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Price Distributions and Competition

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Abstract: Considerable evidence demonstrates that significant dispersion exists in the prices charged for seemingly homogeneous goods. This paper adopts a simple, flexible equilibrium model of search to investigate the way the market structure influences price dispersion. Using the noisy search approach, the paper demonstrates the effects of having a single large, price-leading firm with multiple outlets and a competitive fringe of small firms with one retail outlet each.

JEL classification: D40, L7

Key words: price dispersion, consumer search, market structure

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Empirical research has established that there is significant dispersion in prices charged by retail outlets for seemingly homogeneous goods.¹ Indeed, price dispersion appears to be pervasive in that it is observed in markets with very different structures. The objective here is to investigate the way in which the underlying structure of the market can affect equilibrium price distributions.

A relatively large literature attempts to explain how price distributions can arise as part of a market equilibrium. In search models², a large number of firms post prices and consumers pay a cost to observe the prices offered. Most contributions in this literature generate non-degenerate equilibrium price distributions by assuming that firms and/or consumers differ in some way. (See, for example, Reinganum, 1976; Varian, 1980; Carlson and McAfee, 1983; Rob, 1985; Stahl, 1989; Spulber 1989; Jansen and Moraga 2004.) These models have led to new and important insights. Nevertheless, their particular nature makes them difficult to generalize to other environments.

The Burdett and Judd (1983) model of noisy search differs somewhat. When both consumers and firms are homogeneous, the unique equilibrium is characterized by a continuous distribution of price offers. This paper illustrates the flexibility of Burdett-Judd approach. We consider a market structure with one large firm having multiple outlets and a competitive fringe consisting of many small firms each with a single outlet. Single outlet firms offer a continuous distribution of prices, and the multiple outlet firm, the recognized market leader, offers the highest price in the market. Not surprisingly, market power generates higher average prices, making consumers worse off. Prices at the multiple outlet firm tend to be higher than prices charged if there were only single outlet firms. The high price from the market leader confers benefits

¹See, for example, Stigler, (1961); Pratt, Wise and Zeckhauser, (1979); Carlson and Pescatrice, (1980); Villas-Boas, (1995); Warner and Barsky, (1995); and Sorensen, (2000). This dispersion, somewhat surprisingly, appears to persist despite the recent rise of e-commerce. Although consumers with a connected computer can now observe a huge number of prices for any particular good at a relatively small cost in time and energy, evidence to date demonstrates that price dispersions for similar goods sold on the web are significant and stable. (See, for example, Brynjolfsson and Smith, 2000; Baye and Morgan, 2001; and Baye, Morgan and Scholten, 2001.)

²So called clearing house models offer an alternative approach to price dispersion.

on all outlets.

The market structure considered here has become part of the standard ‘tool-kit’ in industrial organization (Carlton and Perloff, 1999, p. 63-65, 107-118). There are, of course, a wide variety of additional settings and topics. The more general message here is that the broad framework offers a flexible and robust platform on which to study many other classical problems in industrial organization.

The Framework

Suppose there is a fixed continuum of consumers and retail outlets. One large firm has proportion θ of all outlets and many small firms have a single outlet each. All outlets sell a homogeneous, indivisible good. A consumer who purchases at price p , enjoys utility $z - p$. Before the market opens, each firm posts a price which cannot be changed once set. The marginal cost of production is constant. Without loss of generality, assume marginal cost is zero.

All large firm outlets offer the same price - x . Let $F(p)$ denote the probability a single outlet firm offers a price no greater than p . Further, let p_ℓ and p_h indicate the infimum and supremum of the support of F with $z \geq p_h \geq p_\ell \geq 0$ for any F .

As distributions with mass points will be considered, the following notation convention is used. Given distribution function F , assume for any p

$$F(p) = \lim_{\varepsilon \rightarrow 0, \varepsilon > 0} F(p - \varepsilon) + \mu(p)$$

where $\mu(p)$ indicates the mass of firms offering price p , if such a mass exists. If $\mu(p) = 0$ for all p , the distribution is said to be continuous.

