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Home Bias in Financial Markets: Robust Satisficing with Info Gaps

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Abstract: The observed patterns of equity portfolio allocation around the world are at odds with predictions from a capital asset pricing model (CAPM). What has come to be called the "home-bias" phenomenon is that investors tend to hold a disproportionately large share of their equity portfolio in home country stocks as compared with predictions of the CAPM. This paper provides an explanation of the home-bias phenomenon based on information-gap decision theory. The decision concept that is used here is that profit is satisficed and robustness to uncertainty is maximized rather than expected profit being maximized. Furthermore, uncertainty is modeled nonprobabilistically with info-gap models of uncertainty, which can be viewed as a possible quantification of Knightian uncertainty.

JEL classification: D81, F30, G11, G15

Key words: equity home bias, Knightian uncertainty

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1 Introduction

The observed patterns of equity portfolio allocation around the world are at odds with predictions from the Capital Asset Price Models (CAPM) of Sharpe [16] and Lintner [13]. What has come to be called the 'home bias' phenomenon is that investors tend to hold a disproportionately large share of their equity portfolio in home country stocks, as compared with predictions of the CAPM. This paper provides an explanation of the home bias phenomenon based on information-gap decision theory. The decision concept which is used here is that profit is satisficed and robustness-to-uncertainty is maximized, rather than expected profit being maximized. Furthermore, uncertainty is modelled non-probabilistically with info-gap models of uncertainty.

French and Poterba [6, 7] quantify the magnitude of the home bias for a number of countries and conclude that the size of the home bias is substantial, which led to the terms *Home Bias Puzzle* or *French and Poterba Puzzle*. For more details see Jeske [10].

In their calculations, French and Poterba assume that the investor maximizes the expected return decremented with a variance-based risk term. Let $w = (w_{\mathrm{h}}, w_{\mathrm{f}})'$ represent the vector of home and foreign investments, let $\widetilde{\mu}$ be the expected returns for home and foreign investments, and let Σ be the covariance matrix of returns. In the French and Poterba model, w is chosen to maximize $U(w) = w'\widetilde{\mu} - \frac{\lambda}{2} w'\Sigma w$ where λ is chosen to weight the risk term. An extremum of $U(w)$ occurs when $\widetilde{\mu} = \lambda \Sigma w$.

Let w^{o} denote the observed portfolio weights for a particular country, so that the expected returns consistent with CAPM profit maximization would be $\widetilde{\mu}^{\mathrm{o}} = \lambda \Sigma w^{\mathrm{o}}$. For the same country, let w^{mc} denote the portfolio weights based on world market capitalization, without any home bias effect. That is, $w_{\mathrm{h}}^{\mathrm{mc}}$ is the fraction of the world market capitalization ascribable to the country in question. For example, $w_{\mathrm{h}}^{\mathrm{mc}} = 0.475$ for the US. Without any home bias, one would expect that 47.5% of US investments would be in US assets. The vector of expected returns consistent with the CAPM, based on world market capitalization without home bias, is $\widetilde{\mu}^{\mathrm{mc}} = \lambda \Sigma w^{\mathrm{mc}}$. French and Poterba observed, for a wide range of countries, that:

$$\widetilde{\mu}_{\mathrm{h}}^{\mathrm{o}} > \widetilde{\mu}_{\mathrm{h}}^{\mathrm{mc}} \quad \text{and} \quad \widetilde{\mu}_{\mathrm{f}}^{\mathrm{o}} < \widetilde{\mu}_{\mathrm{f}}^{\mathrm{mc}} \tag{1}$$

The returns vectors $\widetilde{\mu}^{\mathrm{o}}$ and $\widetilde{\mu}^{\mathrm{mc}}$ are not observed market values, but rather *calculated* values which are consistent with the CAPM decision model and observed portfolio weights. If the investor's behavior is rightly modelled by the CAPM theory, then $\widetilde{\mu}^{\mathrm{o}}$ and $\widetilde{\mu}^{\mathrm{mc}}$ can be interpreted as returns vectors which are anticipated or perceived by the investor. Relations (1) imply that investors perceive the domestic assets to perform better, and the foreign assets to perform worse, than the bias-free portfolio would indicate. French and Poterba assess these perceptions with the following 'shadow cost':

$$s = \underbrace{(\widetilde{\mu}_{\mathrm{h}}^{\mathrm{o}} - \widetilde{\mu}_{\mathrm{h}}^{\mathrm{mc}})}_{\substack{\text{perceived} \\ \text{advantage of} \\ \text{domestic equity}}} + \underbrace{(\widetilde{\mu}_{\mathrm{f}}^{\mathrm{mc}} - \widetilde{\mu}_{\mathrm{f}}^{\mathrm{o}})}_{\substack{\text{perceived} \\ \text{disadvantage of} \\ \text{foreign equity}}} \tag{2}$$

The shadow cost expresses how much an annual management fee for foreign assets must exceed the management fee for domestic assets, in order to induce the investor away from foreign assets, and towards domestic assets, according to the CAPM analysis. Most OECD countries display shadow costs between 200 and 500 basis points, and the values for some OECD countries are much higher. Annual management fees of several hundred basis points are unrealistically high for financial markets.

This paper evaluates whether a model in which investors act as robust satisficers of profit, rather than as profit maximizers, is able to account for the home bias effect. In particular, we explore whether an info-gap robust-satisficing decision rule requires lower shadow costs (than required by the CAPM model) in order to account for the observed home bias phenomenon.

2 Robust Satisficing with an Info-gap Model

In this section we formulate a robust-satisficing decision function motivated by severe information gaps facing the decision maker. Our notation is as follows.

$w \in \Re^N$ is a vector of investment weight-fractions. w_i is the fraction of the total budget which is invested in stock i, and $\sum_{i=1}^N w_i = 1$. w is chosen by the decision maker.

$\mu \in \Re^N$ is the profit vector, so $1 + \mu_i$ dollars are earned per dollar invested in stock i. μ is unknown when the investment is made, though the anticipated profit vector $\widetilde{\mu}$ is known. The profit from investment, normalized to a unit budget, is $w'\mu$. The anticipated rate of profit is $w'\widetilde{\mu}$.

π_{c} is a reservation profit: the lowest acceptable value of the unknown profit $w'\mu$, per dollar invested. For instance, $\pi_{\mathrm{c}} = 0.02$ means that 2% capital gain is necessary for 'survival'. Likewise, $\pi_{\mathrm{c}} = 0$ means that breaking even is adequate.

The decision maker has very limited information about the variability of the profit vector μ, and is unable to verify a probability distribution for μ. In lieu of this, the uncertain variation of μ is represented by an **info-gap model**, which is a **family of nested sets** of μ-vectors: $\mathcal{U}(\alpha, \widetilde{\mu})$, $\alpha \geq 0$. All sets contain the known, anticipated, profit vector, $\widetilde{\mu}$, and the range of variation of μ increases as α grows. That is, an info-gap model of uncertainty displays the properties of 'nesting' and 'contraction' [3]:

$$\text{Nesting:} \qquad \alpha < \alpha' \quad \text{implies} \quad \mathcal{U}(\alpha, \widetilde{\mu}) \subset \mathcal{U}(\alpha', \widetilde{\mu}) \tag{3}$$

$$\text{Contraction:} \qquad \mathcal{U}(0, \widetilde{\mu}) = \{\widetilde{\mu}\} \tag{4}$$

Relation (3) imbues α with its meaning as an 'horizon of uncertainty': the range of variability of μ increases as α grows. Combining relations (3) and (4) shows that the anticipated profit vector $\widetilde{\mu}$ is always possible, and $\widetilde{\mu}$ is the only possibility when there is no uncertainty ($\alpha = 0$).

Info-gap models express very sparse knowledge about uncertainty. An info-gap model represents uncertainty at two levels: for any positive value of α, the specific realization of μ from among the possibilities in $\mathcal{U}(\alpha, \widetilde{\mu})$, is unknown. In addition, the horizon of uncertainty, α, is unknown and unbounded so there is no known worst (or best) case. There is no measure-theoretic aspect to an info-gap model: there are no density functions of probability, plausibility, membership, etc. There are many types of info-gap models [3], and we will see one example shortly.

Since the profit vector μ is unknown, the decision maker cannot directly maximize the profit on investment, $w'\mu$. Furthermore, since the probability distribution of μ is also unknown, the decision maker cannot maximize the expected profit. What can be attempted is to *satisfice the profit* — guarantee adequate reward — and *maximize the robustness* to the information-gaps which surround μ. This strategy of **robust satisficing** will now be explained, based on an info-gap model for uncertainty in the profit.

