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Redistribution and Fiscal Policy

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Abstract: This paper studies the optimal behavior of a democratic government in its use of fiscal policies to redistribute income. I present a stochastic dynamic general equilibrium model with heterogeneous agents to analyze (1) the differences between the effects on the optimal tax rate of permanent and nonpermanent perturbations and (2) the relationship between initial inequality and both steady-state levy and income distribution. In addition, the optimal fiscal policy for the transition is calculated. The analysis leads me to three main conclusions. First, there are no important differences between how taxes respond to a permanent or nonpermanent perturbation. Second, the initial inequality has a huge effect on both actual levy and actual income distribution. And finally, the Chari, Christiano, and Kehoe (1992) result, i.e., taxes on labor are roughly constant over the business cycle, holds only if the productivity ratio is constant. In addition, the model implies a positive correlation between inequality and tax rate, just as in the basic literature.

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Redistribution and Fiscal Policy

1. Introduction

The main concern of this paper is to assess the optimal behavior of a democratic government in its use of fiscal policies to redistribute income. This problem, and similar issues, has been studied recently in the literature by Perotti(1993), Persson and Tabellini(1994), Krusell, Rios-Rull and Quadrini(1997), and Krusell and Rios-Rull (1999).

Perotti (1993) has used a non-overlapping generations model to study the effects of income distribution on growth when agents vote over the degree of redistribution and an externality on human capital is the source of growth. Persson and Tabellini (1994) have used an overlapping generations model to study the same problem when the driving force of growth is an externality in physical capital. Krusell, Rios-Rull and Quadrini(1997) and Krusell and Rios-Rull (1999) have worked on the effects of inequality on fiscal policies in a recursive framework.

It is important to note that Perotti(1993) and Persson and Tabellini(1994) were interested in the effects of inequality on growth through the fiscal channel, while Krusell, Rios-Rull and Quadrini(1997) and Krusell and Rios-Rull (1999) studied the effects of inequality on policies. This paper follows the second approach.

One limitation of these models is that they only calculate the non-stochastic stationary equilibrium. This restriction limits the analysis in, at least, two dimensions. It is not possible to study the difference between the effects of permanent and non-permanent perturbations. In addition, the consequences of the initial conditions on both the steady-state and transition policies cannot be analyzed.

Concerning the first limitation, Hall (1988) has shown that permanent and non-permanent shocks have very different implications on the intertemporal substitution of consumption. However, it remains unclear whether this result holds in an optimal taxation environment with heterogenous agents.

Moreover, the study of convergence issues has become an important aspect of economic theory. However, previous research in optimal distribution has ignored the effects of initial inequality on steady-state policies and income distribution.

In order to examine these effects, this paper presents a stochastic dynamic equilibrium model with heterogenous agents to analyze both the differences between the effects of permanent and non-permanent perturbations on the optimal tax rate and the relationship between initial inequality and steady-state variables. In addition, the optimal fiscal policy for the transition will be calculated.

Model and Solution Method Description The environment is a stochastic dynamic general equilibrium model with three agents: two infinitely lived consumers with different skill levels and a government that maximizes the median voter utility[1]. We assume that the fraction of each consumer in the economy is constant over time and that the fraction of low skill level households is bigger than the fraction of high skill level households. Consumers make decisions over consumption and leisure, thus there is neither capital nor endogenous growth. The government sets proportional income taxes and transfers, both equal between consumers, and is able to borrow and lend in a complete markets environment. Technology is linear and separable on agent's labor. There is also a random shock that affects consumer skill level.

Two versions of the model will be presented. In the first version, the perturbations will affect aggregate productivity in a classical RBC way; the skill ratio will be constant. This version will be called the **Symmetric Shock Model**. In the second one, the shock will only affect the lower ratio skill level; the skill ratio will not be constant. This case will be labeled as the **Asymmetric Shock Model**.

Since the identity of the median voter does not change over time and a full commitment technology is assumed, a Ramsey problem in the Arrow-Debreu sense will be defined. The most numerous consumer will play the role of the planner and he/she will optimize overall possible sequences of future variables.

Given the model and the definition of equilibrium, which is discussed later, it is not possible to get a closed form solution for the different policy functions. For this reason, We will use a numerical method that will be described in the appendix.

Results The analysis of the stochastic equilibrium leads us to three main conclusions. First, tax responses to both permanent and non-permanent perturbations are very similar. Second, the initial skills inequality has a huge effect on both actual levy and actual income distribution. And finally, the Chari, Christiano, Kehoe (1992) result- i.e. taxes on labor are roughly constant over the business cycle- holds only if the productivity ratio is constant. In addition, the model implies positive correlation between inequality and tax rate, just as in the basic literature.

The rest of the paper is organized as follows. Section 2 presents the two versions of the model. There we will define and characterize both the equilibrium and the Ramsey problem. Section 3 presents the results, and Section 4 the final remarks.

[1] We will see that we do not need to speak about the median voter, since we will assume one of the types of consumers is majoritarian. However, I will call this one the median voter in order to compare our results with Persson and Tabellini.

2. The Model

The rest of this section is as follows. First, we introduce the **Symmetric Shock Model** and both the equilibrium and the Ramsey problem are defined and characterized. Second, the same is done for the **Asymmetric Shock Model**. Finally, we highlight the differences between the two models.

2.1. The Symmetric Shock Model

We study a dynamic stochastic general equilibrium model with two types of households, household type "h" and household type "l", and a government that maximizes the utility of the median voter. Household type h has measure γ and household type l has measure $1-\gamma$, where $\gamma \in (0, 0.5)$. This fact implies that a type l household is the median voter. We consider and economy without capital and with a single final good, y_t, that is produced using elastically suplied labor in the following way:

$$y_t = (\gamma(1-x_{h,t})\phi_h + (1-\gamma)(1-x_{l,t})\phi_l)\theta_t \tag{1}$$

where $(1-x_{i,t})$ is the amount of labor supplied by a household of type $i \in I \equiv \{h, l\}$, $\theta_t\phi_i$ is its marginal product and θ_t is a aggregate productivity shock following a Markov process:

$$\ln \theta_t = \rho \ln \theta_{t-1} + \varepsilon_t \qquad |\rho| < 1 \qquad \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$$

As the reader can observe, in this environment the aggregate productivity shocks, θ_t, have not effect on the ratio between household type l and household h marginal products, $\frac{\theta_t\phi_l}{\theta_t\phi_h}$. This is why we call this set up the **Symmetric Shock Model**.

Households Consumers derive utility from consumption and leisure. The household's type $i \in I$ objective function is:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, x_{i,t}) \tag{2}$$

where U is strictly increasing and concave on its two arguments. Consumer type i is endowed with an unit of time which is devoted to work and leisure. Besides, the household can lent to or borrow from other households or the government using a full array of contigent one period bonds that complete the markets. Thus, a type i household faces the following budget constraint every period:

$$c_{i,t} + \int p_t(\theta) b_{i,t}(\theta) d\theta = (1-\tau_t)\omega_{i,t}(1-x_{i,t}) + b_{i,t-1}(\theta_t) + T_t \tag{3}$$

taking as given θ_0 and $b_{i,-1}$. Where $c_{i,t}$ denotes the consumption level of the type i consumer at t, $\omega_{i,t}$ denotes the hourly wage rate of household type i at t, $x_{i,t}$ denotes leisure of household type i at t, $p_t(\theta)$ is the price at t of a bond that pays a unit of the final good at $t+1$ if the aggregate productivity shock is θ, $b_{i,t}(\theta)$ is the type i consumer demand at t for bonds that pay a unit of the final good at $t+1$ if the aggregate productivity shock is θ, T_t is the level of transfers fixed by the government at t, τ_t is the level of labor taxes fixed by the government at t. In addtion, there are upper and lower bounds for $b_{i,t}$ large enough not to bind in equilibrium but finite to avoid Ponzi games.

Government Government maximizes median voter's utility, i.e. consumer's of type l utility, subject to the following sequence of budget constraints:

$$T_t + b_{t-1}(\theta_t) = \tau_t(\gamma(1-x_{h,t})\omega_{h,t} + (1-\gamma)(1-x_{l,t})\omega_{l,t}) + \int p_t(\theta)b_t(\theta)d\theta \tag{4}$$

taking as given θ_0 and b_{-1}. Where $b_t(\theta)$ is the government demand at t for bonds that pay a unit of the final good at $t+1$ if the aggregate productivity shock is θ. In addtion, there are upper and lower bounds for b_t large enough not to bind in equilibrium but finite to avoid Ponzi games.