Consumers search over prices to maximize expected utility from purchasing the good net of search costs, $c > 0$. A consumer observes one offer with probability $1 - \alpha$ and two offers with probability α . An observed price is the realization of an independent draw from all prices on offer. There is no recall of previously rejected offers. Neither firms nor consumers discount the future.

This type of search has been termed noisy search and can be motivated as follows. In an attempt to find a low price, the consumer pays a cost to observe a price, say by visiting a store. After visiting this store and observing the price, with probability

α this consumer meets a friend who has contacted another store and observed a different price. If this meeting occurs, the friends exchange information by informing each other of the price offered and location of each store visited. The consumer then either purchases from one of these two stores, or pays c and searches again. If a friend is not contacted, the consumer either purchases from the store visited, or search continues.

Single Outlet Firms

A given outlet cannot influence the number of buyers it contacts - consumers search randomly. Hence, a firm maximizes expected profits by maximizing expected profit per consumer visit. Suppose for the moment all firms expect consumers to use effective reservation price $R > 0$. Each single outlet firm takes as given three objects - the price offered by the multiple outlet firm, x , the effective reservation price of consumers, R , and the distribution of prices made by other single outlet firms, F . Given random search, the firm's expected profit per potential customer when it offers price $p \leq R$ can be written as

$$\pi(p|R, x, F) = \begin{cases} p[\gamma + (1 - \gamma)(1 - \theta)(1 - F(p) + \mu(p)/2) + (1 - \gamma)\theta] & \text{for } p < x \\ p[\gamma + (1 - \gamma)(1 - \theta)(1 - F(p) + \mu(p)/2) + (1 - \gamma)\theta/2] & \text{for } p = x \\ p[\gamma + (1 - \gamma)(1 - \theta)(1 - F(p) + \mu(p)/2)] & \text{for } p > x \end{cases} \quad (1)$$

where γ denotes the probability the consumer contacted is one who has only observed one price that period. Note that $\pi(p|R, x, F) = 0$, for $p > R$.

A market equilibrium is defined later. Nevertheless, it will be useful to specify an important element of it at this point. In an equilibrium we require that for given (R, x) , $0 < x \leq R$, there exists a single outlet price distribution (*SPD*), F , such that

$$\pi(p|R, x, F) \begin{cases} = \bar{\pi} \geq 0 & \text{for all } p \text{ on support of } F \\ < \bar{\pi} & \text{otherwise} \end{cases} \quad (2)$$

It is possible to restrict the class of distributions functions that can be a *SPD*.

Claim 1: *Given (R, x) , $0 < x \leq R$, if F is a *SPD*, then F is continuous.*

Proof: $R > 0$ implies that for any given F , when $0 < \alpha < 1$, firms can obtain strictly positive expected profits per customer. Hence, no firm charges $p = 0$.

Assume in equilibrium F has a mass at $p' > 0$. In this case

$$\pi(p'|R, F) = p'\gamma + p'(1 - \gamma)[(1 - F(p')) + \mu(p')/2]$$

Clearly, if a firm offers price $p' - \varepsilon$, $\varepsilon > 0$, then

$$\pi(p' - \varepsilon|R, F) \geq (p' - \varepsilon)\gamma + (p' - \varepsilon)(1 - \gamma)[(1 - F(p')) + \mu(p')]$$

It follows from inspection that $\pi(p' - \varepsilon|R, F) > \pi(p'|R, F)$ for small enough $\varepsilon > 0$ and therefore a contradiction completes the proof. ■

Given Claim 1, (1) can be written as

$$\pi(p|R, x, F) = \begin{cases} p[\gamma + (1 - \gamma)(1 - \theta)(1 - F(p)) + (1 - \gamma)\theta] & \text{for } p < x \\ p[\gamma + (1 - \gamma)(1 - \theta)(1 - F(p)) + (1 - \gamma)\theta/2] & \text{for } p = x \\ p[\gamma + (1 - \gamma)(1 - \theta)(1 - F(p))] & \text{for } p > x \end{cases} \quad (3)$$

The object now is to demonstrate that for given (R, x) , $0 < x \leq R$, there exists a unique SPD . Claim 2 first establishes that at any SPD the supremum of the support is either, $p_h = R$, or $p_h = x$. This result along with (3) can be used to establish the expected profits at any SPD .

