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No. 565

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A Sticky-Dispersed Information Phillips Curve:*

A model with partial and delayed information

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First Draft: November 02, 2008
This Draft: December 22, 2009

Abstract

We study the interaction between dispersed and sticky information by assuming that firms receive private noisy signals about the state in an otherwise standard model of price setting with sticky-information. We show that there exists a unique equilibrium of the incomplete information game induced by the firms' pricing decisions, and derive the resulting Sticky-Dispersed Information (SDI) Phillips curve. The (equilibrium) aggregate price level and the inflation rates we derive depend on all values they have taken in the past. We perform several numerical simulations to evaluate how the Sticky-Dispersed Phillips curve we derive respond to changes in the main parameters of the model.

JEL Classification: D82, D83, E31

Keywords: Sticky information, dispersed information, Phillips curve

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Contents

1 Introduction

Over the last years, there has been renewed interest in the idea pioneered by Lucas (1972) and Phelps (1968) that prices fail to respond quickly to nominal shocks due to the fact agents are imperfectly informed about those shocks. As an example, Mankiw and Reis (2002) suggest that, perhaps due to acquisition costs, information (rather than prices) is *sticky*, i.e., new information is not immediately revealed to agents so that it diffuses slowly in the economy. As a result, although prices are always changing, pricing decisions are not always based on current information, and, consequently, do not respond instantaneously to nominal shocks.

There is also a large literature that assumes that agents have access to timely but heterogenous information about fundamentals. As a result, in the *dispersed-information* models of Morris and Shin (2002), Angeletos and Pavan (2007) and others, prices reflect the interaction among differently informed agents and their heterogenous beliefs about the state and about what *others* know about the state.

In this paper, we study how individual firms set prices when information is both sticky *and* dispersed, and analyze the resulting dynamics for aggregate prices and inflation rates. In our model, the firms' optimal price is a convex combination of the current state of the economy and the aggregate price level. Moreover, as in Mankiw and Reis (2002), only a fraction of firms update their information set at each period. Those who update receive two sources of information: the first piece is the value of all previous periods states, while the second piece is a noisy, idiosyncratic, private signal about the current state of the economy. Since noisy signals are idiosyncratic, the firms that update their information set will have heterogenous information about the state (as in Morris and Shin (2002) and Angeletos and Pavan (20007)). Hence, in our model, *heterogenous* information disseminates slowly in the economy.

As individual prices depend on the current state and the aggregate price level, firms that update their information set must not only form beliefs about the current state but also form beliefs about the other firms' beliefs about current the state, and so on and so forth, so that higher-order beliefs play a key role in our model. A firm's belief about the state depends on it is private signal. Hence, the pricing decisions by firms induce an incomplete information game among them.

In our main result, we prove that there exists a unique equilibrium of such game. The uniqueness of the equilibrium allows us to unequivocally speak about the sticky-dispersed-information (henceforth, SDI) aggregate price level and Phillips curve. The SDI aggregate price level we derive depends on all the prices firms have set in the past. This is so for two reasons. First, there are firms in the economy for which the information set has been last updated in the far past. This is a *direct* effect of sticky information. Second, firms that have just received new information will behave, at least partly, as if they were backward-looking. This happens because of an *strategic* effect: their optimal relative price depends on how they believe all other firms (including those that have outdated information sets) in the economy are setting prices

From aggregate prices, we are able to derive the SDI Phillips curve. Since current aggregate prices depend on all prices set by firms in the past, the current inflation rate will also depend on inflation rates that prevailed in the past. Therefore, in spite of the fact that firms are forward looking in our model, the Phillips curve that results from their interaction displays a non-trivial dependence on inflation rates that prevailed in the past. This is an implication of the stickiness of information in our model and was already present in Mankiw and Reis (2002). In our model, however, in addition to being sticky, information is also noisy and *dispersed*. The fact that information is *noisy* leads a firm that has its information set updated in t to find it optimal to place positive weight on the states from periods $t-j$, $j>0$, to predict the state in period t. Hence, in comparison to an economy à la Mankiw and Reis (2002), the adjustment of prices to shocks will be slower in an economy with noisy information. Through the complementarities in price setting, the *dispersion* of information magnifies such effect.

Our model nests as special cases the complete information model, the dispersed information model and the sticky information model. To better understand the roles played by information stickiness and dispersed information, we decompose our SDI Phillips curve into three benchmark inflation rates that can be obtained as limiting cases of our model: (i) complete-information inflation, (ii) dispersed-information inflation, and (iii) sticky-information inflation.

We study the individual contribution to the SDI Phillips curve of each of the main parameters of our model: (i) Degree of strategic complementarity, (ii) Degree of informational stickiness, (iii) Public information precision, and (iv) Private information precision. First, we analyze the impact of current and past complete-information inflation rates on current SDI inflation. Second, we consider the inflation response to monetary shocks. Finally, we compare the variance of SDI inflation with the variances of complete-information inflation, dispersed-information inflation, and sticky-information inflation.

In addition to the effects discussed above, the introduction of dispersed information in an otherwise standard sticky-information model sheds light on two different issues. First, dispersion in an sticky- information setting generates price and inflation inertia irrespective of assumptions regarding the firms' capacity to predict equilibrium outcomes. Indeed, although they may not have their information sets up to date, the firms in our model correctly predict the equilibrium behavior of their opponents. In spite of correctly predicting the *strategies* (i.e., contingent plans) adopted by the opponents in equilibrium, a firm cannot infer what is the actual price set by them (i.e., the action taken), since it does not observe its opponents' private signals. Hence, a firm that has not updated its information set cannot infer the current state from the behavior of its opponents. This is in contrast to Mankiw and Reis (2002) who, at least for the main numerical experiment presented in their paper, obtain price inertia by (implicitly) assuming that agents cannot condition on equilibrium behavior from the opponents. In fact, in such experiment, there is a (single) nominal shock that only a fraction of the firms observe. Trivially, the prices set by those firms (as well as aggregate prices) will reflect such change in the fundamental. Hence, firms that haven't observed the shock but can predict the equilibrium behavior of the opponents will be able to infer the fundamental from such behavior.[1] It follows that *all* firms should adjust prices in response to a shock.

