n), s)$ is sufficiently increasing in s or the agent's risk aversion is not too large.

Proposition 10 *Assume that each signal has only two possible realizations, or* $I(\sigma_n, s) - I(\sigma_n, s-1)/(|u''(x)/u'(x)|) > D$, *with* D *sufficiently large for all possible* x *and* s, *then* $\eta_l^e < \eta^{FB}$.

This finding, which is in contrast with all the received literature, follows from the fact that incentive constraints associated to less informative signals are less stringent. Indeed, this incentive effect gives rise to a key second best trade-off. On the one hand, by proposition 8, a contract prescribing to gather a signal less informative than the first best one, η_l^{FB}, allows the agent to obtain a less distorted allocation than the one he would obtain under η_l^{FB}. On the other hand, gathering a signal less informative than η_l^{FB} reduces the expected consumption of the agent. The consumption loss due to the choice of a signal η_l less informative η_l^{FB}, however, becomes of second order when the informativeness of the signal that the agent gathers is only slightly reduced. As a consequence, the incentive effect dominates the wealth effect for small distortions of the signal. In equilibrium, this induces competing principals to offer a contract that prescribes to gather a signal less informative than η_l^{FB}. In other words, the effects of the negative externality created by precontractual information acquisition get larger for more informative signals. As under competition these effects ultimately reduce agents expected utility, in equilibrium competing principals issue contracts requiring less information to be gathered after contracting so as to offer better deals to the agents.

7 Opting-out opportunities and multiple consumption dates

For the sake of parsimony, in the previous sections we restricted attention to an environment where agents consume in a single period. More importantly, did not take into account the possibility that agents opt-out from a contract after having observed payoff relevant news, nor we investigated the effects of opting-out opportunities on prvate incentives to gather private information.

In real insurance and credit markets, agents often sign multi-period contracts which allows them to opt-out after one or more periods, possibly at some cost. Both practitioners and economists have emphasized that, while long term-commitment is often impossible or too costly to enforce, opting- out opportunities may lower the profitability of intermediation activities in insurance and credit markets, and may reduce risk-sharing and consumption smoothing opportunities. For instance in competitive markets, discovering good news on their prospective health offer individuals the opportunity to change insurance company if their premiums are not lowered. But, if low-risk individuals opt-out from contracts offering rates which are fair given initial priors, these contracts become unprofitable. There exists a large economic literature, studying the effects of the unenforceability of long term contracts, in environments where payoff relevant information is revealed to the parties in a contract as time elapses. Opportunities of earlier termination of a long term contract, however, affect also the agents' incentives to gather costly private information after entering the contract, since this information can be used to better evaluate the consequences of opting-out decisions.

The main objective of this section is to show that all the main results of the paper continue to hold in settings where agents can use private information to exploit opting out opportunities in the absence of long term commitment .More precisely, we shall show that private incentives to gather information fall short of (respectively exceed) social incentives under the same conditions, either in environments where long term contracts are enforceable but agents may gather private information before contracting or in those where information can be gathered only after contracting but agents can opt-out from the contract after having gathered information. To begin understanding this poi,nt consider a modified version of the game studied in the previous section where :

· Intermediaries offering contracts designed to attract non informed agents can require their contractual parties to sign these contracts at the initial stage $\tau = 0$ (e.g., before the information gathering stages τ_1 and τ_2.)

· Agents who have accepted a contract at $\tau = 0$ can opt out from this contract at $\tau = \tau_1$, and sign at that stage a new contract which prescribes to gather and disclose information before contracting.

In fact it is straightforward to verify that this game has the same set of equilibrium payoffs as the one analyzed in the previous section have exactly . This is just because, a strategy prescribing to sign a contract before gathering information, and to choose after having gathered information whether to opt-out from this contract is equivalent, from the point of view of the agent, to gather information before contracting and decide subsequently which contract to accept. In other words, having access to private information after contracting, instead of before, does not impose any additional constraint to the agent, in the absence of "long term commitment".

The game satisfying the two assumptions above, however, while capturing the essential aspects of the information gathering problem in the presence of opting-out opportunities, exhibits a somewhat awkward feature: termination decisions by agents take place before any payment of that contract has been delivered and agents have undertaken any production or consumption activity. In fact, in most situations where those opportunities are relevant, an agent's usual course of actions is such that, after having signed a contract, he follows contractual prescriptions, receives (or makes) payments and undertake consumption and production activities for some periods, after which he starts considering early exit options. Usually, this is just because agents receive news or, equivalently the cost of gathering news drops, as time goes by. As a consequence, it is natural to wonder whether the homeomorphism stressed above continue to hold in the presence of multiple consumption dates. In fact it is quite straightforward to verify that all the main results of the previous sections generalize to a setting with multiple consumption dates where long term contracts are enforceable and agents can gather pre-contractual information. Throughout the rest of the section, we investigate how the presence of multiple consumption dates and opting-out opportunities affect equilibrium information gathering strategies and contracts.

Consider an environment where risk-averse agents receive a positive random endowment and consume in two dates : either at the investment stage $\tau = \tau_1$, or in a final stage $\tau = \tau_3$. Denote $(w_{\tau 1,\ldots,},w_{\tau S};$ $p_{\tau 1,\ldots,}p_{\tau S})$ the period τ distribution of the agent's endowment, and x_τ $=(x_{\tau 1,\ldots,}x_{\tau S})$ the period τ state contingent consumption, with $x = (x_{\tau_2}, x_{\tau_3})$. To minimize the differences with the environment studied in the previous sections, we shall assume that agents receive returns in the final stage $\tau = \tau_3$ only, and can gather information in two different stages as before. Moreover, signals provide information only on the individual states of the final stage (exactly as in the previous section), while the return function r and the set of signals also satisfy the same properties as in the previous sections. Intermediaries can offer either two-periods ("long term") contracts, which transfer resources across period and states, or one period contracts which prescribe to gather and disclose information, which transfer resources only across period $\tau = 1$ consumption states. Within this setting, the key assumptions will be that agents can sign a long term contract only at the initial stage, e.g., before gathering information; while, at a lsubsequent stage they decide whether to opt-out from their long term contract and accept a new contract which requires to gather and disclose information . A long term contract $b = (\mathbf{z}, \eta, a)$ pays off in both the consumption dates, so that $\mathbf{z} = (z_{\tau_2}(s), z_1(\sigma_n, s))$; differently a short term contract, $b = (z_{\tau_2}, \eta, a)$ contains presciptions on investment and on the agent information gathering policy same but transfers resources only across states of the final period.