Claim 2: *Given (R, x) , $0 < x \leq R$, suppose $F(\cdot|x, R)$ is the associated SPD (provided one exists).*

- (a) *If $F(x|x, R) < 1$ and $x < R$, i.e., if a positive measure of single outlet firms offer a price greater than x , then $p_h = R$ and the expected profits of all single outlet firms is γR .*
- (b) *If $F(x|x, R) > 0$, i.e., if a positive measure of single outlet firms offer a price no greater than x , then $F(x|x, R) - F(x - \varepsilon|x, R) > 0$ for all $\varepsilon > 0$ small enough. Further, the expected profits of all single outlet firms is*

$$x[\gamma + (1 - \gamma)(1 - \theta)(1 - F(x|x, R)) + (1 - \gamma)\theta].$$

Proof: Given $F(x|x, R) < 1$, suppose $p_h < R$. As $F(\cdot|x, R)$ must be continuous, the expected profits of a firm offering p_h is γp_h . However, a firm offering price p such that

$p_h < p \leq R$ obtains expected profits $\gamma p > \gamma p_h$. This leads to a contraction thereby establishing (a).

Given $F(x|x, R) > 0$, suppose there exists a $p_1 < x$ such that $F(p_1) = F(x)$, i.e., no single outlet firm offers a price $p_1 < p \leq x$. The expected profits of a firm offering price p , where $p_1 < p \leq x$, can be written as

$$\pi(p|x, R, F(\cdot|x, R)) = \begin{cases} p[\gamma + (1 - \gamma)(1 - \theta)(1 - F(x|x, R)) + (1 - \gamma)\theta] & \text{for } p_1 < p < x \\ p[\gamma + (1 - \gamma)(1 - \theta)(1 - F(x|x, R)) + (1 - \gamma)\theta/2] & \text{for } p = x \end{cases}$$

It follows immediately that $\pi(p|x, R, F(\cdot|x, R))$ is strictly increasing in p when $p_1 < p < x$. This implies there is a contradiction establishing the first part of (b). It also follows that

$$\pi(p|R, x, F(\cdot|x, R)) \rightarrow x[\gamma + (1 - \gamma)(1 - \theta)(1 - F(x|x, R)) + (1 - \gamma)\theta]$$

as $p \rightarrow x$. Hence, because $F(\cdot|x, R)$ is a *SPD* all firms must make these expected profits. ■

The next Claim establishes that given R ($0 < R \leq z$), there exists two numbers, $\underline{x}(R)$ and $\bar{x}(R)$, such that (a) if $x < \underline{x}(R)$ then at a *SPD* all single outlet firms offer a price more than x and (b) if $x > \bar{x}(R)$, then at a *SPD* all single outlet firms offer a price less than x . This Claim further fully characterizes the unique *SPD* for given R and x in both case (a) and case (b).

Claim 3:

(a) Given R such that $0 < R \leq z$, define $\underline{x}(R)$ by

$$\underline{x}(R) = \gamma R \tag{4}$$

If $x < \underline{x}(R)$, there is a unique *SPD*, $F(\cdot|x, R)$, such that $F(x|x, R) = 0$, i.e., all single outlet firms offer a price greater than x . Further,

$$F(p|x, R) = 1 - \frac{\gamma(R - p)}{(1 - \gamma)(1 - \theta)p} \tag{5}$$

with support $[p_\ell, R]$ such that

$$p_\ell = \frac{\gamma}{(1 - \gamma)(1 - \theta) + \gamma} R \tag{6}$$

(b) Given R such that $0 < R \leq z$, define $\bar{x}(R)$ by

$$\bar{x}(R) = \frac{\gamma R}{\gamma + \theta(1 - \gamma)} > \underline{x}(R) \quad (7)$$

