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Abstract: The last decade has brought about substantial mortgage innovation and increased refinancing. The objective of this paper is to understand the determinants and implications of mortgage choice in the context of a general equilibrium model with incomplete markets. The equilibrium characterization allows us to study the impact of mortgage financing decisions in the productive economy. We show the influence of different contract characteristics such as the down payment requirement, repayment structure, and the amortization schedule for mortgage choice. We find that loan products that allow for low or no down payment or an increasing repayment schedule increase the participation of young and lower-income households. We find evidence that the volume of housing transactions increases when the payment profile is increasing and households have little housing equity. In contrast, we show that loans that allow for a rapid accumulation of home equity can still have positive participation effects without increasing the volatility of the housing market. The model predicts that the expansion of mortgage contracts and refinancing improves risk sharing opportunities for homeowners, but the magnitude varies with each contract.

JEL classification: E0, D58, D91, R21

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1. Introduction

Housing is important for both households and the aggregate economy. At the macro level, housing investment (both residential and nonresidential structures) accounts for about half of all gross private investment. The importance of housing at the household level is more evident since the purchase of a house is typically the largest single consumer transaction. Since houses are expensive, the availability of housing finance is equally important. From an aggregate perspective, approximately eight trillion dollars in mortgage debt is allocated to the financing of housing. For the household, the mortgage decision has implications for expenditure patterns and asset allocations. Recent events have increased the role of housing and mortgage finance in the economy. This is especially apparent given the crisis in the subprime mortgage market. The financial turbulence resulting from this problem has preoccupied traders in the financial markets and policymakers, because of the potential consequences for the performance of the aggregate economy.

There is relatively little research on mortgage choice that examines the ramifications of that choice for both households and the aggregate economy. There is a large literature on mortgage choice in the finance literature.[1] This literature mainly focuses on the choice between adjustable rate and types of fixed rate mortgages. These papers include, Alm and Follain (1984), Dunn and Spatt (1985), Kearl (1979), LeRoy (1996), Stanton and Wallace (1999), and Shilling, Dhillon, and Sirmans (1987). More recently, Campbell and Cocco (2003) examine the role mortgage choice in the household risk management problem. They analyze the between fixed and adjustable rate mortgages in the context of a partial equilibrium dynamic life-cycle model with borrowing constraints and income risk. Given their focus, they do not consider the different dimensions of mortgages or the implications for the aggregate economy.

Given an array of mortgage products, the optimal mortgage choice for a household is a complex problem. Households have to take into consideration many dimensions such as the downpayment, maturity of the contract, repayment structure, the ability to refinance, the possibility of being subject to borrowing constraints, and the evolution of economic variables such as the interest rate, inflation, house appreciation and income growth. For instance, the optimal choice for a buyer moving into the housing market might be different than a homeowner looking to upsize. Therefore, understanding mortgage decisions requires a framework that explicitly acknowledges the heterogeneity of households along the age, income, and wealth dimensions. In addition, these decisions must consider the complexities of the tax code that favors owner-occupied housing with the deductibility of mortgage interest payments and the lack of taxation from the imputed rental income for homeowners. Only in such a framework might we be able to understand the observed mortgage choice across households and its impact in the performance of the overall economy.

The objective of this paper is to understand the determinants of mortgage choice and the implications of these choices for the economy in the context of a general equilibrium model.

[1] Follain (1990) has written a survey of this literature prior to 1990.

The equilibrium characterization is particularly important because it allows us to study the impact of mortgage financing decisions in the productive economy. We solve for equilibrium using a restricted subset of mortgage choices that are not subject to interest rate risk. This limitation does not seem to be major as more than 90 percent of the households use loans with fixed interest rates. The failure to consider variable interest rate products could be important for refinancing. A key determinant of mortgage decisions is the set of loan products available to households. Until the 1990s there were two predominant loan types: an adjustable rate mortgage (ARM) and a 30 year fixed rate mortgage (FRM).[2] However, in the last decade there has been substantial innovation in mortgage markets that has substantially expanded the set of loan products making the choice of mortgage even more complex. These new loan products have changed the nature of the conventional 30 year fixed rate mortgage by eliminating the presence of downpayment constraints and allowing the repayment structure to change over the length of the mortgage. The result is that the introduction of these products has increased opportunities for families that otherwise might be unable to buy a house.

To understand the determinants of housing finance we develop a quantitative equilibrium theory of mortgage choice to investigate the impact of housing tenure (renting vs. owning), as well as the ramifications on the asset allocation in the households' portfolio (financial assets vs. housing). In the model households face uninsurable mortality and labor income risks and make decisions with respect to consumption (goods and housing services), and asset allocation (capital and risky housing investment).[3] The model stresses the dual role of housing as a consumption and risky investment good. Investment in housing differs from real capital since it requires a long-term mortgage loan. Households can choose from a menu of mortgage contracts that differ in downpayment requirement, payment schedule, and maturity so in equilibrium different long-term mortgage loans coexist. House sales are subject to an idiosyncratic capital gains shock that affects the value of the property.[4] Since households are subject to uninsurable income risk, we also consider an economy where we allow them to refinance the terms of the contract. The introduction of mortgage decisions introduce important computation complexity so it becomes infeasible to allow household to choose over a large set of mortgage products, but it is feasible to examine a restrictive set of mortgage products.

In recent years, there has been a number of papers that examined housing in a general equi-

[2]Campbell and Cocco (2003) use a quantitative model of mortgage choice to study the determinant between ARMs and FRMs. They show that ARMs should be attractive to unconstrained households when inflation risk is large relative to interest rate risk, and they should be unattractive to risk averse borrowing constrained households. However, they claim that their theory fails to rationalize the observed mortgage holdings in periods with high spreads.

[3]It is important to note that in an environment with complete markets mortgage decisions are irrelevant. Households can always offset any limitation of the mortgage loan (i.e. downpayment requirement) by borrowing or lending in the asset market. Mortgage choice is meaningful in an environment with incomplete markets and with borrowing constraints.

[4]There has been a lot of discussion about the high growth rates of house prices over the same time period. In this paper we do not seek to explain the joint movement of house price and homeownership. The introduction of idiosyncratic capital gains has the objective of partially capturing the risk associated to investing in real estate upon the sale of the property.

librium framework with heterogeneous agents. Some of these papers are Berkovec and Fullerton (1992), Chambers, Garriga, and Schlagenhauf (2007a), Díaz and Luengo-Prado (2002), Fernandez-Villaverde and Krueger (2002), Gervais (2002), Jeske and Krueger (2005), Li and Yao (2007), Nakajima (2003), Ortalo-Magne and Rady (2006), Sánchez (2007), Sánchez-Marcos and Ríos-Rull (2006). Much of this literature looks at the taxes effects on housing choice or the wealth implications of housing. The papers most related to this paper are Jeske and Krueger who examine the role of Government-Sponsored Enterprises for housing and the macroeconomy, and Chambers, Garriga, and Schlagenhauf (2007a) which tries to account for changes in homeownership in the United States. The emphasis is on decomposing the observed boom in real estate into demographic changes, mortgage innovation, and joint effects. We find that roughly one third of the increase can be attributed to demographic factors, and the remaining to mortgage innovation and joint effects. Although the papers share a similar methodology and framework, this paper focuses on the determinants of mortgage decisions and their implications for the aggregate economy. We explore many dimensions of mortgage contracts such as the repayment schedule, downpayment, amortization, and refinancing that impact mortgage choice, as well as housing tenure, duration, risk sharing decisions.

We can segment the primary findings in the paper into five categories.

- **Aggregate effects:** The model predicts that the introduction of mortgage decisions has a positive effect on ownership and housing consumption but the magnitude depends on the profile of the repayment schedule and the downpayment. We find that the introduction of nontraditional loans with an increasing repayment profile have similar aggregate effects as loans with no downpayment and decreasing repayments, but the distributional effects on participation and housing transactions are different. We show that the aggregate effects are maximized with loans that combine high loan-to-value ratios with low initial mortgage costs. The introduction of nontraditional loans have positive effects on output, but the increase in output only translates in an important increase in consumption when the income effects are large. Otherwise, only the average house size increases in a significant way.

- **Distributional effects:** When we study the determinants of mortgage choice, we find that the decisions are influenced through three dimensions: the downpayment constraint, the repayment profile, and the amortization schedule. In terms of age and income the model suggests a certain separation of mortgage decisions. For example, when downpayment requirements are high, young and poor individuals benefit from using mortgage loans with steep repayment profile since it reduces the initial cost of purchase a house. Individuals with more income or older age prefer mortgages with high downpayment and fast amortization of the principal. A very close substitute to loans with increasing repayment schedules for young and poor individuals are loans with no downpayment and decreasing repayments. When we explore the determinants of downpayment choice we find that the standard 20 percent downpayment is a compromise between 57 percent of the homeowners that prefer a larger downpayment and 36 percent that prefer a lower one.

- **Mobility:** The introduction of nontraditional loans with low initial payments, low downpayments, or both increase the number of transactions in and out of the housing market with respect to the baseline model. For example, in the baseline model there is a relation around 4 to 1 between renters that move into ownership and homeowners that have to sell the house and rent. With the introduction of these low financing products this relation is close to one. The distributional impact on mobility between low initial payment and no downpayment are very different. Loans with low initial costs generate a decreasing distribution by age of individuals leaving the housing market, whereas contracts with no downpayment exhibit a humped-shaped pattern.

- **Consumption smoothing of mortgage choice:** In general, we find that the introduction of mortgage choice improves consumption smoothing because it reduces the coefficient of variation of consumption for homeowners. The reduction is specially important for contracts that allow for a steep profile of repayment and/or a low downpayment.

- **Refinancing:** Finally, we show that the introduction of a refinancing option increases the participation rate and the average house size. It also reduces the number of transactions in the housing market when compared to the baseline without refinancing. We also show that given our specification of preferences, refinancing reduces the coefficient of variation of housing services, but does not affect the variability of consumption.

Beyond policy implications, this paper fills a few important gaps in the modeling of the housing market. First, we employ a model which explicitly models mortgage decisions using contracts which last for several periods. The fact that houses are typically purchased through long duration mortgages is often avoided in other life-cycle models with housing. These long duration loans will have an effect on households ability to accumulate capital assets and smooth income risk. Second, we implement an endogenous rental market where supply and demand is completely driven by household decisions. As a result, we find that our model matches several features of the housing market including: the rate of homeownership, the average house and apartment size, and the age distribution of landlords just to name a few. Thus, we have a developed a model that can be used to address several additional questions about housing.

This paper is organized into five sections. In the first section, we describe the properties of different mortgage contracts. In the second section, we describe the model economy and define equilibrium. The third section discusses the estimation of the model to the US economy. The next section analyzes the performance of the model with a standard mortgage contract, while the final section examines the implications of alternative mortgage contracts.

2. Mortgage Contracts

2.1. Characteristics of mortgage contracts

A mortgage contract is a loan secured by real property. Mortgage lending is the primary mechanism used in most countries to finance the acquisition of residential property. These loans are structured as long-term loans that require periodic payments consisting of an interest payment and a principal payment. There are many types of mortgage loans which can be broadly defined by three main characteristics: the payment structure, the amortization schedule, and the term of the mortgage loan. The payment structure defines the amount and the frequency of mortgage payments. The amortization structure refers to the size of the principal payments over the life of the mortgage. This schedule differs across mortgage loans and it can be increasing, decreasing, or constant. Some contracts allow for no amortization of the principal and full repayment of principal at a given date. Other contracts allow negative amortization usually in the initial periods of the loan. The term or duration usually refers to the maximum length of time given to repay the mortgage loan. The most common durations are 15 and 30 years. In theory, the combination of these three factors allows a large variety of distinct mortgage products to be constructed. Among these, only a subset of products exist in the marketplace.

Understanding mortgage loans is essential to understanding owner-occupied housing. In the United States, according to the *Residential Finance Survey* in 2001, roughly 97 percent of the housing units where purchased through mortgage loans while only 1.6 percent were purchased with cash. Between 1995 and 2005, a substantial change occurred in the structure of the mortgage market in the United States. According to data presented in the *Mortgage Market Statistical Annual,* the market share of nontraditional mortgage contracts has increased since 2000. Nontraditional or alternative mortgage products include interest-only loans, option ARMs, loans that couple extended amortization with balloon payment requirements and other contracts of alternative lending. For example, in 2004 these products accounted for 12.5 percent of the originations. By 2006, the fraction increased to 32.1 percent of new originations. With the share of conventional and conforming loans declining, it is important to examine the structure of mortgage contacts.

2.2. General structure of mortgage contracts

Despite the differences in the observed types of mortgage contracts, all of them have the same fundamental elements: a downpayment, an amortization schedule, an interest payment, and outstanding principal. To characterize the various features of mortgage contracts it is useful to introduce some general notation common to all contracts. Let $z \in \mathcal{Z} = \{1, ..., Z\}$ be a specific type of mortgage loan from the set of available contracts that borrowers can use to purchase a house of size h with a unit price p. A mortgage loan usually requires a downpayment to guarantee that there is some equity in the house. We define $\chi(z) \in \mathbb{R}$ to be the fraction of the house value paid up-front by the homeowners. The term $H_0(z) = \chi(z)ph$ represents the initial amount of equity in the house and $D_0(z) = (1 - \chi(z))ph$ represents the initial debt owed to the lender. At each period, t, the borrower faces a payment amount that depends on the size of the loan, $D_0(z)$, the term of the mortgage, $N(z)$, the mortgage loan interest rate, $r^m(z)$, and repayment structure associated to each mortgage contract z. We denote the mortgage repayment schedule at time t as being determined by the function $m_t(x, z)$ where x is defined by the set $(p, h, \chi(z), N(z), r^m(z))$. This payment can be decomposed into an amortization term, $A_t(z)$, that depends on the amortization schedule of the mortgage loan and an interest term, $I_t(z)$, that depends on the outstanding debt. That is,

$$m_t(x, z) = A_t(z) + I_t(z), \qquad \forall t, \tag{2.1}$$

where the interest payments are calculated by $I_t(z) = r^m(z)D_t(z)$. The law of motion for the level of housing debt $D_t(z)$ can be written as,

$$D_{t+1}(z) = D_t(z) - A_t(z), \qquad \forall t. \tag{2.2}$$

The law of motion for the level of home equity with respect to the loan $H_t(z)$ is

$$H_{t+1}(z) = H_t(z) + A_t(z), \qquad \forall t, \tag{2.3}$$

where $H_0(z) = \chi(z)ph$ denotes the home equity in the initial period.

Notice that this formulation is very general since it allows a 100 percent financing when $\chi(z) = 0$ with a initial loan of $D_0(z) = ph$, and all cash purchase with $\chi(z) = 1$ with no initial loan $D_0(z) = 0$. Some contracts even allow closing costs to be rolled into the loan, so the downpayment fraction could be negative, $\chi(z) < 0$. Next, we will discuss the specifics of primary mortgage contract types such as the standard fixed rate mortgage (FRM), a constant amortization loan (CAM), a balloon payment loan, combo-loans with a financed downpayment (COM), and graduated mortgage payments (GPM).