Market clearing conditions The clearing condition in the bond market is:

$$(1-\gamma)b_{l,t}(\theta) + \gamma b_{h,t}(\theta) = b_t(\theta) \tag{5}$$

and this same condition for $t=-1$ implies that:

$$(1-\gamma)b_{l,-1} + \gamma b_{h,-1} = b_{-1} \tag{6}$$

Since there is not capital, the final good clearing market condition is:

$$(1-\gamma)c_{l,t} + \gamma c_{h,t} = y_t \tag{7}$$

Therefore, given the production function (1) and the final good clearing market condition (7), we can write the economy resource constraint:

$$\gamma c_{h,t} + (1-\gamma)c_{l,t} = (\gamma(1-x_{h,t})\phi_h + (1-\gamma)(1-x_{l,t})\phi_l)\theta_t \tag{8}$$

2.1.1. Competitive Equilibrium

In this section we first describe which is the households' problem and then define a competitive equilibrium.

Household's type i problem is to choose $\{c_{i,t}, x_{i,t}, b_{i,t}(\theta)\}$ that maximizes the objective function (2) subject to the sequence of budget constraints (3) and taking the sequence of wages, taxes, transeferences, prices $\{\omega_{i,t}, \tau_t, T_t, p_t(\theta)\}$, the initial stock of bonds and shock $b_{i,-1}$ and θ_0 as given. The first order conditions with respect to bonds holdings and leisure requiere:

$$p_t(\theta) = \beta \frac{U_{c,i,t+1}(\theta)}{U_{c,i,t}} \Pr(\theta_{t+1} = \theta/\theta_t) \tag{9}$$

$$\frac{U_{x,i,t}}{U_{c,i,t}} = (1 - \tau_t)\omega_t \tag{10}$$

where $U_{c,i,t}$ and $U_{x,i,t}$ are the marginal utilities with respect to consumption and labor respectively.

In this envoirenment a competitive equilibrium is defined as follows:

Definition 1 (Competitive Equilibrium Definition). *Given* $\{\theta_0, b_{l,-1}, b_{h,-1}, b_{-1}\}$ *such that* (6) *holds, a competitive equilibrium is a process for allocations* $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}, (b_{i,t}(\theta))_{i\in I}, b_t(\theta)\}$, *taxes and transfers* $\{\tau_t, T_t\}$ *and prices* $\{p_t(\theta), (\omega_{i,t})_{i\in I}\}$ *such that:*

1. *For each* $i \in I$, $\{c_{i,t}, x_{i,t}, b_{i,t}(\theta)\}$ *maximizes household's utility function* (2) *subject to the budget constraint* (3) *given* $\{\tau_t, T_t\}$, $\{p_t(\theta), \omega_{i,t}\}$, $b_{i,-1}$ *and* θ_0.

2. *For each* $i \in I$
$$\omega_{i,t} = \phi_i \theta_t \tag{11}$$

3. *The goverment budget constraint* (4), *the bonds market clearing condition* (5) *and economy resource constraint* (8) *hold.*

2.1.2. The Ramsey Problem

As mentioned before, government maximizes consumer of type l's utility. The government is aware of consumers answer to policy announcements and takes this reaction into account when its solve its maximitation problem. This is what has been called a Ramsey problem. Hence, the Ramsey problem consists on choosing taxes and transfers that maximize the utility of a household type l over the set of competitive equilibriums defined above. When doing so, the government faces various trade-offs. Consumers of type l derive utitlity from higher transfers but those transfers has to be finance throught taxes or government debt. Taxes affect both consumers symmetrically and they distort labor supply decisions. Higher governmet debt today increases taxes tomorrow. Since the government solves an intertemporal problem it will try to smooth taxes throught time and states of nature.

As widely noticed in the literature, this problem is not time consistent. To avoid dealing with this issue, we assume that the government has some device such it can commit itself to the Ramsey outcome.

Techinically the Ramsey problem consists of maximizing consumer of type l's utility over the set of competetive equilibria. This is equivalent to choose the allocations, taxes, transfers and prices that maximizes consumer of type l's utility over the set of allocations, taxes, transfers and prices that define a competetive equilibria. In general this problem can be very complicated. Thus, the next step it is to define the minimal set of equations that characterize the set of allocations, taxes, transfers and prices that define a competetive equilibria. We do this in the next proposition.

Proposition 1 (Competitive Equilibrium Charaterization). *Given* $\{\theta_0, b_{l,-1}, b_{h,-1}, b_{-1}\}$ *such that* (6) *holds, if the equilibrium is interior and unique, then the equilibrium process for* $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}, (b_{i,t}(\theta))_{i\in I}, b_t(\theta)\}$, $\{\tau_t, T_t\}$ *and* $\{p_t(\theta), (\omega_{i,t})_{i\in I}\}$ *is uniquely determined by the following conditions:*

- $\exists\ \lambda$ *such that*
$$\frac{U_{c,l,t}}{U_{c,h,t}} = \lambda \tag{12}$$
and
$$\frac{U_{x,l,t}\phi_h}{U_{x,h,t}\phi_l} = \lambda \tag{13}$$
holds.

- *The next restriction is satisfied*
$$b_{h,-1} - b_{-1} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} (\Phi_{h,t} - \Phi_t) \tag{14}$$
where
$$\Phi_{i,t} = c_{i,t} - (1 - x_{i,t}) \frac{U_{x,i,t}}{U_{c,i,t}} \qquad \forall i \in I$$
$$\Phi_t = (1 - \frac{U_{x,h,t}}{U_{c,h,t}\theta_t\phi_h})(\gamma(1 - x_{h,t})\phi_h + (1-\gamma)(1 - x_{l,t})\phi_l)\theta_t$$

- *The economy resource constraint* (8) *holds.*

Proof. The proof will be as follows. First, we are going to show that given a sequence for consumption and leisure allocations for both types of consumers $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}\}$ such that the resource constraint (8), the restrictions (12) and (13) and (14) hold for some λ, we can find a sequence for bonds $\{(b_{i,t}(\theta))_{i\in I}, b_t(\theta)\}$, taxes and transfers $\{\tau_t, T_t\}$ and prices $\{p_t(\theta), (\omega_{i,t})_{i\in I}\}$ such that consumers' budget constraint (3) for $\forall i \in I$, the equilibrium wage (11) for $\forall i \in I$, the government budget constraint (4), the bonds market clearing condition (5), the resource constraint (8) and the households' first order conditions (9) and (10) $\forall i \in I$ hold.

At this point, it is important to notice that with concave utility function, and if the equilibrium is interior and unique (as assumed), the solution to the maximization problem of the consumer is uniquely determined by the consumer's budget constraint (3) and the first order conditions (9) and (10), so that the consumer's budget constraint (3) for $\forall i \in I$, the equilibrium wage (11) for $\forall i \in I$, the government budget constraint (4), the bonds market clearing condition (5), the resource constraint (8) and the households' first order conditions (9) and (10) $\forall i \in I$ are necessary and sufficient for competitive equilibrium.

Assume that $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}\}$ and λ are such the resource constraint (8), the restrictions (12) and (13) and (14) hold. Now, we are going to find $\{p_t(\theta)\}_{t=0}^{\infty}$ such that the first order condition (9) holds for $\forall i \in I$.

First, define wages as $\omega_{i,t} = \phi_i \theta_t$ $\forall i \in I$ and $\forall t$, which is (11) for $\forall i \in I$.

Define $\{p_t(\theta)\}_{t=0}^{\infty}$ as:

$$p_t(\theta) = \beta \frac{U_{c,l,t+1}(\theta)}{U_{c,l,t}} \Pr(\theta_{t+1} = \theta/\theta_t) = \beta \frac{U_{c,h,t+1}(\theta)}{U_{c,h,t}} \Pr(\theta_{t+1} = \theta/\theta_t) \tag{15}$$

(which is (9) for $i = l$) then, (12) implies (9) for $i = h$.

Let us now probe that exists $\{\tau_t\}_{t=0}^{\infty}$ such that the first order condition (10) holds for $\forall i \in I$.

Define $\{\tau_t\}_{t=0}^{\infty}$ as:

$$1 - \tau_t = \frac{U_{x,l,t}}{U_{c,l,t}\theta_t\phi_l} = \frac{U_{x,h,t}}{U_{c,h,t}\theta_t\phi_h} \tag{16}$$

then, using (11) for $\forall i \in I$ we can write

$$\frac{U_{x,l,t}}{U_{c,l,t}\theta_t\phi_l} = \frac{U_{x,h,t}}{U_{c,h,t}\theta_t\phi_h} = \omega_{i,t}(1 - \tau_t)$$

(which is (10) for $i = l$) then (12) and (13) imply that is equal to (10) for $i = h$.