If $x > \bar{x}(R)$, there is a unique SPD, $F(\cdot|x, R)$, that implies $F(x|x, R) = 1$, i.e., all single outlet firms offer a price less than x . Further,

$$F(p|x, R) = 1 - \frac{(x - p)[\gamma + (1 - \gamma)\theta]}{p(1 - \gamma)(1 - \theta)} \quad (8)$$

with support $[p_\ell, x]$, where

$$p_\ell = x[\gamma + (1 - \gamma)\theta] \quad (9)$$

Proof: Given (x, R) such that $0 < x < R \leq z$, suppose all single outlet firms offer a price greater than x . From Claim 2(a) it follows that all firms make expected profits γR . Suppose a deviant single outlet firm offers a price $p < x$. Such a firm sells to all consumers who visit and therefore obtains expected profits (per potential customer) $\pi(p|x, R, F(\cdot|x, R)) = p$ for $p < x$. Hence, the deviation is not profitable if $x < \underline{x}(R)$, where $\underline{x}(R)$ is defined in (4). Given $x < \underline{x}(R)$, equal expected profits for all single outlet firms yields

$$\gamma R = p[\gamma + (1 - \gamma)(1 - \theta)(1 - F(p))]$$

Solving implies that $F(\cdot|x, R)$ satisfies (5) and (6).

Given (x, R) such that $0 < x < R \leq z$, suppose all single outlet firms offer a price less than x . From Claim 2(b), it follows that if F is a SPD, then all single outlet firms make expected profits $x[\gamma + (1 - \gamma)\theta]$. When will a firm want to deviate and offer a price greater than x ? Given $x < R$, a firm that offers a price greater than x will offer price R and obtain expected profits γR . Hence, no single outlet firm will select to deviate and offer a price greater than x if $x > \bar{x}(R)$, where $\bar{x}(R)$ is defined in (7). Given $x > \bar{x}(R)$, equal expected profits for all single outlet firms yields

$$p[\gamma + (1 - \gamma)(1 - \theta)(1 - F(x|x, R)) + (1 - \gamma)\theta] = x[\gamma + (1 - \gamma)\theta]$$

for all p on the support of $F(\cdot|x, R)$. Solving implies $F(\cdot|x, R)$ satisfies (8) and (9). ■

The next step is to characterize the unique *SPD* for when the multiple outlet firm charges intermediate prices, i.e., for x between $\underline{x}(R)$ and $\bar{x}(R)$. For such an intermediate x , single outlet firms not surprisingly distribute themselves on either side of the multiple outlet price. However, this distribution contains a gap. There exists a range of prices immediately above x which lead to lower than equilibrium expected profits so that no single outlet firm sets its price in this region.

Claim 4: For fixed $R > 0$ ($0 < R \leq z$), if $x \in (\underline{x}(R), \bar{x}(R))$, then there exists a unique *SPD*, such that

$$\phi(x, R) = \frac{\gamma R - x[\gamma + (1 - \gamma)\theta]}{x(1 - \gamma)(1 - \theta)} \quad (10)$$

indicates the proportion of single outlet firms offering a price greater than x . Further, the unique *SPD*, $F(\cdot|x, R)$, can be written as

$$F(p|x, R) = \begin{cases} \frac{p - \gamma R}{p(1 - \gamma)(1 - \theta)} & \text{for } p_l < p \leq x \\ F(x|x, R) & \text{for } x < p \leq \tilde{p}(x, R) \\ 1 - \frac{(R - p)\gamma}{p(1 - \gamma)(1 - \theta)} & \text{for } \tilde{p}(x, R) < p \leq R \end{cases} \quad (11)$$

where

$$\tilde{p}(x, R) = \frac{\gamma R}{\gamma + (1 - \gamma)(1 - \theta)\phi(x, R)} > x \quad (12)$$

and

$$p_l = \gamma R \quad (13)$$

Proof: Given R ($0 < R < z$), if $\underline{x}(R) < x < \bar{x}(R)$, then we require the expected profits of single outlet firms offering a price greater than x to be the same as that obtained by those offering a price less than x , i.e.,

$$\gamma R = x[\gamma + (1 - \gamma)(1 - \theta)\phi(x, R) + (1 - \gamma)\theta] \quad (14)$$

Solving implies $\phi(x, R)$ must satisfy (10). For those single outlet firms offering a price greater than x , equal profits implies

$$\gamma R = p[\gamma + (1 - \gamma)(1 - \theta)(1 - F(p|x, R))]$$

for all $p > x$ on the support of $F(\cdot|x, R)$. As

$$\phi(x, R) = 1 - F(x|x, R) = 1 - F(\tilde{p}|x, R)$$

this yields $\tilde{p}(x, R)$ in (12). Given γR in (14) it follows that $\tilde{p}(x, R) > x$.

