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Did the Great Inflation Occur Despite Policymaker Commitment to a Taylor Rule?

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Abstract: The authors study the hypothesis that misperceptions of trend productivity growth during the onset of the productivity slowdown in the United States caused much of the great inflation of the 1970s. They use the general equilibrium, sticky price framework of Woodford (2002), augmented with learning using the techniques of Evans and Honkapohja (2001). The authors allow for endogenous investment as well as explicit, exogenous growth in productivity and the labor input. They assume the monetary policymaker is committed to using a Taylor-type policy rule. The authors study how this economy reacts to an unexpected change in the trend productivity growth rate under learning. They find that a substantial portion of the observed increase in inflation during the 1970s can be attributed to this source.

JEL classification: E4, E5

Key words: monetary policy rules, productivity slowdown, learning

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1. INTRODUCTION

1.1. The great inflation and the productivity slowdown

Broadly speaking, U.S. inflation was low in the early 1960s, then high in the 1970s and early 1980s, and then lower again during the last twenty years. Figure 1 shows one measure of the dramatic rise and fall, which is sometimes called the *great inflation*. We investigate the hypothesis that much of the great inflation was due to a misperception on the part of economic actors—both the private sector and the Federal Reserve—concerning the pace of productivity growth. This hypothesis is associated most closely (and most recently) with Orphanides (2000a, 2000b, 2001, 2002). The broad idea is that it was initially very difficult for the economy's participants to detect that the productivity slowdown had occurred—that is, agents had to learn about it. The misperception caused the central bank to overestimate the size of the output gap, leading through a Taylor-type policy rule to lower-than-intended interest rates and, subsequently, higher-than-intended inflation.

Our vehicle for analysis is a version of the general equilibrium, sticky price model of Woodford (2002). We allow for endogenous investment along with explicit, exogenous growth, both of which we view as essential for discussion of this issue. We include learning to capture the idea that it took some time for the economy's participants to evaluate the changing nature of the nation's balanced growth path dictated by the productivity slowdown. Our learning methodology is that of Evans and Honkapohja (2001).

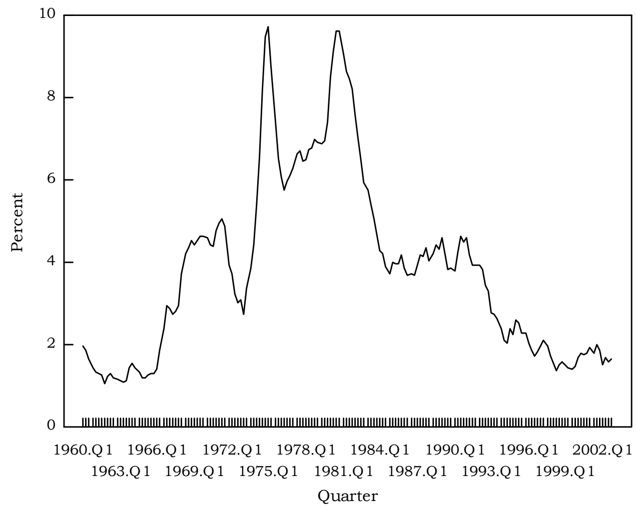
For the purposes of this paper, we use a Perron (1989)-style characterization of the productivity data, in which trend-stationarity is buffeted by rare breaks in trend, occurring perhaps once or twice in the postwar data.[3] This means the process driving productivity growth is actually nonstationary. The agents in our model—both the central bank and the private sector—suspect that such shocks change may occur, and employ learning algorithms to ensure that they will be able to adjust. In this sense the agents in our model are protecting themselves against the possibility of *structural change*—permanent changes in key aspects of the economy, like the pace of productivity growth—by re-estimating their perceived laws of motion for the economy each period. When there is no structural change for a period of time, our systems will simply converge to a small neighborhood of the rational expectations equilibrium, balanced growth path. But when structural change *does* occur, the agents will be able to learn the new balanced growth path. Thus, learning will act as the glue that holds the piecewise balanced growth paths of our model together. Our paper concerns a quantitative assessment of the reaction of key macroeconomic variables to permanent productivity growth shocks in this environment.

Of the many recent hypotheses for the 1970s inflation experience, the "misperceived change-in-trend" view has some of the more jarring policy implications. It

[3] For recent empirical papers concerning trend breaks in productivity, see Bae, Lumsdaine, and Stock (1998) and the survey by Hansen (2001). Our reading of the empirical literature is that it is difficult to reconcile a completely trend-stationary view with the macroeconomic data.

Figure 1. U.S. inflation, 1960-2002

Core PCE inflation from one year earlier.



Personal consumption expenditures inflation rate from the previous year, less food, energy, and insurance payments.

FIG. 1 The U.S. inflation experience includes a sharp increase in observed inflation following the onset of the productivity slowdown in the early 1970s. Excluding volatile components and smoothing the data slightly provides one indication of what might be called medium-run inflation movements. Our analysis is designed to address movements at this frequency.

suggests the possibility that even a determined and knowledgeable central bank could end up with a lot of inflation. The monetary authorities are determined and knowledgeable in the sense that they are committed to using a Taylor-type policy rule that would be optimal or near-optimal in some stationary contexts where structural change never occurs.[4] Since productivity growth is notoriously hard to track, because of noise and measurement problems in the data, and since, at the same time, productivity is critically important for economic growth, it seems quite reasonable that some learning about rare changes in its mean growth rate must occur. Thus it is not out of the question, under this hypothesis, that the inflation experience of the 1970s could be repeated, given the right type of shock. Understanding this shock and what might be done should it occur again would then be a key concern for policymakers.

A backdrop to this issue is the more recent improvement in U.S. productivity

[4] The argument that monetary policy during the 1970s was essentially the same as the policy recommended as optimal or near-optimal in the recent literature has been made by Orphanides; see for instance Orphanides (2002). See the papers in Taylor (1999) for arguments that Taylor-type policy rules provide desirable policy outcomes.

growth during the 1990s, the so-called "new economy." To the extent that private sector actors and policymakers had to again learn about the changing nature of the balanced growth path, this event might have been expected to lead to lower inflation through a Taylor-type policy rule. We provide an assessment of this hypothesis as well.

1.2. Model summary

We use a version of Woodford's (2002) general equilibrium, sticky price model with endogenous investment. We include explicit, exogenous growth in the model, driven by growth in the labor force as well as productivity. We maintain the assumption of firm-specific labor inputs, but we allow for a homogeneous capital input, traded in a perfectly competitive and economy-wide capital market. The main reason for the homogeneous capital assumption is to keep the model relatively simple and comparable with models currently used for policy analysis. The model economy has a well-defined rational expectations equilibrium characterized as a balanced growth path. We assume the economy begins on such a path. Our experiment is to unexpectedly alter the rate of growth of productivity along this path, and allow the economy's actors to adapt to the new rational expectations equilibrium. So long as the system is expectationally stable (which we verify), the disturbance to productivity growth will only temporarily cause the system to depart from the rational expectations equilibrium, as the agents will eventually learn the new equilibrium.

We include learning in the model using the methodology of Evans and Honkapohja (2001). Expectations are formed by agents using well-specified vector autoregressions updated each period as new information becomes available. The regressions are well-specified in the sense that they are consistent with the rational expectations equilibrium law of motion for the economy.

1.3. Main findings

We find that a one-time, unexpected change in productivity growth of the magnitude observed in the early 1970s generates a lot of inflation in our model, arguably a large portion of the persistent inflationary acceleration during this period. Thus our assessment is that the "misperceived-change-in-trend" view has considerable merit, even in the context of a completely micro-founded, forward-looking, general equilibrium model with endogenous investment.

We also find, however, that the inflation generated by the productivity slowdown in this model is far too persistent, as it does not fall rapidly enough by the early 1980s to provide a satisfactory account of the data. The onset of the new economy in the 1990s does lead to disinflation, but the model misses the sharp disinflation of the early 1980s. This suggests to us that a change in the policy rule occurred during the first portion of the Volker era. When we add a modest, unexpected reduction in the target rate of inflation in 1980, which might be viewed as a good approximation to policy developments at that time,[5] then the model can capture a substantial portion of the medium run movements in inflation from 1970 to the present depicted in Figure 1. We also show that other features of the simulated economy, including the pattern of inflation expectations and the pace of output growth, are close to observations from the postwar U.S. data.

We find that an important component of our quantitative results is that we allow both the central bank and the private sector to learn about the change in the balanced growth path. If the private sector has rational expectations while the central bank does not, then they understand more than the central bank both about the shock that has hit the economy and about the nature of the central bank's reaction to that shock. That is, they understand that the central bank is setting the nominal interest rate at "too low" of a level and accordingly they take actions that mitigate the inflation that would otherwise occur. We discuss this and other variations of the model in the results section of the paper.

1.4. Recent related literature

A number of authors have recently put forward formal models offering an explanation for the postwar U.S. inflation experience. Examples include Clarida, Gali, and Gertler (1999), Christiano and Gust (1999), and Albanesi, Chari, and Christiano (2002). The studies of Sargent (1999), Cho, Williams, and Sargent (2002), and Williams (2003) emphasize possession of a misspecified model on the part of policymakers. Our learning methodology is similar to theirs, but in our systems the learning dynamics simply converge to the economy's unique balanced growth path following a disturbance (that is, we have the mean dynamics of Sargent (1999) and Evans and Honkapohja (2001)).

Orphanides (2000a, 2000b, 2001, 2002) has written extensively on the 1970s U.S. inflation experience from the perspective of policymakers at the time. His work suggests that the perceived output gap was quite large during this period, and that this influenced policy appreciably. Lansing (2002) studies the interaction of monetary policy and trend growth changes in a simplified version of Fuhrer and Moore (1995), and finds a modest increase in inflation in response to a permanent technology shock. For research concerning optimal monetary policy, see Tambalotti (2002), who studies policymaker reactions to persistent, but not permanent, shocks to technology, in a model without capital. Tambalotti (2002) does not consider the policy 'mistake' discussed in Orphanides' papers. Nelson and Nikolov (2002) study the productivity slowdown-inflation nexus in the U.K.

[5] In all other respects the policy rule remains unchanged.