The precise timing of actions is now as follows. At the initial stage, $\tau = 0$, principals offer either two periods contracts designed to attract uninformed agents or one period contracts which pay off only in the second consumption date; agents can accept long term contracts only at this stage. Information can be gathered by the agents either at $\tau = \tau_1$, or at $\tau = \tau_2$ with $\tau_1 < \tau_2$. At $\tau = \bar{\tau}$, with $\tau_1 < \bar{\tau} < \tau_2$, an agent can opt out from long term contract and apply for a short term contract; at this stage he can also disclose the information previously acquired to his contractual counterpart. At τ_2, applications for short term contracts are accepted, and agents receive funds and invest; while at the final stage $\tau = \tau_3$, uncertainty is completely resolved, agents receive their endowment and investment returns, and contracts pay off.

To each agent strategy, it corresponds a particular agents state contingent allocation x which yields to the agent the utility :

$$E_{\eta_l} u(x) = \sum_s p_{\tau_1}(s) U_{\tau_1}(x_{\tau_2}(s)) + \sum_{s,n} p_{l\tau_3}(\sigma_n, s) U_{\tau_3}(x_{\tau_3}(\sigma_n, s))$$

Similarly to the single consumption date set-up, in order to characterize competitive equilibria one need to define the second period interim efficient allocations that competing principals can offer to agents disclosing information. Let $x_{i\tau_3}(n, l)$ be the allocation preferred by the agents in the set defined by the the stage τ_3 non negative profit constraint :

$$\sum_{n \in N, s \in S} p_l(s \,|\sigma_n)(x_{\tau_3}(\sigma_n, s) - w_s + r(a_l^*(\sigma_n), s) - c_1(\eta_l)) = 0$$

Let B_i the set of interim efficient contract . If all these contracts are offered at the initial stage, an agent who accepts the long term contract b implementing the allocation $x = (x_{\tau_1}, x_{\tau_3})$, gathers information at τ_1, and decides whether to opt-out from this contract at this stage, obtains the expected utility $E_{\eta_l} u(x, x_{i\tau_3})$ equal to :

$$\max_{l' \in L(l')} \left\{ \sum_s p_{\tau_1}(s) U_{\tau_1}(x_{\tau_1}(s) - \Delta c(\eta_l)) + \sum_{n \in N,} g(\sigma_n) \max \left[\sum_s p_{l\tau_3}(\sigma_n, s) U_{\tau_3}(x_{\tau_3}(\sigma_n, s)), U(x_{i\tau_3}(n, l') \right] \right\}$$

with $L(l') = l$ if $\eta_l > \eta_0$ *and* $L(l') = L$ if $\eta_l > \eta_0$. Thus if all interim efficient contracts are offeerred, a contract that prescribes not to gather information at $\tau = \tau_1$ and implements the allocation x is incentive compatible if and and only if $E_{\eta_l} u(x, x_{i\tau_3}) \leq E_{\eta_l} u(x)$. By using the argument employed first by Khalil and Kremer, that we exploited to characterize the competitive equilibria of the single consumption date environment, one then shows that agents never gather information at $\tau = \tau_1$, since earlier information is costlier and paying agents more conditional on good news (on high realizations of σ_n) always relaxes incentives. Precisely, it is straightforward to verify that these two facts have the following implication. For any strategy profile such that agents accept in the initial period the contract b, which in the absence of early termination would implement the allocation x, is such that $E_{\eta_l} u(x, x_{i\tau_3}) > E_{\eta_l} u(x)$, and makes non negative profit, there exists another contract b' which makes non negative profits and implements x' such that $E_{\eta_l} u(x, x_{i\tau_3}) < E_{\eta_l} u(x')$ and $E_{\eta_l} u(x', x_{i\tau_3}) < E_{\eta_l} u(x')$.

Then, by the same logic used in the previous sections, it follows that in equilibrium agents never opt-out from the contract they sign in the first period, and the competitive equilibrium allocations maximize the agent expected utility under the ex ante non negative profit constraint and the incentive compatibility constraint. Nevertheless, opting out opportunities not only make first best allocations not incentive compatible, but also lead to suboptimal information acquisition choices. Indeed, equilibrium distortions in information gathering activities remain qualitatively the same described in the previous sections. Specifically, this is for the following reason. The characterization of agents' information gathering decisions presented in the previous sections relies only upon the following key properties of the set of incentive compatible allocations :

(P1) If $c_2(\eta_l)$ or $\Delta c(\eta_l)$ are sufficiently small for all l, and $r(\eta_l) - c_2(\eta_l) > -k(\Delta c(\eta_{l'}))$, with $k(\Delta c(\eta_{l'})) > 0$ and sufficiently small for all l, there exists an incentive compatible allocation b which prescribe to gather information after contracting which is preferred to any incentive compatible contract prescribing not to gather information at any stage.