For those single outlet firms offering a price less than x , equal profits implies

$$\gamma R = p[\gamma + (1 - \gamma)(1 - \theta)(1 - F(p|x, R)) + (1 - \gamma)\theta]$$

for all $p < x$ on the support of $F(\cdot|x, R)$. Solving for F and then again for p_l generates the form of the unique *SPD* as presented in (11) and (13). Note, $F(p|x, R) = F(x|x, R)$ for all $p \in [x, \tilde{p}(x, R)]$. ■

The Multiple Outlet Firm

The multiple outlet firm takes as given the reservation price of consumers, R , and the response of single outlet firms as represented by the family of *SPDs* $\{F(\cdot|x, R)\}_{0 < x \leq R}$. Whatever price the multiple outlet firm charges, x , the response by single outlet firms can be described by the unique *SPD*, $F(\cdot|x, R)$. This specification in effect makes the multiple outlet firm a market leader that sets its price before single outlet firms.

At a particular outlet, the multiple outlet firm's expected profits per potential customer, given R and the family $\{F(\cdot|x, R)\}_{0 < x \leq R}$, is

$$\pi^m(x, R) = \begin{cases} x[\gamma + (1 - \gamma)(1 - \theta) + (1 - \gamma)\theta/2] & \text{for } x < \underline{x}(R) \\ x[\gamma + (1 - \gamma)(1 - \theta)\phi(x, R) + (1 - \gamma)\theta/2] & \text{for } \underline{x}(R) \leq x < \bar{x}(R) \\ x[\gamma + (1 - \gamma)\theta/2] & \text{for } \bar{x}(R) \leq x \leq R \end{cases} \quad (15)$$

where again $\phi(x, R)$ equals the proportion of single outlet firms offering a price greater than x . It follows

$$\frac{\partial \pi^m(x, R)}{\partial x} = \begin{cases} \gamma + (1 - \gamma)(1 - \theta) + (1 - \gamma)\theta/2 & \text{for } x < \underline{x}(R) \\ \gamma + (1 - \gamma)(1 - \theta)\phi(x, R) + (1 - \gamma)\theta/2 \\ \quad + x(1 - \gamma)(1 - \theta)\frac{\partial \phi(x, R)}{\partial x} & \text{for } \underline{x}(R) < x < \bar{x}(R) \\ \gamma + (1 - \gamma)\theta/2 & \text{for } \bar{x}(R) \leq x \leq R \end{cases} \quad (16)$$

Note, if $x < \underline{x}(R)$ or $x > \bar{x}(R)$, the above derivative is strictly positive. Further, if $\underline{x}(R) < x < \bar{x}(R)$, then substituting (10) and its derivative into (16) yields

$$\frac{\partial \pi^m(x, R)}{\partial x} = \frac{2\gamma R}{x} - \frac{\theta(1 - \gamma)}{2}$$

Clearly, $\partial^2 \pi^m(x, R)/\partial x^2 < 0$. At both $x = \underline{x}(R)$ and $x = \bar{x}(R)$, $\partial \pi^m(x, R)/\partial x$ is strictly positive and therefore $\partial \pi^m(x, R)/\partial x > 0$ for all $\underline{x}(R) < x < \bar{x}(R)$. It follows from inspection that the multiple outlet firm will maximize its profits by offering price R .

