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Abstract: This paper studies the implications of internal consumption habit for propagation and monetary transmission in New Keynesian dynamic stochastic general equilibrium (NKDSGE) models. We use Bayesian methods to evaluate the role of internal consumption habit in NKDSGE model propagation and monetary transmission. Simulation experiments show that internal consumption habit often improves NKDSGE model fit to output and consumption growth spectra by dampening business cycle periodicity. Nonetheless, habit NKDSGE model fit is vulnerable to nominal rigidity, the choice of monetary policy rule, the frequencies used for evaluation, and spectra identified by permanent productivity shocks.

JEL classification: E10, E20, E32

Key words: habit, New Keynesian, propagation, monetary transmission, Bayesian Monte Carlo

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Please address questions regarding content to Takashi Kano, Faculty of Economics, Graduate School of Economics, University of Tokyo, 7–3–1 Hongo, Tokyo, Japan, tkano@e.u-tokyo.ac.jp, or James M. Nason, Research Department, Federal Reserve Bank of Atlanta, 1000 Peachtree Street, N.E., Atlanta, GA 30309-4470, jim.nason@atl.frb.org.

1. Introduction

It is a 'folk-theorem' of macroeconomics that, "All models are false." A sufficiently rich set of stylized facts will reject a dynamic stochastic general equilibrium (DSGE) model. One response finds optimal moments to evaluate a DSGE model, which follows from Hansen (1982). Another approach focuses on sample moments relevant for students of the business cycle.

This paper takes the latter tack to study the business cycle implications of consumption habit for new Keynesian (NK)DSGE models. These models often rely on the real rigidity of internal consumption habit to obtain a better fit to sample moments.[1] Typical is the NKDSGE model analyzed by Del Negro, Schorfheide, Smets, and Wouters (2007).[2] They find that *external* consumption habit is important for matching the hump-shaped output response to a monetary policy shock. This result contrasts with estimates of NKDSGE models reported by Christiano, Eichenbaum, and Evans (2005). In their NKDSGE models, eliminating *internal* consumption habit matters little for replicating the transmission of monetary policy shocks to output.

Lettau and Uhlig (2000) and Otrok, Ravikumar, and Whiteman (2002) also study consumption habit in DSGE models.[3] Instead of the effect on model fit of habit, their focus is on its unintended consequences. According to Lettau and Uhlig consumption habit may solve asset pricing puzzles, but in real business cycle (RBC) models it creates excess consumption smoothness compared to U.S. data. The reason is that habit drives down the local elasticity of substitution. Otrok, Ravikumar, and Whiteman examine habit with spectral utility functions that break consumption volatility down frequency by frequency. A spectral utility decomposition reveals that households are averse to high-frequency consumption movements under

[1] Consumption habit is first adapted to a growth model by Ryder and Heal (1973). Nason (1988), Sundaresan (1989), and Constantinides (1990) are early attempts at solving risk-free rate and equity premium puzzles with consumption habit. However, Pollak (1976) shows that long-run utility with linear habit describes long-run behavior rather than long-run preferences. Rozen (2008) gives an axiomatic treatment of linear intrinsic habit.

[2] Schmitt-Grohé and Uribe (2007) is an excellent survey of habit in macro and finance; also see Nason (1997).

[3] Other critiques of consumption habit are Dynan (2000) and Kano (2009). Dynan rejects estimated moment conditions restricted by consumption habit on U.S.household panel data. Kano develops an observationally equivalence for current account dynamics for consumption habit and a world interest rate shock in a small open economy model. See Ravina (2007) and Gruber (2004) for evidence that supports consumption habit.

habit which explains its ability to solve risk-free rate and equity premium puzzles.

This paper is inspired by Lettau and Uhlig and Otrok, Ravikumar, and Whiteman to explore the role consumption habit has in NKDSGE model propagation and monetary transmission. We frame NKDSGE model propagation and monetary transmission with output and consumption growth spectral densities (SDs). These moments direct attention to the impact habit has on output and consumption growth periodicity. Our choice of these SDs is also guided by business cycle theory and the permanent income hypothesis (PIH). The PIH predicts a flat consumption growth SD, which Galí (1991) notes is at odds with U.S. data. Cogley and Nason (1995b) observe that DSGE models often cannot match the U.S. output growth SD because it peaks between seven and two years per cycle. They and Nason and Cogley (1994) find many DSGE models fail to replicate output's response to permanent and transitory shocks.

The NKDSGE models are borrowed from Christiano, Eichenbaum, and Evans (CEE). Their NKDSGE models have households whose preferences include (additive) internal consumption habit. This paper ties propagation and monetary transmission driven by internal consumption habit to intertemporal complementarity in future near-dated consumption. Our evidence about propagation and monetary transmission offers a resolution to the conflicting evidence of Del Negro, Schorfheide, Smets, and Wouters and CEE by gauging the fit of habit and non-habit NKDSGE models to output and consumption growth SDs.[4]

In the CEE model, the only disturbance is a transitory monetary policy shock. Besides monetary transmission, this paper also studies propagation in NKDSGE models given a random walk total factor productivity (TFP) shock. With these TFP and monetary policy shocks, a NKDSGE model satisfies long-run monetary neutrality (LRMN).

We invoke LRMN to identify *permanent* and *transitory* output and consumption growth SDs. These moments are computed using structural vector moving averages (SVMAs) of output (or consumption) growth and inflation that are just-identified by LRMN. The NKDSGE models

[4]The appendix shows that focusing on internal consumption habit sacrifices little generality because it and external habit can produce observationally equivalent linear approximate consumption growth dynamics.

predict that SVMAs are driven by current and lagged TFP and monetary policy shocks. Since these shocks are orthogonal at all leads and lags, SVMAs serve to parameterize permanent and transitory output and consumption growth SDs.

We examine a problem presented by Del Negro and Schorfheide (2008) conditional on LRMN. They find that priors can make it difficult to settle on which if any nominal rigidity is key for NKDSGE model fit using aggregate time series and Bayesian estimation methods. Rather than rely on Bayesian estimation of NKDSGE models, this paper explores the match between permanent and transitory output and consumption growth SDs using Bayesian calibrated habit and non-habit NKDSGE models that contain sticky prices and wages, only sticky prices, or just sticky wages. The fit of these NKDSGE models provides evidence about which, if any, of these rigidities matter for propagation and monetary transmission.

The permanent-transitory decomposition also gives us the opportunity to address an issue raised by Dupor, Han, and Tsai (2009). They obtain estimates of NKDSGE model parameters that are sensitive to whether technology or monetary policy shocks are used for identification. This paper explores this issue by asking if Bayesian calibrated NKDSGE models with different combinations of sticky prices and wages fit better to permanent or transitory output and consumption growth SDs.

This paper employs Bayesian calibration and simulation methods to study NKDSGE model propagation and monetary transmission. We adapt the Bayesian approach of DeJong, Ingram, and Whiteman (1996) and Geweke (2007) to conduct model evaluation. Geweke calls this the minimal econometric approach because it relies neither on likelihood-based tools nor arbitrarily focuses on a few moments while ignoring the rest of the predictive density of a NKDSGE model. Instead, the minimal econometric approach uses distributions of moments computed from atheoretic econometric models to link NKDSGE models to observable data.

We apply the minimal econometric approach by using SVMAs to tie NKDSGE models to sample permanent and transitory output and consumption growth SDs. Sample data, a SVMA, its priors, and Markov chain Monte Carlo (MCMC) simulators create posteriors that yield *empir-*

ical distributions of population SDs. *Theoretical* distributions of population SDs are garnered from SVMAs estimated on synthetic data that are simulated from calibrated NKDSGE models whose parameters are drawn from priors. We study propagation and monetary transmission with means of empirical and theoretical SD distributions. NKDSGE model fit is evaluated with the Kolmogorov-Smirnov (*KS*) statistic because it distills a multi-dimensional SD into a scalar. A NKDSGE model earns a good fit if its theoretical *KS* statistic distributions intersect empirical *KS* statistic distributions. This measure of fit constitutes a 'joint test' of NKDSGE model fit because theoretical SDs must match empirical SDs at several frequencies to achieve substantial overlap of empirical and theoretical *KS* statistic distributions.

The rest of the paper is constructed as follows. Section 2 discusses internal consumption habit and NKDSGE models. The Bayesian minimal econometric approach to DSGE model evaluation is reviewed in section 3. Results appear in section 4. Section 5 concludes.

2. Internal Consumption Habit and NKDSGE Models

This section describes household preferences with internal consumption habit, studies internal consumption habit propagation, connects it to intertemporal complementarity in future near-dated consumption, and outlines a NKDSGE model.

2.1 Internal consumption habit

Consumption habit is often superinduced in NKDSGE models to improve fit. This paper adopts additive internal consumption habit. Internal habit operates on lagged household consumption, unlike external habit which assume lags of aggregate consumption appear in utility, of which the (multiplicative) 'catching-up-with-the-Joneses' specification of Abel (1990) is typical. We assume that household preferences are intertemporally separable as well as separable across (net) consumption flow, labor disutility, and real balances

$$\mathcal{U}\left(c_t, c_{t-1}, n_t, \frac{H_t}{P_t}\right) = \ln[c_t - hc_{t-1}] - \frac{n_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} + \ln\left[\frac{H_t}{P_t}\right], \quad 0 < \gamma, \tag{1}$$

where c_t, n_t, H_t, P_t, and H_t/P_t, are household consumption, labor supply, the household's stock of cash at the end of date $t-1$, the aggregate price level, and real balances, respectively. We also maintain that $h \in (0, 1)$ and $0 < c_t - hc_{t-1}$, $\forall\ t$. Since internal habit ties current household consumption choice to its past consumption, the marginal utility of consumption is forward-looking, $\lambda_t = \dfrac{1}{c_t - hc_{t-1}} - \mathbf{E}_t\left\{\dfrac{\beta h}{c_{t+1} - hc_t}\right\}$, where $\beta \in (0, 1)$ is the household discount factor and $\mathbf{E}_t\{\cdot\}$ is the mathematical expectation operator given date t information.[5]

2.2 The internal consumption habit propagation mechanism

Forward-looking marginal utility suggests internal habit acts as propagation mechanism for consumption. We study this mechanism with a log linear approximation of the Euler equation $\lambda_t = \mathbf{E}_t\left\{\lambda_{t+1}R_{t+1}/(1+\pi_{t+1})\right\}$, where R_t is the nominal rate and $1+\pi_{t+1}$ $(= P_{t+1}/P_t)$ is date $t+1$ inflation. The log linear approximation gives a second order stochastic difference equation for demeaned consumption growth, $\widetilde{\Delta c}_t$, whose solution is

$$\widetilde{\Delta c}_t \;=\; \varphi_1 \widetilde{\Delta c}_{t-1} \;+\; \frac{\Psi}{\varphi_2}\sum_{j=0}^{\infty}\varphi_2^{-j}\mathbf{E}_t\tilde{q}_{t+j}, \tag{2}$$

where the stable and unstable roots are $\varphi_1 = h\alpha^{*-1}$ and $\varphi_2 = \alpha^*(\beta h)^{-1}$, α^* is the steady state growth rate of the economy, the demeaned real rate is $\tilde{q}_t = \tilde{R}_t - \dfrac{\pi^*}{1+\pi^*}\tilde{\pi}_t$, π^* is mean inflation, and Ψ is a constant that is nonlinear in model parameters.[6]

We analyze internal consumption habit propagation using the solved linearized Euler equation (2). This is depicted in figure 1 with impulse response functions (IRFs) generated by equation (2) and a one percent shock to $\tilde{q}_t$. The calibration sets $[\beta\ \ \alpha^*]' = [0.993\ \ \exp(0.004)]'$, $h = [0.15\ 0.35\ 0.50\ 0.65\ 0.85]$, and $\tilde{q}_t$ to a quarterly first-order autoregression, AR(1), with a AR1 coefficient of 0.869.[7] Figure 1 shows that at impact $\widetilde{\Delta c}_t$ is driven higher. However, its

[5] Dunn and Singleton (1986), Eichenbaum and Hansen (1990), and Heaton (1995) estimate consumption-based asset pricing models with habit and local substitution through service flows. The adjustment cost hypothesis is rejected in favor of services flows according to their estimates. However, habit appears in the data if local substitutability operates at lower frequencies than the sampling frequency of consumption.

[6] The appendix constructs equation (2), which assumes a unit root TFP shock drives trend consumption.

[7] The real demeaned federal funds rate $\tilde{q}_t$ equals the quarterly nominal federal funds rate net of implicit GDP deflator inflation multiplied by the ratio of its mean to one plus its mean. The SIC selects a AR(1) for $\tilde{q}_t$ over any lag length up to ten on a 1954Q1–2002Q4 sample. The appendix has details.

response falls from about one to 0.11 percent as h rises from 0.15 to 0.85. Figure 1 also displays IRFs that are shifted to the right with higher peaks and slower decay rates as $h \longrightarrow 1$. Thus, as internal habit becomes stronger, it dictates greater utility costs that persuades the household to move longer sequences of future near-dated consumption in tandem.

The internal consumption habit propagation mechanism is also discussed by CEE. They note that in their NKDSGE model, in which h is estimated to be about 0.65, internal consumption habit generates a hump-shaped consumption response to a real rate shock. Figure 1 reveals a similar internal consumption habit propagation mechanism for equation (2) that relies on $h \geq 0.5$ to produce a humped-shaped IRF with a peak at or beyond two quarters. This mechanism contrasts with $h \in (0, 0.5)$ or the non-habit model, $h = 0$, in which a linear approximation of the Euler equation sets $\mathbf{E}_t\left\{\widetilde{\Delta c}_{t+1} - \tilde{q}_{t+1}\right\} = 0$. In these cases, figure 1 suggests that consumption growth dynamics are dominated by the time series properties of $\tilde{q}_t$.

Greater risk aversion is often cited as the reason that consumption habit is a useful real rigidity to improve model fit. This explanation is bound up with consumption habit lowering the (local) elasticity of substitution. An equivalent notion is that consumption habit imposes utility costs on intertemporal consumption choice. For example, as h rises from zero toward one, the household comes to view near-dated consumption as complements rather than substitutes. According to figure 1, this switch creates an economically important internal consumption habit propagation mechanism as h moves past 0.5 and closes in on one.

This paper studies the business cycle implications of internal consumption habit for NKDSGE models. Nonetheless, the results of this paper should extend beyond internal consumption habit to external habit. In the appendix, we show that internal and external habit produce equivalent consumption growth IRFs after impact given $\tilde{q}_t$ is a AR(1).[8] Given this, there is little lost by focusing on internal consumption habit. Also, the appendix finds that the impact response of $\widetilde{\Delta c}_t$ becomes large under external consumption habit as $h \longrightarrow 1$.

[8]The observational equivalence can extend to multiplicative internal and external consumption habit using the onto mapping from additive to multiplicative consumption habit parameters that Dennis (2009) constructs.

2.3 A new Keynesian DSGE model

We adapt the NKDSGE model of CEE. The model contains (a) internal consumption habit, (b) capital adjustment costs, (c) variable capital utilization, (d) fully indexed Calvo-staggered price setting by monopolistic final goods firms, and (e) fully indexed Calvo-staggered wage setting by monopolistic households with heterogenous labor supply.

Households reside on the unit circle with addresses $\ell \in [0, 1]$. The budget constraint of household ℓ is

$$\frac{H_{t+1}}{P_t} + \frac{B_{t+1}}{P_t} + c_t + x_t + a(u_t)k_t + \tau_t \;=\; r_t u_t k_t + \frac{W_t(\ell)}{P_t} n_t(\ell) + \frac{H_t}{P_t} + R_t \frac{B_t}{P_t} + \frac{D_t}{P_t}, \tag{3}$$

where B_{t+1} is the stock of government bonds the household carries from date t into date $t+1$, x_t is investment, k_t is household capital at the end of date t, τ_t is a lump sum government transfer, r_t is the real rental rate of k_t, $W_t(\ell)$ is the nominal wage paid to household ℓ, R_t is the nominal return on B_t, D_t is dividends received from firms, $u_t \in (0, 1)$ is the capital utilization rate, and $a(u_t)$ is its cost function. At the steady state, $u^* = 1$, $a(1) = 0$ and to achieve a determinate solution $\dfrac{a''(1)}{a'(1)} = 1.174$. Note that u_t forces household ℓ to forgo $a(\cdot)$ units of consumption per unit of capital. The CCE adjustment costs specification is placed into the law of motion of household capital

$$k_{t+1} \;=\; (1 - \delta)k_t \;+\; \left[1 - S\left(\frac{1}{\alpha}\frac{x_t}{x_{t-1}}\right)\right] x_t, \quad \delta \in (0, 1), \quad 0 < \alpha, \tag{4}$$

where δ is the capital depreciation rate and α ($= \ln \alpha^*$) is deterministic TFP growth. The cost function $S(\cdot)$ is strictly convex, where $S(1) = S'(1) = 0$ and $S''(1) \equiv \varpi > 0$. In this case, the steady state is independent of the adjustment cost function $S(\cdot)$.

Given k_0, B_0, and c_{-1}, the expected discounted lifetime utility function of household ℓ

$$\mathbf{E}_t \left\{ \sum_{i=0}^{\infty} \beta^i \mathcal{U}\left(c_{t+i},\, c_{t+i-1},\, n_{t+i}(\ell),\, \frac{H_t}{P_t}\right) \right\} \tag{5}$$

is maximized by choosing c_t, k_{t+1}, H_{t+1}, B_{t+1}, and $W_t(\ell)$ subject to period utility (1), budget constraint (3), the law of motion of capital (4), and downward sloping labor demand.

Monopolistically competitive firms produce the final goods that households consume. The consumption aggregator is $c_t = \left[\int_0^1 y_{D,t}(j)^{(\xi-1)/\xi} dj\right]^{\xi/(\xi-1)}$, where $y_{D,t}(j)$ is household final good demand for a firm with address j on the unit interval. Final good firm j maximizes its profits by setting its price $P_t(j)$, subject to $y_{D,t}(j) = [P_t/P_t(j)]^{\xi} Y_{D,t}$, where ξ is the price elasticity, $Y_{D,t}$ is aggregate demand, and the price index is a $P_t = \left[\int_0^1 P_t(j)^{1-\xi}\right]^{1/(1-\xi)}$.

The jth final good firm mixes capital, $K_t(j)$, rented and labor, $N_t(j)$, hired from households (net of fixed cost N_0) with labor-augmenting TFP, A_t, in the constant returns to scale technology, $[u_t K_t(j)]^{\psi} [(N_t(j) - N_0)A_t]^{1-\psi}$, $\psi \in (0, 1)$, to create output, $y_t(j)$. TFP is a random walk with drift, $A_t = A_{t-1}\exp\{\alpha + \varepsilon_t\}$, and ε_t its Gaussian innovation, $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$.

Calvo-staggered price setting restricts a firm to update to optimal price $P_{c,t}$ at probability $1 - \mu_P$. Or with probability μ_P, firms are stuck with date $t-1$ prices scaled by inflation of the same date, π_{t-1}. This gives the price aggregator $P_t = \left[(1-\mu_P)P_{c,t}^{1-\xi} + \mu_P\left(\pi_{t-1}P_{t-1}\right)^{1-\xi}\right]^{1/(1-\xi)}$. Under full price indexation, Calvo-pricing yields the optimal forward-looking price

$$\frac{P_{c,t}}{P_{t-1}} = \left(\frac{\xi}{\xi-1}\right) \frac{\mathbf{E}_t \sum_{i=0}^{\infty} (\beta\mu_P)^i \lambda_{t+i}\phi_{t+i} Y_{D,t+i}\pi_{t+i}^{\xi}}{\mathbf{E}_t \sum_{i=0}^{\infty} (\beta\mu_P)^i \lambda_{t+i} Y_{D,t+i}\pi_{t+i}^{\xi-1}} \tag{6}$$

of a firm able to update its price.

Households offer differentiated labor services to firms in a monopolistic market in which a Calvo staggered nominal wage mechanism operates. We assume the labor supply aggregator $N_t(j) = \left[\int_0^1 n_t(\ell)^{(\theta-1)/\theta} d\ell\right]^{\theta/(\theta-1)}$, where θ is the wage elasticity. Labor market monopoly force firms to face downward sloping labor demand schedules for differentiated labor services, $n_t(\ell) = \left[W_t/W_t(\ell)\right]^{\theta} N_t(j)$, where the nominal wage index is $W_t = \left[\int_0^1 W_t(\ell)^{1-\theta} d\ell\right]^{1/(1-\theta)}$. The nominal wage aggregator is $W_t = \left[(1-\mu_W)W_{c,t}^{1-\theta} + \mu_W\left(\alpha^*\pi_{t-1}W_{t-1}\right)^{1-\theta}\right]^{1/(1-\theta)}$, which has households updating their desired nominal wage $W_{c,t}$ at probability $1-\mu_W$. With probability μ_W, households receive the date $t-1$ nominal wage indexed by steady state TFP growth, $\alpha^* = \exp(\alpha)$, and lagged inflation. In this case, the optimal nominal wage condition is

$$\left[\frac{W_{c,t}}{P_{t-1}}\right]^{1+\theta/\gamma} = \left(\frac{\theta}{\theta-1}\right)\frac{\mathbf{E}_t\sum_{i=0}^{\infty}\left[\beta\mu_W\alpha^{*-\theta(1+1/\gamma)}\right]^i\left[\left[\frac{W_{t+i}}{P_{t+i-1}}\right]^{\theta}N_{t+i}\right]^{1+1/\gamma}}{\mathbf{E}_t\sum_{i=0}^{\infty}\left[\beta\mu_W\alpha^{*(1-\theta)}\right]^i\lambda_{t+i}\left[\frac{W_{t+i}}{P_{t+i-1}}\right]^{\theta}\left[\frac{P_{t+i}}{P_{t+i-1}}\right]^{-1}N_{t+i}}, \tag{7}$$

because households solve a fully indexed Calvo-pricing problem.

We close the NKDSGE model with one of two monetary policy rules. CEE identify monetary policy with a money growth process that is a structural infinite-order moving average, SMA(∞). As CEE note, the SMA(∞) is equivalent to the AR(1) money growth supply rule

$$\ln M_{t+1} - \ln M_t = m_{t+1} = (1-\rho_m)m^* + \rho_m m_t + \mu_t, \quad \left|\rho_m\right| < 1, \quad \mu_t \sim \mathcal{N}\left(0,\, \sigma^2_\mu\right), \tag{8}$$

where m^* is mean money growth and the money growth innovation is μ_t. NKDSGE-AR defines models with the money growth rule (8). Monetary policy is described with the Taylor rule

$$(1-\rho_R\mathbf{L})R_t = (1-\rho_R)\left(R^* + a_\pi\mathbf{E}_t\pi_{t+1} + a_{\tilde{Y}}\tilde{Y}_t\right) + \upsilon_t, \quad \left|\rho_R\right| < 1, \quad \upsilon_t \sim \mathcal{N}\left(0,\, \sigma^2_\upsilon\right), \tag{9}$$

in NKDSGE-TR models, where $R^* = \pi^*/\beta$ and $\pi^* = \exp(m^* - \alpha)$. Under the interest rate rule (9), the monetary authority obeys the 'Taylor' principle, $1 < a_\pi$, and sets $a_{\tilde{Y}} \in (0, 1)$. This assumes the monetary authority computes private sector inflationary expectations, $\mathbf{E}_t\pi_{t+1}$, and mean-zero transitory output, $\tilde{Y}_t$, without inducing measurement errors.

The government finances B_t, interest on B_t, and a lump-sum transfer τ_t with new bond issuance $B_{t+1} - B_t$, lump-sum taxes τ_t, and money creation, $M_{t+1} - M_t$. Under either monetary policy rule, the government budget constraint is $P_t\tau_t = [M_{t+1} - M_t] + [B_{t+1} - (1+R_t)B_t]$. Government debt is in zero net supply, $B_{t+1} = 0$ and the nominal lump-sum transfer equals the monetary transfer, $P_t\tau_t = M_{t+1} - M_t$, along the equilibrium path at all dates t.

Equilibrium requires goods, labor, and money markets clear in the decentralized economy. This occurs when $K_t = k_t$ given $0 < r_t$, $N_t = n_t$ given $0 < W_t$, $M_t = H_t$, and also requires P_t, and R_t are strictly positive and finite. This leads to the aggregate resource constraint, $Y_t = C_t + I_t + a(u_t)K_t$, where aggregate consumption $C_t = c_t$ and aggregate investment $I_t = x_t$. A rational expectations equilibrium equates, on average, firm and household subjective forecasts of r_t and A_t to the objective outcomes generated by the decentralized economy. We add to this list μ_t and R_t, υ_t, P_t, or W_t under the money growth rule (8), the interest rate rule (9), a flexible price regime, or a competitive labor market, respectively.

3. Bayesian Monte Carlo Strategy

This section outlines Bayesian Monte Carlo methods of DeJong, Ingram, and Whiteman (1996) and Geweke (2007) that we use to assess NKDSGE model fit. DeJong, Ingram, and Whiteman (DIW) and Geweke eschew standard calibration and likelihood-based tools because, in their view, a NKDSGE model lacks predictions for all but population moments. We follow their approach and evaluate NKDSGE models with atheoretic econometric models that tie observed sample data to population moments.

3.1 Solution methods and Bayesian calibration of the DSGE models

Several steps are needed to solve and simulate NKDSGE models. The models have a permanent TFP shock, which requires stochastic detrending of optimality and equilibrium conditions before log-linearizing around deterministic steady states. We engage an algorithm of Sims (2002), sketched in the appendix, to solve for linear approximate equilibrium laws of motion of a NKDSGE model. Synthetic samples result from feeding TFP and monetary policy shocks into these equilibrium laws of motion given initial conditions and draws from priors of NKDSGE model parameters.

Priors embed our uncertainty about NKDSGE model parameters, which endow population SDs with theoretical distributions; see Geweke (2007). Table 1 lists these priors. For example,

h has an uninformative prior that is drawn from an uniform distribution with end points 0.05 and 0.95 in table 1. The uninformative prior reflects a belief that any $h \in \left[0.05, 0.95\right]$ is as likely as another. Non-habit NKDSGE models are defined by the degenerate prior $h = 0$.

Priors are also taken from earlier DSGE model studies. We place degenerate priors on $\left[\beta\ \gamma\ \delta\ \alpha\ \psi\right]' = \left[0.9930\ \ 1.3088\ \ 0.0200\ \ 0.0040\ \ 0.3500\right]'$ that are consistent with the Cogley and Nason (1995b) calibration. Uncertainty about $\left[\beta\ \gamma\ \delta\ \alpha\ \psi\right]'$ is captured by 95 percent coverage intervals, which include values in Nason and Cogley (1994), Hall (1996), and Chang, Gomes, and Shorfheide (2002). We set the prior of the investment cost of adjustment parameter ϖ to estimates reported by Bouakez, Cardia, and Ruge-Murcia (2005). An uninformative prior is imposed on the standard deviation of TFP shock innovations, σ_ϵ. The RBC literature suggests that any $\sigma_\epsilon \in [0.0070, 0.0140]$ is equally fair, which motivates our choice of this prior.

There are four sticky price and wage parameters to calibrate. The relevant prior means are $[\xi\ \mu_P\ \theta\ \mu_w]' = \left[8.0\ 0.55\ 15.0\ 0.7\right]'$. The mean of ξ implies a steady state price markup, $\xi/(\xi - 1)$, of 14 percent with a 95 percent coverage interval that runs from 11 to 19 percent. This coverage interval blankets estimates found in Basu and Fernald (1997) and CEE. More uncertainty surrounds the priors of μ_P, θ, and μ_w. For example, Sbordone (2002), Nason and Slotsve (2004), Lindé (2005), and CEE suggest a 95 percent coverage interval for μ_P of $[0.45, 0.65]$. Likewise, a 95 percent coverage interval of $[0.04, 0.25]$ suggests substantial uncertainty around the seven percent prior mean household wage markup, $\theta/(\theta - 1)$. However, the degenerate mean of μ_w and its 95 percent coverage interval reveals stickier nominal wages than prices, as found for example by CEE, but with the same degree of uncertainty.

The money growth rule (8) is calibrated to estimates from a $1954Q1$–$2002Q4$ sample of M1. The estimates are degenerate priors for $\left[m^*\ \rho_m\ \sigma_\mu\right]' = \left[0.015\ 0.627\ 0.006\right]'$. Precision of these estimates yield narrow 95 percent coverage intervals. For ρ_m, the lower end of its interval is near 0.5. CEE note that $\rho_m \approx 0.5$ implies the money growth rule (8) mimics their identified monetary policy shock process.

The calibration of the interest rate rule (9) obeys the Taylor principle and $a_y \in (0, 1)$.

The degenerate prior of a_π is 1.80. We assign a small role to movements in transitory output, $\tilde{Y}$, with a prior mean of 0.05 for a_y. The 95 percent coverage intervals of a_π and a_y rely on estimates that Smets and Wouters (2007) report. The interest rate rule (9) is also calibrated to smooth R_t given a prior mean of 0.65 and a 95 percent coverage interval of [0.55, 0.74]. Ireland (2001) is the source of the prior mean of the standard deviation of the monetary policy shock, $\sigma_\upsilon = 0.0051$, and its 95 percent coverage interval, [0.0031, 0.0072]. We assume all shock innovations are uncorrelated at all leads and lags (*i.e.*, $\mathbf{E}\{\varepsilon_{t+i}\,\upsilon_{t+q}\} = 0$, for all i, q).