The second, and more important, issue relates to policy. In a world in which information is dispersed, the optimal communication policy for a benevolent central banker who has (imperfect) information about the states is far from trivial. On the one hand, any information disclosed by the central banker about the state will have the benefit of allowing the agents to count on an additional piece of information about the state when deciding on their prices. This benefit is particularly relevant when information is sticky for a fraction of firms is setting prices based on outdated information about the current state. On the other hand, since the information disclosed by the central banker is a public signal, agents will place too much weight on any information disclosed by the central banker as this is a public signal (e.g., Morris and Shin (2002), Angeletos and Pavan (2007). We believe the model we put forth in this paper is a suitable framework to study optimal communication policy by central banks when information is heterogenous and sticky.

Related Literature. This work follows a large number of papers that sheds new light into the tradition that dates back to Phelps (1968) and Lucas (1972) of considering the effects of imperfect information on price-setting decisions. Mankiw and Reis (2009) provide the most

[1] The argument here is similar to the one in Rational Expectations Equilibrium models à la Grosmann (1981).

recent survey of aggregate supply under imperfect information, whereas Veldkamp (2009) covers a myriad of topics related to informational asymmetries and information acquisition in macroeconomics and finance. Our paper connects to this broad literature through two specific strands. In our model, (i) information in our model is sticky, as in Mankiw and Reis (2002) and others, and (ii) following Woodford (2002) and Morris and Shin (2002), among others, information is dispersed.

The papers that are the closest to ours are Mankiw and Reis (2009) and Angeletos and La'O (2009). In addition to surveying the most recent literature on the impact of informational frictions on pricing decisions, Mankiw and Reis (2009) compare a partial (dispersed) information model with a delayed (sticky) information model, and derive their common implications.[2] In turn, Angeletos and La'O (2009) introduce dispersed information (and explicitly discuss the role of higher order beliefs) in an otherwise standard setting with sticky prices *à la* Calvo (1983). We depart from Mankiw and Reis (2009) by combining in a single model both dispersed information and informational stickiness, highlighting their joint effects on aggregate prices and inflation rates. To the best of our knowledge, we are the first to offer an integrated approach to study the interaction of dispersion and stickiness on pricing decisions. By focusing on informational stickiness (rather than price stickiness), we complement the analysis of Angeletos and La'O (2009).

Organization. The paper is organized as follows. In section 2, the set-up of the model is described. In section 3, we derive the unique equilibrium of the pricing game played by the firms, and derive the implied aggregate prices and inflation rates. In section 4, we compare our SDI Phillips curve with three benchmarks: the complete information, the sticky-information and the dispersed information Phillips curves. Section 5 calibrates our SDI Phillips curve for different values of the main parameters of the model. Section 6 draws the concluding remarks. All derivations that are not in the text can be found in the Appendix.

2 The Model

The model is a variation of Mankiw and Reis' (2002) sticky information model.[3] There is a continuum of firms, indexed by $i \in [0,1]$, that set prices at every period $t \in \{1,2,...\}$.

[2] The theories of "rational inattention" proposed by Sims (2003, 2009) and "inattentiveness" proposed by Reis (2006a, 2006b), have been used to justify models of dispersed information and sticky information.

[3] Subsequent refinements of the sticky information models can be found in Mankiw and Reis (2009, 2007, 2006) and Reis (2009, 2006a, 2006b).

Although prices can be re-set at no cost at each period, information regarding the state of the economy is made available to the firms infrequently. At period t, only a fraction λ of firms is selected to update their information sets about the current state. For simplicity, the probability of being selected to adjust information sets is the same across firms and independent of history.

We depart from a standard sticky-information model by allowing information to be *heterogeneous* and *dispersed*: a firm that updates its information set receives public information regarding the past states of the economy as well as a *private* signal about the current state.

Pricing Decisions:

Under complete information, any given firm $z \in [0,1]$ set its (log-linear) price $p_t(z)$ equal to the optimal price decision p_t^* given by

$$p_t^* \equiv rP_t + (1-r)\theta_t, \tag{1}$$

where $P_t \equiv \int_0^1 p_t(z)\,dz$ is the aggregate price level, and θ_t is the nominal aggregate demand, the current state of the economy. This pricing rule is standard, and, although we don't do it explicitly, can be derived from a firm's profit maximization problem in a model of monopolistic competition à la Blanchard and Kiyotaki (1987).

Information:

The state θ_t follows a random walk

$$\theta_t = \theta_{t-1} + \epsilon_t, \tag{2}$$

with $\epsilon_t \sim N(0, \alpha^{-1})$.

If firm $z \in [0,1]$ is selected to update its information set in period t, it observes all *previous* periods realizations of the state, $\{\theta_{t-j}, j \geq 1\}$. Moreover, it obtains a noisy private signal about the current state. Denoting such signal by $x_t(z)$, we follow the literature and assume:

$$x_t(z) = \theta_t + \xi_t(z), \tag{3}$$

where $\xi_t(z) \sim N\left(0, \beta^{-1}\right)$, β is the precision of $x_t(z)$, and the error term $\xi_t(z)$ is independent of ϵ_t for all z, t.