(P2) If $I(\sigma_n, s) - I(\sigma_n, s-1)/(|u''(x)/u'(x)|) > D$, with D sufficiently large for all possible x and s, then or all $l > 0$, and for any $x_l(\sigma_n, s) \in IC_{\eta_l}$, there exists $\hat{x}_{l-1}(\sigma_n, s) \in IC_{\eta_{l-1}}$ such that $E_{\eta_l} u(x_l(\sigma_n, s)) < E_{\eta_{l-1}} u(\hat{x}_{l-1}(\sigma_n, s))$ and $E_{\eta_l} x_l(\sigma_n, s) = E_{\eta_l - 1}(\hat{x}_{l-1}(\sigma_n, s))$.

By following exactly the same arguments developped in the proofs of Proposition 6, Lemma 8 allows to show that either **(P1)** or **(P2)** continue to hold in the presence of multiple consumption periods, provided that $r(\eta_l)$ and $I(\sigma_n, s)$ are suitably reinterpreted as expected returns and contingent wealth in τ_3, respectively. This, in turn, leads to establish that the statement of Proposition 6 and 9 continue to hold true in the presence of opting out opportunities. Namely, agents overinvest in information gathering if information has negative operational value, asnd its cost is not too large, while they often underinvest in information when they have access to signals with positive operational value.

8 Concluding remarks

Information acquisition may have double edged welfare consequences, and private and social incentives to acquire information often do not coincide. Gathering public information may generate positive externalities whenever information has operational value for many agents, as it is typically case for innovative ideas (Arrow (1971)) On the other hand, since private information reduces the set of mutually profitable trades that two parties can conclude in the market, a negative contractual externality arises whenever an agent can acquire private information on some payoff relevant variable, before deciding whether to enter in a contract, . According to a well established conventional wisdom, this negative contractual externalities may often lead agents to under-invest in information, at least whenever a sufficiently large part of the news gathered by each of them can be kept private. Under private information, however, the design of appropriate incentive schemes aimed at deterring opportunistic behavior starts to play a central role and, we argue, the Pigouvian logic assuming "action taking behavior" may not be appropriate to investigate the effects of negative contractual externalities. Specifically, this paper characterizes competitive equilibrium outcomes and investigates the properties of these schemes under the assumptions that agents may gather information either before or after contracting. We show that equilibrium contracts, appropriately designed to "minimize" the negative welfare of negative contractual externalities, often lead agents to underinvest in private information gathering activities, provided the informational value of information is not too small. The opposite, result, however may hold true when the quality of the signals available to the agents is relatively low and these signals have a net negative operational value. Both these results contrasts with the received literature.

9 Appendix

Proof of lemma 3 The statement simply follows from the definition of $IC_{\eta_l}(B))$ and from the fact that

$$V(X(n,l\left|B'\right.) = \sum_s \max_{\mathbf{x}\in X(n,l|B')} p_l(s\left|\sigma_n\right.)U(x(\sigma_n,s)) \leq V(X(n,l\left|B''\right.)$$

where $V(X(n,l\left|B''\right.) = \sum_s \max_{\mathbf{x}\in X(n,l|B'')} p_l(s\left|\sigma_n\right.)U(x(\sigma_n,s))$.

Proof of proposition 4 To prove that in equilibrium agents do not gather any signal before contracting, one can follow exactly the same lines of the proof of Khalil-Kremer (1992), for this reason the formal proof of this fact is omitted. It is also immediate to verify that $IC_{\eta_l}(B^e)$ is non empty whenever $c_2(\eta_l) < c_1(\eta_l)$.Consider now an incentive compatible contract $b' \equiv (z,\eta,\ a_l \neq a_l^*)$ with $a_l \neq a_l^*$. Since investment is publicly verifiable and does not enter the incentive constraints, there always exists another contract b''=(z,η,a_l^*) that yields the same utility as b' to the agents and an higher profit to the principal. Second, the continuity of the agent's utility function implies that there exists a small vector ε such that b'' =($z+\varepsilon,\eta,a_l^*$) is preferred by both the agent and the principal to b. Any principal can thus profitably deviate from an equilibrium candidate in which agents accept b' by offering b''.We now show that there exists a Bayesian perfect equilibrium where all principals offer the set $B^i \cup b^e$ of contracts while agents accept b^e. This amounts to prove that that there exists no offer b' whose associated allocation x' belongs to $IC_{\eta_l}(B^e \cup \{b^e\}\left\{b'\right\}) \cap \Pi_{\eta l'}$.and is strictly preferred by the agents to x^e.The proof is by contradiction. By lemma 1, $x' \in IC_{\eta_l}(B^e \cup \{b^e\}\left\{b'\right\}) \cap \Pi_{\eta l'}$, implies $x' \in IC_{\eta_l}(B^e \cup \left\{b'\right\}) \cap \Pi_{\eta l'}$ But, in turn, x' preferred to x^e implies that x^e cannot solve program (3).The proof that the agent obtains the same expected utility in all equilibria, provided that none of the contracts offered makes negative profits if accepted, is done by contradiction as well, and is divided in two parts. First, we show that in equilibrium agents cannot obtain an utility larger than that provided by b^e. Suppose that there exists an equilibrium where the set of contracts B' is offered, the agent gathers the signal $\eta_{l'}$ after contracting, and consumes the allocation x' such that $\sum\limits_{s,n} p_{l'}(s,\sigma_n)U(x'(\sigma_n,s)) > \sum\limits_{s,n} p_{l^e}(s,\sigma_n)U(x^e(\sigma_n,s))$. Since x^e solves the equilibrium program in (3), Lemma 1 implies that a subset of interim efficient contracts is not offered in this equilibrium. Now consider the subset of contracts $B'' = \left\{b''_{nl}\right\}_{n=1,\dots,N}^{l=1,\dots L}$ such that for each l and n, b''_{nl} prescribes to gather η_l and disclose η_l and σ_n before contracting, implements $a_l^*(\sigma_n)$, and the allocation $x''_{nl} = x_i(n,l) - \varepsilon$, with ε positive and sufficiently small. Since preferences are continuous, by lemma 1 one has

$$\sum_{s,n} p_{\hat{l}}(s,\sigma_n)U(x'(\sigma_n,s)) < \sum_n g(\sigma_n)V(X(n,\hat{l}\left|B' \cup B''\right.)$$

This inequality together with the definition of B'' implies that, given any equilibrium candidate where the agent obtains an allocation preferred to x^e, there exists a set of contracts which prescribe to gather pre-contractual information, would be accepted by the agents if offered, and, make positive profits if accepted.

