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Financial Intermediation and Growth: Long Run Consequences of Capital Market Imperfections

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Abstract

The model describes an economy in which banks develop in order to meet the entrepreneurs' demand of capital. Domestic savers can lend in the informal credit market where they have to bear some risk; they can also save in a safe bank account. Banks cannot perfectly check the choices of borrowers, hence they ask for a collateral. Therefore, small firms borrow in the informal market where costly information can be obtained.
The long run steady state is determined by the initial distribution of wealth and aggregate wealth. The economy may eventually stop growing, and the banking system will fail to develop. Alternatively, banks may progressively dominate the financial system and the economy will reach a stable positive rate of growth.

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1. INTRODUCTION

What triggers the development of a modern banking system? This issue is essential to fully understand the process of economic development. Moreover, potential feedback effects on growth are in the heart of policy and academic debates. On the policy side, the financial repression argument, which was formulated first by Mac Kinnon and Shaw (see also Roubini and Sala-i-Martin [1992]), has partially offset a more specific difficulty of development programs in providing cheap credit in rural and poor areas of developing countries. During this period, informal credit institutions have been thriving in many developing countries; this observation has led to numerous microeconomic studies (see for instance the World Bank Economic Review [September 1990]). What is the role of an informal credit market? Why do banks succeed or not in expanding their activities? The paper takes a first step in the direction of analyzing those issues in a dynamic macroeconomic model, where the evolution of financial institutions is the result of agents' decisions, and in turn affects the rate of growth.

The structure of the model is the following. Consider a population of entrepreneurs looking for outside finance. They can borrow either in the "informal" credit market, or from a bank[1]. Information is costly: a moneylender in the informal market can expend time and effort to acquire complete information about the borrower, but it is more difficult to verify the activities of an entrepreneur when the size of the firm increases. On the contrary, banks have to rely on collateral to provide incentives, whereas their average lending cost does not depend on the size of the loan. Nevertheless, banks can perfectly diversify risk which is not possible in the informal credit market: risk averse savers take this feature into account when they decide how to allocate their wealth. Therefore, the following pattern is derived. Small firms seek finance in the informal market whereas banks provide capital to large firms: the current distribution of assets, the resulting demand of capital, and the aggregate supply of capital determine the size of the informal and the banking sectors. The result is obtained for an economy where the informal credit market can only rely on domestic savings, whereas banks can borrow or lend in the international capital markets.

Consequently, the whole development path of the economy is characterized by the initial distribution of assets and aggregate wealth. The link between the size of the banking sector and the rate of growth depends on the relative cost of banks' and moneylenders' activities. However, a necessary condition for long run growth is that a banking system develops. We choose to concentrate on a simple example where two steady states exist: (1) the banking sector does not develop

[1]There is no stock market: we do not concentrate on developed economies.

in the long run and the economy reaches a stable aggregate wealth level. (2) The banking sector progressively develops and eventually dominates the financial system; in that case, the economy grows at a constant and positive rate in the long run.

The model is related to the papers that analyze the effect of the financial structure on the rate of growth (see Pagano [1993] for a survey). Two basic arguments can be identified in this literature. Financial institutions (banks or a stock market) stimulate economic growth because (1) they allow a better risk diversification (Bencinvenga and Smith [1993], Saint Paul [1992]), (2) financial intermediaries are more efficient than individual investors in gathering information (Diamond [1984], Greenwood and Jovanovic [1990], King and Levine [1993b], Berthelemy and Varoudakis [1994], Sussman and Zeira [1995]). Our approach departs from these models in two ways.

First, they concentrate on savers-investors who are willing to diversify risk or to acquire information on the potential investment opportunities. Asymmetric information issues caused by the separation between lenders and borrowers cannot be investigated, which is instead crucial to this model[2]; moreover, it potentially introduces a role for the demand of capital.

Second, self-finance cannot be considered as the only alternative to modern financial institutions: there is considerable evidence of local credit institutions that appear prior to banks (see for instance Besley [1995]). The aggregate effects of informal credit institutions have not been analyzed systematically. However, some scattered pieces of evidence do exist. Biggs (quoted by M.F. Fry [1995, p.350]) remarks that "the emergence of a large and thriving curb market has been enormously important to Taiwan's industrial development (...) The curb market complemented the formal credit market by providing information-intensive, efficient credit facilities (...), helped to mobilize domestic savings by offering high returns (although riskier) on investable funds". Lee and Tsai [1988] (quoted by Fry [1995, p.350]) argue that "in Taiwan, curb markets provided 48% of loans to private businesses in 1964. Although this ratio fell to 27% in 1973, it was back at 48% in 1986". In order to be fair, the analysis must therefore compare banks to informal credit institutions.

The model emphasizes a demand-side effect on financial development based on the accumulation of collateral by entrepreneurs. In that sense, it is also related to the literature that studies the long run effects of the wealth distribution when credit markets are imperfect (Aghion and Bolton [1997], Banerjee and Newman [1991, 1993], Galor and Zeira [1993], Piketty [1997]). As in the models of Aghion

[2]Bencinvenga and Smith [1993] make a similar argument; however, they do not consider distributional effects.

and Bolton and of Piketty, the market imperfection is caused by moral hazard. However, we are mainly interested in the evolution of financial institutions over the process of development, whereas they underline the link between economic growth and wealth distribution.

The model yields several empirical predictions. First, it allows to discuss development policies aimed at stimulating the development of modern banking institutions. Second, we give an insight into the predictions relating inequalities and development; more specifically, the model suggests that enough inequalities must exist initially to trigger the development of banks. However, when the economy is getting richer, inequalities must not be too large or else the development of banks may stop.

The paper is organized as follows: section 2 picks up some basic facts about capital markets in developing countries. In section 3, the model is described. In section 4 loan contracts and agents' decisions are characterized. Section 5 solves for the static equilibrium and the dynamics of the economy. Section 6 is devoted to the discussion of empirical predictions.

2. SEGMENTED MARKETS IN DEVELOPING COUNTRIES

As noticed in the introduction, capital markets in developing countries happen to be segmented, and the banking sector often has to compete with a large curb market[3]. This may be surprising because lending interest rates in the banking sector are substantially below those charged in the informal sector : for instance, Nabi [1988] remarks that in the region of Pakistan he studied, the spread is around 20%.

Beyond explanations based on risk diversification (which is also part of our argument), it seems that the cost of acquiring information is crucial to understand why both markets coexist despite the lower interest rates charged by banks. Hoff and Stiglitz [1990] argue that "in developing countries potential lenders vary greatly in their costs of direct screening and monitoring (...). Thus, village lenders often do considerable monitoring, while banks may find it virtually impossible to do so". The analysis of R.M. Townsend [1995, p.1040] suggests indeed that there may be "a rough correlation between shared information, on the one hand, and the existence of informal markets or quasi-formal institutions, on the other".

[3]Examples of informal and semi-formal institutions are: rotating savings and credit associations (Tontines in Senegal, Kye in Korea, etc.), "pawnshops" in the Philippine, landlords, traders, friends, relatives, etc.

Even if local moneylenders seem to have an advantage over banks in reducing asymmetries of information, this cost may partially explain the usurious interest rates that are observed. Siamwalla and al. [1990] claim that in their sample of Thai villages "the informal sector is competitive, and that high interest rates reflect high monitoring costs, not the scarcity of funds". Aleem [1990] reaches a similar conclusion for a region of Pakistan: he shows that "the costs of screening are substantial - on average, screening costs one day of the lender's time and Rs 20 ($2.02) in transportation expenditures - despite the fact that many of the lenders had been operating in the area for periods in excess of five years".

However, as is pointed out by Nabi [1988] and Besley [1995], maintaining nearly perfect information implies a sufficient proximity between the moneylender and the borrowers, which consequently restricts the size of his operations.

The basic framework laid out in the next section tries to capture these features.

3. STRUCTURE OF THE ECONOMY

There is a large population (a continuum) of agents with identical preferences; the population at time t is described by a distribution function $G_t(w)$ which gives the measure of the population with initial wealth less than w. The size of the population is 1.

3.1. The Agents

Agents live two sub-periods of date t. At the beginning of their life they receive a bequest from their parents. The timing is the following: in sub-period 1, agents face idiosyncratic shocks: with probability α_0 an individual has no investment opportunity and thus must lend his endowment; with a probability $\alpha_1 = 1 - \alpha_0$, he has an investment opportunity (capital is the only input). He can either invest his own wealth, or increase the size of his firm by borrowing. He does not consume in this first sub-period. In sub-period 2, productive shocks are realized, output is produced and loans are paid back. Individuals choose how much to consume and to bequeath. They give birth to a single child, and die[4].There is only one good, which can be consumed or invested in its own production (capital totally depreciates in one period).