The **robustness** of investment profile w is the greatest horizon of uncertainty α at which no realization of the profit vector μ causes a profit $w'\mu$ less than the aspiration π_{c}:

$$\widehat{\alpha}(w, \pi_{\mathrm{c}}) = \max \left\{ \alpha : \min_{\mu \in \mathcal{U}(\alpha, \widetilde{\mu})} w'\mu \geq \pi_{\mathrm{c}} \right\} \tag{5}$$

More robustness is better than less, at the same aspiration for profit π_{c}, so $\widehat{\alpha}(w, \pi_{\mathrm{c}})$ establishes a preference ranking on investments w based on satisficing the profit at π_{c}. The

robust-optimal investment profile, $\widehat{w}(\pi_c)$, is an investment vector which maximizes the robustness, at fixed profit aspiration π_c:

$$\widehat{w}(\pi_c) = \arg\max_{w} \widehat{\alpha}(w, \pi_c) \tag{6}$$

A very important property of the robustness function is that $\widehat{\alpha}(w, \pi_c)$ decreases monotonically as the aspiration π_c increases. This expresses the inexorable trade-off between immunity to uncertainty, and aspiration for profit: large aspirations are more vulnerable than low aspirations. This is an immediate result of the nesting of the info-gap model, and holds for any fixed investment profile w. It can also be proven that this monotonic trade-off holds for the robust-optimal profile. That is, $\widehat{\alpha}(\widehat{w}(\pi_c), \pi_c)$ decreases as π_c increases [2].

We note from eq.(5) that, since w and π_c are both normalized by the budget, the robustness is homogeneous to order zero in the budget: $\widehat{\alpha}(Bw, B\pi_c) = \widehat{\alpha}(w, \pi_c)$ for any $B \neq 0$. In other words, our unit-budget assumption will not influence the evaluation of the robustness, or the choice of a robust-optimal investment profile, so long as π_c is interpreted as a fractional profit on the investment, relative to the total budget.

3 Robustness Function with Ellipsoid-bound Info-gap Model

We will now evaluate the robustness function for a specific choice of the info-gap model of uncertainty in the profit vector.

The analyst selects the specific form of the info-gap model as the most restrictive family of nested sets which is consistent with the knowledge about the uncertain variability. In the present case, the anticipated profit vector $\widetilde{\mu}$ is known. In addition, data are available which express the interdependencies between the variation of the profits of different assets. These data are usually represented by an empirical covariance matrix $\Sigma \in \Re^{N \times N}$, though we do not know the underlying probability distribution.

With this information about the profit vector, a suitable choice is the **ellipsoid-bound info-gap model,** which is a family of nested ellipsoids of μ-vectors:

$$\mathcal{U}(\alpha, \widetilde{\mu}) = \left\{ \mu = \widetilde{\mu} + u : \ u' \Sigma^{-1} u \leq \alpha^2 \right\}, \quad \alpha \geq 0 \tag{7}$$

Σ is a known, real, symmetric, positive definite matrix which determines the shape and orientation of the uncertainty-ellipsoids. The size of these ellipsoids is determined by α, which is the unknown horizon of uncertainty. This family of sets of μ vectors displays the properties of nesting and contraction, eqs.(3) and (4).

Using Lagrange optimization one readily finds the minimum profit, up to uncertainty α, to be:

$$\min_{\mu \in \mathcal{U}(\alpha, \widetilde{\mu})} w' \mu = w' \widetilde{\mu} - \alpha \sqrt{w' \Sigma w} \tag{8}$$

Referring to eq.(5), the robustness of investment w is the greatest value of α at which this minimum profit does not fall below the critical profit π_c. If the anticipated profit, $w'\widetilde{\mu}$, falls short of the critical profit π_c, then the robustness is zero. Otherwise, equating the righthand side of eq.(8) to π_c and solving for α yields the robustness of investment profile w:

$$\widehat{\alpha}(w, \pi_c) = \begin{cases} \dfrac{w' \widetilde{\mu} - \pi_c}{\sqrt{w' \Sigma w}} & \text{if } w' \widetilde{\mu} \geq \pi_c \\ 0 & \text{else} \end{cases} \tag{9}$$

The robust-optimal investment vector, $\widehat{w}(\pi_c)$, and the maximal robustness, $\widehat{\alpha}(\widehat{w}, \pi_c)$, are derived in section 8. Explicit expressions are obtained, and it is shown that, unless all the elements of the anticipated profit vector $\widetilde{\mu}$ are the same, both $\widehat{w}(\pi_c)$ and $\widehat{\alpha}(\widehat{w}, \pi_c)$ vary as $\widetilde{\mu}$ changes.

4 Robust-satisficing Shadow Cost

In this section we derive an explicit analytical expression for the robust-satisficing shadow cost based on the info-gap robustness function for the ellipsoidal info-gap model of profit-uncertainty.

4.1 Calculating the Shadow Cost

We calculate the info-gap shadow cost in a manner analogous to the French and Poterba method summarized in eq.(2): as the perceived advantage of the domestic equity plus the perceived disadvantage of the foreign equity. We calculate two centerpoint profit vectors: $\widetilde{\mu}^{\rm mc}$ is consistent with the portfolio weights corresponding to the world market capitalization, $w^{\rm mc}$, and $\widetilde{\mu}^{\rm o}$ is consistent with the observed portfolio weights, $w^{\rm o}$. The shadow cost is calculated from $\widetilde{\mu}^{\rm mc}$ and $\widetilde{\mu}^{\rm o}$ with eq.(2), which can be expressed:

$$s = (1 \quad -1)\,(\widetilde{\mu}^{\rm o} - \widetilde{\mu}^{\rm mc}) \tag{10}$$

We do this for each country and each year, using the estimated covariance matrix Σ, whose calculation is specified in section 5.2, to define the ellipsoidal info-gap model for each case.

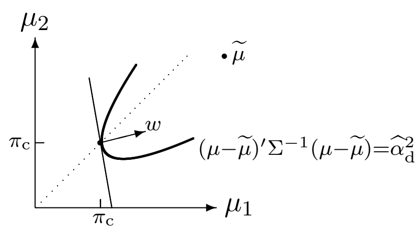

Figure 1. Determining the centerpoint, $\widetilde{\mu}$, given portfolio weights w (two-country case).

We now formulate this more precisely. Let $\widehat{\alpha}_{\rm d}$ represent the demanded robustness, and let $\pi_{\rm c}$ denote the aspired annual profit. (We will not need to choose a $\pi_{\rm c}$ value in order to calculate a shadow cost. The meaning and choice of $\widehat{\alpha}_{\rm d}$ will be discussed in section 4.2.) Let w be the specified vector of portfolio weights (either $w^{\rm mc}$ or $w^{\rm o}$). We seek a centerpoint vector $\widetilde{\mu}$ for which w is the vector of portfolio weights which maximizes the robustness, as in eq.(6), and for which the resulting maximal robustness equals $\widehat{\alpha}_{\rm d}$. As we will explain in section 6.1.1, this robust-optimal configuration can be represented geometrically in μ-space: the ellipsoid, $(\mu - \widetilde{\mu})'\Sigma^{-1}(\mu - \widetilde{\mu}) = \widehat{\alpha}_{\rm d}^2$, is tangent to the hyperplane, $w'\mu = \pi_{\rm c}$, at the point $\mu = \pi_{\rm c}\mathbf{1}$. ($\mathbf{1}$ is the N-vector whose elements are all ones.) This is shown schematically in fig. 1 for the 2-country case. Tangency at the point $\mu = \pi_{\rm c}\mathbf{1}$ requires that the gradient of the ellipsoid is parallel to w:

$$-cw = \Sigma^{-1}(\pi_{\rm c}\mathbf{1} - \widetilde{\mu}) \tag{11}$$

where c is a positive constant which is chosen so that, when this value of $\widetilde{\mu}$ is used in eq.(9), the robustness equals $\widehat{\alpha}_{\rm d}$. After solving for c in this way one finds the centerpoint to be:

$$\widetilde{\mu} = \pi_{\rm c}\mathbf{1} + \frac{\widehat{\alpha}_{\rm d}}{\sqrt{w'\Sigma w}}\Sigma w \tag{12}$$

Using this equation, $\widetilde{\mu}^{\rm o}$ is calculated with $w^{\rm o}$ and $\widetilde{\mu}^{\rm mc}$ is calculated $w^{\rm mc}$. The shadow cost is calculated with $\widetilde{\mu}^{\rm o}$ and $\widetilde{\mu}^{\rm mc}$ in eq.(10). Notice that the shadow cost does not depend upon the profit aspiration, $\pi_{\rm c}$, but that it scales linearly with the demanded robustness, $\widehat{\alpha}_{\rm d}$.

4.2 Choosing Demanded Robustness $\widehat{\alpha}_{\rm d}$

In this section we discuss the choice of the demanded robustness $\widehat{\alpha}_{\rm d}$ to express the investor's aversion to, or proclivity for, risk. Our choice of $\widehat{\alpha}_{\rm d}$ is analogous to the method which French and Poterba use to select the risk aversion coefficient λ discussed in section 1. However, the concepts of risk aversion are quite different.

