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Mimicking Portfolios, Economic Risk Premia, and Tests of Multi-beta Models

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Abstract: This paper considers two alternative formulations of the linear factor model (LFM) with nontraded factors. The first formulation is the traditional LFM, where the estimation of risk premia and alphas is performed by means of a cross-sectional regression of average returns on betas. The second formulation (LFM*) replaces the factors with their projections on the span of excess returns. This formulation requires only time-series regressions for the estimation of risk premia and alphas. We compare the theoretical properties of the two approaches and study the small-sample properties of estimates and test statistics. Our results show that when estimating risk premia and testing multi-beta models, the LFM* formulation should be considered in addition to, or even instead of, the more traditional LFM formulation.

JEL classification: G12

Key words: mimicking portfolios, economic risk premia, multi-beta models

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Introduction

The broad question that this paper wants to address is: How should we construct mimicking portfolios? Implicit in the practice of running cross-sectional regressions (CSR) of returns or average returns on betas (e.g. see Black, Jensen, and Scholes, 1972; and Fama and MacBeth, 1973) there is the construction of unit-beta portfolios, i.e. portfolios with a beta of unity w.r.t. a specific factor and a beta of zero w.r.t. the other relevant factors. Similarly, when returns are regressed against other "characteristics," the CSR coefficients are the returns on portfolios with a specific characteristic of one, and all other characteristics of zero. An alternative approach is that of projecting the factors on the span of returns augmented with a constant. The projection coefficients are the weights of portfolios whose returns have maximum (squared) correlations with the factors. Hence, these portfolio weights are proportional to the hedging-portfolio weights of Merton (1973).[1] Other examples of this second approach are Breeden (1979), Breeden, Gibbons, and Litzenberger (1989), and Lamont (2001).

A related question is: How should we estimate and test linear factor models (LFM), when the factors are economic non-traded factors? In the traditional CSR approach, the average excess returns generated by the unit-beta mimicking portfolios are the factor risk premia and, based on these estimates, we can then estimate the cross-section of pricing errors, or alphas. Alternatively, we can replace the factors in the linear factor model with the excess returns generated by the maximum-correlation mimicking portfolios (LFM* formulation). In this case, the factor risk premia are the average excess returns on the maximum-correlation mimicking portfolios, and the alphas are estimated from the time series as the intercepts of the regressions.

The existing literature does not provide answers to the two questions of how to best construct mimicking portfolios, and how to best estimate and test linear factor models. For example, Huberman, Kandel, and Stambaugh (1987) only provide a theoretical discussion of the different approaches to constructing mimicking portfolios, showing how both unit-beta

[1]Fama (1996) shows that Merton's investors hold overall portfolios that minimize the return variance, for given expected return and covariances with the state variables. Recently, Ferson, Siegel, and Xu (2004) investigate the solution of the optimal portfolio problem in the presence of conditioning information. A special case of their solution is a portfolio with maximum squared unconditional correlation with a factor.

portfolios and maximum-correlation portfolios can in principle replace the factors in linear factor models. Hence, we do not know how the small-sample properties of estimates of risk premia and betas in the LFM and LFM* representations compare. Similarly, we do not know how the actual size and power of tests of multi-beta models in the two formulations compare. In turn, this is the main objective of the present paper.[2]

We start with a discussion clarifying the relation between the LFM and LFM* formulations, and the relation between the unit-beta and maximum-correlation portfolios. In doing so, we provide new insights relative to the existing literature. For example, we show that the alphas resulting from the CSR-GLS estimation of the LFM are the same as the alphas in the LFM* formulation. We also show that the maximum Sharpe ratio attainable from the CSR-GLS unit-beta portfolios is the same as that attainable from the maximum-correlation portfolios.

Following Hansen (1982) and Cochrane (2001), we show how both formulations of the linear factor model, and both estimation approaches (CSR *vs* time-series) can be cast within the unifying framework of GMM. This approach allows us to derive the asymptotic properties of estimates and statistics without the need of restrictive distributional assumptions. We then construct a simulation exercise to assess the properties of the estimators for the two different formulations of the model. The simulations consider a one-factor (C-CAPM) and a two-factor (I-CAPM) economy, both i.i.d. and serially-correlated data, different lengths of the sample (525 *vs* 240 months), and different choices of test assets (ten size-sorted portfolios *vs* the 25 size- and value-sorted portfolios of Fama and French).

The main results from the simulation are as follows. The LFM* estimator of the risk premium performs better than the CSR-GLS estimator in terms of bias, and in terms of the comparison between small-sample and asymptotic properties. When the factor is observed with noise, results are dramatically in favor of the LFM* formulation, relative to both CSR-OLS and CSR-GLS estimators. We also consider the power of the tests, where the

[2]Recently, Kimmel (2003) derives the asymptotic properties of estimates of risk premia on CSR-GLS unit-beta portfolios and on maximum-correlation portfolios. His paper differs from ours because he focuses on asymptotics only, for the case of Gaussian i.i.d. returns, and because he does not address issues of size and power of the test statistics.

size is corrected based on the results of a simulation under the null. Power for the LFM* specification is very similar to power for the LFM specification, CSR-GLS estimator, which, in turn, is higher than for the CSR-OLS estimator.

These results should be considered together with the fact that like the CSR-GLS estimates, but unlike the CSR-OLS estimates, risk-premium estimates in the LFM* formulation are invariant to the "repackaging" of securities (Kandel and Stambaugh, 1995). Hence, we conclude that when estimating risk premia and testing multi-beta models, the LFM* formulation should be considered in addition, or even instead, of the more traditional LFM formulation.

Several papers have studied the properties of CSR risk-premium estimators. Litzenberger and Ramaswamy (1979), for example, document the error-in-variables (EIV) problem arising in this setting, and suggest a correction that asymptotically removes the resulting bias. Amsler and Schmidt (1985) and MacKinlay (1987), on the other hand, perform Monte Carlo exercises focusing on the small-sample properties of test statistics of the CAPM. Shanken (1992) considers the properties of the CSR estimator both when the time dimension is large, and when the cross-sectional dimension is large. Affleck-Graves and Bradfield (1993) perform a simulation exercise to assess the power of the CSR estimator in rejecting the hypothesis of a zero market risk premium, when the economy being simulated is one where the market premium is positive. Kim (1995) provides an asymptotic correction for the EIV problem, which is robust to conditional heteroskedasticity. More recently, Chen and Kan (2004) develop finite-sample corrections for the EIV bias. [3]

Other studies have focused on the comparison between SDF and LFM representations in tests multi-beta models. Cochrane (2001) focuses on tests where the factor is traded, it is the excess return on the market. Kan and Zhou (1999, 2001) and Jagannathan and Wang (2002), on the other hand, consider the case where the factor is not traded. Hence, the present paper is the first paper to provide small-sample evidence for the two alternative formulations of a multi-beta model with non-traded factors.

The paper is organized as follows: Section I illustrates the two alternative formulations

[3] Also see Grauer and Janmaat (2004), for a simulation analysis of the properties of cross-sectional and multi-variate tests of the CAPM, when the true economy is a three-factor Fama and French (1992) economy.

of a linear factor model. Section II discusses the properties of the unit-beta and maximum-correlation mimicking portfolios. Section III derives the maximum Sharpe ratios obtainable from the different types of mimicking portfolios. Section IV derives the moment conditions used in estimation. Section V derives the asymptotic properties of the GMM estimators. Section VI discusses the case where both traded and non-traded factors are present, and where the non-traded factor is measured with noise. Section VII illustrates the data used to calibrate the simulation exercise. Section VIII discusses the results of the simulation. Section IX concludes. Proofs of the analytical results are collected in the Appendix.

I. Multi-beta models

The LFM in its standard formulation is given by

$$r_t = \alpha + \beta^\top[\lambda + y_t - E(y_t)] + e_t, \tag{1}$$

where r_t is an $N \times 1$ vector of returns in *excess* of the risk-free rate, y_t is a $K \times 1$ vector of factor realizations, and e_t is an $N \times 1$ vector of residuals orthogonal to the factors. Under the null, $\alpha = 0$.

An alternative formulation of the LFM obtains when the factors are replaced by the variable component of their projections onto the span of excess returns augmented of a constant. We have

$$y_t = \gamma_0^\star + (\gamma^\star)^\top r_t + \epsilon_t \tag{2}$$

and $y_t^\star \equiv (\gamma^\star)^\top r_t$, where $\gamma^\star = \Sigma_{rr}^{-1}\Sigma_{ry}$.[4] The coefficients $\gamma^\star$ are the weights of portfolios whose returns have the maximum squared correlation with the factors.

We have

$$r_t = \alpha^\star + (\beta^\star)^\top y_t^\star + e_t^\star. \tag{3}$$

[4]Note that $\gamma^\star$ is a set "factor mimicking portfolios" in the terminology of Huberman et al. (1987); i.e. a collection of position of risky assets with returns that can be used in place of the factors for pricing the risky assets. Moreover, note that for the mimicking portfolios to exist we need $(\gamma^\star)^\top 1_k = \Sigma_{yr}\Sigma_{rr}^{-1}1_k \neq 0$, where 1_k is a vector of ones. This condition is equivalent to assuming that the global minimum-variance portfolio has positive systematic risk, see Huberman et al. (1987).

We denote this alternative formulation as LFM*.

The next result establishes the relation between the two representations.

Result 1. Mispricing in the LFM* representation relates to mispricing in the LFM representation as follows:

$$\alpha^\star = [I - (\beta^\star)^\top (\gamma^\star)^\top]\alpha. \tag{4}$$

Hence, if $\alpha = 0$, $\alpha^\star = 0$.[5] On the other hand, if $\alpha^\star = 0$, α is not necessarily zero, since the $N \times N$ matrix $[I - (\beta^\star)^\top (\gamma^\star)^\top]$ is of rank $N - K$. Yet, the only case where $\alpha^\star = 0$, but $\alpha \neq 0$, is when $\alpha = \beta^\top \delta$, but this means that the multi-beta model holds, although with parameters $\lambda + \delta$. Hence, we can conclude that if a multi-beta result holds, then $\alpha^\star = 0$.[6]

The next result obtains the relation between risk premia in the two formulations.

Result 2. We have

$$\lambda^\star = (\gamma^\star)^\top \alpha + \Sigma_{y^\star y^\star} \Sigma_{yy}^{-1} \lambda, \tag{5}$$

[5]Note that the condition that $\alpha^\star = 0$ can be interpreted in terms of a test for intersection: under the null, the addition of individual assets to the portfolios mimicking the factors does not improve the mean-variance trade-off. This result is implicit in the analysis of Huberman et al. (1987). See also DeRoon and Nijman (2001) for a useful review of the literature on spanning and intersection.

[6]The proof that $[I - (\beta^\star)^\top (\gamma^\star)^\top]$ is of rank $N - K$ goes as follows. It is straightforward to show that both $I - (\beta^\star)^\top (\gamma^\star)^\top$ and $(\beta^\star)^\top (\gamma^\star)^\top$ are idempotent matrices. What we want to show is that if $(\beta^\star)^\top (\gamma^\star)^\top$ is idempotent of rank K, then $I - (\beta^\star)^\top (\gamma^\star)^\top$ is idempotent of rank $N - K$. Notice that $(\beta^\star)^\top (\gamma^\star)^\top$ can be written as $\Sigma_{ry}(\Sigma_{yr}\Sigma_{rr}^{-1}\Sigma_{ry})^{-1}\Sigma_{yr}\Sigma_{rr}^{-1}$. Given that Σ_{rr}^{-1} is full rank,

$$\text{rank}[\Sigma_{ry}(\Sigma_{yr}\Sigma_{rr}^{-1}\Sigma_{ry})^{-1}\Sigma_{yr}\Sigma_{rr}^{-1}] = \text{rank}[\Sigma_{ry}(\Sigma_{yr}\Sigma_{rr}^{-1}\Sigma_{ry})^{-1}\Sigma_{yr}].$$

Since $\Sigma_{yr}\Sigma_{rr}^{-1}\Sigma_{ry}$ is symmetric and invertible, $\text{rank}[(\beta^\star)^\top (\gamma^\star)^\top] = \text{rank}[\Sigma_{ry}(\Sigma_{yr}\Sigma_{rr}^{-1}\Sigma_{ry})^{-1}\Sigma_{yr}] = \text{rank}(\Sigma_{yr}) = K$. Now, the null space of $I - (\beta^\star)^\top (\gamma^\star)^\top$ is the set of all $z \neq 0$ such that $[I - (\beta^\star)^\top (\gamma^\star)^\top]z = 0$. But $[I - (\beta^\star)^\top (\gamma^\star)^\top]z = 0$ if and only if $z = (\beta^\star)^\top (\gamma^\star)^\top z$, i.e., z is an eigenvector of $(\beta^\star)^\top (\gamma^\star)^\top$ with eigenvalue equal to 1. Since $(\beta^\star)^\top (\gamma^\star)^\top$ is idempotent, the dimension of all such eigenvectors is equal to $\text{rank}[(\beta^\star)^\top (\gamma^\star)^\top] = K$, which means that $\text{rank}[I - (\beta^\star)^\top (\gamma^\star)^\top] = N - K$. Moreover, the null space of the idempotent matrix $I - (\beta^\star)^\top (\gamma^\star)^\top$ is spanned by linear combinations of the column vectors in $\beta^\top$. Since $\beta^\top$ is of rank K, the set of all $\alpha = \beta^\top \delta$ for $\delta \in \Re^K$ is of dimension K. This set is contained in the null space of $I - (\beta^\star)^\top (\gamma^\star)^\top$ and it must be all of the null space. We thank Raymond Kan for pointing out this issue.

where $\lambda^\star = (\gamma^\star)^\top E(r_t)$.