Individuals are risk averse; preferences over second period consumption c and bequest b are represented by the utility function $U(c,b) = \left(c^{1-\gamma} b^{+\gamma}\right)^a$, where:

[4]This dynastic model has been extensively used in the litterature that studies the links between the wealth distribution and growth: see Banerjee and Newman (1991,1993), Aghion and Bolton (1997), Piketty (1997).

$0 < a < 1$ and: $0 < \gamma < 1$.

Relative risk aversion is constant and the bequest motive is of the "warm-glow" type (Andreoni 1989)[5].Labor supply does not enter in this reduced form of the utility function, and moral hazard is modelled without an explicit effort function.

If w is the final revenue of an individual, then he will consume $(1-\gamma)\cdot w$ and bequeath $\gamma \cdot w$. His expected utility can be written $V(w) = (1-\gamma)^{a\cdot(1-\gamma)} \cdot \gamma^{a\cdot\gamma} \cdot E(w^a) = \Phi \cdot E(w^a)$ (Φ is dropped in the analysis).

3.2. Technologies

There is no aggregate risk. However, the technology is subject to moral hazard as in Holmstrom and Tirole (1997). The entrepreneur can choose between two possible projects: a good project with a high probability of success p, and a bad project with a low probability of success q. Success or failure of the project can be observed at no cost. However the bad project gives a private benefit to the borrower: this is the source of moral hazard. We depart from Holmstrom and Tirole (1997) by assuming that the private benefit is received only if the project is successful[6]. k is the capital invested.

Good project:

$$Y = \begin{cases} Ak \text{ with probability } p \\ 0 \text{ with probability } 1-p \end{cases}$$

Bad project:

$$Y = \begin{cases} Ak \text{ with probability } q, \text{ plus private benefit } \sigma k \\ 0 \text{ with probability } 1-q \end{cases}$$

where $q < p$.

Define R the required rate of return on banks' loans.

Assumptions:

1. $q(A+\sigma) < R < pA$: bad projects are socially inefficient.

2. $p(A-\frac{R}{p})^a < q(A+\sigma-\frac{R}{p})^a$. This guaranties that banks always ask for collateral, as we shall see in section 4.2.

3. $\gamma A < 1$[7]: internal finance cannot lead to an increase in the wealth of an entrepreneurial dynasty. Moreover, the maximum return that savers can receive is bounded by A. Hence, dynasties of savers won't increase their wealth. This

[5] agents care about the amount they bequest, not about the utility of their children.

[6] This assumption is made for technical reasons: it avoids mixing incentive and risk diversification issues.

[7] γA is the return to capital when the project is successful time the bequest rate.

guaranties that a necessary condition for long-run growth (at the individual and macro levels) is that firms increase their size by borrowing.

4. $(\alpha_1 \gamma p \mu + \alpha_0 \nu R)\,\gamma > 1$. The structure of the economy is such that it has the potential to have a positive long run growth rate (μ, to be defined later, is the return to the successful entrepreneur who borrows from a bank).

3.3. Capital Markets

Entrepreneurs willing to borrow have two alternatives: they can either borrow from a bank, or in the "informal" credit market. We call "banks" financial institutions that cannot (perfectly) monitor entrepreneurs: they ask for collateral. "Informal credit institutions" monitor entrepreneurs at a cost that is rising with the size of the project; they cannot rely on collateral.

The Informal Credit Market

Assume that agents are uniformly distributed on a circle[8], such as in the model of Salop, and that anybody can become a local moneylender. The moneylender monitors a potential borrower, and finds an agent willing to lend (if he does not himself). Differentiation is only on the borrowing side, and a moneylender competes with the two nearest local intermediaries when setting interest rates. Let the fixed cost associated with this activity be asymptotically equal to zero. The free entry condition guaranties that the number of intermediaries is close to infinity, and that they make zero profit.

We consider simple debt contracts[9]. The required rate of return to the lender is R' (a lender in the informal market *cannot diversify his portfolio*: the intermediary lends his wealth to only one entrepreneur), and is endogenous. Define R'_b the interest rate paid by borrowers. *The interest rate R' is endogenous.*

An intermediary in the informal market relies on a monitoring technology that solves the moral hazard problem. Wealth cannot be used as a collateral. By paying a cost C ex-ante, the intermediary makes the private benefit σ fall to zero. Hence, if monitoring occurs, the entrepreneur chooses the good project. The monitoring cost is paid by the intermediary ex-ante, and is observable. Consequently the lender knows his expected return on the loan. At that moment he can at no cost refuse to lend (if the money-lender does not monitor the project).

The monitoring technology is modelled in the following way: we assume it is an increasing (quadratic) function of the size of the loan[10]. If the size of the

[8] This is not essential to the model, it simply permits to have a rough spatial framework.

[9] Given the simple structure of investment projects, this is obviously equivalent to any contract specifying a given repayment in the good state.

[10] In their paper, Holmstrom and Tirole (1997) consider a monitoring cost proportional to the size of the project.

project is k, then the total cost is:

$$C(k) = \delta \cdot \frac{(k-w)^2}{2}$$

Assume that the moneylender monitors the project[11]. The zero profit condition gives:

$$p \cdot R'_b = \delta \cdot \frac{(k-w)}{2} + p \cdot R'.$$

The Banking Sector

Entrepreneurs can as well borrow from a bank. Financial intermediaries gather funds, and invest them in the most productive investment opportunities in the economy. As all investment projects are identical, the only point is the incentive issue. Here, we simply assume that banks cannot monitor at all while they can increase the volume of savings gathered at no additional cost.

There is perfect competition among banks. The required rate of return on loans is *exogenous*, equal to R. This is because banks are able, for instance, to borrow in the international capital market at this rate of interest (therefore our economy is small relative to the size of the world). The rate of return on deposits is equal to νR ($\nu < 1$), and it is safe (the number of projects financed is large enough).

Thus savers have two alternatives: they can either lend in the informal credit market, or lend to a bank. The difference is that the interest rate on deposits is safe. Entrepreneurs can borrow from a bank, or directly in the informal credit market. Alternatively, they can decide not to invest in their own project, and become investors.

Assumption 5: borrowers have to pay an entry cost ρ to the banking network. It is a fixed, non monetary cost that can be seen as the effort to be expended in order to apply for a loan from a bank: an entrepreneur has to spend time going to the bank, filling application forms, getting acquainted with the terms of credit, etc. This kind of cost may be quite relevant in the case of developing countries, as suggested by some empirical studies[12]. We do not assume spatial differentiation

[11] The intermediary must be given incentives to monitor. Assume that he has anyway to pay collection costs (to gather capital): this can be done by considering that the cost C is in fact the sum of two costs, one for monitoring,and the second for collecting funds. It is then easy to show that if q is small enough, the moneylender cannot compensate the lender for the increase in risk. Hence, he will fail to get funds. Consequently, monitoring always occurs in equilibrium, and the good project is chosen by the entrepreneur.

[12] In his survey of small Indonesian firms McLeod (1991) gives the following example: "Sutarjo [an entrepreneur] prefers the simplicity of supplier finance to the difficult and time consuming process of obtaining bank loans - even though the latter are nominally cheaper."

of banks.

4. ENTREPRENEURS AND SAVERS DECISIONS

In this section, taking R' and R as given, the decision of savers and entrepreneurs are described. This will later determine the demand and supply of capital in the informal and banking markets.

4.1. Investors (Savers)

Consider an agent who has no choice but to lend his wealth. He has to decide how to allocate his wealth, namely the amount he will lend "informally" (knowing that in this case he has to bear some risk), and the amount he will save on a bank account (and earn a return νR). This is therefore a simple portfolio problem, with two assets: a risky asset with return R' in case of success, and a safe asset with return νR. With constant relative risk aversion, the share of the wealth w invested in the risky asset depends only on the risk aversion coefficient and the two rates of return.

Lemma 1. *Given R' the return in case of success (probability p) in the informal market, and νR the rate of return on deposits, the investor will allocate a proportion $\eta = \eta\left(\overset{+}{R'}, \overset{-}{\nu R}, \overset{+}{p}, \overset{+}{a}\right)$ of his wealth in the informal credit market. We have $\eta = 0$ if $R' = \nu R$.*

Proof. see the appendix for detailed expression.

Define $V_{lend}(w)$ the indirect utility of an investor[13].

4.2. Loan Contracts

Before turning to the choices of a potential entrepreneur (an agent who has an investment opportunity), let us characterize the credit contracts between a borrower and the intermediary (moneylender or bank).