French and Poterba choose λ by assuming that an investor who is unbiased (that is, who diversifies according to market shares) uses an implicit μ given by:

$$\mu^{\rm mc} = \lambda \Sigma w^{\rm mc} \tag{13}$$

They pick a country, for instance the U.S., and choose λ so that the implicit return for that country, $\mu_{\rm us}$, matches a target return, $\mu_{\rm us}^{\rm t}$. For example, this target return would be the average return in the U.S. for the last couple of decades. The 'U.S.' row in eq.(13) is solved for λ as:

$$\lambda = \frac{\mu_{\rm us}^{\rm t}}{\sum_{i=1}^{N} \sigma_{{\rm us},i} w_i^{\rm mc}} \tag{14}$$

In other words, French and Poterba ask: "How risk averse would an unbiased investor have to be in order to have an implicit expected return for the U.S. equal to some observed long-term average return $\mu_{\rm us}^{\rm t}$?". The answer is expressed in the choice of λ. A large value of λ entails large aversion to risk.

We do something analogous in the choice of the demanded robustness $\widehat{\alpha}_{\rm d}$, which expresses how much immunity, to uncertainty in the returns, is required by the investor. We ask: "How much robustness to long-term profit-variability, $\widehat{\alpha}_{\rm d}$, would be required by an investor.

Fig. 4 shows uncertainty ellipses, $\mu' \Sigma^{-1} \mu = \alpha_{\rm d}^2$, for home and foreign assets, μ_1 (horizontal) and μ_2 (vertical), based on covariance matrices for three different years. For instance, the figure for Belgium shows that the value of $\widehat{\alpha}_{\rm d}$ for 1990 (dotted) is chosen to immunize the investor against long-term variability of the profit by about ± 10 percentage points in both foreign and domestic assets. The immunity is less for years 1995 and 2000: about ± 8 percentage points. This level of robustness is quite large, considering that long-term profits are in the range of 2 or 3 percentage points. An investor who chooses $\widehat{\alpha}_{\rm d}$ values like those in this figure is exceedingly, implausibly, risk averse. By comparison, consider the uncertainty ellipses in fig. 6. For Belgium, the demanded robustness, $\widehat{\alpha}_{\rm d}$, is chosen to attain immunity against long-term profit variability of about ± 2.5 percentage points, a more plausible expression of aversion to variability.

5 Empirical Study

5.1 Formulation and Results

We have applied info-gap robust-satisficing decision theory to the analysis of financial investments in 12 industrialized countries: Australia, Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Sweden, Switzerland, UK and USA. We have calculated shadow costs as explained in section 4, based on the optimal robustness function, $\widehat{\alpha}(\widehat{w}(\pi_{\rm c}), \pi_{\rm c})$, defined in eqs.(5) and (6), with the ellipsoid-bound info-gap model for profit-uncertainty in eq.(7). Covariance matrices Σ for monthly data for each country are obtained from the MSCI [14] data base for the continuously growing window from 1987 through 2000, as explained in section 5.2. The observed homeshares are obtained from the FIBV [5] and the IMF [9] data bases. These data for each country contain one home asset and one foreign asset, which are constructed as a capitalization-weighted average of all foreign equity markets.

Fig. 2 shows the CAPM annual shadow costs over time for 12 OECD countries, calculated with the method of French and Poterba as explained in section 1 with $\lambda = 3$. These shadow costs are all higher than plausible values of management fees.

Fig. 3 shows the demanded robustness, $\widehat{\alpha}_{\rm d}$, over time for 12 OECD countries. Each value of $\widehat{\alpha}_{\rm d}$ is chosen so that the resulting info-gap robust-satisficing shadow cost, whose calculation was explained in section 4.1, precisely equals the CAPM shadow cost for the same year and country. Fig. 4 shows profit-uncertainty ellipses, $\mu'\Sigma^{-1}\mu = \widehat{\alpha}_{\rm d}^2$, for home and foreign assets, μ_1 (horizontal) and μ_2 (vertical), based on the $\widehat{\alpha}_{\rm d}$ values from fig. 3. Ellipses for three years are drawn: 1990, 1995 and 2000.

The results in figs. 3 and 4 are re-done in figs. 5 and 6, but now the demanded robustness is chosen to achieve a shadow cost of precisely 0.01 for each country and each year.

The following conclusions can be drawn.

1. From fig. 4, the robustness, $\widehat{\alpha}_{\rm d}$, required to precisely reproduce the CAPM shadow costs, typically entails aversion to long-term profit fluctuations in the neighborhood of:
   (a) ±5% in 3 countries: CAN, UK and USA.
   (b) ±6 to ±8% in 6 countries: AUS, BEL, FRA, GER, NET and SWI.
   (c) ±20% or more in 3 countries: ITA, JAP and SWE.
2. From fig. 6, the robustness, $\widehat{\alpha}_{\rm d}$, required to produce annual shadow costs of 0.01, typically entails aversion to long-term profit fluctuations in the neighborhood of:
   (a) ±1% in 3 countries: AUS, ITA and JAP
   (b) ±2% in 5 countries: BEL, CAN, FRA, GER and SWE.
   (c) ±3% in 3 countries: SWI, UK and USA.
   (d) ±6% in 1 country: NET.
3. From fig. 3, the robustness, $\widehat{\alpha}_{\rm d}$, required to precisely reproduce the CAPM shadow costs, typically is fairly stable over time.
4. From fig. 5, the robustness, $\widehat{\alpha}_{\rm d}$, required to produce annual shadow costs of 0.01, typically fluctuates very substantially over time.

These observations are discussed in section 6.

5.2 Data Appendix

We consider two sets of countries: One consisting of the twelve countries for which we compute the home bias, call it $G12$, and one larger set of 23 countries, call it $G23$, consisting of the original 12 in addition to 11 more countries. For the 11 countries in $G23$ but not in $G12$ we do not calculate home bias, but nevertheless they are included in the computation of the index of the world stock market.

The data sources are as follows:

The international equity investment positions for the $G12$ countries Australia, Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Sweden, Switzerland, UK and USA come from the IFS database (International Financial Statistics) from the International Monetary Fund. The same is true for monthly CPI inflation[1] and monthly exchange rates.

[1] Australia is the only country among the twelve that does not report monthly CPI numbers. The quarterly CPI figures are therefore split evenly among the three months of each quarter. Since most of the variation of real monthly stock returns in the other 11 countries comes from the movements in the index, not in the CPI, this seems very reasonable.

The end-of-year market capitalizations for the years 1990-2000 for $G23$ countries come from the World Federation of Exchanges (FIBV). The 23 countries are the $G12$ countries for which we compute home bias in addition to Brazil, Chile, Hong Kong, Indonesia, South Korea, Spain, Mexico, Singapore, Taiwan, Thailand and Turkey.

Monthly nominal returns for 1987:12 to 2000:12 in $US for the $G23$ countries come from the MSCI database (www.msci.com).

For each of the G12 countries we compute domestic and foreign returns in home currency and deflate them by domestic CPI. The foreign series is a capitalization-weighted average of the other 22 countries.[2] Precisely, the correlation matrices Σ needed for the info-gap ellipsoids are computed for the $G12$ countries and 11 years using returns from 1987:12 to the end of the corresponding year. For example, the Σ for Italy for the year 1995 is computed using returns from 1987:12 up to 1995:12 for both Italian domestic returns and an average of returns from the other 22 countries.

6 Discussion

The following conclusions can be drawn from the results of section 5.1, given the observed and market capitalization portfolio weights, $w^{\rm o}$ and $w^{\rm mc}$, and using the info-gap robust-satisficing decision model.

1. The CAPM shadow costs entail unreasonably high risk aversion for all countries throughout the time interval examined.

2. Considering long-term returns of 4 to 5%, we think that robustness against profit variations of $\pm 3\%$ constitutes plausible aspiration for immunity. For prediction of annual shadow costs equal to 0.01, the robust-satisficing model implies:

   (a) Unreasonably low risk aversion for 3 countries: AUS, ITA and JAP ($\pm 1\%$.)

   (b) Somewhat low risk aversion for 5 countries: BEL, CAN, FRA, GER and SWE ($\pm 2\%$.)

   (c) Plausible risk aversion for 4 countries: NET ($\pm 6\%$), SWI, UK and USA. ($\pm 3\%$.)

3. For the 3 countries, AUS, ITA and JAP, the $\pm 1\%$ could be raised to $\pm 3\%$ by raising the shadow cost, but it would still be substantially lower than the CAPM shadow cost, as witnessed by fig. 4.

4. For the 5 countries, BEL, CAN, FRA, GER and SWE, the value of $\pm 2\%$ could be raised to $\pm 3\%$ by modest increase of the shadow cost, perhaps to 0.015.

5. For NET, the value of $\pm 6\%$ could be reduced to $\pm 3\%$ by further reducing the shadow cost substantially below 0.01.

The implication of these points is that the info-gap robust-satisficing decision model provides a plausible explanation of the observed portfolio weights, while the CAPM model does not. We now proceed to develop some understanding of why this is.

[2] These $G23$ countries comprise between 95% and 97% of world market capitalization during the years considered, so that we can confidently assume the 23 countries taken together are a good approximization to the world stock market.

6.1 Economic Intuition: Why Robust-satisficing Works

6.1.1 Geometrical Introduction

To understand why the info-gap approach finds it easier — measured as the necessary shadow cost — to account for home bias, it is useful to look at the mechanics of the model. The discussion in this section is intuitive and geometrical. A rigorous derivation of the optimal robustness appears in the appendix (section 8).