2. THE ENVIRONMENT

2.1. Exogenous growth

The economy is populated by a continuum of households indexed by h. The size of the aggregate labor force is described by N_t which grows according to

$$N_t = \eta N_{t-1}, \tag{1}$$

with $\eta \geq 1$, and the normalization $N_0 = 1$, so that η is the gross rate of growth in the labor force for the economy. We assume this growth is equally distributed across households h, so that the size of each household also grows at gross rate η. Individual households are of size N_t^h, and so $N_t^h = \eta N_{t-1}^h$, with $\int N_0^h dh = 1 = N_0$.

We also assume explicit technological progress. We take the most standard case by assuming that this progress, defined in terms of efficiency units X_t, affects only labor productivity. Productivity is assumed to grow according to

$$X_t = \gamma X_{t-1}, \tag{2}$$

with $\gamma \geq 1$ and initial normalization $X_0 = 1$. We assume labor productivity improvements apply equally to all households.

2.2. Notation

Given the many changes of variables we use below a few comments about the notation are needed. Because of the exogenous growth, a given variable Y_t may possibly be non-stationary in our model. Aggregate output and aggregate capital, for instance, will grow over time. Where necessary, we denote such a variable in stationary form as $\hat{y}_t = \frac{Y_t}{X_t N_t}$. We will take logarithmic deviations from a steady state for some purposes, and so we define $\tilde{y}_t = \ln\left(\frac{\hat{y}_t}{\bar{y}}\right)$, where $\bar{y}$ is the steady state value of the stationary variable $\hat{y}$. And finally, for our learning model we will want to refer to the logarithm of the steady state component of this deviation separately, and so we denote $y_t = \ln \hat{y}_t$ and $y = \ln \bar{y}$.

2.3. Household decisions

Each household h makes expected utility-maximizing decisions regarding consumption, labor supply, and money-holding, with expected utility given by

$$E_t \sum_{i=0}^{\infty} \beta^i \eta^i \left[\ln C_{t+i}^h - a \int_0^1 \frac{\left[L_{t+i}^h(f)\right]^{1+\nu}}{1+\nu} df + \Psi\left(\frac{M_{t+i}^h}{P_{t+i}}\right)\right]. \tag{3}$$

Here, $\beta \in (0,1)$ is the discount factor, and C_{t+i}^h is an index of household h consumption at date $t+i$. In the economy a continuum of differentiated goods are supplied, indexed by $f \in (0,1)$. Each household consumes some units of each good produced. Each individual good $C_t(f)$ is produced by using capital and a specialized labor input—labor of type f is used to produce the differentiated good indexed by f. We use the standard consumption aggregator

$$C_t^h = \left[\int_0^1 C_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}. \tag{4}$$

Following Woodford (2002), every household h simultaneously supplies all types of firm-specific labor $L_t^h(f)$, with $a > 0$ and $\nu \geq 0$. Households are endowed with one unit of time, and so we require $\int L_t^h(f)\, df \in (0,1)$.

The function $\Psi(\cdot)$ denotes the utility derived from holding real balances M_t^h/P_t, where M_t^h is nominal money balances held by household h at time t, and P_t is the price index associated with C_t^h at time t, defined by

$$P_t = \left[\int_0^1 P_t(f)^{1-\theta} df \right]^{\frac{1}{1-\theta}}, \tag{5}$$

where $P_t(f)$ is the price associated with good $C_t(f)$. We will denote $\tilde{\beta} \equiv \beta\eta$, and call it the 'effective' discount factor.

Each household h faces the budget constraint

$$P_t C_t^h + M_t^h + B_t^h = M_{t-1}^h + (1 + i_{t-1}) B_{t-1}^h + \Gamma_t^h + \int_0^1 N_t^h L_t^h(f) W_t(f)\, df, \tag{6}$$

where Γ_t^h defines the nominal profits from holding a share of each firm in the economy, and $W_t(f)$ is the nominal wage paid by the firm that uses labor of type f. We let B_t^h denote household holdings of financial assets other than money.[6] The short-term nominal interest rate, i_t, is assumed to be controlled directly by the central bank.

We emphasize that under our assumptions the consumption decision of every household h is identical because each household receives the same flow of income and shares the same initial money and other asset holdings. From the asset accumulation decision, we obtain the Euler equation

$$1 + i_t = \tilde{\beta}^{-1} \Lambda_t (E_t \Lambda_{t+1})^{-1}, \tag{7}$$

where

$$\Lambda_t = C_t^{-1}/P_t \tag{8}$$

is the Lagrange multiplier associated with the optimization problem and $C_t = \int C_t^h dh = C_t^h$.

By defining $\hat{\lambda}_t = \Lambda_t P_t (X_t N_t)$ we can express (7) as the stationary equation

$$1 + i_t = \gamma \beta^{-1} \hat{\lambda}_t \left(E_t \Pi_{t+1} \hat{\lambda}_{t+1} \right)^{-1} \tag{9}$$

where $\Pi_t = P_t/P_{t-1}$. The labor supply decision for *each* type of labor f is determined by

$$\frac{a [L_t(f)]^\nu}{N_t} \left(\frac{W_t(f)}{P_t} \right)^{-1} = \Lambda_t P_t \tag{10}$$

[6] Here we assume complete markets; for details, see Woodford (2002), Ch. 2.

where $\frac{W_t(f)}{P_t}$ is the real wage. We assume that the labor market is competitive, so that the households take the wage as given. Equation (10) can also be expressed in stationary form, which is

$$\frac{a\left[L_t\left(f\right)\right]^{\nu}}{\hat{w}_t\left(f\right)} = \hat{\lambda}_t. \tag{11}$$

where $\hat{w}_t\left(f\right)$ represents the real wage per efficiency unit $W_t\left(f\right)/\left(P_tX_t\right)$. We have dropped the subscript h because the labor supply decision is identical across households, given that they have identical preferences and make identical consumption decisions (in particular, $L_t(f) = \int L_t^h\left(f\right)dh = L_t^h\left(f\right)$). Moreover, bond and money holdings will be the same for every agent.

2.4. Firm behavior

Each firm produces a differentiated good and has some market power. As mentioned above, we assume that each good is produced using capital and labor. We assume that capital is homogenous in the economy, while labor is firm-specific. For capital, we assume the existence of a perfectly competitive, economy-wide market. Each firm, in order to produce its differentiated good, uses a different type of labor. The households supply labor hours $L_t\left(f\right) = \int L_t^h\left(f\right)dh$ to the firm producing good f, given the wage offered by the firm. Again, we assume that labor markets are competitive.[7]

Firm f produces the good $Y_t\left(f\right)$ using the technology

$$Y_t\left(f\right) = K_t\left(f\right)^{\alpha}\left[X_tN_tL_t\left(f\right)\right]^{1-\alpha}, \tag{12}$$

where $K_t\left(f\right)$ is the portion of the capital stock used for production at time t by firm f and $L_t\left(f\right)$ is the amount of hours households supply for production. The optimal allocation of household spending across differentiated goods implies the demand curve[8] for each firm f,

$$Y_t\left(f\right) = \left[\frac{P_t\left(f\right)}{P_t}\right]^{-\theta}Y_t, \tag{13}$$

where $\left(\theta-1\right)^{-1}$ denotes firms' markup and where $Y_t = C_t + I_t = \int Y_t\left(f\right)df$, and $I_t = \int I_t\left(f\right)df$ is aggregate investment. Cost minimization then implies the real marginal cost[9] for firm f is

$$S_t\left(f\right) = \frac{W_t\left(f\right)}{P_t}\frac{N_t}{MPL_t\left(f\right)}, \tag{14}$$

where $S_t\left(f\right)$ is real marginal cost and the marginal product of labor is given by

$$MPL_t\left(f\right) = \left(1-\alpha\right)K_t\left(f\right)^{\alpha}\left(X_tN_t\right)^{1-\alpha}L_t\left(f\right)^{-\alpha}. \tag{15}$$

[7] See Woodford (2002) for a discussion.

[8] This is a standard result implied by the consumption aggregator.

[9] We follow Woodford (2002) and Erceg (1997).

The real rental rate of capital is

$$R_t^K = \left(\frac{\alpha}{1-\alpha}\right)\frac{W_t\left(f\right)}{P_t}\frac{N_t L_t\left(f\right)}{K_t\left(f\right)}. \tag{16}$$

Given the assumption of an economy-wide capital market each firm faces the *same* rental price of capital, and thus it is not indexed by f. At the same time, each firm faces a different wage in the labor market for the type of labor needed.

The objective of a firm is to maximize expected future profits. Assuming convex adjustment costs as in Casares and McCallum (2000) and Woodford (2002), profit maximization implies that capital per efficiency unit $\hat{k}_{t+1}$ is chosen according to

$$I'\left(\frac{\gamma\eta\hat{k}_{t+1}\left(f\right)}{\hat{k}_t\left(f\right)}\right) = E_t Q_{t,t+1}\Pi_{t+1}\left[R_{t+1}^K + \frac{\gamma\eta\hat{k}_{t+2}\left(f\right)}{\hat{k}_{t+1}\left(f\right)}I'\left(\frac{\gamma\eta\hat{k}_{t+2}\left(f\right)}{\hat{k}_{t+1}\left(f\right)}\right) - I\left(\frac{\gamma\eta\hat{k}_{t+2}\left(f\right)}{\hat{k}_{t+1}\left(f\right)}\right)\right] \tag{17}$$

where

$$Q_{t,t+1} = \frac{U'\left(\hat{c}_{t+1}\right)P_t}{U'\left(\hat{c}_t\right)P_{t+1}}\frac{\beta}{\gamma} \tag{18}$$

is the stochastic discount factor as defined in Woodford (2002), and

$$I_t\left(f\right) = I\left(\frac{K_{t+1}\left(f\right)}{K_t\left(f\right)}\right)K_t\left(f\right) \tag{19}$$

is total investment expenditure. In particular, the function $I\left(\cdot\right)$ describes the amount of resources needed in order to obtain K_{t+1} units of capital. It has the properties

$$I\left(\gamma\eta\right) = \gamma\eta - 1 + \delta, \tag{20}$$

$$I'\left(\gamma\eta\right) = 1, \tag{21}$$

and

$$I''\left(\gamma\eta\right) = \epsilon_\psi > 0, \tag{22}$$

where ϵ_ψ denotes the cost of adjusting the capital stock.[10]

Finally, we assume Calvo-type sticky prices. Firm f decides its optimal price according to

$$E_t\sum_{j=0}^{\infty}\tilde{\beta}\xi^j Q_{t,t+j}\left[\frac{P_t\left(f\right)}{P_t} - \frac{\theta}{\left(\theta-1\right)}S_{t+j}\left(f\right)\bar{\Pi}^j\right] \times\left(\frac{P_t\left(f\right)}{P_{t+j}}\right)^{-\theta}Y_{t+j} = 0 \tag{23}$$

where ξ is the probability of not changing the price, which is assumed to be equal across firms, and $\bar{\Pi}$ is the gross target inflation rate (we use $\pi_t = \ln\Pi_t$, and $\bar{\pi} =$

[10] See Woodford (2002) for discussion.