4.2.1. Informal Credit Market

As explained before we concentrate on the cases where the moneylender has an incentive to monitor. The timing is the following:

[13] $V_{lend} = \left[p\left(\eta R' w + (1-\eta)\nu R\right)^a + (1-p)\left((1-\eta)\nu R\right)^a\right] \cdot w^a$

(1) The entrepreneur goes to a moneylender and ask for a loan of size $L = k - w$, where k is the desired project size and w the initial wealth of the entrepreneur.

(2) The moneylender gathers savings (he contacts a lender).

(3) The loan is granted, the interest rates are fixed, and the intermediary monitors the entrepreneur.

(4) The project is undertaken, the shock occurs (freely observed), and finally the loan is paid back.

The entrepreneur anticipates the sequence of events, and chooses to ask for a loan of size $L = k - w$ by maximizing his expected utility:

$$Max_k \left[p \left(Ak - R'_b(k-w) \right)^a \right]$$

subject to the zero profit condition for the moneylender:

$$R'_b = \frac{\delta}{2p}(k-w) + R' = \frac{\delta'}{2}(k-w) + R'$$

Lemma 2. *Consider an entrepreneur with initial wealth w. The size k of the project financed in the informal credit market is: $k = w + \frac{(A-R')}{\delta'}$. The expected utility of the entrepreneur is: $V^E_{\inf} = p\left[Aw + \frac{(A-R')^2}{2\delta'}\right]^a$.*

4.2.2. Bank Loan

Assume that the project size is k, w being the collateral (the initial wealth of the entrepreneur). It is straightforward to show that the entrepreneur always wants the largest possible loan. If banks anticipate that the entrepreneur chooses the good project, ex-ante competition among banks guaranties that the interest rate on the loan is $\frac{R}{p}$. The incentive compatibility constraint for the entrepreneur is therefore:

$$pU\left[Ak - \frac{R}{p}(k-w)\right] > qU\left[(A+\sigma)k - \frac{R}{p}(k-w)\right]$$

Define $\lambda = \frac{w}{k}$.

The constraint reduces to

$$F(\lambda) \geq \left(\frac{q}{p}\right)^{\frac{1}{a}}$$

where: $F(\lambda) = 1 - \frac{\sigma}{A+\sigma-R+R\lambda}$.

Figure 1 gives the locus of incentive compatible loan's sizes.

Figure 1 about here.

Under Assumption (2), it is straightforward to show that there exists a lower value $\widehat{\lambda}$ below which the incentive constraint is not met. Therefore, the size of the project is given by: $k = \frac{1}{\widehat{\lambda}} w$.

Lemma 3. *The size of the loan L made by a bank is* $L = k - w = \left(\frac{1}{\widehat{\lambda}} - 1\right) w$. *The net revenue to the entrepreneur, in case of success, is* $\left[A\frac{1}{\widehat{\lambda}} - \frac{R}{p}\left(\frac{1}{\widehat{\lambda}} - 1\right)\right] w = \mu \cdot w$, *where* $\mu' < 0$.

Both are decreasing functions of the required rate of return R. The expected utility is $V_{bank}(w) = p\mu^a \cdot w^a - \rho$.

4.3. Potential Entrepreneurs

Let us now consider a potential entrepreneur with initial wealth w. He faces four possibilities. First, he can undertake his project without borrowing. Second, he can borrow in the informal credit market. Third, he can borrow from a bank. Fourth, he can instead lend his wealth. It is straightforward to show that if he decides to undertake his project, he will always borrow. There are three remaining possibilities. The choice will depend on the initial wealth of the individual, the equilibrium interest rate in the informal market, and the risk aversion parameter.

The intuition is the following: very poor individuals always prefer to borrow to finance their own project. They are willing to take risks because it may get them out of poverty. Lending, even with risk diversification possibilities, is not worthwhile because their initial wealth is too small. Finally they always borrow in the informal credit market, because the bank would accept to lend only a ridiculous amount.

Next, the choice of middle income agent will depend on the interest rate in the informal sector. If it is low, then lending is not worthwhile, even with the risk diversification possibilities given by a bank deposit. They either borrow in the informal market or from a bank, depending on their wealth. If the interest rate is high, it may be worthwhile to lend rather than borrow. Indeed, they were to invest as entrepreneurs, the size of the loan they would obtain in the informal credit market would be too low, and the loan's size from a bank won't be large enough to compensate the fixed cost of going to a bank. Notice that the existence of those two patterns of occupational choices depends on the assumption of the fixed cost ρ. This is consistent with the fact that local lenders are often middle income agents (landlords, traders, etc.).

Finally, rich individuals always prefer to borrow from a bank.

The results are summarized in the following proposition.

Proposition 1. ***(Pattern of Occupational Choices).***

If a<1, there exists $\underline{R}$*' such that:*

1. *If R'<* $\underline{R}$*', there exists* $w_1(R')$ *such that:*

if $w < w_1(R')$*, the entrepreneur borrows in the informal credit market,*

if $w_1(R') \leq w$*, he borrows from a bank.*

2. *If R'≥* $\underline{R}$*', there exist* $w_2(R')$ *and* $w_3(R')$ *such that:*

if $w < w_2(R')$*, the entrepreneur borrows in the informal credit market,*

if $w_2(R') \leq w < w_3(R')$ *, he prefers to lend his wealth,*

if $w_3(R') \leq w$*, he borrows from a bank.*

The formal proof is given in the appendix.

Figures 2 and 3 about here.

Vinf is the expected utility if the entrepreneur borrows in the informal sector, Vlend is the expected utility if he lends, and Vbank the expected utility if he borrows from a bank.

5. Static Equilibrium and Dynamics of the Economy

5.1. Static Equilibrium

IN our economy where banks can borrow in the international capital market, the static equilibrium is simply characterized by an interest rate R' that clears the informal credit market. Let $w_D(R') = Min\left(w_1(R'), w_2(R')\right)$.

Demand in the informal credit market is:

$$D_{\inf}(R') = \alpha_1 \int_0^{w_D(R')} L(R', w) \cdot dG(w) = \alpha_1 G_t\left(w_D(R')\right) \cdot \frac{(A - R')}{\delta'}$$

If R'< $\underline{R}$',funds are provided only by savers. If R'≥ $\underline{Rl}$, funds are also provided by potential entrepreneurs with initial wealth between $w_2(R')$ and $w_3(R')$.

Hence the supply in the informal credit market is:

$$S_{\inf}(R') = \eta\left(R'\right)\left[\alpha_0 W_t + H(R' - \underline{Rl})\alpha_1 \int_{w_2(R')}^{w_3(R')} w dG(w)\right]$$

where H is the function:

$$\begin{aligned} H(x) &= 0 \text{ if } x < 0 \\ H(x) &= 1 \text{ if } x \geq 0 \end{aligned}$$

The equilibrium interest rate in the informal credit market is the solution of:

$$S_{\inf}(R') = D_{\inf}(R')$$

The interesting feature is that the demand for capital in the informal credit market depends on the distribution of wealth. From now one can have an intuition of what happens: demand in the informal credit market is higher the larger is the number of entrepreneurs with low initial collateral. This demand effect makes the distribution of wealth matters for the static equilibrium: if the mass of poor agents (below a certain threshold) increases, the demand shifts out, and the equilibrium interest rate increases.

Figure 4 about here.

The equilibrium interest rate in the informal credit market is likely to be high when capital is scarce and a large proportion of entrepreneurs have low initial wealth. If this rate of interest is large enough, some productive projects may not be undertaken.

In a static model, the size of the informal market is influenced by the distribution of wealth. Let us now turn to the dynamics of the economy. One can already have an idea of the effects that may arise: if the interest rate in the informal credit market is initially high, then small firms may not be able to reach a sufficient size to get financed by banks, hence they will remain small, and demand in the informal sector will remain high. This will constrain the growth of the aggregate stock of capital, and therefore the supply of capital in the informal market at the subsequent period will remain low. Hence a high interest rate in the informal market may be self sustaining. A similar reasoning can be made for a low interest rate.

The next section is devoted to formalize this idea in a dynamic framework.

5.2. The Joint Evolution of the Real Economy and the Financial Structure

In this section we show that long run growth occurs only if the banking sector becomes dominant in the financial sector of the economy. More specifically, we characterize two types of possible long run equilibria, depending on the initial wealth distribution, and initial aggregate wealth. Two initially identical economies, except for the distribution of wealth, may have dramatically different development paths for certain range of aggregate wealth. The economy with the larger proportion of poor entrepreneurs will stagnate, and its banking sector will shrink, whereas if the proportion of poor agents is lower, the economy will reach a stable rate of growth and the banking sector will become dominant. The effect is the one described above: if there are initially a lot of poor entrepreneurs, then the demand for capital in the informal sector will be high, and the equilibrium interest rate will also be high. Accumulation is reduced, which will make this high

interest rate self-sustaining: firms will remain small, and access to the banking sector limited. If on the contrary, the distribution of wealth is more equal, or at least the number of entrepreneurs below a certain level is low, then demand of capital in the informal sector and the interest rate will be relatively low. Firms will grow quicker and gain access to banks' loan. This limits the demand for funds in the informal sector, and, as wealth accumulate, it becomes easier to make this low interest rate sustainable: the economy starts growing steadily, and more and more entrepreneurs borrow in the banking sector.