3.2 Output and consumption moments

We evaluate NKDSGE model fit with output and consumption growth SDs. The SDs are calculated from just-identified SVMAs, which are identified with a LRMN restriction that is embedded in the NKDSGE model of section 2. In this model, LRMN ties the TFP innovation ε_t to the permanent shock. The transitory shock is identified with the money growth innovation μ_t or Taylor rule innovation υ_t. We recover the SVMAs from unrestricted VARs with the Blanchard and Quah (1989) decomposition. The VARs are estimated for $[\Delta \ln Y_t \;\; \Delta \ln P_t]'$ and $[\Delta \ln C_t \;\; \Delta \ln P_t]'$ using 1954Q1–2002Q4 and synthetic samples.[9]

We employ just-identified SVMAs to compute permanent and transitory output and consumption growth SDs. If the Taylor rule (9) is the source of the transitory monetary policy shock (υ_t), the just-identified SVMA is

$$\begin{bmatrix} \Delta \ln Y_t \\ \Delta \ln P_t \end{bmatrix} = \sum_{j=0}^{\infty} \mathbb{B}_j \begin{bmatrix} \varepsilon_{t-j} \\ \upsilon_{t-j} \end{bmatrix}, \quad \text{where} \quad \mathbb{B}_j = \begin{bmatrix} \mathbb{B}_{\Delta Y,\varepsilon,j} & \mathbb{B}_{\Delta Y,\upsilon,j} \\ \mathbb{B}_{\Delta P,\varepsilon,j} & \mathbb{B}_{\Delta P,\upsilon,j} \end{bmatrix}. \tag{10}$$

The elements of $\mathbb{B}_j$ are identified by the LRMN restriction $\mathbb{B}_{\Delta Y,\upsilon}(\mathbf{1}) = 0$ (*i.e.*, output is independent of the Taylor rule shock υ_t at the infinite horizon) and that the TFP shock ε_t and υ_t are orthogonal at all leads and lags; see the appendix for details. These restrictions per-

[9] VAR lag length is chosen using the sample data and likelihood ratio statistics testing down from a maximum of ten lags. These tests settle on a lag length of five for VARs of $[\Delta \ln Y_t \;\; \Delta \ln P_t]'$ and $[\Delta \ln C_t \;\; \Delta \ln P_t]'$.

mit the SVMA (10) to be decomposed into univariate SMA(∞)s of output growth, $\mathbb{B}_{\Delta Y,\varepsilon}(\mathbf{L})\varepsilon_t$ and $\mathbb{B}_{\Delta Y,\upsilon}(\mathbf{L})\upsilon_t$. The former (latter) SMA is the IRF of output growth with respect to the permanent shock, ε_t (transitory shock, υ_t). The SVMA (10) is also a Wold representation of $[\Delta \ln Y_t \;\; \Delta \ln P_t]'$ whose spectrum (at frequency ω) is $S_{[\Delta Y \;\; \Delta P]}(\omega) = (2\pi)^{-1}\Gamma_{[\Delta Y \;\; \Delta P]}\exp(-i\omega)$, where $\Gamma_{[\Delta Y \;\; \Delta P]}(l) = \sum_{j=0}^{\infty}\mathbb{B}_j\mathbb{B}'_{j-l}$. Expanding the convolution $\Gamma_{[\Delta Y \;\; \Delta P]}(l)$ at horizon j gives

$$\mathbb{B}_j\mathbb{B}'_{j-l} \;=\; \begin{bmatrix} \mathbb{B}_{\Delta Y,\varepsilon,j}\mathbb{B}_{\Delta Y,\varepsilon,j-l} + \mathbb{B}_{\Delta Y,\upsilon,j}\mathbb{B}_{\Delta Y,\upsilon,j-l} & \mathbb{B}_{\Delta Y,\varepsilon,j}\mathbb{B}_{\Delta P,\varepsilon,j-l} + \mathbb{B}_{\Delta Y,\upsilon,j}\mathbb{B}_{\Delta P,\upsilon,j-l} \\ \mathbb{B}_{\Delta P,\varepsilon,j}\mathbb{B}_{\Delta Y,\varepsilon,j-l} + \mathbb{B}_{\Delta P,\upsilon,j}\mathbb{B}_{\Delta Y,\upsilon,j-l} & \mathbb{B}_{\Delta P,\varepsilon,j}\mathbb{B}_{\Delta P,\varepsilon,j} + \mathbb{B}_{\Delta P,\upsilon,j}\mathbb{B}_{\Delta P,\upsilon,j-l} \end{bmatrix},$$

whose off-diagonal elements imply output growth and employment cross-covariances and, therefore, co- and quad-spectra, while the upper left diagonal elements contain output growth autocovariances $\mathbb{B}_{\Delta Y,\varepsilon,j}\mathbb{B}_{\Delta Y,\varepsilon,j-l}$ and $\mathbb{B}_{\Delta Y,\upsilon,j}\mathbb{B}_{\Delta Y,\upsilon,j-l}$. The autocovariances suggest treating the univariate output growth SMAs $\mathbb{B}_{\Delta Y,\varepsilon}(\mathbf{L})\varepsilon_t$ and $\mathbb{B}_{\Delta Y,\upsilon}(\mathbf{L})\upsilon_t$ as objects whose innovations are the permanent TFP shock ε_t and transitory Taylor rule shock υ_t. We employ these SMAs to parameterize permanent and transitory output growth SDs.[10] Given the BQ decomposition assumption $\sigma_\iota^2 = 1$, this gives us the output growth SD at frequency ω

$$S_{\Delta Y,\iota}(\omega) \;=\; \frac{1}{2\pi}\left|\mathbb{B}_{\Delta Y,\iota,0} + \mathbb{B}_{\Delta Y,\iota,1}e^{-i\omega} + \mathbb{B}_{\Delta Y,\iota,2}e^{-i2\omega} + \ldots + \mathbb{B}_{\Delta Y,\iota,j}e^{-ij\omega} + \ldots\right|^2, \quad \iota \;=\; \varepsilon,\, \upsilon.$$

Before computing $S_{\Delta Y,\iota}(\omega)$, we truncate its polynomial at $j = 40$, a ten year horizon.

3.3 Bayesian simulation methods

We use MCMC software of Geweke (1999) and McCausland (2004) to create posteriors of SVMAs given priors and a $1954Q1$–$2002Q4$ sample ($T = 196$) of U.S. output, consumption, and price growth.[11] The posteriors contain $\mathcal{J} = 5000$ SVMA parameter vectors that are the basis of empirical, $\mathcal{E}$, permanent and transitory output and consumption growth SD distributions.

[10] The idea of parameterizing permanent and transitory output and consumption growth SDs with a SMA extends ideas found in Akaike (1969) and Parzen (1974).

[11] The software is found at `http://www2.cirano.qc.ac/~bacc`, while the appendix describes the data.

The SVMAs are also engaged to create theoretical, $\mathcal{T}$, distributions of population permanent and transitory output and consumption growth SDs. The $\mathcal{T}$ SD distributions are computed using SVMAs estimated on $\mathcal{J}$ synthetic samples of length $\mathcal{M} \times T$, $\mathcal{M} = 5$, that are simulated from a linearized NKDSGE model conditional on priors placed on its parameters.[12] NKDSGE models are judged on the overlap of $\mathcal{T}$ and $\mathcal{E}$ moment distributions.

3.4 Measures of fit

Our metric for judging the fit of a NKDSGE model begins with Cogley and Nason (1995a). They measure the fit of DSGE models to sample moments using Kolmogorov-Smirnov (*KS*) and Cramer-von Mises (*CvM*) goodness of fit statistics.

This paper employs *KS* statistics, but in the context of Bayesian calibration experiments. The *KS* statistics are centered on the sample output (or consumption) growth SD, $\widehat{I_T}(\omega)$, which is constructed from SVMAs estimated on the actual data. At frequency ω, the jth draw from the ensemble of $\mathcal{E}$ SDs of output growth (or consumption growth) is $I_{\mathcal{E},T,j}(\omega)$. The associated draw from a $\mathcal{T}$ distribution is $I_{\mathcal{T},T,j}(\omega)$. Define the ratio $\mathcal{R}_{\mathcal{D},T,j}(\omega) = \widehat{I_T}(\omega) \ / \ I_{\mathcal{D},T,j}(\omega)$ at replication j, as well as its partial sum $\mathcal{V}_{\mathcal{D},T,j}\left(2\pi q/T\right) = 2\pi \sum_{\ell=1}^{q} \mathcal{R}_{\mathcal{D},T,j}\left(2\pi \ell/T\right)/T$, where $\mathcal{D} = \mathcal{E}$, $\mathcal{T}$. The partial sum serves to construct $\mathcal{B}_{T,\mathcal{D},j}(\kappa) = 0.5\sqrt{2T}\Big[\mathcal{V}_{T,\mathcal{D},j}(\kappa\pi) - \kappa\mathcal{V}_{T,\mathcal{D},j}(\pi)\Big]/\pi$, $\kappa \in [0, 1]$. If the 'partial' differences $\mathcal{B}_{T,\mathcal{D},j}(\cdot)$, $j = 1, \ldots, \mathcal{J}$, are small, the sample and $\mathcal{D}$ spectra are close.[13] Vectors of 'partial' differences $\left\{\mathcal{B}_{T,\mathcal{D},j}(\cdot)\right\}_{j=1}^{J}$ are collected to form $KS_{\mathcal{D},j} = Max\left|\mathcal{B}_{T,\mathcal{D},j}(\kappa)\right|$. Although *KS* statistics measure the distance between sample and $\mathcal{E}$ or $\mathcal{T}$ spectra, we employ distributions of $\mathcal{E}$ and $\mathcal{T}$ *KS* statistics to gauge the fit of the NKDSGE models. NKDSGE model fit is judged on the overlap of $\mathcal{E}$ and $\mathcal{T}$ distributions of *KS* statistics. Substantial overlap of these distributions indicate a good fit for a NKDSGE model.

The *KS* statistic is useful because it collapses a multidimensional SD into a scalar. Thus, NKDSGE model fit is gauged jointly on several frequencies. However, we also include mean $\mathcal{E}$ and $\mathcal{T}$ permanent and transitory output and consumption growth SDs to study NKDSGE

[12] NKDSGE models generate mean theoretical SDs nearly identical to population SDs at $\mathcal{M} = 5$.

[13] Since $\mathcal{V}_T(\omega)$ is the sum of the ratio $\mathcal{R}_T(\omega)$, a linear filter applied to the actual and synthetic data has no effect on $\mathcal{B}_{T,\mathcal{D},j}(\kappa)$. Hence, linear filtering will not alter the *KS* statistics and NKDSGE model evaluation.

model propagation and monetary transmission frequency by frequency.

DIW advocate using the confidence interval criterion (*CIC*) to quantify the intersection of $\mathcal{E}$ and $\mathcal{T}$ distributions. The *CIC* measures the fraction of a $\mathcal{T}$ *KS* distribution that occupies an interval defined by lower and upper quantiles of the relevant $\mathcal{E}$ *KS* distribution, given a $1 - p$ percent confidence level.[14] We set $p = 0.05$. If a habit NKDSGE model yields a $CIC > 0.3$ (as DIW imply in their analysis of RBC models), say, for the transitory output growth SD and the non-habit model's $CIC \leq 0.3$, the data view habit as more plausible for this moment. We also report densities of $\mathcal{E}$ and $\mathcal{T}$ *KS* statistic distributions to examine visually NKDSGE model fit.

We calculate SDs on the entire spectrum and on business cycle horizons from eight to two years per cycle. By restricting attention to business cycle fluctuations, we build on an approach to model evaluation of Diebold, Ohanian, and Berkowitz (1998). Their insight is that a focus on business cycle frequencies matters for DSGE model evaluation when model misspecification (*i.e.*, 'all models are false') corrupts measurement of short- and long-run fluctuations. We address these problems by judging NKDSGE model fit on the business cycle frequencies, which ignores low and high frequency output and consumption growth amplitude and periodicity.

4. Habit and Non-Habit NKDSGE Model Evaluation

This section presents evidence about habit and non-habit NKDSGE model fit to $\mathcal{E}$ permanent and transitory output and consumption growth SDs. Mean $\mathcal{E}$ SDs appear in figure 2. We report *CIC* in table 2. Figures 3–8 give visual evidence about NKDSGE model fit.

4.1 Business cycle moments: Output and consumption growth SDs

Figure 2 contains mean $\mathcal{E}$ permanent and transitory output and consumption growth SDs. The top (bottom) panel of figure 2 contains mean $\mathcal{E}$ permanent (transitory) output and consumption growth SDs. Mean $\mathcal{E}$ output growth SDs appear as solid (blue) lines in figure 2, while consumption growth SDs plots are thicker with '♦' symbols.

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[14]DIW set the *CIC* of $\mathcal{Q}$ to $\frac{1}{1-p}\int_a^b \mathcal{T}(\mathcal{Q}_j)d\,\mathcal{Q}_j$ for a $1 - p$ percent confidence level, where a (b) is the lower $0.5p$ (upper $1 - 0.5p$) quantile. The *CIC* is normalized by $1 - p$ to equal $\int_a^b \mathcal{E}(\mathcal{Q}_j)d\,\mathcal{Q}_j$.

The SDs decompose variation in output and consumption growth frequency by frequency in response to permanent and transitory shocks. The former shock yields mean $\mathcal{E}$ permanent output and consumption growth SDs that display greatest power at frequency zero as shown in the top panel of figure 2. However, the consumption growth SD exhibits only about a third of the amplitude (*i.e.*, volatility) that is found in output growth at the long run. The permanent shock also produces smaller peaks around four years per cycle in output and consumption growth SDs that reveal periodicity in the business cycle frequencies.

The lower panel of figure 2 presents mean transitory $\mathcal{E}$ output and consumption growth SDs. The mean $\mathcal{E}$ transitory output (consumption) growth SD peaks at less than four (eight) years per cycle. At the mean peaks, output growth is almost four times more volatile than consumption growth. However, output and consumption growth display periodicity at the business cycle frequencies with mean peaks between eight and two years per cycle.

We view the mean permanent and transitory $\mathcal{E}$ output and consumption growth SDs as challenges to NKDSGE models.[15] Mean $\mathcal{E}$ consumption growth SDs appear to vary enough at growth and business cycle frequencies to reject the PIH. Thus, NKDSGE models must violate the PIH to match these moments. Output growth SDs confront NKDSGE models with periodicity at low and business cycle frequencies that show these models need economically meaningful propagation and monetary transmission to achieve a good fit.

4.2 Habit and non-habit NKDSGE model fit: Evaluation by CIC

Table 2 reports *CIC* that evaluate NKDSGE model fit. The top panel has *CIC* of habit and non-habit sticky price and wage (baseline), sticky price only (SPrice), and sticky wage only (SWage) NKDSGE-AR models (the money growth rule (8) defines monetary policy).[16] The lower panel includes *CIC* of NKDSGE-TR models (the Taylor rule (9) replaces the money growth rule). Columns headed $\infty : 0$ and $8 : 2$ contain *CIC* that measure the overlap of $\mathcal{E}$ and $\mathcal{T}$ *KS* statistic

[15] When the SVMAs are calculated from VAR(2)s rather than VAR(5)s, $\mathcal{E}$ permanent and transitory output and consumption growth SDs are qualitatively unchanged.

[16] The SWage NKDSGE model requires the degenerate prior $\mu_P = 0$ with fixed markup $\phi = (\xi - 1)/\xi$. When the nominal wage is flexible, households set their optimal wage period by period in SPrice NKDSGE models. The markup in the labor market is fixed at $(\theta - 1)/\theta$, which equals $n^{-1/\gamma}$, given $\mu_W = 0$.

distributions based on the entire spectrum and eight to two years per cycle, respectively.

The lower panel of table 2 includes *CIC* of $\mathcal{E}$ and $\mathcal{T}$ *KS* statistic distributions of output and consumption growth SDs that are tied to NKDSGE-TR models. Habit NKDSGE-TR models yield *CIC* of 0.3 or more in 14 of 24 simulation experiments, but non-habit NKDSGE-TR models are responsible for only seven *CIC* ≥ 0.3. When habit and non-habit NKDSGE-TR models generate these *CIC* on the same SD, non-habit model *CIC* are larger only in two of seven cases. We view these results as evidence that internal consumption habit improves NKDSGE-TR model fit.[17] It also worth noting that it is difficult to choose between the fit of baseline, SPrice, and SWage habit NKDSGE-TR models to $\mathcal{E}$ transitory SDs using the *CIC*.

The NKDSGE-AR models are less successful at replicating the $\mathcal{E}$ *KS* statistic distributions. The upper panel of table 2 contains only five *CIC* ≥ 0.3 of the 48 entries. The SPrice habit NKDSGE-AR model is responsible for four of these *CIC*.

A striking feature of table 2 is that the fit of the baseline habit NKDSGE-AR model is dominated by the baseline habit NKDSGE-TR model. The baseline habit NKDSGE-TR model better replicates $\mathcal{E}$ transitory SDs compared to the baseline habit NKDSGE-AR model. Nevertheless, baseline NKDSGE models fail to propagate TFP innovations into output and consumption growth amplitude and periodicity that match $\mathcal{E}$ permanent SD distributions. The relevant *CIC* are less than 0.3 in the first two rows of the top and bottom panels of table 2.

Table 2 also provides information about the impact of sticky prices on NKDSGE model fit. Only SPrice habit NKDSGE models yield *CIC* > 0.3 for *KS* statistic distributions of the permanent output and consumption growth SDs. However, the comparisons must be limited to eight to two years per cycle for these models to generate *CIC* of this size. Thus there is evidence that internal consumption habit and fully indexed Calvo staggered pricing combine to propagate TFP shocks into economically meaningful output and consumption growth periodicity, but only at the business cycle frequencies.

[17] The *CIC* of table 2 are nearly unchanged either using the *CvM* statistic instead of the *KS* statistic or replacing the uniform prior of h with a prior drawn from a beta distribution with mean, standard deviation, and 95 percent coverage interval of 0.65, 0.15, and [0.38, 0.88]. These *CIC* are found in the appendix.

The SWage NKDSGE models only duplicate the $\mathcal{E}$ transitory output and consumption growth SDs if monetary policy is defined by the Taylor rule (9). The last two rows of table 2 contain *CIC* that imply a good fit for the SWage habit NKDSGE-TR model to $\mathcal{E}$ transitory SDs on the entire spectrum. The relevant $CIC \geq 0.36$. When internal consumption habit is missing from the SWage NKDSGE-TR model and the match to $\mathcal{E}$ transitory SDs is constrained to eight to two years per cycle, the $CIC \geq 0.45$. The same NKDSGE model cannot replicate these moments over the entire spectrum with $CIC \leq 0.13$. The SWage NKDSGE-AR models also produce $\mathcal{T}$ *KS* statistic densities of transitory SDs that have little overlap with their $\mathcal{E}$ counterparts (*i.e.*, $CIC \leq 0.22$). These results indicate that sticky wages combined with the Taylor rule (9) create monetary transmission in the habit NKDSGE model.

4.3 Baseline habit NKDSGE model fit: The role of monetary policy rules

This section expands on the *CIC* of table 2 by presenting additional evidence about the fit of baseline NKDSGE models to $\mathcal{E}$ permanent and transitory output and consumption growth SDs. We plot mean $\mathcal{E}$ and $\mathcal{T}$ permanent and transitory SDs and *KS* statistic densities of the baseline NKDSGE models in the first column of figures 3 and 4. The second (third) column contains densities of *KS* statistics that are constructed over the entire spectrum (constrained to eight to two years per cycle). The *KS* statistic densities also appear with the relevant *CIC*. From top to bottom, the rows of figures 3 and 4 present results for permanent output, transitory output, permanent consumption, and transitory consumption growth SDs. We denote mean $\mathcal{E}$ SDs and *KS* statistic densities with (blue) solid lines, mean $\mathcal{T}$ non-habit SDs and *KS* statistic densities with (green) dashed lines, and mean $\mathcal{T}$ habit SDs and *KS* statistic densities with (red) dot-dash lines in figures 3 and 4. The four remaining figures employ the same layout.

Baseline NKDSGE models fail to match $\mathcal{E}$ permanent output and consumption growth SDs. The poor fit is reflected in $\mathcal{T}$ *KS* statistic densities that are flat or far to the right of the relevant $\mathcal{E}$ densities as seen in the second and third columns of the odd numbered rows of figures 3 and 4. The lack of overlap of these $\mathcal{E}$ and $\mathcal{T}$ *KS* densities are consistent with mean $\mathcal{T}$ permanent SDs that often peak between eight and four years per cycle while mean $\mathcal{E}$

permanent SDs slowly decay from the low to business cycle frequencies in the odd numbered windows of the first column of figures 3 and 4. Thus, the combination of internal consumption habit or not, sticky prices and wages, and either monetary policy rule fails to propagate TFP shocks into the low and business cycle frequencies to match $\mathcal{E}$ permanent SDs.

The choice of monetary policy rule matters for baseline NKDSGE model fit to the $\mathcal{E}$ transitory SDs. The baseline habit NKDSGE-TR model responds to a monetary policy shock by dampening output and consumption growth volatility between eight and two years a cycle by a factor of four or more as shown in the first column of the even numbered rows of figure 4. This moves mean $\mathcal{T}$ transitory SDs of the baseline habit NKDSGE-TR model closer to mean $\mathcal{E}$ transitory SDs within the business cycle frequencies. These moments are not matched by the habit baseline NKDGSE-AR model because it yields mean $\mathcal{T}$ transitory SDs with business cycle periodicity in the first column of figure 3 that are far from mean $\mathcal{E}$ transitory SDs. As a consequence, the habit baseline NKDGSE-AR model produce $\mathcal{T}$ KS statistic densities of transitory SDs that fail to overlap $\mathcal{E}$ KS statistic densities in the second and third columns of the even numbered rows of figure 3. The same plots in figure 4 display greater overlap of $\mathcal{E}$ and $\mathcal{T}$ KS statistic densities. Thus, combining internal consumption habit and the Taylor rule (9) pushes the baseline NKDSGE model closer to $\mathcal{E}$ transitory output and consumption growth SDs.

In summary, internal consumption habit works with the Taylor rule (9) to improve the fit of the baseline NKDGSE model to $\mathcal{E}$ transitory SDs by generating business cycle periodicity and flattening $\mathcal{T}$ high frequency amplitude. Poole (1970) obtains a similar result by showing that an interest rate rule damps output fluctuations in a sticky price Keynesian macro model when monetary shocks are less volatile than real shocks. Thus, we have a resolution of the disparate NKDSGE model estimates of Del Negro, Schorfheide, Smets, and Wouters (2007) and CEE. It is the Del Negro, Schorfheide, Smets, and Wouters combination of consumption habit and a Taylor rule that helps the baseline NKDSGE model better match $\mathcal{E}$ transitory SDs compared to the baseline habit NKDSGE-AR model. These results also can be interpreted in light of Otrok, Ravikumar, and Whiteman (2002). They show that habit creates a distaste by households for high frequency consumption fluctuations. This distaste is consistent with internal consumption habit creating intertemporal complementarity that operates in the business cycle frequencies and, in conjunction with the Taylor rule (9), is a source of monetary transmission in the baseline habit NKDSGE-TR model.

4.4 NKDSGE model propagation and transmission: Habit and nominal rigidities

This section studies the role of internal consumption habit and the nominal rigidities of sticky prices and wages in propagation and monetary transmission. Erceg, Henderson, and Levin (2000) recognize that sticky prices and wages matter for monetary policy evaluation. However, sticky prices and wages must propagate TFP shocks and transmit monetary shocks to the real economy in this case. This suggests we judge the fit of habit NKDSGE models with and without sticky prices and sticky wages while remembering that these models also include capacity utilization and investment adjustment costs.

Table 2 shows that stripping out sticky nominal wages ($\mu_W = 0$) or sticky prices ($\mu_P = 0$) have disparate effects on NKDSGE models. Retaining sticky prices as the only nominal rigidity leads habit SPrice NKDSGE models to match better to $\mathcal{E}$ permanent output and consumption growth SDs than either baseline or SWage NKDSGE models. However, the improved fit is achieved only on business cycle frequencies. The SWage NKDSGE models have difficulties matching these moments, but not $\mathcal{E}$ transitory output and consumption growth SDs.

Removing nominal wage stickiness conveys a propagation mechanism to SPrice NKDSGE models. The propagation mechanism pushes SPrice NKDSGE models closer to $\mathcal{E}$ permanent output and consumption growth SDs in the business cycle frequencies. Figures 5 and 6 present visual evidence about the ability of SPrice NKDSGE-AR and NKDSGE-TR models to propagate TFP shocks from eight to two years per cycle. This evidence appears in the first and third rows of the third column of figures 5 and 6 as $\mathcal{E}$ and $\mathcal{T}$ KS statistic densities that display substantial overlap. When the fit is extended to the entire spectrum, $\mathcal{T}$ KS statistic densities have smaller peaks with tails far to the right relative to the associated $\mathcal{E}$ KS statistic densities. Note that when limited to eight to two years per cycle, SPrice non-habit NKDSGE models also are able to match the $\mathcal{E}$ permanent output growth SD.

The monetary policy rules contribute different propagation mechanisms to the SPrice habit NKDSGE model. The first column of figure 5 displays the mean $\mathcal{T}$ permanent output and consumption growth SDs of the SPrice habit NKDSGE-AR model. These $\mathcal{T}$ SDs fall from frequency zero into a lesser peak around three years per cycle before a sharp loss of power at two years per cycle. Compare this to the slow decay from the low into the business cycle frequencies of mean $\mathcal{T}$ permanent SDs generated by the SPrice habit NKDSGE-TR model in the first column of figure 6. In either case, mean $\mathcal{T}$ permanent SDs are always above mean $\mathcal{E}$ permanent SDs, except at low and high frequencies which signals excess theoretical output and consumption growth volatility at the business cycle frequencies. However, the Taylor rule (9) yields relatively less volatility at these frequencies in the SPrice habit NKDSGE model while the SPrice habit NKDSGE-AR model creates business cycle periodicity in mean $\mathcal{T}$ permanent SDs resembling that found in mean $\mathcal{E}$ permanent SDs.

There are also differences in monetary transmission across SPrice habit NKDSGE-AR and NKDSGE-TR models. The latter model matches $\mathcal{E}$ transitory output and consumption growth SDs over the entire spectrum given the overlap of $\mathcal{E}$ and $\mathcal{T}$ KS statistic densities in the even numbered rows of the second column of figure 6. The SPrice habit NKDSGE-AR model is also successful at duplicating transitory $\mathcal{E}$ SDs if limited to eight to two years per cycle. This evidence is provided by the third column of the second and fourth rows of figure 5 that depict substantial overlap of $\mathcal{E}$ and $\mathcal{T}$ KS densities. The third column of these rows in figure 6 shows that the SPrice habit NKDSGE-TR model fits at least as well to the transitory $\mathcal{E}$ SDs when the comparison is only at the business cycle frequencies. Nonetheless, only the Taylor rule shock υ_t is transmitted by the SPrice non-habit NKDSGE model into fluctuations that match the $\mathcal{E}$ transitory SDs across the entire spectrum.

The SPrice habit NKDSGE models generate mean $\mathcal{T}$ transitory output and consumption growth SDs that reflect the good match to $\mathcal{E}$ transitory SDs found in figures 5 and 6. The first column of figure 6 shows that the SPrice habit NKDSGE-TR model damps mean $\mathcal{T}$ transitory

SDs at the business cycle frequencies. Thus, these moments are near mean $\mathcal{E}$ transitory SDs. Since the SPrice habit NKDSGE-AR model generates amplitude that is expressed as periodicity around two years per cycle in its mean $\mathcal{T}$ transitory SDs, as seen in the first column of figure 5, these moments and mean $\mathcal{E}$ transitory SDs are not as close.

Next, we study the implications of nominal sticky wages for permanent and transitory output and consumption growth fluctuations. Figures 7 and 8 report results for the SWage NKDSGE-AR and NKDSGE-TR models. The evidence is that these models have problems matching $\mathcal{E}$ permanent output and consumption growth SDs, but that the SWage habit NKDSGE-TR model produces a good match to $\mathcal{E}$ transitory SDs.

The SWage NKDSGE models yield a poor match to $\mathcal{E}$ permanent SDs. Figures 7 and 8 reveal, in the first and third windows of their first column, that mean $\mathcal{T}$ permanent output and consumption growth SDs have peaks in the business cycle frequencies not observed in mean $\mathcal{E}$ permanent SDs. Without sticky prices, habit NKDSGE models produce excess business cycle volatility and periodicity in response to permanent TFP shocks. The distance between mean $\mathcal{E}$ and $\mathcal{T}$ permanent SDs is mirrored by the lack of overlap of $\mathcal{T}$ and $\mathcal{E}$ KS statistic densities in the second and third columns of the odd number rows of figures 7 and 8.

The even number rows of figure 8 testify to the good fit the SWage habit NKDSGE-TR model has to $\mathcal{E}$ transitory output and consumption growth SDs. This model produces mean $\mathcal{T}$ transitory SDs with maximum power at business cycle frequencies consistent with that found for mean $\mathcal{E}$ transitory SDs, as seen in the second and fourth windows of the first column of figure 8. The $\mathcal{E}$ and $\mathcal{T}$ transitory SD distributions map into $\mathcal{E}$ and $\mathcal{T}$ KS statistic densities that display substantial overlap over the entire spectrum or when constrained to eight to two years per cycle. The SWage habit NKDSGE-AR model is unable to match mean $\mathcal{E}$ transitory SDs either on the entire spectrum or when limited to eight to two years per cycle as shown in the second and third columns of the even numbered rows of figure 7.

