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Abstract: This paper sets up a model to account for differences in total factor productivity due to differences in enforcement of contracts. Vertical specialization generates the need for intra-period credit, because final goods producers cannot pay their intermediate goods suppliers before they produce their final good. The paper shows that if there are enforcement problems, the capital distribution is skewed in the sense that intermediate goods producers operate at lower capital levels and higher marginal products of capital than final goods producers. This wedge is created by the price for intermediate goods, which is lower in economies with bad enforcement. For this reason, the high-productivity firms in the intermediate goods sector have no incentive to grow and the low-productivity firms in the final goods sector, benefiting from low intermediate goods prices, have no incentive to shrink, which causes productivity to be lower in countries with bad enforcement.

JEL classification: E10, E23

Key words: vertical specialization, limited enforcement, productivity

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1 Introduction

Why are certain countries poor and stay that way? The neoclassical growth model predicts that poor countries, even if living in autarky from the rest of the world, should rapidly catch up to industrialized countries. In the presence of perfect international capital markets this process would even take place instantaneously as capital flows to the countries with low capital endowment and high interest rates in order to equalize interest rates across countries.

All this requires that technology across countries is identical. Prescott (1997) therefore raises the point that a theory explaining differences in production functions across nations, in particular differences in total factor productivity, is needed. This paper attempts to develop a theory of differences in TFP that have their origin in incomplete contract enforcement.

We set up a dynamic general equilibrium model to examine the role played by incomplete enforcement of intra-period trade credit contracts. Output is produced in a sequence of intermediate production stages. Due to a benefit to specialization every firm concentrates its activity to the production of one single stage. Firms in each stage except the initial one purchase the intermediate good of the previous stage using trade credit and sell their output to the firms in the next stage, receiving trade credit of that firm in exchange. That is, rather than looking at enforcement problems pertaining to long term borrowing contracts we look at default risk within the chain of intra-period trade credit.

A real world example for this chain of credit would be the two intermediate goods: iron ore and steel, and the final good automobiles. Iron ore is an input in the steel production and steel is an input in the automobile production. The model implicitly assumes that there is a benefit to specialization, that is, every firm in this economy specializes in only one sector of the economy. Car producers, therefore, would never buy iron ore to produce the steel themselves. Instead they purchase steel in the intermediate goods market.

There is, by the way, plenty of evidence that vertical specialization in the real world is far more advanced than the relatively coarse grid of intermediate stages iron ore - steel - automobiles. Kei-Mu Yi (2001) shows that there is specialization even across countries that can account for the recent boost to international trade. He shows that effects of declines in tariffs are magnified if intermediate goods cross the border several times. Given that producers are even willing to pay tariffs by shipping goods across borders several times clearly indicates that there must be some payoff to specialization.

Why do firms use trade credit? Notice that in a standard macro model with a sequence of intermediate goods as described above there would be no borrowing and lending other than potential long term borrowing contracts for financing of firms. In this paper however, I assume that the intra-period timing of production stages requires intra-period debt contracts. To stay with the iron ore-steel-automobile example, the stage 1 producer hands over 100 units of iron ore to the steel producer. Let the relative price of iron ore to cars be 0.2, then the steel producer pays with a promise to deliver 20 cars. Next, the steel producer produces 100 units of steel and sells it to the car manufacturer at a relative price of 0.5, that is, he gets a promise from the car manufacturer over 50 cars. Next, the car manufacturer uses the steel to make 100 cars. He then gives 50 cars to the steel manufacturer who in turn gives 20 cars to the iron ore producer. In the model there is default risk on the obligations of the automobile producer and the steel producer. More precisely, contracts have to be designed to be individually rational, that is, in such a way that the car manufacturer does not decide to run away without paying 50 cars to the steel producer and the steel producer does not run away without handing over 20 cars as payment for the iron ore he received.

The timing of production creates debt relationships that would not exist in a macro model where one assumes that factor payments, production, revenues, and profits all occur simultaneously. In my model the firm that produces first, receives its factor payments and therefore, its profits, last. Note that the total amount of intra-period debt outstanding increases with the number of production stages. Intra-period debt can reach any multiple of final production, if we add enough stages. Suppose there were N different stages and the relative price of stage i is i/N. The trade credit of a firm in sector i it owes to the sector $i-1$ firms is then $(i-1)/N$. The total amount of debt relative to the final product is then $\sum_{i=1}^{N}(i-1)/N = (N-1)/2$. This is quite intuitive: The more stages we use for production the more leveraged firms become with respect to their intermediate goods purchases, and consequently the amount of credit relative to output increases.

There is some evidence that trade credit in fact plays a big role in the real world, in particular in developing countries. Cardoso (2002) points out that of all firms in Mexico that use credit, 40% use trade credit as their main source of financing. According to Ickes (1998) promissory notes, called *veksels*, are very wide spread in Russia. Large enterprises like natural gas or oil producers purchase inputs like machinery with notes denominated in natural gas or oil for a number of reasons: First, the exact same timing problem as mentioned above applies and the veksel gives the ability to pay for inputs **before** the firm's own production and revenue take place. Second, promissory notes are the perfect tax evasion vehicles for firms that face essentially 100% marginal tax rates, which is the case in many developing countries. Third, firms might not be able to get large cash loans from a bank due the bank's fear that funds are diverted offshore and never paid back.

The main theoretical result is that with identical initial capital endowments a country with poor enforcement will end up in a lower steady state than a country with effective enforcement. The risk of default on credit induces two distinct channels of inefficiency: First, and most obvious, firms in the sectors that use intermediate goods and purchase them with trade credit might operate below capacity because they cannot get as much trade credit to purchase intermediate goods as they want to. Second, the pricing of intermediate goods might create perverse incentives: Intermediate goods producers operate with low capital levels and have a large marginal product, however they do not accumulate more capital because the retail price for intermediate goods is going to be low in an economy with bad enforcement. On the other hand final goods producers over-accumulate capital: They operate with large amounts of capital at low marginal product, and they are willing to do so, because the markup between intermediate goods and final goods is large.