Hence, the risk premia in the two formulations are the same only under the null and if the factors are traded (in which case $\Sigma_{y^\star y^\star} = \Sigma_{yy}$).

II. Cross-sectional regressions and mimicking portfolios

The standard approach of running cross-sectional regressions (CSR) of average excess returns on betas leads to the risk-premium coefficients

$$\tilde{\lambda} = (\beta W \beta^\top)^{-1} \beta W E(r_t). \tag{6}$$

The CSR coefficients minimize the quadratic form

$$[E(r_t) - \beta^\top \tilde{\lambda}]^\top W [E(r_t) - \beta^\top \tilde{\lambda}]. \tag{7}$$

Two common choices for the weighting matrix W are $W = I$ and $W = \Sigma_{ee}^{-1}$. The first choice corresponds to an OLS regression; the second choice corresponds to a GLS regression. Alternatively we can choose $W = \Sigma_{rr}^{-1}$.[7]

Interestingly, we obtain the same CSR coefficients for $W = \Sigma_{rr}^{-1}$ as for $W = \Sigma_{ee}^{-1}$.[8] This result proves to be useful when we consider an alternative interpretation of the CSR coefficients. First, we can recognize that the $K \times N$ matrix

$$\tilde{\gamma}^\top = (\beta W \beta^\top)^{-1} \beta W \tag{8}$$

is a matrix of K sets of N portfolio weights. Each portfolio has unit beta w.r.t. the chosen

[7]Note that while the choice of $W = \Sigma_{ee}^{-1}$ or $W = \Sigma_{rr}^{-1}$ leads to factor mimicking portfolios (Huberman et al., 1987), the choice of $W = I$ does not.

[8]The intuition for this result is straightforward: minimizing the total portfolio variance with a constraint on systematic risk is equivalent to minimizing the idiosyncratic portfolio variance with a constraint on systematic risk. This result is stated, but not proved, by Shanken (1985). In turn, he references his unpublished dissertation for a proof.

factor and zero betas w.r.t. all other factors included the LFM:[9]

$$\tilde{\gamma}^\top \beta^\top = (\beta W \beta^\top)^{-1} \beta W \beta^\top = I. \tag{9}$$

We can then show how the CSR portfolio weights correspond to the solution of a different minimization problem.

Result 3. The CSR portfolio weights are the solution to the minimization problem

$$\begin{aligned} &\min_{\gamma_k} \gamma_k^\top W^{-1} \gamma_k \\ &\text{s.t. } \beta \gamma_k = s_k, \end{aligned} \tag{10}$$

where $\tilde{\gamma}_k$ is the $N \times 1$ vector of portfolio weights; and s_k is a vector with the $k-$th element equal to one, and zeros elsewhere.[10]

Hence, when $W = I$, the CSR portfolio weights minimize the length of the vector of portfolio weights, subject to the unit-beta constraint. When $W = \Sigma_{rr}^{-1}$, the CSR portfolio weights minimize the variance of the mimicking portfolio returns, subject to the unit-beta constraint. This second set of portfolio weights is equal to the set of portfolio weights implicit in the GLS-CSR coefficients $(\beta \Sigma_{ee}^{-1} \beta^\top)^{-1} \beta \Sigma_{ee}^{-1} E(r_t)$.

Note that the maximum-correlation portfolios are proportional to the solution of the minimization of $\gamma_k^\top \Sigma_{rr}^{-1} \gamma_k$ w.r.t. γ_k, subject to the *single* constraint $\Sigma_{y_k r} \gamma_k = s_{kk} \neq 0$. Hence, both unit-beta, CSR-GLS portfolios and maximum-correlation portfolios minimize the variance of the portfolio returns subject to constraints on covariances. The difference is that the unit-beta portfolios are subject to constraints on covariances with *all* the other factors, whereas the maximum-correlation portfolios are only subject to a single constraint on the covariance with the factor being tracked.

One additional property of the unit-beta portfolio weights is that, for $W = \Sigma_{rr}^{-1}$ (or $W = \Sigma_{ee}^{-1}$), and in the case of a single factor, the weights are proportional to the maximum-

[9] This property is worth noting in the context of characteristics-based explanations of the cross-section of asset returns. When excess returns are regressed on characteristics, the corresponding CSR coefficients are the excess returns on portfolios with a specific characteristic equal to one, and all other characteristics equal to zero. See, for example, Fama (1976) and Fama and French (1992).

[10] The special case of this minimization problem for $W = \Sigma_{rr}^{-1}$ is characterized in Huberman et al. (1987).

correlation mimicking-portfolio weights. In fact

$$\tilde{\gamma}^\top = (\beta\Sigma_{rr}^{-1}\beta^\top)^{-1}\beta\Sigma_{rr}^{-1} = (\Sigma_{yr}\Sigma_{rr}^{-1}\Sigma_{ry}/\sigma_y^2)^{-1}\Sigma_{yr}\Sigma_{rr}^{-1}. \tag{11}$$

Since $(\Sigma_{yr}\Sigma_{rr}^{-1}\Sigma_{ry}/\sigma_y^2)$ in the one-factor case is a scalar, $\tilde{\gamma}^\top$ above is proportional to $\Sigma_{yr}\Sigma_{rr}^{-1} = (\gamma^\star)^\top$.[11] In the general case of K factors, on the other hand, the unit-beta mimicking-portfolio weights $\tilde{\gamma}$ are *linear combinations* of the maximum-correlation mimicking-portfolio weights $\gamma^\star$.

Turning now to risk premia, the average excess returns on the unit-beta portfolios $\tilde{\lambda}$ coincide with the λ parameters only under the null:

$$\tilde{\lambda} = \tilde{\gamma}^\top E(r_t) = (\beta W\beta^\top)^{-1}\beta W E(r_t) = \lambda + (\beta W\beta^\top)^{-1}\beta W\alpha. \tag{12}$$

In other words, under the alternative, the expected excess returns on the unit-beta portfolios do not have any direct relation with the λ parameters. Moreover, the risk premia $\tilde{\lambda}$ depend on the choice of weighting matrix and are generally sensitive to the repackaging of securities.

Similarly, under the alternative, the alphas associated with the CSR premia differ from the true alphas:

$$\tilde{\alpha} = E(r_t) - \beta^\top(\beta W\beta^\top)^{-1}\beta W E(r_t) = [I - \beta^\top(\beta W\beta^\top)^{-1}\beta W]\alpha. \tag{13}$$

Hence, $\alpha = 0$ implies $\tilde{\alpha} = 0$, and $\alpha \neq 0$ implies $\tilde{\alpha} \neq 0$. On the other hand, $\tilde{\alpha} = 0$ does not imply $\alpha = 0$ since the $K \times K$ matrix $I - \beta^\top(\beta W\beta^\top)^{-1}\beta W$ is of rank $N - K$. As in Result 1, though, the null space of the idempotent matrix $I - \beta^\top(\beta W\beta^\top)^{-1}\beta W$ is spanned by linear combinations of the columns of $\beta^\top$.[12] Hence, we can conclude that if a multi-beta result holds, then $\tilde{\alpha} = 0$.

The next result shows how the alphas associated with the CSR premia equal the alphas of the LFM* representation when $W = \Sigma_{ee}^{-1}$ (or $W = \Sigma_{rr}^{-1}$).

Result 4. When $W = \Sigma_{ee}^{-1}$, then $\alpha^\star = \tilde{\alpha}$.

[11] This property is exploited by Breeden, Gibbons, and Litzenberger (1989), who construct a maximum-correlation portfolio for consumption growth by regressing consumption growth on asset returns.

[12] These two statements can be proved using the same arguments used for the matrix $I - (\beta^\star)^\top(\gamma^\star)^\top$.

III. Mimicking portfolios and Sharpe ratios

Consider the unit-beta mimicking portfolios. The maximum squared Sharpe ratio from investment in the unit-beta mimicking portfolios is given by[13]

$$E(r_t)^\top \tilde{\gamma}^\top (\tilde{\gamma}^\top \Sigma_{rr} \tilde{\gamma})^{-1} \tilde{\gamma} E(r_t) = E(r_t)^\top W \beta^\top (\beta W \Sigma_{rr} W \beta)^{-1} \beta^\top W E(r_t). \tag{14}$$

Using the decomposition $E(r_t) = \tilde{\alpha} + \beta^\top \tilde{\lambda}$, where $\beta W \tilde{\alpha} = 0$ by construction ($\tilde{\alpha}$ is the vector of residuals of a weighted least squares regression of the expected excess returns on the betas), the maximum squared Sharpe ratio simplifies to

$$\tilde{\lambda}^\top (\beta W \beta^\top)(\beta W \Sigma_{rr} W \beta^\top)^{-1} (\beta^\top W \beta) \tilde{\lambda}. \tag{15}$$

Note that $(\beta \Sigma_{ee}^{-1} \beta^\top)^{-1} \beta \Sigma_{ee}^{-1} = (\beta \Sigma_{rr}^{-1} \beta^\top)^{-1} \beta \Sigma_{rr}^{-1}$ and the unit-beta portfolio weights are the same for the two choices of weighting matrix $W = \Sigma_{ee}^{-1}$ and $W = \Sigma_{rr}^{-1}$. Substituting $W = \Sigma_{rr}^{-1}$ in the expression above, we obtain

$$\tilde{\lambda}^\top \beta \Sigma_{rr}^{-1} \beta^\top \tilde{\lambda}. \tag{16}$$

Hence, in the special case where $W = \Sigma_{ee}^{-1}$, if the multi-beta result holds ($\lambda = \tilde{\lambda}$ and $\beta^\top \tilde{\lambda} = E(r_t)$) then the maximum squared Sharpe ratio attainable from investment in the unit-beta portfolios equals the maximum squared Sharpe ratio attainable from investment in all the assets available.

Consider now the maximum-correlation mimicking portfolios. The maximum squared Sharpe ratio from investing in the maximum-correlation mimicking portfolios is given by

$$E(y_t^\star)^\top \Sigma_{y^\star y^\star}^{-1} E(y_t^\star) = E(r_t)^\top \gamma^\star ((\gamma^\star)^\top \Sigma_{rr} \gamma^\star)^{-1} (\gamma^\star)^\top E(r_t). \tag{17}$$

Using again the decomposition $E(r_t) = \tilde{\alpha} + \beta^\top \tilde{\lambda}$ and the fact that $\Sigma_{yr} W \tilde{\alpha} = 0$,[14] we have

$$E(y_t^\star)^\top \Sigma_{y^\star y^\star}^{-1} E(y_t^\star) = \tilde{\lambda}^\top \beta \Sigma_{rr}^{-1} \beta^\top \tilde{\lambda}, \tag{18}$$

[13] Note that this statement is technically true only if the covariance matrix of excess returns equals the covariance matrix of *returns*.

[14] We have

$$\tilde{\alpha} \equiv E(r_t) - \beta^\top \tilde{\lambda} = E(r_t) - \Sigma_{ry} (\Sigma_{yr} W \Sigma_{ry})^{-1} \Sigma_{yr} W E(r_t) = [I - \Sigma_{ry} (\Sigma_{yr} W \Sigma_{ry})^{-1} \Sigma_{yr} W] E(r_t).$$

Hence, it easy to verify that $\Sigma_{yr} W \tilde{\alpha} = 0$.

which, not surprisingly, is the same as the maximum squared Sharpe ratio from investing in the unit-beta mimicking portfolios, for $W = \Sigma_{ee}^{-1}$.[15] Again, under the null, the investor is able to achieve the same Sharpe ratio from investing in the maximum-correlation mimicking portfolios as from investing in the original assets.

IV. Moment conditions

In the following, we illustrate the moment conditions imposed in the estimation of the two formulations of the linear factor model.

A. LFM

Let z_t denote the data and θ the parameter vector. The moment conditions $E[f(z_t, \theta)] = 0$ for the LFM representation are given by:

$$E(r_t - \beta_0 - \beta^\top y_t) = 0 \tag{19}$$

$$E[(r_t - \beta_0 - \beta^\top y_t) \otimes y_t] = 0 \tag{20}$$

$$E(r_t - \beta^\top \lambda) = 0. \tag{21}$$

Following Hansen (1982), we set a linear combination of the moment conditions equal to zero, i.e. $a\hat{E}[f(z_t, \hat{\theta})] = 0$, where

$$a = \begin{bmatrix} I_{N(K+1)} & 0_{N(K+1),N} \\ 0_{K,N(K+1)} & \hat{\beta}W \end{bmatrix}. \tag{22}$$

We consider two weighting matrices: $W = I$ and $W = \hat{\Sigma}_{ee}^{-1}$

The resulting λ estimates are given by

$$\hat{\lambda} = (\hat{\beta}W\hat{\beta}^\top)^{-1}\hat{\beta}W\hat{E}(r_t). \tag{23}$$

For $W = \hat{\Sigma}_{ee}^{-1}$ (or $W = \hat{\Sigma}_{rr}^{-1}$), the resulting λ estimates coincide with those of the *optimal* GMM estimator (in the i.i.d. case),

$$a = \frac{\partial \hat{E}[f(z_t, \theta)]^\top}{\partial \theta}\Big|_{\theta=\hat{\theta}} \hat{\Sigma}_{ff}^{-1}, \tag{24}$$

[15] Remember that the two sets of mimicking portfolios are linear transformations of each other.

conditional on $\hat{\beta}$.