This section reports that SPrice and SWage habit NKDSGE models have propagation and monetary transmission mechanisms that are economically meaningful. We find that $\mathcal{E}$ transitory output and consumption growth SDs are duplicated over the entire spectrum by SPrice and SWage habit NKDSGE-TR models. Only SPrice habit NKDSGE-AR and NKDSGE-TR models propagate TFP shock innovations into $\mathcal{T}$ permanent output and consumption growth SD distributions that replicate $\mathcal{E}$ permanent SDs distributions. However, this match occurs only at the business cycle frequencies.

Internal consumption habit contributes to propagation and monetary transmission in NKDSGE models by inducing intertemporal consumption complementarity. Propagation and monetary transmission in habit NKDSGE models produce $\mathcal{T}$ output and consumption growth SD that match $\mathcal{E}$ output and consumption growth SD, but there are subtleties to this fit. The fit is vulnerable to the specification of nominal rigidities in the habit NKDSGE models. Baseline, SPrice, and SWage habit NKDSGE models duplicate $\mathcal{E}$ transitory output and consumption growth SDs. However, the baseline and SWage habit NKDSGE models require the Taylor rule (9) to achieve this match while the SPrice habit NKDSGE models replicate $\mathcal{E}$ transitory SDs with either monetary policy rule. Nonetheless, we find that it is difficult to choose which combination of sticky prices and wages in the habit NKDSGE models best replicate $\mathcal{E}$ transitory SDs. These results affirm Del Negro and Schorfheide (2008). On the other hand when output and consumption growth SDs are identified by a permanent TFP shock, only the SPrice habit NKDSGE models duplicate $\mathcal{E}$ permanent SDs. It is worth mentioning again that the match between $\mathcal{E}$ and $\mathcal{T}$ permanent SDs only arises on the business cycle frequencies. Our results about the disparate fit of the habit NKDSGE models to the $\mathcal{E}$ and $\mathcal{T}$ permanent and transitory SDs are in the spirit of Dupor, Han, and Tsai (2009). They report estimates of NKDSGE model that differ across identifications tied to permanent TFP or transitory monetary policy shocks. Although their estimates indicate little habit persistence and a lack of price stickiness under the TFP shock identification, our evidence indicates that habit NKDSGE model fit is sensitive to the shock that drives output and consumption growth fluctuations.

5. Conclusion

This paper studies the business cycle implications of internal consumption habit for new Keynesian dynamic stochastic general equilibrium (NKDSGE) models. We examine the fit of 12 NKDSGE models that have different combinations of internal consumption habit, Calvo staggered prices and nominal wages, along with several other real rigidities. The NKDSGE models are confronted with output and consumption growth spectral densities (SDs) identified by permanent productivity and transitory monetary shocks.

The fit of habit and non-habit NKDSGE models is explored using Bayesian calibration and Monte Carlo methods. The evidence favors retaining internal consumption habit in NKDSGE models because this real rigidity often pushes theoretical permanent and transitory output growth SDs closer to the associated empirical SDs. This confirms Del Negro, Schorfheide, Smets, and Wouters (2007) who argue that consumption habit moves NKDSGE models closer to output dynamics, but is not consistent with Christiano, Eichenbaum, and Evans (2005).

Nonetheless, the Bayesian simulation experiments reveal that internal consumption habit has subtle effects on NKDSGE model fit. We find a poor fit for NKDSGE models to output and consumption growth SDs identified by the permanent productivity shock with one exception. These moments are replicated by habit NKDSGE models that have been stripped of sticky wages if the evaluation is limited to the business cycle frequencies. The habit NKDSGE models have more success at matching SDs identified by a Taylor rule shock than by a money growth rule shock. This fit is about the same whether the habit NKDSGE model with a Taylor rule combines sticky prices and wages or strips out one of these nominal rigidities.

Our results raise issues about the manner in which consumption habit is often handled within NKDSGE models. Internal consumption habit is often treated as if it is deeply founded in household preferences rather than as a reduced-form real friction that improves model fit. Alternatives to this view are found in Chetty and Sziedl (2005) and Ravn, Schmitt-Grohé, and Uribe (2006) who develop micro foundations for consumption habit. Also, Rozen (2008) provides valuable insights with axioms for intrinsic habit. We suspect that including these ideas in NKDSGE models will become an important part of business cycle research.

This paper reports that there are vulnerabilities in NKDSGE model fit. The fit is compromised by focusing on permanent output and consumption growth SDs instead of transitory SDs. This issue is explored by Dupor, Han, and Tsai (2009) and Del Negro and Schorfheide (2008). Dupor, Han, and Tsai obtain limited information NKDSGE model estimates that show how the moments used for identification affect inference about sticky prices and wages. In contrast, Del Negro and Schorfheide argue that Bayesian likelihood methods and aggregate data cannot distinguish between competing nominal rigidities in NKDSGE models. Our results suggest that both views have merit as explanations for NKDSGE model fit. Along with their work, we hope this paper inspires research about the role real and nominal rigidities play in NKDSGE model propagation and monetary transmission.

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Table 1: Bayesian Calibration of NKDSGE Models

		Prior Distribution	Mean	Standard Deviation	95 Percent Cover Interval
h	Internal Consumption Habit	Uniform	—	—	[0.0500, 0.9500]
β	H'hold Subjective Discount	Beta	0.9930	0.0020	[0.9886, 0.9964]
γ	Labor Supply Elasticity	Normal	1.3088	0.3196	[0.7831, 1.8345]
δ	Depreciation Rate	Beta	0.0200	0.0045	[0.0122, 0.0297]
α	Deterministic Growth Rate	Normal	0.0040	0.0015	[0.0015, 0.0065]
ϖ	Capital Adjustment Costs	Normal	4.7710	1.0260	[3.0834, 6.4586]
ψ	Capital's Share of Output	Beta	0.3500	0.0500	[0.2554, 0.4509]
σ_ϵ	TFP Growth Shock Std.	Uniform	—	—	[0.0070, 0.0140]
ξ	Final Good Dmd Elasticity	Normal	8.0000	1.1000	[6.1907, 9.8093]
μ_P	No Price Change Probability	Beta	0.5500	0.0500	[0.4513, 0.6468]
θ	Labor Demand Elasticity	Normal	15.0000	3.0800	[8.9633, 21.0367]
μ_W	No Wage Change Probability	Beta	0.7000	0.0500	[0.5978, 0.7931]
m^*	$\Delta \ln M$ Mean	Normal	0.0152	0.0006	[0.0142, 0.0162]
ρ_m	$\Delta \ln M$ AR1 Coef.	Beta	0.6278	0.0549	[0.5355, 0.7162]
σ_μ	$\Delta \ln M$ Shock Std.	Normal	0.0064	0.0008	[0.0048, 0.0080]
a_π	Taylor Rule $\mathbf{E}_t \pi_{t+1}$ Coef.	Normal	1.8000	0.2000	[1.4710, 2.1290]
$a_{\hat{Y}}$	Taylor Rule $\hat{Y}_t$ Coef.	Normal	0.1000	0.0243	[0.0524, 0.1476]
ρ_R	Taylor Rule AR1 Coef.	Beta	0.6490	0.0579	[0.5512, 0.7417]
σ_υ	Taylor Rule Shock Std.	Normal	0.0051	0.0013	[0.0031, 0.0072]

The calibration relies on existing DSGE model literature; see the text for details. For a non-informative prior, the right most column contains the lower and upper end points of the uniform distribution. When the prior is based on the beta distribution, its two parameters are $a = \overline{\Gamma}_{i,n}\left[(1-\overline{\Gamma}_{i,n})\overline{\Gamma}_{i,n}/STD(\Gamma_{i,n})^2 - 1\right]$ and $b = a(1-\overline{\Gamma}_{i,n})/\overline{\Gamma}_{i,n}$, where $\overline{\Gamma}_{i,n}$ is the degenerate prior of the ith element of the parameter vector of model $n = 1,\ldots,4$, and its standard deviation is $STD(\Gamma_{i,n})$.

Table 2: *CIC* of Kolmogorov-Smirnov Statistics

Model	ΔY w/r/t Trend Sh'k $\infty:0$	$8:2$	ΔY w/r/t Transitory Sh'k $\infty:0$	$8:2$	ΔC w/r/t Trend Sh'k $\infty:0$	$8:2$	ΔC w/r/t Transitory Sh'k $\infty:0$	$8:2$
NKDSGE-AR								
Baseline								
Non-Habit	0.01	0.03	0.00	0.01	0.00	0.04	0.00	0.00
Habit	0.01	0.07	0.11	0.11	0.05	0.26	0.15	0.21
SPrice								
Non-Habit	0.03	0.30	0.00	0.21	0.01	0.23	0.00	0.05
Habit	0.06	0.42	0.18	0.62	0.15	0.55	0.26	0.54
SWage								
Non-Habit	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.01
Habit	0.00	0.02	0.19	0.22	0.03	0.14	0.16	0.24
NKDSGE-TR								
Baseline								
Non-Habit	0.00	0.00	0.13	0.73	0.00	0.00	0.02	0.41
Habit	0.00	0.04	0.54	0.75	0.05	0.19	0.37	0.73
SPrice								
Non-Habit	0.02	0.52	0.01	0.79	0.00	0.24	0.00	0.54
Habit	0.10	0.67	0.30	0.77	0.15	0.57	0.35	0.81
SWage								
Non-Habit	0.00	0.00	0.13	0.74	0.00	0.00	0.01	0.45
Habit	0.00	0.04	0.44	0.66	0.04	0.16	0.36	0.74

NKDSGE-AR and NKDSGE-TR denote the NKDSGE model with the AR(1) money supply rule (8) and the Taylor rule (9), respectively. Baseline NKDSGE models include sticky prices and sticky wages. The acronyms SPrice and SWage represent NKDSGE models with only sticky prices or sticky nominal wages, respectively. The column heading $\infty : 0$ ($8 : 2$) indicates that *CIC* measure the intersection of distributions of *KS* statistics computed over the entire spectrum (from eight to two years per cycle).

FIGURE 1: ΔC RESPONSE TO REAL INTEREST RATE SHOCK

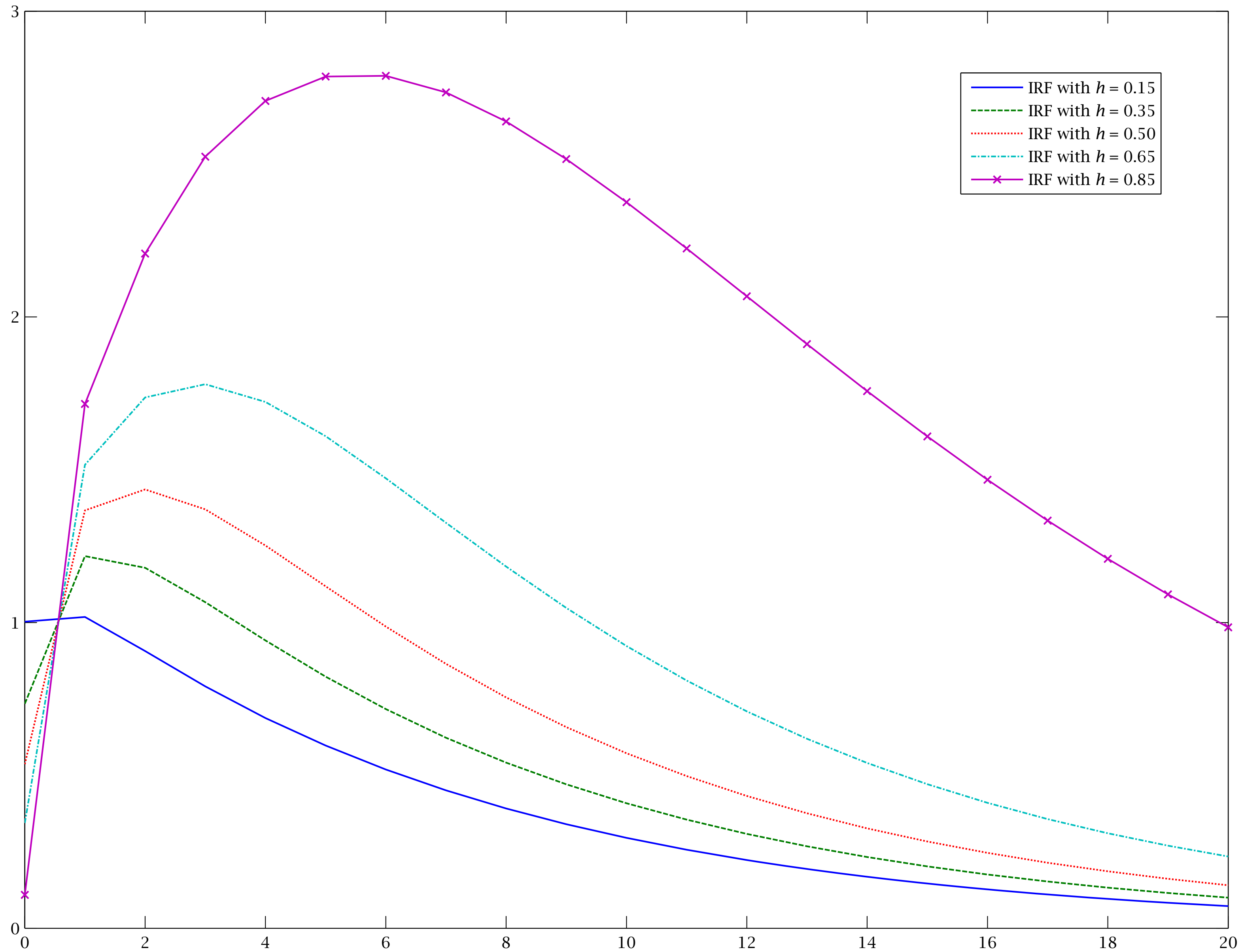


Figure 2: Mean Structural $\mathcal{E}$ Spectra of ΔY and ΔC

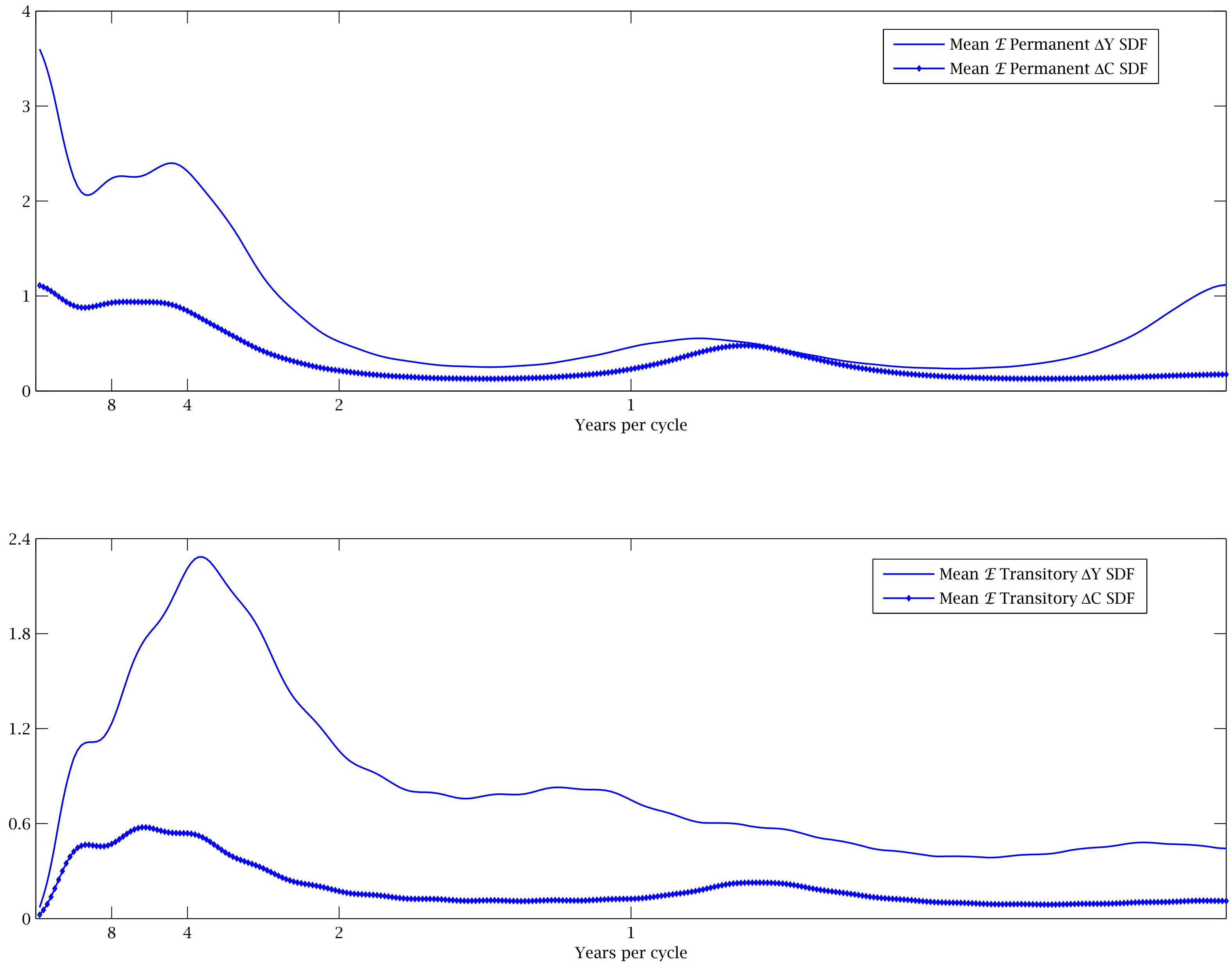


Figure 3: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with AR(1) Money Growth Rule

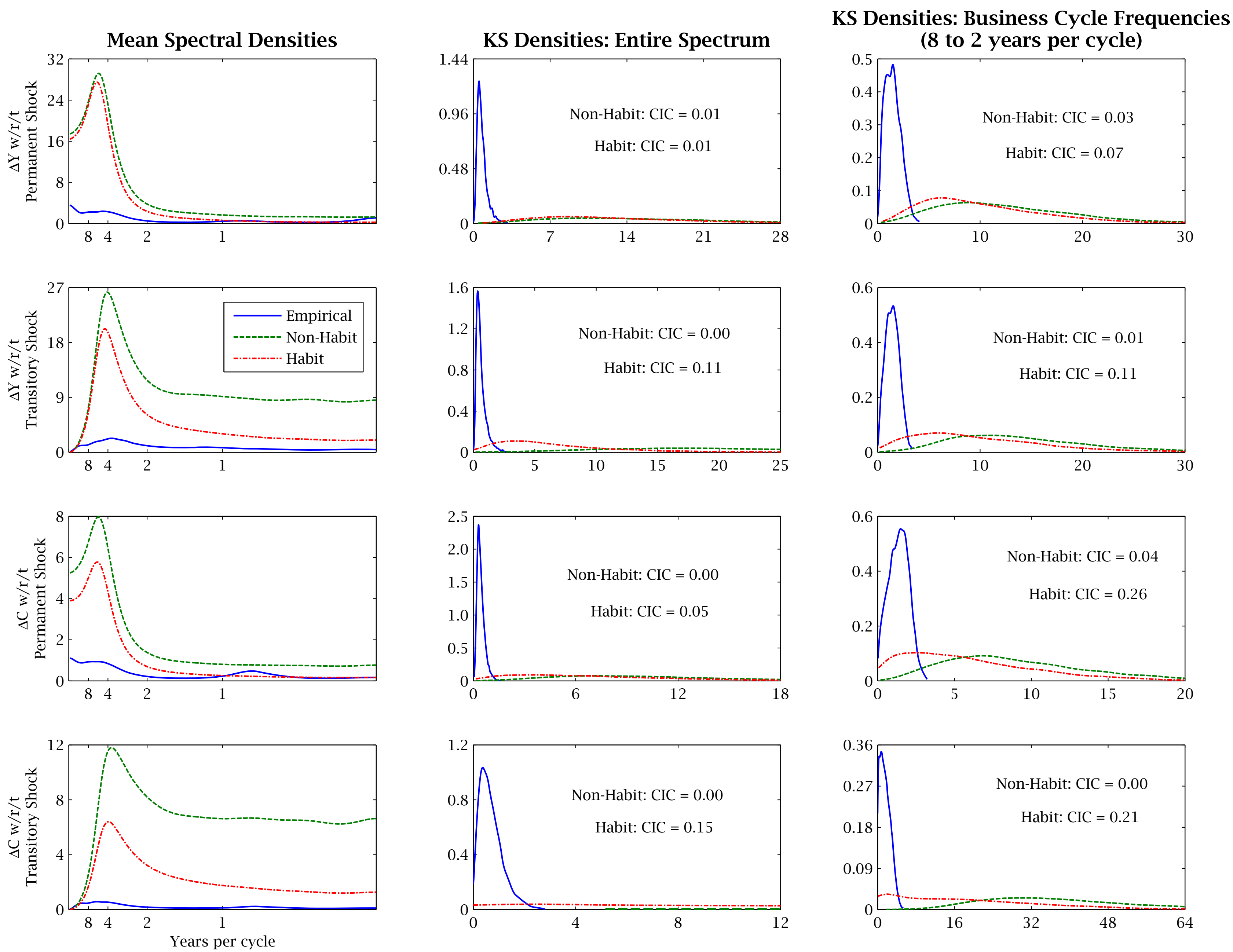


Figure 4: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with Taylor Rule

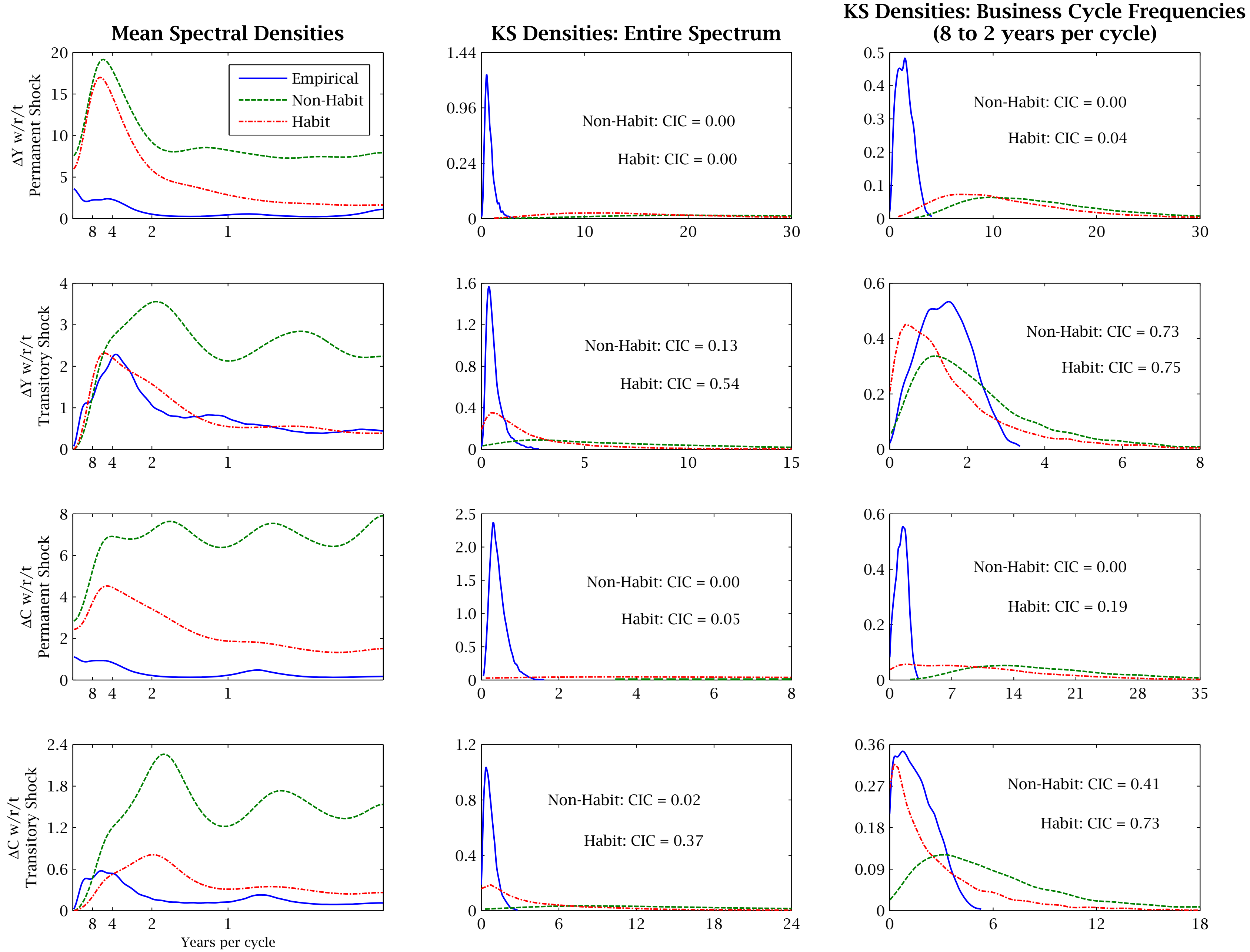


FIGURE 5: MEAN STRUCTURAL $\mathcal{E}$ AND $\mathcal{T}$ SDs AND KS DENSITIES FOR NKDSGE MODELS WITH AR(1) MONEY GROWTH RULE AND ONLY STICKY PRICES

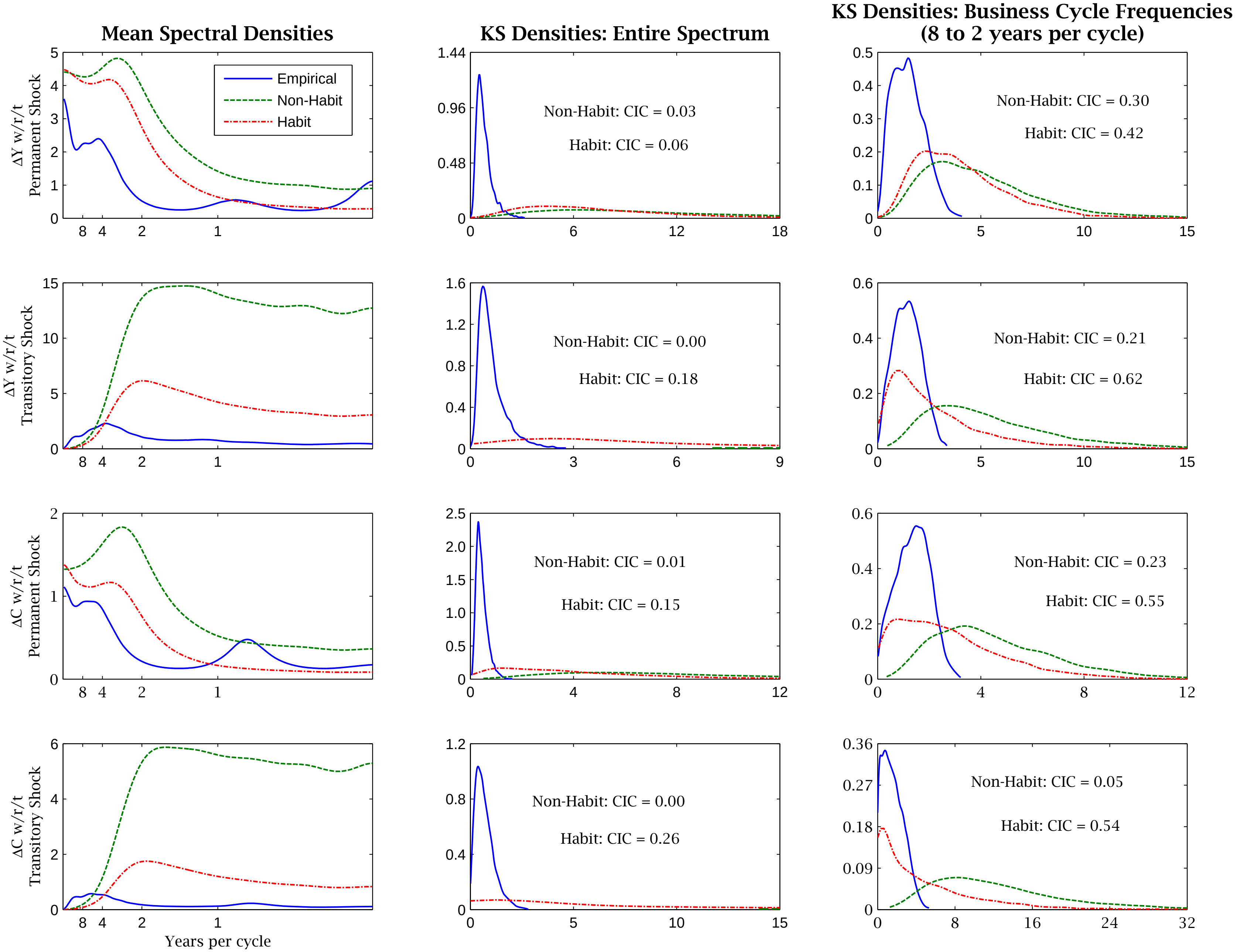


Figure 6: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Taylor Rule and only Sticky Prices