This paper is related to Quintin's (2000) work. In his paper, entrepreneurs can be characterized by a two-dimensional vector consisting of their productivity and their credit-worthiness which is modelled endogenously. In the benchmark economy with perfect enforcement, only the most productive individuals are able to get financing in order to run a firm, regardless of their credit-worthiness. With incomplete enforcement, however, there is an inefficient allocation of firm financing: Relatively unproductive firms exist, just because their entrepreneurs happened to be credit-worthy, and relatively productive entrepreneurs who happen to be not credit-worthy cannot run a firm and have work in the labor force instead.

The remainder of the paper is organized as follows: Section 2 introduces the model and defines equilibrium. Section 3 derives results about steady states including a numerical example. A theoretical contribution of this baseline model is that steady states so not have to be unique. Therefore, sections 4 shows how to pin down a unique steady state by introducing capital markets. Section 5 concludes the paper.

2 Model

The model I consider is an extremely stylized environment with infinite horizon and no uncertainty. Time is discrete and indexed by $t = 0, 1, 2, ...$

The economy is populated by a measure one of infinitely-lived households who have preferences over consumption streams $\{c_t\}_{t=0,1,2,...}$ representable as:

$$U = \sum_{t=1}^{\infty} \beta^t u\left(c_t\right)$$

where $0 < \beta < 1$ and u satisfies the usual Inada conditions. Each household has a production technology and an initial capital and labor endowment (k_0, l_0). Labor is assumed to be fixed at $l_t = 1$ for all households and all periods. Initially, I assume that production factors are non-tradable, even though this assumption will be relaxed later. That is, every household starts with an initial capital endowment k_0 and labor endowment $l = 1$. Then over time households can accumulate more capital if they desire, but only through personal savings, not through borrowing.

For simplicity I assume there are only two sectors, one intermediate goods sector, called sector 1, and one final goods sector, called sector 2. This model, therefore, is even simpler than the 3 goods economy mentioned in the introduction, but it still delivers the desired results, at least qualitatively.

The output of a sector 1 firm is:

$$y_1 = f(k)$$

where f satisfies the usual assumptions: $f(0) = 0, f'(0) = \infty, f'(\infty) = 0, f''(k) < 0$. Output in sector 2, the final goods sector, is a function of both capital and intermediate goods x_1 :

$$y_2 = y_2(x_1, k) = \min\{x_1, f(k)\}$$

That is, in order to produce one unit of the final good, a firm has to purchase at least one unit of the intermediate good. Moreover, firms face a borrowing constraint of the following form:

$$x_1 \leq h(k)$$

The function h is given exogenously and specifies - as a function of a firm's capital - how much credit it can get. The function h is assumed to be increasing in the capital level, reflecting the assumption that larger firms are able to get more trade credit.[1]

[1] This appears to be in line with real world data. Gertler and Gilchrist (1994) point out that there is a strong positive correlation between firm size and access to capital markets. They do, however, also concede that the size may not be the actual driving force in this correlation. It is also possible that other factors affecting credit-worthyness are simply correlated with firm size.

Additional assumptions on h will be necessary later in this section, once we introduce some more structure.

The within-period timing works as follows. First, firms decide in which sector they want to produce. Notice that firms make this decision every period, that is, they can switch between sectors over time. Second, sector 1 produces its output. Third, Sector 2 purchases the intermediate good for price p where p is the per unit price relative to the final product. Sector 2 producers, however cannot pay for the intermediate good right away, but instead hand over IOUs denominated in the final good. These advances are like those of the Russian oil producers mentioned in the introduction would do. Fourth, sector 2 produces the final good and pays off its trade credit to sector 1. Fifth, both sectors make their savings versus consumption decision. Notice that there is no market for final goods.

The equilibrium concept used in this paper is that of recursive competitive equilibrium and the precise definition can be found in Appendix 1. Just for reference purposes, here is the firm's problem in sequential form:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to:

$$\begin{aligned} c_t + k_{t+1} &\leq \max\{p_t f(k_t), (1-p_t)\min\{h(k), f(k)\}\} + (1-\delta) k_t \\ c_t, k_{t+1} &\geq 0,\ k_0 \text{ given} \end{aligned}$$

That is, sector 2 households can get trade credit of at most $h(k)$. Given p_t, the price for intermediate goods, the firm makes its choice between the two sectors, and makes the savings versus consumption decision. The function h is chosen in such a way that there is a critical capital level $0 \leq \bar{k} \leq \infty$ with

$$\begin{aligned} h(k) &\geq f(k) \text{ for } k \geq \bar{k} \\ h(k) &< f(k) \text{ for } k < \bar{k} \end{aligned}$$

which means that for capital levels low enough the trade credit constraint always binds.

3 Equilibrium Properties: Steady States

This section is going to characterize steady state equilibria. Let's start with the case of no enforcement problems as a benchmark:

3.1 The case of no enforcement problems: $\bar{k} = 0$

Proposition 1 *If $\bar{k} = 0$ then $p^\star = \frac{1}{2} \forall t$.*

Proof. In either case $p^\star < \frac{1}{2}$ and $p^\star > \frac{1}{2}$ market clearing would be violated because all producers would either operate in sector 2 (if $p^\star < \frac{1}{2}$) or sector 1 (if $p^\star > \frac{1}{2}$). ■

Proposition 2 *The steady state capital level is equal for all firms and given by:*

$$f'\left(\hat{k}\right) = 2\left(\frac{1}{\beta} - 1 + \delta\right)$$

Proof. Then the Euler condition is

$$\beta \frac{u'(c')}{u'(c)} = \frac{1}{\frac{1}{2} f'(k') + 1 - \delta}$$

The steady state capital level is then identical to the one in a standard neoclassical growth model as stated above. ■