The dynamics of the economy are as follows: starting with an initial wealth distribution $G_0(w)$, one determines the equilibrium interest rate $R'_0 = r_0(G_0)$ in the informal credit market as explained in the previous section. From this equilibrium interest rate, one defines the transition functions $w_1^i = f_0^i(w_0^i)$ for the initial wealth of each dynasty i between date 0 and date 1. This set of transition functions in turn gives the initial wealth distribution at date 1 $G_1(w) = g_0(G_0)$. Hence the dynamics of the economy is defined by the infinite sequence $(R'_t, G_t)_{t=0}^{\infty}$ of interest rates and cumulative functions that characterize the wealth distribution at each date.

The infinite sequence of wealth distribution functions follows a Markov process. Let P_t be the transition function that defines this Markov process at date t. This transition function is obtained from individual transition functions and characterizes the transitions into any interval of wealth between dates t and t+1: $P_t(w_0; [w_1, w_2])$ gives the probability of having initial wealth at date t+1 in the interval $[w_1, w_2]$ if the initial wealth is w_0.

5.2.1. Individual Transition Functions

Consider a dynasty i at date t. Let w_t^i be the initial wealth of the individual of the dynasty at date t, and w_{t+1}^i the initial wealth of the dynasty at date t+1.

1. **If the individual is a saver:**
   $w_{t+1}^i = \gamma\left[\eta_t R'_t + (1-\eta_t)\nu R\right] \cdot w_t^i$ with probability p
   $w_{t+1}^i = \gamma\left[(1-\eta_t)\nu R\right] \cdot w_t^i$ with probability $1-p$
2. **If the individual is a potential entrepreneur**:
   - *If R'<$\underline{R'}$, he always undertakes his project:*
     (a) If $w < w_1(R')$, he borrows in the informal credit market:
     $w_{t+1}^i = \gamma\left[Aw_t + \frac{(A-R')^2}{2\delta'}\right]$ with probability $p, 0$ otherwise.
     (b) If $w \geq w_1(R')$, he borrows from a bank:

$w^i_{t+1} = \gamma\mu(R) \cdot w^i_t$ with probability $p, 0$ otherwise.

- *If R'$\geq \underline{R'}$, we have the intermediate range where the agent chooses to lend:*

  (a) If $w < w_2(R')$, he borrows in the informal credit market:
  $w^i_{t+1} = \gamma\left[Aw_t + \frac{(A-R')^2}{2\delta'}\right]$ with probability p, 0 otherwise.
  (b) If $w_2(R') \leq w \leq w_3(R')$, he prefers to lend:
  $w^i_{t+1} = \gamma\left[\eta R' + (1-\eta)\nu R\right] \cdot w^i_t$ with probability p
  $w^i_{t+1} = \gamma\left[(1-\eta)\nu R\right] \cdot w^i_t$ with probability $1-p$
  (c) If $w_3(R') \leq w$ he borrows from a bank:
  $w^i_{t+1} = \gamma\mu(R) \cdot w^i_t$ with probability p, 0 otherwise.

5.2.2. Long Run Development or Stagnation

In this section, we establish three results. First, we provide sufficient conditions for the "relevance" of the distribution of wealth: there exists a range of aggregate wealth such that the development path of an economy starting in this range will be determined by the distribution of wealth: if inequalities are large enough the economy will end up stagnating, and if inequalities are small enough, the economy will go on growing. The intuition is simple: large inequalities may be sufficient to make a high interest rate self sustaining in the informal market, which may make the access to the banking sector impossible for poor entrepreneurs. On the contrary, if the proportion of poor entrepreneurs is low, the interest rate in the informal sector will be low: entrepreneurs will gain access to the banking sector in a few periods.This theorem is relevant for "middle-income" countries.

Second, if we consider a poor country, it is clear that there must be some inequality initially to make sure that the banking sector does exist: if this is not the case, we may have an economy where only informal credit institutions exist: in that case, the economy will reach a stable aggregate wealth level.

Third, development is guarantied above a certain aggregate wealth level: whatever the wealth distribution is, the supply of capital is high enough to deter the existence of a poverty trap for poor entrepreneurs.

Before turning to the first theorem which is the core of the paper, we need first to establish the existence of different configurations for the transition functions of entrepreneurs.

Lemma 4. *There exists $\overline{\overline{R'}}$ and $\underline{\underline{R'}}$, where: $\overline{\overline{R'}} > \underline{\underline{R'}}$, such that:*

If $R' > \overline{\overline{R'}}$, middle income potential entrepreneurs prefer to become lenders and poor entrepreneurs cannot get access to the banking sector. ***(Figure 5 about here)***

If $R' < \underline{\underline{R}}'$, potential entrepreneurs always undertake their project and get access to the banking sector in only one period. ***(Figure 6 about here).***

The proof is given in the appendix.

Theorem 1

There exist a range of parameters and aggregate wealth levels W_1 and W_2 such that, if $W_1 < W_0 < W_2$:

(1) We can have $D_0(\underline{\underline{R'}}) < S_0(\underline{\underline{R'}})$ or $D_0(\overline{\overline{R'}}) \geq S_0(\overline{\overline{R'}})$, depending on the initial distribution of wealth.

(2) If $D_0(\underline{\underline{R'}}) < S_0(\underline{\underline{R'}})$ then $D_t(\underline{\underline{R'}}) < S_t(\underline{\underline{R'}})$ for all t. The economy exhibits a positive long run rate of growth, and the volume of loans financed in the informal sector becomes negligible relative to the aggregate wealth. The long run steady state (positive rate of growth, interest rate, and stationary wealth distribution) is unique.

(3) If $D_0(\overline{\overline{R'}}) \geq S_0(\overline{\overline{R'}})$ then $D_t(\overline{\overline{R'}}) \geq S_t(\overline{\overline{R'}})$ for all t. The banking sector finances a negligible number of project, the informal sector is the dominant source of credit, and the economy stagnates. The steady state is unique (positive aggregate wealth level, interest rate, and stationary wealth distribution).

Proof. The formal proof is given in the appendix. Heuristically, we proceed in two steps.

First Step: first, we look for conditions under which a low interest rate (below $\underline{\underline{\text{R'}}}$) is self-sustaining from one period to the next one. The condition is simple: entrepreneurs financing from banks must on average increase their wealth from one period to the next one. Second we look for conditions under which a high interest rate (above$\overline{\overline{R'}}$) is self-sustaining. The condition is that the positive effect on aggregate wealth from entrepreneurs financing their project from banks is bounded by some function of parameters. The reason is that, otherwise, the demand effect could be dominated by a supply effect (the demand in the informal credit market remains high because poor entrepreneurs remain in the bottom of the distribution),: indeed the volume of loans made by banks may sufficiently increase aggregate wealth to make supply in the informal market (remember that supply is increasing with aggregate wealth) at the next period large enough to compensate for the important demand. This means that the depressing effect of a high interest rate in the informal sector must dominate the positive effect of bank loans on accumulation. This condition is guaranteed if aggregate wealth is not too large. Finally,we just need to check that with this upper bound on aggregate wealth at date 0, we can have the two cases (R'_0 either lower than $\underline{\underline{\text{R'}}}$ or either greater than $\overline{\overline{R'}}$), depending on the distribution of wealth.

Second Step: we know that two initially identical economies, except the distribution of wealth, each starting with one of the two cases described above, will have different development paths. However, we have to check for the existence of long run stationary equilibria. In the case where R' remains above $\overline{\overline{R'}}$, convergence can be shown directly,

using the fact that in the long run, the volume of projects financed by banks is asymptotically equal to zero. With our functional choices, the dynamics of the rate of interest and aggregate wealth can be then studied separately from the one of the distribution of wealth. Next, one shows the existence of a unique stationary distribution. In the case where R' remains below $\underline{\underline{R'}}$, convergence can be shown using general results on Markov Processes. ■

Theorem 2

Assume that initially all entrepreneurs borrow in the informal credit market (this is a poor economy). There exists a range of parameters such that the economy converges to a stable aggregate wealth level and such that a banking sector never appears. This equilibrium is determined by W_{inf}, R'_{inf}, *and* G_{inf}, *with* $W_{\text{inf}} < W_1$, *respectively the long run aggregate wealth, interest rate and distribution of wealth.*

Theorem 3

There exists $\overline{W} > W_2$ *such that if* $W_0 > \overline{W}$ *the economy reaches a stable and positive long run rate of growth independently of the initial wealth distribution.*

The central result of this section (theorem 1) can be compared to the one obtained by Gilles Saint Paul [1992]. He studies the relationship between the development of a stock market and the rate of growth of an economy. He shows that there exists a range of initial stock of capital where the growth path is indeterminate: a stock market may develop or not, leading to long run growth or stagnation. Here we have a "multiplicity" result if the initial aggregate wealth is below a certain level. However, the existence of several development paths is explained by an economic variable, that is the wealth distribution. We argue that it matters because credit markets are imperfect: the way firms finance their growth depends on their available collateral. As in the model of Saint Paul, as well as in the one of Galor and Zeira [1993], the existence of a fixed cost is necessary for this dynamic property.