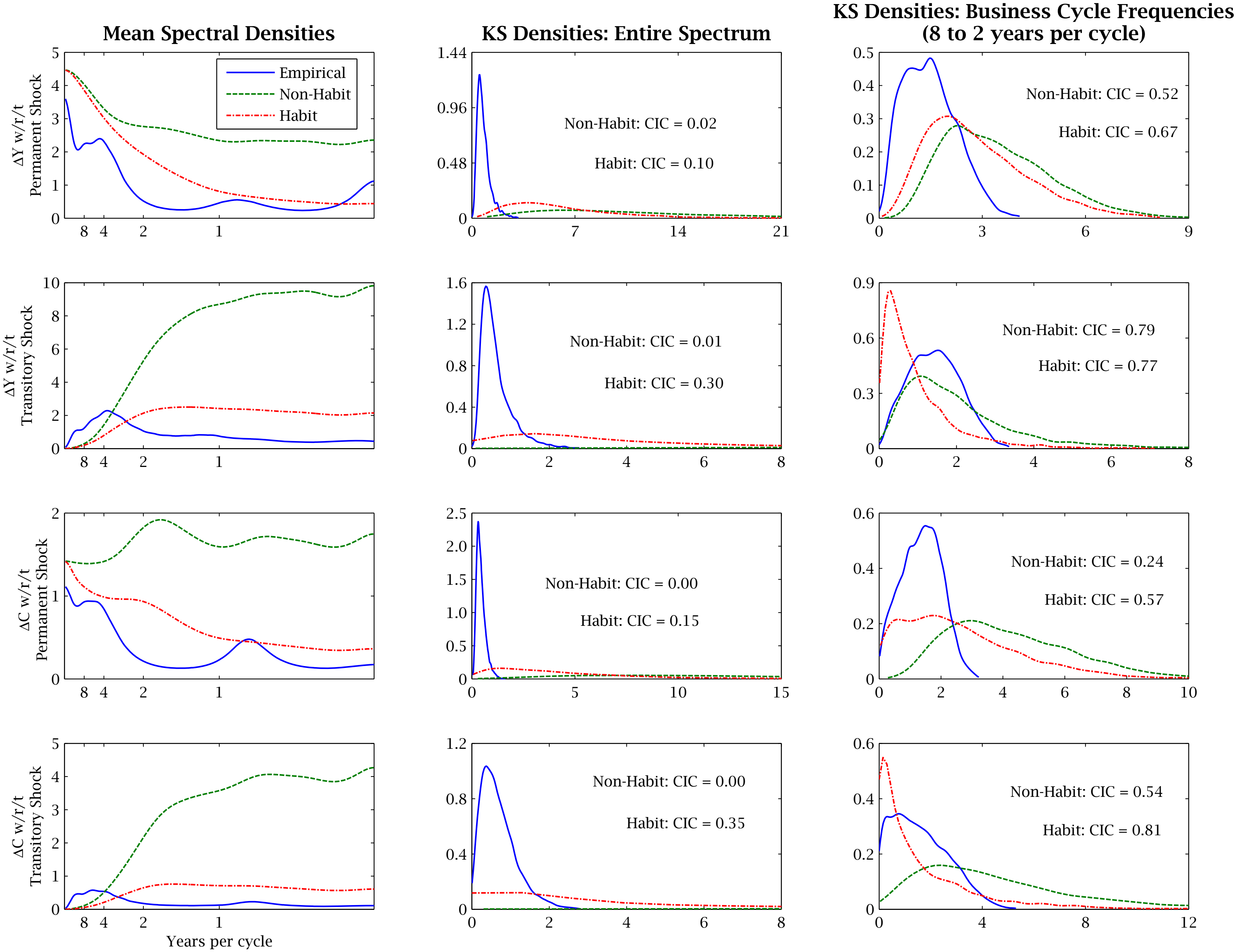


Figure 7: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with AR(1) Money Growth Rule and Only Sticky Wages

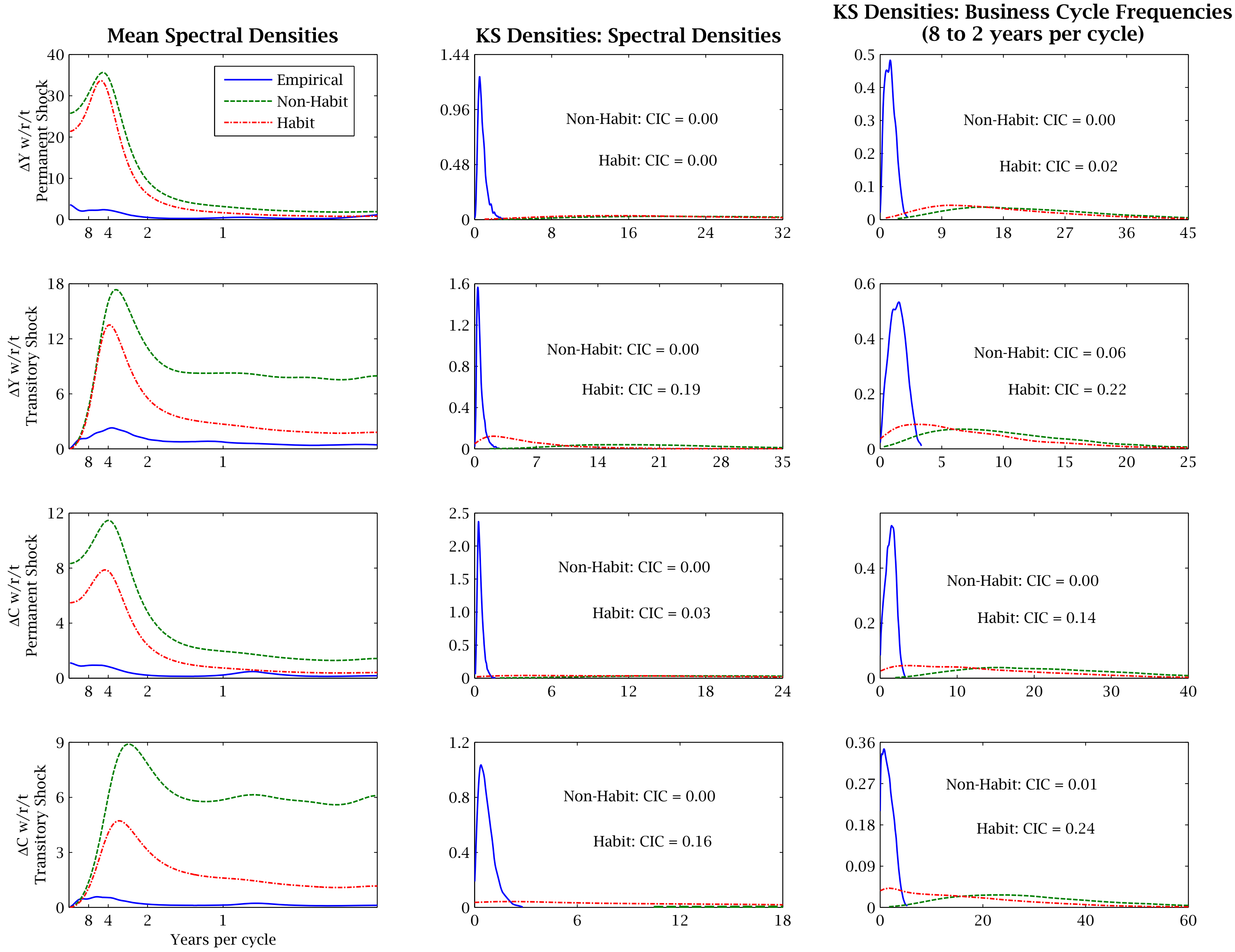
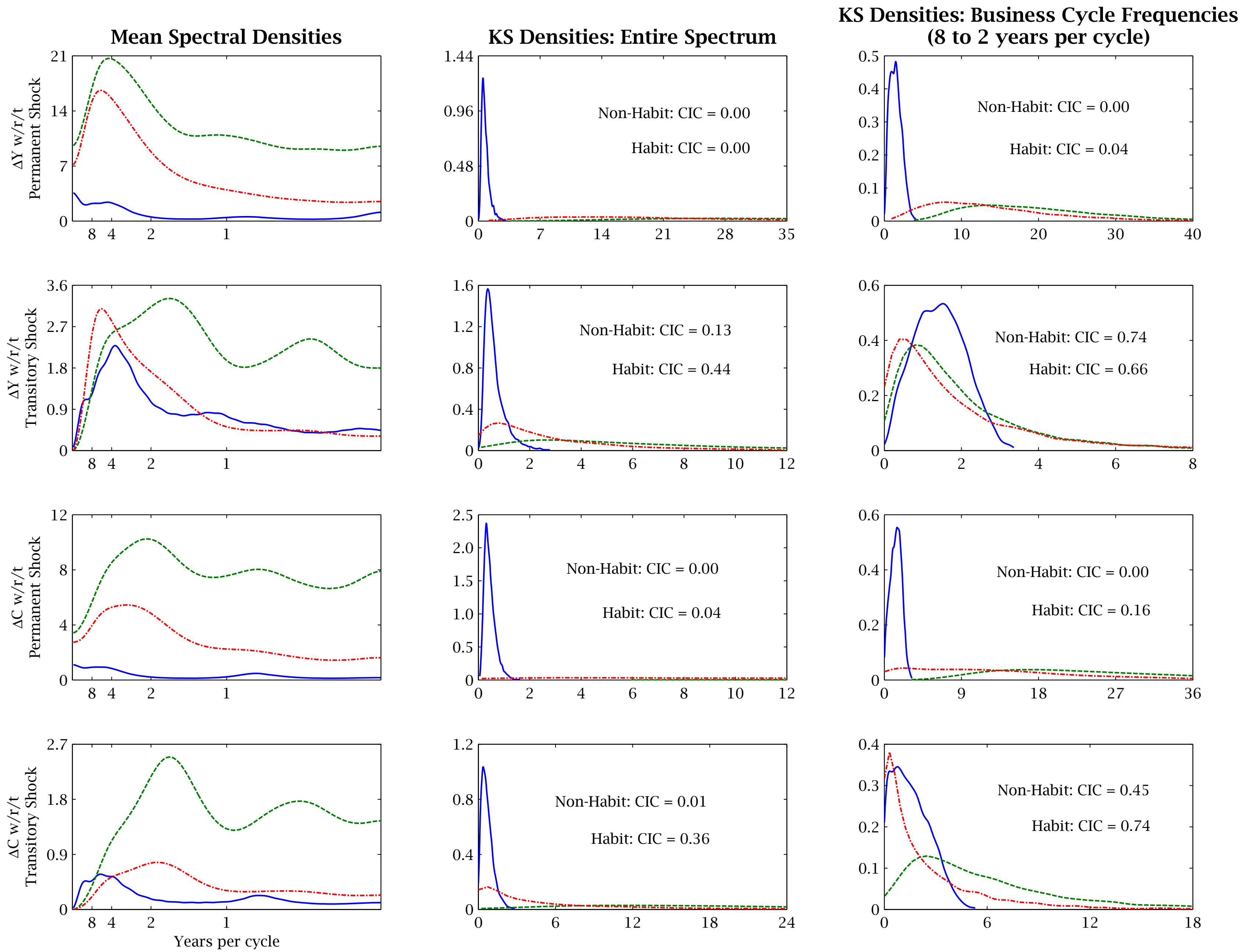


Figure 8: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Taylor Rule and only Sticky Wages



Appendix

The appendix consists of five sections. The sample data is described in section **A0**. **A1** completes our discussion of the internal consumption habit propagation mechanism. We present optimality and equilibrium conditions of the baseline habit new Keynesian dynamic stochastic general equilibrium (NKDSGE) model in section **A2**, along with stochastically detrended, steady state, and linearized versions of these equations. This section also outlines the algorithm applied to solved the linearized NKDSGE models. Section **A3** gives instructions to identify and estimate infinite order structural vector moving averages, SVMA(∞)s. Next, we adapt existing literature to show that the SVMA(∞)s retrieve the economic shocks of the NKDSGE models. This is followed by formulas to compute the identified output and consumption growth spectral densities. Section **A4** ends the appendix with a summary of NKDSGE model fit using Cramer-von Mises goodness of fit statistics or a β prior for the habit parameter h.

A0. Data Sources and Construction

This section sketches the $1954Q1$–$2002Q4$ sample data. The source of the data is FRED-II maintained by the Federal Reserve Bank of St. Louis at *http://research.stlouisfed.org/fred2/*. Mnemonics appear in parentheses. The NIPA data are real chained 1996 billion dollars and seasonally adjusted at annual rates. The consumption series equals *Real Personal Consumption Expenditures on Nondurables* (PCNDGC96) plus *Real Personal Consumption Expenditures on Services* (PCESVC96). Investment is constructed by adding together *Real Personal Consumption Expenditures on Durables* (PCDGCC96), *Real Gross Private Domestic Investment* (GPDIC1), *Real National Defense Gross Investment* (DGIC96), and *Real Federal Nondefense Gross Investment* (NDGIC96). Government spending subtracts *Real National Defense Gross Investment* plus *Real Federal Nondefense Gross Investment* from *Real Government Consumption Expenditures and Gross Investment* (GCEC1). Output equals the sum of consumption, investment and government spending. Aggregate quantities are divided by *Civilian Labor Force* (CLF16OV) to create per capita series. Since the *Civilian Labor Force* is monthly, temporal aggregation produces quarterly observations. Finally, the money stock is equated with the seasonally adjusted, *St.*

Louis Adjusted Monetary Base (AMBSL). This monthly series is temporally aggregated to obtain a quarterly series and also made per capita.

A1. Consumption Dynamics under Internal and External Consumption Habits

This sections studies the propagation mechanism of additive internal consumption habit. We also show that subsequent to log linearization additive internal and external consumption habit produce observationally equivalent consumption growth dynamics up to a normalization on the impact shock of the AR(1) real rate.

A1.1 The Internal Consumption Habit Propagation Mechanism

Section 2.2 of the paper presents a calibration exercise that discusses the additive internal consumption habit propagation mechanism. The discussion begins with the Euler equation

$$\lambda_t = \mathbf{E}_t \left\{ \frac{\lambda_{t+1} R_{t+1}}{1+\pi_{t+1}} \right\}, \qquad (\mathcal{A}1.1)$$

where the forward-looking marginal utility of consumption is $\lambda_t = \dfrac{1}{c_t - hc_{t-1}} - \mathbf{E}_t\left\{\dfrac{\beta h}{c_{t+1} - hc_t}\right\}$, h is the habit parameter, c_t is household consumption, β is the household discount factor, the mathematical expectations operator conditional on date t information is $\mathbf{E}_t\{\cdot\}$, R_t is the nominal rate, and $1 + \pi_{t+1}$ $(= P_{t+1}/P_t)$ is date $t+1$ inflation. Given a random walk (with drift) drives total factor productivity (TFP) A_t, the Euler equation ($\mathcal{A}1.1$) and λ_t are stochastically detrended according to

$$\hat{\lambda}_t = \mathbf{E}_t \left\{ \frac{\hat{\lambda}_{t+1} R_{t+1}}{\alpha_{t+1}(1+\pi_{t+1})} \right\}, \qquad (\mathcal{A}1.2)$$

and

$$\hat{\lambda}_t = \frac{\alpha_t}{\alpha_t \hat{c}_t - h\hat{c}_{t-1}} - \mathbf{E}_t\left\{\frac{\beta h}{\alpha_{t+1}\hat{c}_{t+1} - h\hat{c}_t}\right\}, \qquad (\mathcal{A}1.3)$$

where $\hat{\lambda}_t \equiv A_t\lambda_t$ and $\alpha_t = A_t/A_{t-1} = \exp(\alpha + \varepsilon_t)$, $\alpha > 0$, and ε_t is the mean zero, homoskedastic TFP shock innovation. Since household consumption and the marginal utility of consumption are stationary, the Euler equation ($\mathcal{A}1.2$) and marginal utility function ($\mathcal{A}1.3$) can be log linearized around the means (*i.e.* steady state) of $\hat{\lambda}_t$, $\hat{c}_t$, R_t, and π_t. The result is

$$\tilde{\lambda}_t = \mathbf{E}_t\left\{\tilde{\lambda}_{t+1} - \varepsilon_{t+1} + \tilde{R}_{t+1} - \frac{\pi^*}{1+\pi^*}\tilde{\pi}_{t+1}\right\}, \qquad (\mathcal{A}1.4)$$

and

$$(\alpha^* - \beta h)(\alpha^* - h)\tilde{\lambda}_t = \alpha^*\beta h\mathbf{E}_t\tilde{c}_{t+1} - (\beta h^2 + \alpha^{*2})\tilde{c}_t + \alpha^* h\tilde{c}_{t-1} - \alpha^*\beta h\mathbf{E}_t\varepsilon_{t+1} + \alpha^* h\varepsilon_t, \ (\mathcal{A}1.5)$$

where, for example, $\tilde{c}_t = \ln\hat{c}_t - \ln c^*$ or $\tilde{R}_t = \ln R_t - \ln R^*$, and $\alpha^* = \exp(\alpha)$ is the deterministic TFP growth rate. We combine equations ($\mathcal{A}1.4$) and ($\mathcal{A}1.5$) to obtain

$$\begin{aligned}\alpha^*\beta h\mathbf{E}_t\left\{\Delta\tilde{c}_{t+2} + \varepsilon_{t+2}\right\} - (\beta h^2 + \alpha^{*2})\mathbf{E}_t\left\{\Delta\tilde{c}_{t+1} + \varepsilon_{t+1}\right\} + \alpha^* h(\Delta\tilde{c}_t + \varepsilon_t) \\ = -(\alpha^* - \beta h)(\alpha^* - h)\mathbf{E}_t\tilde{q}_{t+1},\end{aligned} \qquad (\mathcal{A}1.6)$$

where the demeaned real rate is $\tilde{q}_t = \tilde{R}_t - \dfrac{\pi^*}{1+\pi^*}\tilde{\pi}_t$. By exploiting stochastic detrending, the linearized Euler equation ($\mathcal{A}1.6$) can be written as a second-order expectational stochastic difference equation in demeaned household consumption growth

$$\alpha^*\beta h\mathbf{E}_t\widetilde{\Delta c}_{t+2} - (\beta h^2 + \alpha^{*2})\mathbf{E}_t\widetilde{\Delta c}_{t+1} + \alpha^* h\widetilde{\Delta c}_t = -(\alpha^* - \beta h)(\alpha^* - h)\mathbf{E}_t\tilde{q}_{t+1}, \qquad (\mathcal{A}1.7)$$

where $\widetilde{\Delta c}_t = \Delta\ln\hat{c}_t + \varepsilon_t$ denotes demeaned household consumption growth.

We solve equation ($\mathcal{A}1.7$) to obtain the backward-looking stable root $\varphi_1 = h\alpha^{*-1}$ and forward-looking unstable root $\varphi_2 = \alpha^*/(\beta h)$. These roots are exploited by the lag polynomial $-\mathbf{L}^{-1}(1 - \varphi_1\mathbf{L})(1 - \varphi_2^{-1}\mathbf{L}^{-1})\varphi_2\alpha^*\beta h\widetilde{\Delta c}_t$, which is an alternative to the left side of equation ($\mathcal{A}1.7$). After applying the lag polynomial, we have

$$\left(1 - \frac{h}{\alpha^*}\mathbf{L}\right)\widetilde{\Delta c}_t \;=\; \frac{(\alpha^* - \beta h)(\alpha^* - h)}{\alpha^{*2}} \sum_{j=0}^{\infty} \left(\frac{\beta h}{\alpha^*}\right)^j \mathbf{E}_t \tilde{q}_{t+j}, \qquad (\mathcal{A}1.8)$$

which is the unique (*i.e.*, sunspot free) solution of the second-order stochastic difference equation ($\mathcal{A}$1.7). This solution is equation (2) of the paper, where $\Psi = \frac{(\alpha^* - \beta h)(\alpha^* - h)}{\alpha^* \beta h}$. Equation ($\mathcal{A}$1.8) is forward-looking in the expected discounted present value of $\tilde{q}_t$ and backward-looking in the lag of demeaned consumption growth. Assume $\tilde{q}_t$ is a AR(1) with persistence parameter ρ_q. In this case, the Wiener-Kolmogorov formulas alter equation ($\mathcal{A}$1.8) to

$$\left(1 - \frac{h}{\alpha^*}\mathbf{L}\right)\widetilde{\Delta c}_t \;=\; \frac{(\alpha^* - \beta h)(\alpha^* - h)}{\alpha^*(\alpha^* - \rho_q \beta h)}\tilde{q}_t. \qquad (\mathcal{A}1.9)$$

We employ equation ($\mathcal{A}$1.9), put h on the grid $[0.15\ 0.35\ 0.50\ 0.65\ 0.85]$, calibrate $[\beta\ \ \alpha^*]' = [0.993\ \ \exp(0.004)]'$, and estimate a AR(1) for $\tilde{q}_t$ to generate the impulse response functions plotted in figure 1.

The real federal funds rate $\tilde{q}_t$ is measured with the demeaned quarterly nominal federal funds rate and demeaned implicit GDP deflator inflation. The latter is multiplied by the ratio of its mean to one plus its mean and subtracted from the former to create the real federal funds rate $\tilde{q}_t$ on a 1954Q1–2002Q4 sample. Although likelihood ratio tests and the Hannan-Quinn criterion suggest a AR(3), we settle on a AR(1) using the SIC against AR(2) to AR(10) specifications. On the 1954Q1–2002Q4 sample, OLS estimates of the AR(1) of $\tilde{q}_t$ are $\rho_q = 0.8687$ and the standard error of the regression is 1.2059.

A1.2 An Observational Equivalence Result for Internal and External Consumption Habits

We show in this section that internal and external additive consumption habit preferences $\ln[c_t - hc_{t-1}]$ yield observationally equivalent log linearized Euler equations up to a normalization of the AR(1) real rate, $\tilde{q}_t$. This habit specification is found in the NKDSGE models that Christiano, Eichenbaum, and Evans (2005) estimate. An alternative NKDSGE model is estimated by Smets and Wouter (2007), but their preference specification contains external rather than internal consumption habit. The economic content of estimates of linearized NKDSGE model estimates appear to be unaffected by the choice of consumption habit specification according to Dennis (2009). He also notes that the mapping from additive to multiplicative (*i.e.* 'keeping up with the Jones') consumption habit parameters is onto only in this direction.

Under additive external consumption habit, the marginal utility of consumption is purely backward-looking. After stochastic detrending, $\widehat{\lambda}_{t,ECH} = \dfrac{\alpha_t}{\alpha_t \hat{c}_t - h\hat{c}_{t-1}}$, where *ECH* denotes external consumption habit. The log linearized Euler equation ($\mathcal{A}$1.4) becomes

$$\alpha^* \tilde{c}_t - h\tilde{c}_{t-1} + h\varepsilon_t \; = \; \mathbf{E}_t\Big\{\alpha^* \tilde{c}_{t+1} - h\tilde{c}_t + \alpha^* \varepsilon_{t+1} + (\alpha^* - h)\tilde{q}_{t+1}\Big\}, \qquad (\mathcal{A}1.10)$$

with $(\alpha^* - h)\widetilde{\lambda}_{t,ECH} = -\alpha^* \tilde{c}_t + \alpha^* \tilde{c}_{t-1} - h\varepsilon_t$. A bit of rearranging transforms the linearized Euler equation ($\mathcal{A}$1.10) into the first-order expectational stochastic difference equation

$$\mathbf{E}_t\Big\{\Delta\tilde{c}_{t+1} + \varepsilon_{t+1}\Big\} \; = \; \frac{h}{\alpha^*}\left(\Delta\tilde{c}_t + \varepsilon_t\right) \; - \; \frac{\alpha^* - h}{\alpha^*}\mathbf{E}_t \tilde{q}_{t+1}. \qquad (\mathcal{A}1.11)$$

Given a mean zero, homoskedastic expectational error ϑ_t, the first-order stochastic difference equation ($\mathcal{A}$1.11) can be written

$$\left(1 - \frac{h}{\alpha^*}\mathbf{L}\right)\widetilde{\Delta c}_t \; = \; \frac{\alpha^* - h}{\alpha^*}\tilde{q}_t + \vartheta_t, \qquad (\mathcal{A}1.12)$$

which represents reduced-form consumption growth dynamics under additive ECH.

Equations ($\mathcal{A}$1.9) and ($\mathcal{A}$1.12) produce observationally equivalent dynamics in $\widetilde{\Delta c}_t$ up to the impact coefficient on $\tilde{q}_t$ given it is a AR(1). The dynamics are equivalent because equations ($\mathcal{A}$1.9) and ($\mathcal{A}$1.12) share the same leading autoregressive root, which equals h/α^*. Thus across additive internal and external consumption habit, a shock to $\tilde{q}_t$ generates identical responses in $\widetilde{\Delta c}_t$ beyond impact. Only at impact can internal and external consumption habit yield disparate responses in $\widetilde{\Delta c}_t$ to an innovation in $\tilde{q}_t$. As $h \longrightarrow 1$ the impact responses of $\widetilde{\Delta c}_t$ differ by a factor of 12 for internal and external consumption habit at the calibration of section 2.2, but the impact responses converge as $h \longrightarrow 0$.

A2. Solving the Habit NKDSGE Models

This section presents the optimality and equilibrium conditions of the baseline habit NKDSGE models, the stochastically detrended versions of these conditions, the steady state of this economy, the log linearized optimality and equilibrium conditions, and solution method invoked to compute a multivariate linear approximate equilibrium law of motion.

A2.1 Optimality and equilibrium conditions

The baseline habit NKDSGE models have first-order necessary conditions (FONCs) that are restricted by the primitives of preferences, technology, market structure, and monetary policy regime. The FONCs imply optimality and equilibrium conditions that must be satisfied by any candidate equilibrium time series. The optimality and equilibrium conditions are

$$\lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta h \mathbf{E}_t \left\{ \frac{1}{C_{t+1} - hC_t} \right\}, \tag{$\mathcal{A}$2.1}$$

$$\frac{1-q_t}{q_t} + S\left(\frac{X_t}{\alpha^* X_{t-1}}\right) + S'\left(\frac{X_t}{\alpha^* X_{t-1}}\right)\frac{X_t}{\alpha^* X_{t-1}} = \frac{\beta}{\alpha^*}\mathbf{E}_t\left\{\frac{\lambda_{t+1}q_{t+1}}{\lambda_t q_t} S'\left(\frac{X_{t+1}}{\alpha^* X_t}\right)\left[\frac{X_{t+1}}{X_t}\right]^2\right\}, \tag{$\mathcal{A}$2.2}$$

$$q_t = \beta \mathbf{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t}\left[\psi u_{t+1}\phi_{t+1}\frac{Y_{A,t+1}}{K_{t+1}} - a(u_{t+1}) + q_{t+1}(1-\delta)\right]\right\}, \tag{$\mathcal{A}$2.3}$$

$$\frac{\lambda_t}{P_t} = \beta \mathbf{E}_t \frac{\lambda_{t+1}}{P_{t+1}} R_{t+1}, \tag{$\mathcal{A}$2.4}$$

$$\frac{\lambda_t}{P_t} = \beta \mathbf{E}_t \left\{ \frac{\lambda_{t+1}}{P_{t+1}} + \frac{1}{M_{t+1}} \right\}, \tag{$\mathcal{A}$2.5}$$

$$a'(u_t) = \psi \phi_t \frac{Y_{A,t}}{K_t}, \tag{$\mathcal{A}$2.6}$$

$$\frac{W_t}{P_t} = \phi_t (1-\psi) \frac{Y_{A,t}}{N_t - N_0}, \tag{$\mathcal{A}$2.7}$$

$$\frac{Y_{A,t}}{Y_{D,t}} = \left(\frac{P_{A,t}}{P_t}\right)^{-\xi}, \tag{$\mathcal{A}$2.8}$$

$$\frac{P_{c,t}}{P_{t-1}} = \left(\frac{\xi}{\xi-1}\right) \frac{\mathbf{E}_t \sum_{i=0}^{\infty} \left[\beta \mu_P\right]^i \lambda_{t+i} \phi_{t+i} Y_{D,t+i} \left[\frac{P_{t+i}}{P_{t+i-1}}\right]^{\xi}}{\mathbf{E}_t \sum_{i=0}^{\infty} \left[\beta \mu_P\right]^i \lambda_{t+i} Y_{D,t+i} \left[\frac{P_{t+i}}{P_{t+i-1}}\right]^{\xi-1}}, \tag{$\mathcal{A}$2.9}$$

$$\frac{N_t}{n_t} = \left(\frac{W_{D,t}}{W_t}\right)^{-\xi}, \tag{$\mathcal{A}$2.10}$$

$$\left[\frac{W_{c,t}}{P_{t-1}}\right]^{1+\theta/\gamma} = \left(\frac{\theta}{\theta-1}\right) \frac{\mathbf{E}_t \sum_{i=0}^{\infty} \left[\beta \mu_W \alpha^{*-\theta(1+1/\gamma)}\right]^i \left[\left[\frac{W_{t+i}}{P_{t+i-1}}\right]^{\theta} N_{t+i}\right]^{1+1/\gamma}}{\mathbf{E}_t \sum_{i=0}^{\infty} \left[\beta \mu_W \alpha^{*(1-\theta)}\right]^i \lambda_{t+i} \left[\frac{W_{t+i}}{P_{t+i-1}}\right]^{\theta} \left[\frac{P_{t+i}}{P_{t+i-1}}\right]^{-1} N_{t+i}}, \tag{$\mathcal{A}$2.11}$$

$$K_{t+1} = (1-\delta)K_t + \left[1 - S\left(\frac{X_t}{\alpha^* X_{t-1}}\right)\right] X_t, \tag{$\mathcal{A}$2.12}$$

$$Y_{D,t} = C_t + X_t + a(u_t)K_t, \tag{$\mathcal{A}$2.13}$$

$$Y_{A,t} = \left[u_t K_t\right]^{\psi}\left[(N_t - N_0)A_t\right]^{1-\psi}, \tag{$\mathcal{A}$2.14}$$

$$P_t^{1-\xi} = \mu_P \left[\frac{P_{t-1}}{P_{t-2}} P_{t-1}\right]^{1-\xi} + (1-\mu_P)P_{c,t}^{1-\xi}, \tag{$\mathcal{A}$2.15}$$

$$P_{A,t}^{-\xi} = \mu_P \left[\frac{P_{A,t-1}}{P_{A,t-2}} P_{A,t-1}\right]^{-\xi} + (1-\mu_P)P_{c,t}^{-\xi}, \tag{$\mathcal{A}$2.16}$$

$$W_t^{1-\theta} = \mu_W \left(\alpha * \frac{P_{t-1}}{P_{t-2}} W_{t-1}\right)^{1-\theta} + (1-\mu_W)W_{c,t}^{1-\theta}, \tag{$\mathcal{A}$2.17}$$

and

$$W_{D,t}^{-\theta} = \mu_W \left(\alpha * \frac{P_{t-1}}{P_{t-2}} W_{D,t-1}\right)^{-\theta} + (1-\mu_W)W_{c,t}^{-\theta}, \tag{$\mathcal{A}$2.18}$$

where λ_t, C_t, q_t, $S(\cdot)$, X_t, u_t, r_t, δ, $a(u_t)$, P_t, M_t, $Y_{A,t}$, $P_{A,t}$, ξ, $Y_{D,t}$, ϕ_t, ψ, K_t, W_t, $P_{c,t}$, μ_P, θ, μ_W, $W_{c,t}$, $W_{D,t}$, and γ denote the marginal utility of consumption, aggregate consumption, the shadow price of capital (per unit of consumption), the investment growth cost function, the deterministic TFP growth rate, aggregate investment, capital utilization rate, the rental rate of capital, the depreciation rate of capital, the household cost of capital utilization, the aggregate (demand) price level, the aggregate money stock at the end of date $t-1$, aggregate output, the aggregate supply price level, the price elasticity, aggregate demand, real marginal cost, capital's share of output, the aggregate capital stock at the end of date $t-1$, the aggregate nominal wage, the firm's optimal date t price, the fraction of firms forced to update their price at the previous period's inflation rate, the wage elasticity, the fraction of households forced to update their nominal wage at the previous period's inflation rate, the household's optimal date t nominal wage, the aggregate demand nominal wage, and the inverse of the Frisch labor supply elasticity, respectively.