Define $\{T_t\}_{t=0}^{\infty}$ as:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} T_t = -b_{-1} + E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} \Phi_t \tag{17}$$

Then, restriction (14) implies

$$b_{h,-1} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} (\Phi_{h,t} - T_t) \tag{18}$$

and the initial bonds market clearing condition, (6), implies

$$b_{l,-1} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} (\Phi_{l,t} - T_t) \tag{19}$$

Define household type h demand for bonds $\{b_{h,t}\}_{t=0}^{\infty}$ as:

$$b_{h,t} = E_{t+1} \sum_{j=0}^{\infty} \beta^j \frac{U_{c,h,t+1+j}}{U_{c,h,t+1}} \left(\Phi_{h,t+j+1} - T_{t+j+1}\right) \tag{20}$$

household type l demand for bonds $\{b_{l,t}\}_{t=0}^{\infty}$ as:

$$b_{l,t} = E_{t+1} \sum_{j=0}^{\infty} \beta^j \frac{U_{c,h,t+1+j}}{U_{c,h,t+1}} \left(\Phi_{l,t+j+1} - T_{t+j+1}\right) \tag{21}$$

and government demand for bonds $\{b_t\}_{t=0}^{\infty}$ as:

$$b_t = E_{t+1} \sum_{j=0}^{\infty} \beta^j \frac{U_{c,h,t+1+j}}{U_{c,h,t+1}} \left(\Phi_{t+j+1} - T_{t+j+1}\right) \tag{22}$$

Notice that the three bonds demand definitions (20), (21) and (22) are such that bonds market clearing condition (5) holds. Now, using the definition of prices (15) and taxes (16), the condition (18) and the definition of bonds demand (20) we can write:

$$b_{h,t} = w_{h,t+1} + E_{t+1} \left[\beta \frac{U_{c,h,t+2}}{U_{c,h,t+1}} E_{t+2} \sum_{t=1}^{\infty} \beta^{t-1} \frac{U_{c,h,t+2+j}}{U_{c,h,t+2}} \Phi_{h,t+j+2} \right] = w_{h,t+1} + \int p_{t+1}(\theta) b_{h,t+1}(\theta)\, d\theta$$

for $t \geq -1$, what it means that the household's type h budget constraint (3) (the sequence of budget constraints for consumer type h) holds. Using a similar procedure with (17) and (19) we can show that the household's type l budget constraint (3) for $i = l$ (the sequence of budget constraints for consumer type l) and the government budget constraint (4) (the sequence of government budget constraints) also hold.

Second, we are going to probe that given a sequence for consumption, leisure and bonds $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}, (b_{i,t}(\theta))_{i\in I}, b_t(\theta)\}$, taxes and transfers $\{\tau_t, T_t\}$ and prices $\{p_t(\theta), (\omega_{i,t})_{i\in I}\}$ such that consumers' budget constraint (3) for $\forall i \in I$, the equilibrium wage (11) for $\forall i \in I$, the government budget constraint (4), the bonds market clearing condition (5), the resource constraint (8) and the households' first order conditions (9) and (10) $\forall i \in I$ hold we can find a λ such that the restrictions (12), (13) and (14) hold.

From (9) for $\forall i \in I$, we obtain that

$$\frac{U_{c,l,t}}{U_{c,h,t}} = \frac{U_{c,l,t+1}(\theta)}{U_{c,h,t+1}(\theta)} \quad \forall t, \theta$$

i.e. the ratio marginal utilities of consumption at t is equal to the ratio at $t+1$ with probability one. By induction,

$$\frac{U_{c,l,0}}{U_{c,h,0}} = \frac{U_{c,l,t}}{U_{c,h,t}} \ \forall t$$

but, since $b_{h,-1}$, $b_{l,-1}$ and θ_0 are given, we can define λ to be

$$\lambda \equiv \frac{U_{c,l,0}}{U_{c,h,0}}$$

such that (12) holds.

Then, note that from (10) and (11) for $\forall i \in I$ we have

$$\frac{U_{x,l,t}\phi_h}{U_{x,h,t}\phi_l} = \frac{U_{c,l,t}}{U_{c,h,t}}$$

that together with (12) imply (13).

Finally, using (9) and (11) for $i = h$ in the consumer type h budget constraint (3) and in the government budget constraint (4) and solving recursively both restrictions we get

$$b_{h,-1} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} (c_{h,t} - (1-\tau_t)(1-x_{h,t})\phi_h\theta_t - T_t)$$

$$b_{-1} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} (\tau_t\theta_t (\gamma(1-x_{h,t})\phi_h + (1-\gamma)(1-x_{l,t})\phi_l) - T_t)$$

If we convine these two equations with (10) for $i = h$ we get (14). ■

Proposition 1 implies two important features of a competitive equilibrium. First, if we assume separable between consumption and leisure utility function, (12) and (13) imply that both consumption and hours worked ratios between the two types of households are constant through time and realizations of the productivity shock. Second, λ, and, consequently, the two mentioned ratios, depend on the whole productivity shock sequence, and not only its actual realization. As we will see in the numerical exercice to be presented in the next section, this implies the intial conditions, i.e. θ_0, b_{-1}, $b_{l,-1}$ and $b_{h,-1}$, are going to very important on today's households consumption and hours worked optimal choices and, therefore, on the optimal fiscal policy.

But the most important implication of proposition 1 is the following: The economy resource constraint (8) and the restrictions (12), (13) and (14) are necesary and sufficient for *competitive equilibrium*. Hence, for each sequence of $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}\}$ and λ, such that (8) and the restrictions (12), (13) and (14) hold, there exists allocations, taxes, transfers and prices such that they define a competitive equilibria. Therefore, the Ramsey Problem will be to choose $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}\}$ and λ such that maximize type l consumer's utility subject to (8), (12), (13) and (14).

Formally, the Ramsey problem becomes:

$$\underset{\{(c_{i,t})_{i\in I},(x_{i,t})_{i\in I}\},\lambda}{Max} \quad E_0 \sum_{t=0}^{\infty} \beta^t U(c_{l,t}, x_{l,t})$$

subject to:

$$\frac{U_{c,l,t}}{U_{c,h,t}} = \lambda$$

$$\frac{U_{x,l,t}\phi_h}{U_{x,h,t}\phi_l} = \lambda$$

$$b_{h,-1} - b_{-1} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} (\Phi_{h,t} - \Phi_t)$$

$$\gamma c_{h,t} + (1-\gamma) c_{l,t} = (\gamma(1 - x_{h,t}) + (1-\gamma)(1 - x_{l,t})\phi_l)\theta_t$$

where θ_0, $b_{h,-1}$ and b_{-1} are given.

At this point, we would like to remark that we are not aware of any paper that has characterized and solved this problem in the way we have done here. Garcia-Milà et. al. (2001) have a similar Ramsey problem but they do not solve for the optimal Ramsey allocation. Instead, they calibrate λ to some data features. In what follows, first we are going to solve for the optimal Ramsey allocation. Then, we will perform some numerical exercices to understand which are the main features of the optimal Ramsey allocation.

To do that we assume some functional form for preferences:

$$U(c_{i,t}, x_{i,t}) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} + \frac{x_{i,t}^{1-\sigma}}{1-\sigma}$$