6. POLICY IMPLICATIONS AND CONCLUDING REMARKS

6.1. Wealth Distribution and Growth

Providing a detailed characterization of the evolution of the wealth distribution along different development paths is beyond the scope of the paper. However we reach an interesting conclusion, namely that the impact of inequality on future growth depends on the level of development attained by the economy. In poor countries, inequalities must be large enough to trigger the set up of modern banking institutions (theorem 3), otherwise nobody is willing to borrow from a bank. But this process must be sustainable. This is possible only if there are always new borrowers in the banking sector, this will be true if, when the economy starts to develop, inequalities are not too important (theorem 1), or else the banking sector may stop growing. Consequently, redistributive policies should be designed accordingly, and the impact on long run growth will be complex even in this model that tackles only one dimension of the problem.

The models of Aghion and Bolton [1997], and of Piketty [1997] both show that wealth redistribution has a impact because it reduces productive inefficiencies (caused by moral hazard). The suggested redistribution is from rich to poor individuals. In the model of Aghion and Bolton, it reduces productive inefficiencies along the convergence path. However, redistribution must occur at each period, as it has no long run effects[14]. The mechanism is the following: because of decreasing returns to capital, the incentive problem appears only for low wealth individuals (firms), and vanishes once the firm has reached a given size. Consequently, redistribution from rich to poor agents is not neutral in the aggregate: the disincentive effect on rich agents is more than compensated by the positive incentive effect on poor agents. The mechanism on which we focus is different.

Here redistribution has long-run effects[15] because it can permanently affect the first stage of development of firms for all periods, by lowering permanently the interest rate in the informal market. It is essential to notice that it is because small firms may choose (optimally) not to apply for loans and remain small (through the effect on the interest rate R'), and because this trade-off is not relevant for large firms, that redistribution has a role to play. It is not related to different incentive problems with respect to initial wealth: informal institutions are anyway more efficient than banks in financing small firms. *Redistribution, in order to have a positive impact, is not necessarily from rich to poor individuals.*

[14] In the model of Piketty, more similar to ours, redistribution needs to occur only once.

[15] It affects also the current rate of growth, but we choose not to focus on this dimension in this version of the model.

6.2. Policy Implications

The model suggests that a policy aiming at enhancing the access to the banking sector for poor entrepreneurs is beneficial to economic development. Needless to say, such policies are difficult to implement. Governments and international institutions have devoted huge efforts in the second half of the century to stimulate capital accumulation among the poor individuals. Poverty alleviating programs, directed credit policies, and development financial institutions make sense only if one thinks that access to credit for poor agents is difficult, that commercial banks are naturally reluctant to lend without palpable guarantees[16], and that *lack of access to credit is detrimental to long run growth*, because valuable investment opportunities are not exploited[17]. We won't discuss here the success or failure of those policies. Moral hazard is however the argument that is used to justify the encountered difficulties. "Financial substitution, and fungibility in general, enables borrowers of targeted loans to do what is largely in their own self interest, regardless of the policy maker's objective" (Adams, [1992]). At the same time, informal credit institutions have been thriving in developing countries, and they seem to be working quite well.

What kind of policy will have positive long run effects on the rate of growth? The model suggests that access to the banking sector should be stimulated among the poor. One possibility is to subsidize bank loans for entrepreneurs who would otherwise borrow in the informal credit market. This is a way to facilitate the access to the banking sector and to avoid situations such as the one illustrated in figure 5. This measure can be financed by taxing the revenue of large firms. The short run effect on the rate of growth is ambiguous because large firms will borrow less (taxing revenue is equivalent to a decrease in the productivity A, which exacerbates the moral hazard issue, and increases the rationing), however the long run effect is clear: this measure will unambiguously decrease the rate of interest in the informal market. This conclusion is of course subject to a limitation, which is the assumption of an exogenous rate of interest in the banking sector[18].

A second possibility is to impose ceilings on deposit rates (assuming that the banks cannot extract rents). Ceilings on deposit rates have been extensively used in developing countries. The rationale is that it is a way to provide cheap finance

[16] "Providing rural financial services is expensive, as evidenced by the unwillingness of many commercial banks in Africa to do it without being forced". (Adams, 1992)

[17] The essence of this issue lies in this remark: "Entrepreneurs with potential production opportunities may lack resources of their own...Those with substantial endowments may lack 'internal' production opportunities" (Mc Kinnon).

[18] See Tressel [1999] for a similar analysis in a closed economy.

to firms that otherwise may not choose to borrow because of the prohibitive cost of capital. This strategy has been criticized on the ground that it decreases the quality of projects undertaken (Mc Kinnon, 1973); however, because of adverse selection, the quality of the pool of applicants may as well increase following a fall in lending rates (Stiglitz and Weiss, 1981).

In our model, such a policy will have two effects: (1) the cost of capital for borrowers in the banking sector decreases, which stimulates the access for poor entrepreneurs.(2) As the return on deposits decreases, savers reallocate their capital from the banking sector to the informal credit market; this does affect the supply of capital in the banking sector, therefore the effect is positive: the supply of capital in the informal credit market increases, which reduces the equilibrium rate of interest there. Both effects goes in the same direction: they stimulate the demand in the banking sector, and makes situations such as in figure 5 less likely. The limitation of the analysis is the same as before: the supply of capital in the banking sector is perfectly elastic. In a closed economy, an additional effect arises: the supply of capital in the banking sector falls, which may have dramatic consequences (Tressel [1999]).

Finally, the banking sector should not be enhanced by making informal credit institutions illegal. In the model, a policy that makes informal institutions illegal[19], may have short run positive effects (if there are no alternatives to bank borrowing, then more individuals will choose to borrow), *but in the long run a poverty trap at the individual level appears, and eventually all individuals fall in this trap: the economy "collapses". Banks cannot perform their role because they cannot enforce loan repayment if no collateral is provided.* Informal credit institutions have therefore a role to play.

6.3. Concluding Remarks

The aim of the model is to make a first step in the direction of analyzing the development of banking institutions in developing countries, when poor agents can also borrow from informal credit institutions. Informal credit institutions can lend without asking for a collateral; however they cannot lend large amounts of capital, and they cannot diversify risk. Banks on the contrary offer a safe return to savers and are able to gather large amounts of capital without incurring additional costs. But they cannot perfectly verify the behavior of borrowers. Hence they ask for a collateral. We argue that this effect caused by imperfect information is essential to fully understand the evolution of modern banking institutions in

[19]In Malaysia, Rotating Savings and Credit Associations are illegal; however, they are very active. This suggests that they are the effect of a strong demand.

developing countries.

This reliance on palpable assets makes the distribution of wealth relevant. The analysis broadly suggests that some inequalities are necessary initially to trigger the set up of modern banking institutions; however, the banking sector will go on developing if there is a constant flow of new borrowers: inequalities must not be large once the economy has started to develop.

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Appendix

- **Proof of Lemma 1**

An investor chooses the proportion of his wealth η to be allocated to the informal credit market:

$$Max_{\eta} \left[\widetilde{p} U \left(\eta R' w + (1-\eta) \nu R w \right) + (1-\widetilde{p}) U \left((1-\eta) \nu R w \right) \right]$$

The solution can be written:

$$\eta = \frac{(\beta - 1)\,\nu R}{\Delta R + R\beta}$$

where: $\beta = \left[\frac{\widetilde{p}\Delta R}{(1-\widetilde{p})\nu R}\right]^{\frac{1}{1-a}}$.