A symmetric equilibrium is imposed on the markets in which final good firms and households have monopolistic and monopsonistic power. Along the symmetric equilibrium path, firms i and j choose the same commitment price $P_{c,t} = P_{i,t} = P_{j,t}$. The same restriction is placed on the nominal wages $W_{c,t} = W_{\ell,t} = W_{\wp,t}$ of households ℓ and $\wp$. The optimality conditions ($\mathcal{A}$2.9) and ($\mathcal{A}$2.11) reflect the impact of the symmetric equilibrium assumptions. Rather than $P_{i,t}$ and $W_{\ell,t}$, the symmetric equilibrium impose the final good price $P_{c,t}$ and nominal wage $W_{c,t}$ on the optimality conditions ($\mathcal{A}$2.9) and ($\mathcal{A}$2.11).

The impulse vector consists of TFP and monetary policy shocks. We assume TFP, $\ln A_t$, is a random walk with drift

$$\ln A_t = \alpha + \ln A_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right). \tag{$\mathcal{A}$2.19}$$

The monetary policy shock is either the innovation μ_t of the first order autoregression, AR(1), money growth supply rule

$$m_{t+1} = (1-\rho_m)m^* + \rho_m m_t + \mu_t, \quad \left|\rho_m\right| < 1, \quad \mu_t \sim \mathcal{N}\left(0, \sigma_\mu^2\right), \tag{$\mathcal{A}$2.20}$$

of the NKDSGE-AR model, where m^* is the steady state money growth rate, or the innovation υ_t to the interest rate smoothing Taylor rule

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left(R^* + a_\pi \mathbf{E}_t\left\{\frac{P_{t+1}}{P_t}\right\} + a_{\tilde{Y}}\tilde{Y}_t\right) + \upsilon_t, \ \left|\rho_R\right| < 1, \ \upsilon_t \sim \mathcal{N}\left(0, \sigma_\upsilon^2\right), \tag{$\mathcal{A}$2.21}$$

of the NKDSGE-TR model, where the steady state nominal rate R^* equals steady state inflation deflated by the household discount factor, π^*/β, π^* is the differential of steady state money growth and deterministic TFP growth, $\exp(m^* - \alpha)$, and $\widetilde{Y}_t$ is the output gap (*i.e.*, deviations of output from its trend). The TFP and money growth (or Taylor rule) innovations are assumed to be uncorrelated at leads and lags, $\mathbf{E}\{\varepsilon_{t+i}\,\mu_{t+j}\} = 0$, (or $\mathbf{E}\{\varepsilon_{t+i}\,\upsilon_{t+j}\} = 0$) for all i, j.

Equations ($\mathcal{A}2.1$)–($\mathcal{A}2.11$) are the optimality conditions of the baseline habit NKDSGE model. Internal consumption habit creates the forward-looking marginal utility of current consumption, which is restated by equation ($\mathcal{A}2.1$). Equation ($\mathcal{A}2.2$) sets the cost of adding one unit of aggregate investment, X_t, to its discounted expected benefit. The cost is represented by the ratio of the cost of installing a unit of investment to the market value of extant capital (*i.e.*, the inverse of Tobin's q), $(1 - q_t)/q_t$, plus the total cost of installing a unit of investment and the marginal cost of adding a unit of capital at the investment growth rate, X_t/X_{t-1}, net of steady state growth α^*. The expected benefit equals the foregone marginal cost of future investment valued at the pricing kernel, $\beta\lambda_{t+1}/\lambda_t$, which is weighted by the change in the price of capital. The Euler equation of capital ($\mathcal{A}2.3$) equates the price of increasing the capital stock by one unit to the discounted expected return on the service flow of that unit of capital net of the cost of capital services (or utilization) plus the net value of the unit of capital after production evaluated at the pricing kernel. The riskless bond is priced in the Euler equation ($\mathcal{A}2.4$). The dynamics of the purchasing power of money is described by the Euler equation ($\mathcal{A}2.5$), where money is valued at the marginal utility of consumption. Equations ($\mathcal{A}2.4$) and ($\mathcal{A}2.5$) yield the money demand function of the baseline habit NKDGSE model. Equation ($\mathcal{A}2.6$) is an intratemporal optimality condition that forces the marginal capital utilization rate to match the marginal product of capital, which equals the rental rate of capital. Final good firm labor demand is tied down by the intratemporal optimality condition ($\mathcal{A}2.7$). The ratio of aggregate supply to aggregate demand is connected to the ratio of the alternative aggregate price level to the aggregate price level raised to the negative of the price elasticity by equation ($\mathcal{A}2.8$). Equation ($\mathcal{A}2.9$) specifies optimal pricing of a monopolistically competitive final good firm. This decision is restricted by the Calvo staggered price technology, the firm's discount factor, real marginal cost, aggregate demand, and full indexation to lagged inflation of those firms unable to obtain their optimal price at date t. Aggregate labor demand is equated to aggregate labor supply in equation ($\mathcal{A}$2.10) up to the ratio of the aggregate nominal wage indices raised to the negative of the nominal wage elasticity. The optimal nominal wage decision is characterized by equation ($\mathcal{A}$2.11). The household settles on its optimal nominal wage by balancing the discounted expected disutility of labor supply to the benefits of greater real labor income in marginal utility of consumption units (*i.e.*, the marginal rate of substitution between the expected discounted lifetime disutility of work to the expected discounted value of permanent income). Note that these costs and benefits are affected by the wage and labor supply elasticities, and that those households unable to update their date t nominal wage reset using lagged inflation.

Equilibrium conditions are given by equations ($\mathcal{A}$2.12)–($\mathcal{A}$2.18) for the baseline habit NKDSGE model. Equation ($\mathcal{A}$2.12) is the law of motion of capital with capital adjustment costs. Aggregate demand equals its constituent parts according to equation ($\mathcal{A}$2.13). The constant returns to scale aggregate technology is found in equation ($\mathcal{A}$2.14). Equations ($\mathcal{A}$2.15), ($\mathcal{A}$2.16), ($\mathcal{A}$2.17), and ($\mathcal{A}$2.18) are the laws of motion of the aggregate price levels and nominal wages under Calvo price and nominal wage setting with full indexation.

The laws of motion ($\mathcal{A}$2.16) and ($\mathcal{A}$2.18) are added to avoid the curse of dimensionality. Under Calvo staggered price and nominal wage setting, Yun (1996) points out that the price and nominal wage aggregators ($\mathcal{A}$2.15) and ($\mathcal{A}$2.17) place the histories P_t and W_t (from date t = 0) into the state vector of the baseline habit NKDSGE model. The reason is the histories of P_t and W_t drive the process that restrict $P_{C,t}$ and $W_{C,t}$ along any candidate equilibrium path. The aggregate supply price and aggregate demand nominal wage laws of motion ($\mathcal{A}$2.16) and ($\mathcal{A}$2.18) are used to replace $P_{C,t}$ and $W_{C,t}$ with $P_{A,t}$ and $W_{D,t}$ in the state vector. This leaves the state vector with P_t, $P_{A,t}$, W_t, and $W_{D,t}$ rather than their histories.

A2.2 Stochastically detrended optimality and equilibrium conditions

The NKDSGE models contain a permanent technology shock A_t. Since this shock is a random walk (with drift), stochastic detrending renders the equilibrium path of state and other endogenous variables stationary. Stochastic detrending consists of $\widehat{C}_t \equiv C_t/A_t$, $\widehat{X}_t \equiv X_t/A_t$, $\widehat{Y}_{j,t} \equiv Y_{j,t}/A_t$, $j = A, D$, $\widehat{K}_{t+1} \equiv K_{t+1}/A_t$, $\widehat{P}_t \equiv P_t A_t/M_t$, $\widehat{P}_{i,t} \equiv P_{i,t}A_t/M_t$, $i = A, c$, $\widehat{W}_t \equiv W_t/M_t$, and $\widehat{W}_{c,t} \equiv W_{\wp,t}/M_t$, $\wp = D, c$. Applying these definitions to equations ($\mathcal{A}$2.1)–($\mathcal{A}$2.18) yields the stochastically detrended optimality and equilibrium conditions

$$\widehat{\lambda}_t = \frac{\alpha_t}{\alpha_t\widehat{C}_t - h\widehat{C}_{t-1}} - \beta h \mathbf{E}_t\left\{\frac{1}{\alpha_{t+1}\widehat{C}_{t+1} - h\widehat{C}_t}\right\}, \tag{$\mathcal{A}$2.22}$$

$$\frac{1-q_t}{q_t} + S\left(\frac{\alpha_t\widehat{X}_t}{\alpha^*\widehat{X}_{t-1}}\right) + S'\left(\frac{\alpha_t\widehat{X}_t}{\alpha^*\widehat{X}_{t-1}}\right)\frac{\alpha_t\widehat{X}_t}{\alpha^*\widehat{X}_{t-1}}$$

$$= \frac{\beta}{\alpha^*}\mathbf{E}_t\left\{\alpha_{t+1}\frac{q_{t+1}\widehat{\lambda}_{t+1}}{q_t\widehat{\lambda}_t}S'\left(\frac{\alpha_{t+1}\widehat{X}_{t+1}}{\alpha^*\widehat{X}_t}\right)\left[\frac{\widehat{X}_{t+1}}{\widehat{X}_t}\right]^2\right\}, \tag{$\mathcal{A}$2.23}$$

$$q_t = \beta\mathbf{E}_t\left\{\frac{\widehat{\lambda}_{t+1}}{\widehat{\lambda}_t}\left[\psi u_{t+1}\phi_{t+1}\frac{\widehat{Y}_{t+1}}{\widehat{K}_{t+1}} + \frac{q_{t+1}[1-\delta] - a(u_{t+1})}{\alpha_{t+1}}\right]\right\}, \tag{$\mathcal{A}$2.24}$$

$$\frac{\widehat{\lambda}_t}{\widehat{P}_t} = \beta\mathbf{E}_t\left\{\frac{\widehat{\lambda}_{t+1}}{\widehat{P}_{t+1}}\frac{R_{t+1}}{\exp(m_{t+1})}\right\}, \tag{$\mathcal{A}$2.25}$$

$$\frac{\widehat{\lambda}_t}{\widehat{P}_t} = \beta\mathbf{E}_t\left\{\left[\frac{\widehat{\lambda}_{t+1}}{\widehat{P}_{t+1}} + 1\right]\exp(-m_{t+1})\right\}, \tag{$\mathcal{A}$2.26}$$

$$a'(u_t) = \psi\phi_t\alpha_t\frac{\widehat{Y}_t}{\widehat{K}_t}, \tag{$\mathcal{A}$2.27}$$

$$\frac{\widehat{W}_t}{\hat{P}_t} = (1-\psi)\phi_t \frac{\hat{Y}_t}{N_t - N_0}, \tag{$\mathcal{A}$2.28}$$

$$\frac{\hat{Y}_{A,t}}{\hat{Y}_{D,t}} = \left(\frac{\hat{P}_{A,t}}{\hat{P}_t}\right)^{-\xi}, \tag{$\mathcal{A}$2.29}$$

$$\exp(m_t - \varepsilon_t)\frac{\hat{P}_{c,t}}{\hat{P}_{t-1}} = \left(\frac{\xi}{\xi-1}\right)\frac{\mathbf{E}_t \sum_{i=0}^{\infty}(\beta\mu_P)^i\,\hat{\lambda}_{t+i}\phi_{t+i}\hat{Y}_{D,t+i}\Big[\exp(m_{t+i}-\varepsilon_{t+i})\frac{\hat{P}_{t+i}}{\hat{P}_{t+i-1}}\Big]^{\xi}}{\mathbf{E}_t \sum_{i=0}^{\infty}(\beta\mu_P)^i\,\hat{\lambda}_{t+i}\hat{Y}_{D,t+i}\Big[\exp(m_{t+i}-\varepsilon_{t+i})\frac{\hat{P}_{t+i}}{\hat{P}_{t+i-1}}\Big]^{\xi-1}}, \tag{$\mathcal{A}$2.30}$$

$$\frac{N_t}{n_t} = \left(\frac{\widehat{W}_{D,t}}{\widehat{W}_t}\right)^{-\xi}, \tag{$\mathcal{A}$2.31}$$

$$\begin{aligned}\left[\exp(m_t)\frac{\widehat{W}_{c,t}}{\hat{P}_{t-1}}\right]^{1+\theta/\gamma} &= \frac{\theta}{\theta-1}\\ &\times\frac{\mathbf{E}_t \sum_{i=0}^{\infty}(\beta\mu_W)^i \exp(\theta(1+1/\gamma)(m_{t+i}+\sum_{j=1}^{i}\varepsilon_{t+j-1})\left[\left[\frac{\widehat{W}_{t+i}}{\hat{P}_{t+i-1}}\right]^{\theta}N_{t+i}\right]^{1+1/\gamma}}{\mathbf{E}_t \sum_{i=0}^{\infty}(\beta\mu_W)^i\,\lambda_{t+i}\exp(-(1-\theta)(m_{t+i}+\sum_{j=1}^{i}\varepsilon_{t+j-1}))\left[\frac{\widehat{W}_{t+i}}{\hat{P}_{t+i-1}}\right]^{\theta}\left[\frac{\hat{P}_{t+i}}{\hat{P}_{t+i-1}}\right]^{-1}N_{t+i}},\end{aligned} \tag{$\mathcal{A}$2.32}$$

$$\hat{K}_{t+1} = \frac{(1-\delta)\hat{K}_t}{\alpha_t} + \left[1 - S\left(\frac{\alpha_t\hat{X}_t}{\alpha^*\hat{X}_{t-1}}\right)\right]\hat{X}_t, \tag{$\mathcal{A}$2.33}$$

$$\widehat{Y}_t = \widehat{C}_t + \widehat{X}_t + \frac{a(u_t)\widehat{K}_t}{\alpha_t}, \tag{$\mathcal{A}$2.34}$$

$$\widehat{Y}_t = \left[u_t \frac{\widehat{K}_t}{\alpha_t}\right]^{\psi} \left[N_t - N_O\right]^{1-\psi}, \tag{$\mathcal{A}$2.35}$$

$$\widehat{P}_t^{1-\xi} = \mu_P \left[\exp(-m_t + m_{t-1} + \varepsilon_t - \varepsilon_{t-1}) \frac{\widehat{P}_{t-1}}{\widehat{P}_{t-2}} \widehat{P}_{t-1}\right]^{1-\xi} + (1-\mu_P)\widehat{P}_{c,t}^{1-\xi}, \tag{$\mathcal{A}$2.36}$$

$$\widehat{P}_{A,t}^{-\xi} = \mu_P \left[\exp(-m_t + m_{t-1} + \varepsilon_t - \varepsilon_{t-1}) \frac{\widehat{P}_{A,t-1}}{\widehat{P}_{A,t-2}} \widehat{P}_{A,t-1}\right]^{-\xi} + (1-\mu_P)\widehat{P}_{c,t}^{-\xi}, \tag{$\mathcal{A}$2.37}$$

$$\widehat{W}_t^{1-\theta} = \mu_W \left[\exp(-m_t + m_{t-1} - \varepsilon_{t-1}) \frac{\widehat{P}_{t-1}}{\widehat{P}_{t-2}} \widehat{W}_{t-1}\right]^{1-\theta} + (1-\mu_W)\widehat{W}_{c,t}^{1-\theta}, \tag{$\mathcal{A}$2.38}$$

and

$$\widehat{W}_{D,t}^{-\theta} = \mu_W \left[\exp(-m_t + m_{t-1} - \varepsilon_{t-1}) \frac{\widehat{P}_{t-1}}{\widehat{P}_{t-2}} \widehat{W}_{D,t-1}\right]^{-\theta} + (1-\mu_W)\widehat{W}_{c,t}^{-\theta}, \tag{$\mathcal{A}$2.39}$$

where it is understood in equation ($\mathcal{A}$2.32) that at $i = 0$ the sum $\sum_{j=1}^{i} \varepsilon_{t+j-1}$ equals one. Equations ($\mathcal{A}$2.22)–($\mathcal{A}$2.39) constitute the basis of the steady state equilibrium and the first-order linear approximation of the baseline habit NKDSGE models.

A2.3 Deterministic steady state

Let λ^*, C^*, Y^*, X^*, N^*, K^*, q^*, W^*, r^*, P^*, u^*, ϕ^*, and R^* denote deterministic steady state values of the corresponding endogenous variables. The steady state equilibrium rests on $u^* = 1$, $a(1) = 0$, and $S(1) = S'(1) = 0$, which is consistent with Christiano, Eichenbaum, and Evans (2005). Given these assumptions, the following equations characterize the deterministic steady state of the stochastically detrended system ($\mathcal{A}$2.22)–($\mathcal{A}$2.39)

$$C^*\lambda^* = \frac{\alpha^* - \beta h}{\alpha^* - h}, \tag{$\mathcal{A}$2.40}$$

$$q^* = 1, \tag{$\mathcal{A}$2.41}$$

$$\frac{K^*}{Y^*} = \frac{\beta\alpha^*\psi\phi^*}{\alpha^* - \beta(1-\delta)}, \tag{$\mathcal{A}$2.42}$$

$$R^* = \frac{\exp(m^*)}{\beta}, \tag{$\mathcal{A}$2.43}$$

$$\frac{\lambda^*}{P^*} = \frac{\beta}{\exp(m^*) - \beta}, \tag{$\mathcal{A}$2.44}$$

$$a'(1) = \psi\phi^*\alpha^*\frac{Y^*}{K^*}, \tag{$\mathcal{A}$2.45}$$

$$\frac{W^*}{P^*} = (1-\psi)\phi^*\frac{Y^*}{N^* - N_0}, \tag{$\mathcal{A}$2.46}$$

$$\phi^* = \frac{\xi - 1}{\xi}, \tag{$\mathcal{A}$2.47}$$

$$\frac{W^*}{P^*} = \left(\frac{\theta}{\theta - 1}\right) \frac{N^{*1/\gamma}}{\lambda^*}, \tag{$\mathcal{A}$2.48}$$

$$\frac{X^*}{K^*} = 1 - \frac{(1 - \delta)}{\alpha^*}, \tag{$\mathcal{A}$2.49}$$

$$Y^* = C^* + X^*, \tag{$\mathcal{A}$2.50}$$

and

$$Y^* = \left[\frac{K^*}{\alpha^*}\right]^{\psi} \left[N^* - N_O\right]^{1-\psi}. \tag{$\mathcal{A}$2.51}$$

Note that equations ($\mathcal{A}$2.46), ($\mathcal{A}$2.48), and ($\mathcal{A}$2.51) imply that the solution for N^* is nonlinear. Also, at the steady state equilibrium, $P^* = P_A^* = P_c^*$ and $W^* = W_D^* = W_c^*$.

A2.4 Log-linearized baseline habit NKDSGE models

We log linearize the optimality and equilibrium conditions of the baseline NKDSGE models in this section. The log linear approximations (*i.e.*, first-order Taylor expansions) of the stochastically detrended system ($\mathcal{A}$2.22)–($\mathcal{A}$2.39) are around the deterministic steady state given by equations ($\mathcal{A}$2.40)–($\mathcal{A}$2.51). The approximations exploit, for example, the definitions $\widetilde{C}_t = \ln \hat{C}_t - \ln C^*$ or $\widetilde{N}_t = \ln N_t - \ln N^*$.

A symmetric equilibrium has several implications for the log linear approximation of the baseline habit NKDSGE models. Subsequent to log linearizing around the steady state, the aggregate price indices are equated $\widetilde{P}_t = \widetilde{P}_{A,t}$, as are the aggregate nominal wages $\widetilde{W}_t = \widetilde{W}_{D,t}$, given $P_0 = P_{A,0}$ and $W_0 = W_{D,0}$. This further reduces the dimension of the state vector.

Log linearizing the stochastically detrended system ($\mathcal{A}$2.22)–($\mathcal{A}$2.39) yields the linear approximate optimality and equilibrium conditions of the baseline habit NKDSGE model. The relevant conditions are

$$(\alpha^* - h)(\alpha^* - \beta h)\widetilde{\lambda}_t = \beta\alpha^* h\mathbf{E}_t\widetilde{C}_{t+1} - (\beta h^2 + \alpha^{*2})\widetilde{C}_t + \alpha^* h(\widetilde{C}_{t-1} - \varepsilon_t), \qquad (\mathcal{A}2.52)$$

$$\beta\varpi\mathbf{E}_t\widetilde{X}_{t+1} - (1+\beta)\varpi\widetilde{X}_t + \varpi\widetilde{X}_{t-1} + \widetilde{q}_t = \varpi\varepsilon_t, \qquad (\mathcal{A}2.53)$$

$$\widetilde{q}_t + \widetilde{\lambda}_t = \mathbf{E}_t\left\{\widetilde{\lambda}_{t+1} + \beta\psi\phi^*\frac{Y^*}{K^*}\left[\widetilde{\phi}_{t+1} + \widetilde{Y}_{t+1} - \widetilde{K}_{t+1}\right] + \beta\frac{1-\delta}{\alpha^*}\widetilde{q}_{t+1}\right\}, \qquad (\mathcal{A}2.54)$$

$$\widetilde{\lambda}_t - \widetilde{P}_t = \mathbf{E}_t\left\{\widetilde{\lambda}_{t+1} - \widetilde{P}_{t+1} + \widetilde{R}_{t+1}\right\} - \widetilde{m}_{t+1}, \qquad (\mathcal{A}2.55)$$

$$\widetilde{\lambda}_t - \widetilde{P}_t = \frac{\lambda^*}{\lambda^* + P^*}\mathbf{E}_t\left\{\widetilde{\lambda}_{t+1} - \widetilde{P}_{t+1}\right\} - \widetilde{m}_{t+1}, \qquad (\mathcal{A}2.56)$$

$$\varrho\widetilde{u}_t = \widetilde{\phi}_t + \widetilde{Y}_t - \widetilde{K}_t + \varepsilon_t, \qquad (\mathcal{A}2.57)$$

$$\widetilde{W}_t - \widetilde{P}_t = \widetilde{\phi}_t + \widetilde{Y}_t - \frac{N^*}{N^* - N_0}\widetilde{N}_t, \tag{$\mathcal{A}$2.58}$$

$$\begin{aligned} \mu_P(1+\beta)\widetilde{\pi}_t \;&=\; \beta\mu_P \mathbf{E}_t \widetilde{\pi}_{t+1} + \mu_P \widetilde{\pi}_{t-1} \\ &+\; (1-\mu_P)(1-\beta\mu_P)\widetilde{\phi}_t + \beta\mu_P \widetilde{m}_{t+1} - \mu_P(1+\beta)(\widetilde{m}_t - \varepsilon_t) + \mu_P(\widetilde{m}_{t-1} - \varepsilon_{t-1}), \end{aligned} \tag{$\mathcal{A}$2.59}$$

$$\begin{aligned} \left[1 + \beta\mu_W^2 - \frac{\theta(1-\mu_W)(1-\beta\mu_W)}{\theta+\gamma}\right]\widetilde{W}_t \;&=\; \beta\mu_W \mathbf{E}_t \widetilde{W}_{t+1} + \mu_W \widetilde{W}_{t-1} + \left[\frac{(1-\mu_W)(1-\beta\mu_W)}{\theta+\gamma}\right]\widetilde{N}_t \\ -\left[\frac{\gamma(1-\mu_W)(1-\beta\mu_W)}{\theta+\gamma}\right](\widetilde{\lambda}_t - \widetilde{P}_t) &- \beta\mu_W \widetilde{\pi}_t + \mu_W \widetilde{\pi}_{t-1} + \beta\mu_W \widetilde{m}_{t+1} - (1+\beta)\mu_W \widetilde{m}_t + \mu_W \widetilde{m}_{t-1} \\ &+\; \beta\mu_W \varepsilon_t - \mu_W \varepsilon_{t-1}, \end{aligned} \tag{$\mathcal{A}$2.60}$$

$$\widetilde{K}_{t+1} = \frac{(1-\delta)}{\alpha^*}\left(\widetilde{K}_t - \varepsilon_t\right) + \frac{X^*}{K^*}\widetilde{X}_t, \tag{$\mathcal{A}$2.61}$$

$$\widetilde{Y}_t = \frac{C^*}{Y^*}\widetilde{C}_t + \frac{X^*}{Y^*}\widetilde{X}_t + \psi\phi^*\widetilde{u}_t, \tag{$\mathcal{A}$2.62}$$

$$\widetilde{Y}_t = \psi\left(\widetilde{u}_t + \widetilde{K}_t\right) + (1-\psi)\frac{N^*}{N^* - N_0}\widetilde{N}_t - \psi\varepsilon_t, \tag{$\mathcal{A}$2.63}$$

and

$$\widetilde{m}_{t+1} \;=\; \rho_m \widetilde{m}_t + \mu_t, \tag{$\mathcal{A}$2.64}$$

for NKDSGE-AR rule models, or for NKDSGE-TR models the interest rate rule

$$(1-\rho_R\mathbf{L})\widetilde{R}_t \;=\; (1-\rho_R)\left(a_\pi \mathbf{E}_t\widetilde{\pi}_{t+1} + a_\pi\widetilde{m}_{t+1} + a_y\widetilde{Y}_t\right) \;+\; \upsilon_t, \tag{$\mathcal{A}$2.65}$$

where $\varrho \equiv \dfrac{a''(1)}{a'(1)}$ $(= 1.174)$ and $\widetilde{\pi}_t \equiv \widetilde{P}_t - \widetilde{P}_{t-1}$. The linear approximate habit NKDSGE-TR model consists of the linear stochastic difference equations ($\mathcal{A}$2.52)–($\mathcal{A}$2.63) and ($\mathcal{A}$2.65) with the unknowns $\widetilde{\lambda}_t$, $\widetilde{C}_t$, $\widetilde{X}_t$, $\widetilde{q}_t$, $\widetilde{Y}_t$, $\widetilde{K}_{t+1}$, $\widetilde{R}_t$, $\widetilde{u}_t$, $\widetilde{\phi}_t$, $\widetilde{N}_t$, $\widetilde{P}_t$, and $\widetilde{W}_t$. When the AR(1) money growth rule ($\mathcal{A}$2.64) replaces the Taylor rule ($\mathcal{A}$2.65) in the system of linear stochastic difference equations that approximate the baseline habit NKDSGE-AR model, the linearized detrended bond Euler equation ($\mathcal{A}$2.55) can be dropped along with the demeaned nominal rate $\widetilde{R}_t$.