If this is the case, we can use the resource constraint (8), and the restrictions (12), (13) to write $c_{l,t}$, $x_{l,t}$ and $x_{h,t}$ as a function of $c_{h,t}$, θ_t and λ, in the following way:

$$c_{l,t}(c_{h,t}, \lambda) = c_{h,t} \lambda^{-\frac{1}{\sigma}}$$

$$x_{l,t}\left(c_{h,t},\lambda,\theta_t\right)=\frac{(\gamma\phi_h+(1-\gamma)\phi_l)-\frac{\left(\gamma+(1-\gamma)\lambda^{-\frac{1}{\sigma}}\right)}{\theta_t}c_{h,t}}{\left(\gamma\phi_h\left(\frac{\phi_h}{\lambda\phi_l}\right)^{-\frac{1}{\sigma}}+(1-\gamma)\phi_l\right)}$$

$$x_{h,t}\left(c_{h,t},\lambda,\theta_t\right)=\left(\frac{\phi_h}{\lambda\phi_l}\right)^{-\frac{1}{\sigma}}\frac{(\gamma\phi_h+(1-\gamma)\phi_l)-\frac{\left(\gamma+(1-\gamma)\lambda^{-\frac{1}{\sigma}}\right)}{\theta_t}c_{h,t}}{\left(\gamma\phi_h\left(\frac{\phi_h}{\lambda\phi_l}\right)^{-\frac{1}{\sigma}}+(1-\gamma)\phi_l\right)}$$

at the same time, and since taxes, τ_t, are a function of λ and $c_{h,t}$ we can use (10) for $i=h$ to write:

$$\tau_t\left(c_{h,t},\lambda,\theta_t\right)=1-\frac{x_{h,t}\left(c_{h,t},\lambda,\theta_t\right)^{-\sigma}}{c_{h,t}^{-\sigma}\theta_t\phi_h}$$

so, we can write

$$\tau_t\left(c_{h,t},\lambda,\theta_t\right)=1-\left(\frac{\phi_h}{\lambda\phi_l}\right)\left[\frac{\frac{(\gamma\phi_h+(1-\gamma)\phi_l)}{\theta_t^{-\frac{1}{\sigma}}\phi_h^{-\frac{1}{\sigma}}c_{h,t}}-\left(\gamma+(1-\gamma)\lambda^{-\frac{1}{\sigma}}\right)\theta_t^{\frac{1}{\sigma}-1}\phi_h^{\frac{1}{\sigma}}}{\gamma\phi_h\left(\frac{\phi_h}{\lambda\phi_l}\right)^{-\frac{1}{\sigma}}+(1-\gamma)\phi_l}\right]^{-\sigma}\tag{23}$$

This is going to be the most important object of study in this work. Now onwards, this function will be referred to as the policy function.

At this point, it is important to note that the assumption of separability between consumption and leisure decisions allows us to relate λ to the equilibrium income distribution. From (12) we have:

$$\lambda=\frac{c_{h,t}^{\sigma}}{c_{l,t}^{\sigma}}$$

Thus, if $\lambda>1$, higher λ implies more income inequality. From this point, λ will be referred to as the income distribution parameter.

We can simplify the Ramsey problem as:

$$\underset{\{c_{h,t}\},\lambda}{Max}\quad E_0\sum_{t=0}^{\infty}\beta^t U(c_{l,t}\left(c_{h,t},\lambda\right),x_{l,t}\left(c_{h,t},\lambda,\theta_t\right))$$

subject to:

$$b_{h,-1}-b_{-1}=E_0\sum_{t=0}^{\infty}\beta^t\frac{U_{c,h,t}}{U_{c,h,0}}(\Phi_{h,t}\left(c_{h,t},\lambda,\theta_t\right)-\Phi_t\left(c_{h,t},\lambda,\theta_t\right))\tag{24}$$

where θ_0, $b_{h,-1}$ and b_{-1} are given.

If an optimal policy exists and it is interior, the optimal allocations must satisfy the government's first order conditions with respect to $c_{h,t}$ and λ and the restriction (24).

Let η be the langrangian multiplier of (24). Then, the first order conditions of the Ramsey

Problem are with respect to $c_{h,t}$ and λ are:

$$c_{l,t}^{-\sigma}\frac{\partial c_{l,t}}{\partial c_{h,t}}+x_{l,t}^{-\sigma}\frac{\partial x_{l,t}}{\partial c_{h,t}}+\eta\beta^{t}\frac{c_{l,t}^{-\sigma-1}}{c_{h,0}^{-\sigma}}(\frac{\partial \Phi_{t}}{\partial c_{h,t}}-\frac{\partial \Phi_{h,t}}{\partial c_{h,t}})=0 \tag{25}$$

$$E_{0}\sum_{t=0}^{\infty}\beta^{t}\left[\left(c_{l,t}^{-\sigma}\frac{\partial c_{l,t}}{\partial \lambda}+x_{l,t}^{-\sigma}\frac{\partial x_{l,t}}{\partial \lambda}\right)+\eta\frac{c_{h,t}^{-\sigma}}{c_{h,0}^{-\sigma}}(\frac{\partial \Phi_{t}}{\partial \lambda}-\frac{\partial \Phi_{h,t}}{\partial \lambda})\right]=0 \tag{26}$$

Thus, given the optimal λ and η, $c_{h,t}$ only depends on the contemporaneous shock θ_t and it has the same correlation properties as the former.

Given (25), (26) and (24) the solution to the Ramsey problem can be written as:

$$\eta=\eta(\phi_l,\theta_0)$$

$$\lambda=\lambda(\phi_l,\theta_0)$$

$$c_{h,t}=c_h(\phi_l,\theta_t,\theta_0)$$

2.2. The Asymmetric Shock Model

Now we are going to introduce some asymmetry in the way the aggregate productivity shock, θ_t, affects agents marginal productivity (or wage), $\omega_{i,t}$. In the model described in section 2.1 the ratio of hourly wages was not affected by the aggregate productivity shock, θ_t. In this new version of the model, the aggregate productivity shock, θ_t, only affects type l consumer's marginal productivity (or wage), $\omega_{l,t}$, so it affects the ratio of marginal productivities, or wages. This is the reason why we call this set up the **Asymmetric shock model**. The arising differences are:

1. The production function

$$y_t=\gamma(1-x_{h,t})\phi_h+(1-\gamma)(1-x_{l,t})\phi_l\theta_t \tag{27}$$

2. Consumers' type i problem

$$\underset{\{c_{i,t},x_{i,t}\}_{t=0}^{\infty}}{Max}\quad E_0\sum_{t=0}^{\infty}\beta^t U(c_{i,t},x_{i,t}) \tag{28}$$

subject to

$$c_{i,t}+\int p_t(\theta)b_{i,t}(\theta)d\theta=(1-\tau_t)\omega_{i,t}(1-x_{i,t})+b_{i,t-1}(\theta_t)+T_t \tag{29}$$

given $b_{i,-1}$ and θ_0.

3. Government's restriction

$$T_t + b_{t-1}(\theta_t) = \tau_t(\gamma(1 - x_{h,t})\omega_{h,t} + (1-\gamma)(1 - x_{l,t})\omega_{l,t}) + \int p_t(\theta)b_t(\theta)d\theta \tag{30}$$

4. The bonds market clearing conditions are as in the **symmetric shock model**.

5. The economy resource constraint

$$\gamma c_{h,t} + (1-\gamma)c_{l,t} = \gamma(1 - x_{h,t})\phi_h + (1-\gamma)(1 - x_{l,t})\phi_l\theta_t \tag{31}$$

In this case a competitive equilibrium is defined as:

Definition 2. *Given* $\{\theta_0, b_{l,-1}, b_{h,-1}, b_{-1}\}$ *such that* (6) *holds, a competitive equilibrium is a process for allocations* $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}, (b_{i,t}(\theta))_{i\in I}, b_t(\theta)\}$*, taxes and transfers* $\{\tau_t, T_t\}$ *and prices* $\{p_t(\theta), (\omega_{i,t})_{i\in I}\}$ *such that:*

1. *For each* $i \in I$, $\{c_{i,t}, x_{i,t}, b_{i,t}(\theta)\}$ *maximizes household's utility function* (28) *subject to the budget constraint* (29) *given* $\{\tau_t, T_t\}$, $\{p_t(\theta), \omega_{i,t}\}$, $b_{i,-1}$ and θ_0.

2. *The equilibrium wages are as follows*

$$\omega_{l,t} = \phi_l\theta_t$$

$$\omega_{h,t} = \phi_h$$

3. *The goverment budget constraint* (30)*, the bonds market clearing condition* (5) *and the economy resource constraint* (31) *hold.*

Two are the main differences with the **symmetric shock model.** First, the equilibrium wages. As noted, in the **symmetric shock model**, the productivity shock affects both households' wages simmetrically. In the **asymmetric shock model** that is not the case anymore and the productivity shock only affects household's l wage. Second, both the production function (27) and the economy resource constraint (31) reflect this same fact, since the productivity shock only affects household's l marginal productivity.

As before, we have to characterize the equilibrium. This is done in the following proposition:

Proposition 2. *Given* $\{\theta_0, b_{l,-1}, b_{h,-1}, b_{-1}\}$ *such that* (6) *holds, if the equilibrium is interior and unique, then the equilibrium process for* $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}, (b_{i,t}(\theta))_{i\in I}, b_t(\theta)\}$, $\{\tau_t, T_t\}$ *and* $\{p_t(\theta), (\omega_{i,t})_{i\in I}\}$ *is uniquely determined by the following conditions:*

- $\exists\ \lambda$ *such that*

$$\frac{U_{c,l,t}}{U_{c,h,t}} = \lambda$$

and

$$\frac{U_{x,l,t}\phi_h}{U_{x,h,t}\phi_l} = \lambda\theta_t$$

holds.