2.3. Fixed Rate Mortgage

Fixed rate mortgages (FRM) are considered the "standard" loan product used to finance the purchase a house. This loan product is characterized by a constant mortgage payment over the term of the mortgage, $m(x, z_{FRM}) = m_1(x, z_{FRM}) = ... = m_N(x, z_{FRM})$. The constant mortgage payment results in an increasing amortization schedule of the principal and a decreasing schedule for interest payments. Formally,

$$m(x, z_{FRM}) = A_t(z_{FRM}) + I_t(z_{FRM}),$$

and satisfies

$$m(x, z_{FRM}) = \lambda D_0(z_{FRM}), \tag{2.4}$$

where $\lambda = r^m[1 - (1 + r^m)^{-N}]^{-1}$. Since the outstanding debt decreases over time, $D_0(z_{FRM}) > ... > D_N(z_{FRM})$, the contract front loads the interest rate payments $I_t(z_{FRM}) = r^m(z_{FRM})D_t(z_{FRM})$, and back loads the capital or principal payments given by

$$A_t(z_{FRM}) = \lambda D_0(z_{FRM}) - r^m(z_{FRM})D_t(z_{FRM}).$$

The level of debt is reduced by the repayment each period

$$D_{t+1}(z_{FRM}) = (1 + r^m)D_t(z_{FRM}) - m(x, z_{FRM}), \qquad \forall t, \tag{2.5}$$

and the equity in the house increases each period by the mortgage payment net of interest. $H_{t+1}(z_{FRM}) = H_t(z_{FRM}) + [m(x, z_{FRM}) - r^m(z_{FRM})D_t(z_{FRM})], \qquad \forall t.$

2.4. Constant Amortization Mortgage

One of the features of the fixed rate mortgage is that households accrue little equity early in the mortgage due the front loaded interest payments. A contract that does not have this unattractive feature is the constant amortization mortgage (CAM). This loan product assumes constant contributions to the amortization schedule, $A_t(z_{CAM}) = A_{t+1}(z_{CAM}) = A(z_{CAM})$, but since the interest repayment schedule depends on the size of outstanding level of debt, $D_t(z_{CAM})$, and the loan term, N, the mortgage payments $m_t(x, z_{CAM})$ are no longer constant. Formally, the constant amortization terms is calculated as

$$A(z_{CAM}) = \frac{D_0(z_{CAM})}{N} = \frac{(1-\chi)ph}{N}.$$

Under this contract, mortgage payments $m_t(x, z_{CAM})$ decrease over time

$$m_t(x, z_{CAM}) = \frac{D_0(z_{CAM})}{N} + r^m(z_{CAM})D_t(z_{CAM}).$$

The law of motion for the outstanding level of debt and home equity are represented by

$$D_{t+1}(z_{CAM}) = D_t(z_{CAM}) - \frac{D_0(z_{CAM})}{N}, \qquad \forall t,$$

and

$$H_{t+1}(z_{CAM}) = H_t(z_{CAM}) + \frac{D_0(z_{CAM})}{N}, \qquad \forall t.$$

2.5. Balloon and Interest Only Mortgages

At the other end of the spectrum we have mortgage contracts with very little or no amortization along the term of the mortgage. One example is the balloon loan (BAL) where all the principal borrowed is paid in full the last period, N. This product is popular in times where mortgage rates are high and home buyers anticipate lower future mortgage rates. In addition, homeowners who expect to stay in their home for a short duration may find this attractive because the lack of principal payments reduces the total mortgage payments. The amortization schedule can be written as:

$$A_t(z_{BAL}) = \begin{cases} 0 & \forall t < N \\ (1-\chi)ph & t = N \end{cases}.$$

All the mortgage payments, except the last one, reflect interest rate payments $I_t\,(z_{BAL}) = r^m(z_{BAL})D_0(z_{BAL})$. The mortgage payment for this contract is:

$$m_t(x, z_{BAL}) = \begin{cases} I_t(z_{BAL}) & \forall t < N \\ (1+r^m)D_0(z_{BAL)} & t = N \end{cases},$$

where $D_0(z_{BAL}) = (1-\chi)ph$. The evolution of the outstanding level of debt can be written as

$$D_{t+1}(z_{BAL}) = \begin{cases} D_t(z_{BAL}), & \forall t < N \\ 0, & t = N. \end{cases}.$$

The other example is the interest only loan, (BALI). With this mortgage contract the homeowner never accrues more equity in the house that the initial downpayment. In this case, $A_t(z_{BALI}) = 0$ and $m_t(x, z_{BALI}) = I_t(z_{BALI}) = r^m D_0(z_{BALI})$ for all t. With this mortgage the homeowner is effectively renting the property from the lender and the interest payments are the effective rental cost.

With these two type of mortgage loans since there is no additional equity is accrued in the property the mortgage payments are the lowest. In this situation, the homeowner is fully levered with the bank and maximizes the return from housing investment when capital gains are realized. In the presence of mortgage interest deductions this contract becomes very attractive since the government subsidizes all your effective rental cost.

2.6. Graduate Mortgage Payments

In an environment with high housing prices, another product that may be of interest to first time buyers is the graduated payment mortgage (GPM) where mortgage payments grow over time. This product could be attractive to first time buyers as mortgage payments are initially lower than payments in a standard contract. With this contract, mortgage payments increase with income over time keeping housing expenses stable as a fraction of income. Of course, this product increases the lender's risk exposure because the borrower builds equity in the home at a slower rate than the standard contract which may explain the lack of popularity of this product.[5] The repayment schedule depends on the growth rate of these payments. We consider two different cases that differ on the growth rate of mortgage payments.

1. **Geometric Growth:** In this type of contract, mortgage payments evolve according to a constant geometric growth rate given by

$$m_{t+1}(x, z_{GPMG}) = (1+g)m_t(x, z_{GPMG}),$$

[5] In 1974 Congress authorized an experimental FHA insurance program for GPM's. In this program, negative amoritization was permitted, but required higher downpayments so that the outstanding principal balance would never be greater during the life of the mortgage than would be permitted for a standard mortgage insured by FHA. Activity under this program and successor programs has been limited.

where $g > 0$. Consequently, the amortization term and interest payments are also growing. Formally,

$$m_t(x, z_{GPMG}) = A_t(z_{GPMG}) + I_t(z_{GPMG}),$$

with the initial mortgage payments being,

$$m_0(x, z_{GPMG}) = \lambda_g D_0(z_{GPMG}),$$

where $\lambda_g = (r^m - g)[1 - (1 + r^m)^{-N}]^{-1}$. The law of motion for the level of debt satisfies

$$D_{t+1}(z_{GPMG}) = (1 + r^m(z_{GPMG})D_t(z_{GPMG}) - (1 + g)^t m_0(x, z_{GPMG}),$$

and the amortization term is $A_t(z_{GPMG}) = \lambda_g D_0(z_{GPMG}) - r^m D_t(z_{GPMG})$.

2. **Arithmetic Growth:** In this case, the mortgage payment grows at a constant nominal amount $\triangle = m_1(x, z_{GPMA}) - m_0(x, z_{GPMA})$. The law of motion for the repayment schedule is

$$m_{t+1}(x, z_{GPMA}) = m_0(x, z_{GPMA}) + t \cdot \triangle,$$

The initial payment is calculated as usual, and is given by

$$m_0(x, z_{GPMA}) = \frac{[D_0(z_{GMPA}) + \frac{\triangle N}{r^m}]r^m}{[1 - (1 + r^m)^{-N}]} - \triangle(\frac{1}{r^m} + N).$$

The law of motion for the outstanding debt is

$$D_{t+1}(z_{GPMA}) = (1 + r^m)D_t(z_{GMPA}) - (m_0(x, z_{GPMA}) + t \times \triangle).$$

In this case the amortization term is $A_t = (m_0(x) + t \times \triangle) - r^m D_t$.

2.7. Combo or Piggyback Loan

In the late 1990's the combo or Piggyback loan, (COM), became a popular loan product as way to avoid large downpayment requirements and personal mortgage insurance (PMI).[6] This loan product amounts to the use two different loans. The primary loan covers a fraction of the total purchase, $D_1(z_{COM}) = (1 - \chi)ph$, with a payment schedule, $m_t^1(x, z_{COM})$, and maturity, N_1. The second loan partially or fully covers the downpayment amount, $D_2(z_{COM}) = \varkappa\chi ph$, where $\varkappa \in (0, 1]$ and represents the fraction of downpayment financed by the second loan. The second loan has an interest premium $r_2^m = r_1^m + \zeta$ (where $\zeta > 0$), a mortgage payment $m_t^2(x, z_{COM})$,

[6] Government sponsored mortgage agencies initiated the use of this product in the late 1990's and this product became popular in private mortgage markets between 2001 and 2002.

and a maturity $N_2 \leq N_1$. In this case

$$m_t(x, z_{COM}) = \begin{cases} m^1(x, z_{COM}) + m^2(x, z_{COM}) \text{ when } N_2 \leq t \leq N_1 \\ \\ m^1(x, z_{COM}) \text{ when } t < N_2 \end{cases},$$

the laws of motion for both loans, and home equity are computed as in the mortgage with constant repayment. There are different type of combo loans offered in the industry, for example a "80-15-5" implies a primary loan for 80 percent of the value, a secondary loan for 15 percent, and a 5 percent downpayment. Another special case is the so-called "no downpayment" or a "80-20-0" that corresponds to the traditional loan-to-value rate of 80 percent using a second loan for the 20 percent downpayment.

3. Equilibrium Model of Mortgage Choice

The model economy is comprised of households, a representative firm, a financial intermediary and a government sector. In this section, we discuss each of these elements in detail as and define the market clearing conditions. The formal definition of the recursive equilibrium for this model appears in an appendix.

3.1. Households

The household sector is populated by overlapping generations of *ex-ante* identical households that face mortality risk and uninsurable labor earning uncertainty. Household age is denoted by j where each household lives a maximum of J periods. The survival probability conditional of being alive at age j is given by $\psi_{j+1} \in [0, 1]$, with $\psi_1 = 1$, and $\psi_{J+1} = 0$. Preferences are defined over consumption goods c and housing services s. Bundles of goods are ranked according to an index function $u : \mathbb{R}^2_+ \rightarrow \mathbb{R}$. The function $u(c, s)$ satisfies $u_i > 0$ and $u_{ii} < 0$ with respect to each good $i = c, s$, and $u_{ij} > 0$. The utility function satisfies the standard Inada conditions. Household preferences are given by the expected value of a discounted $\beta > 0$ sum of momentary utility functions:

$$E \sum_{j=1}^{J} \psi_{j+1} \beta^{j-1} u(c_j, s_j). \tag{3.1}$$

Besides consumption (goods and housing service) decisions, households make portfolio decisions to smooth out income uncertainty. We consider two distinct assets : a riskless financial asset denoted by $a' \in \mathcal{A}$ with a net return r, and a risky housing durable good denoted by $h' \in \mathcal{H}$ with a market price p where the prime is used to denote future variables. In addition to being an investment good, housing provides services according to the linear technology function $s = g(h') = h'$ which is bounded by the size of the investment $s \leq h'$. In addition, housing investment is financed through long-term mortgage contracts and is subject to transaction costs.

Household income is stochastic during working years, $j < j^*$, and depends on a number of factors. Basic wage income is denote by w. In addition, households earnings depend on age. This factor is denoted as υ_j and introduces a life-cycle pattern to earnings. The remaining factor is the idiosyncratic, stochastic factor, $\epsilon \in \mathcal{E}$ which is drawn from a probability space, and evolves according to the transition law $\Pi_{\epsilon,\epsilon'}$. During the retirement years, $j \geq j^*$, a household receives a retirement benefit from the government equal to θ. In addition to labor earnings and financial wealth $(1+r)a$, households with a positive housing investment can earn rental income by supplying housing services to the rental market, $R(h'-s)$ where R denotes the rental price. To receive rental income households have to pay a fixed cost $\varpi > 0$ to enter this market.

Households are subject to a progressive income tax represented by a function $T(ay)$ where ay denotes households' adjusted gross income ay.[7] The importance of including a progressive income code is to understand and account for the interaction between mortgage choice and the tax code. Clearly, changes in the tax code and limits on deductions are likely to impact the choice of mortgage. Adjusted income ay is defined as

$$ay(a,h',s,\epsilon,j,\upsilon_j;q) = \begin{cases} w\epsilon\upsilon_j + ra + R(h'-s) - \Phi, & if \quad j < j^*, \\ \\ \theta + ra + R(h'-s) - \Phi, & if \quad j \geq j^*. \end{cases} \tag{3.2}$$

where $q = \{p, R, r, r^m\}$ represents a price vector and Φ represents deductions to gross income. Notice that the tax system treats owner-occupied and rental occupied housing asymmetrically as rental housing services are taxed while the imputed service flow from owner-occupied is not. The deduction of mortgage payments for owner-occupied housing introduces another asymmetry. After tax income (excluding rental income) is defined as:

$$y(a,h',s,\epsilon,j,\upsilon_j;q) = \begin{cases} (1-\tau_p)w\epsilon\upsilon_j + (1+r)a + tr - T(ay), & if \quad j < j^*, \\ \\ \theta + (1+r)a + tr - T(ay), & if \quad j \geq j^*. \end{cases} \tag{3.3}$$

where τ_p represents the social security contributions used to finance the social security system. In the presence of mortality risk and missing annuity markets we assume borrowing constraints $a' \geq 0$, to prevent households from dying with negative wealth. The proceeds from households that die and have a positive housing investment and/or asset position are redistributed to the living households as a lump-sum transfer, tr. We also assume that households are born with initial wealth dependent on their initial income level.[8]

As we have previously mentioned, housing investment requires long-term financing through mortgage contracts. Since we focus on recursive equilibrium we want to summarize all the relevant information of these long-term mortgage contracts with a finite number of state variables.

[7] We assume standard properties of a progressive tax function such as differentiability $T'(ay) > 0$ and $T''(ay) < 0$, where $T(ay)/ay > 0$ represents the average income tax.

[8] The purpose of this assumption is to account for the fact that some of the youngest household's who purchase housing have some wealth. Failure to allow for the initial asset position creates a bias against the purchase of homes in the earliest age cohorts.

In a stationary environment, the housing stock, h, the type of mortgage contract, z, and remaining length of the mortgage, n, are sufficient to recover all the relevant information such as mortgage payment, remaining liability, and equity in the house.[9]

Individuals make decisions over consumption goods, housing services, mortgage contract type, and investment in assets and housing. The household's current period budget constraint depends on the household's asset holdings, the current housing investment, the remaining length of the mortgage, labor income shock, and households age. We can isolate five possible optimization problems that the household solves. The value function for a household is described by the state vector which depends the entering asset position, a, the prior period housing position, h, the number of periods remaining on an existing mortgage, n, mortgage contract type, z,the value of the current period idiosyncratic shock ϵ and age, j. We will always characterize the value function by the order of state variables $v(a, h, n, z, \epsilon, j)$. We can think of the household as being in one of five situations with respect to yesterday's and today's housing investment position.

1. **Renter yesterday** $(h = 0)$ **and renter today** $(h' = 0)$: Consider a household that does not own a house at the start of the period, $h = n = z = 0$, and decides to continue renting housing services in the current period, $h^{'} = n' = z' = 0$. The decision problem in recursive form can be expressed as:

$$v(a, 0, 0, 0, \epsilon, j) = \max_{(c,s,a')\in\mathbb{R}_+} \left\{ u(c, s) + \beta\psi_{j+1} \sum_{\epsilon'\in\mathcal{E}} \pi(\epsilon, \epsilon')v(a', 0, 0, 0, \epsilon', j + 1) \right\}, \tag{3.4}$$

$$s.t. \quad c + a' + Rs = y(a, h', s, \epsilon, \upsilon_j, j; q),$$

where Rs denote the cost of housing services purchased in the rental market. Their is no restriction on the size of housing services rented.[10] The restriction in the choice set indicates that asset markets are incomplete since short-selling is precluded and only an noncontingent claim on capital is traded.