3.2 The case of $\bar{k} > 0$: Enforcement constraint is binding

Now I draw my attention to the interesting case where the economy faces enforcement constraints. A steady state is characterized by four variables: The two sectors' capital levels $k_1^\star, k_2^\star$, the intermediate goods price $p^\star$, and the share of firms $\mu^\star$ working in sector 1, leaving $1 - \mu^\star$ working in sector 2. Three equalities have to hold: The Euler condition for sector 1 firms in steady state:

$$\frac{1}{\beta} - 1 + \delta = p^\star f'(k_1^\star)$$

For sector 2 firms the Euler condition takes the following form:

$$\frac{1}{\beta} - 1 + \delta \left\{ \begin{array}{c} = (1 - p^\star) h'(k_2^\star) \\ \in [(1 - p^\star) f'(k_2^\star), (1 - p^\star) h'(k_2^\star)] \\ = (1 - p^\star) f'(k_2^\star) \end{array} \right\} \left\{ \begin{array}{c} \text{if } h(k_2^\star) < f(k_2^\star) \\ \text{if } h(k_2^\star) = f(k_2^\star) \\ \text{if } h(k_2^\star) > f(k_2^\star) \end{array} \right\}$$

and a market clearing condition:

$$\mu^\star f(k_1^\star) = (1 - \mu^\star) \min \{h(k_2^\star), f(k_2^\star)\}$$

In addition, firms must prefer to produce in their sector, that is, a firm holding capital $k_i^\star$ must find it in its best interest to produce in sector i rather than in sector $3-i$:

$$\begin{aligned} p^\star f\left(k_1^\star\right) &\geq \left(1-p^\star\right)\min\left\{h\left(k_1^\star\right), f\left(k_1^\star\right)\right\} \\ \left(1-p^\star\right)\min\left\{h\left(k_2^\star\right), f\left(k_2^\star\right)\right\} &\geq p^\star f\left(k_2^\star\right) \end{aligned}$$

Clearly, the three equations and two inequalities are not enough to pin down the steady state allocation. Later in this section we show that there are indeed multiple steady states.

The numerical examples in this section all use the following functional forms:

$$\begin{aligned} y_1 &= Ak^\alpha \\ y_2 &= \min\left\{qk^\gamma, Ak^\alpha\right\} \\ U &= \sum_{t=0}^{\infty}\beta^t\frac{c_t^{1-\sigma}-1}{1-\sigma} \end{aligned}$$

with $\sigma, A, q > 0, 1 \geq \gamma > \alpha$. That is, for $k \leq \bar{k} = (A/q)^{1/(\gamma-\alpha)}$, sector 2 firms are borrowing constrained. They could potentially produce Ak^α, but they only get trade credit for qk^γ which limits their production to exactly qk^γ. One can think of A as being the TFP in the production and q being the TFP in the enforcement technology. Evidently, larger firms can borrow more in absolute terms, but also the *share* of trade credit relative to its own production capacity becomes larger: $\frac{q}{A}k^{\gamma-\alpha}$ is an increasing function, starting at zero. This reflects the intuition that a firm like General Motors would find it hard to run away and hide one quarter worth of car production. On the other hand Jeske Inc. might find it very easy to run away with the proceeds of one quarter and hide on the Cayman Islands. This is also consistent with Ickes (1998) who points out that the ability to issue promissory notes depends on size: Only large firms like oil, gas or railway companies are usually able to issue them. Hence, qualitatively this parametrization has the same flavor as a model with endogenous borrowing constraints where firms have the opportunity to run away with a certain portion of their capital.

Figure 1 plots profit as a function of capital if $p < \frac{1}{2}$, that is, in the case where in the absence of trade credit constraints producing in sector 2 would always dominate sector 1. However, since firms do face a borrowing constraint it is actually optimal to produce in sector 1 for low enough capital levels. For capital levels in the intermediate region between the two kink points firms would choose to produce in sector 2 but output is constrained by function h. For capital levels beyond the second kink firms produce in sector 2 without a binding credit constraint. For example, if sector 1 builds engines and sector 2 builds cars, then firms with very little capital (to the left of the first kink point) would rather produce engines themselves. Prices for engines may be low and therefore profit margins for car manufacturers are high, but a small firm could only get credit for very few engines, and therefore can only produce very few cars. Slightly larger firms, who have capital in the intermediate region (between the two kink points) purchase engines via trade credit, even though not as many as