$\ln \bar{\Pi}$ for the net inflation rate and the net inflation target, respectively). This price setting equation is consistent with the hypothesis that firms not choosing the optimal price in period t adjust their prices according to the following rule of thumb

$$P_t(f) = \bar{\Pi} P_{t-1}(f), \tag{24}$$

which corresponds to automatic adjustment by the amount of the inflation target. This updating rule is chosen to keep the model simple. It could be objected that in periods of high inflation firms should change prices every period according to past inflation, as suggested by Christiano, *et al.*, (2001). We choose instead to keep the inertia in the inflation rate generated by the model under rational expectations as low as possible, in order to be able to better assess the role of learning in generating persistence.

2.5. The linearized model

We wish to linearize the model about the nonstochastic balanced growth path in order to be able to apply the learning methodology of Evans and Honkapohja (2001). We begin by considering household behavior. Concerning the labor supply we obtain

$$\tilde{w}_t(f) = \nu \tilde{L}_t(f) - \tilde{\lambda}_t \tag{25}$$

where we recall that $\tilde{x}_t$ denotes a logarithmic deviation of a stationary variable $\hat{x}_t$ from the deterministic balanced growth path value $\bar{x}$. The Euler equation becomes

$$\tilde{\lambda}_t = E_t(\tilde{\imath}_t - \tilde{\pi}_{t+1}) + E\tilde{\lambda}_{t+1}. \tag{26}$$

Linearizing equations (14)-(16) implies that, after some algebra, we can express the real rental price of capital as

$$\tilde{R}_t^K = \rho_y \tilde{y}_t(f) - \rho_k \tilde{k}_t(f) - \tilde{\lambda}_t. \tag{27}$$

By inserting (27) in the linearized (17) and averaging across firms, we find the following approximation for capital dynamics

$$\tilde{\lambda}_t + \epsilon_\psi \left(\tilde{k}_{t+1} - \tilde{k}_t\right) = \frac{\beta}{\gamma}(1-\delta) E_t \tilde{\lambda}_{t+1} + \left[1 - \frac{\beta}{\gamma}(1-\delta)\right] E_t \left(\rho_y \tilde{y}_{t+1} - \rho_k \tilde{k}_{t+1}\right) + \frac{\beta}{\gamma}\epsilon_\psi E_t \left(\tilde{k}_{t+2} - \tilde{k}_{t+1}\right). \tag{28}$$

Furthermore, linearizing (19) we obtain

$$\tilde{I}_t(f) = \frac{\bar{k}}{\bar{y}} \left[\gamma\eta \tilde{k}_{t+1}(f) - (1-\delta)\tilde{k}_t(f)\right], \tag{29}$$

where $\tilde{I}$ denotes deviations of investment per effective worker from its steady state, as a percentage of output per effective worker. Using (29) in the economy's resource constraint, averaging across firms, and substituting for consumption in (8) after linearizing we obtain

$$\tilde{\lambda}_t = -\frac{\bar{y}}{\bar{c}}\left[\tilde{y}_t - \frac{\bar{k}}{\bar{y}}\left(\gamma\eta\tilde{k}_{t+1} - (1-\delta)\,\tilde{k}_t\right)\right]. \tag{30}$$

In addition, with some algebraic manipulation we can express average real marginal cost as

$$\tilde{S}_t = \omega(\tilde{y}_t - \tilde{k}_t) + \nu\tilde{k}_t - \tilde{\lambda}_t, \tag{31}$$

where $\omega = \omega_p + \omega_w$ and $\omega_p = \frac{\alpha}{1-\alpha}$ and $\omega_w = \frac{\nu}{1-\alpha}$. Substituting (30) in (31) we obtain

$$\tilde{S}_t = \left(\omega + \sigma^{-1}\right)\tilde{y}_t - \sigma^{-1}\frac{\bar{k}}{\bar{y}}\gamma\eta\tilde{k}_{t+1} + \left[\sigma^{-1}\frac{\bar{k}}{\bar{y}}(1-\delta) - (\omega - \nu)\right]\tilde{k}_t, \tag{32}$$

where, following Woodford (2002), we define σ as the intertemporal elasticity of substitution in consumption (which is equal to one here because consumption preferences are logarithmic) times $\bar{c}/\bar{y}$. The linearized price setting equation is

$$E_t\sum_{j=0}^{\infty}\tilde{\beta}^j\left[\tilde{P}_t^* - \tilde{S}_{t+j}(f) + \left(\sum_{i=1}^{j}\bar{\pi} - \sum_{i=1}^{j}\pi_{t+i}\right)\right] = 0 \tag{33}$$

where P_t^* is the optimal relative price. In Appendix A we show how the real marginal cost can be expressed just as a function of output and the average marginal cost only, that is,

$$\tilde{S}_t = \bar{\omega}\left(\tilde{y}_t - \tilde{y}_t(f)\right) + \tilde{S}_t(f) \tag{34}$$

where

$$\bar{\omega} = \frac{\rho_y - \rho_k}{\rho_k}. \tag{35}$$

In addition, linearizing (5) and inserting the optimal price we obtain

$$\tilde{P}_t^* = \frac{\xi}{1-\xi}\tilde{\pi}_t. \tag{36}$$

Finally, using the demand for output of firm f and substituting (36) in the price setting equation we get a Phillips curve, which has the standard form

$$\tilde{\pi}_t = \psi\tilde{S}_t + \tilde{\beta}E_t\tilde{\pi}_{t+1} \tag{37}$$

where

$$\psi = \left(\frac{1-\xi}{\xi}\right)\left(\frac{1-\xi\tilde{\beta}}{1+\theta\bar{\omega}}\right). \tag{38}$$

2.6. The central bank

We assume that monetary policy is conducted according to a time-invariant, Taylor-type policy rule. The central bank sets the short-term nominal interest rate in response to deviations of inflation and output growth from the inflation target and the long-run growth trend, respectively. The rule is given by

$$i_t = \rho i_{t-1} + (1-\rho)\left[E_t i + \phi_\pi (\pi_t - \bar{\pi}) + \phi_y (g_{yt} - E_t g_y)\right] \tag{39}$$

where i_t is the net nominal interest rate, $\bar{\pi}$ is the central bank's inflation target, ρ, ϕ_π, and ϕ_y are parameters, $g_{yt} = y_t - y_{t-1} + \ln \gamma\eta$ is the observed output growth rate, $E_t g_y$ is the central bank's estimate of the growth trend, and $E_t i$ is the central bank's estimate of the long-run level of the nominal interest rate. We stress that to make this rule operational in our framework, the bank needs the estimates $E_t g_y$ and $E_t i$. This latter term depends on the long-run real rate of interest, which will change when the productivity growth rate changes. The hypothesis we wish to investigate works as follows. As the productivity slowdown hits the economy, the bank observes a decrease in the current growth rate of output while its current estimates of the growth trend and the real interest rate remain unchanged. Hence, the bank perceives a negative "output growth gap" and reduces the nominal interest rate.

The choice of a rule that responds to the output growth gap instead of to the output gap is dictated by an important consideration in a model with explicit exogenous growth: The steady state level of output per efficiency unit is negatively related to the growth rate of productivity (see the calculation in Appendix B). Hence, if the bank were assumed to respond to deviations of output per efficiency unit from its estimated steady state value, a decrease in the productivity growth rate would actually lead the central bank to *increase* the interest rate, not to decrease it. The effect would go in the wrong direction from the perspective of evaluating the "misperceived-change-in-trend" hypothesis. The problem is avoided if the bank is assumed to respond to deviations of the output growth rate from the long-run trend growth rate.

Orphanides (2001) argues that a Taylor-type rule that reacts to the output growth gap may be more stabilizing than a rule that responds to the output gap, in the sense that the implied policy mistake is smaller. This conclusion does not necessary hold in the present model. In fact, the result of Orphanides (2001) is based on the fact that the growth rule is still based on estimates of potential output. He argues that a rule that responds to output growth would be more stabilizing because the mistakes from the estimation of potential output would 'cancel out.' This does not happen in our case because the central bank is *directly estimating the trend growth rate*. Its mistakes will therefore be autocorrelated.

2.7. The complete model

2.7.1. Deviations form versus levels form

We can now express the model in a relatively compact form, and briefly compare it to standard models without investment dynamics. However, the deviations from steady state form of the model is still not sufficient for our purposes, because we want to introduce learning in the next section. When agents learn, they do not know the steady state values of each economic variable. In order to force them to learn these values when the productivity growth rate changes, we rearrange the logarithms of steady state values into a constant term in a “log-levels” form for each equation. In the model with learning, the constant terms will be estimated recursively by the agents each period.[11]

2.7.2. IS equation

Putting together the equations (26) and (30) implies

$$\tilde{y}_t = -\sigma E_t\left(\tilde{\imath}_t - \tilde{\pi}_{t+1}\right) + E_t\tilde{y}_{t+1} - E_t\frac{\bar{k}}{\bar{y}}\left(\gamma\eta\tilde{k}_{t+2} - (1-\delta)\tilde{k}_{t+1}\right) + \frac{\bar{k}}{\bar{y}}\left(\gamma\eta\tilde{k}_{t+1} - (1-\delta)\tilde{k}_t\right), \quad (40)$$

which can be also written in terms of investment

$$\tilde{y}_t = -\sigma^{-1}E_t\left(\tilde{\imath}_t - \tilde{\pi}_{t+1}\right) + E_t\tilde{y}_{t+1} - E_t\tilde{I}_{t+1} + \tilde{I}_t \quad (41)$$

as deviations from steady state. This equation can be expressed in log-levels form as[12]

$$y_t = \sigma\left(E_t i - \pi\right) - \sigma E_t\left(i_t - \pi_{t+1}\right) + E_t y_{t+1} - E_t\frac{\bar{k}}{\bar{y}}\left[\gamma\eta k_{t+2} - (1-\delta)k_{t+1}\right] + \frac{\bar{k}}{\bar{y}}\left[\gamma\eta k_{t+1} - (1-\delta)k_t\right]. \quad (42)$$