- **Proof of Proposition 1**

The expected utility of borrowing from a bank is:

$$V_{bank} = p\,(\mu w)^a - \rho$$

Informal borrowing gives:

$$V_{\text{inf}}^E = p\left[Aw + \frac{(A - R')^2}{2\delta'}\right]^a$$

w_1 is defined by: $p\,(\mu w_1)^a - \rho = p\left[Aw_1 + \frac{(A-R')^2}{2\delta'}\right]^a$
Lending gives the utility:

$$V_{lend} = \left[p\left(\eta R' w + (1-\eta)\nu R\right)^a + (1-p)\left((1-\eta)\nu R\right)^a\right] \cdot w^a$$

It is evident that if there exists ζ sucht that if $R' > A - \zeta$ lending becomes more valuable than borrowing from the informal sector for some values of w. $\underline{\text{R'}}$ is defined by:

$$V_{bank}\left(w_1(\underline{Rl})\right) = V_{lend}\left(w_1(\underline{Rl})\right)$$

- **Proof of Theorem 1**

  Preliminary conditions (Lemma 4):

  1 First, we show that depending on the interest rate in the informal sector, we can have the crossing of the transition function of (E) financing from the informal market with the 45° line. This property is probably stronger than what we actually need for the proof of the theorem, however it makes it simpler. A sufficient condition is indeed that when R' is above a certain value, poor entrepreneurs that are successful over several periods eventually fall in the region $[w_2, w_3]$ where lending is more interesting than borrowing. This stops the accumulation process for the dynasty. Clearly, one anyway needs restriction on parameters in any case; thus considering a more restrictive case is not of importance. Indeed, even in a more general case, the proof will depend on the existence of a fixed opportunity cost (otherwise the region $[w_2, w_3]$ does not exist).

1. Define $\widetilde{w}$ the wealth level such that :$\widetilde{w} = \gamma\left[A\widetilde{w} + \frac{(A-R')^2}{2\delta'}\right]$.
   The transition function of the entrepreneur will cross the 45° line if and only if:
   $$\widetilde{w} < w_1$$
   or:
   $$V_{bank}(\widetilde{w}) < V_{\inf}(\widetilde{w})$$
   From the definition of $\widetilde{w}$ above, the condition is equivalent to:
   $$\frac{p\left(\gamma^a\mu - 1\right)}{\left(2\delta'(1-\gamma A)\right)^a}\left(A - R'\right)^a < \rho$$
   This is obviously true as R' tends to A (its upper bound). Now we are intrested in the cases where this property does not always arise. Hence, we just need to check that this is not true for the lower value of R'. A sufficient condition is therefore:
   $$H = \frac{p\left(\gamma^a\mu - 1\right)}{\left(2\delta'(1-\gamma A)\right)^a}\left(A - \nu R\right)^a > \rho \mathbf{(1)}$$
   Assume (1) is met. Hence there exists $R'_1 \in]\nu R; A[$ such that:
   $$R' \leq R'_1 \Leftrightarrow \widetilde{w} \geq w_1$$

2 Now we show that if R' is not too large, an entrepreneur can have access to bank loans in only one period. For this we just need to check that for some parameters, an entrepreneur with zero initial wealth may have access to bank loans in one period if he is successful. A sufficient condition is that:
   $$w_1(R'_{t+1}) < \frac{\gamma\left(A - R'_t\right)^2}{2\delta'}$$
   which is equivalent to:
   $$V_{bank,t+1}\left(\frac{\gamma\left(A - R'_t\right)^2}{2\delta'}\right) < V_{\inf,t+1}\left(\frac{\gamma\left(A - R'_t\right)^2}{2\delta'}\right)$$
   By using the fact that R'$_{t+1}$>νR, the condition reduces to:
   $$p\left[\mu^a - A^a\right] \cdot \frac{\gamma^a\left(A - R'\right)^{2a}}{\left(2\delta'\right)^a} > \rho + p \cdot \frac{\left(A - \nu R\right)^{2a}}{\left(2\delta'\right)^a}$$

This cannot be verified for large values of R' (R' tends to A). In order to have this possibility, we just need to assume:

$$p\left[\mu^a-(1+A)^a\right]\cdot\frac{\gamma^a\left(A-\nu R\right)^{2a}}{\left(2\delta'\right)^a}>\rho \textbf{(2)}$$

1. Hence there exists R_2' such that:

$$R'\leq R_2'\Leftrightarrow \text{ transition to bank loans takes exactly one period}$$

It is straighforward to see that (1) and (2) can be jointly verified, as in both cases one just needs to have ρ small enough, given other parameters. Moreover $\mu^a-(1+A)^a>0$ and $\gamma^a\mu>1$ if R is not too large and A is large enough (take for example $\gamma A\lessapprox 1$.)

3 Define $\overline{\overline{R'}}=Max\left\{\underline{R'},R_1',R_2'\right\}$ and $\underline{\underline{R'}}=Min\left\{\underline{R'},R_1',R_2'\right\}$: Consequently, if R'<$\underline{\underline{R'}}$, we have the case where entrepreneurs always undertake their project, and get access to bank loans in only one period; if R'> $\overline{\overline{R'}}$, then middle income entrepreneurs prefer to lend and poor entrepreneurs cannot escape the bottom of the distribution.

Conditions under which a low interest rate (below $\underline{\underline{R'}}$) is sustainable from one period to the next one:

Assume that at date t, we have:

$$D_{\inf,t}\left(\underline{\underline{R'}}\right)<S_{\inf,t}\left(\underline{\underline{R'}}\right)\ \text{(a)}$$

where:

$D_{\inf,t}\left(\underline{\underline{R'}}\right)=\left(\frac{A-\underline{\underline{R'}}}{\delta'}\right)\alpha_1 G_t\left(w_1(\underline{\underline{R'}})\right)$

$S_{\inf,t}\left(\underline{\underline{R'}}\right)=\alpha_0\eta(\underline{\underline{R'}}).W_t$

Aggregate wealth at period t+1 is equal to (time subscripts are dropped for R' and derived parameters):

$$\begin{aligned}W_{t+1} &= \alpha_0\int_0^\infty\left(\gamma p\left[\eta R'+(1-\eta)\nu R\right]w+\gamma(1-p)\left[(1-\eta)\nu R\right]w\right)dG_t(w)\\ &\quad+\alpha_1\gamma p\cdot\mu\int_0^\infty w dG_t(w)+\alpha_1\gamma p\frac{\left(A-R'\right)^2}{2\delta'}G_t(w_1)-\alpha_1\gamma p\left(\mu-A\right)\int_0^{w_1}w dG_t(w)\end{aligned}$$

Define:

$\varepsilon' = (\gamma p \left[\eta R' + (1-\eta)\nu R\right] + \gamma(1-p)\left[(1-\eta)\nu R\right])$

Hence aggregate wealth is:

$$W_{t+1} = \left(\alpha_0 \varepsilon' + \alpha_1 \gamma p \cdot \mu\right) W_t + \alpha_1 \gamma p \frac{(A-R')^2}{2\delta'} G_t(w_1) - \alpha_1 \gamma p \left(\mu - A\right) \int_0^{w_1} w dG_t(w)$$

where the second part measures the relative average efficiency of the informal market with respect to the formal market over its range of action.

Demand in the informal market at date t+1 can be written the following way:

define $w_{1,0,s} = \frac{w_1}{\gamma(\eta R' + (1-\eta)\nu R)}$ and $w_{1,0,f} = \frac{w_1}{\gamma((1-\eta)\nu R)}$.

Agents fall in the interval $[0; w_1]$ at period t+1 if and only if at period t they are unsuccessful entrepreneurs or savers in some range depending on the success or failure of the project they financed:

$$D_{\inf,t+1}\left(\underline{\underline{R'}}\right) = \left(\frac{A - \underline{\underline{R'}}}{\delta'}\right) \alpha_1 G_{t+1}\left(w_1(\underline{\underline{R'}})\right)$$

with:

$$\begin{aligned} G_{t+1}(w_1) &= \alpha_1(1-p) + \alpha_0 \left(pG_t(w_{1,0,s}) + (1-p)G_t(w_{1,0,f})\right) \\ &= \alpha_1(1-p) + \alpha_0 H_t \end{aligned}$$

And:

$$S_{\inf,t+1}\left(\underline{\underline{R'}}\right) = \alpha_0 \eta\left(\underline{\underline{R'}}\right) W_{t+1}$$

We look for a condition that garanties that

$$D_{\inf,t+1}\left(\underline{\underline{R'}}\right) < S_{\inf,t+1}\left(\underline{\underline{R'}}\right)$$

with the assumption that this is true at date t, and using the equation giving the evolution of aggregate wealth.

$\int_0^{w_1} w dG_t(w)$ can be bounded from above by: $w_1\left(G_t(w_1) - (1-p)\right)$.