A2.5 Solving the baseline habit NKDSGE models

This section describes the solution method we apply to solve the linear stochastic difference equations that approximate the NKDSGE models. Consider the baseline habit NKDSGE-TR model. For this model, the vector of endogenous variables is

$$H_t = \left[\widetilde{\lambda}_t \;\; \widetilde{C}_t \;\; \widetilde{X}_t \;\; \widetilde{q}_t \;\; \widetilde{Y}_t \;\; \widetilde{K}_{t+1} \;\; \widetilde{R}_t \;\; \widetilde{u}_t \;\; \widetilde{\phi}_t \;\; \widetilde{N}_t \;\; \widetilde{P}_t \;\; \widetilde{W}_t \;\; \widetilde{m}_{t+1}\right]'.$$

Next define the expectational forecast errors $\vartheta_{\widetilde{\lambda},t+1} = \widetilde{\lambda}_{t+1} - E_t\widetilde{\lambda}_{t+1}$, $\vartheta_{\widetilde{C},t+1} = \widetilde{C}_{t+1} - E_t\widetilde{C}_{t+1}$, $\vartheta_{\widetilde{X},t} = \widetilde{X}_{t+1} - E_t\widetilde{X}_{t+1}$, $\vartheta_{\widetilde{q},t+1} = \widetilde{q}_{t+1} - E_t\widetilde{q}_{t+1}$, $\vartheta_{\widetilde{Y},t+1} = \widetilde{Y}_{t+1} - E_t\widetilde{Y}_{t+1}$, $\vartheta_{\widetilde{u},t+1} = \widetilde{u}_{t+1} - E_t\widetilde{u}_{t+1}$,

$\vartheta_{\tilde{\Phi},t+1} = \tilde{\Phi}_{t+1} - E_t\tilde{\Phi}_{t+1}$, $\vartheta_{\tilde{P},t} = \tilde{P}_{t+1} - E_t\tilde{P}_{t+1}$, and $\vartheta_{\widetilde{W},t+1} = \widetilde{W}_{t+1} - E_t\widetilde{W}_{t+1}$. Collect these forecast errors into the vector ϑ_{t+1}. We use H_t and ϑ_t, the linear approximate optimality and equilibrium conditions ($\mathcal{A}$2.52)–($\mathcal{A}$2.63) and the Taylor rule ($\mathcal{A}$2.65) to form the multivariate first-order stochastic difference equation system of the baseline habit NKDSGE-TR model

$$\mathbb{G}_0\mathcal{H}_t = \mathbb{G}_1\mathcal{H}_{t-1} + \mathbb{V}\zeta_t + \mathbb{K}\vartheta_t, \tag{$\mathcal{A}$2.66}$$

where $\mathcal{H} = [H_t \ \ \mathbf{E}_t H_{t+1}]'$ and $\zeta_t = [\varepsilon_t \ \ \upsilon_t]'$ (or when monetary policy is defined by the AR(1) money growth rule ($\mathcal{A}$2.64), $\zeta_t = [\varepsilon_t \ \ \mu_t]'$). It is understood that $\mathbf{E}_t H_{t+1}$ contains only those elements of H_t that enter equations ($\mathcal{A}$2.52)–($\mathcal{A}$2.63) as one-step ahead expectations. The matrices $\mathbb{G}_0$, $\mathbb{G}_1$, and $\mathbb{V}$ contain cross-equation restriction embedded in the optimality and equilibrium conditions ($\mathcal{A}$2.52)–($\mathcal{A}$2.63), and the Taylor rule ($\mathcal{A}$2.65).

Sims (2002) studies and solves multivariate linear rational expectations models that match ($\mathcal{A}$2.66). His solution algorithm taps the QZ (or generalized complex Schur) decomposition of matrices $\mathbb{G}_0$ and $\mathbb{G}_1$. The QZ decomposition employs $\mathcal{Q}'\mathcal{F}\mathcal{Z}' = \mathbb{G}_0$ and $\mathcal{Q}'\mathcal{O}\mathcal{Z}' = \mathbb{G}_1$, where $\mathcal{Q}'\mathcal{Q} = \mathcal{Z}'\mathcal{Z} = \mathbf{I}$ and matrices $\mathcal{F}$ and $\mathcal{O}$ are upper triangular. Matrices $\mathcal{Q}$, $\mathcal{Z}$, $\mathcal{F}$ and $\mathcal{O}$ are possibly complex. Let $\mathcal{D}_t = \mathcal{Z}'\mathcal{H}_t$ and premultiply equation ($\mathcal{A}$2.66) by $\mathcal{Q}$ to obtain

$$\begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} \\ \mathbf{0} & \mathcal{F}_{22} \end{bmatrix} \begin{bmatrix} \mathcal{D}_{1,t} \\ \mathcal{D}_{2,t} \end{bmatrix} = \begin{bmatrix} \mathcal{O}_{11} & \mathcal{O}_{12} \\ \mathbf{0} & \mathcal{O}_{22} \end{bmatrix} \begin{bmatrix} \mathcal{D}_{1,t-1} \\ \mathcal{D}_{2,t-1} \end{bmatrix} + \begin{bmatrix} \mathcal{Q}_{1\cdot} \\ \mathcal{Q}_{2\cdot} \end{bmatrix} \left(\mathbb{V}\zeta_t + \mathbb{K}\vartheta_t\right), \tag{$\mathcal{A}$2.67}$$

where $\mathcal{Q}_{j\cdot}$ denotes the jth block of rows of $\mathcal{Q}$. Although the QZ decomposition of $\mathbb{G}_0$ and $\mathbb{G}_1$ never fails to exist, these decompositions are not unique. Nonetheless, generalized eigenvalues of $\mathcal{F}$ and $\mathcal{O}$ can be unique if infinite values are allowed and zero eigenvalues for $\mathbb{G}_0$ and $\mathbb{G}_1$ are ruled out. Denote the generalized eigenvalues of $\mathcal{F}$ and $\mathcal{O}$ as $f_{ii}^{-1}o_{ii}$. These eigenvalues are ordered to partition the system ($\mathcal{A}$2.67) in such a way to place only explosive elements in $\mathcal{D}_{2,t}$. The 'reduced form' process of $\mathcal{D}_{2,t}$ is the second row of the system ($\mathcal{A}$2.67), which is written

$$\mathcal{D}_{2,t} = \mathcal{M}\mathcal{D}_{2,t-1} + \mathcal{M}\mathcal{O}_{22}^{-1}\mathcal{Q}_{2\cdot}\left(\mathbb{V}\zeta_t + \mathbb{K}\vartheta_t\right), \tag{$\mathcal{A}$2.68}$$

where $\mathcal{M} \equiv \mathcal{F}_{22}^{-1}\mathcal{O}_{22}$. Forward iteration of equation ($\mathcal{A}$2.68) gives

$$\mathcal{D}_{2,t} = -\sum_{i=0}^{\infty} \mathcal{M}^{-i}\mathcal{O}_{22}^{-1}\mathcal{Q}_{2\cdot}\left(\mathbb{V}\zeta_{t+i+1} + \mathbb{K}\vartheta_{t+i+1}\right), \tag{$\mathcal{A}$2.69}$$

where the transversality condition $\mathcal{M}^{-i}\mathcal{D}_{2,t+i}$, $i \longrightarrow \infty$ holds.

Extrinsic or sunspot equilibria are excluded from the solution of the present value ($\mathcal{A}$2.69) of $\mathcal{D}_{2,t}$. The present value invokes a no sunspot result because the expectation error vector ϑ_t has no impact on $\mathbb{G}_0$ and $\mathbb{G}_1$. The implications is that $\mathcal{D}_{2,t}$ belongs only to the date t information set (*i.e.*, it includes only the intrinsic shocks of ζ_t), which mean that

$$\mathbf{E}_t \sum_{i=0}^{\infty} \mathcal{M}^{-i}\mathcal{O}_{22}^{-1}\mathcal{Q}_{2\cdot}\mathbb{V}\zeta_{t+i+1} = \sum_{i=0}^{\infty} \mathcal{M}^{-i}\mathcal{O}_{22}^{-1}\mathcal{Q}_{2\cdot}\left(\mathbb{V}\zeta_{t+i+1} + \mathbb{K}\vartheta_{t+i+1}\right),$$

For an intrinsic equilibrium to exist, Sims (2002) shows that the necessary and sufficient conditions are that the set of equations $\mathcal{Q}_{2\cdot}\mathbb{V}\zeta_{t+1} + \mathcal{Q}_{2\cdot}\mathbb{K}\vartheta_{t+1}$ equal a column vector of zeros. A solution is available for the multivariate first order system ($\mathcal{A}$2.66) if (and only if) the column space of $Q_{2\cdot}\mathbb{V}$ is contained in that of $Q_{2\cdot}\mathbb{K}$. Given ζ_t is uncorrelated, the solution follows immediately. This is not true for the NKDSGE-AR models. When the intrinsic shocks are serially correlated, $\mathcal{Q}_{2\cdot}\mathbb{K}\vartheta_t$ is calculated from information in $\mathcal{Q}_{2\cdot}\mathbb{V}\zeta_t$.

Suppose that an intrinsic solution exists. When there is no sunspot equilibria, the row space of $\mathcal{Q}_{1\cdot}\mathbb{K}$ is contained in that of $\mathcal{Q}_{2\cdot}\mathbb{K}$. This is a necessary and sufficient condition for uniqueness of the solution of the linear approximate system ($\mathcal{A}$2.66), as shown by Sims (2002). He suggests working with a matrix Φ that yields $\mathcal{Q}_{1\cdot}\mathbb{K} = \Phi Q_{2\cdot}\mathbb{K}$. By premultiplying equation ($\mathcal{A}$2.67) with $[\mathbf{I} \quad -\Phi]$, combining this with equation ($\mathcal{A}$2.68), and noting that this wipes out the expectational forecast errors ϑ_t, we have

$$\mathcal{F}_{11}\mathcal{D}_{1,t} + (\mathcal{F}_{12} - \Phi\mathcal{F}_{22})\mathcal{D}_{2,t} = \mathcal{O}_{11}\mathcal{D}_{1,t-1} + (\mathcal{O}_{12} - \Phi\mathcal{O}_{22})\mathcal{D}_{2,t-1} + (Q_{1\cdot} - \Phi Q_{2\cdot})\mathbb{V}\zeta_t.$$

Stacking these equations on top of the equations of ($\mathcal{A}$2.69) produces

$$\begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} - \Phi\mathcal{F}_{22} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathcal{D}_{1,t} \\ \mathcal{D}_{2,t} \end{bmatrix} = \begin{bmatrix} \mathcal{O}_{11} & \mathcal{O}_{12} - \Phi\mathcal{O}_{22} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathcal{D}_{1,t-1} \\ \mathcal{D}_{2,t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} Q_{1\cdot} - \Phi Q_{2\cdot} \\ \mathbf{0} \end{bmatrix} \mathbb{V}\zeta_t + \begin{bmatrix} \mathbf{0} \\ \mathbf{E}_t \sum_{i=0}^{\infty} \mathcal{M}^{-i}\mathcal{O}_{22}^{-1}\mathcal{Q}_{2\cdot}\mathbb{V}\zeta_{t+i+1} \end{bmatrix}.$$

This matrix system maps into the unique intrinsic solution for $\mathcal{H}_t$

$$\mathcal{H}_t = \Theta_{\mathcal{H}}\mathcal{H}_{t-1} + \Theta_{\zeta}\zeta_t, \qquad (\mathcal{A}2.70)$$

where

$$\Theta_{\mathcal{H}} = \mathcal{Z} \begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} - \Phi\mathcal{F}_{22} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathcal{O}_{11} & \mathcal{O}_{12} - \Phi\mathcal{O}_{22} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathcal{Z}'$$

and

$$\Theta_{\zeta} = \mathcal{Z} \begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} - \Phi\mathcal{F}_{22} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} Q_{1\cdot} - \Phi Q_{2\cdot} \\ \mathbf{0} \end{bmatrix}.$$

We employ the system of first-order stochastic difference equations ($\mathcal{A}$2.70) to produce linear approximate solutions for the NKDSGE models. These solutions generate synthetic data sets that are inputs into our Bayesian simulation experiments.

A3. Estimating SVMAs, Checking Their ABC and Ds, and Spectral Density Computation

This section fills in a few gaps about the methods used to evaluate the NKDSGE models. We review the Blanchard and Quah (1989) decomposition and apply it to vector autoregressions (VARs) of output growth (or consumption growth) and inflation. These VARs are identified with a long-run monetary neutrality restriction (LRMN) that the level of output or consumption is independent of monetary shocks at $t \longrightarrow \infty$. The LRMN restriction yields SVMA(∞), processes of output (or consumption growth) and inflation. We show that it retrieves the TFP and monetary policy shock innovations of the NKDSGE models as in the ABCs and Ds of Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (2007). The SVMA(∞) also provides a map to permanent and transitory output and consumption growth spectral densities (SDs). This section ends with a review of several approaches that are available to compute SDs.

A3.1 VARs and SVMAs

The SVMAs are constructed from VARs of $\mathcal{X}_t = [\Delta \ln Y_t \;\; \Delta \ln P_t]'$ or $[\Delta \ln C_t \;\; \Delta \ln P_t]'$ and the LRMN restriction using the Blanchard and Quah (1989) decomposition. The unrestricted joint probability distribution of $\mathcal{X}_t$ is approximated by the finite-order VAR

$$\mathcal{X}_t = \mathbb{A}(\mathbf{L})\mathcal{X}_{t-1} + e_t, \quad \mathbb{A}(\mathbf{L}) = \sum_{j=1}^{p} \mathbb{A}_j \mathbf{L}^j, \qquad (\mathcal{A}3.1)$$

where constants are ignored, the forecast errors $e_t = \mathcal{X}_t - \mathbf{E}\left\{\mathcal{X}_t \,\middle|\, \mathcal{X}_{t-1}, \mathcal{X}_{t-2}, \ldots, \mathcal{X}_{t-p}\right\}$ are Gaussian, and its covariance matrix is Σ.

The lag length p is chosen by a general-to-specific test procedure. The procedure begins with the null hypothesis of $p = 9$ against the alternative of ten lags, if the null is not rejected, set

p to eight, repeat the test but against the alternative of a VAR(9). This process is repeated until either the null is rejected or until the null is a VAR(1) against a two lag specification. A likelihood ratio statistic is used to compare the competing hypotheses on a output (or consumption) growth-inflation sample that is described in section **A1**. For either $\mathcal{X}_t = [\Delta \ln Y_t \;\; \Delta \ln P_t]'$ or $[\Delta \ln C_t \;\; \Delta \ln P_t]'$, we find that $p = 5$.

The unrestricted VAR of ($\mathcal{A}$3.1) is invertible whether estimated or under the NKDSGE models. Inverting this VAR yields the reduced form VMA(∞), $\mathcal{X}_t = \left[\mathbf{I} - \mathbb{A}(\mathbf{L})\right]^{-1} e_t$, or Wold representation of $\mathcal{X}_t$, $\mathbb{C}(\mathbf{L})e_t$, where $\mathbb{C}(\mathbf{L}) = \sum_{i=0}^{\infty} \mathbb{C}_i \mathbf{L}^i$ and the reduced form impact matrix $\mathbb{C}_0 = \mathbf{I}$. The corresponding SVMA(∞) is

$$\mathcal{X}_t = \mathbb{B}(\mathbf{L})\varsigma_t, \quad \varsigma_t \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}\right), \tag{$\mathcal{A}$3.2}$$

which summarizes equation (10) of the paper. The NKDSGE models predict that in the long run the levels of output and consumption are independent of monetary policy innovations (*i.e.*, the money growth rule innovation μ_t or Taylor rule innovation υ_t). This is the LRMN restriction, which forces the upper right element of $\mathbb{B}(\mathbf{1})$ to be zero, or $\sum_{j=0}^{\infty} \mathbb{B}_{j,1,2} = \mathbb{B}(\mathbf{1})_{1,2} = 0$. The SVMA ($\mathcal{A}$3.2) and the reduced form VMA(∞) also restrict $e_t = \mathbb{B}_0\varsigma_t$ and $\mathbb{B}_j = \mathbb{A}_j\mathbb{B}_0$. Note that once estimates of the four unknown elements of the structural impact response matrix $\mathbb{B}_0$ are available, we can compute the SVMA of ($\mathcal{A}$3.2) from the reduced form VMA(∞).

Our goal is to recover the four unknown coefficients of $\mathbb{B}_0$. The map from the structural shocks to the reduced form errors, $e_t = \mathbb{B}_0\varsigma_t$, and the covariances matrices of e_t and ς_t place three restrictions on the four unknowns of $\mathbb{B}_0$. These three restrictions present us with three nonlinear equations that follow from expanding $\Sigma = \mathbb{B}_0\mathbb{B}_0'$ to

$$\begin{aligned} \Sigma_{1,1} &= \mathbb{B}_{0,1,1}^2 + \mathbb{B}_{0,1,2}^2, \\ \Sigma_{1,2} &= \mathbb{B}_{0,1,1}\mathbb{B}_{0,2,1} + \mathbb{B}_{0,1,2}\mathbb{B}_{0,2,2}, \\ \Sigma_{2,2} &= \mathbb{B}_{0,2,1}^2 + \mathbb{B}_{0,2,2}^2. \end{aligned} \tag{$\mathcal{A}$3.3}$$

The remaining restriction is found by summing both sides of $\mathbb{B}_j = \mathbb{A}_j\mathbb{B}_0$ from $j \geq 0$, which leads to $\mathbb{B}(\mathbf{1}) = \mathbb{C}(\mathbf{1})\mathbb{B}_0$. The LRMN restriction forces

$$\mathbb{C}(\mathbf{1})_{1,1}\mathbb{B}_{0,1,2} + \mathbb{C}(\mathbf{1})_{1,2}\mathbb{B}_{0,2,2} = 0, \tag{$\mathcal{A}$3.4}$$

which is a fourth nonlinear equation. We solve the four nonlinear equations ($\mathcal{A}$3.3) and ($\mathcal{A}$3.4) to calculate estimates of the four unknown coefficients of $\mathbb{B}_0$.

Markov chain Monte Carlo simulations of the SVMA(∞) of equation ($\mathcal{A}$3.2) engage the BACC software of Geweke (1999) and McCausland (2004). The simulators need priors that are obtained, only in part, from ordinary least squares (OLS) estimates of the reduced form VAR(5) of equation ($\mathcal{A}$3.1). These estimates and related covariance matrices are the prior information used to generate $\mathcal{J}$ (= 5,000) posterior draws of the reduced form VAR(5) coefficients. Next we calculate the reduced form VMA(∞) and apply the BQ decomposition by imposing the LRMN restriction to recover the SVMA(∞) of equation ($\mathcal{A}$3.2). The $\mathcal{J}$ samples of the $\mathbb{B}(\mathbf{L})$s are the basis of the empirical, $\mathcal{E}$, distributions of the permanent and transitory output and consumption growth spectral densities. The theoretical, $\mathcal{T}$, distributions of these moments are estimated in the same manner, but on synthetic samples generated by NKDSGE models.

We treat synthetic samples generated by Markov chain Monte Carlo simulations of the unrestricted VAR ($\mathcal{A}$3.1) and the NKDSGE models in the same way, with one caveat. The exception is that although the off diagonal elements of the NKDSGE model structural shock covariance matrix $\Xi = \mathbf{E}\left\{\zeta_t\zeta_t'\right\}$ are zero, its diagonal elements are not unity. The NKDSGE model SVMAs are normalized for the Blanchard and Quah (BQ) decomposition with a correction that relies on the Choleski decomposition of Ξ, $\Xi^{1/2}$. Given $\mathbb{D}(\mathbf{L})$ is the infinite-order lag polynomial matrix of the theoretical SVMA(∞), the normalization is $\mathbb{D}(\mathbf{L})\ \Xi^{1/2}$. This normalization is imposed by the BQ decomposition on the ensemble of $\mathcal{J}$ theoretical SVMAs that are created from synthetic time series of length $\mathcal{M} \times T$ obtained from Bayesian simulations of the NKDSGE models.

A3.2 The ABCs and Ds of the NKDSGE models, LRMN, and SVARs

This section shows that the SVMA(∞) of equation ($\mathcal{A}$3.2) retrieves the economic shocks of a NKDSGE model. This involves restating a result from Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (2007). They study a condition that equates the shocks identified by an econometric model to those of a DSGE model. We exploit their condition to tie the shocks of a structural VAR (SVAR) identified by LRMN to the NKDSGE model shocks ζ_t.

Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (FVRRSW) construct a VAR(∞) driven by DSGE model shocks to expose the condition that links these shocks to those identified by a SVAR(∞). The baseline habit NKDSGE Taylor rule model yields the VAR(∞)

$$\mathcal{X}_t = \Gamma_{\mathcal{H}} \sum_{j=0}^{\infty} \left[\Theta_{\mathcal{H}} - \Theta_{\zeta}\Gamma_{\zeta}^{-1}\Gamma_{\mathcal{H}}\right]^j \Theta_{\zeta}\Gamma_{\zeta}^{-1}\mathcal{X}_{t-j-1} + \Gamma_{\zeta}\zeta_t, \tag{$\mathcal{A}$3.5}$$

which combines the equilibrium law of motion ($\mathcal{A}$2.70), the system

$$\mathcal{X}_t = \Gamma_{\mathcal{H}}\mathcal{H}_{t-1} + \Gamma_{\zeta}\zeta_t, \tag{$\mathcal{A}$3.6}$$

that relates the observables of $\mathcal{X}_t$ to $\mathcal{H}_t$ and ζ_t, and several steps described by FVRRSW. Note that Γ_{ζ} is square and its inverse is taken to exist. FVRRSW also examine

$$\mathcal{H}_t = \sum_{j=0}^{\infty} \left[\Theta_{\mathcal{H}} - \Theta_{\zeta}\Gamma_{\zeta}^{-1}\Gamma_{\mathcal{H}}\right]^j \Theta_{\zeta}\Gamma_{\zeta}^{-1}\mathcal{X}_{t-j}, \tag{$\mathcal{A}$3.7}$$

which results from passing Γ_{ζ}^{-1} through equation ($\mathcal{A}$3.6), substituting it into the equilibrium law of motion ($\mathcal{A}$2.70), and rearranging terms. Equation ($\mathcal{A}$3.7) recovers the state vector $\mathcal{H}_t$ from the history of $\mathcal{X}_{t-j}$, which consists of observed variables (*i.e.*, there are no latent state variables), if (and only if) the eigenvalues of $\Theta_{\mathcal{H}} - \Theta_{\zeta}\Gamma_{\zeta}^{-1}\Gamma_{\mathcal{H}}$ are strictly less than one in modulus. This is the condition FVRRSW require to equate shocks identified by a SVAR to the NKDSGE model shocks ζ_t. Given the FVRRSW condition is satisfied by $\Theta_{\mathcal{H}} - \Theta_{\zeta}\Gamma_{\zeta}^{-1}\Gamma_{\mathcal{H}}$, the coefficients of the lag polynomial implied by $\left[\mathbf{I} - \left(\Theta_{\mathcal{H}} - \Theta_{\zeta}\Gamma_{\zeta}^{-1}\Gamma_{\mathcal{H}}\right)\mathbf{L}\right]$ also fulfill the needs of square summability.

By also assuming that $\Gamma_\zeta \zeta_t$ is orthogonal to $\mathcal{X}_{t-j-1}$ (j = 0, 1, ..., ∞), equation ($\mathcal{A}$3.5) can be interpreted as the theoretical VAR(∞) of $\mathcal{X}_t$.

We rely on LRMN for identification of the SVMA(∞) of equation ($\mathcal{A}$3.2). This complicates the problem of using the FVRRSW condition to connect NKDSGE model shock innovations ζ_t to innovations identified by an econometric model. A solution is to exploit an approach of King and Watson (1997) that imposes the LRMN restriction on the SVAR(∞)

$$\begin{bmatrix} 1 & -\Lambda_{\Delta Y,\Delta P,0} \\ -\Lambda_{\Delta P,\Delta Y,0} & 1 \end{bmatrix} \mathcal{X}_t = \begin{bmatrix} \Lambda_{\Delta Y,\Delta Y}(\mathbf{L}) & \Lambda_{\Delta Y,\Delta P}(\mathbf{L}) \\ \Lambda_{\Delta P,\Delta Y}(\mathbf{L}) & \Lambda_{\Delta P,\Delta P}(\mathbf{L}) \end{bmatrix} \mathcal{X}_{t-1} + \begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \end{bmatrix}, \qquad (\mathcal{A}3.8)$$

where the impact matrix Λ_0 is nonsingular, $\Lambda(\mathbf{L})$ summarizes the lag polynomial attached to $\mathcal{X}_{t-1}$, $\eta_t = \left[\eta_{1,t} \;\; \eta_{2,t}\right]'$, Ω is the diagonal covariance matrix of $\mathbf{E}\left\{\eta_t \eta_t'\right\}$, $\mathbf{E}\eta_t = \mathbf{0}$, and $\mathbf{E}\left\{\eta_t \eta_{t-i}'\right\}$ = $\mathbf{0}$, for all non-zero i.

King and Watson (1997) are interested in identifying and estimating SVARs with impact and long run restrictions. We focus on the latter type of restriction to identify the SVAR of ($\mathcal{A}$3.8) with LRMN. The identification relies on the response of the level of output to a permanent change in the nominal shock $\eta_{2,t}$, which is

$$\mathcal{L}_{\Delta Y,\Delta P} = \frac{\Lambda_{\Delta Y,\Delta P,0} + \Lambda_{\Delta Y,\Delta P}(\mathbf{1})}{1 - \Lambda_{\Delta Y,\Delta Y}(\mathbf{1})}.$$

This ratio is zero when LRMN holds because it measures the long run response of output to a monetary shock. Following King and Watson, the LRMN restriction is imposed on the structural VAR of ($\mathcal{A}$3.8) by rewriting its top equation as

$$\Delta \ln Y_t = [\Lambda_{\Delta Y,\Delta P,0} + \Lambda_{\Delta Y,\Delta P}(\mathbf{1})]\Delta \ln P_t + \Lambda_{\Delta Y,\Delta Y}(\mathbf{1})\Delta \ln Y_{t-1}$$

$$+ \Psi_{\Delta Y,\Delta Y}(\mathbf{L})\Delta^2 \ln Y_{t-1} + \Psi_{\Delta Y,\Delta P}(\mathbf{L})\Delta^2 \ln P_{t-1} + \eta_{1,t},$$

where, for example, $\Psi_{\Delta Y,\Delta P,i} = -\sum_{s=i+1}^{\infty} \Lambda_{\Delta Y,\Delta P,s}$. Next, multiply and divide the first term after the equality by $\mathcal{L}_{\Delta Y,\Delta P}$ to produce

$$\Delta \ln Y_t - \mathcal{L}_{\Delta Y,\Delta P} \Delta \ln P_t = \Lambda_{\Delta Y,\Delta Y}(\mathbf{1})[\Delta \ln Y_{t-1} - \mathcal{L}_{\Delta Y,\Delta P} \Delta \ln P_t]$$

$$+ \Psi_{\Delta Y,\Delta Y}(\mathbf{L})\Delta^2 \ln Y_{t-1} + \Psi_{\Delta Y,\Delta P}(\mathbf{L})\Delta^2 \ln P_{t-1} + \eta_{1,t},$$

or under LRMN

$$\Delta \ln Y_t = \Lambda_{\Delta Y,\Delta Y}(\mathbf{1})\Delta \ln Y_{t-1} + \Psi_{\Delta Y,\Delta Y}(\mathbf{L})\Delta^2 \ln Y_{t-1} + \Psi_{\Delta Y,\Delta P}(\mathbf{L})\Delta^2 \ln P_{t-1} + \eta_{1,t}.$$

The previous equation and the bottom equation of ($\mathcal{A}$3.8) form a just-identified SVAR from which $\eta_{1,t}$ and $\eta_{2,t}$ can be computed. An estimator of these shocks does not rely on identifying either impact coefficient $\Lambda_{\Delta Y,\Delta P,0}$ or $\Lambda_{\Delta P,\Delta Y,0}$. Rather the former coefficient is obtained from $\mathcal{L}_{\Delta Y,\Delta P,0} = 0$ given $\Lambda_{\Delta Y,\Delta Y}(\mathbf{1})$, $\Psi_{\Delta Y,\Delta Y}(\mathbf{L})$ and $\Psi_{\Delta Y,\Delta P}(\mathbf{L})$, while the latter coefficient is obtained from the bottom equation of ($\mathcal{A}$3.8). King and Watson (1997) suggest using an instrumental variable estimator with $\eta_{1,t}$ serving as the additional instrument. Instead, we apply the BQ decomposition of equations ($\mathcal{A}$3.3) and ($\mathcal{A}$3.4) to synthetic samples of $\mathcal{X}_t$, rather estimate SVAR(∞)s.