This equation differs from the standard version of the forward-looking IS curve in that investment appears. An expected increase in investment has a negative effect on consumption and therefore output. We emphasize that, in order to preserve the symmetry between the central bank and the private sector's information set, we assume that the households need an estimate of the long run nominal interest rate in order to take consumption decisions (that is, $E_t i$ in the first term on the right hand side). Writing this equation is in simpler notation we get

$$y_t = a_{00} + a_{01}E_t i + a_{02}E_t\left(i_t - \pi_{t+1}\right) + a_{03}E_t y_{t+1} - a_{04}E_t\left(k_{t+2} - k_{t+1}\right) + a_{05}\left(k_{t+1} - k_t\right), \quad (43)$$

where

$$a_{00} = -\sigma\pi, \quad (44)$$

$$a_{01} = \sigma, \quad (45)$$

$$a_{02} = \sigma, \quad (46)$$

$$a_{03} = 1, \quad (47)$$

$$a_{04} = \frac{\bar{k}}{\bar{y}}\gamma\eta, \quad (48)$$

[11] If we did not do this, we would in effect be telling the agents the value of the new steady state in the event of a structural change. This does not seem very reasonable, and would in any event be contrary to the hypothesis we are trying to investigate.

[12] Here we use our notation $y_t = \ln\hat{y}_t$ and $y = \ln\bar{y}$.

and

$$a_{05} = \frac{\bar{k}}{\bar{y}} (1 - \delta). \tag{49}$$

2.7.3. Phillips curve

Substituting equation (32) for the average marginal cost in (37) we obtain

$$\tilde{\pi}_t = \psi \left[\left(\omega + \sigma^{-1} \right) \tilde{y}_t - \sigma^{-1} \tilde{I}_t - (\omega - \nu) \tilde{k}_t \right] + \tilde{\beta} E_t \tilde{\pi}_{t+1}, \tag{50}$$

where the marginal cost depends also on investment. Expressed in terms of capital only, this equation becomes

$$\begin{aligned} \tilde{\pi}_t = \psi \left(\omega + \sigma^{-1} \right) \tilde{y}_t - \psi \sigma^{-1} \frac{\bar{k}}{\bar{y}} \gamma \eta \tilde{k}_{t+1} \\ + \psi \left[\sigma^{-1} \frac{\bar{k}}{\bar{y}} (1 - \delta) - (\omega - \nu) \right] \tilde{k}_t + \tilde{\beta} E_t \tilde{\pi}_{t+1}. \end{aligned} \tag{51}$$

Following the same process as for the IS curve we can define the equation in log-levels as

$$\pi_t = a_{10} + a_{11} y_t + a_{12} k_{t+1} + a_{13} k_t + a_{14} E_t \pi_{t+1}, \tag{52}$$

with

$$a_{10} = \left(1 - \tilde{\beta} \right) \bar{\pi} - a_{11} y - (a_{12} + a_{13}) k, \tag{53}$$

$$a_{11} = \psi \left(\omega + \sigma^{-1} \right), \tag{54}$$

$$a_{12} = -\psi \sigma^{-1} \frac{\bar{k}}{\bar{y}}, \tag{55}$$

$$a_{13} = \psi \left[\sigma^{-1} \frac{\bar{k}}{\bar{y}} (1 - \delta) - (\omega - \nu) \right], \tag{56}$$

and

$$a_{14} = \tilde{\beta}. \tag{57}$$

2.7.4. Capital equation

Substituting (30) in (28) we obtain

$$\begin{aligned} \epsilon_\psi \left(\tilde{k}_{t+1} - \tilde{k}_t \right) = \\ \frac{\beta}{\gamma} (1 - \delta) E_t \left(-\sigma^{-1} \left[\tilde{y}_{t+1} - \frac{\bar{k}}{\bar{y}} \left(\gamma \eta \tilde{k}_{t+2} - (1 - \delta) \tilde{k}_{t+1} \right) \right] \right) \\ + \sigma^{-1} \left[\tilde{y}_t - \frac{\bar{k}}{\bar{y}} \left(\gamma \eta \tilde{k}_{t+1} - (1 - \delta) \tilde{k}_t \right) \right] \\ + \left[1 - \frac{\beta}{\gamma} (1 - \delta) \right] E_t \left(\rho_y \tilde{y}_{t+1} - \rho_k \tilde{k}_{t+1} \right) \\ + \frac{\beta}{\gamma} \epsilon_\psi E_t \left(\tilde{k}_{t+2} - \tilde{k}_{t+1} \right), \end{aligned} \tag{58}$$

which, in log-levels, can be expressed as

$$k_t = a_{20} + a_{21}E_t y_{t+1} + a_{22}E_t k_{t+2} + a_{23}y_t + a_{24}k_t \tag{59}$$

where

$$\begin{aligned}
a_{20} &= (1 - a_{22} - a_{24})\,k - (a_{21} + a_{23})\,y, &(60)\\
a_{21} &= \bar{a}^{-1}\left[-\frac{\beta\sigma\,(1-\delta)}{\gamma}\left(\sigma^{-1} + \rho_y\right) + \rho_y\right], &(61)\\
a_{22} &= \bar{a}^{-1}\left(\frac{\beta}{\gamma}\frac{(1-\delta)\,\sigma^{-1}k\gamma\eta}{y} + \frac{\beta}{\gamma}\epsilon_\psi\right), &(62)\\
a_{23} &= \bar{a}^{-1}\sigma^{-1}, &(63)\\
a_{24} &= \bar{a}^{-1}\left[\epsilon_\psi + \sigma^{-1}\frac{\bar{k}}{\bar{y}}\,(1-\delta)\right], &(64)
\end{aligned}$$

and

$$\bar{a} = \epsilon_\psi + \sigma^{-1}\frac{\bar{k}}{\bar{y}}\gamma\eta - \left[-\frac{\beta}{\gamma}\,(\gamma\eta - \delta)\left(\sigma^{-1}\frac{\bar{k}}{\bar{y}}\,(1-\delta) + \rho_k\right) + \frac{\beta}{\gamma}\epsilon_\psi + \rho_k\right]. \tag{65}$$

3. LEARNING

3.1. Two-sided learning

3.1.1. Learning by the private sector

The model can be re-written to a four dimensional system of equations expressed as

$$V_t = B_1 + B_2\begin{bmatrix} E_{t-1}^{cb}g_y \\ E_{t-1}^{cb}i \end{bmatrix} + B_3E_{t-1}^{ps}V_{t+1} + B_4V_{t-1} + B_5\epsilon_t, \tag{66}$$

where $V_t = [y_t,\, \pi_t,\, k_{t+1},\, i_t]'$, E_{t-1}^{cb} denotes the expectation operator for the central bank and E_{t-1}^{ps} is the expectation operator for the private sector, ϵ_t is a normally distributed *i.i.d* shock, and the matrices B_i, $i = 1, 2, 3, 4, 5$ are conformable. The private sector makes forecasts concerning future inflation, output and capital (in efficiency units) in order to take consumption and production decisions. The central bank requires an estimate of the trend growth rate of output to assess the magnitude of the output growth gap. Both the central bank and the private sector households require an estimate of the steady state value of the nominal interest rate in order to make decisions. We will require this estimate to be the *same* for the central bank and the private sector.

Let us first consider private sector expectations. Following Evans and Honkapohja (2001), market participants have a correct (in form) model of the economy, known as the *perceived law of motion*, or PLM. They believe the economic variables in the economy evolve according to

$$V_t = \Omega_0 + \Omega_1 V_{t-1} + e_t, \tag{67}$$

where e_t is an unobservable *i.i.d.* shock. The agents must estimate the elements of the matrices Ω_0 and Ω_1 recursively. We stress that the presence of the constant implies that the agents do not know the steady state value of V_t. Under this PLM the agents form expectations according to[13]

$$E_{t-1}V_t = \Omega_{0,t-1} + \Omega_{1,t-1}V_{t-1}, \tag{68}$$

and

$$E_{t-1}V_{t+1} = \Omega_{0,t-1} + \Omega_{1,t-1}\Omega_{0,t-1} + \Omega_{1,t-1}^2 V_{t-1}. \tag{69}$$

We assume that the agents update the estimates of the VAR parameters using new observations available each period on V_t. We assume the agents use stochastic gradient learning, so that the parameters are recursively estimated according to

$$\vartheta_t = \vartheta_{t-1} + \zeta Z_{t-1}\left(V_t - \vartheta_{t-1}' Z_{t-1}\right)' \tag{70}$$

where $\vartheta_t' = (\Omega_{0,t}, \Omega_{1,t.})$, $Z_t = [1, y_t, \pi_t, k_{t+1}, i_t]$, and ζ is the gain.

We chose the stochastic gradient specification for two main reasons. First, it may be viewed as more plausible in a behavioral sense, as it is less complex than recursive least squares. Second, we found in the simulations that under recursive least squares, the system quite often leaves the basin of attraction of the rational expectations equilibrium and diverges, even if it is, technically, locally stable. In other words, the basin of attraction is quantitatively small. In order to achieve stability of the learning process under recursive least squares, we needed to use an extremely small value for the gain parameter, which in turn slowed down the learning process to an empirically implausible rate.

As we have stressed, the private sector agents in the model also need an estimate of the long run nominal interest rate. Since they use the same estimate as the central bank, we discuss it in the next section.