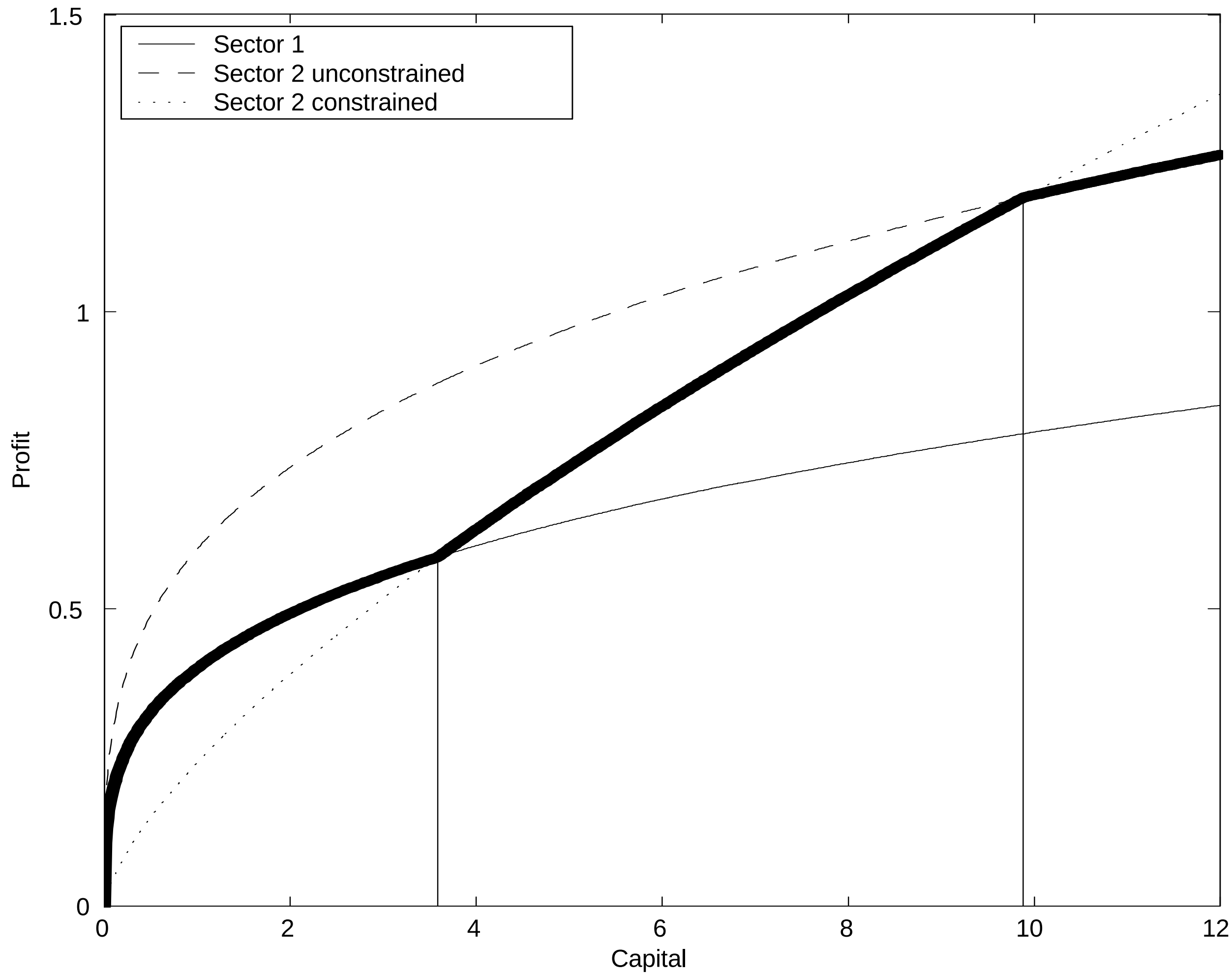

Figure 1: Profit in the two sectors: Max Profit in bold.

they would like to. They could produce more cars but are able to only buy a limited number of engines. Therefore, part of their capital is effectively idle. Due to the low price for engines however, this is still more profitable than producing engines themselves. Finally, large firms who are unconstrained choose between producing $f(k)$ engines and $f(k)$ cars. They go for car production because of the low price for engines.

Next one can show an interesting result about the source of the reduction in steady state output of economies with poor enforcement compared to the perfect enforcement case. Apart from the obvious inefficiency due to sector 2 firms producing $h(k) < f(k)$, there is another channel for inefficiency:

Proposition 3 *There are steady state equilibria with output, consumption, and welfare lower than in an economy with perfect enforcement in which $f(k_2^\star) = h(k_2^\star)$, that is, there is no waste due to strictly binding borrowing constraints.*

Proof. By example: Set

$$\begin{aligned} A &= 1.00, \alpha = 0.30 \\ q &= 0.50, \gamma = 0.80 \end{aligned}$$

At price $p^\star = 0.15$ steady state capital levels are $k_1^\star = 0.1099, k_2^\star = \bar{k} = 4.000$. Market clearing dictates that a share of $\mu^\star = 0.7462$ of the firms works in sector 1 and the remaining $1-\mu^\star = 0.2538$ work in sector 2. Output in steady state is 0.3847 compared to 0.4319 in the economy with perfect enforcement, even though sector 2 firms have a borrowing constraint which is only weakly binding: $y_2 = f(k_2^\star) = h(k_2^\star)$. ■

This result demonstrates that the inefficiency occurs even in situations in which $f(k_2^\star) = h(k_2^\star)$, that is, in steady state sector 2 firms operate at full capacity and the borrowing constraint is not limiting the amount of output produced. The lower output in this case is due to the low price for intermediate goods: As mentioned in the introduction, Sector 1 producers have a high marginal product but do not accumulate more capital because of the low price they receive for their product. On the other hand, capital is wasted in the final goods sector because it is operating at high capital levels with low marginal products. Essentially one can think of this as an externality: The marginal product - which is what a social planner would care about - is different from the marginal profit of capital due to a price $p < \frac{1}{2}$, in particular, the social marginal product of sector 2 capital is very low, but since intermediate goods are so inexpensive the personal marginal profit of capital is high and capital is over-accumulated in sector 2. To be precise:

$$
\begin{aligned}
p^\star f'(k_1^\star) &= (1-p^\star) f'(k_2^\star) \text{ and } p^\star < \frac{1}{2} \\
&\Rightarrow MPK_1 = f'(k_1^\star) > f'(k_2^\star) = MPK_2
\end{aligned}
$$

Another way to see the inefficiency is the following: If $k_{pe} = \left(\frac{\frac{1}{2}A\alpha}{\frac{1}{\beta}-1+\delta}\right)^{\frac{1}{1-\alpha}}$ is the steady state capital level with perfect enforcement, one can find cases where $k_1^\star < k_{pe} < k_2^\star$. Given that the production function f is concave it is easy to construct an example where final goods output is higher in the perfect enforcement case. Notice however that this is not a direct application of Jensen's inequality since in general it is not the case that $\mu^\star k_1^\star + (1-\mu^\star)\, k_2^\star = k_{pe}$.