- *The next restriction is satisfied*

$$b_{h,-1} - b_{-1} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} (\Phi_{h,t} - \Phi_t)$$

where

$$\Phi_{i,t} = c_{i,t} - (1 - x_{i,t}) \frac{U_{x,i,t}}{U_{c,i,t}} \qquad \forall i \in I$$

$$\Phi_t = (1 - \frac{U_{x,h,t}}{U_{c,h,t}\phi_h})(\gamma(1 - x_{h,t})\phi_h + (1 - \gamma)(1 - x_{l,t})\phi_l\theta_t)$$

- *The economy resource constraint* (31) *holds.*

The equilibrium characterization is also different from the **symmetric** case. The most important difference is the follwoing: The ratio of marginal utility of leisure is not constant anymore, $U_{x,l,t}\phi_h / (U_{x,h,t}\phi_l) = \lambda\theta_t$. Therefore, at least for the separable utility function used in the numerical exercice that follows, while consumption ratio between the two types of households is constant, hours worked does not need to be. When, θ_t is high hours worked by households type l decreses with respect to those worked by households type h.

In this **asymmetric** case we can rewrite (23) as

$$\tau_t (c_{h,t}, \lambda, \theta_t) = 1 - \left(\frac{\phi_h}{\lambda\phi_l\theta_t}\right) \left(\frac{\frac{(\gamma\phi_h + (1-\gamma)\phi_l\theta_t)}{\theta_t^{-\frac{1}{\sigma}} \phi_h^{-\frac{1}{\sigma}} c_{h,t}} - \left(\gamma + (1-\gamma)\lambda^{-\frac{1}{\sigma}}\right) \theta_t^{\frac{1}{\sigma}} \phi_h^{\frac{1}{\sigma}}}{\gamma\phi_h \left(\frac{\phi_h}{\lambda\phi_l\theta_t}\right)^{-\frac{1}{\sigma}} + (1-\gamma)\phi_l\theta_t} \right)^{-\sigma} \tag{32}$$

3. Results

In the following two subsections, the two versions of the model are used to analyze the relationship between the skill ratio and both the tax rate and the income distribution. Since closed form solutions are not available, we solve the models using numerical simulations. The parameter values choice we consider is very similar to that used in the business cycle literature. For the **symmetric model** we use the following parameter values $(\gamma, \beta, \sigma, \sigma_\varepsilon, \rho) = (0.35, 0.95, 2, 0.1, 0.9)$[2]. For the **asymmetric model** the choice is $(\gamma, \beta, \sigma, \sigma_\varepsilon, \rho) = (0.35, 0.95, 2, 0.01, 0.99985)$. As the reader should notice the only differences are in parameter values that describe the stochastic process. Those imply a less volatile and more persistence process in the **asymmetric model**. This is because when studing the **asymmetric case** we are mainly going to be interested on convergence issues, so business cycle flutuations are not going to be very important.

In section 3.1, we examine the connection between the skill ratio and the tax rate. First, we solve for the optimal policy function of the **symmetric model** to answer the following three questions:

- How are the skill ratio and the average tax rate related?
- Given the skill ratio, what is the effect of the business cycle on the tax rate?
- Are the effects of permanent and non-permanent shocks on the tax rate different?

The main conclusions are:

- The lower the skill ratio, the higher the average levy.
- Chari, Christiano, Kehoe (1992) result- i.e. taxes on labor are roughly constant over the business cycle- holds only if productivity ratio is constant.
- Permanent and non-permanent shocks effects on fiscal policy are very alike.
- There is no fiscal convergence, even when the skill ratio does converge.

  Second, we solve the for the optimal policy function of the asymmetric model to answer the two following questions:

- Are the effects of permanent and non-permanent shocks on the tax rate different?
- Does the initial skill ratio affect the actual optimal tax rate?

In the second subsection, 3.2, the analysis of the relation between the skill ratio and the income distribution is performed. Hence, we attempt to respond to the following queries:

- How are the skill ratio and the income distribution related?
- Is the initial skill ratio significant for the actual distribution?

[2] Our aim is to study optimal fiscal policy and so much to match the data. This is why we do not calibrate the model to get close to the data. We would like to remark that our qualitative results are robust to different calibrations.

ϕ_l^j	$E(\tau^s(\phi_l^j,\theta_t,\theta_0))$
0.7	0.30875
0.825	0.18159
0.95	0.05419

Table 1: Average Tax Rate, as a Function of Consumer "l"'s Marginal Productivity

In this case, the answers are:

- The lower the skill ratio, the more unequal the income distribution.
- The lower the initial skill ratio, the more unequal the income distribution.

Just note that for the stochastic process we use a Markov Change with unconditional mean equal to one. Let us use "s" as superindex for the symmetric model policy function (1), and "as" for the asymmetric one, (32).

3.1. The Skill Ratio and the Tax Rate

3.1.1. Inequality and The Average Tax Rate

Now the effects of permanent changes in the skill ratio on the average optimal tax rate are analyzed. To understand this relation, the average tax level of three identical economies (except by productivity of type l consumer) are compared.

Using (23), this analysis can be formally written as the determination of:

$$E(\tau^s(.,\theta_t,1))$$

i.e. the unconditional mean of the tax level as a function of ϕ_l. Consider three economies indexed by $j \in \{1,2,3\}$ and let $\tau^s(\phi_l^j,\theta_t,1)$ be the policy function associated with economy j where $\phi_l^1 = 0.7$, $\phi_l^2 = 0.825$ and $\phi_l^3 = 0.95$, when θ_t occurs.

As shown in the table 1, the higher ϕ_l, the lower the tax. This is a classical result in the literature.

3.1.2. Business Cycle and Fiscal Policy

Chari, Christiano, Kehoe (1992) examine optimal fiscal policy over the business cycle in the case of homogenous agents. This new setup allows two extensions of their analysis. In the first place, it lets us repeat their exercise in the case of heterogenous agents, and secondly, it permits an extension when the cycle affects the skill ratio.

First, we use the symmetric model to study the consequences of the business cycles on the tax rate. As before, using (23), this analysis can formally be written as the study of the next function

$$\tau^s(\phi_l, ., 1) \tag{33}$$

Second, we exploit the asymmetric model to study how perturbations of the skill ratio affect the tax rate. In this case, the following function is analyzed

$$\tau^{as}(\phi_l, ., 1) \tag{34}$$

Note that the difference between the last two equations is the superindex.

Figure 1 plots both functions. As it can be seen, (33), does not have a very notable upward slope, i.e. taxes are slightly procyclical. On the other hand, (34) has a very significant downward slope.

These two results brings us to the following conclusion: the Chari, Christiano, Kehoe (1992) result- i.e. taxes on labor are roughly constant over the business cycle- holds only if productivity ratio is constant. Thus, optimal tax rate should smooth distortions over time only if the skill ratio does not change.

The intuition for the differences between (33) and (34) is as follows. Type l consumer sets the fiscal policy. She gets half of the difference between type h consumer 's taxes and her own. As a result, she is going to increase the tax level until both type h and l marginal payments are equal. In the symmetric case, shocks do not affect the skill ratio; both type h and l marginal payments are affected in the same way, so taxes do not move. In the asymmetric model, shocks do affect the skill ratio; both type h and l marginal payments are affected asymmetrically, so taxes do move to compensate.

Permanent versus Non-permanent Shocks Now, we analyze whether there are differences between tax policies facing permanent and non-permanent shocks. Consider the following definitions of permanent and non-permanent perturbations environments:

Definition 3. *A permanent perturbation environment holds if* $\theta_t = \theta_0 \ \forall t$.

Definition 4. *A non-permanent perturbation environment holds if it is not permanent.*

Let us use the subindex p for the case of permanent shock environment. Since the non-permanent environment is the one used until now, we will not use any subindex. Thus, we study the difference between the following two functions

$$\tau_p^s(\phi_l, ., .)$$

$$\tau^{s}(\phi_l, ., 1)$$

Note that the first corresponds to a permanent shock and the second to a non-permanent shock. The results are reported in figure 2. There is not much difference between these two policy functions. As noted before, in the symmetric model perturbations correspond to business cycle shocks. Thus, for perturbations in the range (0.8,1.2) both functions are very similar, with differences of less than $\pm 2\%$ over the tax rate at the mean of the perturbation (remember that the mean of the process is 1).

Considering now the asymmetric model, we compare the following functions

$$\tau_p^{as}(\phi_l, ., .)$$

$$\tau^{as}(\phi_l, ., 1)$$

The results are reported in figure 3. The optimal policy functions for the asymmetric model are also alike.