Whenever the number of principals is large, at least one of them can profitably deviate from the equilibrium candidate by offering those contracts.Assume now that there exists an equilibrium where the set of contracts $\hat{B}$ is offered, while the agent accepts $\hat{b} \in \hat{B}$, gathers the signal $\eta_{\hat{l}}$ after contracting, and consumes the allocation $\hat{x}$ such that $\sum_{s,n} p_{\hat{l}}(s, \sigma_n) U(\hat{x}(\sigma_n, s)) < \sum_{s,n} p_{l^e}(s, \sigma_n) U(x^e(\sigma_n, s))$. Since $\hat{x}(\sigma_n, s)$ is not a first best allocation, the incentive compatibility condition $\sum_n g(\sigma_n) V(X(n, \hat{l} \,\middle|\, \hat{B}) \leq \sum_{s,n} p_{\hat{l}}(s, \sigma_n) U(\hat{x}(\sigma_n, s))$ must hold as equality in equilibrium. Suppose now that all contracts in $\hat{B}/\hat{b}$ make non negative profits if taken by some informed agent. By definition of B^i one has

$$\sum_n g(\sigma_n) V(X(n, \hat{l} \,\middle|\, \hat{B}) \leq \sum_n g(\sigma_n) V(X(n, \hat{l} \,\middle|\, B^i \cup \left\{\hat{b}\right\}).$$

But then the continuity of the utility function U implies that there exists a contract $b' = (z', a_l^*(\sigma_n), \eta_l^e)$, implementing the allocation $x^e - \varepsilon$, with ε positive and sufficiently small, such that $\sum_{s,n} p_{\hat{l}}(s, \sigma_n) U(x^e(\sigma_n, s) - \varepsilon) > \sum_n g(\sigma_n) V(X(n, \hat{l} \,\middle|\, \hat{B} \cup \left\{\hat{b}\right\} \cup \{b'\})$. As a consequence, $\hat{B}$ must necessarily contain a contract which prescribes to gather precontractual information and makes negative profit if accepted. Were this not true, some principal could profitably deviate from the equilibrium candidate by offering a contract which allows the agent to consume the allocation $x^e - \varepsilon$.

Proof of proposition 5 After straightforward algebraic manipulations, the first order conditions of the equilibrium program for the case where $\eta_l > \eta_0$ can be rewritten as

$$U_x(x(\sigma_n, s)) = \lambda \text{ for all } (\sigma_n, s) \text{ such that } z(\sigma_n, s) > 0$$

and

$$U_x(x(\sigma_n, s) - \mu_l U_x(x(\sigma_n, s) - \Delta c(\eta_l)) = \lambda \text{ for all } (\sigma_n, s) \text{ such that } z(\sigma_n, s) < 0$$

where λ and μ_l are, respectively, the Lagrangian multipliers associated to the non negative profit and the incentive constraints of the equilibrium program. From these conditions, it follows that, if $\boldsymbol{\eta}_l^e$ is such that $l \geq 1$, $x(\sigma_n, s)$ is constant for all (σ_n, s) such that $z(\sigma_n, s) > 0$. The conditions above also imply $x(\sigma_n, s) > x(\sigma_{n'}, s')$ for any pair of vectors (σ_n, s) and $(\sigma_{n'}, s')$ such that $z(\sigma_n, s) > 0$, and $(\sigma_{n'}, s')$ such that $z(\sigma_{n'}, s') < 0$.

Moreover the derivative of the function $G(x(\sigma_n, s)) = U_x(x(\sigma_n, s) - \mu_l U_x(x(\sigma_n, s) - \Delta c(\eta_l))$ may change sign at most once, given the strict concavity of U, Hence there exists at most two values $\tilde{x}$ and $\underline{x}$ of $x(\sigma_n, s)$ satisfying $G(x(\sigma_n, s)) = 0$. Finally, the derivative of the function $G_x(x(\sigma_n, s))$ is strictly positive if $\xi(x) > -k$ or $\Delta c < \zeta$, with ζ sufficiently small. Hence, in this case, $\tilde{x} = \underline{x}$.$\square$

Proof of proposition 6 Consider the more constrained version, of the equilibrium program (3) in which the additional constraint $\eta_l = \hat{\eta}_l$ is imposed. Let denote $\hat{V}(c_2(\hat{\eta}_l))$ the value function of this program parametrized by the cost $c_2(\hat{\eta}_l)$ of acquiring $\hat{\eta}_l$ after contracting . Let $V_e = \max_l \hat{V}(c_2(\eta_l))$

be the expected utility that the agent obtains in equilibrium.