Relative to the approach of using the optimal weighting matrix as in Jagannathan and Wang (2002) and Kan and Zhou (1999, 2001), we see two main advantages to our approach. First, all estimates are obtained in closed-form, which is especially useful in the simulation exercise. Second, the estimates of the risk premium parameters λ are the same as those obtained with the traditional CSR approach.

B. LFM*

In the LFM* formulation, the moment conditions are given by

$$E[y_t - \gamma_0^\star - (\gamma^\star)^\top r_t] = 0 \tag{25}$$

$$E\{[y_t - \gamma_0^\star - (\gamma^\star)^\top r_t] \otimes r_t\} = 0 \tag{26}$$

$$E[(\gamma^\star)^\top r_t - \lambda^\star] = 0 \tag{27}$$

$$E[r_t - \alpha^\star - (\beta^\star)^\top y_t^\star] = 0 \tag{28}$$

$$E\{[r_t - \alpha^\star - (\beta^\star)^\top y_t^\star] \otimes y_t^\star\} = 0. \tag{29}$$

In this case, the estimates are obtained by exactly-identified GMM ($a = I$).

One potential disadvantage of this second approach is that we now have a total of $K(N+1)+K+N(K+1)$ to estimate, to be compared with $K+N(K+1)$ parameters for the LFM representation. Hence, one concern in the simulation analysis will be to see if the larger number of parameters to estimate affects the small-sample properties of the estimates.

V. Asymptotics

For both representations, the asymptotic covariance matrix of the estimates is given by

$$\text{Cov}(\hat{\theta}) = \frac{1}{T}(ad)^{-1}aSa^\top[(ad)^{-1}]^\top, \tag{30}$$

where

$$d = \frac{\partial \hat{E}[f(z_t, \theta)]}{\partial \theta'}|_{\theta=\hat{\theta}} \tag{31}$$

and[16]

$$S = \sum_{j=-\infty}^{\infty} \hat{E}[f(z_t, \hat{\theta}) f(z_{t-j}, \hat{\theta})^\top]. \tag{32}$$

The covariance matrix of the estimates is used to test the joint significance of the $\alpha^\star$ estimates in the LFM* representation. In the LFM representation, on the other hand, the α estimates are the averages of the last set of K moment conditions. Hence, the joint significance of the α estimates is tested based on the covariance matrix

$$\text{Cov}\{\hat{E}[f(z_t, \hat{\theta})]\} = \frac{1}{T}[I - d(ad)^{-1}a]S[I - d(ad)^{-1}a]^\top. \tag{33}$$

VI. Special cases

A. Traded factors

When a subset y_{1t} of the factors y_t are traded, the analytical results of the previous sections, as well as the estimation approach, are slightly changed. This is because we include Shanken's (1992) restriction that the risk premium-estimates for the traded factors are simply the time-series averages of the traded factors.[17]

[16]It is worth noting that for the LFM* representation, the matrix S does not have full rank $K + KN + K + N + NK$, but it has rank equal to $(K + KN + K + N + NK) - K - K^2$. The reason for this is that one can construct K linear combinations of the excess returns that exactly replicate the excess returns on the maximum-correlation mimicking portfolios. Hence, for K linear combinations of the excess returns, the residuals of the regression on the excess returns on the maximum-correlation mimicking portfolios are identically zero. This also means that the K^2 products of these residuals with the excess returns on the maximum-correlation mimicking portfolios are zero. (A formal proof of the previous statements is in an a separate appendix available from the authors upon request.) One way to get around this problem would be to differentiate the set of the test assets from the set of assets used to construct the mimicking portfolios. Indeed, this is the approach used in the maximum-likelihood estimation of Breeden, Gibbons, and Litzenberger (1989). Note, though, that in our GMM setting the singularity of S does not present a problem in estimation, since it does not need to be inverted. Moreover, when it comes to the covariance matrix of the $\alpha^\star$ estimates, which we do need to invert, that matrix is in fact invertible.

[17]It is easy to show (see, for example, Cochrane, 2001) how the CSR-GLS estimator of the risk premium on a traded factor is simply the time-series average of the factor, when the traded factor is added to the set

Let

$$r_{2t} \equiv r_t - \beta_1^\top y_{1t}, \tag{34}$$

where β_1 are the coefficients of a projection of r_t onto the span of y_{1t}.[18] Also, let β_2 denote the coefficients of the projection of r_{2t} onto the augmented span of the non-traded factors; and let $\gamma_2^\star$ denote the coefficients of the projection of the non-traded factors onto the augmented span of r_{2t}.

The LFM representation leads to the mimicking-portfolio coefficients

$$\tilde{\gamma}_2^\top = (\beta_2 W \beta_2^\top)^{-1} \beta_2 W \tag{35}$$

and the risk premia

$$\tilde{\lambda}_2 = (\beta_2 W \beta_2^\top)^{-1} \beta_2 W E(r_{2t}). \tag{36}$$

The LFM* representation leads to the risk premia

$$\lambda_2^\star = (\gamma_2^\star)^\top E(r_{2t}). \tag{37}$$

Hence, all the analytical results of the previous sections still hold, but where r_t is replaced by r_{2t}; β is replaced by β_2; y_t is replaced by y_{2t}; and $\gamma^\star$ is replaced by $\gamma_2^\star$.

When it comes to estimation, the same type of modifications take place. For the LFM specification, we have the orthogonality conditions

$$\begin{aligned} E(r_t - \beta_1^\top y_{1t}) &= 0 & (38)\\ E(r_{2t} - \beta_0 - \beta_2^\top y_{2t}) &= 0 & (39)\\ E[(r_{2t} - \beta_0 - \beta_2^\top y_{2t}) \otimes y_{2t}] &= 0 & (40)\\ E(r_{2t} - \beta_2^\top \lambda_2) &= 0. & (41) \end{aligned}$$

of test assets. Given that the GLS estimator is asymptotically efficient (Shanken, 1992), this means that the efficient estimate of the premium on a traded factor is its time-series average.

[18]It is worth noting that the analysis that follows would have gone through for slightly different definitions of r_{2t}. First, β_1 could have been the coefficients of a projection including an intercept. Second, β_1 could have been a subset of the coefficients of a projection of r_t on both y_{1t} and y_{2t}. The current approach was chosen because it leads to the smallest number of extra parameters needed for the estimation.

In constructing the a matrix, $\hat{\beta}_2$ replaces $\hat{\beta}$, while W can either equal I or $\hat{\Sigma}_{ee}^{-1}$. For the LFM* formulation, we have the orthogonality conditions

$$\begin{aligned} E(r_t - \beta_1^\top y_{1t}) &= 0 && (42)\\ E[y_{2t} - \gamma_0^\star - (\gamma_2^\star)^\top r_{2t}] &= 0 && (43)\\ E\{[y_{2t} - \gamma_0^\star - (\gamma_2^\star)^\top r_{2t}] \otimes r_{2t}\} &= 0 && (44)\\ E(y_{2t}^\star - \mu_{y_2^\star}) &= 0 && (45)\\ E[r_{2t} - \alpha^\star - (\beta_2^\star)^\top y_{2t}^\star] &= 0 && (46)\\ E\{[r_t - \alpha^\star - (\beta_2^\star)^\top y_{2t}^\star] \otimes y_{2t}^\star\} &= 0. && (47) \end{aligned}$$

B. Noisy factors

It is straightforward to show that in the case of a single noisy factor, the CSR-OLS estimator leads to inconsistent estimates of the risk premium λ.[19] Assume $\hat{y}_t = y_t + \epsilon_t$, where $E(y_t\epsilon_t) = 0$ and $E(r_t\epsilon_t) = 0$.

We have

$$\text{Plim } \hat{\beta} = \frac{\text{Var}(y_t)}{\text{Var}(\hat{y}_t)}\beta. \quad (48)$$

Consider the CSR λ estimator for $W = I$. We have

$$\text{Plim } \hat{\lambda} = \frac{\text{Var}(\hat{y}_t)}{\text{Var}(y_t)}\lambda. \quad (49)$$

In other words, the attenuation bias in the β estimates translates into an upward bias in the λ estimates. On the other hand, it is immediate to show that the $\gamma^\star$ and $\lambda^\star$ estimates are still consistent.

VII. Data

We calibrate the simulation experiment using monthly data from 03:1959 to 2002:11 (525 obs.). We consider two sets of test assets: ten size-sorted portfolios (value-weighted (VW)

[19] See Kan and Zhou (1999) for a similar discussion.

NYSE-AMEX-NASDAQ decile portfolios, from CRSP); and the 25 Fama-French portfolios (from French's web page). We consider the following three factors: log real per-capita consumption growth (nondurable goods and services, CITIBASE); the VW NYSE-AMEX-NASDAQ index return in excess of the risk-free rate (CRSP); and the change in the dividend yield on the composite S&P500, (CITIBASE). As a proxy for the risk-free rate in our economies we use the nominal one-month Treasury bill rate from CRSP.

VIII. Results

A. Bootstrap experiment

We simulate the economy 10,000 times. We consider two economies: a one-factor C-CAPM economy and a two-factor I-CAPM economy. We consider a linearized version of the consumption-based pricing kernel and the corresponding Euler equation:

$$E[r_t(b_0 + b_1 y_{ct})] = 0,$$

where y_{ct} is log-consumption growth. We have

$$E(r_t) = -\beta^\top \frac{b_1 \mathrm{Var}(y_{ct})}{E(b_0 + b_1 y_{ct})} \equiv \beta^\top \lambda.$$

The economies are first simulated under the null (the α parameters are set to zero). We also perform simulations under the alternative ($\alpha \neq 0$) to assess power. Parameter estimates of the LFM are estimated from the data, using GMM with $W = I$. We implement both an i.i.d and a block bootstrap (blocks of three monthly observations; see Cochrane, 2001). In the bootstrap, we simulate jointly the factors and the residuals from the estimated LFM. Hence, factors and residuals are orthogonal, but not independent. We consider two lengths of the data set: the full sample of 525 observations and a shorter sample of 240 observations (the first 240 observations in our sample). For both economies we consider two choices of assets, the ten size portfolios and the 25 Fama-French portfolios. When we investigate the properties of estimates of the LFM, we consider estimates based on the two weighting matrices $W = I$ and $W = \Sigma_{ee}^{-1}$. Estimates of the parameters of the LFM* are obtained by exactly-identified GMM, and they do not require a choice of weighting matrix. Asymptotic statistics are obtained assuming no serial correlation or serial correlation of order 3, with Newey-West adjustment.

B. Estimates of economic risk premia

Table I reports results for the one-factor (C-CAPM) case. Panels A and B report statistics for the λ estimates, while Panel C reports results for the $\lambda^\star$ estimates.

The table presents population values for the risk premia, average risk-premium estimates, biases, and root-mean-squared errors (RMSEs). For ease of comparison, we report risk premia as percentages of the population standard deviation of the corresponding mimicking-portfolio excess return. Given the results of Section IV, this means that the reference values (which are the population Sharpe ratios of the mimicking portfolios) for the CSR-GLS estimates and the $\lambda^\star$ estimates are the same; whereas the reference values for CSR-OLS estimates and CSR-GLS/$\lambda^\star$ estimates differ.[20] Biases and RMSEs are also reported as percentages of the population standard deviation of the mimicking-portfolio excess return.

The table also reports averages and standard deviations of asymptotic t-ratios on the risk-premium estimates. The t-ratios are computed using the population risk premium as the reference value. Finally, the table reports the ratios between the average asymptotic t-ratio on the risk-premium estimate and the standard deviation of the estimate across simulations.

Consider first results for the estimates of λ, for the i.i.d. bootstrap (Table I, Panel A). OLS estimates are biased away from zero, with biases ranging between 0.39 and 1.51. On the other hand, GLS estimates are biased towards zero, with biases ranging between -10.62 and -2.88. RMSEs are substantial, but similar, for the two estimators, ranging between 5.28 and 11.72. As one would expect, reducing the length of the sample increases biases and RMSEs. Using the Fama-French, rather than the size portfolios, leads to smaller (larger) biases and RMSEs for the OLS (GLS) estimates.

We can compare the results above to the results obtained by Chen and Kan (2004), in a similar setting. Chen and Kan derive analytically the small-sample bias and standard deviation of CSR estimates for the OLS and GLS cases, under the assumption of i.i.d., Gaussian,

[20] The population OLS and GLS risk premium estimates are the same, but the standard deviations of the mimicking-portfolio excess return differ.

returns and factor; and they perform a simulation exercise for non-normal (student-t) returns and factor. They consider the case where the factor is calibrated to consumption growth, and they also consider both size-sorted and size- and value-sorted test portfolios. Several of their results are consistent with ours. They find that absolute biases are larger for the GLS than the OLS estimates, that the GLS estimates are consistently biased towards zero, and that the sign of the bias for the OLS estimates can be either positive or negative, depending on the calibration. As to the volatility of the estimates, they find that the GLS estimates are less volatile than the OLS estimates, which is consistent with our finding that RMSEs are similar for the two approaches (the lower volatility compensates the higher absolute bias of the GLS estimates).