A useful measure for comparing the severity of the distortionary effect coming from the borrowing constraint is what I want to call the implicit TFP, defined as follows:

Definition 4 *Suppose an economy using aggregate capital level K produces aggregate output Y. The* ***implicit TFP*** *(total factor productivity) is defined as the TFP necessary to produce Y with capital level K in the absence of borrowing constraints:*

$$A_{impl} = Y/K^\alpha$$

For example, in the perfect enforcement economy the implicit TFP would be $\frac{1}{2}A$. The further apart A_{impl} and $\frac{1}{2}A$ are, the stronger is the effect of the credit constraint in steady state.

Next, let's go through a numerical example to see how macro-economic variables depend on the choice of parameters. We use the same example as in the previous proposition, but with varying values for the parameter q which can be viewed as a measure for the enforcement technology. High values for q make the credit constraint less binding, whereas low values of q cause the region in which sector 2 firms are credit constrained to become larger.

Moreover, the example demonstrates that there can be multiple steady states. Consequently, for each variable we compute the minimum and the maximum value relative to the perfect enforcement case. Numerically this is easy to do: On a grid between 0 and $\frac{1}{2}$ for the price p one can compute the steady state capital levels in the two sectors by using the Euler conditions. After that one has to check that firms with capital $k_i^\star$ would actually prefer producing in sector i. The distribution of capital, that is, the value for μ follows directly from the market clearing condition.

Figure 2 plots the result from the exercise. For low enough values of q both output and capital are lower than in the perfect enforcement case and also increasing in q. Surprisingly, there are cases where steady state capital and even output levels are higher for incomplete enforcement economies. The reason for this is over-investment in sector 2 capital. Intermediate goods prices are for low values of q which causes high profit margins and low marginal product of capital in sector 2. Notice that welfare is still lower in economies with bad enforcement and the reason is that too much capital is being used so that consumption is lower in steady state.

Not surprisingly the implicit TFP is strictly increasing in q, that is, even in the region where output is higher than in the perfect enforcement case, aggregate capital input is higher by such a large margin that Y/K^α is pushed below the implicit TFP of an economy without enforcement problems.

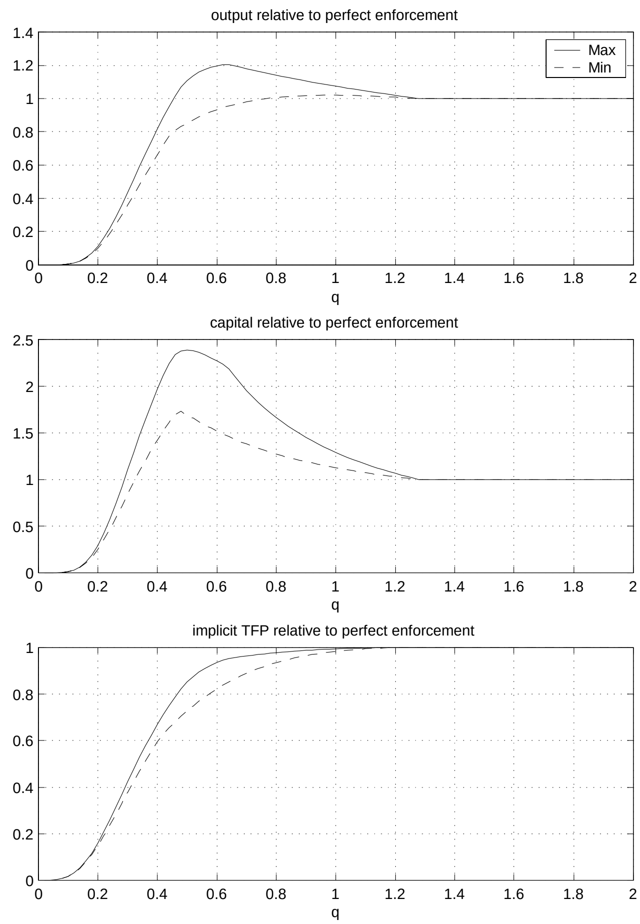

Figure 2: Output, capital, impl. TFP as functions of enforcement parameter q.

4 How to pin down a unique steady state

For the remainder of the paper it is assumed that the function h takes a particular form:

$$h(k) = qk, \; q > 2(1/\beta - 1 + \delta)$$

that is, firms can get trade credit up to a certain share of their capital holdings. This assumption simplifies the analysis because now it is certain that in steady state either the first best outcome is achieved where constraints are not binding and both sectors operate at the same capital level, or firms operate at different capital levels and sector 2 firms operate at the kink point $k^{\star}$, where $h(k) = f(k)$. This happens because the Euler condition in sector 2 can never be satisfied in the linear part of the profit function:

$$(1-p)\frac{\partial h(k)}{\partial k} = (1-p)q < \frac{q}{2} < 1/\beta - 1 + \delta$$

The reason for multiplicity of steady states is of course the non-concavity of the profit function. Because of this there is an incentive to borrow and lend capital, but not necessarily between firms of different capital levels but among firms with the same capital level, namely those producing at levels in the non-concave region of the profit function. Firms of different capital levels have no incentive to borrow and lend in steady state because $\frac{\partial \text{profit}}{\partial k}$ is equalized between the two sectors. However, if firms have a k such that

$$\exists k_{low}, k_{high} : k_{low} < k < k_{high}$$

and

$$\begin{aligned} \pi(k) &< \lambda\pi(k_{low}) + (1-\lambda)\pi(k_{high}) \\ \text{where } \lambda &= \frac{k_{high} - k_1}{k_{high} - k_{low}} \end{aligned}$$

they could be better off by pooling their capital and opening a share of λ firms of size k_{low} and $(1-\lambda)$ firms of size k_{high}. This can also be viewed as share λ of firms each lending $(k - k_{low})$ units of capital at interest rate $\frac{\pi(k_{high})-\pi(k_{low})}{k_{high}-k_{low}}$. One can easily check that at the end of the period all firms make identical profit $\lambda\pi(k_{low})+(1-\lambda)\pi(k_{high})$ under this arrangement.