3.1.2. Learning by the central bank

Simple method Under the simple method, we assume that the central bank estimates the trend of output growth by minimizing the squared deviations of output growth from the constant trend. This leads to the following recursive estimate of g_y,

$$\begin{aligned} \breve{g}_{y,t} &= \breve{g}_{y,t-1} + \zeta\left(g_{y,t} - \breve{g}_{y,t-1}\right) \\ &= \breve{g}_{y,t-1} + \zeta\left(y_t - y_{t-1} + \ln\gamma\eta - \breve{g}_{y,t-1}\right) \end{aligned} \tag{71}$$

where $\breve{g}_{y,t}$ is the estimate of the growth rate at time t. Under this method the central bank does not efficiently use all the information available to estimate the growth trend. It might use other relevant variables to improve its estimates, as under the

[13] When studying the learning process we follow Evans and Honkapohja (2001) and assume that the economic agents take expectations using $t-1$ information. This is the "dating of expectations" issue that often arises in learning environments. Evans and Honkapohja (2001) have viewed $(t-1)$-dating as more realistic in a learning environment. The assumption of t-dating under learning can be employed at the cost of some complications, but then date t variables are being used to form expectations and are also being determined by the system at date t. The simultaneity is common in models solved under rational expectations, but is less satisfactory in a learning environment.

model consistent method outlined below. But in order to do that the central bank needs to know more details about the model of the economy. The simple estimation procedure does not require such precise knowledge about the economy. In this sense, the simple method keeps the assumption of bounded rationality applicable to the central bank.

Finally, both the central bank and the private sector need an estimate of the long run nominal interest rate. In order to simplify the analysis, we assume that the agents use the steady state relation between the growth rate of output and the nominal interest rate in order to estimate the long run nominal rate. From Appendix B, we know that in steady state

$$\ln \gamma - \ln \beta + \bar{\pi} = i. \tag{72}$$

Adding and subtracting $\ln \eta$ on the left hand side we get

$$g_y - \ln \eta\beta + \bar{\pi} = i. \tag{73}$$

Hence, both the central bank and the private sector can use the following estimate of the long run nominal interest rate[14]

$$E_{t-1} i = \breve{g}_{y_{t-1}} - \ln \eta\beta + \bar{\pi}. \tag{74}$$

Inserting the expectations of both private sector and the bank in (66) we obtain the *actual law of motion* (ALM)

$$V_t = \breve{B}_1 + \breve{B}_2 \breve{g}_{y,t-1} + B_3 \left[\Omega_{0,t-1} + \Omega_{1,t-1}\Omega_{0,t-1} + \Omega_{1,t-1}^2 V_{t-1} \right] + B_4 V_{t-1} + B_5 \epsilon_t \tag{75}$$

where $\breve{B}_1$ and $\breve{B}_{12}$ are suitable transformations of B_1 and B_2. The dynamics of the economy is characterized by (75), (71) and (70).[15]

Model consistent method The learning algorithm (71) is assumed to approximate the actual estimation process of the Federal Reserve in the seventies. It is reasonable to ask what would be the effects on the economy if the bank used more sources of information to estimate the output growth trend. At the rational expectations equilibrium, it is possible to express the growth rate of output as a function of past output, capital and the interest rate. We can then assume that to be the perceived low of motion of the growth rate of output:

$$g_{yt} = \kappa_0 + \kappa_1 y_{t-1} + \kappa_2 k_{t-1} + \kappa_4 i_{t-1}. \tag{76}$$

[14] We are implicitly assuming that the agents have already learned the value $-\ln \eta\beta + \bar{\pi}$. This is a constant which does not change when the rate of productivity growth changes. We start our systems on a balanced growth path, and so this seems like a reasonable assumption. In some simulations we will change the inflation target $\bar{\pi}$ unexpectedly. We are assuming that once this change is announced agents immediately adjust their nominal interest rate target downward.

[15] In this version of the paper we do not provide a formal proof for the stability of this system under learning. However, we did verify stability with simulations.

By estimating this equation the central bank can form an estimate of the trend growth rate by considering its steady state

$$g_y = \kappa_0 + \kappa_1 y + \kappa_2 k + \kappa_4 i. \tag{77}$$

Naturally, the bank does not know the steady state values of the system and needs also an estimate for them. This can be done by estimating ϑ_t (thus using the same information as the private sector does). We also assume that the bank and the agents employ (74) together with the model consistent estimate of the output growth rate to estimate the long run nominal interest rate. The actual law of motion of the economy under these assumptions can be described in compact notation

$$\tilde{V}_t = C_1 + C_2 E_{t-1}\tilde{V} + C_3 E_{t-1}\tilde{V}_{t+1} + C_4 \tilde{V}_{t-1} + C_5 \epsilon_t \tag{78}$$

where $\tilde{V}_t = [y_t, \pi_t, k_t, i_t, g_{yt}]$ and

$$E_{t-1}\tilde{V} = \left(I - \tilde{\Omega}_{1,t-1}\right)\tilde{\Omega}_{0,t-1} \tag{79}$$

where $\tilde{\Omega}$ includes the estimation of (76). Inserting the expectations we get[16]

$$\begin{aligned}\tilde{V}_t = C_1 + C_2 \left(I - \tilde{\Omega}_{1,t-1}\right)^{-1} \tilde{\Omega}_{0,t-1} + C_3 \tilde{\Omega}_{0,t-1} + C_3 \tilde{\Omega}_{1,t-1} \tilde{V}_{t-1} + \\ C_4 \tilde{\Omega}_{0,t-1} + C_4 \tilde{\Omega}_{1,t-1} \tilde{\Omega}_{0,t-1} + C_4 \tilde{\Omega}^2_{1,t-1} \tilde{V}_{t-1} + C_5 \tilde{V}_{t-1} + C_5 \epsilon_t,\end{aligned} \tag{80}$$

where C_i, $i = 1, 2, 3, 4, 5$, denotes a conformable matrix.

3.2. One-sided learning

In this section we consider the case in which the only the central bank is learning.[17] The implications of this assumption in the present model are strong. Rational expectations on the part of the private sector means that each firm can observe not only the change in its own productivity but also the change in productivity in all other firms. Also, each household in taking consumption and investment decisions is assumed to perfectly monitor the change in productivity. Hence the average household knows more than the Federal Reserve about firm productivity.

Nevertheless, we want to compare the dynamics of inflation, the output growth gap, and inflation expectations in this case to see whether it is more in line with the data. Under the hypothesis of rational expectations of the private sector, the model can be written as

$$V_t = D_1 + D_2 E^{cb}_{t-1} g_y + D_3 E^{ps}_{t-1} V_{t+1} + D_4 V_{t-1} + D_5 \epsilon_t \tag{81}$$

where the central bank and the private sector form different expectations. The central bank is assumed to be learning about the output growth rate and the long-run

[16] Again, in this version of the paper the stability of the system is not proven but is verified with simulations.

[17] This assumption was used by Lansing (2001).

nominal interest rate, using the same learning algorithm described in the previous section.

In order to find the rational forecast of the private sector we need a guess for the law of motion of the economy

$$V_t = \Omega_0^{ps} + \Omega_1^{ps} V_{t-1} + e_t \tag{82}$$

where e_t is perceived as an *i.i.d.* disturbance. Forecasts by the private sector are then given by

$$\begin{aligned} E_{t-1}^{ps} V_{t+1} &= \Omega_0^{ps} + \Omega_1^{ps} \left(\Omega_0^{ps} + \Omega_1^{ps} V_{t-1} \right) & (83) \\ &= \left(\Omega_0^{ps} + \Omega_1^{ps} \Omega_0^{ps} + \left(\Omega_1^{ps} \right)^2 V_{t-1} \right) & (84) \end{aligned}$$

Inserting (83) in (81) we find the actual law of motion

$$\begin{aligned} V_t = {} & D_1 + D_2 E_{t-1}^{cb} g_y + \\ & D_3 \left(\Omega_0^{ps} + \Omega_1^{ps} \Omega_0^{ps} + \left(\Omega_1^{ps} \right)^2 V_{t-1} \right) + D_4 V_{t-1} + D_5 \epsilon_t. \end{aligned} \tag{85}$$

Using the method of undetermined coefficients we obtain

$$\Omega_1^{ps^*} = \left(D_3 \Omega_1^{ps^*} \right)^2 + D_4, \tag{86}$$

and

$$\Omega_0^{ps} = D_1 + D_2 E_{t-1}^{cb} g_y + D_3 \left(\Omega_0^{ps} + \Omega_1^{ps} \Omega_0^{ps} \right), \tag{87}$$

which gives

$$\Omega_0^{ps^*} = \left(I - D_3 - D_3 \Omega_1^{ps^*} \right)^{-1} \left(D_1 + D_2 E_{t-1}^{cb} g_y \right), \tag{88}$$

where the (*) indicates the rational expectations solution coefficients.

The expression (86) is a matrix of coefficients that is independent of the learning process of the central bank. But the matrix of constants, equation (88), depends on the central bank's estimates of the long run growth rate of output. In fact, rational expectations on the part of the private sector implies not only perfect information about the productivity change but also perfect information about the mistakes of the central bank. The actual law of motion of the economy under real time learning is described by the following equation

$$\begin{aligned} V_t = {} & D_1 + D_2 E_{t-1}^{cb.} g_{y,t-1} + \\ & D_3 \left(\Omega_{0t-1}^{ps*} + \Omega_1^{ps*} \Omega_{0t-1}^{ps*} + \left(\Omega_1^{ps*} \right)^2 V_{t-1} \right) + D_4 V_{t-1} + D_5 \epsilon_t. \end{aligned} \tag{89}$$

The dynamics of the economy are determined by (89) and by the learning algorithm of the central bank.

4. QUANTITATIVE DYNAMICS IN THE BASELINE ECONOMY

4.1. The baseline economy

We have outlined several possible versions of the model. In in order to organize the discussion, we will begin by presenting results for a baseline case. We think of the baseline economy as having two-sided learning—both the central bank and the private sector learn. We assume the simple learning method (as opposed to the model consistent method) for the central bank, and we use the calibration given below. Later in the paper, we consider variations on this baseline and show how results are affected.

4.2. Calibration

We follow Woodford (2002) for the calibration of the following parameters of the model to quarterly data. We assume the disutility from labor to be nearly linear, assigning $\nu = 0.11$, and we set $a = 1.25$. We also calibrate the discount factor $\beta = 0.994$. Concerning the production side, we calibrate the capital share in the production function $\alpha = 0.25$, the depreciation rate of capital $\delta = 0.012$, and the adjustment cost coefficient $\epsilon_{\psi} = 3$. Also, we set $\theta = 7.88$, which implies a mark-up about 15 percent. In contrast to Woodford (2002), we set the probability of not changing the price $\xi = 0.78$, which is in line with the literature but higher than Woodford's choice. This is a consequence of our assumption of homogeneous capital, and lowers the degree of persistence. Nevertheless, our parameterization implies that firms re-optimize their price every 4.5 quarters, which is not implausible. We stress that firms *do* change their prices every quarter in the model, even if not optimally at each date. These parameters imply that $\omega = 0.47$ and $\bar{\omega} = 0.08$.