Moreover, using (a):

$$\left(\frac{A - \underline{\underline{R'}}}{\delta'}\right) \alpha_1 G_t\left(w_1(\underline{\underline{R'}})\right) < \alpha_0 \eta(\underline{\underline{R'}}).W_t$$

a sufficient condition is:

$$\begin{aligned} & \frac{A-\underline{\underline{\underline{R'}}}}{\delta'} \cdot \alpha_1 \left(\alpha_1(1-p)+\alpha_0 H_t\right) \\ < \; & G_t(w_1)\left[\frac{A-\underline{\underline{\underline{R'}}}}{\delta'}\alpha_1\left(\varepsilon'+\alpha_1\gamma p\mu\right)\right] - \alpha_0\eta\alpha_1\gamma p\left(\mu-A\right)w_1\left(G_t(w_1)-(1-p)\right) \\ & +\alpha_0\eta\alpha_1\gamma p\frac{\left(A-R'\right)^2}{2\delta'}G_t(w_1) \end{aligned}$$

By using: $G_t(w_1) > 1-p$ and: $H_t < 1$, a sufficient condition is:

$$(1-p)\,\alpha_1\left(\gamma p\mu - 1\right) > \alpha_0\left(1-(1-p)\varepsilon_t'\right)$$

As $\varepsilon_t' > \nu R$, it reduces to:

$$(1-p)\,\alpha_1\left(\gamma p\mu - 1\right) > \alpha_0\left(1-(1-p)\nu R\right) \quad \text{(S1)}$$

It can be interpreted the following way: $\gamma p\mu$ measures the growth effect of banking finance,it must be the case that an entrepreneur financing from a bank earn enough to leave a higher wealth at his child than what he received. Anyway growth enhancing effects appear only if $\left(\alpha_1\gamma p\mu+\alpha_0\nu R\right)\gamma > 1$ (it is straighforwrd to show that it is possible by taking numerical examples: clear if α_0 goes to zero). Thus what is required is that there are enough potential entrepreneurs in the economy and that collateral requirement by banks are not too important, which means that banks are sufficiently efficient.

Conditions under which a high interest rate (above $\overline{\overline{R'}}$) is sustainable from one period to the next one:

Assume at date t:

$$D_{\text{inf},t}\left(\overline{\overline{R'}}\right) > S_{\text{inf},t}\left(\overline{\overline{R'}}\right)$$

The method is similar. The differences with the previous case are first that poor entrepreneurs don't escape the bottom of the distribution, and second that potential entrepreneurs prefer to be lenders. Hence, aggregate wealth has a slighlty more complex to write:

$W_{t+1} = \left(\alpha_0\varepsilon' + \alpha_1\gamma p\cdot\mu\right)W_t + \alpha_1\gamma p\frac{\left(A-R'\right)^2}{2\delta'}G_t(w_2) - \alpha_1\gamma p\left(\mu-A\right)\int_0^{w_2} w dG_t(w)$
$-\alpha_1\left(\gamma p\mu-\varepsilon'\right)\int_{w_2}^{w_3} w dG_t(w).$

The demand is:

$D_{\text{inf},t}\left(\overline{\overline{R'}}\right) = \left(\frac{A-\overline{\overline{R'}}}{\delta'}\right)\alpha_1 G_t\left(w_2(\overline{\overline{R'}})\right)$

The supply of funds:

$S_{\inf,t}\left(\overline{\overline{R'}}\right) = \eta(\overline{\overline{R'}}).\left(\alpha_0 W_t + \alpha_1 \int_{w_2}^{w_3} w dG_t(w)\right)$

Applying a similar procedure as before, and using the fact that demand of capital in the informal market has necessarily increased (all agents who were in the interval $\left[0, w_2(\overline{\overline{R'}})\right]$ stayed in that interval of wealth, plus savers who fall in this interval in the same way as before) one gets the sufficient condition:

$$\alpha_1 (1-p) \frac{A - \overline{\overline{R'}}}{\delta'} > \alpha_0 \eta \gamma p \mu \cdot \int_{w_3}^{\infty} w dG_t(w) \text{ (S2)}$$

The interpretation is the following: the negative demand effect must overcome the positive supply effect on aggregate wealth that comes from entrepreneurs borrowing from banks.

$\int_{w_3}^{\infty} w dG_t(w)$ can be bounded from above by: $\overline{w_t} M_t$, where $\overline{w_t}$ and M_t are respectively the higher wealth level at date t and the masse of agents above w_3 at date t. By a recursivity argument, one shows that if it is true at date 0, then it remains true at all following dates (using the fact that there has been no entry in this upper region in the precedent dates), if $\alpha_1 (\gamma p \mu) (1 - (1-p)\alpha_1) < 1$, which limits the choices for α_1. Combining S1 with this condition, one gets that the set of possibilities for α_1 is not empty if, given that $\nu R > 1$, p is greater or equal to .5 . This defines a lower bound for α_1. To meet the other conditions, enough degrees of freedom are left (which are the size of the collateral and the bequest motive, for the size of the collateral, one can make it as small as requireb by changing A and σ, which can be understood as an efficiency effect of banks: as already mentioned, σ can well be understood as the private benefits left to the entrepreneurs once the bank has monitored his project.)

Initial Conditions:

Finally one needs to verify that both cases are possible at date 0 for a given aggregate wealth. This can be done by considering two possible wealth distributions. One where agents either have zero wealth or are rich, which gives the second case. And one where a proportion of agents have wealth equal to $w_1(\underline{\underline{R'}})$, for which the result is straighforward. However, by taking this case (initial aggregate wealth equals to $w_1(\underline{\underline{R'}})$), one then needs to check if it is consistent with the conditions above). We get the following condition: $\alpha_1 \frac{A - \overline{\overline{R'}}}{\delta'} > (1-\alpha_1)\eta w_1(\underline{\underline{R'}})$. But we are left with one degree of freedom, which is ρ: by decreasing this cost we can make $w_1(\underline{\underline{R'}})$ verifying this condition.

Convergence Results:

Finally one has to prove that those two types of economies converge to a unique steady state (notice that we do not try to characterize other possible economies where the properties on the interest rate may not be true for each period: this is not the aim of the paper).

The proof of theorem 4.1 is based on Remark 1 of Theorem 2 by Hopenhayn and Prescott (Econometrica, Nov 1992). The difficult point is that we are dealing with non time homegenous Markov Processes . Indeed the transition functions depends on the equilibrium interest rate R', and are therefore non stationary. However, the convergence property does not stricto senso require stationarity.

Basically, we need to check two properties:

1. The Markov process P must be increasing, which is straight forward to verify: a initially poorer dynasty will on average end up with a lower wealth than an initially richer dynasty from one period to the next one.
2. It must verify a mixing condition. Formally, we have to check the following condition: there exists w'' , $\varepsilon > 0$ and n such that, for all t, a dynasty with initial wealth equals to zero, will reach a wealth level greater or equal to w'' in n period with a probability greater than ε. And conversely, a dynasty initially very reach (w close to infinity), will end up after n periods with a wealth level below w'' with a probability greater than ε.

Let us concentrate on the economy where we know that the interest rate remains below $\underline{\underline{\mathrm{R}}}$'. Here the mixing condition is easy to verify: one just needs to choose $w'' = w_1\left(\underline{\underline{R\prime}}\right)$: an entrepreneur with initial wealth equals ot zero reach a wealth level above this threshold with probability p in one period. Conversely, a very rich agent has always a fixed probability $\alpha_1 (1-p)$ of falling at zero in one period. Thus, given the initial wealth distribution the economy will converge to a unique steady states, that is: wealth distribution, interest rate in the informal market, and rate of growth.

Now the economy where the interest rate remains above $\overline{\overline{R'}}$. In this case we acnnot directly apply the theorem as the mixing condition is not met (because of the poverty trap). However, convergence can be shown directly: indeed, in this economy, the interest rate remains always above $\overline{\overline{R'}}$. This means that there is no entry of agents in the upper part of the distribution. However, any entrepreneur has always a fixed probability of going bankrupt, and consequently falling back to zero wealth. And dynasties of savers don't accumulate. Therefore, in the long run all agents fall back in the bottom of the distribution. We can thus restrict the support of the wealth distribution to be $[0, w_2]$, and either apply the theorem,

or directly check that the dynamics of the interest rate and aggregate wealth can be tracked down independently of the distribution of wealth, as in the traditional economy. Next, one can show the existence of a unique long run wealth distribution corresponding to the equlibrium interest rate and aggregate wealth.