Naturally, the k_{low}, k_{high} will be chosen in order to maximize the profit from pooling capital. That is, the maximum profit function can be thought of as the minimal concave function among those uniformly greater or equal than the original profit function. This function is depicted in Figure 3. Since h is linear, k_{high} is given by the intersection of the h and f functions, that is, the point at which the borrowing constraint is just weakly binding. k_{low} is given by the capital level such that the marginal profit in sector 1 is equal to the interest rate in the pooling region.

The next result establishes that when firms can trade capital, then the steady state capital usage is pinned down uniquely.

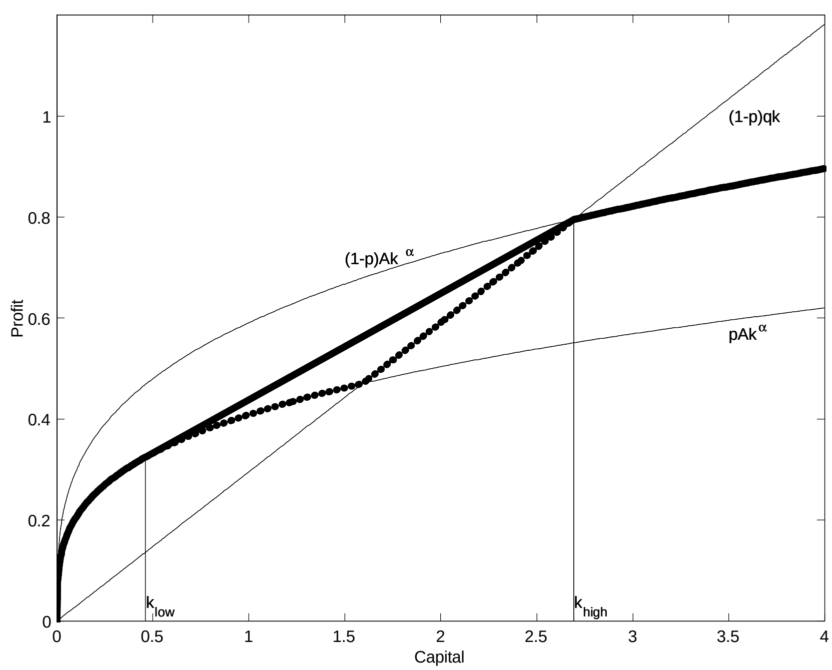

Figure 3: Max Profit with pooling in bold. Without pooling: Dotted line.

Proposition 5 *If firms can borrow and lend capital among each other there is a unique price* $p^\star$ *such that either the credit constraint is not binding, in which case* $p^\star = 1/2$ *and there are* $k_{low} = k_{high} \geq \bar{k}$ *such that* $f'(k_{low}) = 1/\beta - 1 + \delta$*, or* $k_{high} = \bar{k}$*, and there is a* $k_{low} < k_{high}$ *such that the slope of the linear portion in the profit function in figure 3 is equal to* $1/\beta - 1 + \delta$*.*

Proof. In the appendix. ■

Notice that the k_{low}, k_{high} in the proposition are the capital levels **used** in production. The result still allows for multiplicity of capital **ownership**, though. In other words, uniqueness applies only to the macro-economic variables output, aggregate capital level, and the price level but not to the capital distribution.

Going through the same type of numerical exercise as in the previous section, this time with $\gamma = 1.0$, we can again plot output, capital, and implicit TFP relative to the perfect enforcement case as a function of the enforcement parameter q, but now for the one unique steady state. This is done in Figure 4.[2] Qualitatively, the results in the case of a unique steady state are similar to those in the previous section. Initially, capital is increasing in q, because as enforcement becomes better, both the

[2] The reader may notice that in the graphs the lower bound on q is $(1/\beta - 1 + \delta) \approx 0.21$, whereas earlier in the section it was assumed that $q > 2\,(1/\beta - 1 + \delta)$. This lower bound is extremely tight because it only assumed that $p \in (0, 1/2)$, whereas in the numerical example $p \to 0$ fast enough as $q \to (1/\beta - 1 + \delta)$ so that for $q \in [(1/\beta - 1 + \delta), 2\,(1/\beta - 1 + \delta)]$, the linear section in the profit function is still steeper than $(1/\beta - 1 + \delta)$.

share of firms working in sector 2 increases and the increasing price for intermediate goods causes the capital used in sector 1 to increase. After a certain point, at roughly $q = 0.30$, total capital decreases, because $\bar{k} = \left(\frac{A}{q}\right)^{\frac{1}{1-\alpha}}$, the amount of capital used in sector 2, is decreasing in q and this effect dominates the previous two. For low enough values of q output can be made arbitrarily small relative to perfect enforcement. As before, however, output can also be higher than in the perfect enforcement case, which comes at a price of much higher capital levels. Consequently, implicit TFP is below the one, as long as credit constraints cause an intermediate goods price of less than one half. Welfare is still lower in economies with weak enforcement, though. This can be easily seen in the graphs: Output is larger by about up to 15% but it comes at the cost of capital levels that are higher by about 100%. Depreciation therefore pushes the amount of output dedicated to consumption below its level under perfect enforcement.

5 Conclusion

To use the language of Prescott (1997), this paper develops a theory of differences in TFP through differences in enforcement of contracts. Precisely, this paper looks at incomplete enforcement of intra-period debt, which can be thought of as trade credit. The necessity for this intra-period borrowing comes from the timing of the production process: Sector 1 output is produced first and it is employed as an intermediate good