2. **Renter yesterday** $(h = 0)$ **and homeowner today** $(h' > 0)$: In this situation, we consider a households that rented the previous period $h = 0$, and chooses to purchase a house

[9] It should be pointed out that h, z, and n are sufficient information to identify information about a contract even when mortgage loans have different maturities $N(z)$ and interest rates $r^m(z)$. In our recursive notation, households begin with a mortgage obligation $D(N(z), z) = (1 - \chi(z))ph'$ that diminshes every period. The mortgage payment each period are calculated using $m(x, z) = A(n, z) + I(n, z)$, where the interest payments are calculated by $I(n, z) = r^m(z)D(n, z)$. The law of motion for the level of housing debt $D(n, z)$ can be written as,

$$D(n - 1, z) = (1 + r^m(z))D(n, z) - m(x, z).$$

The law of motion for home equity increases with mortgage payments. That is

$$H(n - 1, z) = H(n, z) + [m(x, z) - r^m(z)D(n, z)],$$

where $H(N, z) = \chi(z)ph'$ denotes the home equity in the initial period.

[10] Other housing papers impose some limits in the size of rental-occupied housing. In this paper, renters can consumer any size of housing services.

in the current period, $h^{'} > 0$. The housing investment requires a choice of mortgage $z' \in \mathcal{Z}$ to finance an initial expenditure of $(\phi_b + \chi(z'))ph'$ where ϕ_b represents a transaction cost parameter and $\chi(z')$ denotes the downpayment fraction associated to mortgage z'.[11] The period mortgage payment is $m(x, z')$ where $x = (p, h', \chi(z'), N(z'), r^m(z'))$ In this model housing is a consumption and investment good where housing services can be transacted in the market. To participate in the rental market each period as a landlord, households have to pay a fixed operating cost, $\varpi > 0$.[12] For these households, housing consumption satisfies $s < h'$ receiving rental income $R(h' - s)$.[13] Otherwise, the optimal housing consumption is entirely determined by the housing stock $s = h'$. In order to incorporate this decision into the choice problem we introduce an indicator variable, I_r, that takes on the value of unity when the household chooses to be a landlord, and zero otherwise. Formally:

$$v(a, 0, 0, 0, \epsilon, j) = \max_{\substack{(c,s,a',h') \in \mathbb{R}_+ \\ z' \in \mathcal{Z},\ I_r \in \{0,1\}}} \left\{ u(c, s) + \beta\psi_{j+1} \sum_{\epsilon' \in \mathcal{E}} \pi(\epsilon, \epsilon') v(a', h', N(z) - 1, z', \epsilon', j + 1) \right\}, \tag{3.5}$$

$$\begin{aligned} s.t. \quad c + a' + (\phi_b + \chi(z'))ph' + m(x, z') + x(h', s) &= y(a, h', s, \epsilon, v_j, j; q) \\ &+ I_r \left[R(h' - s) - \varpi \right], \\ s &\leq h'. \end{aligned}$$

Owning property requires a maintenance expense each period. The total maintenance cost depends on the choice to supply rental property. If homeowners choose not to supply housing services to the rental market, (i.e., $I_r = 0$) then $s = h'$ and the maintenance expense is given by $x(h', s) = \delta_o ph'$ where δ_o represents the depreciation rate of owner-occupied housing. Alternatively, a households can choose to supply housing services to the rental market, (i.e., $I_r = 1$). In this case, maintenance expense depends on the amount of housing supplied to the rental market and their own consumption and is defined as $x(h', s) = \delta_o ps + \delta_r p(h' - s)$, where δ_r represents the depreciation rate of rental-housing. The presence of moral hazard associated with renting property implies that there is an spread in depreciation rates ($\triangle\delta = \delta_r - \delta_o > 0$) that reduces the implicit cost of owner-

[11] For computational reasons χ is not a choice variable in the model. The endogenous choice of downpayment would require keeping track of an additional state for the downpayment choice since this decision is dynamic. A higher downpayment today reduces both current and future mortgage payments.

[12] The decision to supply rental property is entwined with the decision to invest in housing. The separation of housing consumption services and housing investment allows the rental market to be formalize while keeping the state space relatively tractable. Introducing two different housing stocks such as owner-occupied and rental-occupied would require an additional portfolio choice making the problem computationally infeasible.

As a result, all the landlords are homeowners but the not the other way around. Nevertheless, the American Housing Survey reports that the fraction of individuals that report to receive rental income and rental the house they occupied is almost zero.

[13] This formulation imples that a household that leases property uses a mortgage with a downpayment of χ percent of the value of the property. Although this may seem to be an unrealistic assumption, the POMS Survey reports that 81.1 percent of rental property owners used some sort of mortgage financing in financing the acquisition of rental property.

occupied consumption. The choice of rental supply is complex because landlords not only take into account the maintenance expense, but the tax provisions with respect to rental-income. For a more detailed analysis of the tax treatment of homeowners and landlords, see Chambers, Garriga and Schlagenhauf (2007).

3. **Homeowner yesterday** $(h > 0)$ **and renter today** $(h' = 0)$: In this situation we consider a household that is selling the property $h > 0$ to become a renter in the current period $h' = 0$. [14] The decision to sell property reveals why housing is a risky investment. At the moment of sales, the household is subject to an idiosyncratic capital gain or amenity shock, $\xi \in \Xi$. This shocks impacts the selling value of the property by change the size of the housing investment.[15] This shock is not revealed until the house is sold. We assume this shock is i.i.d. and discrete. The unconditional probability of the shock is π_ξ. The optimization problem for this situation is:

$$v(a,h,n,z,\epsilon,j) = \max_{(c_\xi,s_\xi,a'_\xi)\in\mathbb{R}_+} \left\{ \sum_{\xi\in\Xi} \pi_\xi [u(c_\xi,s_\xi) + \beta\psi_{j+1} \sum_{\epsilon'\in\mathcal{E}} \pi(\epsilon,\epsilon')v(a'_\xi,0,0,0,\epsilon',j+1)] \right\}, \tag{3.6}$$

$$s.t. \quad c_\xi + a'_\xi + Rs_\xi = y(a,h',s,\epsilon,\upsilon_j,j;q) + [(1-\phi_s)p\xi h - D(n,z)],$$

The optimal choice depends on the income received from selling the property, $p\xi h$, net of transactions costs from selling, ϕ_s, and remaining principle $D(n,z)$ which depends on the mortgage balance ($n = 0$ if the mortgage is paid off or $n > 0$ if positive balance remains) associated to the chosen contract z.[16] Notice that the consumption of goods, housing services, and savings are conditioned on the idiosyncratic shock since net income depends on the realization of ξ.

4. **Homeowner yesterday** $(h > 0)$ **and homeowner today** $(h' > 0)$: The last cases deal with a household that enters the period with a housing investment and decides to continue to have a housing investment position. The critical issue is whether the household decides to change their housing position.

   (a) Homeowner maintains housing size

   If the household decides to maintain their housing investment position, $h = h'$, the

[14] In the last period, all households must sell h, rent housing services and consume all their assets, a, as a bequest motive is not in the model. In the last period, $h' = a' = 0$.

[15] The idiosyncratic capital gain shock introduces a form of risk into the housing investment decision without having to introduce an aggregate shock. Adding aggregate uncertainty is not compuationally feasible in this model at this time. This shock can be thought of as what happens to a property if the surrounding neighborhood deteriorates or improves. This change would be reflected in the house value at the time of sale. An additional advantage of the formulation is that it eliminates the necessity of matching buyers and sellers as any buyer can always purchase a brand new home with independence of the shock received by the seller.

[16] Since our analysis will be conducted at the steady state, other than the differences between buying and selling transaction costs, there are no differences in the purchase and selling prices of housing.

optimization problem can be written as:

$$v(a,h,n,z,\epsilon,j) = \max_{\substack{(c,s,a')\in\mathbb{R}_+ \\ I_r\in\{0,1\}}} \left\{ u(c,s) + \beta\psi_{j+1} \sum_{\epsilon'\in\mathcal{E}} \pi(\epsilon,\epsilon') v(a',h',n-1,z,\epsilon',j+1) \right\} \tag{3.7}$$

$$s.t. \quad c + a' + m(x,z) + x(h',s) = y(a,h',s,\epsilon,\upsilon_j,j;q) + I_r \left[R(g(h')-s) - \varpi \right],$$
$$s \le h',$$

where $n' = \max\{n-1,0\}$. If $n' > 0$,a mortgage payment is required. The decision on the amount of housing services to consume and thus maintenance expenses depends on choice of paying a fixed cost ϖ to become a landlord. It should also be pointed out that this formulation does not allow the household to change mortgage type which is equivalent to an assumption of no refinancing. Later we will relax this assumption an examine the refinancing issue. We initially preclude this option so we can get a clear picture of the implication of different mortgage types for the purchase of housing.

(b) Homeowner changes housing size

If the household decides to either up-size or down-size their housing investment position, $(h \neq h', h > 0, h' > 0)$, the household problem becomes

$$v(a,h,n,z,\epsilon,j) = \tag{3.8}$$
$$\max_{\substack{(c_\xi,s_\xi,a'_\xi,h'_\xi)\in\mathbb{R}_+ \\ z'\in\mathcal{Z},\ I_r\in\{0,1\}}} \left\{ \sum_{\xi\in\Xi} \pi_\xi [u(c_\xi,s_\xi) + \beta\psi_{j+1} \sum_{\epsilon'\in\mathcal{E}} \pi(\epsilon,\epsilon') v(a'_\xi,h'_\xi,N-1,z',\epsilon',j+1)] \right\}$$

This constraint accounts for the additional income from selling their home (net of transaction costs, $\phi_s p \xi h$, and remaining principle, $D(n,z)$), the cost of buying a new home with mortgage product z', as well as the amenity shock associated with the sale of the home. Just as in case 3, optimal choices depends on the realization of the idiosyncratic shock ξ. In this case, savings and household investment depend on the shock.

3.2. The Financial Intermediary

The financial intermediary is a zero profit business. The firm receives the deposits of the households, a' and uses these funds to make loans to firms and households. Firms take out loans of capital to produce goods and households require long-term mortgages to finance the investment in housing. They receive mortgage payments from homeowners, principal payments from individuals who sell their home with remaining principle on their mortgage, and principle payments from individuals who unexpectedly die. The financial intermediary's balance sheet determines the equilibrium condition in the asset market.

3.3. The Production Sector

The production sector is relatively standard. Firms produce according to a constant returns to scale technology $Y = f(K, L)$ where K and L are aggregate inputs of capital and labor, respectively. We assume that capital depreciates at the rate $\delta > 0$ each period. Firms' output can be used for consumption, capital investment or housing purposes.

3.4. Government

In this economy, the government engages in a number of activities: financing some exogenous government expenditure, providing retirement benefits through a social security program, and redistributing the wealth of those individuals who die unexpectedly. We assume that the financing of government expenditures and social security are managed under different budgets

In the general budget constraint, revenues are generated from the taxation of adjusted income. We have previously defined $T(ay)$ as the tax obligations given certain adjusted income. We define $t(a, h, n, z, \epsilon, j)$ to be the tax obligations of a representative household based on their state space. In this situation government revenue is given by

$$G = T = \int \mu_j t(a, h, n, \epsilon, j)\Phi(da \times dh \times dn \times dz \times d\epsilon \times dj), \tag{3.9}$$

and thus government expenditure is determined by the amount of revenue collected from the income taxation.

The government provides social security benefits to retired households. The benefit, θ, is based on some fraction, $\overline{\theta}$, of the average income of workers. These payments are financed by taxing the wage income of employed households at the rate τ_p. Since this policy is self-financing, the tax rate depends on the replacement ratio parameter $\overline{\theta}$. The social security benefit is defined as:

$$\theta \equiv \overline{\theta} \frac{\sum_{j=1}^{j^*-1} \sum_i \mu_j (w v_j \epsilon_i)}{\sum_{j=1}^{j^*-1} \mu_j}$$

where μ_j is the size of the age j cohorts. The social security budget constraint is:

$$\tau_p \sum_{j=1}^{j^*-1} \sum_i (\mu_j w v_j \epsilon) = \theta \sum_{j^*}^{J} \mu_j. \tag{3.10}$$

The final role of the government is to collect the physical and housing assets of those individual who unexpectedly die. Both of these assets are sold and any outstanding debt on housing is paid off. The remaining value of these assets is distributed to the surviving households as a lump-sum payment, tr. This transfer can be defined as

$$tr = \frac{Tr}{1-\mu_1}$$

where Tr is the aggregate (net) value of assets accumulated over the state space from unexpected death and is defined as[17]

$$\begin{aligned} Tr = &\int \mu_j(1-\psi_j)a(a,h,n,z,\epsilon,j)\Phi(da \times dh \times dn \times dz \times d\epsilon \times \{2,..,J\})+ \\ &\sum_{\xi\in\Xi} \pi_\xi \int \mu_j(1-\psi_j)[(1-\phi_s)p\xi h(a,h,n,z,\epsilon,j)- \\ &D(a,h,n,z,\epsilon,j)]\Phi(da \times dh \times dn \times dz \times d\epsilon \times \{2,..,J\}). \end{aligned} \quad (3.11)$$

3.5. Market Clearing Conditions

This economy has four markets: the asset market, labor market, the rental of housing services market, and the goods market. All markets are assumed to be competitive.

The market clearing condition in the goods market is given by:

$$C + K' - (1-\delta)K + I_H + G + \Upsilon = F(K,L) \quad (3.12)$$

where C, K, G, I_H, and Υ represent aggregate consumption, aggregate investment in real capital, aggregate government spending, aggregate housing investment, and various transaction costs, respectively. Each of these aggregates are define more formally in the appendix where the recursive stationary equilibrium is defined.

In the labor market, the equilibrium wage is determined by the marginal product of labor

$$w = F_2(K,L) \quad (3.13)$$

where labor is supplied inelastically in the model and determined by$L = \sum_{j=1}^{j^*-1} \sum_\epsilon \mu_j v_j \epsilon$.

The asset market clearing condition is complicated by the presence of mortgages, unexpected death, and idiosyncratic capital gain shocks. In order to simply the notation, let $\Phi(a,h,n,z,\epsilon,j)$ determines the measure of individuals in a given point in the state space $\Lambda \equiv (a,h,n,z,\epsilon,j)$.In addition, define $I_s(\Lambda)$ to be an indicator function that is equal to one when a housing investment position is sold and zero otherwise. This function will help in identifying when idiosyncratic capital gain shocks are present. The equilibrium condition in the asset market is:

[17] The new generation receives a lump sum transfer as we endow these individuals with capital assets observed in the data. The aggregate mass of households of age 1 is μ_1 and the total population is normalized to unity.

$$
\begin{aligned}
K' = &\int_{I_s(\Lambda)=0} \mu_j a'(\Lambda)\Phi(d\Lambda) + \int_{I_s(\Lambda)=1} \sum_{\xi\in\Xi} \pi_\xi \mu_j a'_\xi(\Lambda)\Phi(d\Lambda) \\
&- \int_{I_s(\Lambda)=0} \mu_j (1-\chi) p h'(\Lambda)\Phi(d\Lambda) - \int_{I_s(\Lambda)=1} \sum_{\xi\in\Xi} \pi_\xi \mu_j (1-\chi) p h'_\xi(\Lambda)\Phi(d\Lambda) \qquad (3.14) \\
&+ \int_{I_s(\Lambda)=0} \mu_j m(x,z)\Phi(d\Lambda) + \int_{I_s(\Lambda)=1} \sum_{\xi\in\Xi} \pi_\xi \mu_j m(x,z)\Phi(d\Lambda) \\
&- \int_{I_s(\Lambda)=0} \mu_j D(\Lambda)\Phi(d\Lambda) - \int_{I_s(\Lambda)=1} \mu_j (1-\psi_j) D(\Lambda)\Phi(d\Lambda)
\end{aligned}
$$

where $\Phi(d\Lambda) \equiv \Phi(da \times dh \times dn \times dz \times d\epsilon \times dj)$.