In the two-country version, a graphical representation combines all the variables of interest: profit aspiration π_c, investment weights w, and robustness $\widehat{\alpha}$. Fig. 7 does exactly that in $(\mu_1,\, \mu_2)$ space.

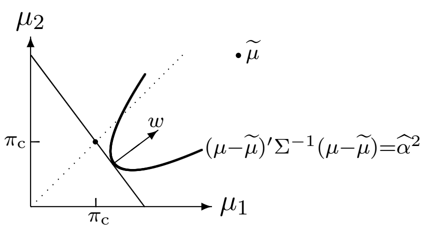

Figure 7. Determining the robustness $\widehat{\alpha}(w, \pi_c)$ (two-country case).

The dotted line lies at 45 degree to the axes. The solid line plots all $(\mu_1,\, \mu_2)$ points for which $\mu^T w = \pi_c$ for given aspiration π_c and given portfolio weights w. Consequently, the solid line has slope $-w_1/w_2$ and crosses the point $(\pi_c,\, \pi_c)$ on the 45 degree line. If $w'\widetilde{\mu} < \pi_c$ then the robustness for this portfolio is zero (second line of eq.(9)). Thus consider w and π_c such that $\widetilde{\mu}$ is above the solid line. The robustness $\widehat{\alpha}\,(w, \pi_c)$ is given by the α in $\mathcal{U}\,(\alpha, \pi_c)$ for which the ellipse is tangent to the solid line. In that case the value of $\arg\min_{\mu\in\mathcal{U}(\widehat{\alpha},\pi_c)} \left\{ w'\mu :\ w^T\mu \geq \pi_c \right\}$ is given by the tangency point, or in other words where the gradient $2\Sigma^{-1}\,(\mu - \tilde{\mu})$ of the ellipsoid function is a multiple of the vector w as in fig. 7.

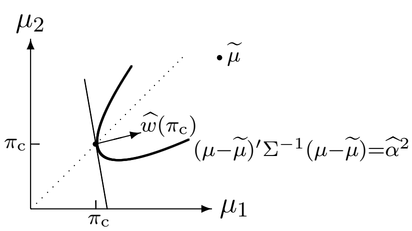

Figure 8. Determining the robustness-optimizing portfolio $\widehat{w}$ (two-country case).

With fig. 7 we have demonstrated the graphical determination of the robustness of given portfolio weights w. To find the robustness-optimizing portfolio weights $\widehat{w}\,(\pi_c)$ we can again employ the graphical representation as shown in fig. 8. Find the α such that the ellipse intersects the 45 degree line at (π_c, π_c). Then it is easy to see that $\widehat{w}(\pi_c)$ is the normalized gradient of the ellipsoid function at (π_c, π_c). Changing w would rotate the straight line around the point (π_c, π_c) which would cut into the ellipse and thus lower the robustness. Hence, $\widehat{w}$ maximizes robustness.[3]

6.1.2 Shadow Costs: Algebraic Formulation

Equipped with these geometrical tools we can see how the info-gap robust-satisficing decision model can explain large home bias with lower shadow cost than in the CAPM approach. The basic intuition is that the very steep part of the ellipse results in a nearly horizontal robust-optimal investment vector $\widehat{w}$ as in fig. 8, which corresponds to a high home share. We now develop this intuition more quantitatively.

[3]This argument is valid if both elements of $\widehat{w}$ are positive. See section 8 for a general derivation of $\widehat{\alpha}(\widehat{w}, \pi_c)$.

The shadow cost from French and Poterba in a two-country example can be derived graphically in a manner similar to the info-gap derivation in figs. 7 and 8. As explained in section 1, the first-order conditions for an extremum of the profit, decremented with a risk-aversion term, require that profit, μ, and investment weights, w, be related by:

$$\mu = \lambda \Sigma w \tag{15}$$

Taking w, Σ, and C as given, consider the following optimization problem, which will underlie our geometrical interpretation of the CAPM approach in section 6.1.3:

$$\max_{\mu} w'\mu \quad \text{subject to} \quad \mu'\Sigma^{-1}\mu = C^2 \tag{16}$$

The first-order condition that comes out of this problem is:

$$\mu = \frac{1}{2\eta}\Sigma w \tag{17}$$

where η is the Lagrange multiplier on the constraint. Comparing eqs.(15) and (17) we see that, in the CAPM approach, the implicit expected return vector μ satisfies the first-order conditions of the optimization in eq.(16) if $\lambda = \frac{1}{2\eta}$. Using the constraint to solve for η, together with some algebraic manipulations, one can show that:

$$C = \sqrt{\lambda w'\mu} \tag{18}$$

$$= \lambda\sqrt{w'\Sigma w} \tag{19}$$

6.1.3 Shadow Costs: Geometric Formulation

The graphical representation of the CAPM shadow cost is portrayed in fig. 9 with two ellipses in (μ_1, μ_2) space, defined by $C^2 = \mu'\Sigma^{-1}\mu$, one with $C_{\text{o}} = \sqrt{\lambda w^{\text{o}\prime}\mu^{\text{o}}}$ and one with $C_{\text{mc}} = \sqrt{\lambda w^{\text{mc}\prime}\mu^{\text{mc}}}$. This is a critical difference between the CAPM and the info-gap approaches. CAPM uses two ellipses of different sizes, C_{o} and C_{mc}. The info-gap approach uses 2 ellipses of the same size, $\widehat{\alpha}_{\text{d}}$. As explained in section 4.2, $\widehat{\alpha}_{\text{d}}$ is homologous to the CAPM risk-aversion parameter λ which takes a single value. (We will discuss risk aversion again at the end of this section.)

As indicated in fig. 9, the implicit expected return vector, μ^{o}, consistent with the observed portfolio weights, w^{o}, is the tangency point to the C_{o} ellipse of the hyperplane defined by w^{o}. Similarly, the implicit market capitalization returns, μ^{mc}, is the tangency point of the w^{mc} hyperplane to the C_{mc} ellipse. The CAPM shadow cost is the sum of the coordinate distances of the two tangency points as specified in eq.(2) and shown by the length of the dotted lines in fig. 9. Furthermore, if $w_1^{\text{o}} \gg w_1^{\text{mc}}$ and $\Sigma_{11} \gg \Sigma_{22}$ then eq.(19) can be used to show that $C^{\text{o}} \gg C^{\text{mc}}$. In that case the horizontal distance between the two points $\tilde{\mu}^{\text{o}}$ and $\tilde{\mu}^{\text{mc}}$ can be very large, contributing to a big shadow cost in the CAPM framework.

In info-gap approach, on the other hand, one begins with two ellipses, both of size $\widehat{\alpha}_{\text{d}}$, one centered at $\widetilde{\mu}^{\text{o}}$ and one centered at $\widetilde{\mu}^{\text{mc}}$, as in figs. 8 and 10. The centerpoints, $\widetilde{\mu}^{\text{o}}$ and $\widetilde{\mu}^{\text{mc}}$, are fixed as described in eq.(12) of section 4.1. One finds that $\widehat{\alpha}_{\text{d}}$ is substantially less than C_{o} and C_{mc}. That is, the info-gap ellipse is smaller than both of the CAPM ellipses. The shadow cost is the sum of the coordinate distance between $\widetilde{\mu}^{\text{mc}}$ and $\widetilde{\mu}^{\text{o}}$, as indicated by the dotted lines in fig. 10.

Let us summarize figs. 9 and 10. We have implemented the info-gap robust-satisficing theory in parallel to the CAPM theory. In particular, the CAPM risk-aversion parameter λ is transposed into the info-gap demanded robustness $\widehat{\alpha}_{\text{d}}$. Each parameter, λ and $\widehat{\alpha}_{\text{d}}$, is single-valued in its respective theory. In addition, we have developed a geometrical explanation of

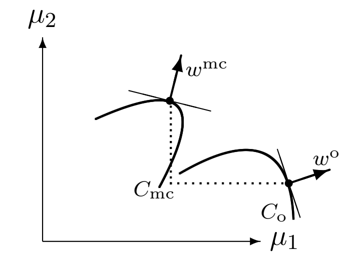

Figure 9. Geometrical explanation of the CAPM shadow cost: the sum of the two dotted lines.

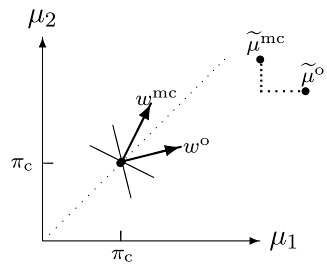

Figure 10. Geometrical explanation of the info-gap shadow cost: the sum of the two dotted lines.

the CAPM shadow cost which entails ellipsoidal sets of profit vectors μ which are homologous to the uncertainty sets of the ellipsoidal info-gap model. The shadow cost is less in the info-gap than the CAPM method because both info-gap ellipses are of the same size, $\widehat{\alpha}_{\mathrm{d}}$, which is smaller than both of the CAPM ellipse-sizes, C_{o} and C_{mc}.