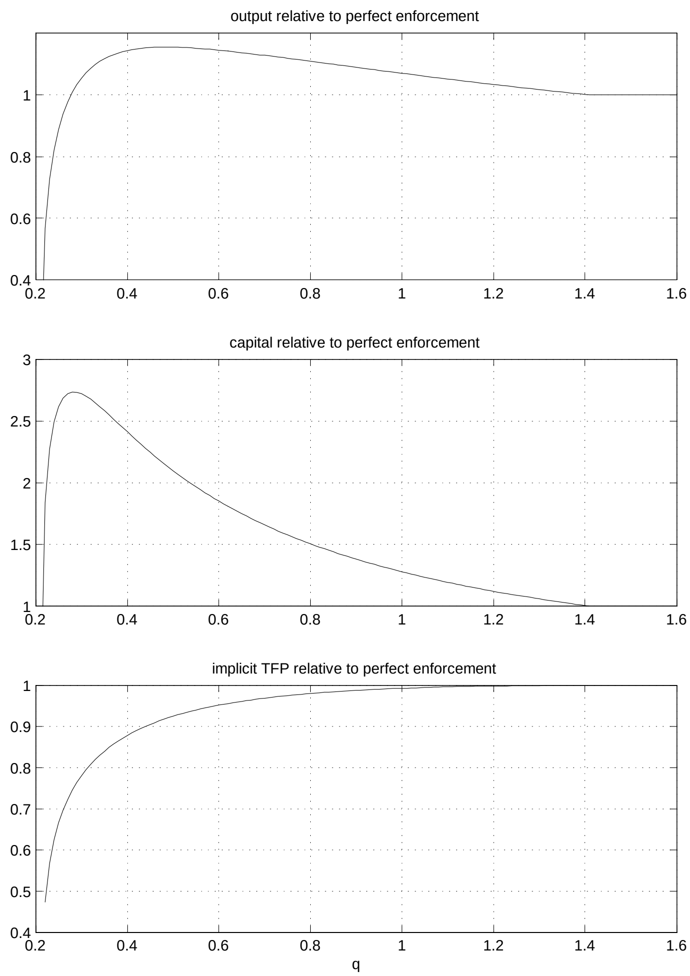

Figure 4: Output, capital, and implicit TFP as a function of q.

in the sector 2 production process, so that sector 2 has an intra-period financing need to pay for intermediate goods before its own production takes place.

If there is risk of default on this intra-period credit we can show a number of results: Prices for intermediate goods are lower in economies with imperfect enforcement of intra-period credit. This causes a skewed capital distribution with many small firms operating at high marginal products of capital and a few large firms that are very unproductive. The reason is that the credit constraint encourages sector 2 producers to operate at relatively large capital levels, and the low productivity is offset by high profit margins in that sector. The misallocation of capital from small firms into one sector that operates at low productivity causes TFP to be lower than in the perfect enforcement economy.

In the benchmark economy without capital markets, the profit function is non-concave, which gives rise to the possibility of multiple steady states. Opening capital markets goes half-way in solving this problem: Aggregate values in steady state, including capital used in the two sectors, are uniquely pinned down. However, the capital ownership distribution is still not uniquely determined.

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6 Appendix 1: Equilibrium definition

In order to define a recursive equilibrium, here is some preliminary work: Let A be the set of possible capital holdings (for example $A = \mathcal{R}_{++}$ would be an adequate choice) and $\mathcal{B}(A)$ be the Borel σ-algebra of A. Let $\mathcal{M}$ be the set of all probability measures on the measurable space $M = (A, \mathcal{B}(A))$. Let $\Phi \in \mathcal{M}$ denote the distribution of firms' capital. Now the household's problem in recursive formulation is:

$$v(k, \Phi) = \max_{c \geq 0, k' \in A} u(c) + \beta v(p', k', \Phi')$$

subject to:

$$\begin{aligned} c + k' &\leq \max\{p(\Phi) f(k), (1 - p(\Phi)) \min\{h(k), f(k)\}\} + (1 - \delta) k \\ \Phi' &= H(\Phi) \end{aligned}$$

where $H : \mathcal{M} \rightarrow \mathcal{M}$ is the law of motion for the capital distribution, and the function $p(\Phi)$ is the market clearing price for capital distribution Φ.

Finally we can specify what an equilibrium is:

Definition 6 *A **recursive equilibrium** is:*

- *A value function $v : A \times \mathcal{M} \rightarrow \mathcal{R}$.*
- *Policy functions for next period's capital $k' : A \times \mathcal{M} \rightarrow \mathcal{R}$ and for consumption $c : A \times \mathcal{M} \rightarrow \mathcal{R}$.*

- *A pricing function* $p : \mathcal{M} \to \mathcal{R}$.

- *A law of motion for capital distributions:* $H : \mathcal{M} \to \mathcal{M}$.

  such that:

- v, k', c *are measurable with respect to* $\mathcal{B}(A)$.

- v *satisfies the Bellman equation.*

- k', c *and the sector choices are the optimal decision rules.*

- *Decision rule* k' *generates the aggregate law of motion for capital.*

- *Sector choices generate market clearing in intermediate and final goods.*

Remark 7 *This equilibrium definition is a little bit sloppy, because in equilibrium it is possible that market clearing can only be achieved with randomization across the two sectors. This would be the case if there is a point mass of firms being indifferent between the two sectors, and these firms would potentially have to randomize between the two sectors in order for markets to clear. To address this eventuality one would have to introduce another policy function* $s : A \times \mathcal{M} \to [0, 1]$ *that specifies the probability of choosing sector 1, but the notation would get rather messy.*

7 Appendix 2: Uniqueness of steady state

In the economy with the (weakly) concave profit function steady state capital levels that firms **own** may not be unique, however the capital levels **used in production** are unique. We need to prove that there is a unique 3-tuple (p, k_{low}, k_{high}) such that either the first-best outcome is a steady state:

$$\begin{aligned} k_{low} &= k_{high}, \\ p &= \frac{1}{2}, \\ pf'(k_{low}) &= \frac{1}{\beta} - 1 + \delta \end{aligned}$$

or we are in the contained case:

$$k_{low} < k_{high} = \left(\frac{A}{q}\right)^{\frac{1}{1-\alpha}}, \ p < \frac{1}{2}$$

where k_{low}, k_{high} are the endpoints of the linear portion in the concave profit function from above.