We assume that the monetary authority uses the same Taylor-type policy rule for the whole sample. For our baseline simulations we choose a value of $\rho = 0.2$, in line with the estimate of Erceg and Levin (2001) for a similar rule. We set ϕ_{π} to 1.5, as in the standard Taylor rule and the coefficient on output growth, $\phi_y = 0.5$, consistent with choice of Woodford (2002) for a similar rule. This is also consistent with the assumption that the Federal Reserve emphasized output stabilization in the seventies (see, for instance, the evidence in Orphanides (2001)). We set the central bank's inflation target to 4 percent, the approximate level of inflation before the onset of the productivity slowdown.

We calibrate the change in productivity using the estimated trend under learning calculated by Bullard and Duffy (2002). They find a productivity break in the third quarter of 1973. Productivity growth falls (in annual rates) from 2.47 percent to 1.21 percent. They also estimate the growth rate of the labor force. For the period that we consider they find a constant labor force growth rate of 1.88 percent. This leads to a change in the aggregate output trend growth rate from 4.36 percent to 3.1 percent, at an annual rate, following the productivity slowdown. We also include an increase in productivity growth (the "new economy") beginning in the third quarter

Preferences and technology								Gain
ν	β	ξ	α	δ	ϵ_ψ	θ	a	ζ
0.11	0.994	0.78	0.25	0.012	3.0	7.88	1.25	0.03

Policy rule			Growth factors			
ρ	ϕ_π	ϕ_y	γ_0	γ_1	γ_2	η
0.2	1.5	0.5	1.00612	1.00301	1.00462	1.00467

TABLE 1
The benchmark calibration.

of 1993. Bullard and Duffy (2002) estimate an increase in productivity growth to an annual rate of 1.86 percent at that date. This implies an output growth rate of 3.75 percent. The corresponding quarterly values for γ_i, $i = 0, 1, 2$ (corresponding to the gross rate of productivity growth before the productivity slowdown, after the productivity slowdown, and after the onset of the new economy) are given in Table 1.

We calibrate the gain ζ (assumed to be the same for both the central bank and the private sector) to 0.03. This yields a plausible speed of learning, implying that the central bank almost fully detects the productivity break by 1980.

Since our goal is to study the transitional behavior output growth and inflation, we follow the methodology of Bullard and Duffy (2002) and reduce the noise to almost zero (we append a shock with standard deviation 0.00001 to each equation).[18]

4.3. The response to a monetary policy shock under rational expectations

We checked the impulse-response of key variables to a monetary policy shock. Assuming rational expectations and using the baseline calibration for the economy, the model replicated almost exactly the impulse-response functions obtained by Woodford (2002) under the assumption of firm-specific capital.

4.4. Results for the baseline economy

4.4.1. Inflation

We first discuss the effects on output growth and inflation of an unexpected slowdown in productivity of the magnitude observed during the 1970s. We begin with the inflation process. As shown in Figure 2, inflation increases from 4 percent (the steady state) in 1970 to more than 7 percent in 1976—that is, the benchmark economy generates an increase in inflation peaking at more than 300 basis points in response to the productivity slowdown. For comparison purposes, we have plotted

[18] An alternative would be to include fundamental shocks to the economy, say to technology and monetary policy, simulate with changes in trend, and average the result. Of course, we would have to take a stand on the nature of these shocks. In this version of the paper we do not pursue this strategy.

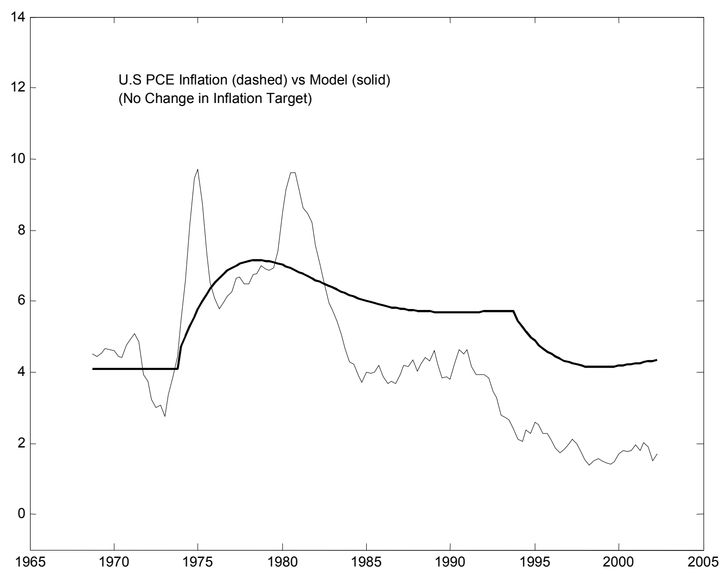


FIG. 2 The inflation dynamics in the baseline model versus the U.S. data. Inflation increases significantly in response to the productivity slowdown, but remains persistently high.

the core PCE inflation data from Figure 1.[19] The inflation rate stays higher after 1980 in the model than it does in the data. It does not fall sharply until the second productivity growth shock, which is the beginning of the "new economy" in 1993. The failure of inflation to fall sufficiently[20] depends in part on how aggressive the Taylor rule is concerning the output growth gap—that is, how much weight the central bank puts on the output growth component of the rule. Still, for any plausible parameterization we did not observe the dramatic fall in inflation observed in the data. We conclude that, according to this model, the productivity slowdown could have sparked much of the observed increase in inflation during the 1970s, but that, without other structural changes, the inflation rate would have stayed high for many more years.

If we take the view that the model is a reasonable approximation of the economy, this might be evidence that the policy rule changed in some way after 1979.[21] We consider one possible change, namely that the central bank changes the inflation

[19] This result is quite robust to changes in the choice of the gain ζ.

[20] Despite the appearances of the figure, it would eventually converge back to the steady state rate of 4 percent. But that process takes many years.

[21] Possible changes could be a downward shift in the inflation target, as in Huh and Lansing (1999) and Erceg and Levin (2001), or a change in the parameters of the Taylor rule. Orphanides (2001) suggests that the coefficient on the output gap has decreased after the 1980. Also, Clarida, Gali and Gertler (1999) find that the inflation coefficient is higher in the post-Volker sample. The model tells a story that is consistent with those facts.

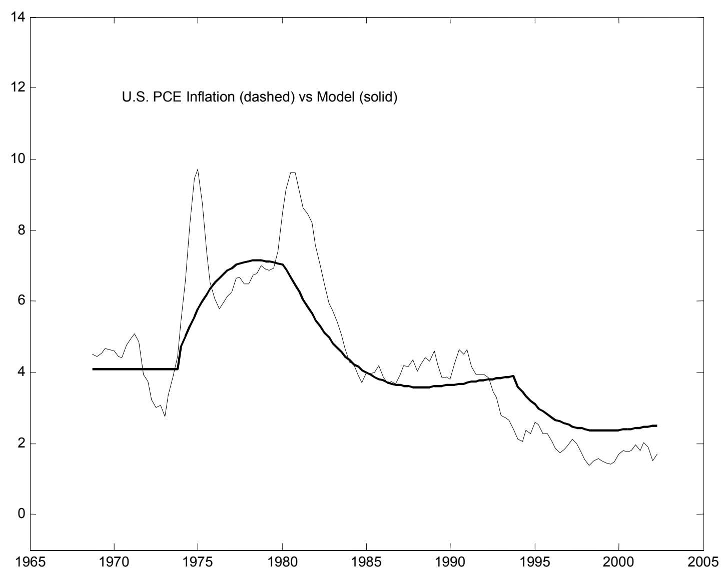


FIG. 3 If the central bank unexpectedly lowers the inflation target to two percent in 1980, the inflation dynamics begin to approximate the data quite well.

target to 2 percent in 1980. As Figure 3 demonstrates, this change helps capture aspects of the Volker disinflation.

4.4.2. Output growth

After the productivity slowdown, output growth decreases. Figure 4 (at the end of the paper) shows the real output growth trend implied by the baseline model with a change in the inflation target against the actual real GDP growth rates in the U.S. data. In general, the growth trend from the model tends to track the longer run behavior of output growth in the data quite well. Perhaps not surprisingly, actual output growth rates are quite volatile compared to the trends coming from the model.

4.4.3. Inflation expectations

The model captures the behavior of actual versus expected inflation surprisingly well, as shown in Figures 5 and 6. Figure 5 displays inflation and inflation expectations data for the U.S. The expected inflation series is expected GDP deflator inflation four quarters ahead as measured by the median projection from the Survey of Professional Forecasters. The inflation series is GDP deflator inflation (from NIPA). This inflation data is a different measure that what is shown in Figure 1, in order to keep the measured expectations matched with the actual inflation rate the professional forecasters were asked to predict. As is well known, inflation expectations were consistently below actual inflation until the early 1980s, and then stay consistently above during much of the remainder of the sample. Figure 6 shows inflation and inflation expectations from the model. These inflation expectations are in line with the data in the sense that inflation expectations are too low in the 1970s, and then too high in the remainder of the sample. Our theory suggests that this feature of the data is exactly what we should expect to observe if economic actors have to learn about structural changes in productivity.

4.4.4. Policy mistakes

Figure 7 shows the misperception of the output growth gap, on which the policy "mistakes" of the central bank are based. The central bank takes time to detect the decreased trend in output growth following the productivity slowdown. Hence, observing slow output growth it conjectures a lower output growth gap than in the actual data. This implies a lower interest rate and a higher inflation rate than would otherwise occur. This effect is reinforced by the higher inflation expectations of the private sector.