So what does it mean that the CAPM uncertainty sets are greater than the info-gap maximal-robustness set? Some insight is obtained by comparing the info-gap and CAPM concepts of risk aversion. In the info-gap robust-satisficing theory, $\widehat{\alpha}_{\mathrm{d}}$ is the robustness, to profit uncertainty, which the investor demands. A larger value of $\widehat{\alpha}_{\mathrm{d}}$ is chosen as the investor's risk-aversion increases. In the CAPM theory, λ is the risk-aversion weight in the utility function. Like $\widehat{\alpha}_{\mathrm{d}}$, a larger λ is chosen as the investor's risk-aversion increases. A robust-satisficing investor, looking at the CAPM sets and thinking in robust-satisficing terms, would say: "What a large $\widehat{\alpha}_{\mathrm{d}}$. How risk averse!" Conversely, a CAPM investor, looking at the robust-satisficing sets and thinking in CAPM terms, would say: "What a small λ. How risk-loving!" In fact, both assessments are wrong, because each is based on a mistaken understanding of the other's mental model. To the CAPM investor, the info-gap robust-satisficer looks risk-loving because the profit is being satisficed rather than maximized. To the robust-satisficer, the CAPM investor looks risk averse because the profit is being maximized rather than satisficed.

Further understanding may be obtained by comparing the CAPM and the info-gap robust-satisficing decision models. The CAPM investor **knows** the returns vector, μ. This would allow the investor to choose the portfolio weights w so as to **maximize the return,** $\mu' w$. However, this objective function, $\mu' w$, is decremented with its weighted **portfolio-variance,** $\lambda w' \Sigma w$, to express aversion to risky fluctuation of the returns.

In contrast, the info-gap robust-satisficing investor **does not know** the returns vector, μ. Consequently the investor chooses the portfolio weights w so as to **satisfice the return,** $\mu' w$. Since satisficing (unlike optimizing) generally has an infinity of solutions, the robust-satisficing investor has an additional degree of freedom, which is used to **maximize the robustness.**

6.2 Knightian Uncertainty, Bounded Rationality, and Info-gap Robust Satisficing

Our explanation of the home bias puzzle is based on using the info-gap robustness function to *satisfice* the profit and to *maximize* the immunity to uncertainty. This decision model provides one possible quantification of Knightian uncertainty combined with Simon's conception of bounded rationality, as we will now explain. See also [3, chap. 12].

Frank Knight emphasized the fundamental distinction between probabilistically measurable uncertainty, which he called risk, and unmeasurable and uninsurable uncertainty, which he referred to as " 'true' uncertainty" [11, p.20; 12, p.120]. It is uncertainty, rather than risk, which Knight saw as underlying the greatest challenges to economic agents and analysts [11, pp.231–232]. Other writers have also noted the distinction between uncertainty and risk, notably Samuelson [15, pp.503–504] and Arrow [1, p.19].

An info-gap model of uncertainty is a family of nested sets $\mathcal{U}(\alpha, \widetilde{u})$, $\alpha \geq 0$. The elements of these sets represent events (in our analysis: realizations of the profit vector μ). At any given value of the horizon of uncertainty, α, the set $\mathcal{U}(\alpha, \widetilde{u})$ specifies a range of possible variation of the events. Since the horizon of uncertainty is unknown and unbounded, there is no worst case. Since no measure functions of probability (or plausibility, or belief, etc.) are specified by an info-gap model, the analyst cannot calculate statistical expectations and cannot probabilistically insure against the unknown contingencies identified in the info-gap model.

An info-gap model of uncertainty is a more extreme departure from probabilistic tradition than proposed by Gilboa and Schmeidler [8] and developed by Epstein and Wang [4] and others, who replace a single prior probability distribution with a set of distributions. In our formulation, preferences are generated without any distribution functions at all.

Knight never provided a quantification of his conception of 'true uncertainty', and such a quantification may not be unique. It would seem, however, that the highly unstructured and minimalistic quantification of uncertainty entailed in an info-gap model corresponds reasonably well to Knight's conception of uncertainty.

'Satisficing' is the satisfaction of minimal requirements or specifications: making the performance good enough, as distinct from optimizing the performance. Herbert Simon has pointed out that 'comparative advantage', whether in biological or economic survival, is a satisficing strategy (don't optimize, just beat the competition) [18, p.69]. Simon recognized the role of satisficing as a means for dealing with the "limitations upon the capacities and complexity of the organism, and [for] taking account of the fact that the environments to which it must adapt possess properties that permit further simplication [sic] of its choice mechanisms." [17, p.129]. Simon refers to this combination of organismic and environmental limitations as 'bounded rationality' [19].

The info-gap robustness function, $\widehat{\alpha}(w, \pi_c)$ in eq.(5), evaluates the immunity, of investment w, to the Knightian uncertainty of an info-gap model, while attempting to satisfice the profit at the level π_c. The robustness function generates preferences on the investment alternatives w. While profit is satisficed rather than maximized, what *is* maximized, resulting in the robust-optimal investment $\widehat{w}(\pi_c)$ in eq.(6), is the immunity to uncertainty.

Decision makers can respond in many ways to Knightian uncertainty and to bounded rationality. One possible response is to employ the info-gap robustness function. The epistemic requirements are no greater than those needed for selecting an info-gap model, which constitutes a quantification of Knightian uncertainty. The robustness function explicitly satisfices rather than optimizes the profit. Our study of the home bias phenomenon suggests that robustness optimization may not exceed the capability of "boundedly rational" decision makers.

7 Appendix: Partial Geometric Derivation of Robust-optimal Investment

In this section we outline a geometrical derivation of the investment vector $\widehat{w}(\pi_c)$ which maximizes the robustness for the ellipsoid-bound info-gap model. We consider the general N-country case, so w and μ are N-vectors, and w satisfies $w'\mathbf{1} = 1$ where $\mathbf{1}$ is the N-vector whose elements are all ones. Let $H(w, \pi_c)$ denote the hyperplane in μ-space defined by $w'\mu = \pi_c$. We first derive the robustness function, $\widehat{\alpha}(w, \pi_c)$, for fixed portfolio w and aspiration π_c. We then derive the robustness-maximizing portfolio, $\widehat{w}(\pi_c)$, given aspiration π_c. We then derive the aspiration and portfolio as functions of the robustness, $\pi_c(\widehat{\alpha})$ and $\widehat{w}(\widehat{\alpha})$.

Robustness function $\widehat{\alpha}(w, \pi_c)$. Examination of eq.(5) shows that, if the anticipated profit vector, $\widetilde{\mu}$, does not yield more than the aspired profit, $w'\widetilde{\mu} \le \pi_c$, then the robustness vanishes: $\widehat{\alpha}(w, \pi_c) = 0$. In other words, the robustness is positive if and only if $\widetilde{\mu}$ is above the hyperplane $H(w, \pi_c)$.

So, for $\widetilde{\mu}$ above $H(w, \pi_c)$, find the value of α at which the ellipsoid in μ-space, $\alpha^2 = (\mu - \widetilde{\mu})' \Sigma^{-1} (\mu - \widetilde{\mu})$, is tangent to $H(w, \pi_c)$. (See fig. 7 for a 2-dimensional illustration.) The geometric realization of the definition of the robustness in eq.(5) is that this value of α is the robustness. At this point of tangency between the hyperplane and the ellipsoid, the outward normal of the ellipsoid is $2\Sigma^{-1} (\mu - \widetilde{\mu})$, while the normal to the hyperplane is w. That is:

$$cw = 2\Sigma^{-1} (\mu - \widetilde{\mu}) \tag{20}$$

where c is a non-zero constant which guarantees that this μ lies on the boundary of the ellipsoid, resulting in:

$$\mu = \widetilde{\mu} - \frac{\alpha}{\sqrt{w'\Sigma w}} \Sigma w \tag{21}$$

Since $w'\mu = \pi_c$ for this μ which lies on the hyperplane $H(w, \pi_c)$, this value of α (which is the robustness), becomes the expression in eq.(9).

Robustness-maximizing portfolio $\widehat{w}(\pi_c)$. The central fact is that the hyperplane $H(w, \pi_c)$ contains the point $\mu = \pi_c\mathbf{1}$ for any choice of the portfolio w. The next point is that the ellipsoid $\alpha^2 = (\mu - \widetilde{\mu})' \Sigma^{-1} (\mu - \widetilde{\mu})$ is tangent to $H(w, \pi_c)$ when α equals $\widehat{\alpha}(w, \pi_c)$, as we have explained in connection with eqs.(20) and (21). It results that the greatest possible robustness, for aspiration π_c, is the value of α for which the ellipsoid is tangent to the hyperplane at the point $\mu = \pi_c\mathbf{1}$ (see fig. 8 for a 2-dimensional illustration).