As a result, if one defines

$$\Theta_{t-j} = \{\theta_{t-k}\}_{k=j}^{\infty}, \tag{4}$$

at period t, the information set of a firm z that was selected to update its information j

periods ago is

$$I_{t-j}(z) = \{x_{t-j}(z), \Theta_{t-j-1}\}. \tag{5}$$

3 Equilibrium

Using (1), the best response for a firm z that was selected to update its information j periods ago – and, therefore, has $I_{t-j}(z)$ as its information set – is its forecast of p_t^*, given the available information $I_{t-j}(z)$ and the equilibrium behavior of its opponents:

$$p_{j,t}(z) = E\left[p_t^* \mid I_{t-j}(z), p_{-j,t}(.)\right]. \tag{6}$$

Denoting by Λ_{t-j} the set of firms that last updated its information set at period $t-j$, we can express the aggregate price level P_t as

$$\begin{aligned} P_t &= \int_{\cup_{j=0}^{\infty}\Lambda_{t-j}} p_t(z)\, dz \\ &= \sum_{j=0}^{\infty} \int_{\Lambda_{t-j}} E\left[p_t^* \mid I_{t-j}(z), p_{-j,t}(.)\right] dz. \end{aligned} \tag{7}$$

Since the optimal price p_t^* is a convex combination of the state, θ_t, and the aggregate price level, firm z needs to forecast the state of the economy *and* the pricing behavior of the other firms in the economy. The pricing behavior of each of these firms, in turn, depends on their own forecast of the other firms' aggregate behavior. It follows that firm z must not only forecast the state of the economy but also, to predict the behavior of the other firms in the economy, must make forecasts of these firms' forecasts about the state, forecasts about the forecasts of these firms forecasts about the state, and so on and so forth. In other words, higher order beliefs will play a key role in the derivation of an equilibrium in our model.

Indeed, if one defines the average k-th order belief about the current state recursively as follows:

$$\bar{E}^k[\theta_t] = \begin{cases} \theta_t, & k = 0 \\ \sum_{j=0}^{\infty} \int_{\Lambda_{t-j}} E\left[\bar{E}^{k-1}[\theta_t] \mid I_{t-j}(z)\right] dz, & k \geq 1 \end{cases} \tag{8}$$

we have:

Proposition 1 *In equilibrium, the aggregate price level is*

$$P_t = (1-r) \sum_{k=1}^{\infty} r^{k-1} \bar{E}^k[\theta_t]. \tag{9}$$

3.1 Computing the Equilibrium

In this section, we derive the unique equilibrium of the pricing game played by the firms. Following Morris and Shin (2002), we do this in two steps. We first derive an equilibrium for which the aggregate price level is a linear function of fundamentals. We then establish, using Proposition 1, that this linear equilibrium is the unique equilibrium of our game.

3.1.1 Prior Distribution

In the Appendix, we show that, given the distribution of the private signals and the process $\{\theta_t\}$ implied by (2), a firm z that updated its information set in period $t-j$ makes use of the variables $x_{t-j}(z) = \theta_{t-j} + \xi_{t-j}(z)$ and $\theta_{t-j-1} = \theta_{t-j} - \epsilon_{t-j}$, to form the following belief about the current state θ_{t-j}:

$$\theta_{t-j} \mid I_{t-j}(z) \sim N\left((1-\delta)\, x_{t-j}(z) + \delta\theta_{t-j-1}, (\alpha+\beta)^{-1}\right), \tag{10}$$

where

$$\delta \equiv \frac{\alpha}{\alpha+\beta} \in (0,1)\,. \tag{11}$$

Hence, a firm that updated its information set in $t-j$ expects the current state to be a convex combination of the private signal $x_{t-j}(z)$ and a (semi) public signal θ_{t-j-1} – the only relevant piece of information that comes from learning all previous states $\{\theta_{t-j-k}\}_{k\geq 1}$.[4] The relative weights given to $x_{t-j}(z)$ and θ_{t-j-1} when the firm computes the expected value of state θ_{t-j} depend on the precision of such signals.

Using (2), one has that, for $m \leq j$,

$$\theta_{t-m} = \theta_{t-j} + \textstyle\sum_{k=0}^{j-m-1} \epsilon_{t-m-k}. \tag{12}$$

Thus, the expectation of a firm z that last updated its information set at $t-j$ about θ is

$$E\left[\theta_{t-m} \mid I_{t-j}(z)\right] = \begin{cases} E\left[\theta_{t-j} \mid I_{t-j}(z)\right] = (1-\delta)\, x_{t-j}(z) + \delta\theta_{t-j-1} & : m \leq j \\ \theta_{t-m} & : m > j \end{cases}. \tag{13}$$

In words, a firm that last updated its information set in period $t-j$ expects that all future values of the fundamental θ will be the same as the expected value of the fundamental at the

[4] θ_{t-j-1} is the only piece of information in $\Theta_{t-j} = \{\theta_{t-j-k}\}_{k=1}^{\infty}$ the firm needs to use because the state's process is Markovian.

period $t-j$. Moreover, since at the moment it adjusts its information set the firm observes all previous states, the firm will know for sure the value of θ_{t-m} for $m > j$.

3.1.2 Linear Equilibrium

To derive the linear equilibrium, we adopt a standard guess and verify approach. We assume that the (equilibrium) aggregate price level is linear and then show that the implied best responses for the individual firms indeed lead to linear aggregate prices.

Toward that, assume that

$$P_t = \sum_{j=0}^{\infty} c_j \theta_{t-j}. \tag{14}$$

for some constants c_j, $j \geq 0$.