Part (i) The proof is by contradiction. Assume that no information is gathered in equilibrium, neither before nor after contracting; risk aversion, implies that the equilibrium allocation x_e is such that $x_e(\sigma_n, s) = x_e(s)$ for all σ_n. Moreover, as in equilibrium the incentive constraint is met as equality (e.g. the agent is indifferent between gathering and not gathering information before contracting), there exists a signal, $\eta_{l'}$, such that

$$\sum p(s)U(x_e(s)) = \sum g(\sigma_n) \max \left\{ \sum p_{l'}(s \,|\sigma_n)U(x_e(\sigma_n, s)), U(x_i(n,l) \right\}$$

Now consider a contract b' that prescribes to gather $\eta_{l'}$ after contracting and implements the allocation $x' = x_e$. By construction this contract is incentive compatible since it yields an expected utility equal to :

$$V' = \sum p_{l'}(\sigma_n, s)U(x'(\sigma_n, s)) = \sum p_0(s)U(x_e(s)) = V_e.$$

Now, for $c_2(\eta_{l'}) = 0$, one has $\hat{V}_l(c_2(\eta_{l'})) \geq V' = Ve$, with $\hat{V}_l(c_2(\eta_{l'})) > V'$ if x_e does not solve the equilibrium program (3) under the additional constraint $\eta_l = \eta_{l'}$. In addition, exactly the same argument used in proposition (5) implies that the solution x' of this program is such that $x'(\sigma_{n'}, s) \neq x'(\sigma_n, s)$ for at least a pair (n, n'). As a consequence, $x' \neq x_e$, and $V_l(c_2(\eta_{l'})) > Ve$ for $c_2(\eta_{l'}) = 0$. Moreover, since $V_l(c_2(\eta_{l'})$ is continuous in $c_2(\eta_{l'})$, one also has $V_l(c_2(\eta_{l'})) > Ve$ for any $c_2(\eta_{l'})$ sufficiently small. But this implies that agents must necessarily gather information in equilibrium for $c_2(\eta_l)$ sufficiently small.

Part (ii) Let $IC(\eta_l, \Delta c)$ be the set of incentive compatible contracts, parametrized with respect to $\Delta c(\eta_l)$, which prescribe to gather η_l after contracting. For all positive φ, and for all $l > 0$, there exists a sufficiently small value of Δc such that $IC_l(\Delta c(\eta_l)) \subset \eth(\varphi, x^i)$ where $\eth(\varphi, x^i)$ is the Euclidean open ball of diameter φ, centered on the allocation x^i such $x^i(\sigma_n, s) = x_i(n, l)$ for all s. Moreover, for all positive ψ, there exists Δc sufficiently small such that $IC(\eta_0, \Delta c(\eta_l)) \subset \eth(\psi, w)$, where $\eth(\psi, w + r_0)$ is the Euclidean open ball centered on the point $w + r_0 = (..., w_s + r(a_0^*(\sigma_n, s)), ...)$ with diameter ψ. Since the agents prefer x_i to any allocation in $IC(\eta_0, \Delta c(\eta_l))$ whenever $\Delta c(\eta_l)$ sufficiently small, they must necessarily acquire information in equilibrium if $r(\eta_l) - c_2(\eta_l) \geq 0$ for all l. By continuity, for $\Delta c(\eta_l)$ sufficiently small for all l, this remains true whenever $r(\eta_l) - c_2(\eta_l) > -k_l(\Delta c(\eta_l))$, where $k_l(\Delta c(\eta_l))$ is a decreasing function taking positive, but sufficiently small values, in all its domain.

Part (iii) Consider a perfectly informative signal η_l. Note that for any contract b' which prescribes η_l and condition payoffs on both signals realizations and states on nature , there exists another contract b'' whose payoff are contingent only on s which implement the same allocation as b'.Suppose now the equilibrium contract b_e prescribes the agent to gather, after contracting, the perfectly informative signal η_l, and that this signal is such that : $r(\eta_l) - c_2(\eta_l) < 0$; and let x_e be the equilibrium allocation. It is without loss of generality to assume $x_e(\sigma_n, s) = x_e(s)$ for all σ_n. Incentive compatibility then requires :

$$\sum p_s U(x_e(s)) = \sum g(\sigma_n) \max \left\{ \sum p_l(s \,|\sigma_n)U(x_e(\sigma_n, s)), U(x_i(n,l) \right\}.$$

Now consider the contract that b' which prescribes not to gather information at any stage and implements the allocation $x'(\sigma_n, s)$= $x_e(s)$ for all $s \neq S$ and $x'(\sigma_n, S)$=$x_e(s) - (r(\eta_l) - c_2(\eta_l))/P_S$.By construction, one has :

$$\sum p_s U(x'(s)) > \sum g(\sigma_n) \max \left\{ \sum p_l(s\,|\sigma_n) U(x'(\sigma_n, s)), U(x_i(n,l) \right\}.$$

Moreover, since by assumption for all signals $\eta_{l'}$ with $l' > 0$, and for each n, there exists s such that $p_l(s/\sigma_n) \leq 1 - \delta_1$, we also have

$$\sum p_s U(x'(s)) > \max_{l' \in L} \sum g(\sigma_n) \max \left\{ \sum p_{l'}(s\,|\sigma_n) U(x'(\sigma_n, s)), U(x_i(n,l') \right\}.$$

for δ_1 sufficiently small. Thus b' is incentive compatible, makes positive profits and is preferred to b_e. It follows that b_e cannot be the contract accepted by the agents in equilibrium. Finally, a careful continuity argument allows to extend this result and to prove that the equilibrium contract will never prescribe to gather a signal $\eta_{l'}$ with $l' > 0$ such that for each n there exists s such that $p_l(s/\sigma_n) \leq 1 - \delta_1$, and $r(\eta_{l'}) - c_2(\eta_{l'}) < 0$.