When the ellipsoid is tangent to $H(w, \pi_c)$, the normal to the ellipsoid is proportional to w. When this occurs at $\mu = \pi_c\mathbf{1}$, the value of w, which is the portfolio which maximizes the robustness, is found to be:

$$\widehat{w}(\pi_c) = \frac{1}{\mathbf{1}'\Sigma^{-1} (\pi_c\mathbf{1} - \widetilde{\mu})} \Sigma^{-1} (\pi_c\mathbf{1} - \widetilde{\mu}) \tag{22}$$

Substituting this into eq.(9) results in the maximal robustness for aspiration π_c:[4]

$$\widehat{\alpha}(\widehat{w}(\pi_c), \pi_c) = \sqrt{(\pi_c\mathbf{1} - \widetilde{\mu})' \Sigma^{-1} (\pi_c\mathbf{1} - \widetilde{\mu})} \tag{23}$$

8 Appendix: Algebraic Derivation of Robust-optimal Investment

In this section we outline the derivation of the investment vector $\widehat{w}(\pi_c)$ which maximizes the robustness for the ellipsoid-bound info-gap model, eq.(9). It is convenient to consider two

[4] Something is wrong. Eq.(22) results in a robustness which is the negative of eq.(23).

cases.

1. $\widetilde{\mu} = m_0 \mathbf{1}$, where m_0 is a scalar.

2. $\widetilde{\mu} \neq m_0 \mathbf{1}$, where m_0 is a scalar. We proceed in two steps:

   (a) Find w to minimize $w'\Sigma w$ subject to $w'\mathbf{1} = 1$ (budget constraint; $\mathbf{1}$ is an N-vector of ones) and $w'\widetilde{\mu} = \kappa$ where κ is a constant no less than $\pi_{\rm c}$. Denote the result $\widehat{w}(\kappa)$.

   (b) Find κ to maximize $\widehat{\alpha}(\widehat{w}(\kappa), \pi_{\rm c})$ in eq.(9).

We first consider case 1. $\widetilde{\mu} = m_0 \mathbf{1}$ and, by the budget constraint, $w'\mathbf{1} = 1$, so $w'\widetilde{\mu} = m_0$. So, from eq.(9), the robustness is zero unless $m_0 \geq \pi_{\rm c}$. Consequently, w maximizes the robustness if w is chosen as a solution of the following optimization problem:

$$\min w'\Sigma w \quad \text{subject to} \quad w'\mathbf{1} = 1 \tag{24}$$

Lagrange optimzation immediately results in the following robust-optimal investment vector and corresponding maximal robustness:

$$\widehat{w} = \frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \Sigma^{-1}\mathbf{1} \tag{25}$$

$$\widehat{\alpha}(\widehat{w}, \pi_{\rm c}) = (m_0 - \pi_{\rm c})\sqrt{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \tag{26}$$

We note that the robust-optimal investment vector, $\widehat{w}$, is independent of the anticipated profit vector $\widetilde{\mu}$. The maximal robustness, $\widehat{\alpha}(\widehat{w}, \pi_{\rm c})$ varies affinely with the magnitude, m_0, of the anticipated profit vector.

We now consider case 2, for which $\widetilde{\mu} \neq m_0 \mathbf{1}$, where m_0 is a scalar.

Step 2a. We use Lagrange optimization to solve the following problem, for any value of $\kappa \geq \pi_{\rm c}$:

$$\min w'\Sigma w \quad \text{subject to} \quad w'\widetilde{\mu} = \kappa, \quad w'\mathbf{1} = 1 \tag{27}$$

The result is:

$$\widehat{w}(\kappa) = \frac{1}{hf - g^2} \Sigma^{-1} \left[(f\widetilde{\mu} - g\mathbf{1})\kappa + h\mathbf{1} - g\widetilde{\mu} \right] \tag{28}$$

where:

$$f = \mathbf{1}'\Sigma^{-1}\mathbf{1}, \ g = \widetilde{\mu}'\Sigma^{-1}\mathbf{1}, \ h = \widetilde{\mu}'\Sigma^{-1}\widetilde{\mu} \tag{29}$$

Some manipulation reveals that eq.(28) can be expressed:

$$\widehat{w}(\kappa) = \overbrace{\frac{1}{\widetilde{\mu}'\Sigma^{-1}(\widetilde{\mu}\mathbf{1}' - \mathbf{1}\widetilde{\mu}')\Sigma^{-1}\mathbf{1}}}^{\theta} \overbrace{\left[\Sigma^{-1}(\widetilde{\mu}\mathbf{1}' - \mathbf{1}\widetilde{\mu}')\Sigma^{-1} \right]}^{\Xi} (\mathbf{1}\kappa - \widetilde{\mu}) \tag{30}$$

which defines the scalar θ and the matrix Ξ, neither of which depends upon κ.

Step 2b. From eqs.(9) and (30), and since $\widehat{w}'(\kappa)\widetilde{\mu} = \kappa$ and $\kappa \geq \pi_{\rm c}$, we express the robustness as a function of κ:

$$\widehat{\alpha}(\widehat{w}(\kappa), \pi_{\rm c}) = \frac{\kappa - \pi_{\rm c}}{\theta\sqrt{(\mathbf{1}\kappa - \widetilde{\mu})'\Omega(\mathbf{1}\kappa - \widetilde{\mu})}} \tag{31}$$

where we have defined $\Omega = \Xi'\Sigma\Xi$.

$\widehat{\alpha}(\widehat{w}(\kappa), \pi_c)$ and $\widehat{\alpha}^2(\widehat{w}(\kappa), \pi_c)$ reach extrema at the same values of κ. Analysis of $\widehat{\alpha}^2(\widehat{w}(\kappa), \pi_c)$ reveals that its only extremum occurs at:

$$\widehat{\kappa} = \frac{(\pi_c \mathbf{1} - \widetilde{\mu})' \Omega \widetilde{\mu}}{(\pi_c \mathbf{1} - \widetilde{\mu})' \Omega \mathbf{1}} \tag{32}$$

provided that this is no less than π_c; otherwise the extremum occurs at $\kappa = \infty$.

From eq.(31) we see that:

$$\widehat{\alpha}(\widehat{w}(\kappa = \pi_c), \pi_c) = 0, \quad \text{and} \quad \lim_{\kappa \to \infty} \widehat{\alpha}(\widehat{w}(\kappa), \pi_c) > 0 \tag{33}$$

Hence the extremum of $\widehat{\alpha}(\widehat{w}(\kappa), \pi_c)$, obtained with $\widehat{\kappa}$ of eq.(32), must be a maximum.

In **summary** of case 2: the robust-optimal investment vector, $\widehat{w}(\widehat{\kappa})$, is obtained by combining eqs.(30) and (32). The maximal robustness, $\widehat{\alpha}(\widehat{w}(\widehat{\kappa}), \pi_c)$, is obtained by combining eqs.(31) and (32). We note that $\widehat{\kappa}$ varies nonlinearly with the magnitude of the anticipated profit vector $\widetilde{\mu}$. Consequently both the optimal investment, $\widehat{w}(\widehat{\kappa})$, and the maximal robustness, $\widehat{\alpha}(\widehat{w}(\widehat{\kappa}), \pi_c)$, vary as $\widetilde{\mu}$ changes. This is unlike case 1, for which the optimal investment is independent of $\widetilde{\mu}$ and the maximal robustness varies affinely with the magnitude of $\widetilde{\mu}$.

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Figure Captions

1. Determining the centerpoint, $\widetilde{\mu}$, given portfolio weights w (two-country case).
2. CAPM annual shadow costs versus year.
3. Demanded robustness, $\widehat{\alpha}_{\rm d}$, versus year. Shadow cost at each point equals the corresponding CAPM value.
4. Profit-uncertainty ellipses, μ_2 (vertical) vs. μ_1 (horizontal). Shadow cost at each point equals the corresponding CAPM value. Year 1990: dotted; 1995: dashed; 2000: solid.
5. Demanded robustness, $\widehat{\alpha}_{\rm d}$, versus year. Shadow cost at each point is 0.01 (annual).
6. Profit-uncertainty ellipses, μ_2 (vertical) vs. μ_1 (horizontal). Shadow cost at each point is 0.01 (annual). Year 1990: dotted; 1995: dashed; 2000: solid.
7. Determining the robustness $\widehat{\alpha}(w, \pi_{\rm c})$ (two-country case).
8. Determining the robustness-optimizing portfolio $\widehat{w}$ (two-country case).
9. Geometrical explanation of the CAPM shadow cost: the sum of the two dotted lines.
10. Geometrical explanation of the info-gap shadow cost: the sum of the two dotted lines.