If n consumers are searching for a low price, then $[2\alpha + (1 - \alpha)]n = (1 + \alpha)n$ is the expected number of offers observed by consumers. It follows that $\gamma = (1 - \alpha)/(1 + \alpha)$ denotes the probability a firm contacts a consumer who only observes one price (given a contact is made). Therefore, in equilibrium the multiple outlet firms charge price R , and the distribution of prices charged by the single outlet firms is given by

$$F(p|R) = 1 - \frac{(x - p)[1 - \alpha + 2\alpha\theta]}{2p\alpha(1 - \theta)} \quad (17)$$

where the support of F is $[p_\ell, R]$

$$p_\ell = \frac{[1 - \alpha + 2\alpha\theta]}{(1 + \alpha)} R$$

Consumer Behavior

Before discussing consumer behavior, we first define an equilibrium

Definition: A market equilibrium is $(R^*, x^*, \{F(\cdot|x, R^*)\}_{0 < x \leq R^*})$, where

- (a) x^* maximizes the multiple outlet firm's expected profits given R^* and $\{F(\cdot|x, R^*)\}_{0 < x \leq R^*}$
- (b) the SPD, $F(\cdot|x^*, R^*)$, describes the optimal pricing behavior of single outlet firms, given x^* and R^*
- (c) R^* is the reservation price of any consumer, given x^* and $F(\cdot|x^*, R^*)$

So far we have shown that, given R , equilibrium conditions (a) and (b) are satisfied when the multiple outlet firm offers price R , and the equilibrium response of single outlet firms is given by the SPD, $F(\cdot|R, R)$. Each consumer takes F and x as given. Without loss of generality assume that (a) and (b) are satisfied and therefore consumers take R and $F(\cdot|R, R)$ as given for some R , where $0 < R \leq z$.

As search is random,

- (a) $(1 - \alpha)(1 - \theta)$ denotes the probability the consumer receives only one offer which is from a single outlet firm,
- (b) $2\alpha(1 - \theta)\theta$ denotes the probability the consumer receives one offer from a single outlet firm and one from the multiple outlet firm; and
- (c) $\alpha(1 - \theta)^2$ denotes the probability the consumer receives two independent offers each from a single outlet firm.

Given the multiple outlet firm charges price R with probability one, with probability $(1 - \alpha)(1 - \theta) + 2\alpha\theta(1 - \theta)$ the consumers lowest price observed is a random draw from $F(\cdot|R, R)$, whereas $\alpha(1 - \theta)^2$ indicates the probability the lowest price observed is a random draw from $G = 1 - (1 - F)^2$. Hence, with probability

$$\kappa = 1 - [(1 - \alpha)(1 - \theta) + 2\alpha\theta(1 - \theta) + \alpha(1 - \theta)^2]$$

the lowest price observed is R . Suppose all firms expect consumers to use effective reservation price, R , the multiple outlet firm charges price R , and $F(\cdot|R, R)$ is the *SPD*. In this case the expected return to a consumer from search can be written as

$$\begin{aligned} V = & -c + [(1 - \alpha)(1 - \theta) + 2\alpha\theta(1 - \theta)] \left[\int_{p_\ell}^Q (z - p) dF(p|R, R) + (1 - F(Q|R, R))V \right] \\ & + \alpha(1 - \theta)^2 \left[\int_{p_\ell}^Q (z - p) dG(p|R, R) + (1 - G(Q|R, R))V \right] + \kappa \max\{V, z - R\} \end{aligned}$$

where Q is the reservation price used that satisfies $z - Q = V$. Integrating by parts and manipulating yields

$$\begin{aligned} V = & -c + [(1 - \alpha)(1 - \theta) + 2\alpha\theta(1 - \theta)] \int_{p_\ell}^Q F(p|R, R) dp + \alpha(1 - \theta)^2 \int_{p_\ell}^Q G(p|R, R) dp \\ & + (1 - \kappa)V + \kappa \max\{V, z - R\} \end{aligned}$$

As $\max\{V, z - R\} = V + \max\{Q - R, 0\}$, we have

$$\begin{aligned} c = & [(1 - \alpha)(1 - \theta) + 2\alpha\theta(1 - \theta)] \int_{p_\ell}^Q F(p|R, R) dp \\ & + \alpha(1 - \theta)^2 \int_{p_\ell}^Q G(p|R, R) dp + \kappa \max\{Q - R, 0\} \end{aligned}$$

for any R ($0 < R \leq z$). In equilibrium we require either $Q = R$, for some $R \leq z$, or $Q > R$, for all $R \leq z$. Define $\Gamma(R)$ by

$$\begin{aligned}\Gamma(R) &= [(1 - \alpha)(1 - \theta) + 2\alpha\theta(1 - \theta)] \int_{p_\ell}^R F(p|R, R) dp \\ &\quad + \alpha(1 - \theta)^2 \int_{p_\ell}^R F(p|R, R)[2 - F(p|R, R)] dp\end{aligned}$$