3.1.3. The Initial skill ratio and the actual tax rate: The Non-Fiscal Convergence.

Consider a set of economies that, starting with different skill ratio levels, converge to the same one. How does the initial skill ratio affect the actual tax rate? In other words, is there convergence on tax rate? The answer is no.

Let us consider four versions of the asymmetric model with index $j \in \{1, 2, 3, 4\}$. Let

$$\tau^{as}(\phi_l, \theta_t, \theta_0^j)$$

be the policy function associated with economy j and let $\theta_0^1 = 0.5$, $\theta_0^2 = 0.7$, $\theta_0^3 = 0.8$ and $\theta_0^4 = 0.9$.

It is important to stress the following three points:

- Type h productivity is fixed, and lower θ_0 means lower initial type l productivity.
- Type l productivity grows over time (since $\theta_0^j < 1 \ \forall j \in \{1, 2, 3, 4\}$).
- Asymptotically, all economies converge to the same skill ratio.

The results are reported in figure 4. As we can see, the lower θ_0, the higher the taxes. Consequently, even assuming skill ratio convergence, there is not fiscal policy convergence.

Since complete markets are assumed, an intertemporal substitution of consumption argument can explain why the lower the initial inequality level, the higher the taxes in the long run. Consumer l wants to smooth consumption, so she increases consumption today

ϕ_l^j	λ
0.7	1.5478
0.825	1.2949
0.95	1.0773

Table 2: Income Distribution Parameter as a Function of Consumer "l"'s Marginal Productivity

via long run taxes. The lower the initial skill level the higher taxes she needs tomorrow. In addition, labor supply's elasticity prevents an excessive increase in tomorrow's taxes, so the initial productivity gap across economies cannot be totally offset.

3.2. The Skill Ratio and the Income Distribution

3.2.1. Inequality and income distribution

Now, the effects of permanent changes in the skill ratio on income distribution are analyzed. The same three economies used in section 3.1.1 are used here, but in this case the function to analyze is

$$\lambda^s = \lambda^s(., 1)$$

Table 2 reports the results for each j. Logically, the lower the skill ratio the higher λ since that means higher inequality in consumption. This means that fiscal policy cannot totally compensate for differences in skill level, even in the case that the poorest agent chooses the taxes. This is because taxes are distortionary and labor supply is elastic.

3.2.2. The Initial skill ratio and the actual income distribution: The Non-income distribution Convergence

Consider a set of economies that, starting with different skill ratio levels, converge to the same one. How does the initial skill ratio affect the income distribution? In other words, is there convergence on income distribution? The answer is also no. In other words, let us analyze the following function

$$\lambda^s = \lambda^s(\phi_l, .)$$

We will use the same four economies used in 3.1.3. As we can see, the lower θ_0, the higher the taxes. Hence, although there is productivity ratio convergence, this does not apply to fiscal policy. λ, the income distribution parameter, does not converge either (see table 3).

θ_0^j	λ
0.5	2.15
0.7	1.8
0.8	1.6
0.9	1.2

Table 3: Income Distribution Parameter as a Function of Initial Inequality Level, in the Asymmetric Shocks Model

The lower θ_0, the lower λ.

As mentioned before, type l consumer determines tax policy. We assume complete markets, thus she can finance transfers today with taxes tomorrow. Thus, an intertemporal substitution of consumption argument can explain why the lower the initial inequality level, the higher the taxes in the long run. In addition, labor supply's elasticity prevents an excessive increase in tomorrow's taxes, so the initial productivity gap across economies cannot be totally offset. The last point shows the importance of elasticity of labor supply[3] to get non-convergence on income distribution.

4. Conclusion

Most of the papers on fiscal policy and income distribution only define and compute the non-stochastic stationary equilibrium. This equilibrium concept is sometimes useful because, together with some assumptions, it permits us to get closed forms solutions for the policy functions (see Persson and Tabellini (1994)). On the other hand, the use of this equilibrium concept limits the results in two dimensions: [1] it is not possible to study the difference between the effects of permanent and non-permanent perturbations; [2] the consequences of the initial conditions on both the steady-state and transition policies cannot be analyzed.

This paper is an attempt to address these two issues. The main conclusions are as follows. First, tax responses to both permanent and non-permanent perturbations are very similar. Second, the initial skills inequality has a huge effect on both actual levy and the actual income distribution. And finally, the Chari, Christiano, Kehoe (1992) result- i.e. taxes on labor are roughly constant over the business cycle- holds only if productivity ratio is constant.

[3] As noted before, this point is missing in Persson and Tabellini(1994).

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5. Appendix

5.1. Numerical Algorithm

We am going to describe the method used for the **symmetric model**.

We need to solve (24), (25) and (26).

Step 1 Set θ_0.

Step 2 Guess λ and η.

Step 3 Generate one realization, 5000 periods long, of the Markov Chain. Let

$$A = \begin{bmatrix} \theta_0 & \theta_1 & \theta_2 & ... & \theta_{4999} \end{bmatrix}$$

Step 4 Generate 100 realization, 21 periods long, of the Markov Chain. Let

$$B = \begin{bmatrix} \theta_0 & \widehat{\theta}_1^1 & \widehat{\theta}_2^1 & ... & \widehat{\theta}_{20}^1 \\ \theta_0 & \widehat{\theta}_1^2 & \widehat{\theta}_2^2 & ... & \widehat{\theta}_{20}^2 \\ ... & ... & ... & ... & ... \\ \theta_0 & \widehat{\theta}_1^{100} & \widehat{\theta}_2^{100} & ... & \widehat{\theta}_{20}^{100} \end{bmatrix}$$

Step 5 Solve, using (25), $c_h(\theta_i; \lambda, \eta)$ for each one of $\theta_i \in A$.

Step 6 Let

$$y_j = \sum_{t=j}^{5000} \beta^j [c_h(\theta_j; \lambda, \eta)^{-\sigma} \frac{\partial c_l(\theta_j; \lambda, \eta)}{\partial \lambda} + x_l(\theta_j; \lambda, \eta)^{-\sigma} \frac{\partial x_l(\theta_j; \lambda, \eta)}{\partial \lambda} +$$

$$+\eta \frac{c_h(\theta_j; \lambda, \eta)^{-\sigma}}{c_h(\theta_0; \lambda, \eta)^{-\sigma}} (\frac{\partial \Phi(\theta_j; \lambda, \eta)}{\partial \lambda} - \frac{\partial \Phi_h(\theta_j; \lambda, \eta)}{\partial \lambda})]$$

Let $Y = \begin{bmatrix} y_1 & y_2 & ... & y_{2500} \end{bmatrix}$. Let $X_1 = \begin{bmatrix} \theta_0 & \theta_2 & ... & \theta_{2499} \end{bmatrix}$. Let $X_2 = \begin{bmatrix} \theta_0^2 & \theta_1^2 & ... & \theta_{2499}^2 \end{bmatrix}$. Using the standard OLS method, estimate the parameters of

$$Y = \mu + \beta_1 X_1 + \beta_2 X_2 + \epsilon$$

Note that, given these estimations, we can write

$$E_t(y_{t+1}/\theta_t) \simeq \widehat{\mu} + \widehat{\beta}_1 \theta_t + \widehat{\beta}_2 \theta_t^2$$

Step 7 Repeat the last step for

$$\widetilde{y}_j = \sum_{t=j}^{5000} \beta^j \frac{c_h(\theta_j;\lambda,\eta)^{-\sigma}}{c_h(\theta_0;\lambda,\eta)^{-\sigma}} (\Phi(\theta_j;\lambda,\eta) - \Phi_h(\theta_j;\lambda,\eta))$$

Let

$$E_t(\widetilde{y}_{t+1}/\theta_t) \simeq \widehat{\pi} + \widehat{\zeta}_1\theta_t + \widehat{\zeta}_2\theta_t^2$$

Step 8 Solve, using (25), $c_h(\widehat{\theta}_i^j, \lambda, \eta)$ for each one of $\widehat{\theta}_i^j \in B$.