Proof of lemma 8 For all $\bar{n} = 2, ..., N$, let define the probability distribution $\gamma_{\bar{n},l}$ as :

$$\gamma_{\bar{n},l} \equiv (\frac{\sum_{n=\bar{n}}^{N} p_l(s_1, \sigma_n)}{P(\bar{n})}, ..., \frac{\sum_{n=\bar{n}}^{N} p_l(s_m, \sigma_n)}{P(\bar{n})}, ..., \frac{\sum_{n=\bar{n}}^{N} p_l(s_S, \sigma_n))}{P(\bar{n})}$$

where $P(\bar{n}) = \sum_{n \geq \bar{n}} p(\sigma_n)$. The proof is divided in two steps. The first, preliminary step proves that $(\gamma_{\bar{n},l}(s_m)/\gamma_{\bar{n},l+1}(s_m))$ is increasing in m, for all $\bar{n}$; which is, $\gamma_{\bar{n},l}$ satisfies the monotone likelihood ratio property. Since $p_l(s_m, \sigma_n) = p_l(\sigma_n/s_m)p(s_m)$, $\gamma_{\bar{n},l+1}(s_m)/\gamma_{\bar{n},l}(s_m) = \sum_{n \geq \bar{n}} [p_{l+1}(\sigma_n/s_m)/p_l(\sigma_n/s_m)]$,

$$sign \left[\frac{\gamma_{\bar{n},l}(s_{m+1})}{\gamma_{\bar{n},l+1}(s_{m+1})} - \frac{\gamma_{\bar{n},l}(s_m)}{\gamma_{\bar{n},l+1}(s_m)} \right] = sign \left[\frac{\sum_{n \geq \bar{n}} p_l(\sigma_n/s_{m+1})}{\sum_{n \geq \bar{n}} p_{l+1}(\sigma_n/s_{m+1})} - \frac{\sum_{n \geq \bar{n}} p_l(\sigma_n/s_m)}{\sum_{n \geq \bar{n}} p_{l+1}(\sigma_n/s_m)} \right]$$

Moreover, as signals are ordered by the Blackwell sufficiency criterion, $p_l(\sigma_n/s_m) = \sum_k b_{nk}^l p_{l+1}(\sigma_k/s_m)$, where b_{mn}^l is the entry corresponding to the $m - th$ row and the $n - th$ column of the stocastic matrix B_l. It follows that

$$\frac{\sum_{n \geq \bar{n}} p_l(\sigma_n/s_{m+1})}{\sum_{k \geq \bar{n}} p_{l+1}(\sigma_k/s_{m+1})} - \frac{\sum_{n \geq \bar{n}} p_l(\sigma_n/s_m)}{\sum_{k \geq \bar{n}} p_{l+1}(\sigma_k/s_m)} =$$

$$\frac{\sum_{n \geq \bar{n}} b_{nk}^l \sum_{k=1}^{N} p_{l+1}(\sigma_k/s_{m+1})}{\sum_{k \geq \bar{n}} p_{l+1}(\sigma_k/s_{m+1})} - \frac{\sum_{n \geq \bar{n}} b_{nk}^l \sum_{k=1}^{N} p_{l+1}(\sigma_k/s_m)}{\sum_{k \geq \bar{n}} p_{l+1}(\sigma_k/s_m)} =$$

$$= \frac{\sum_{n \geq \bar{n}} b_{nk}^l \sum_{k=1}^{\bar{n}-1} p_{l+1}(\sigma_k/s_{m+1})}{\sum_{k \geq \bar{n}} p_{l+1}(\sigma_k/s_{m+1})} - \frac{\sum_{n \geq \bar{n}} b_{nk}^l \sum_{k=1}^{\bar{n}-1} p_{l+1}(\sigma_k/s_m)}{\sum_{k \geq \bar{n}} p_{l+1}(\sigma_k/s_m)} =$$

$$\sum_{n\geq\bar{n}}\sum_{k=1}^{\bar{n}-1}\left[\frac{b_{nk}^{l}p_{l+1}(\sigma_k/s_{m+1})}{\sum_{k\geq\bar{n}}p_{l+1}(\sigma_k/s_{m+1})}-\frac{b_{nk}^{l}p_{l+1}(\sigma_k/s_m)}{\sum_{k\geq\bar{n}}p_{l+1}(\sigma_k/s_m)}\right]$$

Since $p_l(\cdot/s)$ satisfies the monotone likelihood ratio property, the above expression is positive for all m. We shall now complete the proof by showing that $(\gamma_{\bar{n},l}(s_{m+1})/\gamma_{\bar{n},l+1}(s_{m+1}))$ increasing in m for all $\bar{n}$, implies

$$\sum_{s,n\geq\bar{n}}p_{l+1}(s_m,\sigma_n)(I(a(\sigma_n),s_m))>\sum_{s,n\geq\bar{n}}p_k(s_m,\sigma_n\,|\bar{n})(I(a(\sigma_n),s_m)>\sum_{s,n\geq\bar{n}}p_l(s_m,\sigma_n)(I(a(\sigma_n),s_m)$$

where for each $\bar{n}=2,...,N$, $p_k(s_m,\sigma_n\,|\bar{n})$ are the entries of the $\bar{n}\times S$ matrix of joint probabilities $P_k(\bar{n})$ such that :

$$p_k(s_m,\sigma_n\,|\bar{n})=p_l(s_m,\sigma_n)\text{ for all }n\geq\bar{n}+1;$$

$$p_k(s_m,\sigma_{\bar{n}}\,|\bar{n})=p_{l+1}(s_m,\sigma_{\bar{n}})+\sum_{n=\bar{n}+1}^{N}p_{l+1}(s_m,\sigma_n)-\sum_{n=\bar{n}+1}^{N}p_l(s_m,\sigma_n).$$

$\sum_{s,n\geq\bar{n}}p_k(s_m,\sigma_n\,|\bar{n})(I(a(\sigma_n),s_m)>\sum_{s,n\geq\bar{n}}p_l(s_m,\sigma_n)(I(a(\sigma_n),s_m)$ follows from $(\gamma_{\bar{n},l}(s_m)/\gamma_{\bar{n},l+1}(s_m))$ increasing in in m for all $\bar{n}$

It is immediate to verify that $\sum_{s,n\geq\bar{n}}p_{l+1}(s_m,\sigma_n)(I(a(\sigma_n),s_m))>\sum_{s,n\geq\bar{n}}p_k(s_m,\sigma_n\,|\bar{n})(I(a(\sigma_n),s_m)$ for $\bar{n}=N-1$. We shall prove by induction that this inequality holds for all n.