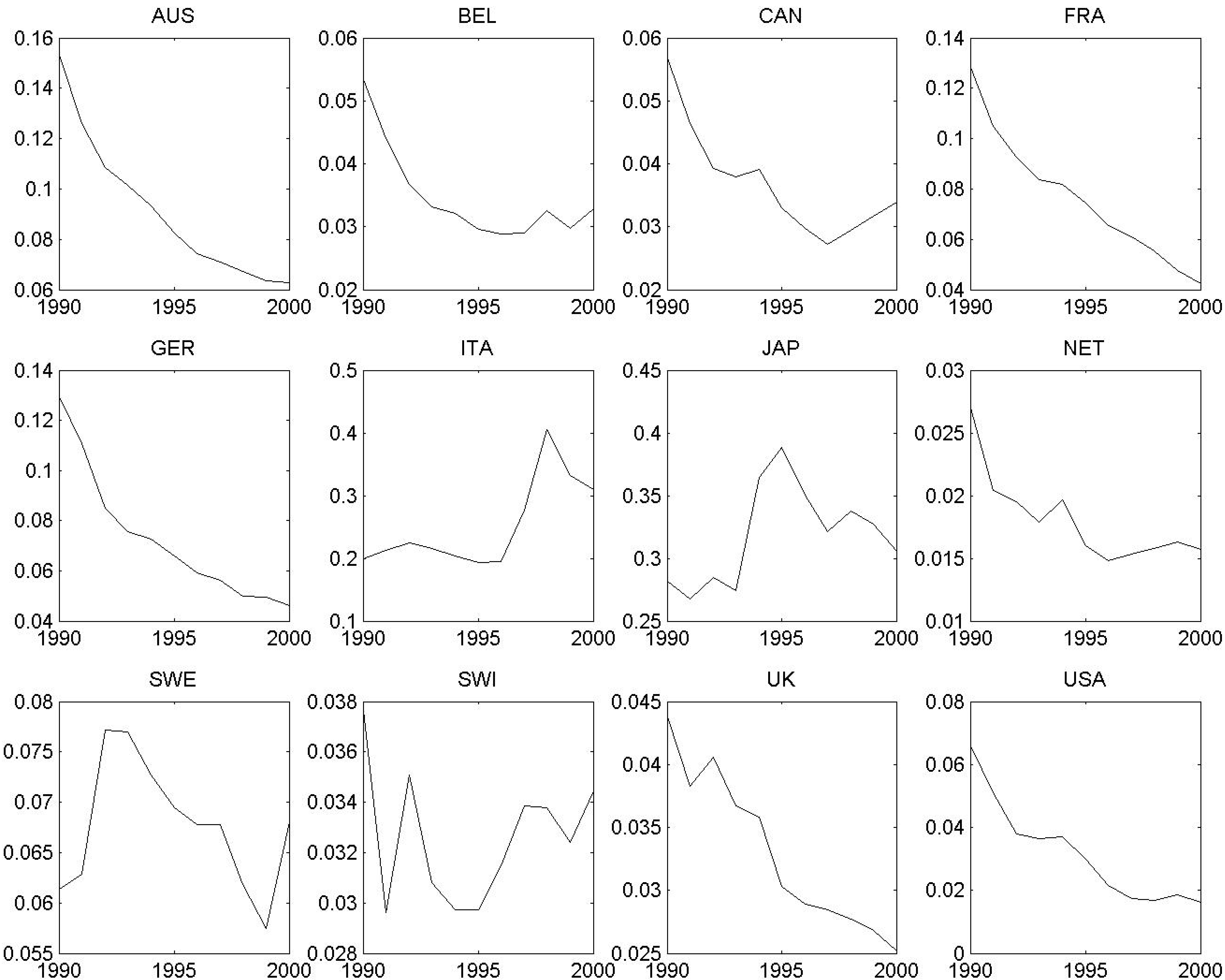
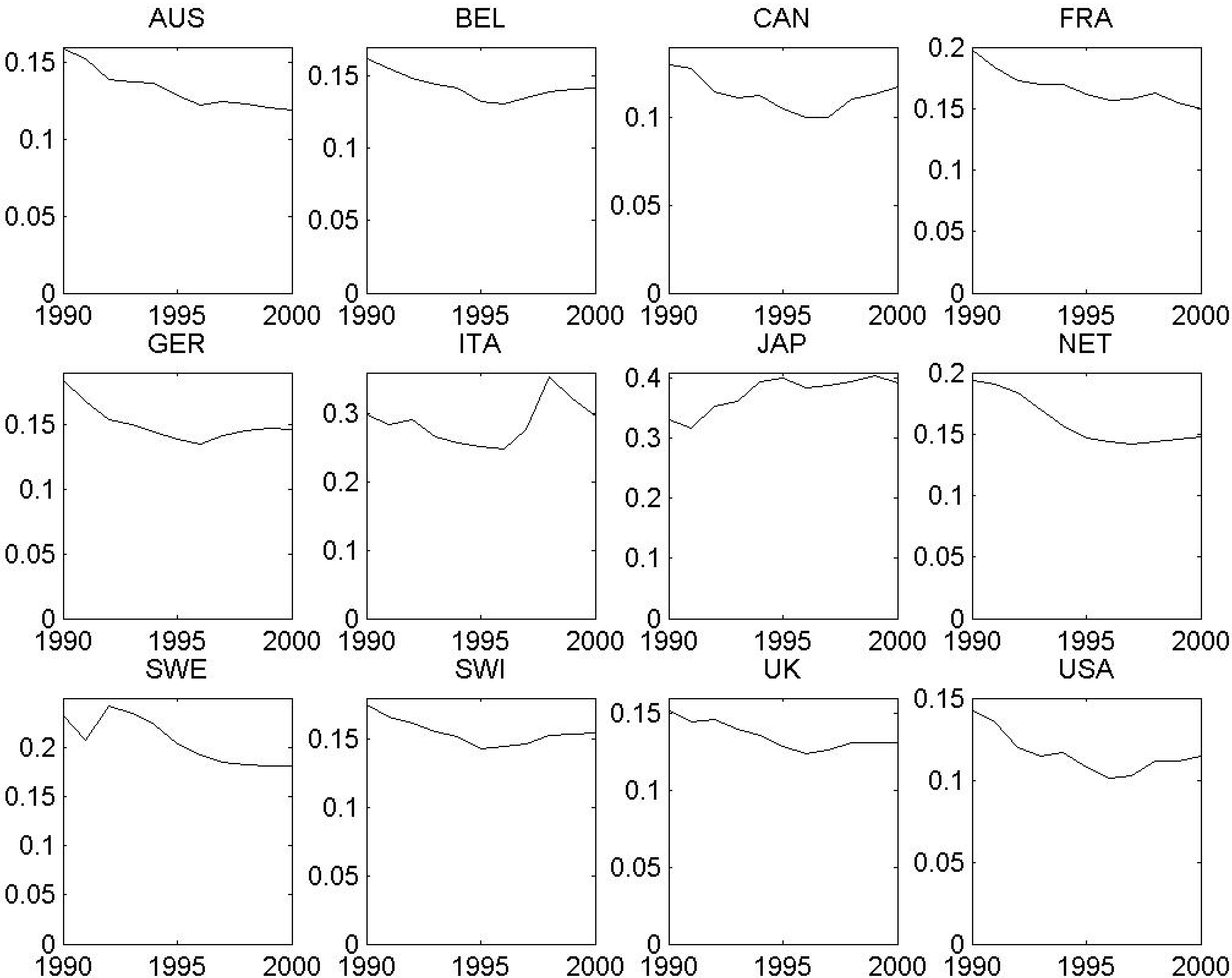