Proof of Theorem 2

By using the equilibrium condition in the informal credit market and the transition functions of dynasties, one gets:

$W_{t+1} = \gamma \left[\alpha_1 pA + (1-\alpha_1)\nu R\left(1-\eta_t\right) + (1-\alpha_1)\eta_t pA\right].W_t - \gamma\frac{(1-\alpha_1)^2}{2}p\frac{\delta'}{\alpha_1}\eta_t^2.W_t^2$

The long run steady state for the aggregate wealth and the interest rate are given by:

$W^*_{\inf} = \frac{\gamma[\alpha_1 pA+(1-\alpha_1)\nu R+(1-\alpha_1)\eta^*[pA-\nu R]]-1}{\frac{(1-\alpha_1)^2}{2}p\frac{\delta'}{\alpha_1}\eta^{*2}}$

and:

$\alpha_1\frac{A-R'^*}{\delta'} = \eta^*\left(1-\alpha_1\right)W^*_{\inf}$

The solution exists if:

$[\alpha_1 pA + (1-\alpha_1)\nu R] < 1$

and: $[\alpha_1 pA + (1-\alpha_1)\nu R + (1-\alpha_1)\eta^*(pA-\nu R)] > 1$

Tedious algebra shows that it is not inconsistent with other assumptions; especially the second condition above defines a lower bound for α_1 which does not go against the conditions of theorem 1.

FIGURES

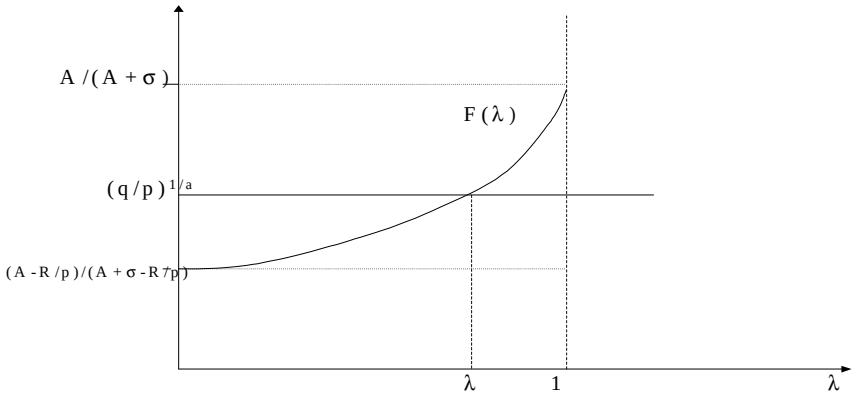

Figure 1: Incentive Compatibility Constraint on Loan's Size.

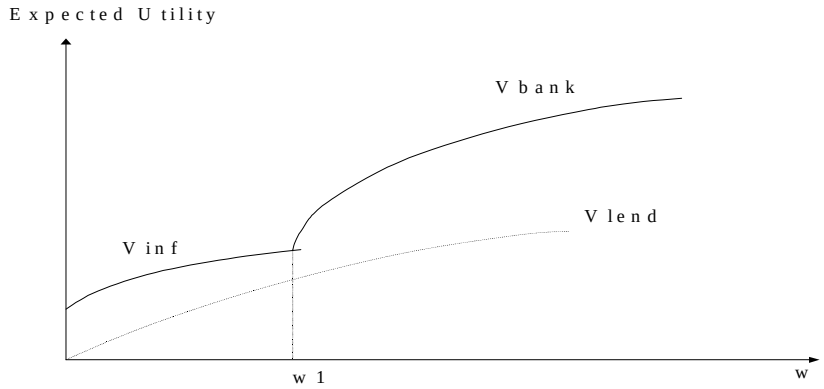

Figure 2: Occupational Choice if R'<R'

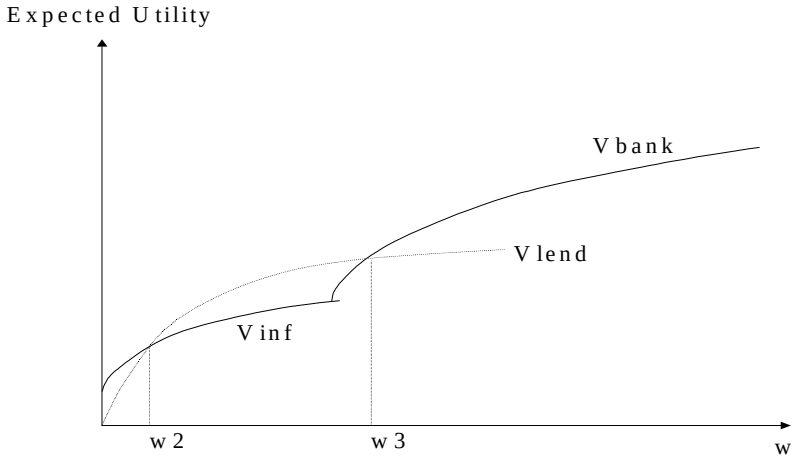

Figure 3: Occupational Choice if R'>R'

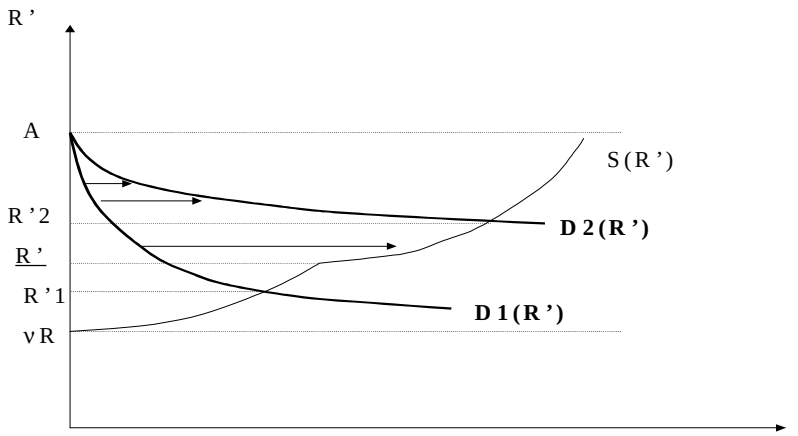

Figure 4: Effect of an increase in the proportion of poor individuals on the equilibrium interest rate in the informal sector.

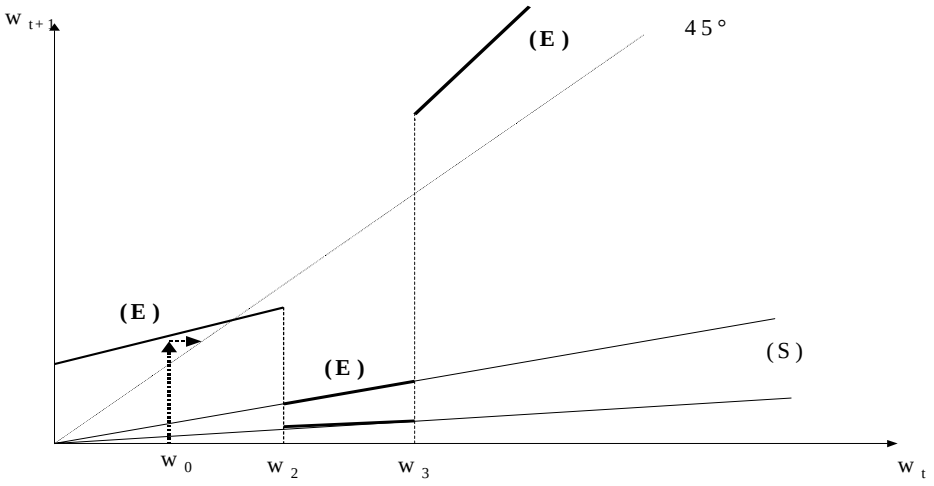

Figure 5: Transition Functions for Entrepreneurs (E) and Savers (S) when R’>$\overline{\overline{R'}}$.

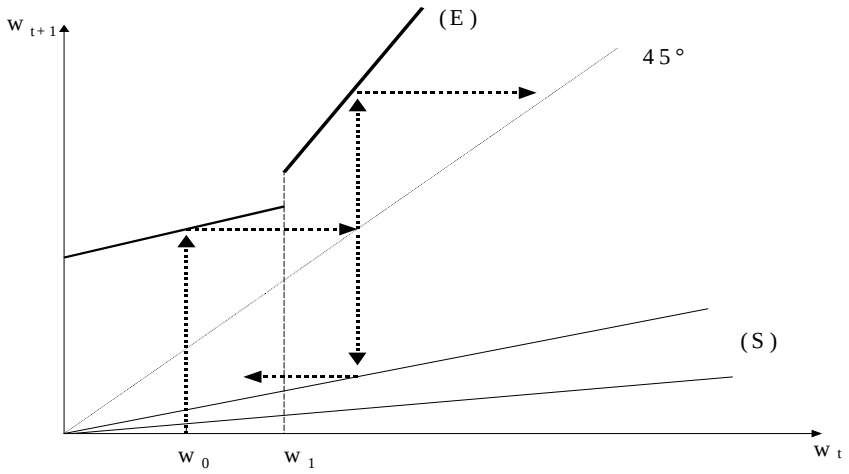

Figure 6: Transition Functions for Entrepreneurs (E) and Savers (S) when R'<$\underline{\underline{R}}$'.