The left hand side of this equation indicates the total amount of capital available to loan to firms, while the right hand side measures the sources of this capital. The first line on the right hand side of the equation captures the savings deposited by households to the financial intermediary. The first of these terms measures household deposits if the housing position is not sold while the second term on this line allows the deposit decision to be impacted by the idiosyncratic capital gain shock when the housing position is sold. From the total of household deposits, new mortgage loans must be subtracted. The second line on the right hand side measures new mortgages, and allows for differences created by idiosyncratic capital gains shocks. The third line measures an additional source of loanable funds as mortgage payments received by the financial intermediary. This includes payments received by first-time buyers and existing homeowners who continue to make payments on their mortgage, as well as those homeowners who sell property and have a new mortgage payment which is affected by the idiosyncratic capital gain shock. The last line on the right hand side of the equation captures the repayment of remaining mortgage principle from households who sell their house as well as the repayment of outstanding debt of households who unexpectedly die with outstanding principle.

In this model, the rental market is endogenous. Individuals who cannot afford to buy a house must purchase or rent housing services. Rental property is supplied by those individuals that have a positive housing investment position and pay the fixed cost $\varpi > 0$ to supply rental property, (i.e., $h' - s > 0$). Households who supply housing services receive $R(h' - s)$ gross rental income. The rental price R adjusts to equate the aggregate demand for housing services with the aggregate supply of rental services. Before defining the market clearing equation, some additional notation is needed for the sake of simplification. The rental market equilibrium condition is:

$$
\begin{aligned}
&\int_{I_s(\Lambda)=0} \mu_j [h'(\Lambda) - s(\Lambda)]\Phi(d\Lambda) + \int_{I_s(\Lambda)=1} \sum_{\xi\in\Xi} \pi_\xi \mu_j [h'_\xi(\Lambda) - s_\xi(\Lambda)]\Phi(d\Lambda) = \qquad (3.15) \\
&\int_{I_s(\Lambda)=0} \mu_j s(\Lambda)\Phi(d\Lambda) + \int_{I_s(\Lambda)=1} \sum_{\xi\in\Xi} \pi_\xi [\mu_j s_\xi(\Lambda)\Phi(d\Lambda)
\end{aligned}
$$

The left hand side of the question measures the supply of housing services while the right hand side measures the demand for housing services. On both sides of the equation, home sellers are differentiated from non-sellers by recognizing that rental choices for home sellers are contingent on the realization of the capital gain shock, ξ.

4. Parameterization

We parameterize the model to match some key moments of the U.S. economy. This strategy allows us to specify a limited number of parameter values while estimating the remaining parameters as an exercise in exactly-identified Generalized Method of Moments. With the parameterized model, we will evaluate the impact of different mortgage contracts across various dimensions.

4.1. Demographics

Each period in the model is taken to be three years. Individuals enter the labor force at age 20 (model period 1) and potentially live till age 86 (model period 23). Retirement is assumed to be mandatory at age 65 (model period 16). Individuals survive to the next period with probability $\psi_{j+1.}$These probabilities are set at survival rates observed in 1994, and the data are from the National Center for Health Statistics, *United States Life Tables*, 1994. The size of the age specific cohorts, μ_j, need to be specified. Because of our focus on steady state equilibrium, these shares must be consistent with the stationary population distribution. As a result, these shares are determined from $\mu_j = \psi_j \mu_{j-1}/(1+\rho)$ for $j = 2, 3, ..., J$ and $\sum_{j=1}^{J} \mu_j = 1$, where ρ denotes the population growth rate. Using the resident population as the measure of the population, the annual growth rate is set at 1.2 percent.

4.2. Preferences and Technology

The choice of preferences is based on empirical evidence. Jeske (2005) documents that the housing service/consumption ratio increases by age. He points out that a constant relative risk aversion momentary utility function that allows the consumption of housing and nonhousing services as arguments has the implication that the housing service to consumption ratio stays constant over the life cycle.[18] We assume that preferences over the consumption of goods and housing services can be represented by the period utility function:

$$U(c, s) = \gamma \frac{c^{1-\sigma_1}}{1-\sigma_1} + (1-\gamma) \frac{s^{1-\sigma_2}}{1-\sigma_2}$$

The coefficients, σ_1, and σ_2, determine the curvature of the utility function with respect to consumption and housing services. The relative ratio of σ_1 and σ_2 determines the growth rate of the housing to consumption ratio. A larger curvature in consumption relative to the

[18] We also find that such a momentary utility function generates insufficient movements in housing position as well as introducing some counterfactional implications for the rental market.

curvature in housing services implies that the marginal utility of consumption exhibits relatively faster diminishing returns. When household income increases over the life-cycle (or different idiosyncratic labor income shocks), a larger fraction of resources are allocated to housing services. We set $\sigma_2 = 1$ and $\sigma_2 = 3$ to match the observed average growth rate while the preference parameter γ is estimated.

The choice of technology is relatively standard. We assume that the aggregate production function is Cobb-Douglas with:

$$F(K, L) = K^{\alpha} L^{1-\alpha}$$

The capital share parameter is set to 0.29. This value is calculated by dividing private fixed assets plus the stock of consumer durables less the stock of residential structures by output plus the service flows from consumer durables less the service flow from housing.[19] Since the firm's output can be used either for consumption, housing investment, or capital good investment, the relative price of housing, p, is equal to one.

4.3. Endowments

Workers are assumed to have an inelastic labor supply, but the effective quality of their supplied labor depends on two components. One component is an age-specific, v_j, and is designed to capture the "hump" in life cycle earnings. We use data from U.S. Bureau of the Census, "Money, Income of Households, Families, and Persons in the Unites Stated, 1994," *Current Population Reports*, Series P-60 to construct this variable. The other component captures the stochastic component of earnings and is based on Storesletten, Telmer and Yaron (2004). We discretize this income process into a five state Markov chain using the methodology presented in Tauchen (1986). The values we report reflect the three year horizon employed in the model. As a result, the efficiency values associated with each possible productivity value ϵ are

$$\epsilon \in \mathcal{E} = \{4.41, 3.51, 2.88, 2.37, 1.89\}$$

and the transition matrix is:

$$\pi = \begin{bmatrix} 0.47 & 0.33 & 0.14 & 0.05 & 0.01 \\ 0.29 & 0.33 & 0.23 & 0.11 & 0.03 \\ 0.12 & 0.23 & 0.29 & 0.24 & 0.12 \\ 0.03 & 0.11 & 0.23 & 0.33 & 0.29 \\ 0.01 & 0.05 & 0.14 & 0.33 & 0.47 \end{bmatrix}.$$

Each household is born with an initial asset position. The purpose of this assumption is to account for the fact that some of the youngest households who purchase housing have some wealth. Failure to allow for this initial asset distribution creates a bias against the purchase of

[19] A data appendix is available that details the calculation of this parameter as well as other parameters used in the paper.

homes in the earliest age cohorts. As a result we use the asset distribution observed in *Panel Study on Income Dynamics* (PSID) to match the initial distribution of wealth for the cohort of age 20 to 23. Each income state has assigned the corresponding level of assets to match the nonhousing wealth to earnings ratio.

4.4. Housing

The housing market introduces a number of parameters. The purchase of a house requires a mortgage and downpayment. In this paper we focus on the 30 year fixed rate mortgage as the benchmark mortgage. As a result of the assumption that a period is three years, we set the mortgage length, N, to ten periods. The downpayment requirement, χ, is set to twenty percent matching facts from the American Housing Survey. Buying and selling property is subject to transaction costs. We assume that all these costs are paid by the buyer and set $\phi_s = 0$ and $\phi_b = 0.06$.

Because of the lumpy nature of housing, the specification of the second point in the housing grid has important ramifications. This grid point, $\underline{h}$, determines the minimum house size, and has implications for the timing of the purchase of housing investment, wealth portfolio decisions and homeownership rate. To avoid having the choice of this variable having inadvertent implications for the results, we determine the size of this grid point as part of the estimation problem.

As previously explained, housing depreciates at rates which depend on whether the property is owner-occupied or rented. The values for δ_o and δ_r are estimated.

We used data from the 1995 *American Housing Survey* to quantify the i.i.d. capital gains shock. To calculate the probability distribution for this shock we measure capital gains based on the purchase price of the property and what the property owner believes to be the current market value. This ratio is adjusted by the holding length to express the appreciation in annualized terms. Then we estimate a kernel density and discretize the density in three even partitions. The average annualized price changes ranging from lowest to highest are -0.19, -0.04, and 0.31 where the expected capital gain is about 2.5 percent. Appropriate adjustments were made for our model where a period corresponds to a three year period.[20]

4.5. Government and the Income Tax Function

The government has three functions in the model. Income is provided to retired individuals through a social security program. The social security budget constraint involves two parameters: the replacement ratio, $\overline{\theta}$, and the social security tax rate. We set the replacement ratio to be thirty percent and solve for the payroll tax rate consistent with the budget constraint. In this case, the payroll tax is 5.25 percent.

[20] To test the robustness of the results based on data from the *American Housing Survey,* we employed a similar approach using 1995 Tax Roll Data for Duval County in Florida. Jacksonville is the major city in Duval County. This data follows real estate properties as opposed to individuals. We calculated annualized capital gains based on actual sales. We found very similar estimates for the capital gains shock using this data source.

Government spending is financed through income taxation. To get an accurate assessment of housing policy wedges, we want the income tax code to be a good approximation of the actual U.S. tax code. Gouveia and Strauss (1994) estimated a functional form for the US federal income tax code that is theoretically motivated by the equal sacrifice principle. The actual tax paid by a household, $T(ay)$, is based on adjusted gross income, and is determined by the functional form

$$T(ay) = \eta_0(ay - (ay^{-\eta_1} + \eta_2)^{\frac{-1}{\eta_1}}),$$

where (η_0, η_1, η_2) are policy parameters. The marginal income tax rate is

$$T'(ay) = \eta_0(1 - (1 + \eta_2 y^{\eta_1})^{-\frac{1}{\eta_1} - 1}).$$

This functional form is very flexible and allows lump-sum taxes ($\eta_1 = -1$), proportional ($\eta_1 \rightarrow 0$), or progressive taxes ($\eta_1 > 0$) as special cases. The parameter η_0 is a scaling factor that determines the level of the tax brackets and the marginal tax rate but does not impact the curvature of the tax function. The parameter η_2 depends on units of measurement used to measure income and determines the size of income deduction. Gouveia and Strauss estimate the policy parameters and find that $\eta_0 = 0.258$, $\eta_1 = 0.768$, and $\eta_2 = 0.003710$. In the benchmark economy we use the same parameter estimates used by Gouveia and Strauss for η_1 but η_2 is set to 0.3710 to accommodate the model measurement units. The parameter η_0 is determined in the estimation section to pin-down the share of federal revenue in GDP. It is important to note that in the various experiments we conduct, we hold η_1 and η_2 constant but allow η_0 to change in each experiment so that the revenue/output ratio remains constant. Following the provisions of the current income tax code, we allow mortgage interest payments and maintenance expenses for rental property to be deducted from income that is taxable. In addition, rental income is taxable, but the imputed rental value of owner-occupied housing is not.

4.6. Estimation

We estimate seven parameters using an exactly-identified Method of Moments approach. The parameters that need to be estimated are the depreciation rate of the capital stock, δ, the depreciation rate for rental units, δ_r, the depreciation rate for ownership units, δ_o, the relative importance of consumption goods to housing services, γ, the discount rate, β, the size of the smallest housing investment position, and the tax function parameter, η_0. We identify these parameter values so that the resulting aggregate statistics in the model economy are equal to seven targets observed in the U.S. economy.

1. **Wealth to gross domestic product ratio** (K/Y) : We find the target is the ratio of capital to gross domestic product (GDP) which is about 2.541, (annualized value) for the period 1958-2001 where we define the capital stock as private fixed assets plus the stock of consumer durables less the stock of residential structures so as to be consistent with capital in the model. We measure GDP to be consistent with output in the model. That

is, output is measured as reported GDP plus service flows from consumer durables less the service flow from housing.[21]

2. **Housing stock to Fixed capital stock ratio** (H/K) : In this ratio the housing capital stock is defined as the value of fixed assets in owner and tenant residential property. The housing stock data is from the fixed asset tables of the Bureau of Economic Analysis. We find the ratio of the housing stock to nonhousing capital stock to be 0.43.

3. **Housing Investment to Housing Stock ratio** (x_H/H) : The ratio of the investment in residential structures to housing capital stock is targeted at 0.04.

4. **Housing services to consumption of goods ratio** (Rs_c/c) : The targeted housing consumption to nonhousing consumption is also based on NIPA data where housing services are defined as personal consumption expenditure for housing and nonhousing consumption is defined as nondurable and services consumption expenditures net of housing expenditures. The targeted ratio for 1994 is 0.23, but the number does not vary greatly over the period 1990-2000. This value is from Jeske(2005).

5. **Fixed capital investment to GDP ratio** $(\delta K/Y)$: The fifth target is the investment in capital goods to output ratio which is 0.135.

6. **Homeownership Ratio:** This target is based on data from the *American Housing Survey* for 1994 and is equal to 64.0 percent.

7. **Government expenditure to output ratio** $(T(ay)/Y)$: The final target using NIPA data is the government expenditure-output ratio. We define government expenditure as federal government expenditures. The parameter η_0 is endogenously determined when solving the model to target the 7.4 percent ratio of federal government expenditure-GDP observed in 1994.[22]

Table 1 summarizes the parameter estimates and the empirical targets. The moments and the parameter values are presented in annual terms.

[21] We estimated service flows using procedures outlines in Cooley and Prescott (1995).

[22] The Gouveia and Strauss tax function was estimated for the period 1979-1989. As our model is calibrated for the period 1994-1996, we acknowledge some inconsistency. However, since our focus is on the importance of various margins impacted by housing policy, we do not feel this inconsistency is a major problem.

Table 1: Method of Moments Estimates (values in annual terms)

Statistic	Parameter	Moment	Model	% Error
1) Ratio of wealth to gross domestic product	$\beta = 0.976$	2.541	2.5446	0.143
2) Ratio of housing stock to Fixed capital stock	$\delta_o = 0.034$	0.430	0.4266	-0.792
3) Housing Investment to Housing Stock ratio	$\delta_r = 0.075$	0.040	0.0403	-0.388
4) Ratio housing services to consumption of goods	$\gamma = 0.954$	0.230	0.2291	-0.411
5) Ratio fixed capital investment to GDP	$\delta_k = 0.043$	0.135	0.1353	0.339
6) Homeownership Rate	$\underline{h} = 1.473$	0.640	0.6370	-0.468
7) Government expenditure to output ratio	$\eta_0 = 0.197$	0.074	0.0742	-0.005

The implied targets generated by the model solution are within one percent error for all the observed targets. The estimation of the structural parameters is not separated from the computation of equilibrium (household's optimization problem and market clearing). That includes three additional nonlinear equations (asset market, government budget constraint, and accidental bequest) to include in the distance minimization routine that have be satisfied in conjunction with the moments observed in the data.

4.7. Model Evaluation

The baseline economy is estimated to match certain key features of the US economy in 1994. Since we want to use the model to evaluate mortgage contract choice, it is important to briefly evaluate the performance of the model. In this section, we examine whether the model generates reasonable patterns of participation in the owner-occupied market, housing consumption, and financial portfolio decisions. A starting point is to inquire whether the model generates a reasonable homeownership rate. Since the aggregate homeownership rate is a target in the estimation problem, we can check to see if the model generates a reasonable amount of "first-time buyers" which we define as households owning a home and being under the age of 35. Data indicates that 37.3 percent of households in this age cohort are homeowners. The model generates a participation rate of 37.6 percent. In Table 2, we present the homeownership rate across the age and income distributions. As can be seen, the observed homeownership rate has a hump shaped behavior with the highest rate occurring in the 65-74 age cohort. The model generates a very similar pattern. It should be pointed out that the under prediction of the oldest cohort is a result of the assumption that households must rent in the final period. Data indicates a rising homeownership rate in income, and the model generates a similar profile. However, the profile generated by the model is steeper.