In such case, the optimal price for a firm that last updated information at $t-m$ is

$$\begin{aligned}
p_t &= E\left[(1-r)\,\theta_t + rP_t \mid I_{t-m}\right] \\
&= (1-r)\,E\left[\theta_t \mid I_{t-m}\right] + r\sum_{j=0}^{\infty} c_j E\left[\theta_{t-j} \mid I_{t-m}\right] \\
&= (1-r)\,E\left[\theta_t \mid I_{t-m}\right] + r\sum_{j=0}^{m} c_j E\left[\theta_{t-j} \mid I_{t-m}\right] + r\sum_{j=m+1}^{\infty} c_j E\left[\theta_{t-j} \mid I_{t-m}\right] \\
&= \left[1 - r\,(1-C_m)\right]\left[(1-\delta)\,x_{t-m} + \delta\theta_{t-m-1}\right] + r\sum_{j=m+1}^{\infty} c_j \theta_{t-j} \\
&= (1-\delta)\left[1 - r\,(1-C_m)\right] x_{t-m} + \delta\left[1 - r\,(1-C_{m+1})\right]\theta_{t-m-1} + r\sum_{j=m+2}^{\infty} c_j \theta_{t-j},
\end{aligned}$$

where

$$C_m \equiv \sum_{j=0}^{m} c_j.$$

Aggregating such individual prices and using (7), we get

$$\begin{aligned}
P_t &= \sum_{m=0}^{\infty} \int_{\Lambda_{t-m}} \left[1 - r\,(1-C_m)\right]\left[(1-\delta)\,x_{t-m} + \delta\theta_{t-m-1}\right] + r\sum_{j=m+1}^{\infty} c_j \theta_{t-j} dz \\
&= \lambda\sum_{m=0}^{\infty} (1-\lambda)^m \left\{\left[1 - r\,(1-C_m)\right]\left[(1-\delta)\,\theta_{t-m} + \delta\theta_{t-m-1}\right] + r\sum_{j=m+1}^{\infty} c_j \theta_{t-j}\right\} \\
&= \lambda\sum_{m=0}^{\infty} (1-\lambda)^m \left\{\left[1 - r\,(1-C_m)\right]\left[(1-\delta)\,\theta_{t-m} + \delta\theta_{t-m-1}\right]\right\} \\
&\quad + r\sum_{m=0}^{\infty} c_m \left[1 - (1-\lambda)^m\right]\theta_{t-m}.
\end{aligned}$$

Note that the above equality can be re-written as

$$
\begin{aligned}
(1-r)\,P_t &= \lambda\,(1-\delta)\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m\left[1-r\,(1-C_m)\right]\theta_{t-m} \\
&\quad +\lambda\delta\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m\left[1-r\,(1-C_m)\right]\theta_{t-m-1} \\
&\quad -r\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m c_m\theta_{t-m},
\end{aligned}
$$

so that the implied aggregate price will be linear in the values of the fundamental, as assumed in (14).

Matching coefficients, we obtain

$$
c_k \equiv \begin{cases} \frac{\lambda(1-\delta)(1-r)}{1-r\lambda(1-\delta)} & \text{if } k=0 \\ \frac{\lambda(1-r)\phi(1-\lambda)^{k-1}}{\left[1-r\left[1-\phi(1-\lambda)^{k-1}\right]\right]\left[1-r\left[1-\phi(1-\lambda)^{k}\right]\right]} & \text{if } k\geq 1, \end{cases} \tag{15}
$$

where

$$
\begin{aligned}
\phi &= 1-\lambda\,(1-\delta)\,, \\
C_\infty &\equiv \lim_{m\to\infty}\textstyle\sum_{j=0}^{m} c_j = 1.
\end{aligned}
$$

We have then shown:

Proposition 2 (Linear Equilibrium) *There exists an equilibrium in which the aggregate price level in period t, P_t, are linear in the states $\{\theta_{t-j}\}_{j=0}^{\infty}$.*

3.1.3 Uniqueness of Equilibrium: Beliefs

As shown in Proposition (1), an alternative way to describe the aggregate price level in period t is through a weighed average of all (average) higher order beliefs about the state θ_t. In this section, we derive such beliefs and establish that the implied aggregate price level will be identical to the one derived in Proposition (2). This will establish that the linear equilibrium is unique.

First Order Beliefs:

Using (13), we are able to compute (8) for the case $k=1$.

$$
\bar{E}^1\left[\theta_t\right] = \lambda\textstyle\sum_{j=0}^{\infty}(1-\lambda)^j\left[(1-\delta)\,\theta_{t-j}+\delta\theta_{t-j-1}\right]. \tag{16}
$$

Higher Order Beliefs:

In the Appendix, we use (16) and the recursion (8) to derive the following useful result:

Lemma 1 *The average k-th order forecast of the state is given by*

$$\bar{E}^k[\theta_t] = \lambda \sum_{m=0}^{\infty} (1-\lambda)^m [\kappa_{m,k}\theta_{t-m} + \delta_{m,k}\theta_{t-m-1}], \tag{17}$$

with the weights $(\kappa_{m,k}, \delta_{m,k})$ are recursive defined for $k \geq 1$

$$\begin{bmatrix} \kappa_{m,k+1} \\ \delta_{m,k+1} \end{bmatrix} = \begin{bmatrix} (1-\delta) \\ \delta \end{bmatrix} [1-(1-\lambda)^m]^k + A_m \begin{bmatrix} \kappa_{m,k} \\ \delta_{m,k} \end{bmatrix},$$

where the matrix A_m is given by

$$A_m \equiv \begin{bmatrix} [(1-\delta)\left[1-(1-\lambda)^{m+1}\right] + \delta[1-(1-\lambda)^m]] & 0 \\ \delta\left[\left[1-(1-\lambda)^{m+1}\right] - [1-(1-\lambda)^m]\right] & \left[1-(1-\lambda)^{m+1}\right] \end{bmatrix},$$

and the initial weights are $(\kappa_{1,k}, \delta_{1,k}) \equiv (1-\delta, \delta)$.