By construction we have,

$$\sum_{m,n\geq\bar{n}=N-m-1}p_{l+1}(s_m,\sigma_n)(I(a(\sigma_n),s_m))-\sum_{m,n\geq\bar{n}==N-m-1}p_k(s_m,\sigma_n\,|\bar{n})(I(a(\sigma_n),s_m)=$$

$$\sum_{m,n\geq\bar{n}=N-m}p_{l+1}(s_m,\sigma_n)(I(a(\sigma_n),s_m))-\sum_{m,n\geq\bar{n}=N-m}p_k(s_m,\sigma_n\,|\bar{n})(I(a(\sigma_n),s_m)+$$

$$\sum_m p_{l+1}(s_m,\sigma_{N-m-1})(I(a(\sigma_{N-m-1}),s_m))-\sum_m p_k(s_m,\sigma_{N-m-1}\,|\bar{n}=N-m-1)(I(a(\sigma_{N-m-1}),s_m)+$$

$$-[-\sum_m p_k(s_m,\sigma_{N-m}\,|\bar{n}=N-m)(I(a(\sigma_{n=N-m}),s_m)+\sum_m p_k(s_m,\sigma_{N-m}\,|\bar{n}=N-m-1)(I(a(\sigma_{n=N-m}),s_m)$$

$$=\sum_{m,n\geq\bar{n}=N-m}p_{l+1}(s_m,\sigma_n)(I(a(\sigma_n),s_m))-\sum_{m,n\geq\bar{n}=N-m}p_k(s_m,\sigma_n\,|\bar{n})(I(a(\sigma_n),s_m)+$$

$$\sum_m p_{l+1}(s_m, \sigma_{n=N-m-1})(I(a(\sigma_{n=N-m-1}), s_m))+$$

$$-\sum_m \left(p_{l+1}(s_m, \sigma_{N-m-1}) + \sum_{n=N-m}^{N} p_{l+1}(s_m, \sigma_n) - \sum_{n=N-m}^{N} p_l(s_m, \sigma_n) \right) (I(a(\sigma_{n\doteq N-m-1}), s_m)+$$

$$-[-\sum_m \left(p_{l+1}(s_m, \sigma_{N-m}) + \sum_{n=N-m+1}^{N} p_{l+1}(s_m, \sigma_n) - \sum_{n=N-m+1}^{N} p_l(s_m, \sigma_n) \right) (I(a(\sigma_{N-m}), s_m)+$$

$$+\sum_m p_l(s_m, \sigma_{N-m} | \bar{n} = N-m-1)(I(a(\sigma_{N-m}), s_m) =$$

$$\sum_{m,n\geq\overline{n}=N-m} p_{l+1}(s_m, \sigma_n)(I(a(\sigma_n), s_m)) - \sum_{m,n\geq\overline{n}=N-m} p_k(s_m, \sigma_n | \bar{n})(I(a(\sigma_n), s_m)+$$

$$-\sum_m \left(\sum_{n=N-m}^{N} p_{l+1}(s_m, \sigma_n) - \sum_{n=N-m}^{N} p_l(s_m, \sigma_n) \right) (I(a(\sigma_{N-m-1}), s_m)+$$

$$-[-\sum_m \left(\sum_{n=N-m}^{N} p_{l+1}(s_m, \sigma_n) - \sum_{n=N-m}^{N} p_l(s_m, \sigma_n) \right) (I(a(\sigma_{N-m}), s_m) =$$

$$\sum_{m,n\geq\overline{n}=N-m} p_{l+1}(s_m, \sigma_n)(I(a(\sigma_n), s_m)) - \sum_{m,n\geq\overline{n}=N-m} p_k(s_m, \sigma_n | \bar{n})(I(a(\sigma_n), s_m)+$$

$$-\sum_m \left(\sum_{n=N-m}^{N} p_{l+1}(s_m, \sigma_n) - \sum_{n=N-m}^{N} p_l(s_m, \sigma_n) \right) [I(a(\sigma_{N-m-1}), s_m) - I(a(\sigma_{N-m}), s_m)]$$

since $[I(a(\sigma_{n\doteq N-m-1}), s_m) - I(a(\sigma_{n=N-m}), s_m)]$ is increasing in m and $\gamma_{l+1}(m) = \sum_{n=N-m}^{N} p_{l+1}(s_m, \sigma_n)$ dominates in the sense of first order stocastc dominance $\gamma_l(m) = \sum_{n=N-m}^{N} p_l(s_m, \sigma_n)$ the second term is positive, thus we can conclude that if

$$\sum_{s,n\geq\overline{n}} p_{l+1}(s_m, \sigma_n)(I(a(\sigma_n), s_m)) > \sum_{s,n\geq\overline{n}} p_k(s_m, \sigma_n | \bar{n})(I(a(\sigma_n), s_m)$$

holds for $\bar{n} = N - m$ it also holds for $N - m - 1$.