One discrepancy between our results and Chen and Kan's (2004) is the higher (absolute) bias in the GLS estimates going from the size to the Fama-French portfolios. Chen and Kan, on the other hand, report a reduction in bias. In order to understand this difference, we have to note two features of the Fama-French, relative to the size portfolios: first, population consumption betas tend to differ more from one portfolio to the other; second, for several portfolios, the LFM captures little variability of the returns, and betas are estimated with more volatility across simulations. Chen and Kan show analytically that the absolute bias in GLS estimates decreases with the cross-sectional dispersion of the population betas and increases with the variance of the beta estimates. In our setting, it is the second effect to dominate for the GLS estimates. In Chen and Kan it is the first effect to dominate, because they use a slightly different measure of consumption, that does not include services.

Asymptotic t-ratios have means that are substantially different from 0, ranging between -2.40 and -0.12. The standard deviations of the asymptotic t-ratios also deviate substantially from the theoretical value of one, ranging between 0.93 and 1.25. Reducing the length of the sample leads to larger deviations of the small-sample properties of the t-ratios relative to their asymptotic counterparts. In line with the bias results, using the Fama-French portfolios, rather than the size portfolios, leads to average asymptotic t-ratios that are more strongly negative for the GLS estimator. The GLS estimator has larger absolute biases in asymptotic t-ratios than the OLS estimator; while the ratios between asymptotic and empirical standard deviations are mainly lower than one for both choices of weighting matrix, ranging between

0.94 and 1.01. Finally, the choice of lags in the computation of asymptotic statistics makes essentially no difference.

Results for the case of block bootstrap (Panel B) are quite similar. In this case, though, the MA(3) adjustment in computing asymptotic statistics makes more of a difference, leading to small-sample properties of t-ratios that are slightly closer to the theoretical properties.

Results for the estimates of $\lambda^\star$ (Table I, Panel C) should be compared with the results for the CSR-GLS (the population Sharpe ratios for the two mimicking portfolios are the same). There is an advantage in using the LFM* formulation in terms of bias: absolute biases of the $\lambda^\star$ estimates are always substantially smaller than the biases of the corresponding CSR-GLS estimates, for both i.i.d. and block bootstrap. For example, in the i.i.d. bootstrap, 240 months, and Fama-French portfolios, the bias for the CSR-GLS estimator is -10.62, whereas it is only -1.97 for the $\lambda^\star$ estimator (the reference value is 17.96). Notice that this is the first paper to show that the estimates of $\lambda^\star$ are biased towards zero, in small samples, a feature that the estimates of $\lambda^\star$ share with the CSR-GLS estimates of λ. Finally, RMSEs are roughly the same magnitude as those for the GLS estimates.

T-ratios are negatively biased: biases are similar to the OLS estimates and much less pronounced than for the GLS estimates. The standard deviations of the t-ratios are consistently less than one, and the ratios between asymptotic standard errors and empirical standard errors are consistently larger than one (again, similar to the OLS estimates, but the opposite of the GLS estimates).

Table II reports the same results for the multi-factor (I-CAPM) case. Panels A and B report results for the λ estimates, while Panel C reports results for the $\lambda^\star$ estimates. In this case, we focus on the risk premia on the non-traded factor: changes in the dividend yield.

Consider first the LFM formulation. As in the one-factor case, the GLS estimates are biased towards zero (the bias is positive, while the true value of the parameter is negative); on the other hand, the bias in the OLS estimates changes sign depending on the set of assets. Again, (absolute) biases are more pronounced for the GLS estimates, ranging between 2.07 and 8.72; while biases range between -0.72 and 0.68 for the OLS estimates. RMSEs are again substantial for both sets of estimates and of similar magnitude, ranging between 4.84 and 10.38. Biases in t-ratios are now consistently positive for both estimators, but again more pronounced for the GLS estimator.

For the LFM* formulation, biases are now positive, i.e. the $\lambda^\star$ estimates are again biased towards zero. Again, biases are consistently smaller than for the corresponding GLS estimates. For example, in the i.i.d. bootstrap, 240 months, and Fama-French portfolios, the bias for the CSR-GLS estimator is 8.64, whereas it is only 2.93 for the $\lambda^\star$ estimator (the reference value is -16.92). RMSEs are comparable to those for the GLS estimates. Biases in t-ratios are also positive, roughly comparable to those of the OLS estimates, and less pronounced than for the GLS estimates.

In summary, in the comparison between CSR-OLS and CSR-GLS estimates, the OLS estimates perform better in terms of bias of estimates and small-sample behavior of t-ratios. In the comparison between CSR-GLS estimates and time-series estimates of $\lambda^\star$, the LFM* approach has advantages in terms of biases and t-ratios. The comparison between CSR-OLS estimates and LFM* approach is less straightforward, without a clear advantage for one approach or the other.

Our comparison of the small-sample properties of OLS and GLS estimates complements Cochrane's (2001) discussion. Cochrane recognizes how the CSR-GLS estimates are related to the minimization of the pricing error for ex-post minimum-variance portfolios.[21] These portfolios may involve large long and short positions, and may be relatively uninteresting portfolios to price. In addition, it is likely that in small samples the composition of minimum-variance portfolios will depart substantially from the population value. Our simulation results support Cochrane's conjecture.

C. Noisy factor

Table III shows results for the one-factor case, when the factor is observed with noise. We focus on one scenario only: i.i.d. bootstrap, full sample, size portfolios, and no Newey-

[21] Indeed, the CSR-GLS estimate minimizes $\alpha^\top \Sigma_{rr}^{-1} \alpha$. It is easy to see that this quantity is proportional to the alpha of the tangency portfolio. In fact, the weights of the tangency portfolio are proportional to $\Sigma_{rr}^{-1} E(r)$. Hence, using the CSR-GLS approach, the alpha of the tangency portfolio is proportional to $E(r)^\top \Sigma_{rr}^{-1} \tilde{\alpha} = \tilde{\lambda}^\top \beta \Sigma_{rr}^{-1} \tilde{\alpha} + \tilde{\alpha}^\top \Sigma_{rr}^{-1} \tilde{\alpha} = \tilde{\alpha}^\top \Sigma_{rr}^{-1} \tilde{\alpha}$, ($\beta \Sigma_{rr}^{-1} \tilde{\alpha} = 0$, by construction), the quantity minimized by the GLS estimator.

West adjustment of the statistics. Following Kan and Zhou (1999), we consider noise with standard deviation equal to 0.5, 1, 1.5, and 2 times the standard deviation of the factor.

Note that a setting where the issue of noise in measuring the factor arises naturally is that of tests of heteregenous-agent models; see, for example, Brav, Constantinides and Geczy (2002), Jacobs and Wang (2004), and Balduzzi and Yao (2004). In this type of analysis, the realizations of the factors are the moments of the cross-sectional distribution of consumption growth. Since the cross-sections used in these studies are relatively small, the cross-sectional moments can be estimated with a great deal of noise.[22]

The table presents reference values of the risk premia, biases, RMSEs, and ratios between average asymptotic standard errors and empirical standard deviations of the estimates.

In this setting, the results are dramatically in favor of the LFM* formulation over both CSR-OLS and CSR-GLS approaches. Moreover, the OLS approach always fares worse than the GLS approach. Bias and RMSE monotonically increases with noise for the CSR λ estimates. For example, when noise is twice as large as the signal, the biases for the CSR-OLS and CSR-GLS estimates are 49 and 19, respectively; and RMSEs are as high as 80 and 26. These values are to be compared with reference values of 11.57 and 15.86 for the OLS and GLS estimates, respectively. On the other hand, the bias for the $\lambda^\star$ estimates never exceeds 0.13 in absolute value, with a maximum RMSE of 9.10.

D. Estimates of betas

We analyze the finite sample properties of β and $\beta^\star$. While the β estimates should be unbiased, the $\beta^\star$ estimates are subject to the EIV problem arising from the estimation of the

[22] For example, in Balduzzi and Yao, the average size of the cross-section for households with financial assets in excess of \$2,000 is 266. The monthly standard deviation of the cross-sectional average of log-consumption growth is 0.0258, while the average cross-sectional variance of consumption growth is 0.0581. If log-consumption growth is uncorrelated in the cross-section, and if 0.0581 is indicative of the true cross-sectional variance, then $\sqrt{0.0581}/\sqrt{266} = 0.0148$ is indicative of the estimation noise around the average log-consumption growth. Hence, if we also assume that the true cross-sectional average of log-consumption growth is unrelated to the estimation error, then the noise-to-signal ratio is $0.0148/\sqrt{0.0258^2 - 0.0148^2} = 0.699$. The same calculations for the sample of households with assets in excess of \$10,000 give a noise-to-signal ratio of 0.811.

weights of the maximum-correlation mimicking portfolio. We conduct simulations to assess the magnitude of this bias.

Table IV reports results for the C-CAPM case. We report the same statistics as in Table I, averaged across assets. For ease of comparison, both β and $\beta^\star$ estimates are scaled by the ratio between the population standard deviations of the factor and of the dependent variable. Hence, they have the dimension of correlation coefficients.

Panel A shows results for the estimates of β. Biases are essentially non-existent and RMSEs are also very modest. Similarly unbiased are, on average, the t-ratios. The volatilities of the t-ratios are close to one, and the ratios between average asymptotic standard errors and empirical standard errors are also close to one.

Panel B shows results for the estimates of $\beta^\star$. Biases are negative (this is the well-known "attenuation bias" associated with the EIV problem) and substantial, ranging between -30 and -12. RMSEs are also substantial, as high as 32. These values are to be compared with reference values of 69 and 57, for the size and Fama-French portfolios, respectively. Finally, biases in t-ratios are negative and pronounced, and there are also substantial discrepancies between small-sample and asymptotic volatilities of t-ratios.

Table V reports results for the I-CAPM. In this case, we focus on the betas for the non-traded factor. As in the single-factor case, biases and RMSEs are much more pronounced for the $\beta^\star$ estimates than for the β estimates.[23]

E. Size and power of the tests

We investigate by simulation the size and power properties of the Wald-style tests for the one-factor and two-factor models. The main contribution consists in the analysis of the size and power properties of tests in the context of the LFM* representation.[24]

[23]Note that in this case average biases in the $\beta^\star$ estimates are positive, rather than negative. This is again consistent with the attenuation bias, since the true average reference value is negative.

[24]The question of the power of tests of multi-beta models when the original factors are replaced by mimicking-portfolio returns is raised, for example, by Huberman et al. (1987).

E.1. Size

Table VI reports the theoretical and actual sizes of the Wald-style test for the one-factor model. Panels A and B report results for the LFM specification, for the full sample and for the short sample of 240 observations, respectively. Panel C reports results for the LFM* specification.

Consider first the results for the LFM specification, full sample, i.i.d. bootstrap. The actual size is generally close to the theoretical value, although there are systematic discrepancies. The CSR-OLS approach generally leads to under-rejections, while the CSR-GLS approach leads to over-rejections; moreover, using the Fama-French portfolios leads to higher rejection rates than using the size-sorted portfolios. Using the serial-correlation adjustment reduces rejection rates somewhat. For example, in the case of i.i.d. bootstrap and size portfolios, when the theoretical size is 50%, the actual size is 43% for the OLS estimator. The size increases to 54% for the GLS estimator; and to 47% for the Fama-French portfolios, OLS estimator. When the statistic is adjusted for serial correlation, the actual size falls to 42% for the OLS estimator.

Results are similar for the block bootstrap and for the shorter sample of 240 months. Indeed, rejection rates are quite similar for the two sample lengths.

When we consider the LFM* specification, overall we see over-rejections. Over-rejections are again more pronounced for the Fama-French portfolios. Rejections are somewhat more pronounced with the MA adjustment. For example, in the case in the case of i.i.d. bootstrap and size portfolios, when the theoretical size is 50%, the actual size is 63%; the size increases to 82% for the Fama-French portfolios; the size increase to 65% for the MA(3) adjustment. These effects are true for both i.i.d. and block bootstrap, and for both sample lengths.

Table VII presents results for the I-CAPM and is organized in the same way as Table VI. The same general patterns arise as in the one-factor specification.

E.2. Power

Table VIII (organized similarly to Tables VI and VII) reports rejection rates for the Wald test under the alternative, where the size is adjusted using the bootstrap results of Table

VI: we compute the 10%, 5%, and 1% quantiles of the empirical distribution of the Wald statistic under the null, and we compute the percentage of times the Wald statistic exceeds the corresponding quantile, when the economy is simulated under the alternative.