Plugging (17) into the expression for the aggregate price level P_t, (9), we get, after a few manipulations, the following expression for the aggregate price level:

$$\begin{aligned} P_t &= (1-r)\sum_{k=1}^{\infty} r^{k-1}\bar{E}^k[\theta_t] \\ &= \sum_{m=0}^{\infty} K_m [(1-\Delta_m)\theta_{t-m} + \Delta_m\theta_{t-m-1}], \end{aligned} \tag{18}$$

where the weights K_m and Δ_m are

$$\begin{aligned} K_m &\equiv \frac{(1-r)\lambda(1-\lambda)^m}{(1-r[1-(1-\lambda)^m])\left(1-r\left[1-(1-\lambda)^{m+1}\right]\right)}, \\ \Delta_m &\equiv \frac{\delta[1-r[1-(1-\lambda)^m]]}{1-r\left[(1-\delta)\left[1-(1-\lambda)^{m+1}\right]+\delta[1-(1-\lambda)^m]\right]}. \end{aligned}$$

Comparing the coefficients above with the $\{c_j\}_{j=0}^{\infty}$ defined in (15), for

$$\begin{aligned} &c_0 \text{ with } K_0(1-\Delta_0), \\ &c_k \text{ with } K_{m-1}\Delta_{m-1} + K_m(1-\Delta_m),\ m \geq 1, \end{aligned}$$

one sees that the aggregate price level implied by (18) is exactly the same as the one derived in Proposition (2).

Having shown that the equilibrium is unique, we can unequivocally speak about *the* Phillips curve of our economy. Denoting the inflation rate by π_t, by taking first differences of equation (18), we can write our **sticky-dispersed-information Phillips curve** as

$$\pi_t = \textstyle\sum_{m=0}^{\infty} K_m \left[(1-\Delta_m)(\theta_{t-m} - \theta_{t-m-1}) + \Delta_m (\theta_{t-m-1} - \theta_{t-m-2})\right]. \tag{19}$$

We summarize all the discussion above in the following result:

Proposition 3 *In an economy in which information is sticky and dispersed, and the state follows (2), there is a unique equilibrium in the pricing game played by the firms. In such equilibrium, the aggregate price level is given by*

$$P_t = \textstyle\sum_{m=0}^{\infty} K_m \left[(1-\Delta_m)\,\theta_{t-m} + \Delta_m \theta_{t-m-1}\right], \tag{20}$$

and the SDI Phillips curve is given by

$$\pi_t = \textstyle\sum_{m=0}^{\infty} K_m \left[(1-\Delta_m)(\theta_{t-m} - \theta_{t-m-1}) + \Delta_m (\theta_{t-m-1} - \theta_{t-m-2})\right], \tag{21}$$

where

$$K_m \equiv \frac{(1-r)\,\lambda\,(1-\lambda)^m}{\left(1-r\left[1-(1-\lambda)^m\right]\right)\left(1-r\left[1-(1-\lambda)^{m+1}\right]\right)}, \tag{22}$$

$$\Delta_m \equiv \frac{\delta\left[1-r\left[1-(1-\lambda)^m\right]\right]}{1-r\left[(1-\delta)\left[1-(1-\lambda)^{m+1}\right]+\delta\left[1-(1-\lambda)^m\right]\right]}. \tag{23}$$

Note that the current aggregate price level P_t depends on all the prices firms have set in the past. This is so for two reasons. First, there are firms in the economy for which the information set has been last updated in the far past. This is a *direct* effect of sticky information. Second, even firms that have just adjusted their information set will be, at least partly, backward-looking. This happens because of an *strategic* effect: their optimal relative price depends on how they believe all other firms (including those that have outdated information sets) in the economy are setting prices. The direct and strategic effects of sticky information are captured by the terms K_m.

It is immediate that, since current aggregate prices depend on all prices set by firms in the past, the current inflation rate will also depend on inflation rates that prevailed in the past. Therefore, in spite of the fact that firms are forward looking in our model, the Phillips curve that results from their interaction displays a non-trivial dependence on inflation rates that prevailed in the past. This is an implication of the stickiness of information in our model and was already present in Mankiw and Reis (2002).

In our model, however, on top of being sticky, information is also *disperse*. The effect of dispersion is captured by the positive weight given to the state in period θ_{t-m-1} by a firm that has its information set updated in $t-m$. As the private signal the firm observes is noisy, it is always optimal to place some weight on past states to forecast the current state. Hence, in comparison to an economy à la Mankiw and Reis (2002), the adjustment of prices to shocks will be slower in an economy with disperse information.

Also, and perhaps more importantly, the introduction of dispersion in an sticky information model allows us to generate price and inflation inertia irrespective of assumptions regarding the firms' capacity to predict equilibrium outcomes. Indeed, although they may not have their information sets up to date, the firms in our model correctly predict the equilibrium behavior of their opponents. In spite of correctly predicting the *strategies* (i.e., contingent plans) adopted by the opponents in equilibrium, a firm cannot infer what is the actual price set by them (i.e., the action taken), since it cannot observe its opponents' private signals. Hence, a firm that hasn't updated its information set cannot infer the current state from the behavior of its opponents.

This is in contrast to Mankiw and Reis (2002) who, in order to obtain price and information inertia in a model with sticky but non-dispersed information, (implicitly) assume that agents cannot condition on equilibrium behavior from the opponents. In fact, in their main experiment, there is a (single) nominal shock that only a fraction of the firms observe. Trivially, the prices set by those firms (as well as aggregate prices) will reflect such change in the fundamental. Hence, a firm that hasn't observed the shock but can predict the equilibrium behavior of the opponents will be able to infer the fundamental from such behavior.[5] It follows that *all* firms will adjust prices in response.

4 Benchmarks for the SDI Phillips Curve

Our model nests the dispersed information model ($\lambda = 1$) and the sticky information model ($\beta^{-1} \to 0$) as special cases. In order to understand the properties of the SDI Phillips curve, in what follows, we compare it to those two benchmarks as well as to the Phillips curve implied by the complete information case.

[5] The argument here is similar to the one in Rational Expectations Equilibrium models à la Grosmann (1981).