One way to think about how important the policy mistake is for the great inflation is to consider a case where only the private sector is learning. In this case the bank is assumed to actually observe the change in productivity, therefore setting the 'correct' interest rate according to the Taylor-type policy rule. We found that the resulting inflation is generally lower, but still, the productivity slowdown has a significant inflationary impact on the economy. In addition, the high inflation is persistent. This example suggests that, even without a policy mistake, by following a Taylor-type policy rule the central bank would not have avoided relatively high inflation during the 1970s. The results also suggest the possibility that the central bank may be able to stabilize inflation and output by responding not only to deviations of inflation and output from trend but also to a measure of private sector expectations.[22]

4.5. Only the central bank learns

Our results for the baseline economy are based on the assumption that both the private sector and the central bank are learning. In this subsection we consider the case where only the central bank learns, and the private sector has rational expectations, in order to assess how important it is for our results that we assume the private sector must learn about structural changes as well. It turns out that this version of the model does not match the data as well. First, under the assumption that the private sector fully adjusts to the decrease in the real rate implied by the

[22] This policy suggestion has been made by Evans and Honkapohja (2001), based on a different argument. They find that a policy rule that responds to expectations implies more robustness of the RE equilibrium to expectational errors. Here we suggest that by responding to private sector's expectations the bank may be able to stabilize inflation more effectively in the face of structural change.

productivity slowdown, the model predicts a *decrease* in inflation rather than an increase! This because the agents *know* that the central bank is using a constant which is too high in the Taylor rule. We also simulate the model assuming: 1) that the households have to estimate the real rate but otherwise they have rational expectations, and, 2) that both the bank and the private sector observe the change in the real interest rate. This latter assumption is common in the literature.[23]

Under these two latter assumptions, the model with only central bank learning predicts a smaller inflation, peaking about 100 basis points higher following the productivity slowdown. Given that the private sector is assumed to know the mistake of the central bank about the output growth gap, inflation expectations are predicted to be higher than actual inflation, which conflicts with the evidence presented in Figure 5. In addition, the model does not predict the downturn in the growth rate of output that is present in the data.

This leads us to the conclusion that the hypothesis of both the central bank and the private sector learning is the most appropriate to capture the behavior of the U.S. economy after the productivity slowdown.

4.6. Model consistent growth estimates by the central bank

In our baseline economy, the central bank uses the simple method to estimate a trend growth rate for the economy. In this subsection we describe how our findings change if the central bank uses the model-consistent estimator of the growth rate. First of all, and perhaps not surprisingly, such an estimator seems to be more efficient in estimating changes in the trend rate of output growth. In general, it takes less time to learn about most of the change in trend under the model consistent method. In addition, the inflation rate is lower through the whole sample. Nevertheless, as Figure 8 suggests, the long run behavior of the inflation process seems to be quite consistent with the data. This suggests that even a more efficient use of information than we have assumed in our baseline economy would not have avoided most of the inflation observed in the U.S. data.

5. DISCUSSION

We have analyzed the effects of permanent changes in productivity trends on inflation, when the economy's actors must learn about the changes in trend and the central bank is committed to using a Taylor-type policy rule. We find that a productivity slowdown of the magnitude observed in the 1970s causes a significant and persistent rise in inflation in the model economy, peaking at more than 300 basis points. A permanent increase in the rate of productivity growth—the "new economy" of the 1990s—then causes a reduction in inflation. These effects alone are not sufficient to provide a qualitative match to the U.S. inflation experience, because of the sharp decline in inflation observed in the early 1980s. However, by adding an unexpected reduction in the inflation target of the central bank,

[23] See, for example, Orphanides (2001).

we were able to provide a qualitative match for the data. We conclude that the misperceived-change-in-trend hypothesis has considerable merit in explaining the medium-run dynamics of inflation in the U.S. since 1970.

We think the policy conclusions from this exercise are quite important. Our analysis suggests that, should a shock of the magnitude of the productivity slowdown occur again in the future, it could generate a considerable inflation disturbance, even if policymakers do their best to remain committed to a Taylor-type policy rule and to estimate the changing growth trends in the economy. Thus, when evaluating Taylor-type policy rules, an additional criterion might be, How well does the rule insulate the economy in the event of structural change?

We have analyzed this economy under a hypothesis of rare, permanent shocks to productivity growth. We think this is a good characterization of the data based on our reading of the econometric literature concerning structural change. Agents protect themselves against the possibility of such a rare shock by employing a version of constant gain learning. Because the system is stable under learning, the agents can then adapt to the new rational expectations equilibrium following a structural break. We view this approach as one model-consistent method of addressing issues like this. There is another method, which we think is also interesting, but ultimately less satisfactory. That method retains the rational expectations assumption, and models the permanent shocks that appear to be in the data as a regime-switching process. Agents understand that there are two (or more) regimes, and rationally infer which regime they are in and how likely they are to transit to an alternative regime when making decisions. In this approach, the rational expectations assumption is completely consistent with the model, and the dynamics of the economy are completely characterized by a rational expectations equilibrium. Again, we think this is an interesting approach. The drawback is that the agents in the model must have specific alternative regimes in mind, along with the associated transition probabilities, when making decisions. Thus, as usual, the informational demands of the rational expectations assumption are stringent. In reality, there are many possible alternative regimes, most of which have rarely, if ever, occurred. Under the learning approach we have used here, the agents are in some sense prepared to adapt to *any* type of structural change that might occur in the economy, so long as it is not so disruptive as to destabilize the system. Therefore, we think the approach we have outlined here provides a reasonable modelling strategy.

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APPENDIX A: DERIVATION OF THE MARGINAL COST EQUATION

From the linearized equations we have

$$\tilde{S}_t(f) = \tilde{w}_t(f) - \alpha\tilde{k}_t(f) + \alpha\tilde{L}_t(f) \tag{90}$$

as well as

$$\tilde{R}_t^K = \tilde{w}_t(f) + \tilde{L}_t(f) - \tilde{k}_t(f), \tag{91}$$

$$\tilde{y}_t(f) = \alpha\tilde{k}_t(f) + (1-\alpha)\tilde{L}_t(f), \tag{92}$$

and

$$\tilde{w}_t(f) = \nu\tilde{L}_t(f) - \tilde{\lambda}_t. \tag{93}$$

Using the latter equations we obtain

$$\tilde{S}_t(f) = \omega(\tilde{y}_t(f) - \tilde{k}_t(f)) + \nu\tilde{k}_t(f) - \tilde{\lambda}_t, \tag{94}$$

where $\omega = \omega_p + \omega_w$, $\omega_p = \frac{\alpha}{1-\alpha}$, and $\omega_w = \frac{\nu}{1-\alpha}$. The average real marginal cost can thus be expressed as

$$\tilde{S}_t = \omega(\tilde{y}_t - \tilde{k}_t) + \nu\tilde{k}_t - \tilde{\lambda}_t. \tag{95}$$

Also, the equation for the real rental price of capital becomes

$$\tilde{R}_t^K = \omega_w \tilde{y}_t(f) + \tilde{c}_t + \frac{\omega_w}{\nu}\tilde{y}_t(f) - \nu\left(\frac{\omega_w}{\nu} - 1\right)\tilde{k}_t(f) - \omega_p\tilde{k}_t(f) - \tilde{k}_t(f). \tag{96}$$

Simplifying this expression we get

$$\begin{aligned} \tilde{R}_t^K &= \omega_w\left(\frac{\nu+1}{\nu}\right)\tilde{y}_t(f) - \left[\nu\left(\frac{\omega_w}{\nu} - 1\right) + \omega_p + 1\right]\tilde{k}_t(f) - \tilde{\lambda}_t \\ &= (\omega+1)\left(\tilde{y}_t(f) - \tilde{k}_t(f)\right) - \nu\tilde{k}_t(f) - \tilde{\lambda}_t \\ &= \rho_y\tilde{y}_t(f) - \rho_k\tilde{k}_t(f) - \tilde{\lambda}_t \end{aligned} \tag{97}$$

where $\rho_y = (\omega + 1)$ and $\rho_k = \rho_y - \nu$. Substituting this equation in (94) to eliminate the capital stock variable we can express the marginal cost in terms of firm f output and the average marginal cost, which is equation (34).

APPENDIX B: THE STEADY STATE

The real variables other than labor are expressed in stationary terms (i.e. per effective worker). We begin with

$$\bar{R}^K = \gamma\beta^{-1} - 1 + \delta, \tag{98}$$

where a bar indicates a steady state value. From the investment equation

$$\frac{\bar{I}}{\bar{k}} = \gamma\eta - 1 + \delta. \tag{99}$$

From the Euler equation we have

$$\gamma\beta^{-1} = \frac{1+i}{\bar{\Pi}}. \tag{100}$$

From the evolution of the price index we know that

$$P_t = \left[(1-\xi)\left(P_t^*\right)^{1-\theta} + \xi\left(\bar{\pi}P_{t-1}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{101}$$

Dividing by P_t and and rearranging, the steady state value of p^* is

$$\bar{p}^* = \left[\frac{1-\xi}{1-\xi}\right]^{\frac{1}{1-\theta}} = 1, \tag{102}$$

while the real marginal cost is equal to

$$\bar{s} = \frac{\theta - 1}{\theta}. \tag{103}$$

Form the firm's first order condition, we obtain the capital-labor ratio

$$\frac{\bar{k}}{\bar{L}} = \left(\frac{\alpha\bar{s}}{\gamma\beta^{-1} - 1 + \delta}\right)^{\frac{1}{1-\alpha}} \tag{104}$$

Also, the output-labor ratio can be found from the production function

$$\frac{\bar{y}}{\bar{L}} = \left(\frac{\alpha s}{\gamma\beta^{-1} - 1 + \delta}\right)^{\frac{\alpha}{1-\alpha}}, \tag{105}$$

which implies an inverse relation between productivity and the output-labor ratio. Then $\frac{\bar{y}}{\bar{L}}\frac{\bar{L}}{\bar{k}}$ gives

$$\frac{\bar{y}}{\bar{k}} = \frac{\gamma\beta^{-1} - 1 + \delta}{\alpha\bar{s}}. \tag{106}$$

Also, from the household first order condition we obtain

$$\bar{L} = \left[\frac{\bar{w}}{a\bar{c}}\right]^{\frac{1}{\nu}}, \tag{107}$$

where

$$\bar{w} = (1-\alpha)\bar{s}\left(\frac{\alpha\bar{s}}{\gamma\beta^{-1} - 1 + \delta}\right)^{\frac{\alpha}{1-\alpha}}, \tag{108}$$

and, from the resource constraint

$$\frac{\bar{c}}{\bar{L}} = \frac{\bar{y}}{\bar{L}} - \frac{\bar{k}}{\bar{L}}\frac{\bar{I}}{\bar{k}} \tag{109}$$

$$= \left(\left(\frac{\alpha s}{\gamma\beta^{-1} - 1 + \delta}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\alpha s}{\gamma\beta^{-1} - 1 + \delta}\right)^{\frac{1}{1-\alpha}}(\gamma\eta - 1 + \delta)\right). \tag{110}$$