Substituting (8) for $F(\cdot|R, R)$ and performing the relevant integration establishes that

$$\Gamma(R) = \alpha(1 - \theta)^2 R$$

hence

$$R^* = \min\left\{z, \frac{c}{\alpha(1 - \theta)^2}\right\} \quad (18)$$

This implies we have a unique equilibrium. Using $\gamma = (1 - \alpha)/(1 + \alpha)$, the following Proposition summarizes what has been established above.

Proposition: *There exists a unique equilibrium in which (a) the multiple outlet firm offers price R^* , (b) R^* satisfies (18) and (c) the distribution of prices offered by single outlet firms can be written as*

$$F(p|R^*, R^*) = 1 - \frac{(R^* - p)[1 - \alpha(1 - 2\theta)]}{2p\alpha(1 - \theta)} \quad (19)$$

with support $[p_\ell, R]$ such that

$$p_\ell = \frac{1 - \alpha(1 - 2\theta)}{1 + \alpha} R^* \quad (20)$$

Further, consumers utilize reservation price R^* .

Discussion

Prices are strategic complements. A higher price at any particular outlet increases the expected payoff at those outlets with lower prices thereby generating a positive spillover among outlets. As potential competitors raise their prices, an outlet has an increased incentive to raise its own price.

The multiple outlet firm but not the single outlet firms takes advantage of its size in this situation. Realizing that with probability θ it will capture the spillover from

a higher price at one of its outlets, the multiple outlet firm has the incentive to push up the highest price found in the market (relative to the supremum price in the case where there are only single outlet firms and when $R < z$). The extent to which this occurs depends on the number of outlets operated by the multiple outlet firm. For $z > c/\alpha(1 - \theta)^2$, a larger proportion of outlets in one firm generates a higher price x at those outlets associated with that firm.

In the competitive fringe of single outlet firms, prices also increase with the proportion of outlets in one firm. The distribution of prices given θ stochastically dominates the distribution of prices given θ' if and only if $\theta < \theta'$, i.e. $F_\theta(p|R^*, R^*) < 0$. The relationship between the price distribution and θ decomposes into two complementary effects. Having more firms charging the highest price reduces competition in the competitive fringe and hence induces higher prices among these firms. In addition, if the supremum price increases as well, i.e. $z < c/\alpha(1 - \theta)^2$, fringe prices rise accordingly.

More interestingly, the range of prices is non-monotonic in θ . In particular, if $z > c/\alpha(1 - \theta)^2$, i.e. if there exists a sufficiently small θ , price dispersion (as measured either by the price range between the supremum and the infimum or by differences in percentile price as discussed above) increases with θ until $R = z$. In this case, as θ increases, the rise of the supremum outstrips the rise in the infimum. However, once θ is sufficiently large so that the reservation price is limited by the monopoly price, that is, $R = z$, a higher θ reduces dispersion. In this case, only the infimum increases with θ .

In equilibrium, because the multiple outlet firm but not single outlet firms is privately providing a public good (for all outlets) of higher prices, single outlet firms earn higher expected profits than outlets associated with the market leader. Further note that for $z < c/\alpha(1 - \theta)^2$ outlets, profit per potential customer at the outlets of the market leader are independent of θ . Increasing the number of outlets at the multiple firm outlet only benefits the remaining single outlet firms.

In this noisy search model, although having one large seller and many small sellers does not alter the fundamental source of price dispersion, it does influence pricing in interesting ways. More generally, this model illustrates that noisy search can be

adapted to address in a consistent manner issues fundamental to industrial economics. For example, the approach appears well suited for including advertising, the adoption of new technologies, and market entry and exit. The approach could likewise be expanded to other areas such as international trade, as in Alessandria (2004).

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