4.1 Benchmark 1: Complete-information Inflation

Under complete information, the price of any firm z is

$$p_t(z) = p_t^* \equiv rP_t + (1-r)\,\theta_t.$$

Since firms are identical, they all set the same price. As a result

$$P_t = rP_t + (1-r)\,\theta_t \Rightarrow P_t = \theta_t.$$

Hence, if θ is common knowledge, the equilibrium entails an inflation rate $\pi_{C,t}$ – that we call the *complete-information inflation* – that is equal to the change of states:

$$\pi_{C,t} = \theta_t - \theta_{t-1} \tag{24}$$

4.2 Benchmark 2: Dispersed-information Inflation

If stickiness vanishes ($\lambda = 1$), our results converge to the ones obtained by Morris and Shin (2002) and Angeletos and Pavan (2007). Denoting the inflation rate for the economy without stickiness by $\pi_{D,t}$ (the dispersed information inflation), we have:

$$\pi_{D,t} = (1-\Delta)\,\pi_{C,t} + \Delta\pi_{C,t-1}, \tag{25}$$

so that the inflation rate in period t is a convex combination of the complete information inflations of period t and $t-1$, with the weight on period $t-1$ complete information inflation given by

$$\Delta = c_1 \equiv \frac{\delta}{1-r\,(1-\delta)}, \tag{26}$$

$1-\Delta = c_0$, and $c_k = 0$, $\forall\ k > 1$.[6]

When compared to the full information case, the inflation rate that prevails with dispersed information displays more inertia. Moreover, note that

$$E\left[\pi_{D,t} \mid I_t(z)\right] = (1-\Delta)\,E\left[\pi_{C,t} \mid I_t(z)\right] + \Delta\pi_{C,t-1}.$$

[6] Alternatively, as in Morris and Shin (2002), we can say that inflation in t is a convex combination of the "state/fundamental", $\pi_{C,t}$, and the "public signal", $\pi_{C,t-1}$.

Hence, when information is dispersed, the forecast error

$$\pi_{D,t} - E\left[\pi_{D,t} \mid I_t(z)\right] = (1-\Delta)\left[\pi_{C,t} - E\left[\pi_{C,t} \mid I_t(z)\right]\right]$$

is proportional to the forecast error of the complete information inflation $\pi_{C,t}$.

4.3 Benchmark 3: Sticky-information Inflation

The other polar case occurs when information is sticky but not dispersed ($\delta = 0$). In such case, the Phillips curve we obtain resembles the one in Mankiw and Reis (2002). Denoting the sticky information inflation by $\pi_{S,t}$, we have

$$\pi_{S,t} = \textstyle\sum_{m=0}^{\infty} K_m \pi_{C,t-m}, \tag{27}$$

where inflation is also a function of current and past complete-information inflation, but with the weights K_m in (22) replacing the coefficients c_m defined in (15). Note that, for $m = 0$

$$c_0 \equiv \frac{(1-r)\lambda(1-\delta)}{1-r\lambda(1-\delta)} < \frac{(1-r)\lambda}{1-r\lambda} \equiv K_0$$

because

$$\frac{\partial c_0}{\partial \delta} \equiv \frac{-(1-r)\lambda}{\left[1-r\lambda(1-\delta)\right]^2} < 0.$$

4.4 Benchmark contribution to SDI inflation

We can rewrite our SDI Phillips curve as a combination of the inflation rates that prevail under the three benchmarks cases discussed above. First, note that the SDI inflation π is a function of complete information inflations π_C of current and previous periods. Indeed, using (14) or (20), we obtain

$$\begin{aligned} \pi_t &= \textstyle\sum_{j=0}^{\infty} c_j \pi_{C,t-j} \\ &= \textstyle\sum_{m=0}^{\infty} K_m \left[(1-\Delta_m)\pi_{C,t-m} + \Delta_m \pi_{C,t-m-1}\right]. \end{aligned} \tag{28}$$

Using (21) and (27), we can also relate the SDI inflation to the sticky-information inflation π_S as follows:

$$\pi_t = \pi_{S,t} - \textstyle\sum_{m=0}^{\infty} K_m \Delta_m \left(\pi_{C,t-m} - \pi_{C,t-m-1}\right).$$

Finally, if we combine this last equation with (25), we obtain a decomposition of SDI inflation that includes all the proposed benchmarks

$$\pi_t = \pi_{S,t} + \sum_{m=0}^{\infty} K_m \left(\frac{\Delta_m}{\Delta}\right) [\pi_{D,t-m} - \pi_{C,t-m}] . \quad (29)$$

Thus, compared to the case in which information is sticky, inflation under sticky *and* dispersed information will be higher if, and only if, the dispersed information inflation, $\pi_{D,t-m}$, is on "average" higher than the complete information inflation $\pi_{C,t-m}$.

5 Inflation Behavior under SDI

We now examine how the SDI Phillips curve behaves in response to changes in the main parameters of the model. Making use of the fact that we can write the SDI inflation as a weighted average of all past complete information inflation rates, we start, in Figure 1, by analyzing the impact of period $t-k$ complete information inflation $\pi_{C,t-k}$ on SDI current inflation π_t. After that, in Figure 2, we consider the inflation response to monetary shocks. Finally, in Figure 3, we consider the behavior of SDI's inflation variance as well as the variances of the three benchmarks considered in Section 4: complete-information inflation, dispersed-information inflation, and sticky-information inflation.

To isolate effects, we perform each of the above exercises for different values of the key parameters of the model as listed in Table 1 – (a) Strategic complementarity r, (b) Information stickiness λ, (c) Public information precision α, and (d) Private information precision β.

5.1 Calibration

The model's structural parameters are r, λ α, and β. The baseline values we use for r and λ (see Table 1) are standard and based on Mankiw and Reis (2002). A value of $\lambda = 0.25$ can be interpreted as implying that, on average, firms adjust their information set (and therefore their prices) once a year. This is compatible with the most recent microeconomic evidence on price-setting.[7] The higher the value of r, the more important becomes the aggregate price level (and therefore the strategic interaction component) for (of) the firms's optimal price. We set $\alpha = \beta = 0.5$ as our benchmark value to keep the baseline calibration as neutral as

[7] See, for example, Klenow and Malin (2009).