Table 2: Homeownership Rates by Age and Income

Variable		Homeownership Rate				
by Age Cohorts	**Total**	**20-34**	**35-49**	**50-64**	**65-74**	**75-89**
Data 1994	64.0	40.0	64.5	75.2	79.3	77.4
Baseline Model 1994	63.7	37.5	76.5	86.4	91.3	66.5
by Income Quintiles		**Q1**	**Q2**	**Q3**	**Q4**	**Q5**
Data 1994		46.6	56.1	64.4	75.5	89.1
Baseline Model 1994		32.0	83.9	98.4	100.0	100.0

Data source: Housing Vacancies and Homeownership (CPS/HVS) and American Housing Survey (AHS)

Another dimension of interest is the consumption of housing services. We measure average consumption of housing services by computing the average size of an owner-occupied house. Data from the *American Housing Survey* (AHS) finds the average owner-occupied house is 2,137 square feet. Our model implies an average house size of 2,348 square feet. In Table 3, we report observed housing size by age cohorts. Housing size increases until age 65 when some downsizing begins to appear. The model captures the magnitude and the hump-shaped behavior by age groups. However, some over-prediction of house size is observed.

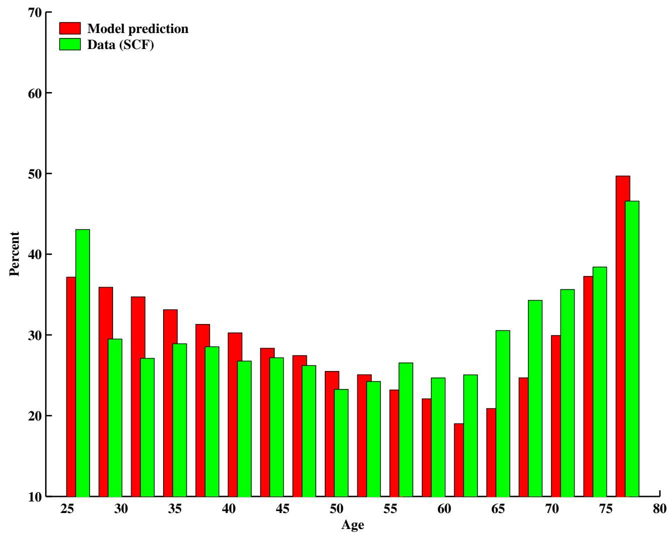
Table 3 : Owner-occupied Housing Consumption

Simulation	Sqft. Owners[1]					
		by Age Cohorts				
	Total	**20-34**	**35-49**	**50-64**	**65-74**	**75-89**
Data 1994	2,137	1,854	2,220	2,301	2,088	2,045
Baseline Model 1994	2,348	2,147	2,297	2,429	2,514	2,362

Data source: American Housing Survey (AHS)

Since households make savings decisions with respect to assets, the portfolio allocations implied by the model can be analyzed. In the model, a households financial portfolio is comprised of asset holding and equity in housing investment. We use data from the 1994 *Survey of Consumer Finances* to determine the importance of housing in household portfolios. We define assets as bond and stock holdings and housing is defined as the respondent's estimated value of their house adjusted for the remaining principle.[23] The data indicates housing makes up a large fraction of a household's portfolio in the youngest age cohorts. This fraction declines as the household ages until around the retirement age, and then increases as households consume their non-housing wealth after retirement. As can be seen in Figure 1, the model generates a very similar pattern.

Figure 1: Housing in the Portfolio by Age



Data source: Survey Consumer Finance (SCF)

5. The Mortgage Decision

Until the mid-nineties the set of mortgage products available was relatively limited. The vast majority of homeowners purchased their residency using a standard 30 year fixed rate loan with a 20 percent downpayment. Over the last decade, as a result of deregulation and innovations in the mortgage industry, private markets have introduced loan products that differ from more traditional mortgage instruments. Each contract type has potentially different dynamic implications resulting in the need to fully understand the impact of housing finance for the productive economy.

We begin by examining mortgage decisions in an environment where homeowners have the choice of financing a house with a 30 year fixed rate contract or one of the other contract types discussed. An analysis with two contracts has the advantage of allowing the effects of an additional mortgage contract to be more transparent. To make all the experiments comparable, we

[23] We acknowledge some inconsistency in the data and the model. The value of housing the SCF includes both the value of the structure and the value of land. Land is not accounted for in the model. Hence, the value of housing in the model reflect soley the value of the structure.

maintain the baseline calibration including the parameters of tax code. This means we allow government revenue to adjust as contract types change.[24] In this paper, we study the implications of mortgage decisions for aggregate and distributional housing market effects, buying and selling patterns, and consumption smoothing. We finalize the section by exploring the implications of downpayment choice.

5.1. Mortgage Contract Choice and Type of Contract

We consider eight different experiments of mortgage choice. We consider four graduated payment mortgage contracts that differ in the nature and the growth rate of the repayment schedule, but the common feature is that loan payments increase over the length of the mortgage. A contract labeled 'low' or 'high' will denote the steepness of the increase in mortgage payments. A GPM contract with an arithmetic structure and noted as 'low' has a step size of .01 while the 'high' case has a step size of .05. If a GPM contract has a geometric structure, we specify growth rate in the 'low' case to be .05, and in the 'high' case .15.

An alternative loan that allows for changing payments over time are balloons or interest only loans. The balloon contracts that we consider are actually a combination of a pure balloon contract for an initial period that rolls into a standard fixed rate mortgage for the remaining years. We consider two different specifications of this loan product. The first contract considers a twelve year (four periods) interest only loan with a twenty downpayment requirement that is rolled into a eighteen year (six periods) FRM contract. The second contract has been motivated by recent events in the subprime crises and modifies the previous loan by allowing a zero downpayment and including the transaction costs associated with the main loan. We also examine a combo or Piggyback mortgage which employs a secondary loan for the downpayment. The main difference of the combo loan contract from the prior contracts is that individuals trade-off a lower downpayment at the expense of higher initial mortgage payments as the household borrows the full amount of the downpayment. In general, the repayment structure of combo loan contracts declines over the length of the mortgage since the second loan has a shorter maturity than the main loan. Despite the high initial payments, this loan product allows households that are downpayment constrained but can afford the mortgage payments to purchase a house. However, combo loan contracts allow for a very slow accumulation of equity. An alternative contract with a decreasing repayment schedule that allows households to accrue equity very fast is the constant amortization loan. In Table 4 we present the aggregate implications of mortgage decisions.

[24] Because of the focus of the paper, we abstracted from the second order effects associated with maintaining the level of government revenue. This also means that a welfare analysis would embed some inconsistencies.

Table 4: Aggregate Effects of Mortgage Choice

Simulation	Ownership Rate	Housing Size Owners	Housing Size Var.	Number of Properties: No Mortgage	Number of Properties: With Mortgage	Share Homeowners with: FRM Mortgage	Share Homeowners with: Second Mortgage
Data (AHS 1994)	64.0	2,137	969	39.3	60.7	-	-
Baseline with FRM	63.7	2,348	816	30.1	69.9	100.0	0.0
GPM-Arithmetic (Low)	64.1	2,412	832	29.8	70.2	69.7	30.3
GPM-Arithmetic (High)	68.3	2,299	1,003	1.7	98.3	45.4	54.6
GPM-Geometric (Low)	64.5	2,450	884	28.8	71.2	68.7	31.3
GPM-Geometric (High)	68.7	2,477	1,188	7.1	92.9	46.6	53.4
Balloon Fixed ($\chi = 0.20$)	65.7	2,467	891	26.7	73.3	65.3	34.7
Balloon Fixed ($\chi = 0.00$)	68.6	2,489	1,270	0.2	99.8	49.9	50.1
Combo 80-20 ($\chi = 0.00$)	68.5	2,413	817	19.2	80.8	67.2	32.8
Const.Amortization	65.5	2,471	896	26.8	73.2	33.9	67.1

The first two columns focus on the aggregate homeownership rate and the ownership rate for households between age 20 and 34. The next two columns examine the impact of mortgage type for housing size. We would like to know whether certain contracts results in larger homes. The last set of columns focus on issues with respect to contract holding patterns.

The model predicts that the introduction of mortgage decisions has a positive effect on ownership and housing consumption but the magnitude depends on the characteristics of the second contract. Two dimensions of mortgage loans that appear to be critical are the profile of repayment schedule and the downpayment requirement. When downpayment requirements are high, individuals benefit from using mortgage loans with a steep, increasing repayment profile since the initial cost of participating in the owner-occupied market is reduced. The increase in the aggregate homeownership rate depends on the rate of payment growth. The faster the rate of increase, the lower the initial mortgage payment, but the higher the future payment obligation. The key is whether these obligations increase faster or slower than earnings. These effects are illustrated in the various GPM (arithmetic and geometric) and the Balloon Fixed contracts. Given the model specification, the parameterization of the step in the arithmetic loan implies a steeper repayment profile initially than in the geometric case. Not surprisingly, when the repayment schedule is flatter the addition of the second mortgage loan has a smaller impact on ownership when compared to the baseline economy. For example, in the GPM contract with an arithmetic structure with a low step increase, the ownership rate increases from 63.7 percent in the baseline economy to 64.1 percent. When the profile is steeper as in the high case, the participation rate increases to 68.3 percent. A similar pattern occurs with different growth in the geometric version of the GPM contract. The flatter repayment schedule generates increases in the average size of an owner-occupied house with roughly one third of homeowners choose this type of contract. The Balloon-FRM with a 20 percent downpayment has very similar effects on homeownership. Since the initial payments are constant for a longer horizon the impact in participation are slightly larger. The average size house is slightly larger with this contract.

An alternative to loans with increasing repayment schedules and higher downpayment requirements are loans with decreasing repayments and no downpayment. The model suggests that the introduction of the combo loan choice with no downpayment has a similar effect on ownership as the in the GPM contracts denoted as 'high'. The aggregate ownership in the combo case is 68.3 and in the 68.5-68.7 percent range depending on the slope of the GPM contract. One difference between the combo contract and the increasing payments contract appears in holding patterns. The steep repayment schedule becomes very attractive for homeowners since 55 percent hold this contract, whereas in the combo loan case only 33 percent will pick this product. The rationale for the result has to be the higher initial payments in the combo loan. All the individuals that can afford to accumulate the downpayment prefer a lower loan-to-value ratio. We will discuss this issue in more detail in the next subsection.

One contract that combines the two features of increasing repayment schedule and no downpayment is the contract denoted as the balloon contract with no downpayment that rolls into a standard fixed rate contract.[25] The combination of these two features result in a high ownership rate (68.6 percent). Homeowners also purchase larger homes as the average size house increases to 2,489 square feet. This loan product is more attractive than the combo loan because the low initial payments as opposed to high, but not as attractive as the GPM-(High) that has even lower initial payments.

All the discussed contracts have the unattractive feature that the amortization of the principal is very slow. The analysis of the constant amortization contract illustrates what happens when the repayment schedule declines over time and the amortization is very fast. The model suggests that the aggregate impact in participation is small (65.5 percent) when compared with the other contracts. However, two thirds of the homeowners choose this product. This result is very interesting. In the presence of uninsurable labor income risk, mortgage contracts that accrued equity earlier allow some homeowners to reduce the utility cost of meeting the mortgage payments every period. This precautionary motive manifests as an implicit preference to have equity in the property. When we study the choice of downpayment we will find similar results, but we purposely delay the discussion to the next section. The model suggests that since the average share of mortgage payments rapidly declines over the life-cycle homeowners choose to purchase large units with this type of contract.

The introduction of mortgage decisions with nontraditional loan products reveals some interesting patterns in the number of properties that are owned free and clear of mortgage obligations. With steep repayment schedules or no downpayment the fraction of housing units without mortgages declines. The drop is mainly accounted by the effects in the patterns of buying and selling

[25] It is interesting to point out that two contracts that have played an important role for the increase in the homeownership rate in the U. S. during the period 2000-2006 are mortgage contracts that have a step function in the payment structure. These are the 80-20 contracts and the '2-28' and '3-27' contracts in the subprime market. The 80-20 product essentially uses a second mortgage to finance the downpayment thus avoiding mortgage interest rate costs. We examined this contract and find the homeownership rate increases in the aggregate and youngest age cohorts to 65.5 and 46.1 percent, respectively. A 3-27 contract involves a three year balloon contract that rolls into a fixed rate contract or a floating rate contract for the remaining 30 years. We introduced this type of contract choice into our model and find the aggregate homeownership rate increases to 70.8 percent. More startling, the homeownership rate for the youngest cohort increases to 68.0 percent.

that we will discuss in the next section. This finding seems to be consistent with the empirical evidence reported by the *American Housing Survey* suggesting that in 1993 roughly 40 percent of the housing units had no mortgage whereas in 2005 this figure had declined to 33 percent. The experiments suggests that mortgage innovation combined with the observed increase in refinancing could account for this observation.

In order to highlight the importance of introducing mortgage choice into the household problem, a single choice environment should be considered. If the model is analyzed where the standard fixed rate contract is replaced with one of the alternative contracts, we find very different results. For example, the model predicts that in a single contract economy the constant amortization mortgage would generate the highest participation rate. In contrast, a GPM with a low geometric growth rate will generate the lowest aggregate rate. A balloon contract that is rolled into a standard mortgage contract would especially hurt younger households. The constant amortization mortgage is the only contract that increases the homeownership for young households and the economy as a whole. These changes are a result of interest rate, payment pattern and downpayment changes that force all households to choose a specific product. With mortgage choice, households choose the contract that is optimal for their state.

5.1.1. Distributional Housing Market Implications

To fully understand mortgage decisions it is important to analyze its determinants at the individual level. In particular, we focus on mortgage holdings and participation rates across household's age and income. The distributional implications based on these characteristics are summarized in Tables 5 and 6.