Define $\bar{k} = \left(\frac{A}{q}\right)^{1/(1-\alpha)}$, the kink point at which the credit constraint for sector 2 is no longer binding. Also call $\rho = \frac{1}{\beta} - 1$.

Case 1: $\frac{1}{2}f'\left(\bar{k}\right) \geq \rho + \delta$. Then the steady state is the same as in first best outcome:

$$\begin{aligned} p &= \frac{1}{2} \\ k_{low} &= k_{high} = \left(\frac{\frac{1}{2}\alpha A}{\rho + \delta}\right)^{\frac{1}{1-\alpha}} \geq \bar{k} \end{aligned}$$

At the same time there can be no steady state with $p < \frac{1}{2}$

$$\begin{aligned} \frac{(1-p) f\left(\bar{k}\right) - pf\left(k_{low}\right)}{\bar{k} - k_{low}} &> \frac{1}{2} \frac{f\left(\bar{k}\right) - f\left(k_{low}\right)}{\bar{k} - k_{low}} \text{ (since } p < \frac{1}{2}) \\ &> \frac{1}{2} f'\left(\bar{k}\right) \text{ (since } f \text{ concave, } k_{low} < \bar{k}) \\ &\geq \rho + \delta. \end{aligned}$$

Case 2: $\frac{1}{2} f'\left(\bar{k}\right) < \rho + \delta$. In this case there can be no steady state with $p = \frac{1}{2}$, because at the proposed capital levels sector 2 firms' credit constraint would be strictly binding. Now all we have to show that there is a unique $p \in \left(0, \frac{1}{2}\right)$ such that the linear part of the profit function has slope $\rho + \delta$. Evidently, $k_{high} = \bar{k}$, as before. Define $k_1(p)$ as the capital level in sector 1 such that marginal profit is equal to $\rho + \delta$:

$$k_1(p) = \left(\frac{p\alpha A}{\rho + \delta}\right)^{\frac{1}{1-\alpha}}$$

Notice, that $k_1(p)$ is equal to k_{low} if and only if p is the steady state price level. Define $i_1(p)$ as the intercept of the tangency line going through the point $(k_1(p), pf(k_1(p)))$ and $i_1(p)$ as the intercept of a line with slope $(\rho + \delta)$ going trough $\left(\bar{k}, (1-p) f\left(\bar{k}\right)\right)$:

$$\begin{aligned} i_1(p) &= pf(k_1(p)) - (\rho + \delta) k_1(p) \\ i_2(p) &= (1-p) f\left(\bar{k}\right) - (\rho + \delta) \bar{k} \end{aligned}$$

Let $y(p)$ be the difference between the two intercepts.

$$\begin{aligned}
y(p) &= i_2(p) - i_1(p) \\
&= (1-p)A\left(\frac{A}{q}\right)^{\frac{\alpha}{1-\alpha}} - (\rho+\delta)\left(\frac{A}{q}\right)^{\frac{1}{1-\alpha}} \\
&\quad -pA\left(\frac{\alpha p A}{\rho+\delta}\right)^{\frac{\alpha}{1-\alpha}} + (\rho+\delta)\left(\frac{p\alpha A}{\rho+\delta}\right)^{\frac{1}{1-\alpha}} \\
&= (1-p)A\left(\frac{A}{q}\right)^{\frac{\alpha}{1-\alpha}} - (\rho+\delta)\left(\frac{A}{q}\right)^{\frac{1}{1-\alpha}} \\
&\quad -p^{1+\frac{\alpha}{1-\alpha}}A^{1+\frac{\alpha}{1-\alpha}}(\rho+\delta)^{-\frac{\alpha}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}} + p^{1+\frac{\alpha}{1-\alpha}}A^{1+\frac{\alpha}{1-\alpha}}(\rho+\delta)^{-\frac{\alpha}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}} \\
&= (1-p)A\left(\frac{A}{q}\right)^{\frac{\alpha}{1-\alpha}} - (\rho+\delta)\left(\frac{A}{q}\right)^{\frac{1}{1-\alpha}} \\
&\quad -p^{\frac{1}{1-\alpha}}A^{\frac{1}{1-\alpha}}(\rho+\delta)^{-\frac{\alpha}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)
\end{aligned}$$

All we have to show is that y has a single root on the interval $\left(0, \frac{1}{2}\right)$

Notice that y is continuous and decreasing in p and:

$$\begin{aligned}
y(0) &= A\bar{k}^{\alpha} - (\rho+\delta)\bar{k} \\
&> f\left(\bar{k}\right) - \frac{q}{2}\bar{k}, \text{ by assumption on } q \\
&> f\left(\bar{k}\right) - q\bar{k} \\
&= f\left(\bar{k}\right) - h\left(\bar{k}\right) \\
&= 0, \text{ by definition of } \bar{k}.
\end{aligned}$$

Also, since f is strictly concave and $k_1(p) \neq \bar{k}$:

$$\begin{aligned}
\frac{1}{2} f\left(\bar{k}\right) &< \frac{1}{2} f\left(k_1\left(\frac{1}{2}\right)\right) + \left(\bar{k} - k_1\left(\frac{1}{2}\right)\right) f'\left(k_1\left(\frac{1}{2}\right)\right) \\
&= \frac{1}{2} f\left(k_1\left(\frac{1}{2}\right)\right) + \left(\bar{k} - k_1\left(\frac{1}{2}\right)\right)(\rho + \delta) \\
&= \frac{1}{2} f\left(k_1\left(\frac{1}{2}\right)\right) - k_1\left(\frac{1}{2}\right)(\rho + \delta) + \bar{k}(\rho + \delta) \\
&= i_1\left(\frac{1}{2}\right) + \frac{1}{2} f\left(\bar{k}\right) - i_2\left(\frac{1}{2}\right)
\end{aligned}$$

and therefore $y\left(\frac{1}{2}\right) < 0$. Consequently, there is a unique steady state price p and therefore unique steady state capital levels k_{low}, k_{high}. QED.