Parameter	Description	Range	Benchmark Value
r	Degree of strategic complementarity	$[0, 1]$	0.90
λ	Degree of informational stickiness	$[0, 1]$	0.25
α	Public information precision	$[0, 1]$	0.50
β	Private information precision	$[0, 1]$	0.50

Table 1: Baseline calibration

possible regarding the importance of public versus private information precision.

To better understand the impact of each individual parameter on the SDI Phillips curve, in what follows, we always keep three of the four key parameters fixed at their benchmark values and vary the fourth one.

5.2 Impact of complete information inflation

We first consider the impact of period $t-k$ complete information inflation $\pi_{C,t-k}$ on the current SDI inflation π_t. Using equation (28), one can readily see that such impact is fully captured by the coefficients $c_j^{\prime}s$ in Equation (15). We plot the results in Figure 1, where each panel shows the effect of changes in one of the four parameters of the model.

Consider Panel (a) of Figure 1. The weight on the current complete information inflation is higher the lower the degree of strategic complementarity, r. As the degree of strategic complementarity rises, the incentive for firms to align prices increases. As a result, even informed firms will attach a higher weight on past information. This leads to a higher impact of *past* complete information on *current* SDI inflation.

Panel (b) of Figure 1 captures the role of informational stickiness on the impact of past full information inflation rates on current SDI inflation. It can be seen that higher values of λ (i.e., smaller degrees of information stickiness) are related to lower weights on past complete information inflation. As the degree of information stickiness increases, however, the share of SDI inflation that comes from the past is higher, since firms have incentives to align prices and, the lower λ, the larger the faction of price setters that are stuck with past information about the state.

The impact of information dispersion on SDI inflation is shown in Panels (c) and (d) of Figure 1. Firms attach more weight on a given piece of information the more precise it is. Consider the case in which public information becomes more precise (α increases)

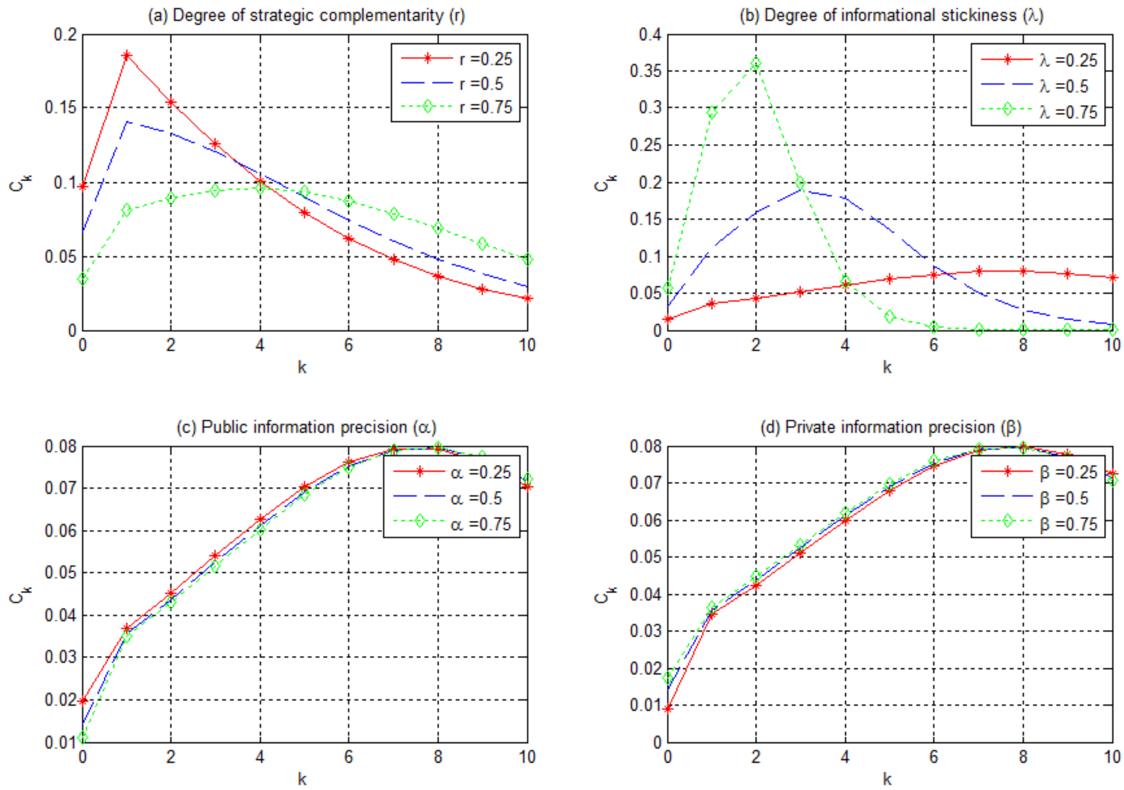

Figure 1: Coefficients $c_j's$ for different values of the parameters $(r, \lambda, \alpha, \beta)$.

and/or private information becomes less precise (β decreases). In such case, $\delta \equiv \alpha / (\alpha + \beta)$ increases, and firms attach more weight to the past since, the larger δ, the more (relatively to their private information) the firms can be confident about past fundamentals being a good source of information about the current fundamental.

5.3 Impulse Response Functions

Figure 2 shows the impulse responses of current SDI inflation, π_t, to a shock in the fundamental process $\{\epsilon_t\}$ in (2).