Table 5: Age Cohort Effects of Mortgage Type

Contract Type[1]	Home Ownership Rate					% Holding FRM				
	20-34	**35-49**	**50-64**	**65-74**	**75-89**	**20-34**	**35-49**	**50-64**	**65-74**	**75-89**
Baseline(FRM)	37.5	76.5	86.4	91.3	66.5	100	100	100	100	100
FRM-GPM(Ar,L)	38.0	76.7	87.5	91.5	65.7	43.2	74.4	83.3	80.7	98.1
FRM-GPM(Ar,H)	54.9	78.1	77.4	82.8	73.3	44.0	47.8	45.4	40.8	52.2
FRM-GPM(Geo-L)	38.6	77.4	87.2	91.7	65.6	42.8	73.9	81.8	78.5	90.7
FRM-GPM(Geo-H)	47.6	81.9	86.3	88.5	71.2	34.4	52.5	51.0	44.7	55.5
FRM-Balloon($\chi = .20$)	39.3	81.0	87.8	92.4	65.7	32.1	72.3	82.3	80.0	90.0
FRM-Balloon($\chi = .00$)	58.2	76.2	74.2	85.7	68.0	46.4	49.3	52.5	47.1	62.6
FRM-Combo($\chi = .00$)	46.7	82.9	85.6	91.0	66.3	55.7	73.5	76.3	57.6	64.2
FRM-CAM	38.6	79.6	88.7	93.6	67.1	58.3	28.3	18.1	30.9	21.5

Consistent with the logic of the previous section, the model reveals that a contract with either a steep repayment schedule or no downpayment has a quantitatively significant impact on the participation rate of young cohorts in the owner-occupied market. In the case of the GPM's (High) the effects are particularly large since the initial mortgage payments are low, whereas in the case of the combo loan the effects are a bit smaller. This is a consequence of the relatively high initial payments. As one could expect, the combination of lower downpayment requirements and a steep repayment schedule has the largest impact. The importance of the second contract is clear when we explore the percent holdings in each contract. Between 55 to 68 percent of the young cohorts prefers the second contract to the conventional 30-year fixed rate mortgage with 20 percent downpayment. In general, the majority of these individuals transition into a FRM contract as they get older, accumulate wealth, and purchase another home. The option of varying the mortgage choice over the life-cycle increases the participation rate of households across the age distribution when compared to the baseline economy. For instance, the GPM with an arithmetic structure and smaller step increase is chosen by 56.8 percent of households under age, but only by 25.6 percent of households in the 35 to 49 age cohort. When the step is increased, a possible unattractive feature of this contract appears. The ownership rate for the youngest households increases to 54.9 percent with 60.3 percent of these households choosing this contract. However, the large increase in payments causes the homeownership rate to fall in the 50-64 and 65-74 age cohorts. In addition, with the rapidly increasing payment structure, households are not transitioning into FRM contracts, since they are exiting the housing market. The geometrically increasing contracts are favored by the youngest cohort. The age cohort homeownership rate with this type of increasing payment structure is similar to those found when a FRM mortgage is the only option. Other contracts such as the constant amortization product exhibit an increasing popularity over the life-cycle, specially for those households that have the resources and choose this instrument to increase their equity position in the house. Those that do not have the resources opt for the standard fixed rate mortgage since it offers relatively lower initial repayments when compared to the constant amortization.

In terms of age the model suggests a certain separation of mortgage choice. Those individuals that are more likely to receive negative income shocks but still can afford to buy a house either choose the loan with the lowest initial repayment schedule or lower downpayment. On the contrary, those that expect to receive positive income shocks tend to choose contracts that increase their equity in the house. Part of this result is due to the fact that income tends to increase over the life-cycle, even for individuals that receive negative shocks. The results are clear when we explore mortgage choice by income quintals.

Table 6: Income Distribution Effects of Mortgage Type

Contract Type[1]	**Home Ownership Rate**					**% Holding FRM**				
	Q1	**Q2**	**Q3**	**Q4**	**Q5**	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**
Baseline(FRM)	32.0	83.9	98.4	100	100	100	100	100	100	100
FRM-GPM(Ar,L)	53.4	92.3	96.6	100	100	68.2	93.7	46.5	100	100
FRM-GPM(Ar,H)	59.6	80.4	99.4	100	100	45.5	31.5	80.1	100	100
FRM-GPM(Geo-L)	53.7	92.6	96.7	100	100	67.3	91.3	41.7	100	100
FRM-GPM(Geo-H)	59.0	87.6	97.5	100	100	46.5	35.9	70.8	97.8	100
FRM-Balloon($\chi = .20$)	55.1	92.3	97.5	100	100	64.1	84.9	48.9	100	100
FRM-Balloon($\chi = .00$)	60.6	71.2	83.4	99.0	100	49.2	31.9	51.1	90.0	100
FRM-Combo($\chi = .00$)	59.9	86.0	96.5	100	100	67.5	58.8	69.0	99.9	100
FRM-CAM	54.6	93.9	97.5	100	100	33.4	20.7	61.2	14.7	0.0

There are three important results that summarize the finding in Table 6. First, the introduction of a second contract with either lower initial payments, a higher loan-to-value ratio, or a combination of both has a positive impact in the participation rate of the lowest income quintal.

Second, the majority of individuals in the two highest income quintals prefer the traditional mortgage contract, or contracts that maximize the equity in the house like in the case of the constant amortization. Third, the decline in participation for the second income quintal relative to the baseline economy in some of the GPM and the Balloon-Fixed with no downpayment can be explained by the increase in payments. Some lower income households are attracted to this products because of the low initial mortgage cost; however, those that receive negative income shocks and cannot meet the payments are forced to exit the market. As a result, the ownership rate falls in the subsequent periods. This drop is consistent with the patterns of ownership by age for the same mortgage contracts. The popularity of these contracts in the third income quintile can be rationalized by individuals that enter in the owner-occupied market after either downsizing or renting for one period. Despite the large magnitude of holdings, the total number of individuals in the third, fourth, and fifth income quintals is very small. These interesting results reveal that some of these nontraditional products can be successful in the short-run to increase the participation rate of young and poor households, but generate some visible swings in the participation rate by age and income. Fortunately, as the effects of idiosyncratic uncertainty mitigate over the life-cycle (the fraction of borrowing constrained households falls after age 40), these individuals can use the same contracts to re-enter the owner-occupied market. This is why the aggregate ownership does not fall. This findings suggests that nontraditional contracts introduce very interesting dynamics in the patterns of buying and selling.

5.1.2. Buying and Selling Implications

This subsection explores the implications of mortgage decisions for housing transactions. Table 7 reports some summary measures of housing transactions. In an environment of no mortgage choice, mobility is limited in our model. We find that only 1.7 percent of homeowners move and those who move, do so for the purposes of upsizing their house. When households are allowed to have mortgage choice, we find an increase in transaction activity. The degree of the transaction activity increase depends on the type of mortgage contract. This transaction activity appears as an increase in households moving to either larger or smaller homes, and is presented in Table 7.

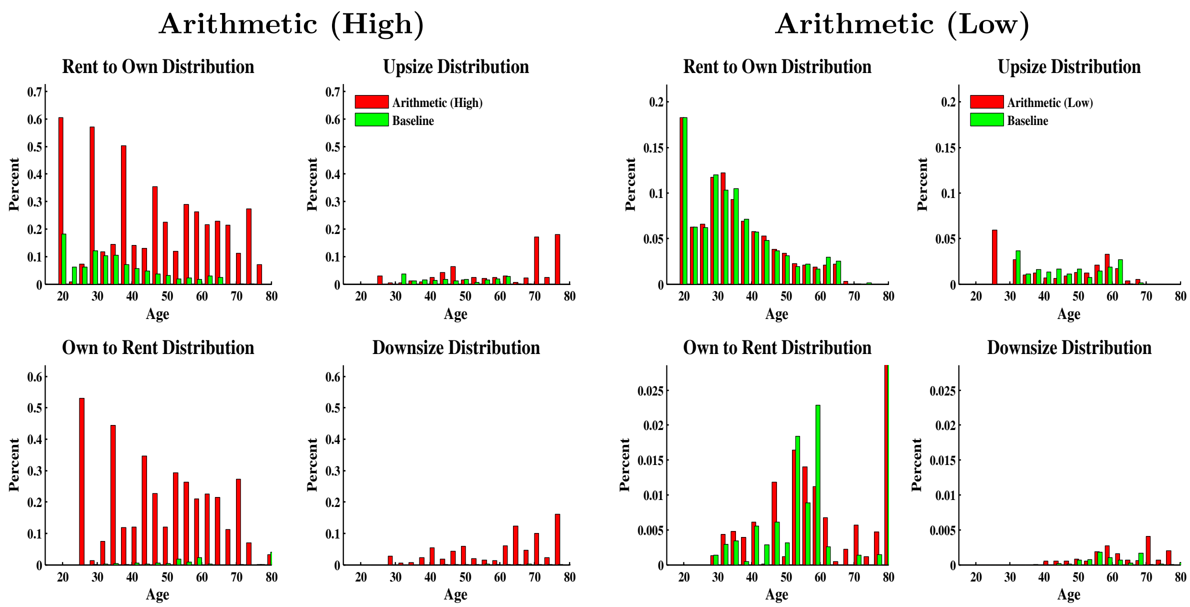
Table 7: Summary Measures for Housing Transactions

	Homeowners			Entry, Exit, and Stay		
Contract Type[1]	**Move**	**% Upsize**	**% Downsize**	**Rent to Own**	**Own to Rent**	**Rent to Rent**
Baseline(FRM)	1.7	97.0	3.0	6.0	1.5	34.8
FRM-GPM (Ar,L)	2.2	95.3	4.7	6.0	1.6	34.3
FRM-GPM (Ar,H)	12.1	46.1	53.9	23.6	19.1	12.6
FRM-GPM (Geo-L)	2.8	89.6	10.4	6.2	1.8	33.8
FRM-GPM (Geo-H)	14.0	55.2	44.8	14.6	10.1	21.1
FRM-Balloon ($\chi = .20$)	3.5	84.6	15.3	6.4	1.9	32.2
FRM-Balloon ($\chi = .00$)	17.8	30.9	69.1	25.5	21.0	10.3
FRM-Combo ($\chi = .00$)	8.7	64.0	36.0	11.0	6.4	25.1
FRM-CAM	3.8	82.1	17.9	6.5	2.0	32.5

The table presents statistics on the fraction of homeowners who choose to change their housing status, as wells as statistics that measure entry and exit decisions. We find two insights from the results. First, the introduction of nontraditional loans - those with low initial payments, low downpayments, or both - have an important impact on mobility. Two contracts, the GPM-Arithmetic (High) and the Balloon-Fixed with zero downpayment, particularly stand out and the combo loan product to a lesser extent. With these contracts, the fraction of homeowners who move is much greater than with the other contracts. In addition, with more flexible contracts the probability of downsizing appears much larger. This suggests that some households purchase large housing units given the relatively low initial financing costs. Those individuals that cannot afford the increase in mortgage obligations are force to downsize or sell. That leads to the second important finding. In the baseline model there is a relation of around 4 to 1 between renters that move into ownership and homeowners that have to sell the house and rent. With the introduction of these low financing products this relation becomes almost 1 to 1. For example, in the Balloon-Fixed contract with zero downpayment, 25.5 percent of the individuals transit from rent to own, and 21 percent transit from own to rent. In the baseline economy these number are 6 and 1.5 percent, respectively. In addition, the number of individuals that remain contiguous renters decreases with nontraditional contracts.

These results are consistent with the observation that the second contract attracts young and poor individuals to participate in the owner-occupied market. One way to illustrate this feature is to present the measures of housing transactions by age. In Figure 2, we present the mobility patterns for GPM contracts with both an arithmetic and geometric payment structure. For each contract, the left two graphs represent household movements from renting into ownership and from ownership to renting. The right size reports upsizing and downsizing activity. Two facts stand out with respect to a GPM contract with an arithmetic payment structure. First, entry into the homeownership state occurs much earlier and in a larger magnitude with this type of GPM contract as compared to the baseline economy. Second, many households that cannot afford the rising mortgage payments become renters a few periods after buying.

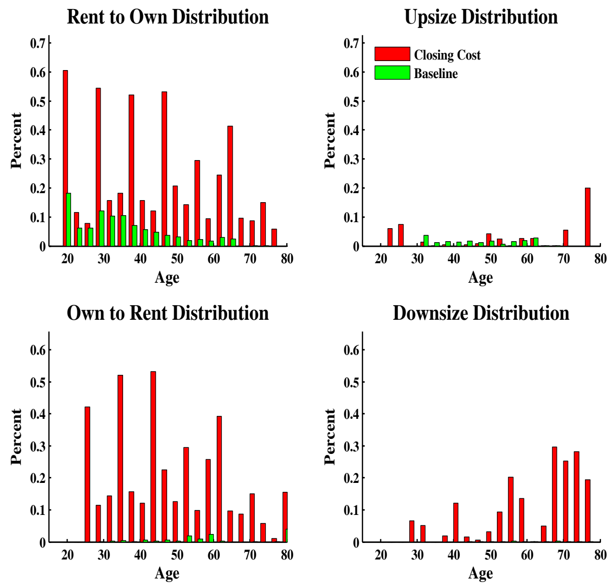
Figure 2: Mobility Patterns with GPM



The distribution of owner into the rental position in the Arithmetic (High) shows that on average 25 to 30 percent of the homeowners cannot afford the increase of mortgage payments after 9 years. The interesting feature is the decline in this pattern over the life-cycle; this occurs because most young individuals have very little equity in the house and they opt to sell. As income rises and homeowners accrue equity in the house and this fraction diminishes. The movements in and out of the housing market are very different when the slope of the repayment schedule is very flat. Despite the change in the scale in the vertical axis, we observe that the distributional patterns in Arithmetic (Low) are very similar to the baseline economy. With a relatively flat repayment schedule most homeowners can afford to meet the increasing payments over the length of the mortgage and the fraction of individuals forced to sell is substantially reduced.

To complete the analysis, Figure 3 presents the mobility patterns for the Balloon-Fixed contract with no downpayment. We observe a significant increase in entry into the housing market followed by a movement out of the market. However, the distribution of individuals that move from ownership to rental is humped-shaped as opposed to declining as in the previous example. This feature is a result of the timing of increase in mortgage payments that increase after 9 years as opposed to every period. After this period, mortgage payments increase from being interest-only to include 100 percent of the house value and the full amount of the closing costs. The movement out of the housing market that appear with this contract is of interest as this contract has similar features to some of the subprime mortgages contracts. In our model, households sell their home to get out of their mortgage contract rather than foreclose. In reality, some of these sales may turn out to be foreclosures.

Figure 3: Mobility Patterns with Balloon-FRM with No Downpayment



5.1.3. Risk Sharing and Aggregate Implications

In economies with incomplete markets and long-term mortgage loans, the introduction of new contracts has some interesting effects on risk sharing that differ from the standard durable good model with a one-period ahead collateralized loan.[26] In the standard model individuals can mitigate labor income risk by changing the house size so consumption can be smoothed. This is possible because the financial obligations do not have long term effects. In models with long-term contracts the decision to purchase a house imposes the obligation to pay the mortgage loan at every period. Consequently, the house payment reduces disposable income and, in the presence of negative income shocks, individuals lose part of the ability to smooth consumption.[27] The introduction of mortgage decisions allows households to choose the housing finance that maximizes their expected discounted utility. For some wealthy individuals with

[26] We have in mind a model where there are no transaction costs and housing wealth ph' and financial wealth $(1+r)a'$ can be summarized by a single state variable such as cash on hand:

$$x' = ph' + (1+r)a',$$

and where the period budget contraint is defined by

$$c + ph' + a' = w + x.$$

and the mortgage constraint is

$$a' \geq -(1-\chi)ph'.$$

[27] In our model ownership provides an alternative mechanism to smooth consumption. Homeowners can pay a fixed cost and supply rental property in the market. They can use the additional rental income to cover the cost of mortgage payments. However, this mechanism is costly.

positive income shocks, this implies contracts that maximize the equity in the house. For low income or young individuals the optimal choice is a contract with increasing payment over the length of the mortgage or high loan-to-value ratios. The result should be a reduction in the variance of consumption for homeowners, but not necessarily in the variance of housing since some of these mortgage loans force some individuals in and out of the housing market.

We can study these effects by computing the coefficient of variation of consumption and housing services for the various mortgage contracts. We also explore the aggregate impact of mortgage decisions by computing the percent change in aggregate output and consumption with respect to the baseline economy. From Table 8 we see that the benchmark economy generates a coefficient of variation of consumption that is 0.113 with renters having a larger coefficient than homeowners. Our measure of variance indicates that the consumption of housing services is 0.487 with renters once again having a larger variance as compared to owners. By themselves these numbers do not have much meaning since they depend on the measurement unit, but the relative numbers indicate whether new contracts allow households to better smooth consumption.