In the case of the LFM specification, the power of the tests is higher for $W = \hat{\Sigma}_{ee}^{-1}$, for the Fama-French portfolio returns, and for the longer sample. For example, consider the case of the i.i.d. bootstrap for the full sample, size-sorted portfolios, and statistics *not* adjusted for serial correlation. When the size is set at 1%, the actual rejection rate is 57% for the OLS approach; the rejection rate increases to 80% for the GLS approach; the size further increases to 98% for the choice of Fama-French test portfolios; and the size decreases to 15% when the sample is shortened to 240 months.

In the case of the LFM* specification, the power is also higher for the Fama-French portfolio returns and for the longer sample. For example, in the case of the i.i.d. bootstrap for the entire sample, size portfolios, when the size is set at 1%, the rejection rate is 78%; the rejection rate increases to 100% for the Fama-French portfolios; the rejection rate falls to 22% for the shorter sample. In general, power for the LFM* specification is very similar to power for the LFM specification, GLS approach.

For the I-CAPM (Table IX, organized similarly to Table VIII) results are similar. In the case of the LFM specification, the power of the tests is generally higher for $W = \Sigma_{ee}^{-1}$, although the differences are now very modest. Power is also higher for tests using the Fama-French portfolios. Power is substantially higher for the longer sample. Power for the LFM* specification is very similar to power for the LFM specification.

Hence, when it comes to power, the CSR-GLS and LFM* approaches have an advantage over the CSR-OLS approach, particularly in the one-factor case. Also, it is a clear advantage to use the larger cross-section of the Fama-French portfolios.

IX. Conclusions

This paper considers two alternative formulations of the linear factor model with non-traded factors. The first formulation is the traditional one, where the estimation of risk premia and alphas is performed by means of a cross-sectional regression of average returns on betas. The second formulation replaces the factors with their projections onto the augmented span of excess returns. This second formulation requires only time-series regressions for the estimation of risk premia and alphas. We compare the theoretical properties of the two approaches and we perform a simulation exercise to assess the small-sample properties of the estimates.

We find that the LFM* formulation of the model leads to estimates of risk premia that are better behaved than those obtained with the CSR-GLS approach. In the case of a single factor is observed with noise, the LFM* risk-premium estimates work much better than both CSR-OLS and CSR-GLS estimates. Power of tests of multi-beta models is similar in the LFM* and LFM/CSR-GLS formulations. In turn, the CSR-GLS approach delivers substantially higher power than the CSR-OLS approach in the one-factor case.

In light of these results, we conclude that when estimating risk premia and testing multi-beta models, the LFM* formulation should be considered in addition, or even instead, of the more traditional LFM formulation.

Appendix

Proof of Result 1. We have

$$\begin{aligned} E(r_t) &= \alpha^\star + (\beta^\star)^\top E(y^\star) \\ &= \alpha^\star + (\beta^\star)^\top (\gamma^\star)^\top E(r_t) \\ &= \alpha^\star + (\beta^\star)^\top (\gamma^\star)^\top \alpha + (\beta^\star)^\top (\gamma^\star)^\top \beta^\top \lambda. \end{aligned} \tag{50}$$

We have

$$\begin{aligned} (\beta^\star)^\top (\gamma^\star)^\top \beta^\top &= \Sigma_{rr}\gamma^\star((\gamma^\star)^\top \Sigma_{rr}\gamma^\star)^{-1}(\gamma^\star)^\top \Sigma_{ry}\Sigma_{yy}^{-1} \\ &= \Sigma_{rr}\Sigma_{rr}^{-1}\Sigma_{ry}(\Sigma_{yr}\Sigma_{rr}^{-1}\Sigma_{rr}\Sigma_{rr}^{-1}\Sigma_{ry})^{-1}\Sigma_{yr}\Sigma_{rr}^{-1}\Sigma_{ry}\Sigma_{yy}^{-1} \\ &= \Sigma_{ry}(\Sigma_{yr}\Sigma_{rr}^{-1}\Sigma_{ry})^{-1}\Sigma_{yr}\Sigma_{rr}^{-1}\Sigma_{ry}\Sigma_{yy}^{-1} \\ &= \Sigma_{ry}\Sigma_{yy}^{-1} \\ &= \beta^\top. \end{aligned} \tag{51}$$

Hence, we write

$$E(r_t) - (\beta^\star)^\top (\gamma^\star)^\top \beta^\top \lambda = E(r_t) - \beta^\top \lambda = \alpha = \alpha^\star + (\beta^\star)^\top (\gamma^\star)^\top \alpha \tag{52}$$

and

$$\alpha - (\beta^\star)^\top (\gamma^\star)^\top \alpha = \alpha^\star, \tag{53}$$

and hence the result.[25]

Proof of Result 2. We have

$$\lambda^\star = (\gamma^\star)^\top E(r_t) = (\gamma^\star)^\top \alpha + (\gamma^\star)^\top \beta^\top \lambda = (\gamma^\star)^\top \alpha + \Sigma_{yr}\Sigma_{rr}^{-1}\Sigma_{ry}\Sigma_{yy}^{-1}\lambda. \tag{54}$$

Note that

$$\text{Var}((\gamma^\star)^\top r_t) = \Sigma_{y^\star y^\star} = \Sigma_{yr}\Sigma_{rr}^{-1}\Sigma_{rr}\Sigma_{rr}^{-1}\Sigma_{ry} = \Sigma_{yr}\Sigma_{rr}^{-1}\Sigma_{ry}. \tag{55}$$

[25] A proof of Result 1 for the case where the projection does not contain a constant is in a separate appendix available from the authors upon request.

Hence,

$$\lambda^\star = (\gamma^\star)^\top \alpha + (\gamma^\star)^\top \beta^\top \lambda = (\gamma^\star)^\top \alpha + \Sigma_{y^\star y^\star} \Sigma_{yy}^{-1} \lambda. \tag{56}$$

Proof of Result 3. Consider the minimization problem

$$\begin{aligned} &\min_{\gamma_k} \frac{1}{2} \gamma_k^\top W^{-1} \gamma_k \\ &\text{s.t. } \beta \gamma_k = s_k. \end{aligned} \tag{57}$$

The lagrangian for the minimization problem is given by

$$\frac{1}{2} \gamma_k^\top W^{-1} \gamma_k + \sum_{j=1}^{K} \phi_{kj} (s_{kj} - \gamma_k^\top \beta_j^\top), \tag{58}$$

where s_{kj} is the j-th element of the vector s_k and β_j is the j-th row of the β matrix. The first-order conditions for a minimum are

$$W^{-1} \gamma_k - \sum_{j=1}^{K} \phi_{kj} \beta_j^\top = 0 \tag{59}$$

$$s_k - \beta \gamma_k = 0. \tag{60}$$

The first set of conditions can be re-written as

$$W^{-1} \gamma_k - \beta^\top \phi_k = 0, \tag{61}$$

where ϕ_k is the $K \times 1$ vector of Lagrange multipliers. Hence, we obtain

$$\gamma_k = W \beta^\top \phi_k. \tag{62}$$

Substituting into the unit-beta constraint, we have

$$s_k = \beta W \beta^\top \phi_k \tag{63}$$

and

$$\phi_k = (\beta W \beta^\top)^{-1} s_k. \tag{64}$$

Substituting in equation (62), we have

$$\tilde{\gamma}_k = W \beta^\top (\beta W \beta^\top)^{-1} s_k. \tag{65}$$

Repeating the exercise for each of the factors, appropriately stacking the vectors of portfolio weights next to each other, and transposing, we obtain

$$\tilde{\gamma}^\top = (\beta W \beta^\top)^{-1} \beta W. \tag{66}$$

Proof of Result 4. Using the fact that $\tilde{\gamma}$ is the same for $W = \Sigma_{ee}^{-1}$ and $W = \Sigma_{rr}^{-1}$, we have

$$\begin{aligned}
\beta^\top(\beta\Sigma_{ee}^{-1}\beta^\top)^{-1}\beta\Sigma_{ee}^{-1}E(r_t) &= \Sigma_{ry}\Sigma_{yy}^{-1}(\beta\Sigma_{rr}^{-1}\beta^\top)^{-1}\beta\Sigma_{rr}^{-1}E(r_t) \\
&= \Sigma_{ry}\Sigma_{yy}^{-1}(\Sigma_{yy}^{-1}\Sigma_{yr}\Sigma_{rr}^{-1}\Sigma_{ry}\Sigma_{yy}^{-1})^{-1}\Sigma_{yy}^{-1}\Sigma_{yr}\Sigma_{rr}^{-1}E(r_t) \\
&= \Sigma_{ry}(\Sigma_{yr}\Sigma_{rr}^{-1}\Sigma_{ry})^{-1}\Sigma_{yr}\Sigma_{rr}^{-1}E(r_t) \\
&= (\beta^\star)^\top\lambda^\star.
\end{aligned} \tag{67}$$

This implies that when $W = \Sigma_{ee}^{-1}$, then $\beta^\top\tilde{\lambda} = (\beta^\star)^\top\lambda^\star$. Hence, we also have $\tilde{\alpha} = E(r_t) - \beta^\top\tilde{\lambda} = E(r_t) - (\beta^\star)^\top\lambda^\star = \alpha^\star$.

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Table I: C-CAPM; λ vs $\lambda^{\star}$

We simulate a one-factor C-CAPM economy 10,000 times under the null (the α parameters are set to zero). Parameter estimates of the LFM are obtained using GMM with $W = I$. In the bootstrap (i.i.d. and block), we simulate jointly the factor and the residuals from the estimated LFM. We consider two lengths of the data set: the full sample of 525 observations (full sample) and a shorter sample of 240 observations (240 months). We consider two choices of assets, the ten size portfolios (size) and the 25 Fama-French portfolios (FF). When we investigate the properties of estimates of the LFM, we consider estimates based on the two weighting matrices $W = I$ and $W = see = \Sigma_{ee}^{-1}$. Estimates of the parameters of the LFM* are obtained by exactly-identified GMM. Asymptotic statistics are obtained assuming no serial correlation or serial correlation of order 3 (with Newey-West adjustment). For each panel, the first row reports the population value of the risk premium as a percentage of the population standard deviation of the excess return on the corresponding mimicking portfolio; the second row reports the average value across simulations of the risk premium as a percentage of the population standard deviation of the excess return on the corresponding mimicking portfolio; the third and fourth rows report absolute bias and root mean square errors (RMSE), respectively; the fifth and sixth rows, in the order, report mean and standard deviation of the t-ratios across simulations; finally, the seventh row reports the ratio between average asymptotic standard errors across simulations and empirical standard deviation.

Panel A: Finite Sample properties of λ (IID Bootstrap)

	Full Sample				240 Months			
	size		**FF**		**size**		**FF**	
	W=I	**W=sse**	**W=I**	**W=sse**	**W=I**	**W=sse**	**W=I**	**W=sse**
λ	11.57	15.86	11.73	17.96	11.57	15.86	11.73	17.96
$\hat{\lambda}$	12.31	12.98	12.12	11.17	13.08	10.44	12.43	7.35
$Bias$	0.74	−2.88	0.39	−6.79	1.51	−5.42	0.70	−10.62
$RMSE$	6.10	5.28	5.48	7.90	11.03	8.11	8.63	11.72
t_{mean}	−0.12 (−0.12)	−0.78 (−0.78)	−0.12 (−0.12)	−1.83 (−1.83)	−0.18 (−0.18)	−1.08 (−1.09)	−0.18 (−0.18)	−2.39 (−2.40)
t_{std}	0.97 (0.97)	1.07 (1.07)	0.97 (0.98)	1.17 (1.17)	0.93 (0.94)	1.10 (1.11)	0.93 (0.94)	1.24 (1.25)
$\frac{AsySE}{EmpStd}$	0.97 (0.96)	0.98 (0.97)	0.98 (0.98)	0.99 (0.98)	0.96 (0.95)	0.95 (0.95)	1.01 (1.01)	0.94 (0.94)

Panel B: Finite Sample properties of λ (Block Bootstrap)

	Full Sample				240 Months			
	size		**FF**		**size**		**FF**	
	W=I	**W=sse**	**W=I**	**W=sse**	**W=I**	**W=sse**	**W=I**	**W=sse**
λ	11.57	15.86	11.73	17.96	11.57	15.86	11.73	17.96
$\hat{\lambda}$	12.10	12.87	12.00	11.22	13.06	10.47	12.54	7.38
$Bias$	0.53	−2.99	0.27	−6.74	1.49	−5.39	0.81	−10.58
$RMSE$	6.20	5.43	5.66	7.90	11.07	8.16	9.22	11.73
t_{mean}	−0.15 (−0.12)	−0.81 (−0.78)	−0.14 (−0.13)	−1.81 (−1.79)	−0.18 (−0.15)	−1.07 (−1.05)	−0.17 (−0.15)	−2.36 (−2.38)
t_{std}	1.04 (0.98)	1.10 (1.07)	1.02 (0.98)	1.17 (1.16)	0.99 (0.94)	1.12 (1.10)	0.98 (0.94)	1.24 (1.25)
$\frac{Asy.\ se}{Emp.\ Std}$	0.93 (0.97)	0.95 (0.99)	0.95 (0.99)	0.97 (0.98)	0.93 (0.96)	0.94 (0.96)	0.96 (0.99)	0.93 (0.93)