The FVRRSW condition enables us to match the shocks of the SVAR of ($\mathcal{A}$3.8) with the NKDSGE shocks ζ_t. This SVAR implies the reduced form VAR(∞)

$$\mathcal{X}_t = \mathbb{S}(\mathbf{L})\mathcal{X}_{t-1} + \nu_t, \quad \mathbb{S}(\mathbf{L}) = \sum_{j=1}^{\infty} \mathbb{S}_j \mathbf{L}^j, \tag{$\mathcal{A}$3.9}$$

is associated with the SVAR of ($\mathcal{A}$3.8), where $\nu_t = \mathcal{X}_t - \mathbf{E}\left\{\mathcal{X}_t \,\middle|\, \mathcal{X}_{t-1}, \mathcal{X}_{t-2}, \ldots, \mathcal{X}_{t-p}, \ldots\right\}$, $\mathbb{S}(\mathbf{L}) = \Lambda_0^{-1}\Lambda(\mathbf{L})$, and $\nu_t = \Lambda_0^{-1}\eta_t$. Equation ($\mathcal{A}$3.9) serves to represent the VAR(∞) of $\mathcal{X}_t$ when the sum from $i = 1, \ldots, \infty$ of $\mathbb{S}^2_{\Delta Y,\Delta Y,i} + \mathbb{S}^2_{\Delta Y,\Delta P,i} + \mathbb{S}^2_{\Delta P,\Delta Y,i} + \mathbb{S}^2_{\Delta P,\Delta Y,i}$ is finite and the orthogonality condition $\mathbf{E}\left\{\nu_t \mathcal{X}'_{t-j}\right\} = \mathbf{0}$, holds for all $j \geq 1$. We can acquire shocks from this reduced form VAR that match those of the baseline habit NKDSGE-TR model when $\Lambda_0 \nu_t = \Gamma_\zeta \zeta_t$. FVRRSW show that the equality links the econometric and NKDSGE model shocks if (and only if) the eigenvalues of $\Theta_{\mathcal{H}} - \Theta_\zeta \Gamma_\zeta^{-1} \Gamma_{\mathcal{H}}$ are strictly less than one in modulus.

The FVRRSW restriction is checked at each of the $\mathcal{J} = 5000$ replications of the Bayesian simulations of the 12 NKDSGE models. The simulations reveal that the NKDSGE models satisfy the FVRRSW restriction on the eigenvalues $\Theta_{\mathcal{H}} - \Theta_\zeta \Gamma_\zeta^{-1} \Gamma_{\mathcal{H}}$ at all $\mathcal{J}$ replications. Thus, the $\mathcal{T}$ SVMA(∞)s always recover the economic shocks of the 12 NKDSGE models.

A3.3 Computing permanent and transitory spectral densities

In Section 3.2, the paper presents the map from the SVMA(∞) of equation ($\mathcal{A}$3.2) to the permanent and transitory output and consumption growth spectral densities (SDs). We reproduce the SD (at frequency ω) that appears at the end of section 3.2 here as

$$S_{\Delta Y,\iota}(\omega) = \frac{1}{2\pi} \sum_{j=0}^{40} \left| \mathbb{B}_{\Delta Y,\iota,j} e^{-ij\omega} \right|^2, \quad \iota = \varepsilon,\, \upsilon, \tag{$\mathcal{A}$3.10}$$

where it is understood that the Bayesian simulations of the NKDSGE models account for the non-unit diagonal elements of Ξ. Although we calculate the permanent and transitory output growth SDs using ($\mathcal{A}$3.10), there are (at least) two other methods available to compute these moments. First, the SD (at frequency ω) can be represented as

$$S_{\Delta Y,\iota}(\omega) = \frac{\mathbb{B}^2_{\Delta Y,\iota,0}}{2\pi} \sum_{j=0}^{40} \left| \frac{\mathbb{B}_{\Delta Y,\iota,j}}{\mathbb{B}_{\Delta Y,\iota,0}} e^{-ij\omega} \right|^2,$$

which leads to the factorization

$$1 + \frac{\mathbb{B}_{\Delta Y,\iota,1}}{\mathbb{B}_{\Delta Y,\iota,0}} z + \frac{\mathbb{B}_{\Delta Y,\iota,2}}{\mathbb{B}_{\Delta Y,\iota,0}} z^2 + \ldots + \frac{\mathbb{B}_{\Delta Y,\iota,40}}{\mathbb{B}_{\Delta Y,\iota,0}} z^{40} \quad = \quad (1 - \chi_{\iota,1} z)(1 - \chi_{\iota,2} z) \cdots (1 - \chi_{\iota,40} z),$$

in terms of the eigenvalues, the $\chi_{\iota,h}$s, of the MA(40) process of output growth with respect to the NKDSGE shocks ε, μ, or υ. The eigenvalue factorization gives

$$S_{\Delta Y,\iota}(\omega) \;=\; \frac{\mathbb{B}^2_{\Delta Y,\iota,0}}{2\pi}\prod_{j=1}^{40}\left[1 + \chi^2_{\iota,j} - 2\chi_{\iota,j}\cos(\omega)\right],$$

which provides a third method to compute the SDs (at frequency ω).

A4. Alternative NKDSGE Model Evaluation

This section discusses two alternative NKDSGE model fit exercises. One alternative evaluates the fit of the NKDSGE models with *CIC* calculated from distributions of the Cramer-von Mises (*CvM*) goodness of fit statistic instead of the Kolmogorov-Smirnov (*KS*) statistic (as in the paper). The *CvM* statistic is computed on the ensembles of permanent and transitory $\mathcal{E}$ and $\mathcal{T}$ output and consumption growth SDs. This section also presents *CIC* based in part from Bayesian simulations of the NKDSGE models in which the uniform prior for the habit parameter h is replaced by a prior drawn from the β distribution. These *CIC* are computed using the Kolmogorov-Smirnov (*KS*) statistic, as in the paper, rather than the *CvM* statistic.

We report results of the alternative NKDSGE model fit exercises in tables A1 and A2 and figures A1–A12. The former table includes *CIC* that measure the overlap of $\mathcal{E}$ and $\mathcal{T}$ *CvM* distributions. Figures A1–A6 reproduce figures 3–8 of the paper, except that densities of $\mathcal{E}$ and $\mathcal{T}$ *CvM* distributions are plotted in the second and third columns of these figures. This sections also presents results that rely on draws from the β distribution to calibrate h in simulations of the NKDSGE models. These results appear in table A2 and figures A7–A12.

Figures A1–A12 are laid out in the same way as figures A3–A8. As a reminder, $\mathcal{E}$ and $\mathcal{T}$ output and consumption growth SDs appear in the first column of figures A1–A12. The second (third) column contains densities of *CvM* or *KS* statistics computed using the entire spectrum (constrained to eight to two years per cycle). The *CvM* or *KS* statistic densities also appear with associated *CIC*. From top to bottom, the rows of figures A1-A12 include results for permanent output, transitory output, permanent consumption, and transitory consumption growth SDs. We denote mean $\mathcal{E}$ SDs and *CvM* or *KS* statistic densities with (blue) solid lines, mean $\mathcal{T}$ non-habit SDs and *CvM* or *KS* statistic densities with (green) dashed lines, and mean $\mathcal{T}$ habit SDs and*CvM* or *KS* statistic densities with (red) dot-dash lines in figures A1–A12.

A4.1 Gauging NKDSGE Model Fit with the Cramer-von Mises Statistic

Table A1 contains *CIC* for six NKDSGE models based on the *CvM* statistic. The *CIC* measure the overlap of distributions of *CvM* statistics computed using the ensemble of permanent and transitory $\mathcal{E}$ and $\mathcal{T}$ output and consumption growth SDs. The *CvM* statistic is

$$CvM_{\mathcal{D},j} = \int_0^1 \mathcal{B}^2_{T,\mathcal{D},j}(\kappa)d\kappa,$$

for $\mathcal{D} = \mathcal{E}, \mathcal{T}$ and replication j of the ensemble of $\mathcal{J}$ (= 5000) $\mathcal{E}$ and $\mathcal{T}$ synthetic samples. Section 3.4 provides details about computing $\mathcal{B}_{T,\mathcal{D},j}(\cdot)$, but to review

$$\mathcal{B}_{T,\mathcal{D},j}(\kappa) \; = \; \frac{\sqrt{2T}}{2\pi}\left[\mathcal{V}_{T,\mathcal{D},j}(\kappa\pi) - \kappa\mathcal{V}_{T,\mathcal{D},j}(\pi)\right],$$

for $\kappa \in [0, 1]$. The partial sum $\mathcal{V}_{\mathcal{D},T,j}\left(2\pi q/T\right) = 2\pi \sum_{\ell=1}^{q} \mathcal{R}_{\mathcal{D},T,j}\left(2\pi\ell/T\right)/T$ and $\mathcal{R}_{\mathcal{D},T,j}(\omega)$ $= \widehat{I_T}(\omega) \;/\; I_{\mathcal{D},T,j}(\omega)$, where the numerator (denominator) is the sample (jth $\mathcal{E}$ or $\mathcal{T}$) output or consumption growth SD at frequency ω.

The fit of the NKDSGE models is qualitatively similar across table A1 and table 2 with two exceptions. First, table 2 shows that the SPrice (*i.e.*, only sticky prices) non-habit NKDSGE-AR model never produces a *KS* statistic based *CIC* larger than 0.30. However, switching to the *CvM* statistic allows this NKDSGE model to produce three *CIC* ≥ 0.39 on the business cycle frequencies in table A1. On the other hand, the $\mathcal{E}$ and $\mathcal{T}$ *CvM* statistics generate *CIC* that indicate the fit of the SPrice habit NKDSGE-AR model dominates the fit of the SPrice non-habit NKDSGE model-AR on the business cycle frequencies. Neither SPrice NKDSGE-AR model matches permanent and transitory $\mathcal{E}$ output and consumption growth SDs on the entire spectrum. Table A1 also depicts a bit weaker match for the SPrice habit NKDSGE-TR model to the transitory $\mathcal{E}$ output growth SD. The *CIC* in question is 0.26 compared to 0.30 for its cousin in table 2.

Figures A1–A6 plot mean $\mathcal{E}$ and $\mathcal{T}$ SDs and densities of ensembles of *CvM* statistics generated by habit and non-habit NKDSGE models. The striking feature of these figures is that $\mathcal{T}$ *CvM* statistic densities are flat even for *CIC* > 0.3 when NKDSGE model fit is gauged on the entire spectrum. For example, the baseline habit NKDSGE-TR model yields a flat *CvM* statistic density in the middle panel of the bottom row of figure A2, but the *CIC* $= 0.38$. The explanation is that the quadratic form of the *CvM* statistic places weight on large deviations between sample SD, $\widehat{I_T}(\omega)$, and $\mathcal{T}$ SD, $I_{\mathcal{T},T,j}(\omega)$, while the supremum of the *KS*, $KS_{\mathcal{T},j} = Max\left|\mathcal{B}_{T,\mathcal{T},j}(\kappa)\right|$, does not. Nonetheless, the *CvM* statistic fails to alter judgements about the fit of habit and non-habit NKDSGE models to permanent and transitory $\mathcal{E}$ output and consumption growth SDs.

A4.2 NKDSGE Model Fit under the β Prior for h

Our choice of a uniform prior for the consumption habit parameter displays only information about the theoretical restriction that h takes values on the open interval between zero and one. We replace this uninformative prior for h with a β prior to assess the impact of using evidence from previous DSGE model studies. The β prior gives h a mean of 0.65, a standard deviation of 0.15, and a 95 percent coverage interval of $[0.3842,\ 0.8765]$. This calibration covers values of h found in Boldrin, Christiano, and Fisher (2001), Francis and Ramey (2005), and Christiano, Eichenbaum, and Evans (2005) among others. The non-habit NKDSGE models remain defined by the degenerate prior $h = 0$.

Table 2 reports *CIC* generated from Bayesian calibration experiments of the NKDSGE model given the β prior for h. We include density plots of *KS* statistic distributions based on permanent and transitory $\mathcal{E}$ and $\mathcal{T}$ output and consumption growth SDs in figures A7–A12. The *CIC* and *KS* statistic densities indicate that the β prior for h produces only minimal changes in NKDSGE model fit. There are instances when this prior for h enhances the match of habit SPrice and SWage NKDSGE-AR models to the transitory $\mathcal{E}$ consumption growth SD, but the improved fit relies on subjective beliefs to calibrate the consumption habit parameter h.

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TABLE A1: *CIC*s OF CRAMER-VON MISES STATISTICS

Model	ΔY w/r/t Trend Sh'k $\infty:0$	ΔY w/r/t Trend Sh'k $8:2$	ΔY w/r/t Transitory Sh'k $\infty:0$	ΔY w/r/t Transitory Sh'k $8:2$	ΔC w/r/t Trend Sh'k $\infty:0$	ΔC w/r/t Trend Sh'k $8:2$	ΔC w/r/t Transitory Sh'k $\infty:0$	ΔC w/r/t Transitory Sh'k $8:2$
NKDSGE-AR								
Baseline								
Non-Habit	0.01	0.03	0.00	0.02	0.00	0.03	0.00	0.00
Habit	0.01	0.04	0.11	0.11	0.04	0.22	0.14	0.22
SPrice								
Non-Habit	0.04	0.43	0.00	0.53	0.01	0.39	0.00	0.23
Habit	0.07	0.53	0.17	0.82	0.13	0.66	0.27	0.69
SWage								
Non-Habit	0.00	0.00	0.00	0.12	0.00	0.00	0.00	0.01
Habit	0.00	0.01	0.19	0.25	0.03	0.13	0.16	0.27
NKDSGE-TR								
Baseline								
Non-Habit	0.00	0.00	0.13	0.82	0.00	0.00	0.02	0.62
Habit	0.00	0.04	0.53	0.77	0.04	0.19	0.38	0.82
SPrice								
Non-Habit	0.03	0.68	0.00	0.94	0.00	0.47	0.00	0.82
Habit	0.11	0.77	0.26	0.76	0.14	0.72	0.37	0.87
SWage								
Non-Habit	0.00	0.00	0.14	0.79	0.00	0.00	0.01	0.66
Habit	0.00	0.04	0.44	0.67	0.04	0.18	0.36	0.82

NKDSGE-AR and NKDSGE-TR denote the NKDSGE model with the AR(1) money supply rule (8) and the Taylor rule (9), respectively. Baseline NKDSGE models include sticky prices and sticky wages. The acronyms SPrice and SWage represent NKDSGE models with only sticky prices or sticky nominal wages, respectively. The column heading $\infty : 0$ ($8 : 2$) indicates that *CIC*s measure the intersection of distributions of *CvM* statistics computed over the entire spectrum (from eight to two years per cycle).

Table A2: CICs of Kolmogorov-Smirnov Statistics Given β prior on h

Model	ΔY w/r/t Trend Sh'k $\infty:0$	ΔY w/r/t Trend Sh'k $8:2$	ΔY w/r/t Transitory Sh'k $\infty:0$	ΔY w/r/t Transitory Sh'k $8:2$	ΔC w/r/t Trend Sh'k $\infty:0$	ΔC w/r/t Trend Sh'k $8:2$	ΔC w/r/t Transitory Sh'k $\infty:0$	ΔC w/r/t Transitory Sh'k $8:2$
NKDSGE-AR								
Baseline								
Non-Habit	0.00	0.00	0.00	0.01	0.00	0.04	0.00	0.00
Habit	0.00	0.02	0.14	0.14	0.05	0.26	0.17	0.29
SPrice								
Non-Habit	0.03	0.30	0.00	0.21	0.01	0.23	0.00	0.05
Habit	0.05	0.47	0.26	0.80	0.15	0.55	0.50	0.84
SWage								
Non-Habit	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.01
Habit	0.00	0.02	0.27	0.28	0.03	0.14	0.18	0.31
NKDSGE-TR								
Baseline								
Non-Habit	0.00	0.00	0.13	0.73	0.00	0.00	0.02	0.41
Habit	0.00	0.05	0.65	0.75	0.04	0.22	0.62	0.88
SPrice								
Non-Habit	0.02	0.52	0.01	0.79	0.00	0.24	0.00	0.54
Habit	0.11	0.72	0.45	0.73	0.17	0.75	0.64	0.90
SWage								
Non-Habit	0.00	0.00	0.13	0.74	0.00	0.00	0.01	0.45
Habit	0.00	0.04	0.51	0.63	0.03	0.17	0.60	0.87

NKDSGE-AR and NKDSGE-TR denote the NKDSGE model with the AR(1) money supply rule (8) and the Taylor rule (9), respectively. Baseline NKDSGE models include sticky prices and sticky wages. The acronyms SPrice and SWage represent NKDSGE models with only sticky prices or sticky nominal wages, respectively. The column heading $\infty:0$ ($8:2$) indicates that CICs measure the intersection of distributions of KS statistics computed over the entire spectrum (from eight to two years per cycle).

Figure A1: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and CvM Densities for Baseline NKDSGE Models with AR(1) Money Growth Rule

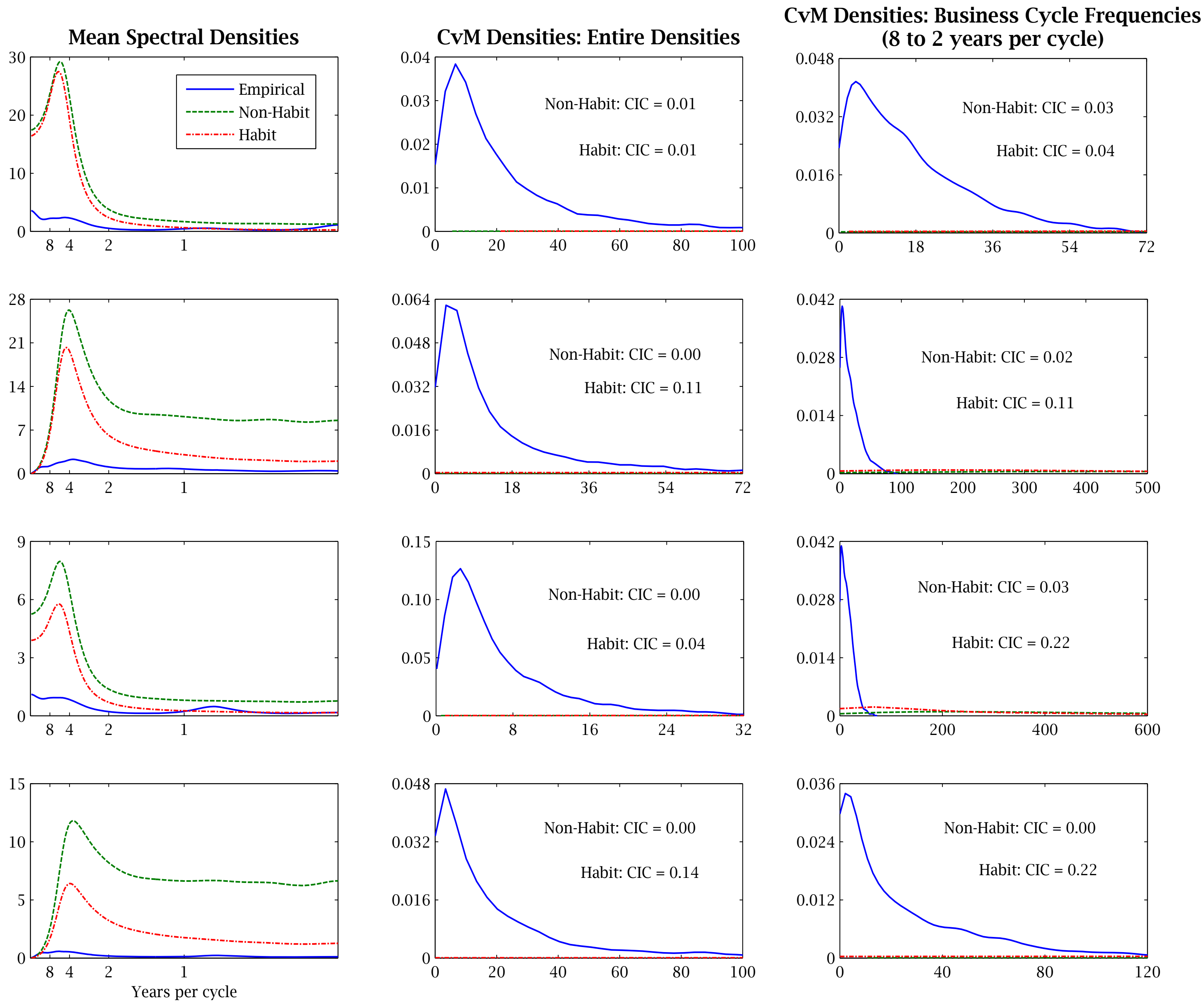

FIGURE A2: MEAN STRUCTURAL $\mathcal{E}$ AND $\mathcal{T}$ SDS AND CVM DENSITIES FOR BASELINE NKDSGE MODELS WITH TAYLOR RULE

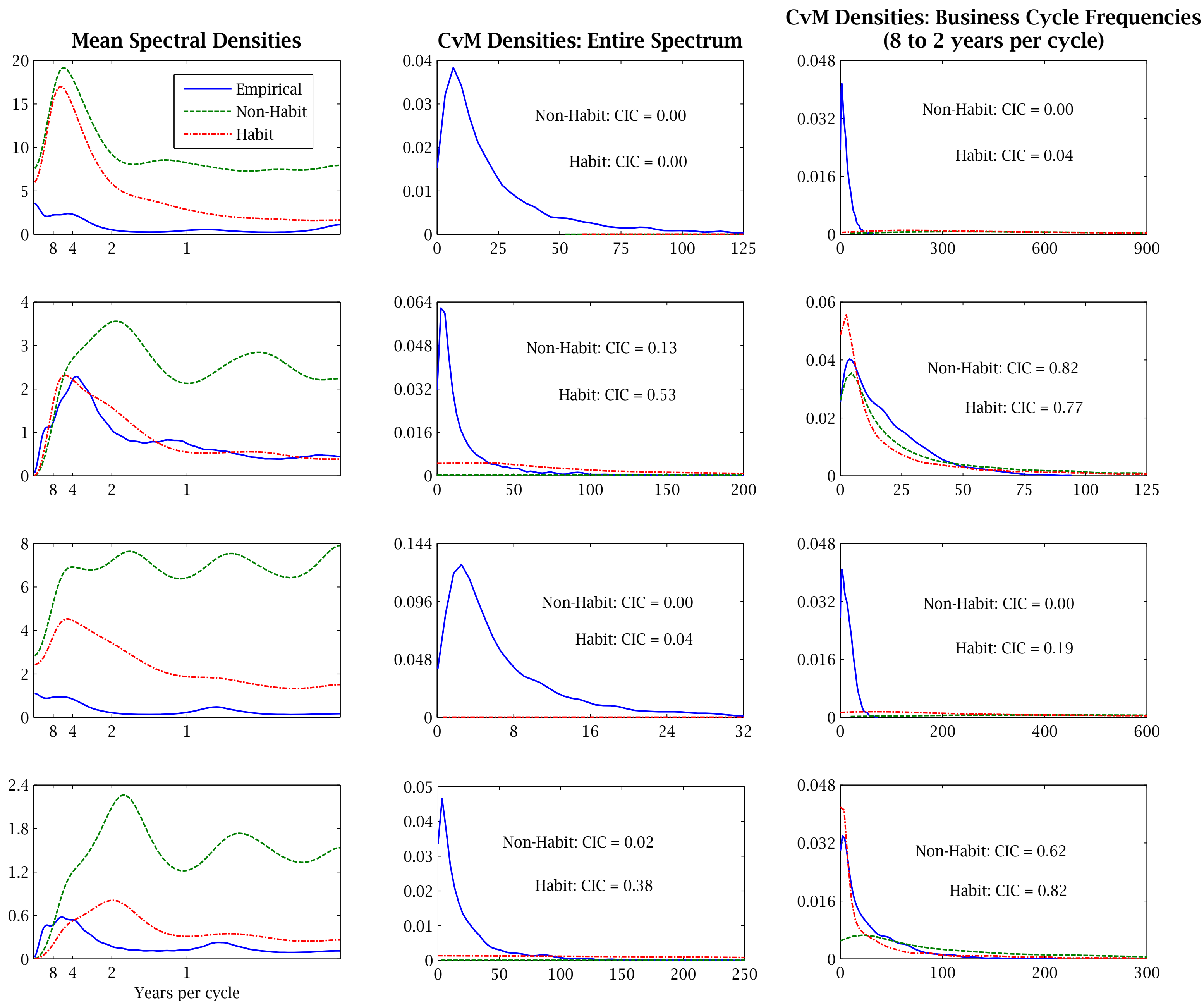

Figure A3: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and CvM Densities for NKDSGE Models with AR(1) Money Growth Rule and Only Sticky Prices

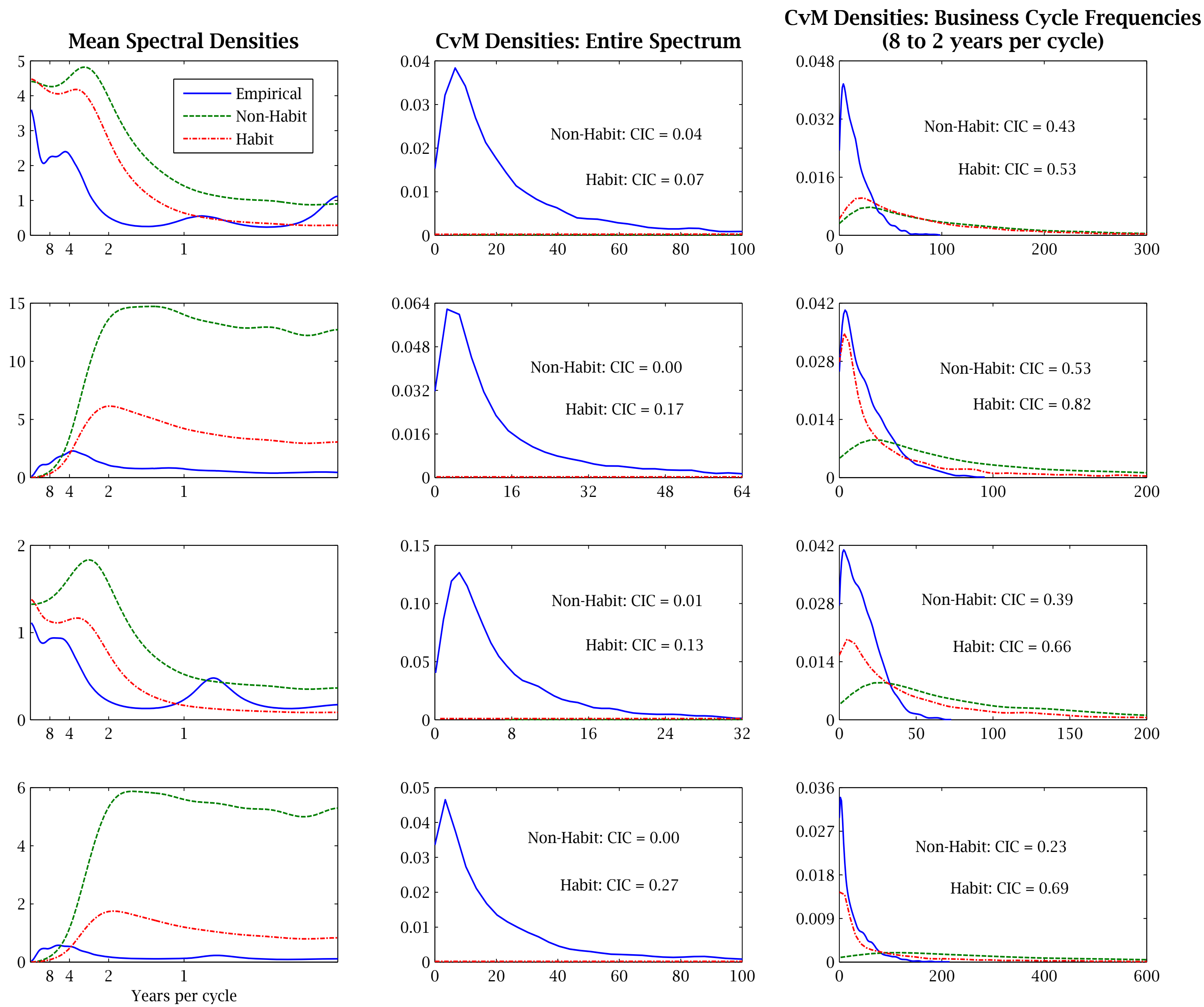

Figure A4: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and CvM Densities for NKDSGE Models with Taylor Rule and only Sticky Prices