Table 8: Effects of Mortgage Choice on Risk Sharing

			Coefficient of Variation					
	Percent Change Aggregate[1]		Consumption			Housing		
Simulation	**Consumption**	**Output**	**Total**	**Owner**	**Renters**	**Total**	**Owner**	**Renters**
Baseline with FRM	-	-	0.113	0.088	0.293	0.487	0.291	3.130
GPM-Arithmetic (Low)	-0.3%	-0.2%	0.115	0.091	0.285	0.507	0.305	3.149
GPM-Arithmetic (High)	2.2%	3.3%	0.098	0.077	0.162	0.474	0.271	1.421
GPM-Geometric (Low)	-0.4%	0.1%	0.112	0.087	0.292	0.504	0.302	3.217
GPM-Geometric (High)	0.21%	2.1%	0.102	0.076	0.228	0.395	0.200	2.318
FRM-Balloon($\chi = .20$)	-0.3%	0.5%	0.116	0.086	0.357	0.515	0.303	3.978
FRM-Balloon($\chi = .00$)	4.7%	5.2%	0.094	0.079	0.143	0.478	0.313	1.040
FRM-Combo($\chi = .00$)	0.1%	1.0%	0.112	0.085	0.276	0.396	0.206	3.114
FRM-CON	0.1%	0.1%	0.111	0.084	0.336	0.482	0.290	3.578

[1]Measures percentage change with respect baseline economy

In general, we find that the introduction of mortgage choice reduces the coefficient of variation of consumption for homeowners. The reduction is especially important for contracts that allow for a steep profile of repayment and/or a low downpayment. The reduction in the variability of consumption for renters is an artifact of the general equilibrium effects that reduce the equilibrium price of rental-occupied housing. Interestingly enough, the model predicts that the contracts that allow for a larger number of transactions in the housing market, also result in smoother consumption of housing services. The sizeable entry and exit decisions observed in the Artithmetic (High) and the Balloon-Fixed contract with no downpayment allow households to reassess the optimal house size, thus reduce consumption of housing services. The introduction of mortgage decisions is far from neutral. The aggregate impact on production depends on the specifics of mortgage contracts. Inspection of Table 9 suggests that the contracts that have a large impact in the aggregate ownership rate have the largest impact in economic activity. Most of this increase can be traced by the increase in the stock of housing (not in the table). For example, with the Arithmetic (High) the stock of housing increases around 6 percent- a value this is similar in magnitude to the housing stock when a combo loan is available and twice the size with a constant amortization contract. However, the increase in aggregate activity does not necessarily generate an increase in aggregate consumption. This only occurs in the simulations where the increase in aggregate output is sufficiently high. Since the model specification assumes that consumption and housing services are imperfect substitutes, the introduction of mortgage choice allows households to buy larger homes (see Table 4) at the expense of consumption. When the income effects are sufficiently large, we observe an increase in the consumption of goods and housing services.

5.2. Downpayment Decisions

As we have seen in the previous section, downpayments are a critical element of mortgage choice since it determines the initial level of equity in the property and the level of the repayment schedule. The empirical evidence reported in Table 9 suggests that there is a large heterogeneity in downpayment choices.

Table 9: Downpayment Choice by Loan Originator

	1995		2001		2003	
Loan Originator	**FHA**	**Others**	**FHA**	**Others**	**FHA**	**Others**
First-time buyer	21.6	29.8	18.1	24.5	16.3	24.1
Repeat buyer	22.0	33.3	22.4	29.1	26.5	28.5
Total	23.2	33.5	19.9	27.4	22.6	27.0

Source: American Housing Survey (AHS)

For example the downpayment choice of individuals that choose government subsidized loans (FHA) choose lower downpayments when compared to non-FHA loans. The downpayment choice for repeated buyers appears to be larger than for first-time buyers. The purpose of this table is to illustrate the large differences in choices, and not to account for the decline in downpayments that is partially due to the introduction of private mortgage insurance (PMI) in the late 1990s and nontraditional loans in the early 2000's.

With incomplete markets this choice becomes relevant, since households that are not downpayment constrained might prefer a lower loan-to-value ratio by reducing the size of the loan. A large downpayment choice χ in a fixed rate mortgage loan reduces the magnitude of the repayment schedule $m(x,z)$ by a factor $\frac{\partial m(x,z)}{\partial \chi} = -\frac{r^m D_0}{[1-(1+r^m)^{-N}]}$. This choice reduces consumption today but increases consumption in the future. The consumer has to consider both the immediate impact as well as for the whole length of the mortgage. This decision with long term contracts differs from the standard housing model that uses a one-period ahead collateralized loan. In this model the downpayment constraint is only relevant for constrained agents. With long-term contracts all the individuals have to balance the cost and benefits of the downpayment choice.

We explore the importance of loan-to-value choice by solving the model with different downpayment choices in a fixed rate setting. In the first experiment, households may choose between a FRM with 20 and 30 percent downpayment. In the second experiment, we add a 10 percent downpayment FRM to the choice problem. The additional downpayment choice should allow for greater consumption smoothing. The aggregate results from the model are summarized in Table 10.

Table 10: Aggregate Effects of Downpayment Choice

	Baseline	20 -30% Downpayment Economy	10-20-30% Downpayment Economy
Homeownership Rate	63.7	65.3	68.5
Share Homeowners with			
No Mortgage	30.2	26.8	9.65
Mortgage	69.8	73.2	90.35
1) 10% Down	-	-	36.7
2) 20% Down	100	19.6	5.6
3) 30% Down	-	80.4	57.7
Homeowners move	1.7	3.9	15.4
Percent upsize	97.0	86.6	60.1
Percent downsize	3.0	13.4	39.9
Homeowners do not move	98.3	96.1	84.6
Renters do not move	34.8	32.7	22.3

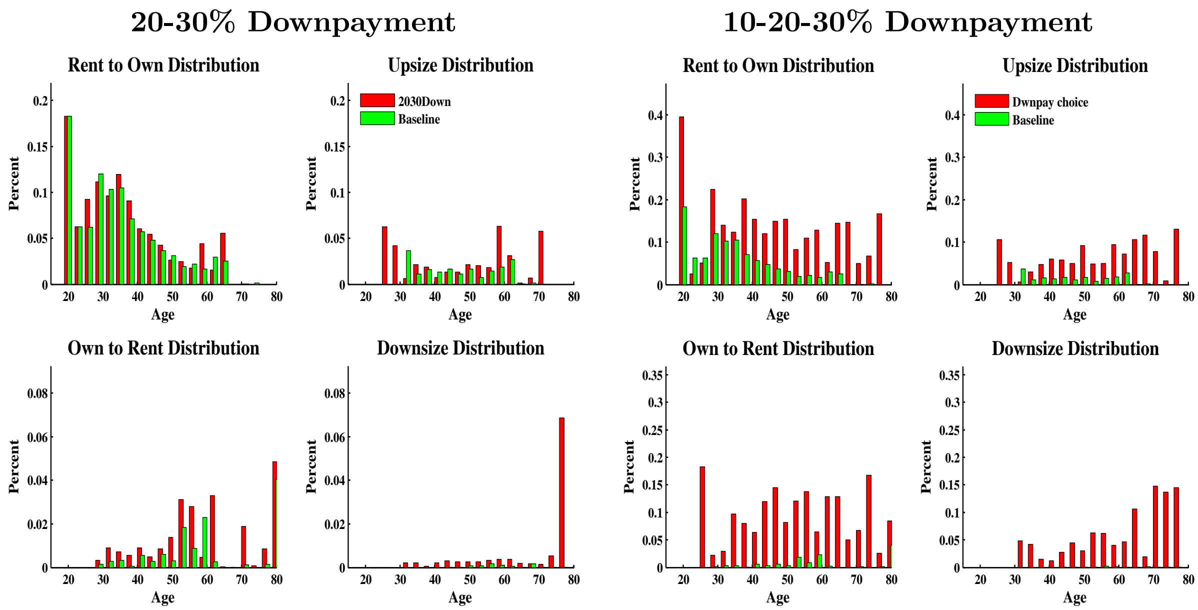
The model predicts that a choice over downpayments can have important quantitative effects for the homeownership rate. By allowing a household a choice between a FRM with a 20 or 30 percent downpayment requirement, the aggregate ownership rate increases by 1.6 basis points over the no choice baseline economy. The interesting finding is that most households, 80 percent, prefer the contract with the larger downpayment. Households who prefer the 20 percent downpayment contract are in the 20-34 and 35-49 age cohorts or are in the lowest income quintal. We do find that nineteen percent of 30 percent downpayment mortgages are held by the youngest age cohort. The attractiveness of the contract with the larger downpayment is not restricted to repeat home buyers.

A simple way to test the impact of expanding the set of downpayment choices on consumption smoothing is to calculate the coefficient of variation of consumption of goods in both economies. The addition of an additional contract reduces this coefficient from 0.113 to 0.110. These values do not separate homeowners from renters. If we condition on housing tenure, the variation in consumption drops 5.6 percent from 0.088 to 0.083 for homeowners. The introduction of additional choice increases the continuation value function for all households since it allows more flexibility in the later stages of the life cycle. The increase in downpayment options has implications on housing transactions. As can be seen in Table 10, transaction activity doubles. The addition of a second mortgage again reduces the fraction of households without a mortgage.

The fact that some households are not able to choose mortgages with a lower downpayment may understate the risk sharing benefits. As a result we add a third FRM contract with 10 percent downpayment into the choice problem. The model indicates that the addition of this third contract generates a large increase in the homeownership rate. The homeownership rate increases to 68.5 percent as compared to 63.7 in the benchmark economy and 65.8 in the two contract choice environment. The larger set of downpayment choices, $\chi \in \{0.10, 0.20, 0.30\}$, has some interesting implications for mortgage choice. We find that 37 percent of homeowners choose a 10 percent downpayment, 6 percent a 20 percent downpayment, and 58 prefer a 30 percent downpayment. The explanation for this choice pattern becomes transparent by looking across the income distribution. A mortgage contract with a ten percent downpayment is the contract of choice for individual's in the lowest two income quintals. Of the households who choose this contract, 96.5 percent are held by households in the lowest two income quintals. Relatively few households hold the 20 percent product. Of those who do hold this product, 66.8 percent are in the lowest income quintal. Higher income households seem to prefer a low loan-to-value ratio to smooth consumption.

The addition of a mortgage contract with a lower downpayment generates additional risk sharing benefits as the coefficient of variation declines to 0.10 and the level of consumption increases. Compared to the reduction in variance when the 30 percent downpayment is introduced into the choice problem, the addition of a ten percent contract generates a larger benefit. The benefit is captured by homeowners rather than renters as the variance of consumption falls for homeowners, but increases for renters. The addition of this third contract results in an additional 6 percent decline in the variance of the consumption for homeowners as compared to the two downpayment choice environment.

Figure 4: Mobility Patterns with Downpayment Choice



Downpayment options do have mobility implications for homeownership. Figure 4 presents mobility patterns when a 20 and 30 percent downpayment choice is available and then mobility patterns when the 10 percent downpayment option is introduced. The increase in mobility that occurs with the addition of the 10 percent contract allows many more households to enter the market. The lower downpayment option generates a larger pattern of buyers early in the life-cycle, but at the same time it also increases the fraction of individuals that have to sell the property and become a renter as well as the fraction of individuals that downsize to a smaller house. This finding is important because it shows that even in the absence of aggregate shocks that change prices, a financial innovation that reduces the minimum downpayment requirement can result in increased turnover in the housing market.

6. Refinancing and Mortgage Choice

In recent years the number of homeowners that have refinanced their home has increased. Homeowners have several motives for refinancing their mortgages. These include better financing opportunities through either lower interest rates or by changing the terms of the mortgage. In addition, some homeowners extract some equity from the property to finance consumption. In our model the interest rate is fixed, and households only realize the idiosyncratic capital gains when they choose to sell the property, so why would they choose to refinance? In the presence of uninsurable income risk and borrowing constraints, some households that receive negative income shocks could benefit by switching to an alternative mortgage contract that allows income risk to be smoothed.[28] Refinancing allows households to reset their mortgage clock, and/or change their equity position in the property. In a sense, refinancing in our model is driven by consumption smoothing through mortgage contracts. The effects of consumption smoothing will depend on the cost of refinancing.

In this section, we allow a homeowner to refinance their mortgage position. This option allows a household to choose a new mortgage contract from the set of contracts $\mathcal{Z}$ rather than continuing on with the existing contract. To accommodate this decision we need to modify the household decision problem along two dimensions. One is to allow an existing homeowner to change to a different mortgage contract while maintaining their current housing investment position. The other modification is to allow a homeowner to change the equity balance in the property when changing the mortgage contract. Formally:

- **Homeowners maintains existing mortgage** $(z = z')$: Households always have the option of continuing with their existing contract. For households that maintain the same mortgage contract the value function is:

$$v_1(a,h,n,z,\epsilon,j) = \max_{\substack{(c,s,a',h')\in\mathbb{R}_+ \\ I_r\in\{0,1\}}} \left\{ u(c,s) + \beta\psi_{j+1} \sum_{\epsilon'\in\mathcal{E}} \pi(\epsilon,\epsilon') v(a',h',n-1,z,\epsilon',j+1) \right\}$$

$$\begin{aligned} s.t. \quad c + a' + m(x,z') + x(h',s) &= y(a,h',s,\epsilon,\upsilon_j,j;q) + I_r\left[R(g(h')-s) - \varpi\right] \\ s &\le h', \end{aligned}$$

where $n' = \max\{N-1, 0\}$. In this situation, the household must make a mortgage payment if $n > 0$. Again, it is important to remark that the decision to consume housing services and maintenance expenses depends on the choice of paying a fixed cost ϖ to become a landlord. The value function $v_1(a,h,n,z,\epsilon,j)$ denotes the optimal value associated with the continuation of the existing mortgage contract.

- **Homeowners refinance** $(z \neq z')$: Households that choose to refinance and maybe change their equity position must solve

$$v_2(a,h,n,z,\epsilon,j) = \max_{\substack{(c,s,a',h')\in\mathbb{R}_+ \\ z'\in\mathcal{Z},\ I_r\in\{0,1\}}} \left\{ u(c,s) + \beta\psi_{j+1} \sum_{\epsilon'\in\mathcal{E}} \pi(\epsilon,\epsilon') v(a',h',N(z)-1,z',\epsilon',j+1) \right\}, \tag{6.1}$$

[28] In most housing models with uninsurable income risk and one-period mortgage contracts, the household objective is to build an optimal h/c ratio. Since accumulating the optimal house size takes time, homeowners increase their housing stock in periods with good income shocks by borrowing. In the presence of negative income shocks, households delay their debt repayment and do not adjust their housing consumption h'. This mechanism allows households to maintain a smooth consumption profile. Our mechanism is somewhat different. Homeowners can obtain their desired house size by borrowing long-term, however, with negative income shocks since $m(x,z)$ and $s = h'$ are fixed, homeowners have to either reduce c, or pay to become a landlord $I_r = 1$ and receive supplemental rental income.

$$s.t. \quad c + a' + \chi(z')ph' + m(x, z') + x(h', s) = y(a, h', s, \epsilon, \upsilon_j, j; q) + I_r \left[R(h' - s) - \varpi\right] + [ph - D(n, z)],$$
$$s \leq h'.$$

where $v_2(a, h, n, z, \epsilon, j)$ is the value function associated with the z' that generates the greatest value function from the set of mortgage contracts $\mathcal{Z}$. This individual is effectively taking a new loan for the amount $\chi(z')ph'$ and can pull out equity amounting to $[ph - D(n, z)]$. The net cost of these two different terms determine whether the homeowners are paying off their house faster, using some of the equity in the house to increase consumption, or just changing the mortgage contract to have a longer maturity.