Inserting this expression into (107) to substitute for $\bar{c}$ and rearranging we get:

$$\bar{L} = \left[\frac{(1-\alpha)\bar{s}\left(\frac{\alpha\bar{s}}{\gamma\beta^{-1}-1+\delta}\right)^{\frac{\alpha}{1-\alpha}}}{a\left(\frac{\alpha s}{\gamma\beta^{-1}-1+\delta}\right)^{\frac{\alpha}{1-\alpha}}\left[1 - \alpha\bar{s}\left(\frac{\gamma\eta-1+\delta}{\gamma\beta^{-1}-1+\delta}\right)\right]}\right]^{\frac{1}{\nu+1}} \tag{111}$$

$$= \left[\frac{(1-\alpha)\bar{s}}{a\left[1 - \alpha\bar{s}\left(\frac{\gamma\eta-1+\delta}{\gamma\beta^{-1}-1+\delta}\right)\right]}\right]^{\frac{1}{\nu+1}}. \tag{112}$$

From this last equation, it is easy to check that, provided $\beta^{-1} < \eta$ (which is verified in our parameterization), a *decrease* in productivity leads to a *increase* in total labor. The steady state output per effective worker is:

$$\bar{y} = \left(\frac{\alpha s}{\gamma\beta^{-1} - 1 + \delta}\right)^{\frac{\alpha}{1-\alpha}}\left[\frac{(1-\alpha)\bar{s}}{a\left[1 - \alpha\bar{s}\left(\frac{\gamma\eta-1+\delta}{\gamma\beta^{-1}-1+\delta}\right)\right]}\right]^{\frac{1}{\nu+1}}$$

$$= \left[\frac{[(1-\alpha)\bar{s}]^{\nu+1}\left(\frac{\alpha s}{\gamma\beta^{-1}-1+\delta}\right)^{\frac{\alpha(\nu+1)}{1-\alpha}}}{a\left[1 - \alpha\bar{s}\left(\frac{\gamma\eta-1+\delta}{\gamma\beta^{-1}-1+\delta}\right)\right]}\right]^{\frac{1}{\nu+1}}.$$

Because both $\bar{L}$ and $\bar{y}/\bar{L}$ are negatively related to productivity, $\bar{y}$ is also negatively related to productivity. Hence, a productivity slowdown will increase $\bar{y}$. Finally, we define $\sigma = \bar{c}/\bar{y}$ which is obtained from $\frac{\bar{c}}{\bar{L}} \left(\frac{\bar{y}}{\bar{L}} \right)^{-1}$.

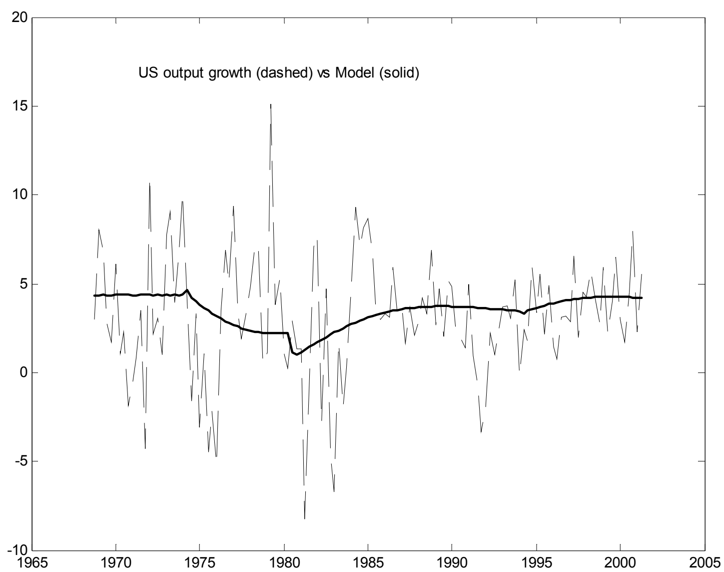

FIG. 4 The output growth trend from the benchmark model with an inflation target change in 1980, as compared to output growth rates in the data. Not surprisingly, output growth rates are highly variable in the data relative to the model.

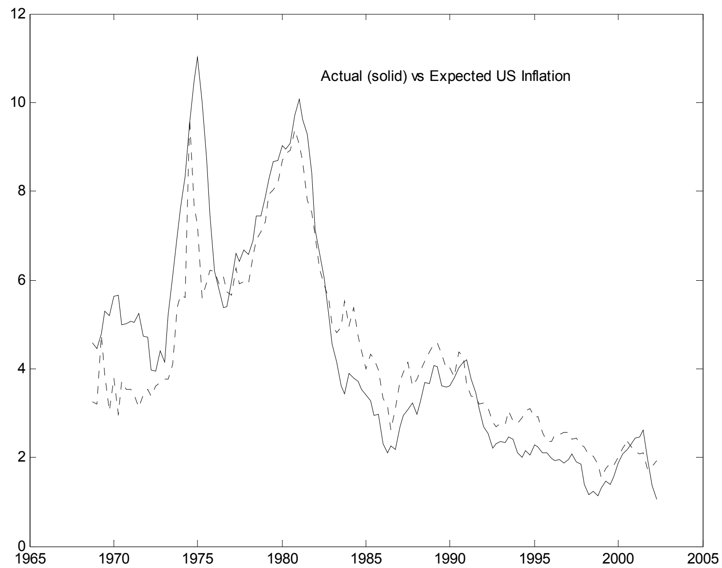

FIG. 5 Actual versus expected inflation in the U.S. data. As is well-known, expectations appear to "lag behind" inflation.

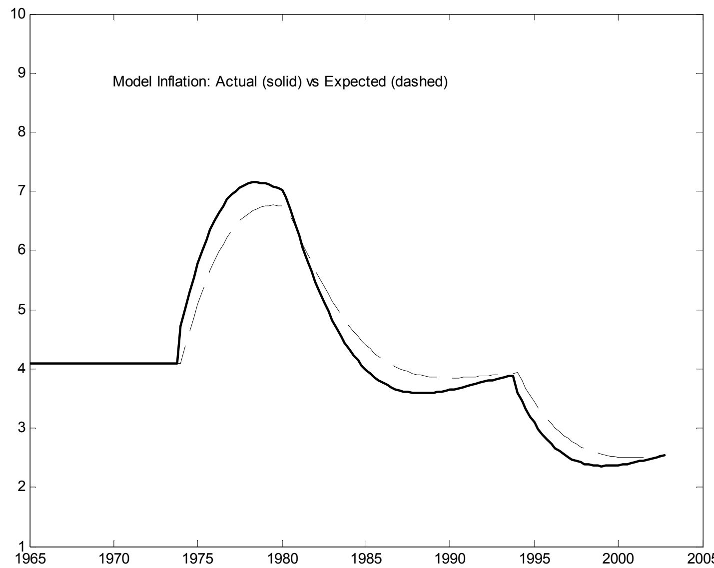

FIG. 6 Actual versus expected inflation in the benchmark model with an inflation target change in 1980. Expectations tend to be too low in the 1970s and too high later in the sample, consistent with the data as shown in Figure 5. The model suggests that this is what one should expect to observe when households are learning about structural productivity changes.

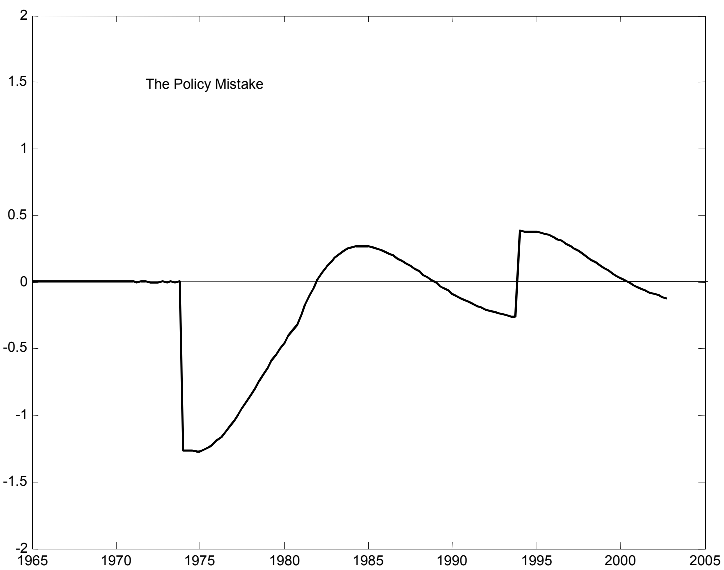

FIG. 7 The policy "mistakes" of the central bank are based on misperceptions of the output growth gap.

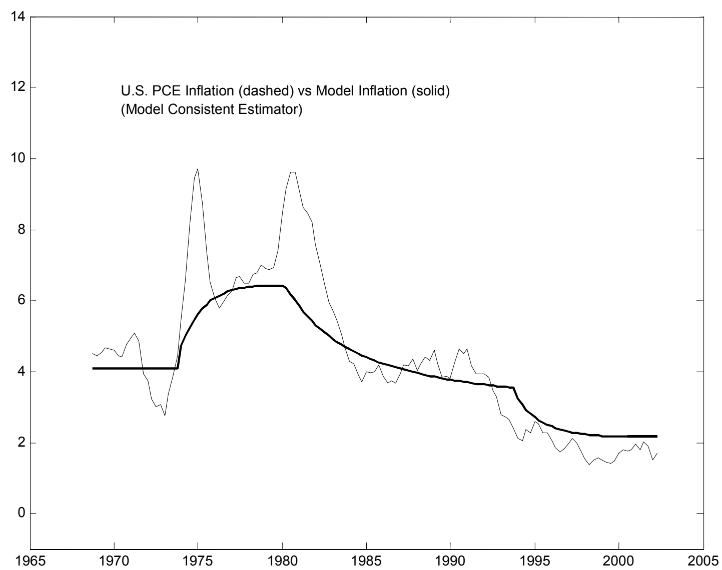

FIG. 8 The behavior of inflation in the model versus the U.S. data in the benchmark economy when the central bank uses a model-consistent estimator of the output growth rate. This example includes the reduction in the inflation target in 1980. The inflation performance is largely the same even when the central bank uses more information.