Proof of proposition 9 By following the same logic developped in Proposition 5 one proves that, for any l, the contract b_l maximizing the agents' expected utility in the set $IC_{\eta_l}(\{b_l\} \cup B^i) \cap \Pi_{\eta_l}$ implements the allocation x such that : $x(s, \sigma_n) = \bar{x}$ for all $(s, \sigma_n) \in \bar{N}\bar{S}$, $x(\sigma_n, s) = \hat{x}$ for all $(s, \sigma_n) \in \hat{N}\hat{S}$, and

$x(\sigma_n, s) = \underline{x}$; for all $(s, \sigma_n) \in \underline{NS}$, for some partition $\wp = \left\{ \bar{N}\bar{S}\, , \hat{N}\hat{S}, \underline{NS} \right\}$ of the set of states' and signals realizations. Moreover, b_l belongs to the frontier of $IC(\eta_l)$:

$$\sum_{n,s} p_l(\sigma_n, s) U(x(\sigma_n, s)) = \max_{l'} \sum_n g(\sigma_n) \max_n \left\{ \sum_s p_{l'}(s\,|\sigma_n) U(x(\sigma_n, s)), U(x_i(\sigma_n)) \right\}$$

It is also straightforward to verify that if the number of states and signals is sufficiently large, as well as in the case where each signal has only two realizations, there exists a contract $b' = (z', \tilde{\eta}, a)$ with $\tilde{\eta} = (\eta_0, \eta_{l-1})$, which implements the allocation x' such that $\sum_{n,s} p_l(\sigma_n, s) U(x(\sigma_n, s)) = \sum_{n,s} p_{l-1}(\sigma_n, s) U(x'(\sigma_n, s))$ and $\sum_{n,s} p_l(\sigma_n, s) x(\sigma_n, s) = \sum_{n,s} p_{l-1}(\sigma_n, s) x'(\sigma_n, s)$

Moreover, since $x_i(n, l) = \sum_s p_l(s\,|\sigma_n)(w_s + r(a_l(\sigma_n^*), s)$ is increasing in n because signals and states are affiliated random variables, there exists $\bar{n}$ such that
$U(x_i(n, l)) \gtreqless \sum_s p_l(s\,|\sigma_n) U(x(\sigma_n, s)$ if and only if $n \gtreqless \bar{n}$.In addition, lemma 8 implies

$$\sum_{n \geq \bar{n}} g(\sigma_n) \sum_s p_l(s\,|\sigma_n)(w_s + r(a_l(\sigma_n^*), s) > \sum_{n \geq \bar{n}} g(\sigma_n) \sum_s p_{l-1}(s\,|\sigma_n)(w_s + r(a_{l-1}(\sigma_n^*), s)$$

These two inqualities imply that if each signal has only two possible realizations, or $(I_s - I_{s-1})/(|u''(x)/u'(x)|) > D$, with D sufficiently large for all possible x and s, one has :

$$\sum_n g(\sigma_n) \max \left\{ \sum_s p_{l-1}(s\,|\sigma_n) U(x'(\sigma_n, s)), U(x_i(n, l-1)) \right\} < .$$

$$\sum_n g(\sigma_n) \max \left\{ \sum_s p_l(s\,|\sigma_n) U(x(\sigma_n, s)), U(x_i(n, l)) \right\}.$$

Proof of proposition 10 We begin by proving that an incentive compatible contract $(z', \eta', a_{l'})$ that prescribes to gather $\eta_{l'} > \eta_l^{FB}$ after contracting is never accepted in equilibrium, if offered. By proposition 9, for any $\eta_l > \eta^{FB}$ there exists another contract $(\hat{z}, \eta_l^{FB}, a_l^*)$ that prescribes η_l^{FB} implements an allocation $\hat{x}$ belonging to the interior of $IC(\eta_l^{FB})$ which satisfies the following inequalities : $\sum_{n,s} p_{l'}(\sigma_n, s) U(x'(\sigma_n, s)) < \sum_{n,s} p_{l^{FB}}(\sigma_n, s) U(\hat{x}(\sigma_n, s))$ and $\sum_{n,s} p_{l'}(\sigma_n, s) x'(\sigma_n, s) > \sum_{n,s} p_{l^{FB}}(\sigma_n, s)\hat{x}(\sigma_n, s)$. As a consequence, starting from any equilibrium candidate such that $(z', \eta', a_{l'})$ is the contract preferred by the agent within the set of existing offers , any intermediary can profitably deviate by offering $(\hat{z} - \varepsilon, \eta_l^{Fb}, a_{l^{FB}})$, where ε is a positive and sufficiently small real number. Such an offer would indeed be accepted by the agent in the subgame following the deviation.

Consider now any contract $b = (z, \eta_l^{FB}, a_l^*)$ which prescibes the first best optimal signal η_l^{FB} and let x be the allocation associated with this contract. By proposition 9, there exists $\hat{b} = (\hat{z}, \hat{\boldsymbol{\eta}}_l, \hat{a}_l)$, with $\hat{\eta}_l$

smaller but sufficiently close to η_l^{FB}, which implements the allocation $\hat{x}$ belonging to the interior of $IC(\hat{\boldsymbol{\eta}}_l)$ and is such that $\sum_{n,s} p_{l^{\Phi B\prime}}(\sigma_n, s)U(x(\sigma_n, s)) < \sum_{n,s} p_{l^{FB}}(\sigma_n, s)U(\hat{x}(\sigma_n, s))$ and $\sum_{n,s} p_{l^{FB}}(\sigma_n, s)x'(\sigma_n, s) > \sum_{n,s} p_{l^{FB}}(\sigma_n, s)\hat{x}(\sigma_n, s)$. Moreover, if $p_{l^{FB}}(\sigma_n, s) - p_{\hat{l}}(\sigma_n, s) < \delta$ with δ sufficiently small, for all σ_n and s, $Er(\eta_l^{FB}) - Er(\hat{\eta}_l) \approx 0$ since the change in the expected return is of second order with respect to δ around the first best optimal signal, η_l^{FB}.The contractual proposal $(\hat{z}, \hat{\eta}_l, \hat{a}_l)$, then is incentive compatible, makes strictly positive profits and yields the agent an higher utility than $(x, \eta_l^{FB}, a_l^{FB})$. As a consequence, $(x, \eta_l^{FB}, a_l^{FB})$ cannot be an equilibrium outcome and the signal that agents gather in equilibrium must necessarily be less informative than η_l^{FB}.

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