Figure 2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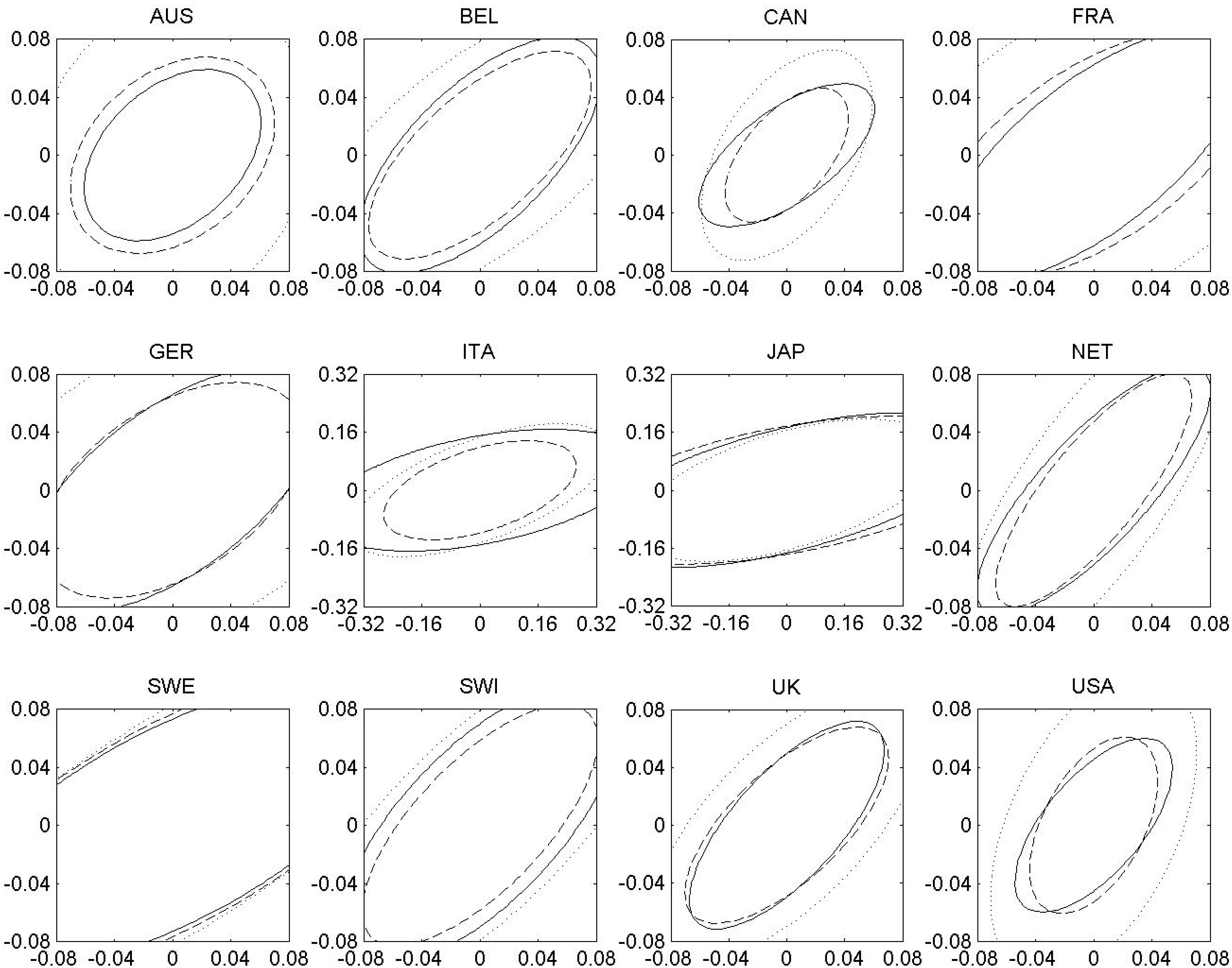
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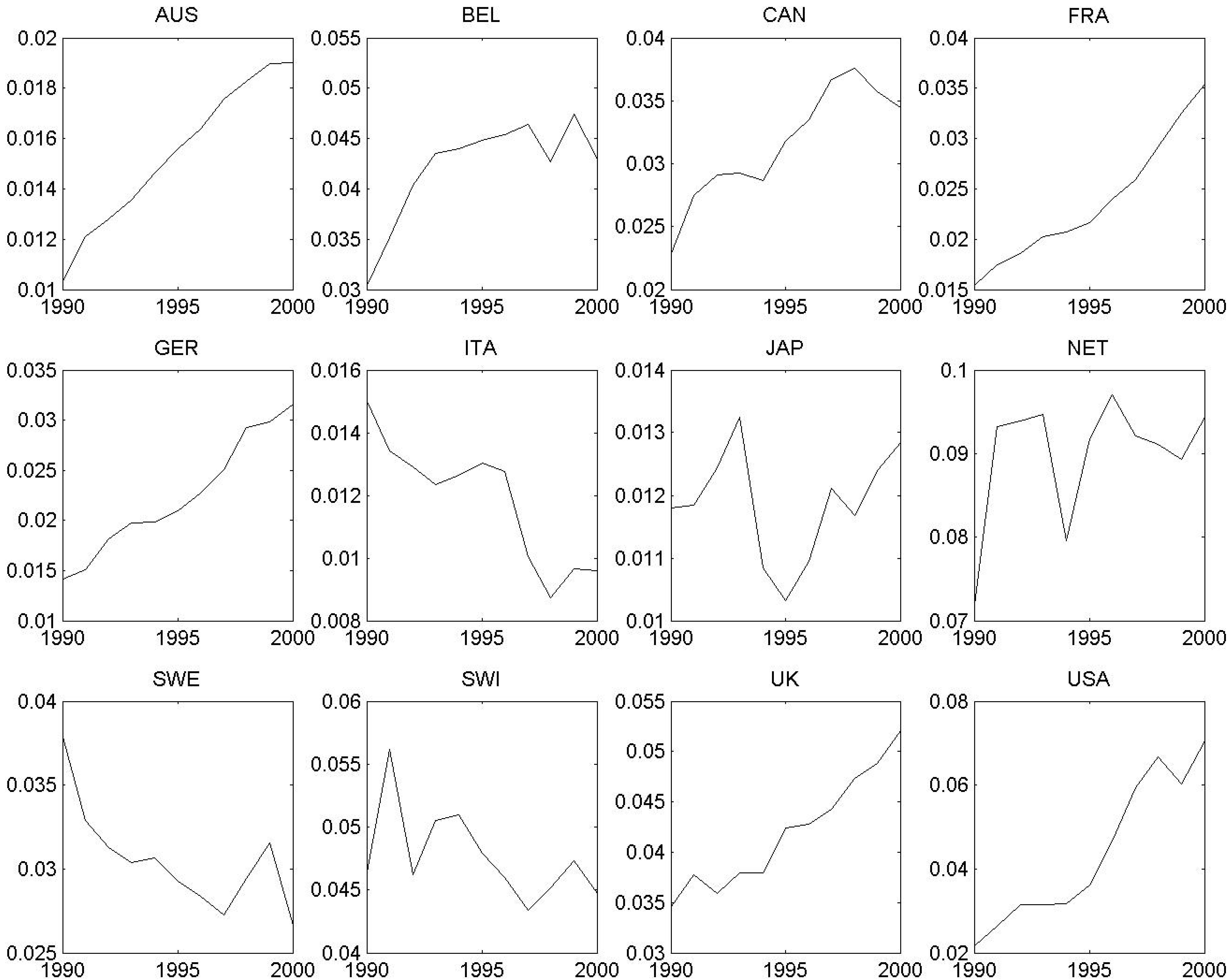

Figure 5

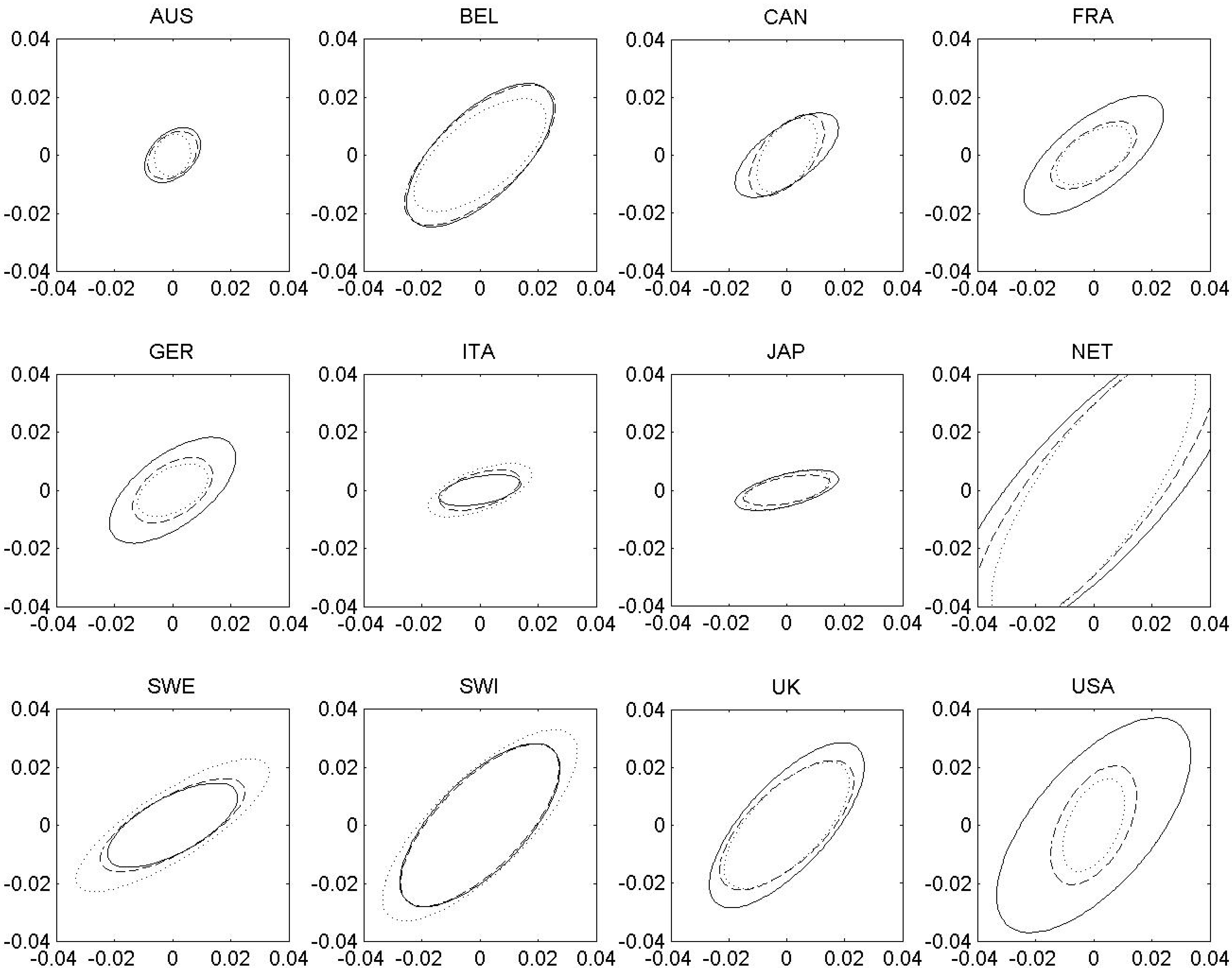

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Figure 6