3 lags NW in parenthesis; **sse** $= \hat{\Sigma}_{ee}^{-1}$

Panel C: Finite Sample properties of $\lambda^\star$

	IID Bootstrap				Block Bootstrap			
	Full Sample		240 Months		Full Sample		240 Months	
	size	**FF**	**size**	**FF**	**Size**	**FF**	**Size**	**FF**
$\lambda^\star$	15.86	17.96	15.86	17.96	15.86	17.96	15.86	17.96
$\hat{\lambda}^\star$	15.75	16.99	15.59	16.01	15.58	17.04	15.55	15.99
$Bias$	-0.11	-0.97	-0.27	-1.95	-0.28	-0.92	-0.31	-1.97
$RMSE$	5.82	6.38	9.32	11.31	5.93	6.72	9.52	11.45
t_{mean}	-0.12 (-0.12)	-0.13 (-0.20)	-0.12 (-0.12)	-0.20 (-0.20)	-0.15 (-0.14)	-0.21 (-0.21)	-0.13 (-0.13)	-0.21 (-0.21)
t_{std}	0.97 (0.98)	0.92 (0.93)	0.92 (0.93)	0.89 (0.90)	0.99 (0.98)	0.98 (0.96)	0.93 (0.93)	0.91 (0.91)
$\frac{AsySE}{EmpStd}$	1.05 (1.04)	1.09 (1.09)	1.09 (1.08)	1.11 (1.11)	1.02 (1.04)	1.03 (1.05)	1.07 (1.07)	1.10 (1.10)

Table II: I-CAPM; λ vs $\lambda^\star$

We simulate a two-factor I-CAPM economy 10,000 times under the null (the α parameters are set to zero). The two factors are the VW NYSE/AMEX/NASDAQ index return and the change in the dividend yield. Parameter estimates of the LFM are obtained using GMM with $W = I$. In the bootstrap (i.i.d. and block), we simulate jointly the factors and the residuals from the estimated LFM. We consider two lengths of the data set: the full sample of 525 observations (full sample) and a shorter sample of 240 observations (240 months). We consider two choices of assets, the ten size portfolios (size) and the 25 Fama-French portfolios (FF). When we investigate the properties of estimates of the LFM, we consider estimates based on the two weighting matrices $W = I$ and $W = see = \Sigma_{ee}^{-1}$. Estimates of the parameters of the LFM* are obtained by exactly-identified GMM. Asymptotic statistics are obtained assuming no serial correlation or serial correlation of order 3 (with Newey-West adjustment). For each panel, the first row reports the population value of the risk premium on the non-traded factor as a percentage of the population standard deviation of the excess return on the corresponding mimicking portfolio (see Section VII.A); the second row reports the average value across simulations of the risk premium on the non-traded factor as a percentage of the population standard deviation of the excess return on the corresponding mimicking portfolio; the third and fourth rows report absolute bias and root mean square errors (RMSE), respectively; the fifth and sixth rows, in the order, report mean and standard deviation of the t-ratios across simulations; finally, the seventh row reports the ratio between average asymptotic standard errors across simulations and empirical standard deviation.

Panel A: Finite Sample properties of λ (IID Bootstrap)

	Full Sample				240 Months			
	size		**FF**		**size**		**FF**	
	W=I	**W=sse**	**W=I**	**W=sse**	**W=I**	**W=sse**	**W=I**	**W=sse**
λ	-7.31	-8.85	-9.36	-16.92	-7.31	-8.85	-9.36	-16.92
$\hat{\lambda}$	-7.58	-6.77	-9.28	-11.94	-8.03	-4.93	-8.77	-8.28
$Bias$	-0.27	2.08	0.08	4.98	-0.72	3.92	0.59	8.64
$RMSE$	4.93	4.69	4.93	6.52	8.25	6.85	6.94	10.30
t_{mean}	0.11 (0.11)	0.61 (0.61)	0.22 (0.22)	1.31 (1.31)	0.15 (0.15)	0.83 (0.84)	0.33 (0.33)	1.78 (1.80)
t_{std}	0.96 (0.97)	1.04 (1.04)	0.98 (0.98)	1.12 (1.12)	0.93 (0.94)	1.04 (1.05)	0.98 (0.99)	1.19 (1.21)
$\frac{AsySE}{EmpStd}$	0.98 (0.97)	0.99 (0.99)	1.02 (1.02)	0.97 (0.96)	0.97 (0.96)	0.99 (0.98)	1.04 (1.03)	0.92 (0.91)

Panel B: Finite Sample properties of λ (Block Bootstrap)

	Full Sample				240 Months			
	size		**FF**		**size**		**FF**	
	W=I	**W=sse**	**W=I**	**W=sse**	**W=I**	**W=sse**	**W=I**	**W=sse**
λ	-7.31	-8.85	-9.36	-16.92	-7.31	-8.85	-9.36	-16.92
$\hat{\lambda}$	-7.47	-6.78	-9.20	-11.96	-7.70	-4.97	-8.68	-8.20
$Bias$	-0.16	2.07	0.16	4.96	-0.39	3.88	0.68	8.72
$RMSE$	5.10	4.84	5.16	6.55	8.51	6.99	7.30	10.38
t_{mean}	0.12 (0.10)	0.61 (0.60)	0.24 (0.22)	1.30 (1.30)	0.17 (0.15)	0.83 (0.82)	0.35 (0.33)	1.80 (1.82)
t_{std}	1.01 (0.98)	1.06 (1.04)	1.04 (1.01)	1.12 (1.12)	0.98 (0.94)	1.06 (1.04)	1.04 (1.01)	1.20 (1.21)
$\frac{Asy.\ se}{Emp.\ Std}$	0.94 (0.97)	0.95 (0.98)	0.97 (0.99)	0.95 (0.95)	0.91 (0.94)	0.95 (0.97)	0.98 (1.00)	0.91 (0.90)

3 lags NW in parenthesis; **sse** $= \hat{\Sigma}_{ee}^{-1}$

Panel C: Finite Sample properties of $\lambda^\star$

	IID Bootstrap				Block Bootstrap			
	Full Sample		240 Months		Full Sample		240 Months	
	size	**FF**	**size**	**FF**	**Size**	**FF**	**Size**	**FF**
$\lambda^\star$	-8.85	-16.92	-8.85	-16.92	-8.85	-16.92	-8.85	-16.92
$\hat{\lambda}^\star$	-8.01	-15.50	-7.06	-13.99	-8.04	-15.52	-7.11	-13.93
$Bias$	0.84	1.42	1.79	2.93	0.81	1.40	1.74	2.99
$RMSE$	4.88	5.66	8.19	9.92	5.07	5.84	8.37	10.06
t_{mean}	0.19 (0.19)	0.28 (0.28)	0.22 (0.22)	0.32 (0.32)	0.18 (0.18)	0.28 (0.28)	0.21 (0.21)	0.32 (0.33)
t_{std}	0.92 (0.92)	0.93 (0.93)	0.86 (0.86)	0.89 (0.90)	0.95 (0.94)	0.96 (0.96)	0.88 (0.88)	0.91 (0.93)
$\frac{AsySE}{EmpStd}$	1.08 (1.07)	1.08 (1.07)	1.14 (1.13)	1.11 (1.11)	1.03 (1.05)	1.04 (1.04)	1.10 (1.10)	1.10 (1.07)

Table III: C-CAPM (Noisy Factor)

λ_1 ($W = I$), λ_2 ($W = \Sigma_{ee}^{-1}$) vs. $\lambda^\star$

We simulate a C-CAPM economy 10,000 times under the null (the α parameters are set to zero). Parameter estimates of the LFM are obtained using GMM with $W = I$. In the i.i.d bootstrap, we simulate jointly the factor and the residuals from the estimated LFM. For each bootstrap replication, we add an i.i.d. Gaussian shock to the factor. The standard deviation of the shock is proportional (constant of proportionality equal to c) to the population standard deviation of the factor. Columns two through six report simulation results for the case of no noise and the case of noise ($c = 0.5, 1, 1.5, 2.0$), respectively. We investigate the properties of estimates of the LFM using the full sample of 525 observations, the ten size portfolios (size) and the two weighting matrices $W = I$ and $W = see = \Sigma_{ee}^{-1}$. Estimates of the parameters of the LFM* are obtained by exactly-identified GMM. Asymptotic statistics are obtained assuming no serial correlation. We report the population value of the risk premium as a percentage of the population standard deviation of the excess return on the corresponding mimicking portfolio, the average value across simulations of the risk premium as a percentage of the population standard deviation of the excess return on the corresponding mimicking portfolio, absolute bias and root mean square errors (RMSE), mean and standard deviation of the t-ratios across simulations, and the ratio between average asymptotic standard errors across simulations and empirical standard deviation.

	no noise	**c=0.50**	**c=1.0**	**c=1.50**	**c=2.0**
λ_1	11.57	11.57	11.57	11.57	11.57
$\hat{\lambda}_1$	12.31	15.50	25.90	42.56	60.52
λ_2	15.86	15.86	15.86	15.86	15.86
$\hat{\lambda}_2$	12.98	15.43	21.44	28.35	34.40
$\lambda^{\star}$	15.86	15.86	15.86	15.86	15.86
$\hat{\lambda}^{\star}$	15.75	15.74	15.74	15.73	15.73
$Bias_{\lambda_1}$	0.74	3.93	14.33	30.99	48.95
$Bias_{\lambda_2}$	-2.88	-0.43	5.58	12.49	18.54
$Bias_{\lambda^{\star}}$	-0.11	-0.12	-0.12	-0.13	-0.13
$RMSE_{\lambda_1}$	6.10	8.87	21.69	46.62	79.97
$RMSE_{\lambda_2}$	5.28	5.41	10.09	17.96	26.28
$RMSE_{\lambda^{\star}}$	5.82	6.04	6.76	7.82	9.10
$(\frac{AsySE}{EmpStd})_{\lambda_1}$	0.97	0.99	0.99	1.04	1.10
$(\frac{AsySE}{EmpStd})_{\lambda_2}$	0.98	1.00	1.01	1.00	0.96
$(\frac{AsySE}{EmpStd})_{\lambda^{\star}}$	1.05	1.06	1.09	1.11	1.13

- size portfolios, full sample, i.i.d bootstrap, 0 lags, shock$\sim N(0, s_n^2)$ with $s_n = c * \sigma_y$

Table IV: C-CAPM; β vs $\beta^\star$

We simulate a one-factor C-CAPM economy 10,000 times. Parameter estimates of the LFM are obtained using GMM with $W = I$. In the bootstrap (i.i.d. and block), we simulate jointly the factor and the residuals from the estimated LFM. We consider two lengths of the data set: the full sample of 525 observations (full sample) and a shorter sample of 240 observations (240 months). We consider two choices of assets, the ten size portfolios (size) and the 25 Fama-French portfolios (FF). Estimates of the parameters of the LFM* are obtained by exactly-identified GMM. Asymptotic statistics are obtained assuming no serial correlation or serial correlation of order 3 (with Newey-West adjustment). Both β and $\beta^\star$ estimates (average βs and $\beta^\star$s across simulations and across assets) are scaled by the ratio between the population standard deviations of the factor and of the excess return. We report population and average values of the betas, absolute bias and root mean square errors (RMSE), mean and standard deviation of the t-ratios across simulations and the ratio between average asymptotic standard errors across simulations and empirical standard deviation.

Panel A: Finite Sample properties of β

	IID Bootstrap				Block Bootstrap			
	Full Sample		240 Months		Full Sample		240 Months	
	size	FF	size	FF	Size	FF	Size	FF
β	17.92	16.27	17.92	16.27	17.92	16.27	17.92	16.27
$\hat{\beta}$	17.80	16.26	17.70	16.29	17.84	16.31	17.69	16.26
$Bias$	-0.12	-0.01	-0.22	0.02	-0.08	0.04	-0.23	-0.01
$RMSE$	4.22	4.24	6.25	6.26	4.04	4.15	5.99	6.13
t_{mean}	−0.01 (−0.01)	−0.02 (−0.02)	−0.01 (−0.01)	−0.02 (−0.02)	−0.00 (−0.00)	−0.01 (−0.00)	−0.03 (−0.02)	−0.03 (−0.03)
t_{std}	1.02 (1.03)	1.02 (1.03)	1.03 (1.04)	1.03 (1.03)	0.98 (1.02)	1.00 (1.02)	0.99 (1.03)	1.01 (1.04)
$\frac{AsySE}{EmpStd}$	0.98 (0.98)	0.98 (0.98)	0.98 (0.98)	0.98 (0.98)	1.03 (0.99)	1.00 (0.98)	1.03 (0.99)	1.00 (0.98)

Panel B: Finite Sample properties of $\beta^{\star}$

	IID Bootstrap				Block Bootstrap			
	Full Sample		240 Months		Full Sample		240 Months	
	size	FF	size	FF	Size	FF	Size	FF
$\beta^{\star}$	69.47	55.67	69.47	55.67	69.47	55.67	69.47	55.67
$\hat{\beta}^{\star}$	57.29	37.00	46.50	26.00	57.63	37.42	46.73	26.39
$Bias$	-12.18	-18.67	-22.97	-29.67	-11.84	-18.25	-22.74	-29.28
$RMSE$	17.73	21.69	27.88	31.63	17.15	21.32	27.19	31.24
t_{mean}	−1.14 (−1.15)	−1.83 (−1.84)	−1.71 (−1.72)	−2.99 (−3.01)	−1.10 (−1.14)	−1.78 (−1.79)	−1.66 (−1.73)	−2.90 (−2.94)
t_{std}	1.15 (1.16)	1.31 (1.32)	1.31 (1.33)	1.72 (1.74)	1.10 (1.15)	1.27 (1.29)	1.24 (1.30)	1.67 (1.68)
$\frac{AsySE}{EmpStd}$	1.07 (1.07)	1.11 (1.11)	1.11 (1.10)	1.10 (1.10)	1.12 (1.09)	1.13 (1.12)	1.17 (1.14)	1.13 (1.12)

3 lags NW in parenthesis

Table V: I-CAPM; β vs $\beta^{\star}$

We simulate a two-factor I-CAPM economy 10,000 times under the null (the α parameters are set to zero). The two factors are the VW NYSE/AMEX/NASDAQ index return and the change in the dividend yield. Parameter estimates of the LFM are obtained using GMM with $W = I$. In the bootstrap (i.i.d. and block), we simulate jointly the factor and the residuals from the estimated LFM. We consider two lengths of the data set: the full sample of 525 observations (full sample) and a shorter sample of 240 observations (240 months). We consider two choices of assets, the ten size portfolios (size) and the 25 Fama-French portfolios (FF). Estimates of the parameters of the LFM* are obtained by exactly-identified GMM. Asymptotic statistics are obtained assuming no serial correlation or serial correlation of order 3 (with Newey-West adjustment). Both β and $\beta^{\star}$ estimates (average βs and $\beta^{\star}$s across simulations and across assets) are scaled by the ratio between the population standard deviations of the factor and of the excess return. We report population and average values of the betas, absolute bias and root mean square errors (RMSE), mean and standard deviation of the t-ratios across simulations and the ratio between average asymptotic standard errors across simulations and empirical standard deviation.