Step 9 Check if

$$\frac{1}{100}\sum_{j=1}^{100}\left(\begin{array}{c} \sum_{i=0}^{19} \beta^i \left(c_h(\widehat{\theta}_i^j;\lambda,\eta)^{-\sigma} \frac{\partial c_l(\widehat{\theta}_i^j;\lambda,\eta)}{\partial\lambda} + x_l(\widehat{\theta}_i^j;\lambda,\eta)^{-\sigma} \frac{\partial x_l(\widehat{\theta}_i^j;\lambda,\eta)}{\partial\lambda} \right) + \\ +\eta \frac{c_h(\widehat{\theta}_i^j;\lambda,\eta)^{-\sigma}}{c_h(\theta_0;\lambda,\eta)^{-\sigma}} \left(\frac{\partial\Phi(\widehat{\theta}_i^j;\lambda,\eta)}{\partial\lambda} - \frac{\partial\Phi_h(\widehat{\theta}_i^j;\lambda,\eta)}{\partial\lambda}\right) + \beta^{20}\left(\widehat{\mu} + \widehat{\beta}_1\widehat{\theta}_i^{20} + \widehat{\beta}_2(\widehat{\theta}_i^{20})^2\right) \end{array}\right) = 0$$

and

$$\frac{1}{100}\sum_{i=1}^{100}\left(\sum_{j=0}^{19}\beta^i \frac{c_h(\widehat{\theta}_i^j;\lambda,\eta)^{-\sigma}}{c_h(\theta_0;\lambda,\eta)^{-\sigma}} (\Phi(\widehat{\theta}_i^j;\lambda,\eta) - \Phi_h(\widehat{\theta}_i^j;\lambda,\eta)) + \beta\left({}^{20}\widehat{\pi} + \widehat{\zeta}_1\widehat{\theta}_i^{20} + \widehat{\zeta}_2(\widehat{\theta}_i^{20})^2\right)\right) = 0$$

hold. If it does not, choose new λ and η, and go to step 2 Note that these two equations are approximations to (24), and (26).