The refinancing decision can be expressed as:

$$v(a, h, n, z, \epsilon, j) = \max[v_1(a, h, n, z, \epsilon, j), v_2(a, h, n, z, \epsilon, j)].$$

Clearly, if $v_2(a, h, n, z, \epsilon, j) > v_1(a, h, n, z, \epsilon, j)$ then refinancing occurs. In our formulation, refinancing is not subject to capital gains shocks. This assumption is done for computational purposes, so we do not have to keep track of changes in the equity of the property that differ from the housing stock[29] This mechanism provides an additional margin to smooth temporary negative income shocks.

Given the strong risk sharing generated from downpayment choice, we focus our attention on the importance of refinancing in the context of downpayment choice. Following the previous section, we assume that households have access to three different downpayment choices $\chi = \{0.10, 0.20, 0.30\}$, but at any point in time they can change the mortgage loan to one with a different downpayment. We assume the cost of refinancing is one percent of the loan value.

Table 11: Aggregate Effects of the Introduction of Refinancing

Simulation	Down Payment	Ownership Total	Ownership 20-34	Housing Size Mean	Housing Size Variance	Properties no Mortgage	Share Downpayment 10%	Share Downpayment 20%	Share Downpayment 30%
Baseline with FRM	20%	63.7	37.5	2348	816.0	30.2	0.0	100.0	0.0
No refinancing	10-20-30%	68.6	46.4	2541	949.7	9.4	36.7	6.0	57.8
Refinancing	10-20-30%	69.4	46.8	2554	952.9	11.3	35.7	5.7	58.6

The introduction of refinancing has additional positive effects on ownership. We find that the homeownership rates increase from 68.6 percent to 69.4 percent when households have access to refinancing options. For households in the 20-34 age range, homeownership rates increase from 46.4 to 47.6 percent. This suggests that in the absence of interest rate risk (or interest rate movements), the option to purchase a house with a low downpayment is more important for ownership than the refinancing option for first time buyers. From a distributional perspective,

[29] This assumption prevents households from pulling out equity associated with capital gains. Since in the model we do not have aggregate shocks, the transitory shocks should not have an effect on the homeowners ability to take on more debt.

the refinancing option does not seem to have implications for the mortgage contract choice. We find that households who receive positive income shocks refinance their mortgage position by choosing mortgage contracts with lower loan-to-value ratios. This behavior appears two ways. First, the number of properties without a mortgage increases by 2 percent when refinancing is allowed. Second, the final column of Table11 reveals that the share of homeowners that choose 30 percent downpayment increases when refinancing is an option. Households are showing a tendency of wanting to build equity or pay off their mortgage debt quickly.

Table 12: Mortgage Switching Matrix

	Future Mortgage Choice			
Current Mortgage Choice	**10%**	**20%**	**30%**	**Total**
10 percent	1.2	5.9	36.0	43.1
20 percent	2.7	11.7	1.6	16.0
30 percent	0.0	4.1	36.9	41.0
Total	3.9	21.7	74.5	100

To understand the dynamics of refinancing, we look at the mobility patterns of mortgage switching. The model predicts that 34.8 percent of the homeowners and roughly 50 percent of the households that stay in the house choose to refinance. The distribution of refinancing suggests that the majority of homeowners move to options with higher downpayments, and only a very small fraction of households choose to move into a mortgage with a lower downpayment requirement. The mobility matrix is summarized in Table 12 where the rows indicate the original mortgage type, z, while the columns denote the new mortgage type chosen, z'. The diagonal values in each matrix represent the fraction of homeowners that choose to maintain the existing mortgage loan. In the model households are allowed to refinance using the same mortgage contract, as a result the diagonals could contain two different types of individuals. However, the mass of individuals that refinance using the same mortgage is zero in the model. For example, consider the 43.1 percent of the homeowners who choose to refinance, starting with a mortgage that has a ten percent downpayment. Of these households, over 75 percent choose to refinance with a 30 percent downpayment. This result indicates that younger and poorer households use a low downpayment mortgage to enter the housing market, and then refinance with loans that have a higher downpayment requirement and lower payments. The 16.0 percent of households who choose to refinance with a twenty percent downpayment tend to be the only cohort that does not favor the 30 percent downpayment. They tend to simply reset their mortgage with the same downpayment to gain access to equity. The large number of households who start with a thirty percent downpayment mortgage and refinance to a similar contract are also likely gaining access to equity.

Table 13: Summary Measures for Housing Transactions

Contract Type[1]	Homeowners Move	% Upsize	% Downsize	Entry, Exit, and Stay Rent to Own	Own to Rent	Rent to Rent
Baseline(FRM)	1.7	97.0	3.0	6.0	1.5	34.8
No refinancing	15.9	59.3	40.7	13.7	9.1	22.3
Refinancing	14.4	61.5	38.5	13.1	8.4	22.2

The presence of a refinancing option offers homeowners an alternative way to smooth positive and negative income shocks as they can better use the equity on the property as partial insurance against income risk. This is especially important for those individuals that purchase a property using their asset holdings to meet the downpayment requirement. These individuals are likely to be more vulnerable to risk. In the absence of refinancing, a negative income shock might force a household to sell their the house, or pay the fixed cost and rent the property. With refinancing, an existing homeowner can extract equity from the house or change the terms of the mortgage and avoid selling the property. As a result, mobility should be reduced. However, the option of refinancing should make owner-occupied housing more attractive to a prospective homeowner, thus increasing mobility. Table 13 examines how housing transactions are affected by refinancing. The model finds that refinancing reduces mobility as the number of homeowners who do not move increases. Of the households who do move, downsizing activity is reduced but some upsizing of houses does occur. This is due to the fact that refinancing provides some insurance to households that choose to buy larger houses. In addition, we observe that the fraction of individuals that choose to exit the housing market is reduced roughly 8 percent. Since individuals leave the ownership state less often than the fraction of individuals that transit in, mobility is also reduced. This suggests that most homeowners use refinancing as an insurance against negative income shocks. Next, we decompose the effects of refinancing by looking at the variability of consumption and housing services.

Table 14: Effects of Mortgage Choice on Risk Sharing

Simulation	Consumption Growth[1]	Coefficient Variation: Consumption Total	Owner	Renters	Housing Total	Owner	Renters
Baseline with FRM	-	0.11	0.09	0.29	0.49	0.29	3.13
No Refinancing	0.45%	0.10	0.08	0.25	0.45	0.25	2.74
Costly Refinancing	1.00%	0.10	0.08	0.26	0.42	0.24	2.82

[1]Percentage change in aggregate consumption with respect baseline

Table 14 measures the consumption smoothing benefits from refinancing. The effects of refinancing in the coefficient of variation in consumption and housing could fall depending on the cost. In the absence of refinancing, but including downpayment choices, the coefficient of variation in consumption is reduced from 0.11 to 0.10. If we condition on ownership, the reduction is of a similar magnitude. The introduction of refinancing has no quantitative effect in the variation of consumption, but it reduces the dispersion in housing consumption. For the average individual the reduction is 14 percent and 17 percent for homeowners. This is mainly accounted for by the findings of Table 13 that suggest that the introduction of refinancing reduces the fraction of individuals that are forced to leave the owner-occupied housing market. Since housing is a luxury good, in the model, it appears that individuals receive a larger payoff by smoothing the consumption of this good. This can be partially explained by the preference specification that assumes that consumption and housing services are imperfect substitutes. With a different elasticity between consumption goods and housing services, or with a different specification of preferences (i.e. homothetic preferences for consumption and housing) the adjustment might be similar in both commodities. However, the objective of the paper is to illustrate the impact of refinancing in mortgage choice, and not to measure the impact in risk sharing. The transmission mechanism in the aggregate economy suggests that refinancing should increase the aggregate consumption between 1 to 1.5 percent when compared to the baseline economy and between 0.5 to 1 percent when compared to the downpayment choice baseline. That suggests that the effects of refinancing are large even in the absence of interest rate movements and house appreciation.

7. Conclusions

A goal of current U.S. housing policy is to increase the homeownership rate. One tool used to achieve this goal has been the reduction in financial restrictions which has lead to greater flexibility in mortgage contracts offered in the market. This paper explores the implications of several different mortgage contracts for tenure and housing investment decisions, and thus the homeownership rate. The analysis was conducted using a quantitative equilibrium model with heterogeneous consumers and liquidity constraints. Our life cycle model is characterized by considering the housing decision as part of the portfolio decision, and allowing households to make discrete choices of whether to own, rent or lease.

The model presents several contributions in the literature of mortgage choice that mainly uses complete markets models to determine the choice between FRM and ARM. In the paper we show that the introduction of long-term mortgage decisions has a positive effect on ownership and housing consumption, but the magnitude depends on the profile of the repayment schedule and the downpayment. We find that contracts with increasing repayment profiles generate similar aggregate effects as that of no downpayment loans. However, the distributional effects on participation and housing transactions are different.

We find that the optimal mortgage choice varies across many individual dimensions such as income and age. For example, when downpayments requirements are high, young and poor individuals benefit from using mortgage loans with an increasing repayment profile since the initial cost of purchase a house is reduced. By contrast, individuals with more income or of an older age prefer mortgages with high downpayment and fast amortization of the principal. We show that a very close substitute to loans with increasing repayment schedules for young and poor individuals are loans with no downpayment and decreasing repayments. The model predicts that the choice of nontraditional loans with low initial payments or no downpayments has an important impact in mobility and it also reduces consumption smoothing.

Our analysis suggests a number of extensions that we are presently investigating. In our model homeowners that cannot afford to meet the payments without violating the non negativity constraint in consumption and are forced to sell. However, this finding suggests a model that allows foreclosures when equity is less that the remaining mortgage debt could be useful to understand episodes of housing default. This is especially true in an environment with stagnant or declining house prices. The model also suggests that mortgage choice is a very complicated subject, and while it would be very useful to have a positive theory of mortgage contracts, the heterogeneity in the decisions as well as the multiple dimensions that mortgage contracts have (i.e., downpayment, amortization schedule, repayment profile) makes it difficult.

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8. Appendix: Definition of Recursive Stationary Equilibrium

We restrict ourselves to stationary equilibria. The individual state variables are asset holdings, a, housing investment holdings, h, mortgage contract type, z, mortgage status, n, labor productivity status, ϵ, and age. The individual state of the economy is completely described by the joint measure Φ over asset positions, housing investment positions, mortgage contract type, mortgage status, productivity state, and age where $\Lambda = (a, h, z, n, \epsilon, j)$. Let $a \in \mathcal{A} \subset \mathbb{R}_+$, $h \in \mathcal{H} \subset \mathbb{R}_+$, z$\in \mathcal{Z} \subset I, n \in \mathcal{N} = (1, 2, ..., N) \in I$, $\epsilon \in \mathcal{E} = \{\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5\} \subset I$, $j \in \mathcal{J} = (1, 2, ..., J) \subset I$, and let $\mathcal{S} = \mathbb{R}_+ \times \mathbb{R}_+ \times \mathcal{Z} \times \mathcal{N} \times \mathcal{E} \times \mathcal{J}$.

Definition (Stationary Equilibrium): Let define I_s to be an indicator function that is equal to one when a housing investment position is sold and zero otherwise. Given a set of time-invariant fiscal policy arrangements $\{G,\tau_y(\eta_0, \eta_1, \eta_2), \tau_p(\overline{\theta})\}$, and initial conditions, *a stationary equilibrium is a collection of value functions,* $v(a, h, z, n, \epsilon, j,)$: $\mathcal{A} \times \mathcal{H} \times \mathcal{Z} \times \mathcal{M} \times \mathcal{E} \times \mathcal{J} \to \mathbb{R}$; *and decision rules for the household,* $\{a', h', z', c, s : S \to \mathbb{R}_+\}$ if $\mathrm{I}_s = 0$ or $\{a'_\xi, h'_\xi, z'_\xi, c_\xi, s_\xi : S \to \mathbb{R}_+\}$ if $\mathrm{I}_s = 1$, aggregate outcomes $\{K, N\}$; *prices* $\{r, p, R, r^m\}$; *stationary population and invariant distribution* $\Phi(a, h, z, n, \epsilon, j)$ *such that*

1. *Given prices,* $\{r, p, R, r^m\}$, *policies, transfers, and initial conditions, the value function* v *and decision rules* c, s, a', *and* h' *solve the consumer's problem as specified in equations (3.4), (3.5), (3.6), (3.7), and (3.8).*
2. Transfers are defined in equation (3.11).
3. The asset market is defined by equation (3.14) clears.
4. The rental market as defined by equation (3.15) clears.
5. The goods market condition is defined as:

$$C + K' - (1 - \delta)K + I_H + G + \Upsilon = F(K, N)$$

where C, $K' - (1 - \delta)K$, I_H, G, Υ represent aggregate consumption expenditures, aggregate investment in fixed capital, aggregate investment in housing goods, government expenditure, and aggregate total transaction costs. These variables are equal to:

$$C = \int_{I_s(\Lambda)=0} \mu_j c(\Lambda)\Phi(d\Lambda) + \int_{I_s(\Lambda)=1} \sum_{\xi\in\Xi} \pi_\xi \mu_j c_\xi(\Lambda)\Phi(d\Lambda)$$

where I_H represents the investment housing goods,

$$I_H = \int_{I_s(\Lambda)=0} \mu_j h'(\Lambda)\Phi(d\Lambda) + \int_{I_s(\Lambda)=1} \sum_{\xi\in\Xi} \pi_\xi \mu_j h'_\xi(\Lambda)\Phi(d\Lambda)$$

$$- [\int \mu_j h(\Lambda)\Phi(d\Lambda) - [\delta_o(\int_{\substack{I_s(\Lambda)=0 \\ s(\Lambda)\geq h'(\Lambda)}} \mu_j h'(\Lambda)\Phi(d\Lambda) + \int_{\substack{I_s(\Lambda)=1 \\ s(\Lambda)\geq h'(\Lambda)}} \sum_{\xi\in\Xi} \pi_\xi \mu_j h'(\Lambda)\Phi(d\Lambda))$$

$$- \delta_r(\int_{\substack{I_s(\Lambda)=0 \\ s(\Lambda)<h'(\Lambda)}} \mu_j h'(\Lambda)\Phi(d\Lambda) + \int_{\substack{I_s(\Lambda)=1 \\ s(\Lambda)<h'(\Lambda)}} \sum_{\xi\in\Xi} \pi_\xi \mu_j h'(\Lambda)\Phi(d\Lambda))]$$

and Υ denotes resources allocated to total transaction and fixed costs,

$$\Upsilon = \int_{I_s(\Lambda)=0} \mu_j \phi_B h'(\Lambda)\Phi(d\Lambda) + \int_{I_s(\Lambda)=1} \sum_{\xi\in\Xi} \pi_\xi \mu_j \phi_B h'(\Lambda)\Phi(d\Lambda)$$

$$+ \varpi \int_{\substack{I_s(\Lambda)=0 \\ I_r(\Lambda)=1}} \mu_j \Phi(d\Lambda) + \varpi \int_{\substack{I_s(\Lambda)=1 \\ I_r(\Lambda)=1}} \sum_{\xi\in\Xi} \pi_\xi \mu_j \phi_B \Phi(d\Lambda)$$

6. The labor market clears where labor demand, as determined by the firm's first order condition, is equal to labor supply.

7. The general government balances as specified by equation (3.9).

8. The social security program is self-financing with the tax rate determined by equation (3.10).

9. Letting T be an operator which maps the set of distributions into itself aggregation requires

$$\Phi'(a', h', z, n-1, \epsilon', j+1) = T(\Phi),$$

and T be consistent with individual decisions. We will restrict ourselves to equilibria which satisfy:

$$\Phi' = T(\Phi)$$

where the function $T : \mathcal{M} \rightarrow \mathcal{M}$.