Panel A: Finite Sample properties of β

	IID Bootstrap				Block Bootstrap			
	Full Sample		240 Months		Full Sample		240 Months	
	size	**FF**	**size**	**FF**	**Size**	**FF**	**Size**	**FF**
β	-11.81	-7.77	-11.81	-7.77	-11.81	-7.77	-11.81	-7.77
$\hat{\beta}$	-11.82	-7.98	-11.86	-8.02	-11.87	-8.03	-12.00	-8.16
$Bias$	-0.01	-0.22	-0.05	-0.25	-0.12	-0.06	-0.19	-0.39
$RMSE$	3.77	3.70	5.64	5.50	3.73	3.67	5.60	5.47
t_{mean}	0.04 (0.04)	-0.02 (-0.02)	0.05 (0.05)	-0.00 (-0.00)	0.03 (0.03)	-0.03 (-0.03)	0.03 (0.04)	-0.02 (-0.02)
t_{std}	1.02 (1.02)	1.01 (1.02)	1.05 (1.06)	1.03 (1.04)	1.01 (1.03)	1.00 (1.03)	1.03 (1.06)	1.03 (1.06)
$\frac{AsySE}{EmpStd}$	0.99 (0.99)	0.99 (0.99)	0.97 (0.96)	0.97 (0.97)	1.00 (0.98)	1.00 (0.98)	0.98 (0.96)	0.98 (0.96)

Panel B: Finite Sample properties of $\beta^{\star}$

	IID Bootstrap				Block Bootstrap			
	Full Sample		240 Months		Full Sample		240 Months	
	size	**FF**	**size**	**FF**	**Size**	**FF**	**Size**	**FF**
$\beta^{\star}$	-57.18	-27.51	-57.18	-27.51	-57.18	-27.51	-57.18	-27.51
$\hat{\beta}^{\star}$	-47.29	-21.65	-38.31	-16.61	-47.29	-21.86	-38.47	-16.87
$Bias$	9.89	5.86	18.87	10.90	9.89	5.65	18.71	10.64
$RMSE$	18.57	12.51	28.85	17.82	18.39	12.30	28.46	17.56
t_{mean}	0.83 (0.84)	0.65 (0.65)	1.29 (1.30)	1.05 (1.06)	0.83 (0.85)	0.63 (0.65)	1.29 (1.32)	1.04 (1.08)
t_{std}	1.09 (1.09)	1.04 (1.04)	1.23 (1.24)	1.13 (1.14)	1.08 (1.10)	1.02 (1.05)	1.23 (1.27)	1.13 (1.17)
$\frac{AsySE}{EmpStd}$	1.08 (1.08)	1.03 (1.03)	1.14 (1.13)	1.03 (1.02)	1.09 (1.08)	1.05 (1.02)	1.15 (1.13)	1.04 (1.01)

3 lags NW in parenthesis

Table VI: Size of the Wald Test (C-CAPM)

We simulate a one-factor C-CAPM economy 10,000 times under the null (the α parameters are set to zero). Parameter estimates of the LFM are obtained using GMM with $W = I$. In the bootstrap (i.i.d. and block), we simulate jointly the factor and the residuals from the estimated LFM. We consider two lengths of the data set: the full sample of 525 observations (full sample) and a shorter sample of 240 observations (240 months). We consider two choices of assets, the ten size portfolios (size) and the 25 Fama-French portfolios (FF). When we investigate the properties of estimates of the LFM, we consider estimates based on the two weighting matrices $W = I$ and $W = see = \Sigma_{ee}^{-1}$. Estimates of the parameters of the LFM* are obtained by exactly-identified GMM. Asymptotic statistics are obtained assuming no serial correlation or serial correlation of order 3 (with Newey-West adjustment). We report the theoretical and actual sizes of the Wald test for the one-factor model. Panels A and B report results for the LFM specification, for the full sample and for the short sample of 240 observations, respectively. Panel C reports results for the LFM* specification. The LFM test statistic should be distributed χ^2_{N-K}, while the LFM test statistic should be distributed χ^2_N.

Panel A: LFM (Full Sample)

	IID Bootstrap				Block Bootstrap			
	size		FF		size		FF	
	W=I	W=sse	W=I	W=sse	W=I	W=sse	W=I	W=sse
Ave. Chisq	8.47 (8.33)	9.57 (9.38)	23.61 (22.61)	27.54 (25.97)	8.03 (8.08)	9.12 (9.10)	23.56 (22.43)	27.56 (25.77)
0.50	0.43 (0.42)	0.54 (0.53)	0.47 (0.42)	0.67 (0.61)	0.39 (0.40)	0.50 (0.51)	0.46 (0.41)	0.66 (0.60)
0.25	0.21 (0.20)	0.30 (0.28)	0.24 (0.19)	0.42 (0.35)	0.18 (0.18)	0.26 (0.25)	0.24 (0.18)	0.42 (0.34)
0.10	0.09 (0.08)	0.14 (0.12)	0.11 (0.07)	0.23 (0.16)	0.07 (0.07)	0.11 (0.10)	0.11 (0.07)	0.23 (0.15)
0.05	0.05 (0.04)	0.07 (0.06)	0.06 (0.03)	0.15 (0.08)	0.04 (0.03)	0.06 (0.06)	0.06 (0.03)	0.15 (0.08)
0.025	0.027 (0.020)	0.040 (0.032)	0.034 (0.014)	0.092 (0.043)	0.020 (0.017)	0.035 (0.028)	0.037 (0.015)	0.094 (0.042)
0.01	0.01 (0.01)	0.02 (0.01)	0.01 (0.00)	0.05 (0.02)	0.01 (0.01)	0.02 (0.01)	0.02 (0.00)	0.05 (0.02)

Panel B: LFM (240 Months)

	IID Bootstrap				Block Bootstrap			
	size		FF		size		FF	
	W=I	W=sse	W=I	W=sse	W=I	W=sse	W=I	W=sse
Ave. Chisq	8.05 (7.79)	9.79 (9.35)	23.01 (20.99)	27.48 (24.26)	7.62 (7.52)	9.40 (9.09)	22.94 (20.76)	27.77 (24.18)
0.50	0.40 (0.39)	0.57 (0.55)	0.46 (0.34)	0.68 (0.55)	0.35 (0.35)	0.52 (0.52)	0.45 (0.33)	0.68 (0.55)
0.25	0.19 (0.17)	0.31 (0.27)	0.23 (0.12)	0.43 (0.24)	0.16 (0.15)	0.29 (0.26)	0.23 (0.11)	0.44 (0.24)
0.10	0.08 (0.06)	0.14 (0.11)	0.10 (0.02)	0.22 (0.07)	0.06 (0.05)	0.13 (0.10)	0.11 (0.02)	0.24 (0.06)
0.05	0.04 (0.02)	0.08 (0.05)	0.05 (0.01)	0.13 (0.02)	0.03 (0.02)	0.07 (0.05)	0.06 (0.01)	0.15 (0.02)
0.025	0.022 (0.012)	0.044 (0.025)	0.027 (0.002)	0.075 (0.004)	0.019 (0.010)	0.038 (0.020)	0.031 (0.002)	0.086 (0.006)
0.01	0.01 (0.00)	0.02 (0.01)	0.01 (0.00)	0.04 (0.00)	0.01 (0.00)	0.02 (0.01)	0.01 (0.00)	0.04 (0.00)

Panel C: LFM*

	IID Bootstrap				Block Bootstrap			
	Full Sample		240 Months		Full Sample		240 Months	
	size	FF	size	FF	size	FF	size	FF
Ave. Chisq	12.01 (12.34)	33.54 (35.66)	11.94 (12.70)	33.59 (38.69)	11.44 (12.05)	33.71 (35.48)	11.50 (12.38)	34.14 (38.72)
0.50	0.63 (0.65)	0.82 (0.85)	0.63 (0.67)	0.81 (0.89)	0.59 (0.63)	0.81 (0.85)	0.59 (0.65)	0.81 (0.88)
0.25	0.40 (0.42)	0.62 (0.69)	0.39 (0.44)	0.62 (0.75)	0.36 (0.40)	0.63 (0.68)	0.36 (0.42)	0.63 (0.74)
0.10	0.21 (0.23)	0.42 (0.49)	0.21 (0.25)	0.42 (0.59)	0.18 (0.21)	0.43 (0.49)	0.19 (0.23)	0.44 (0.58)
0.05	0.14 (0.15)	0.31 (0.38)	0.13 (0.17)	0.31 (0.47)	0.11 (0.13)	0.32 (0.38)	0.11 (0.15)	0.33 (0.47)
0.025	0.082 (0.093)	0.227 (0.293)	0.080 (0.109)	0.227 (0.386)	0.071 (0.087)	0.236 (0.287)	0.075 (0.099)	0.253 (0.388)
0.01	0.04 (0.05)	0.15 (0.20)	0.04 (0.06)	0.15 (0.29)	0.04 (0.05)	0.15 (0.20)	0.04 (0.06)	0.17 (0.30)

3 lags NW in parenthesis; **sse** $= \hat{\Sigma}_{ee}^{-1}$

Table VII: Size of the Wald Test

We simulate a two-factor I-CAPM economy 10,000 times under the null (the α parameters are set to zero). The two factors are the VW NYSE/AMEX/NASDAQ index return and the change in the dividend yield. Parameter estimates of the LFM are obtained using GMM with $W = I$. Parameter estimates of the LFM are obtained using GMM with $W = I$. In the bootstrap (i.i.d. and block), we simulate jointly the factor and the residuals from the estimated LFM. We consider two lengths of the data set: the full sample of 525 observations (full sample) and a shorter sample of 240 observations (240 months). We consider two choices of assets, the ten size portfolios (size) and the 25 Fama-French portfolios (FF). When we investigate the properties of estimates of the LFM, we consider estimates based on the two weighting matrices $W = I$ and $W = see = \Sigma_{ee}^{-1}$. Estimates of the parameters of the LFM* are obtained by exactly-identified GMM. Asymptotic statistics are obtained assuming no serial correlation or serial correlation of order 3 (with Newey-West adjustment). We report the theoretical and actual sizes of the Wald test for the two-factor model. Panels A and B report results for the LFM specification, for the full sample and for the short sample of 240 observations, respectively. Panel C reports results for the LFM* specification. The LFM test statistic should be distributed $\chi^2_{N-K_2}$, where $K_2 = 1$ represents the number of non-traded factors, while the LFM test statistic should be distributed χ^2_N.