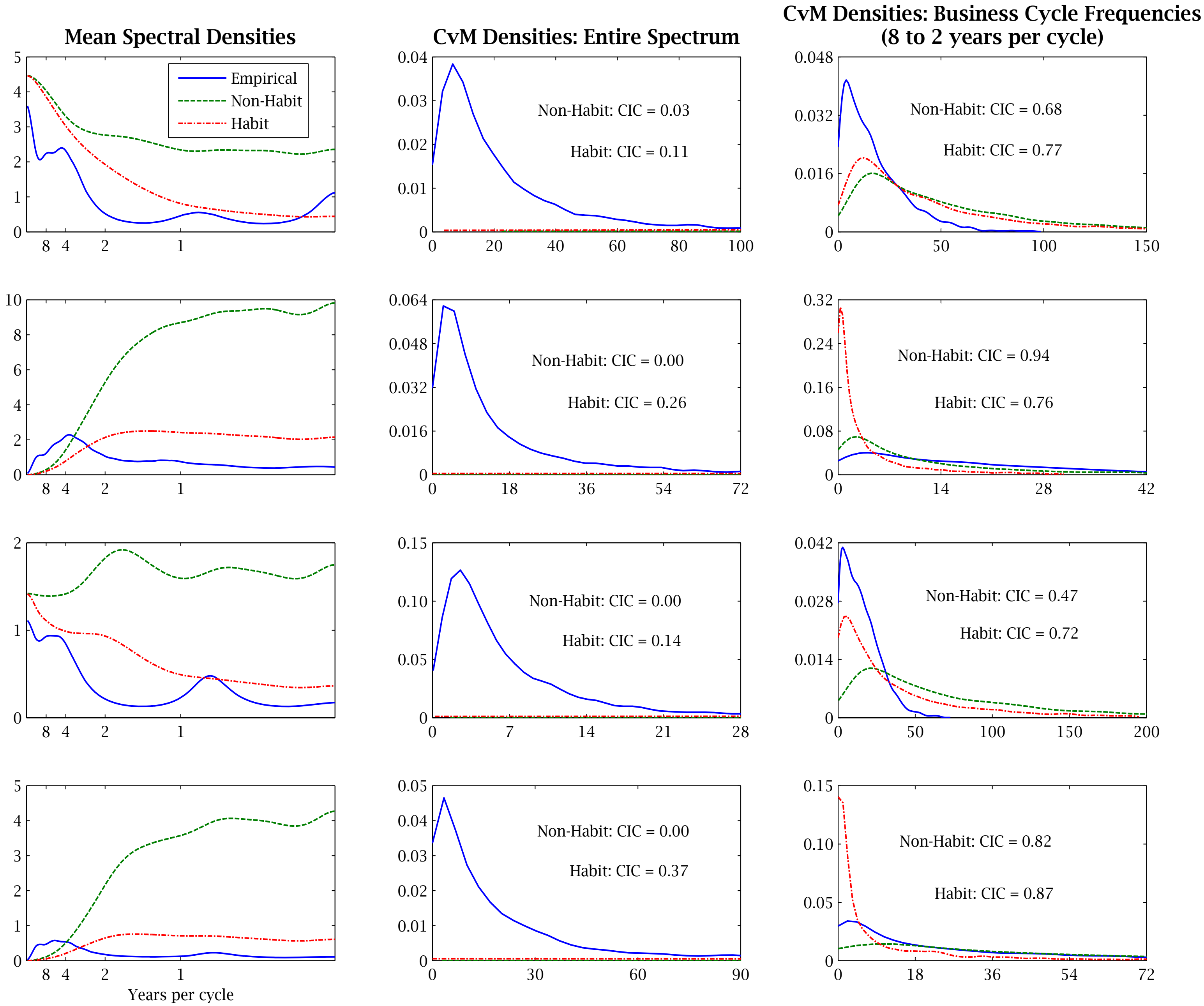

Figure A5: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and CvM Densities for NKDSGE Models with AR(1) Money Growth Rule and Only Sticky Wages

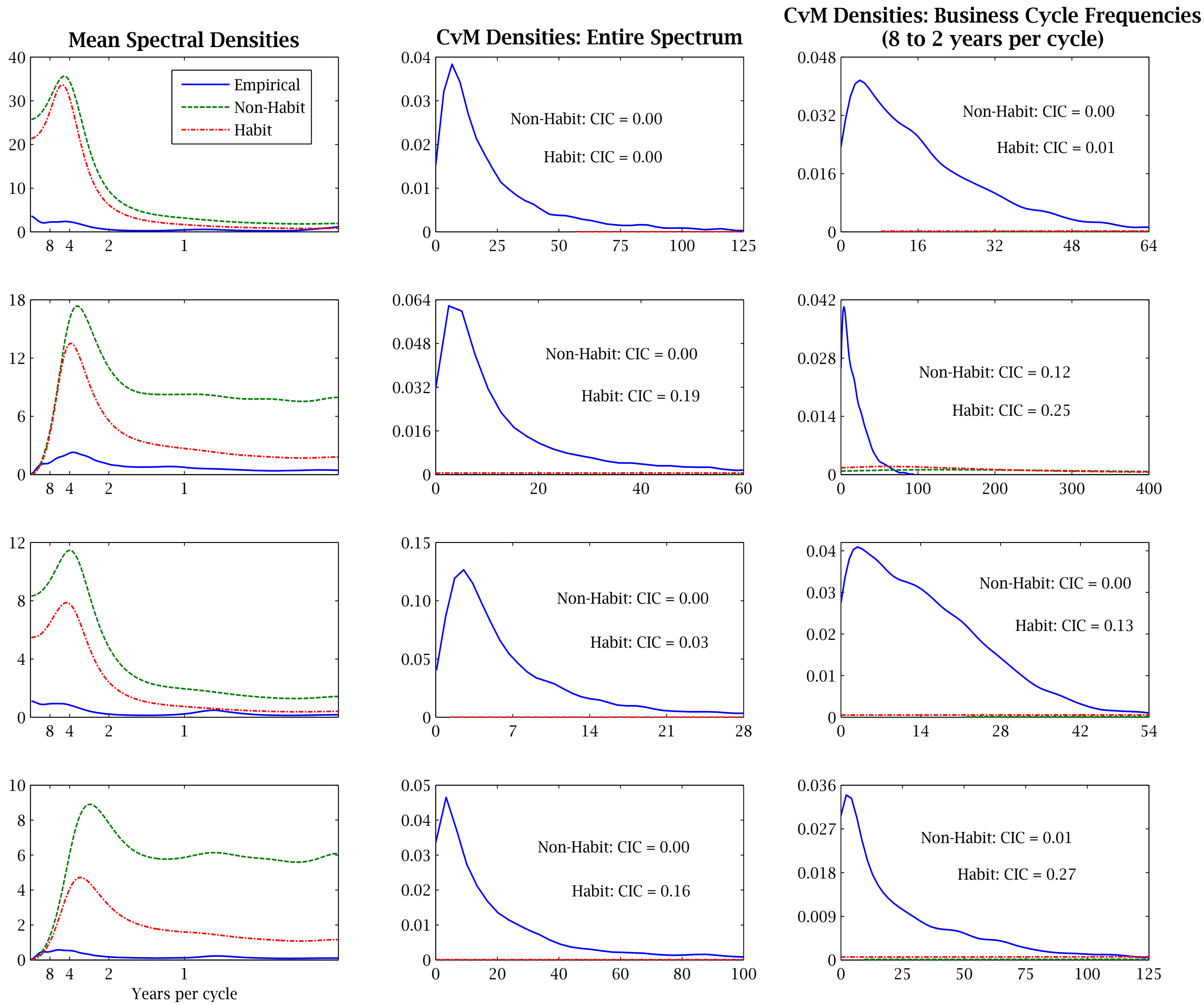

Figure A6: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and CvM Densities for NKDSGE Models with Taylor Rule and only Sticky Wages

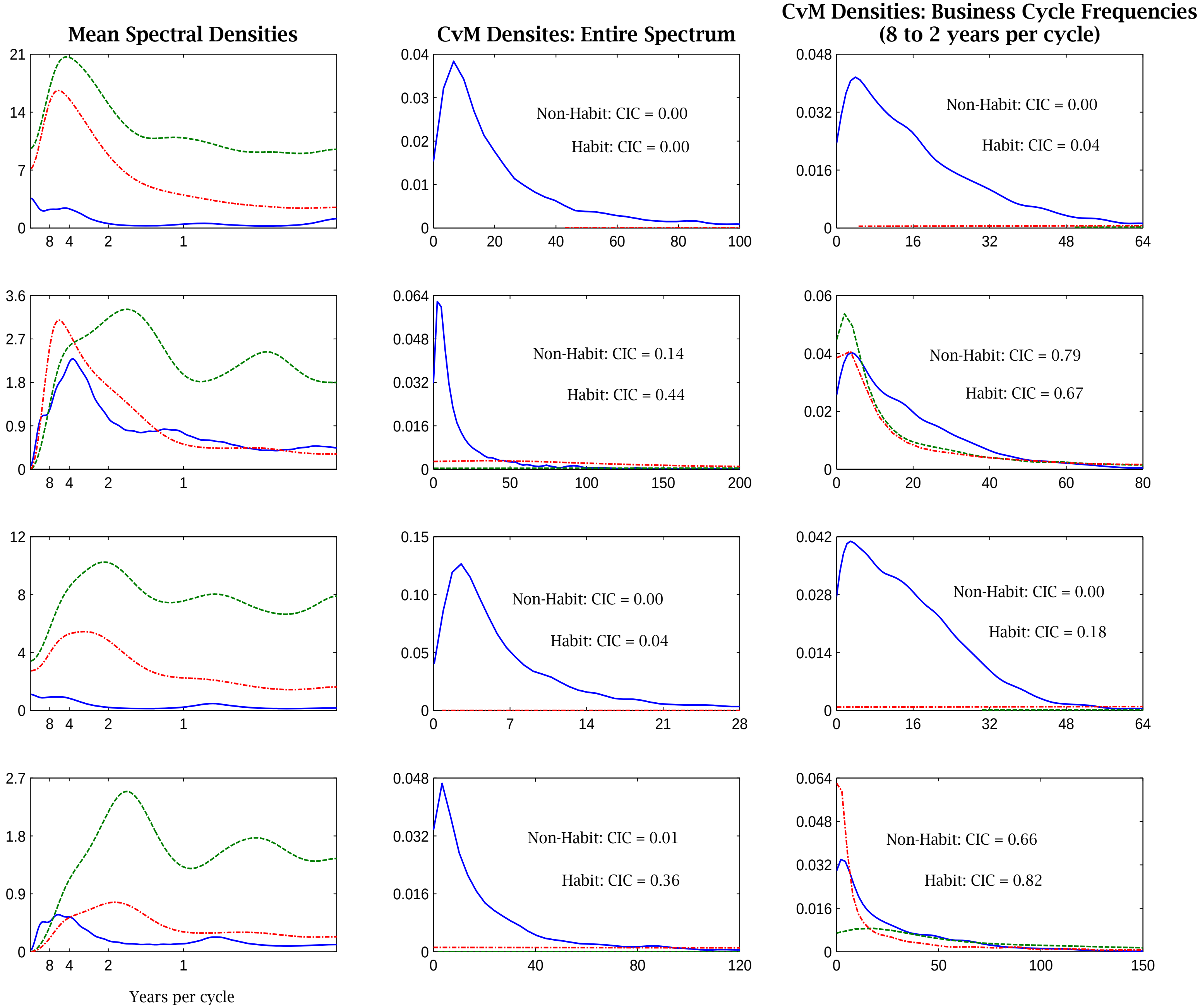

Figure A7: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with AR(1) Money Growth Rule and β Prior for h

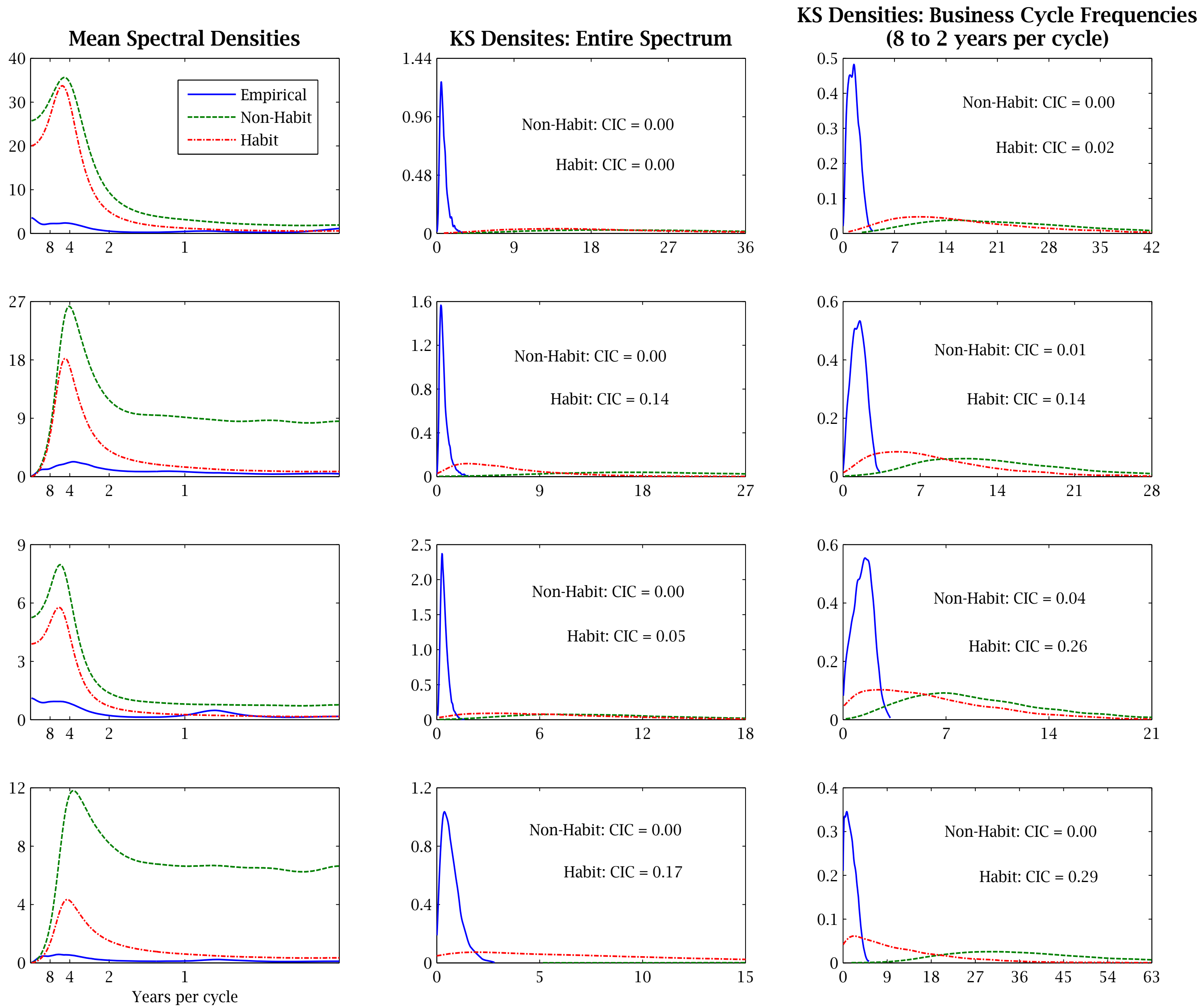

Figure A8: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with Taylor Rule and β Prior for h

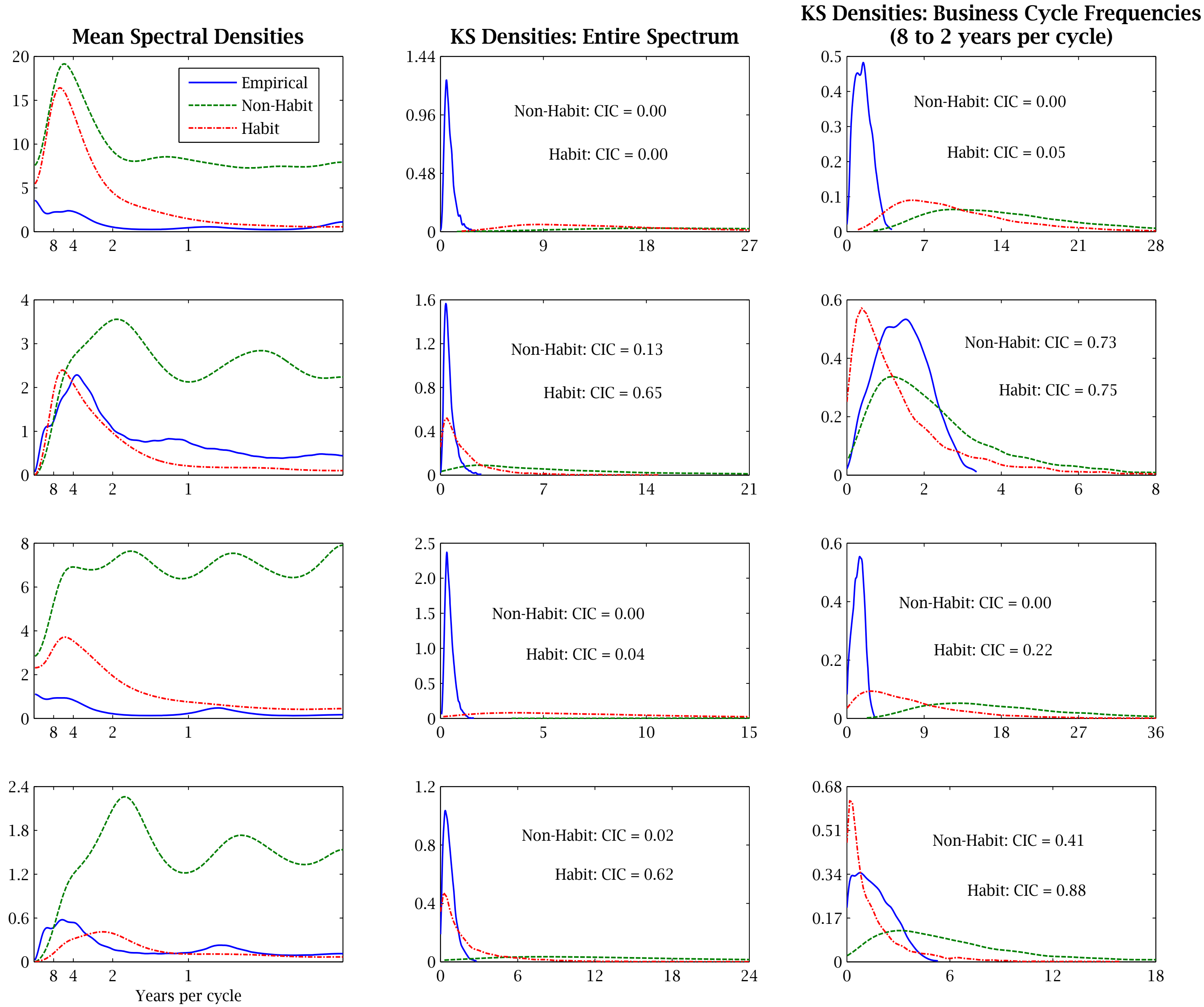

Figure A9: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with AR(1) Money Growth Rule and Only Sticky Prices and β Prior for h

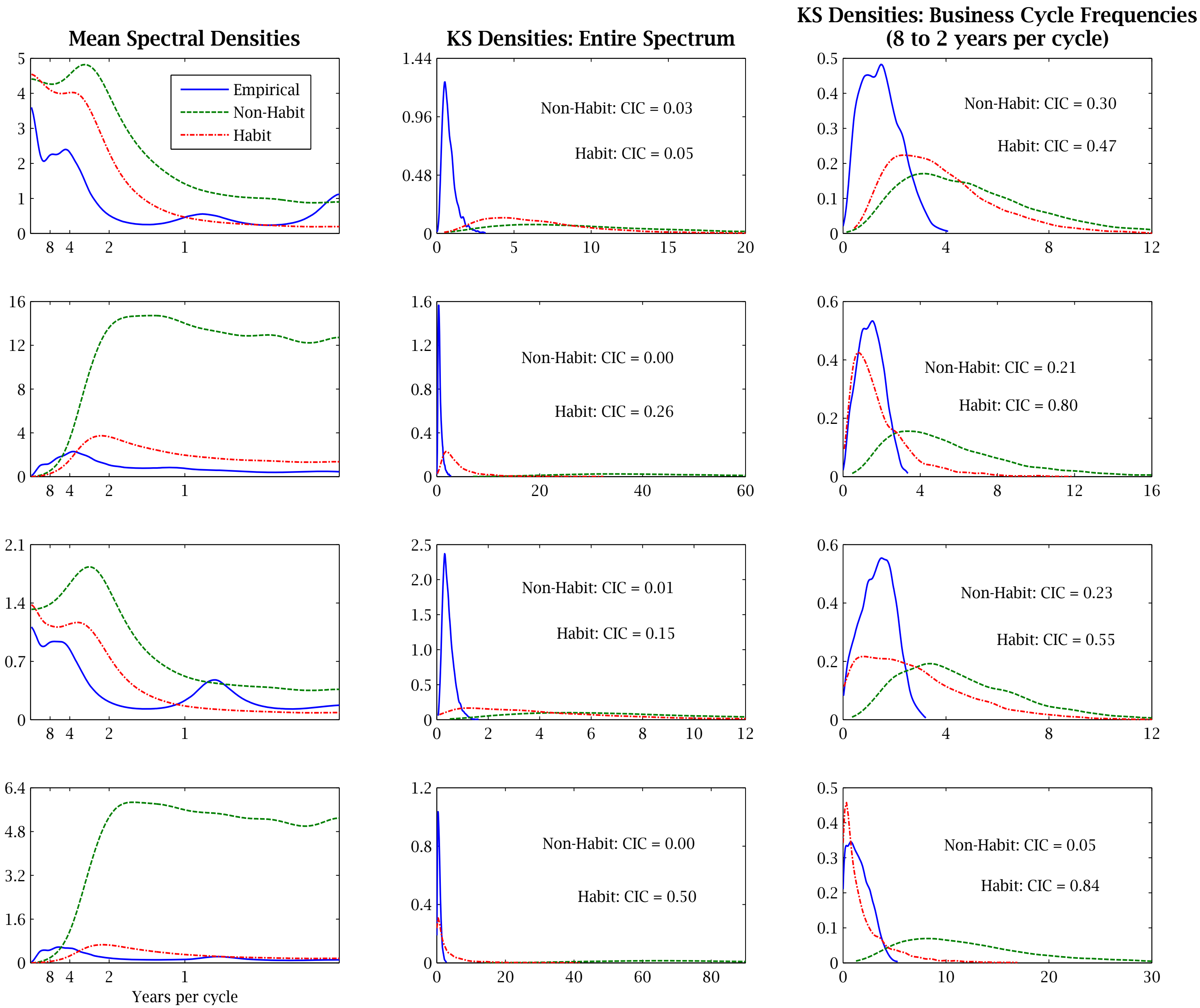

Figure A10: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Taylor Rule and only Sticky Prices and β Prior for h

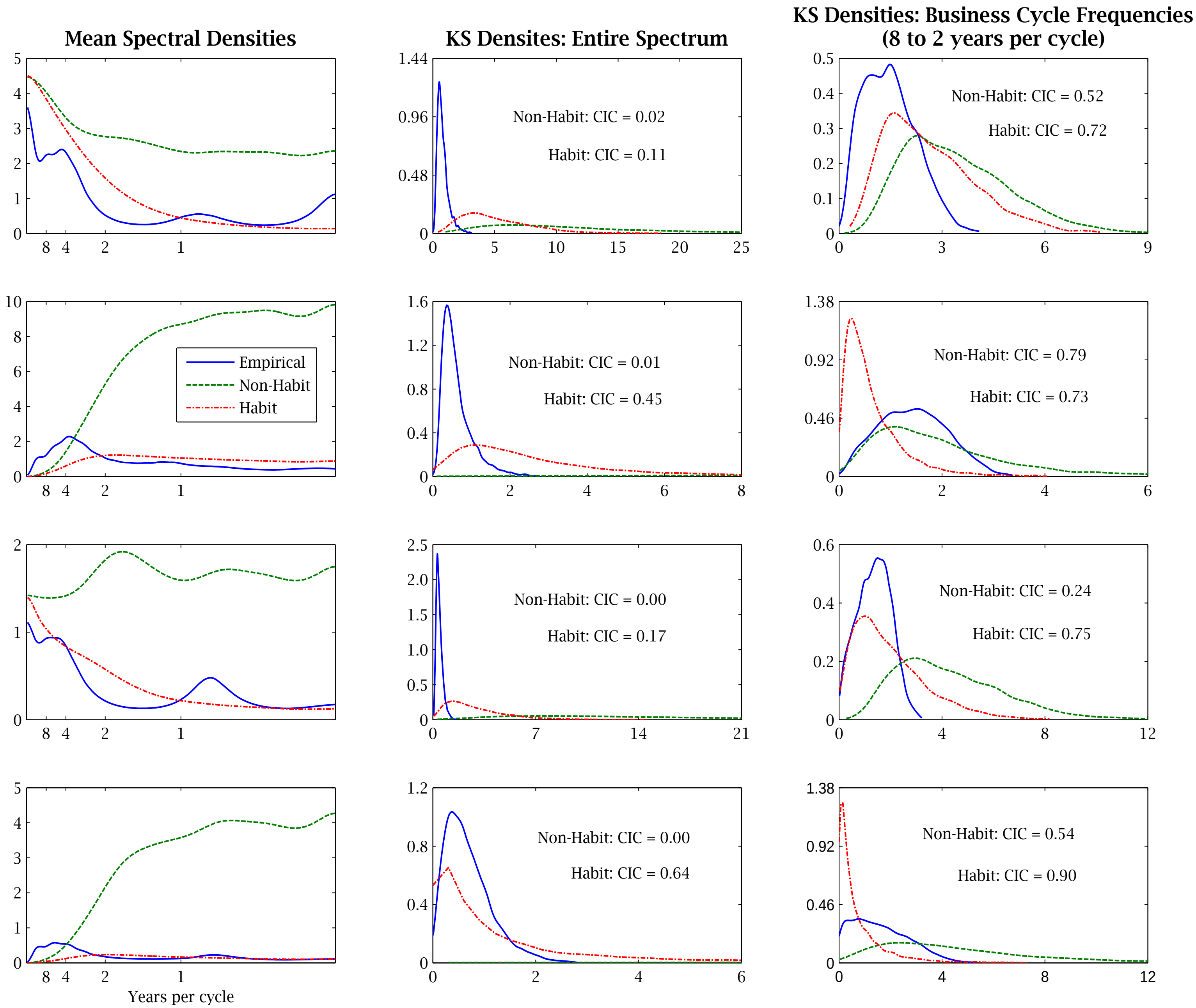

Figure A11: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with AR(1) Money Growth Rule and Only Sticky Wages and β Prior for h

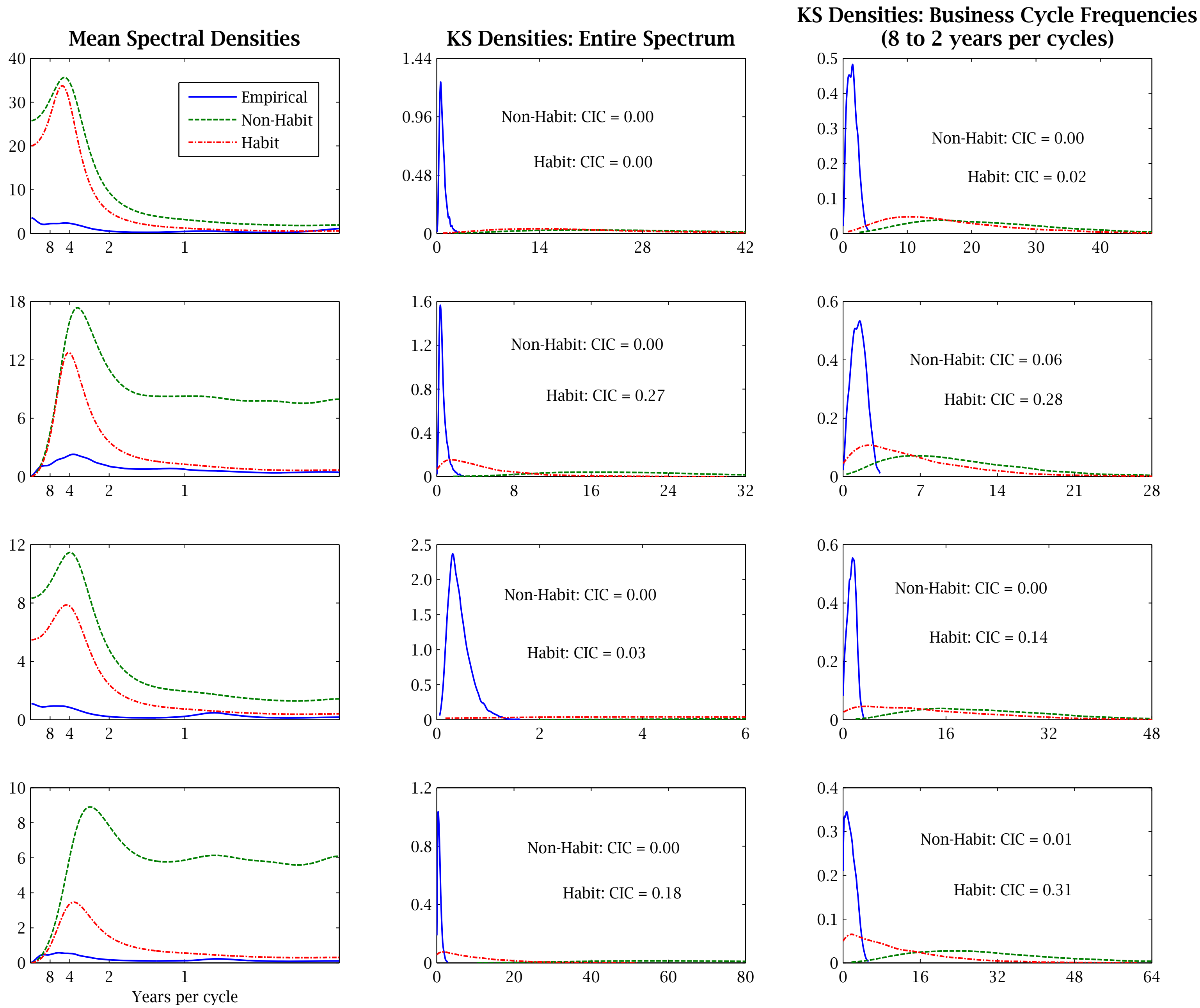

Figure A12: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Taylor Rule and only Sticky Wages and β Prior for h