Fig 1: Policy function comparison between symmetric and asymmetric model

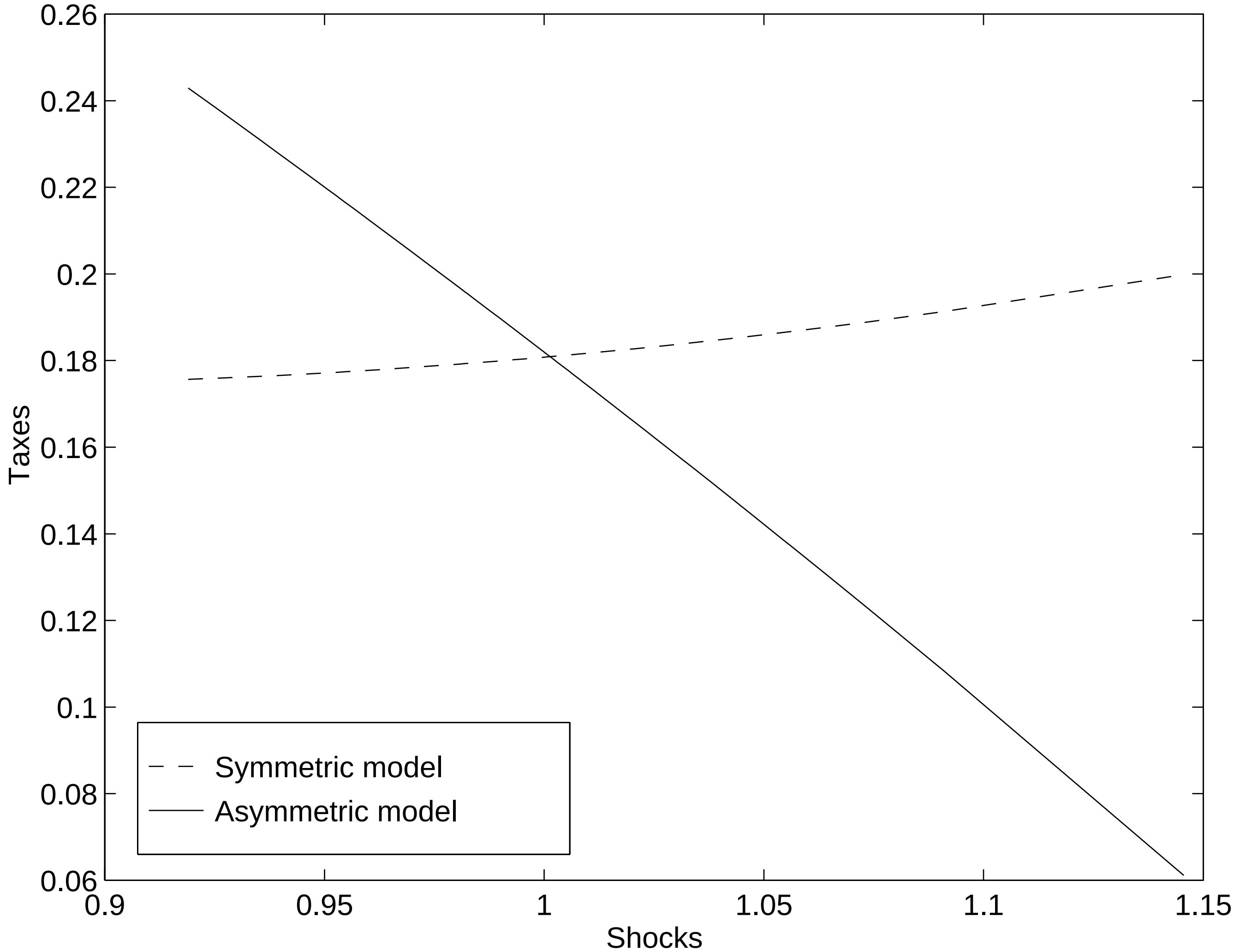


Fig 2: Permanent vs. non permanent perturbation in symmetric model

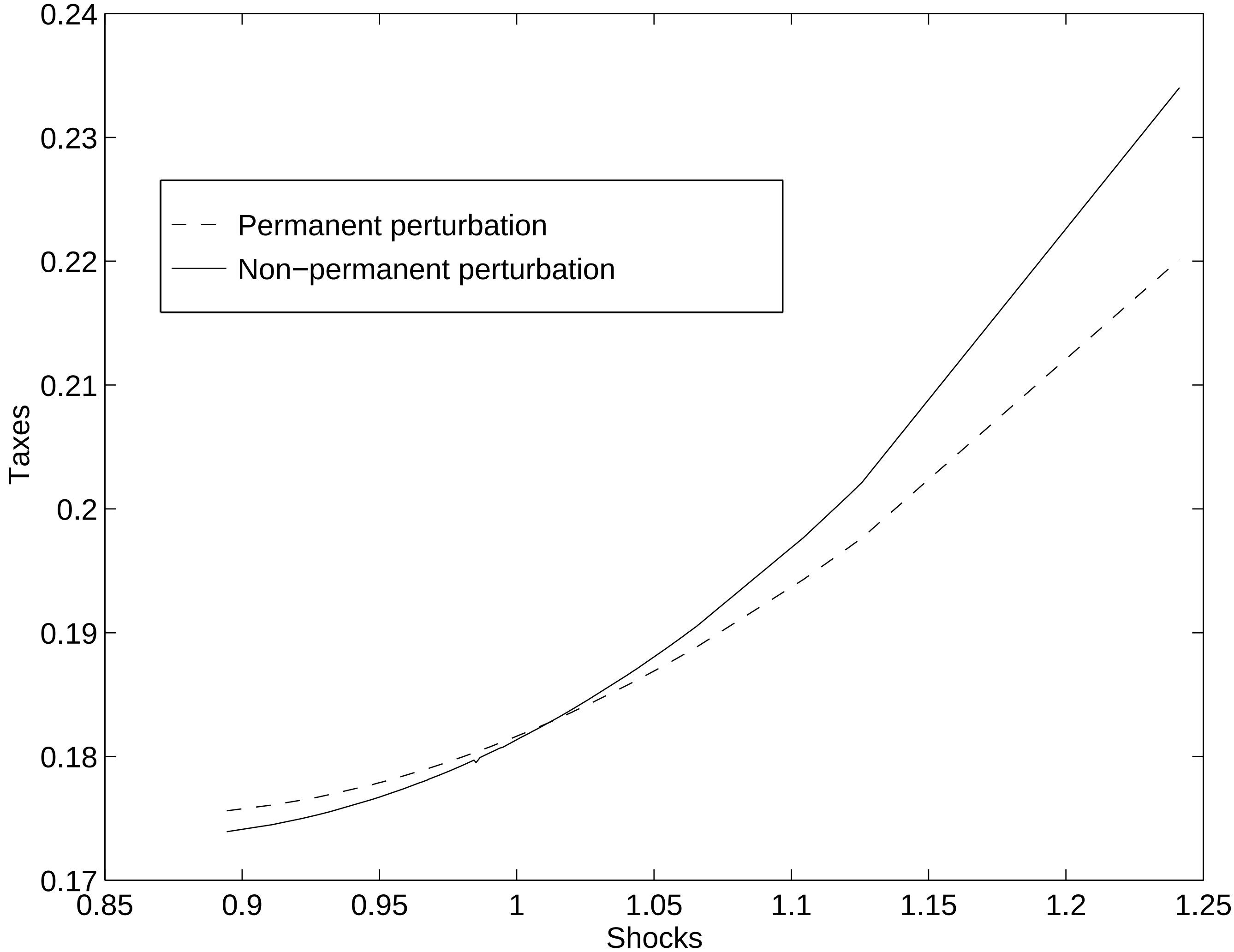
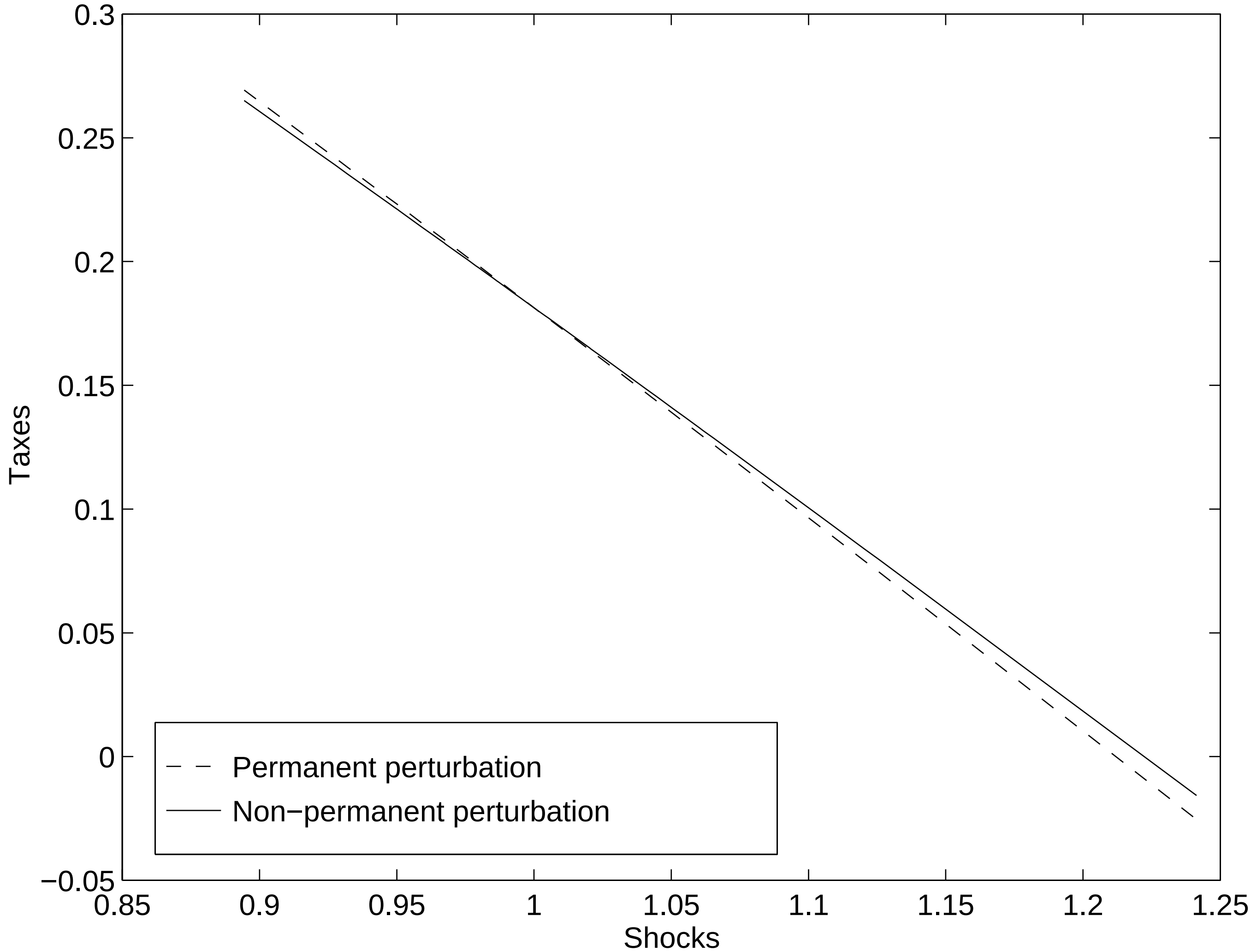


Fig 3: Permanent vs. non permanent perturbation in asymmetric model



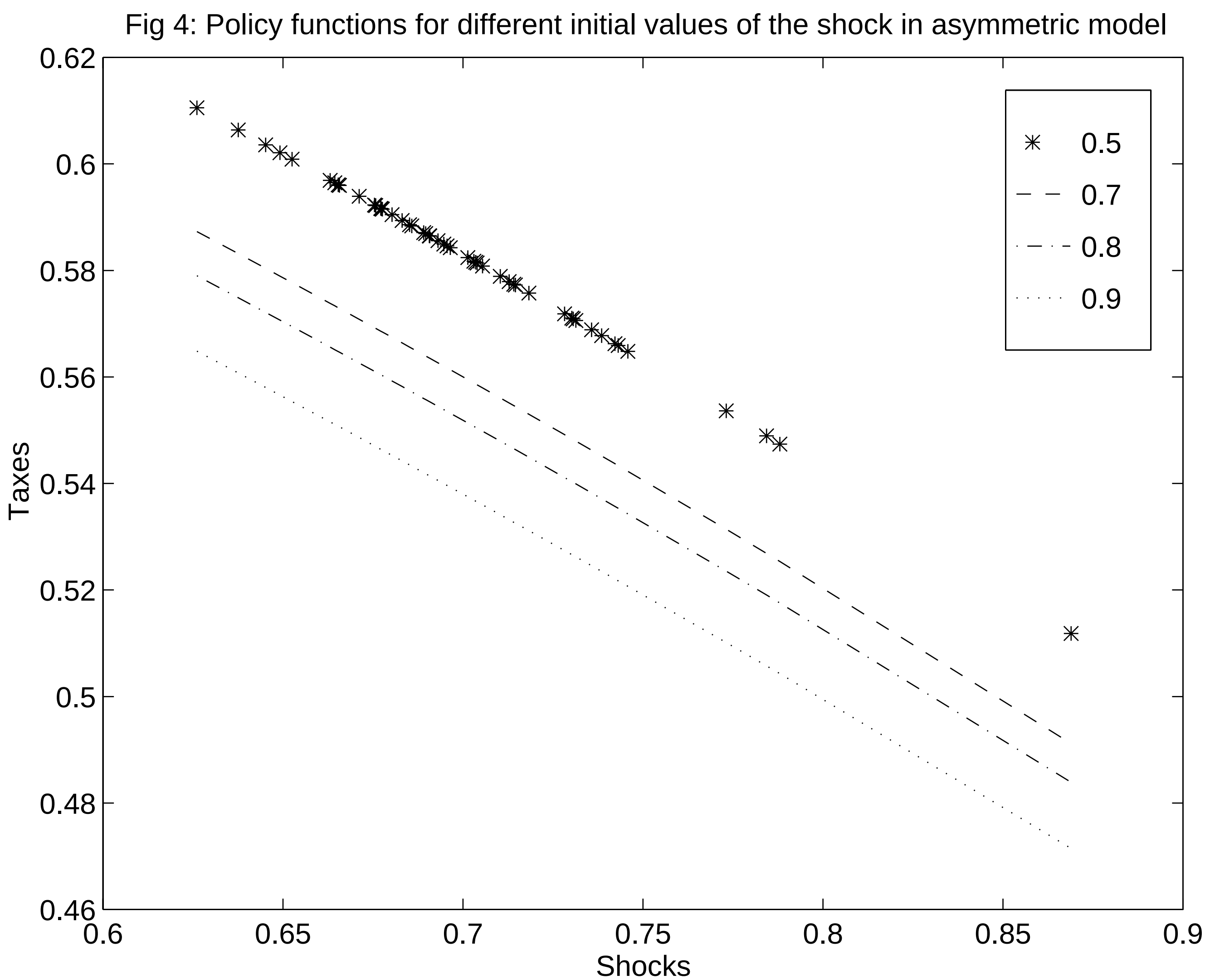


Fig 4: Policy functions for different initial values of the shock in asymmetric model
0.62
0.6
0.58
0.56
0.54
0.52
0.5
0.48
0.46
Taxes
0.6
0.65
0.7
0.75
0.8
0.85
0.9
Shocks
0.5
0.7
0.8
0.9