Panel A: LFM (Full Sample)

	IID Bootstrap				Block Bootstrap			
	size		FF		size		FF	
	W=I	W=sse	W=I	W=sse	W=I	W=sse	W=I	W=sse
Ave. Chisq	8.66 (8.53)	9.27 (9.10)	24.25 (23.20)	26.44 (25.10)	8.31 (8.22)	8.90 (8.76)	24.41 (23.18)	26.56 (24.97)
0.50	0.46 (0.46)	0.52 (0.51)	0.50 (0.45)	0.62 (0.57)	0.43 (0.42)	0.48 (0.48)	0.51 (0.45)	0.62 (0.56)
0.25	0.22 (0.21)	0.27 (0.26)	0.27 (0.22)	0.37 (0.30)	0.19 (0.18)	0.24 (0.23)	0.28 (0.21)	0.37 (0.29)
0.10	0.09 (0.07)	0.12 (0.10)	0.13 (0.08)	0.19 (0.13)	0.07 (0.06)	0.10 (0.09)	0.13 (0.09)	0.19 (0.13)
0.05	0.04 (0.04)	0.06 (0.05)	0.07 (0.04)	0.11 (0.06)	0.04 (0.03)	0.05 (0.04)	0.08 (0.05)	0.12 (0.06)
0.025	0.021 (0.014)	0.033 (0.025)	0.040 (0.019)	0.062 (0.031)	0.020 (0.013)	0.030 (0.020)	0.048 (0.022)	0.072 (0.031)
0.01	0.01 (0.00)	0.01 (0.01)	0.02 (0.01)	0.03 (0.01)	0.01 (0.01)	0.01 (0.01)	0.02 (0.01)	0.04 (0.01)

Panel B: LFM (240 Months)

	IID Bootstrap				Block Bootstrap			
	size		FF		size		FF	
	W=I	W=sse	W=I	W=sse	W=I	W=sse	W=I	W=sse
Ave. Chisq	8.28 (8.02)	9.30 (8.92)	24.39 (22.05)	27.00 (23.92)	8.13 (7.84)	9.11 (8.70)	24.78 (22.16)	27.47 (24.06)
0.50	0.43 (0.41)	0.52 (0.50)	0.52 (0.40)	0.66 (0.53)	0.40 (0.39)	0.50 (0.48)	0.53 (0.40)	0.67 (0.53)
0.25	0.20 (0.17)	0.28 (0.25)	0.28 (0.14)	0.41 (0.21)	0.18 (0.16)	0.26 (0.22)	0.30 (0.16)	0.42 (0.23)
0.10	0.07 (0.05)	0.12 (0.09)	0.13 (0.03)	0.20 (0.05)	0.07 (0.05)	0.11 (0.08)	0.14 (0.04)	0.22 (0.06)
0.05	0.04 (0.02)	0.06 (0.04)	0.07 (0.01)	0.11 (0.01)	0.03 (0.02)	0.06 (0.03)	0.08 (0.01)	0.13 (0.02)
0.025	0.016 (0.007)	0.030 (0.015)	0.036 (0.002)	0.061 (0.004)	0.017 (0.008)	0.030 (0.014)	0.048 (0.004)	0.080 (0.007)
0.01	0.01 (0.00)	0.01 (0.00)	0.01 (0.00)	0.03 (0.00)	0.01 (0.00)	0.01 (0.00)	0.02 (0.00)	0.04 (0.00)

Panel C: LFM*

	IID Bootstrap				Block Bootstrap			
	Full Sample		240 Months		Full Sample		240 Months	
	size	FF	size	FF	size	FF	size	FF
Ave. Chisq	10.87 (11.18)	32.26 (34.33)	10.87 (11.56)	33.40 (38.44)	10.46 (10.75)	32.43 (34.27)	10.70 (11.24)	34.10 (38.96)
0.50	0.56 (0.58)	0.78 (0.82)	0.55 (0.59)	0.81 (0.89)	0.52 (0.55)	0.78 (0.82)	0.53 (0.57)	0.82 (0.89)
0.25	0.32 (0.34)	0.58 (0.65)	0.32 (0.37)	0.62 (0.75)	0.28 (0.31)	0.58 (0.64)	0.31 (0.34)	0.63 (0.75)
0.10	0.16 (0.18)	0.37 (0.45)	0.16 (0.20)	0.42 (0.59)	0.14 (0.15)	0.38 (0.44)	0.15 (0.17)	0.44 (0.59)
0.05	0.09 (0.10)	0.27 (0.34)	0.09 (0.12)	0.31 (0.48)	0.08 (0.09)	0.27 (0.33)	0.09 (0.11)	0.33 (0.48)
0.025	0.051 (0.062)	0.187 (0.251)	0.051 (0.075)	0.225 (0.387)	0.048 (0.051)	0.194 (0.247)	0.053 (0.067)	0.246 (0.392)
0.01	0.03 (0.03)	0.11 (0.17)	0.02 (0.04)	0.14 (0.29)	0.03 (0.03)	0.12 (0.17)	0.03 (0.03)	0.16 (0.30)

3 lags NW in parenthesis; **sse** $= \hat{\Sigma}_{ee}^{-1}$

Table VIII: Power of the Wald Test, C-CAPM

We simulate a one-factor C-CAPM economy 10,000 times under the alternative ($\alpha \neq 0$). Parameter estimates of the LFM are obtained using GMM with $W = I$. In the bootstrap (i.i.d. and block), we simulate jointly the factor and the residuals from the estimated LFM. We consider two lengths of the data set: the full sample of 525 observations (full sample) and a shorter sample of 240 observations (240 months). We consider two choices of assets, the ten size portfolios (size) and the 25 Fama-French portfolios (FF). When we investigate the properties of estimates of the LFM, we consider estimates based on the two weighting matrices $W = I$ and $W = see = \Sigma_{ee}^{-1}$. Estimates of the parameters of the LFM* are obtained by exactly-identified GMM. Asymptotic statistics are obtained assuming no serial correlation or serial correlation of order 3 (with Newey-West adjustment). We report rejection rates for the Wald test under the alternative, where the size is adjusted using the bootstrap results of Table VI. We compute the 10%, 5%, and 1% quantiles of the empirical distribution of the Wald statistic under the null. We then compute the percentage of times the Wald statistic exceeds the corresponding quantile, when the economy is simulated under the alternative. Panels A and B report results for the LFM specification, for the full sample and for the short sample of 240 observations, respectively. Panel C reports results for the LFM* specification.

Panel A: LFM (Full Sample)

	IID Bootstrap				Block Bootstrap			
	size		FF		size		FF	
	W=I	W=sse	W=I	W=sse	W=I	W=sse	W=I	W=sse
Ave. Chisq	24.11 (22.35)	32.35 (29.09)	83.78 (65.28)	104.85 (77.03)	23.83 (22.56)	32.01 (28.66)	83.73 (63.73)	104.95 (75.67)
0.10	0.89 (0.89)	0.97 (0.96)	1.00 (1.00)	1.00 (1.00)	0.92 (0.92)	0.97 (0.97)	1.00 (1.00)	1.00 (1.00)
0.05	0.81 (0.79)	0.93 (0.92)	0.99 (0.99)	1.00 (1.00)	0.84 (0.84)	0.93 (0.93)	0.99 (0.99)	1.00 (1.00)
0.01	0.57 (0.56)	0.80 (0.78)	0.98 (0.98)	1.00 (1.00)	0.57 (0.59)	0.79 (0.78)	0.98 (0.98)	1.00 (1.00)

Panel B: LFM (240 Months)

	IID Bootstrap				Block Bootstrap			
	size		FF		size		FF	
	W=I	W=sse	W=I	W=sse	W=I	W=sse	W=I	W=sse
Ave. Chisq	14.48 (13.26)	19.26 (17.03)	48.85 (36.74)	58.80 (41.80)	14.10 (13.17)	19.05 (16.73)	48.43 (35.82)	59.01 (41.21)
0.10	0.50 (0.48)	0.64 (0.63)	0.88 (0.86)	0.98 (0.97)	0.50 (0.50)	0.64 (0.63)	0.86 (0.84)	0.97 (0.95)
0.05	0.35 (0.35)	0.50 (0.48)	0.82 (0.80)	0.95 (0.92)	0.37 (0.36)	0.50 (0.49)	0.80 (0.77)	0.94 (0.90)
0.01	0.15 (0.13)	0.25 (0.22)	0.66 (0.60)	0.83 (0.78)	0.15 (0.13)	0.25 (0.22)	0.64 (0.56)	0.80 (0.71)

Panel C: LFM*

	IID Bootstrap				Block Bootstrap			
	Full Sample		240 Months		Full Sample		240 Months	
	size	FF	size	FF	size	FF	size	FF
Ave. Chisq	38.27 (39.29)	138.67 (147.30)	23.21 (24.66)	83.14 (95.66)	38.02 (38.35)	139.21 (143.06)	23.09 (24.07)	83.82 (93.23)
0.10	0.96 (0.96)	1.00 (1.00)	0.62 (0.61)	0.98 (0.97)	0.96 (0.96)	1.00 (1.00)	0.62 (0.60)	0.97 (0.95)
0.05	0.92 (0.91)	1.00 (1.00)	0.48 (0.46)	0.96 (0.93)	0.92 (0.91)	1.00 (1.00)	0.48 (0.45)	0.94 (0.90)
0.01	0.78 (0.76)	1.00 (1.00)	0.22 (0.21)	0.84 (0.78)	0.75 (0.72)	1.00 (1.00)	0.22 (0.19)	0.81 (0.71)

3 lags NW in parenthesis; **sse** = $\hat{\Sigma}_{ee}^{-1}$

Table IX: Power of the Wald Test, I-CAPM

We simulate a two-factor I-CAPM economy 10,000 times under the alternative ($\alpha \neq 0$). The two factors are the VW NYSE/AMEX/NASDAQ index return and the change in the dividend yield. Parameter estimates of the LFM are obtained using GMM with $W = I$. In the bootstrap (i.i.d. and block), we simulate jointly the factor and the residuals from the estimated LFM. We consider two lengths of the data set: the full sample of 525 observations (full sample) and a shorter sample of 240 observations (240 months). We consider two choices of assets, the ten size portfolios (size) and the 25 Fama-French portfolios (FF). When we investigate the properties of estimates of the LFM, we consider estimates based on the two weighting matrices $W = I$ and $W = see = \Sigma_{ee}^{-1}$. Estimates of the parameters of the LFM* are obtained by exactly-identified GMM. Asymptotic statistics are obtained assuming no serial correlation or serial correlation of order 3 (with Newey-West adjustment). We report rejection rates for the Wald test under the alternative, where the size is adjusted using the bootstrap results of Table VII. We compute the 10%, 5%, and 1% quantiles of the empirical distribution of the Wald statistic under the null. We then compute the percentage of times the Wald statistic exceeds the corresponding quantile, when the economy is simulated under the alternative. Panels A and B report results for the LFM specification, for the full sample and for the short sample of 240 observations, respectively. Panel C reports results for the LFM* specification.

Panel A: LFM (Full Sample)

	IID Bootstrap				Block Bootstrap			
	size		FF		size		FF	
	W=I	W=sse	W=I	W=sse	W=I	W=sse	W=I	W=sse
Ave. Chisq	25.69 (23.67)	27.91 (25.50)	92.29 (70.20)	101.28 (75.22)	25.61 (23.57)	27.87 (25.37)	92.39 (69.18)	101.83 (74.25)
0.10	0.91 (0.90)	0.91 (0.91)	1.00 (1.00)	1.00 (1.00)	0.92 (0.92)	0.92 (0.92)	1.00 (1.00)	1.00 (1.00)
0.05	0.85 (0.84)	0.85 (0.84)	1.00 (1.00)	1.00 (1.00)	0.87 (0.87)	0.86 (0.86)	1.00 (1.00)	1.00 (1.00)
0.01	0.69 (0.66)	0.68 (0.68)	1.00 (0.99)	1.00 (1.00)	0.66 (0.67)	0.68 (0.69)	1.00 (1.00)	1.00 (1.00)

Panel B: LFM (240 Months)

	IID Bootstrap				Block Bootstrap			
	size		FF		size		FF	
	W=I	W=sse	W=I	W=sse	W=I	W=sse	W=I	W=sse
Ave. Chisq	15.64 (14.20)	17.36 (15.57)	52.25 (38.60)	57.12 (41.06)	15.68 (14.16)	17.43 (15.51)	52.38 (38.05)	57.60 (40.67)
0.10	0.57 (0.55)	0.58 (0.56)	0.93 (0.92)	0.98 (0.97)	0.58 (0.57)	0.59 (0.59)	0.92 (0.90)	0.97 (0.95)
0.05	0.43 (0.42)	0.44 (0.43)	0.89 (0.86)	0.96 (0.93)	0.45 (0.44)	0.45 (0.46)	0.87 (0.82)	0.94 (0.90)
0.01	0.22 (0.21)	0.23 (0.22)	0.74 (0.68)	0.84 (0.77)	0.22 (0.21)	0.23 (0.22)	0.67 (0.59)	0.76 (0.65)

Panel C: LFM*

	IID Bootstrap				Block Bootstrap			
	Full Sample		240 Months		Full Sample		240 Months	
	size	FF	size	FF	size	FF	size	FF
Ave. Chisq	33.28 (34.22)	131.03 (139.18)	20.94 (22.25)	79.05 (90.89)	33.15 (33.89)	131.74 (136.00)	20.96 (22.12)	79.76 (88.97)
0.10	0.93 (0.92)	1.00 (1.00)	0.59 (0.58)	0.98 (0.97)	0.93 (0.93)	1.00 (1.00)	0.60 (0.59)	0.97 (0.94)
0.05	0.88 (0.86)	1.00 (1.00)	0.47 (0.44)	0.95 (0.92)	0.88 (0.88)	1.00 (1.00)	0.47 (0.47)	0.93 (0.88)
0.01	0.71 (0.69)	1.00 (1.00)	0.23 (0.21)	0.83 (0.76)	0.69 (0.70)	1.00 (1.00)	0.21 (0.22)	0.74 (0.63)

3 lags NW in parenthesis; **sse** = $\hat{\Sigma}_{ee}^{-1}$