From Panel (a) of Figure 2, we observe that, as r increases, inflation becomes more inertial. When $r = 0$, the firms' desired prices respond only to the value of the fundamental, θ. In such case, inflation responds quickly to the shock. By contrast, when $0 < r < 1$, firms also care about the overall price level and, therefore, need to consider what information

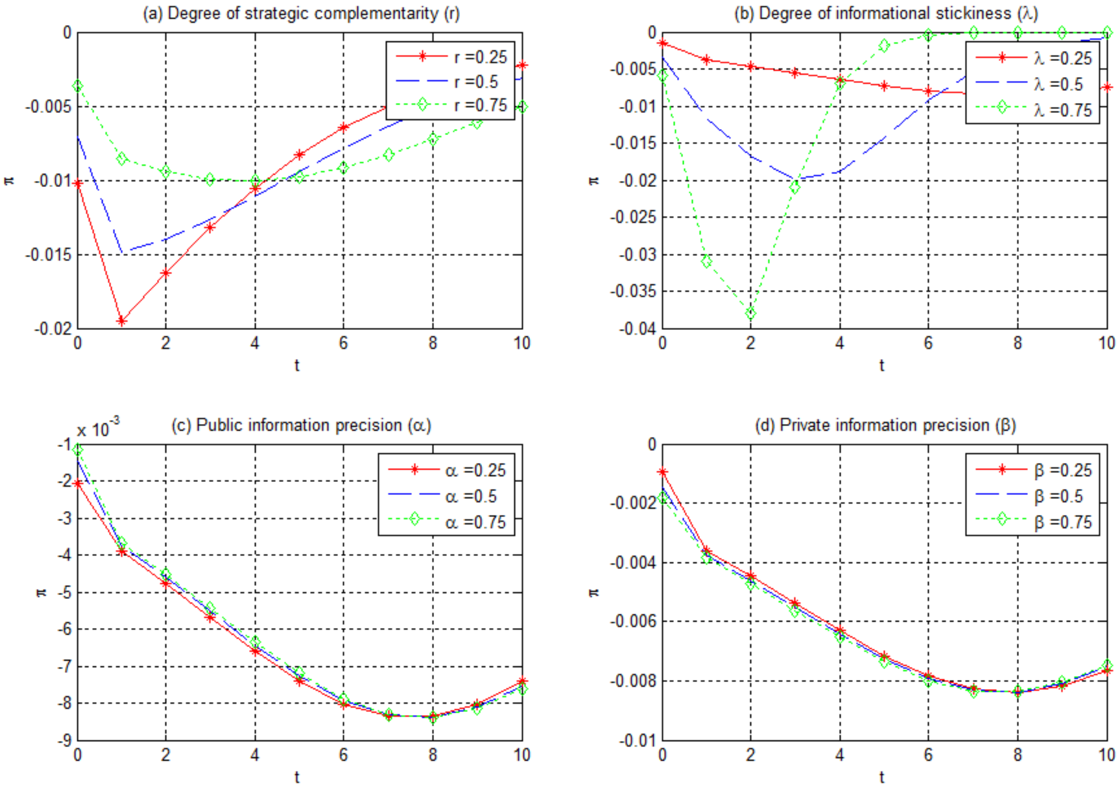

Figure 2: Responses of π_t to a shock in the fundametal process ϵ_t for different values of $(r, \lambda, \alpha, \beta)$.

other firms have. In the SDI model, as well as in the sticky-information model, this strategic complementarity in prices is a source of inflation inertia.

Panel (b) of Figure 2 considers the impact of information stickiness on inflation dynamics. For higher values of λ (smaller degree of information stickiness), inflation not only responds more quickly to a shock in the fundamental but also returns to its pre-shock levels at a faster rate.

Finally, Panels (c) and (d) of Figure 2 show the impact of information dispersion on SDI inflation. Once again, recall that $\delta \equiv \alpha/(\alpha+\beta)$ rises when public information becomes more precise and/or private information becomes less precise. Higher values for δ imply that previous values of θ are relatively more precise signals of the state than the firm's private information. As a result, for large δ, even firms that update their information sets *at* the moment of the shock respond less to such new piece of information.

Also, for a given δ, an additional strategic effect leads the firms to place a larger weight on past information about the state. Indeed, a firm that wishes to align its price to other firms' prices relies more heavily on public information because it is a better predictor of other firms' prices than private information. This effect has been already pointed out by authors such as Morris and Shin (2002), Angeletos and Pavan (2007), and others in related contexts.

5.4 Inflation Variance

We now analyze the variance of inflation under SDI. Using equation (24), we obtain the complete-information inflation variance

$$Var\left[\pi_{C,t}\right] = \alpha^{-1}.$$

From equations (25) and (27), we obtain the variances of dispersed-information inflation and sticky-information inflation

$$\begin{aligned} Var\left[\pi_{D,t}\right] &= \left[(1-\Delta)^2 + \Delta^2\right] Var\left[\pi_{C,t}\right], \\ Var\left[\pi_{S,t}\right] &= \kappa Var\left[\pi_{C,t}\right], \end{aligned}$$

where Δ, defined in (26), is a function of (r, α, β) while

$$\kappa \equiv \textstyle\sum_{j=0}^{\infty} K_j^2$$

is a function of (r, λ), as can be seen by the definition of K_j in (22).

Finally, from equation (28), we obtain the variance of SDI inflation

$$Var\left[\pi_t\right] = \Omega Var\left[\pi_{C,t}\right],$$

where Ω, which is a function of the parameters $(r, \lambda, \alpha, \beta)$, is given by

$$\Omega \equiv \textstyle\sum_{j=0}^{\infty} c_j^2 \in (0,1),$$

where the $c_j's$ are defined in (15).

Notice that the variance of the SDI inflation, $Var\left[\pi_t\right]$, is proportional to the variance of complete information inflation, $Var\left[\pi_{C,t}\right]$. A bit more surprising is the fact that the informational frictions we consider in the model reduce the variance of inflation when compared to

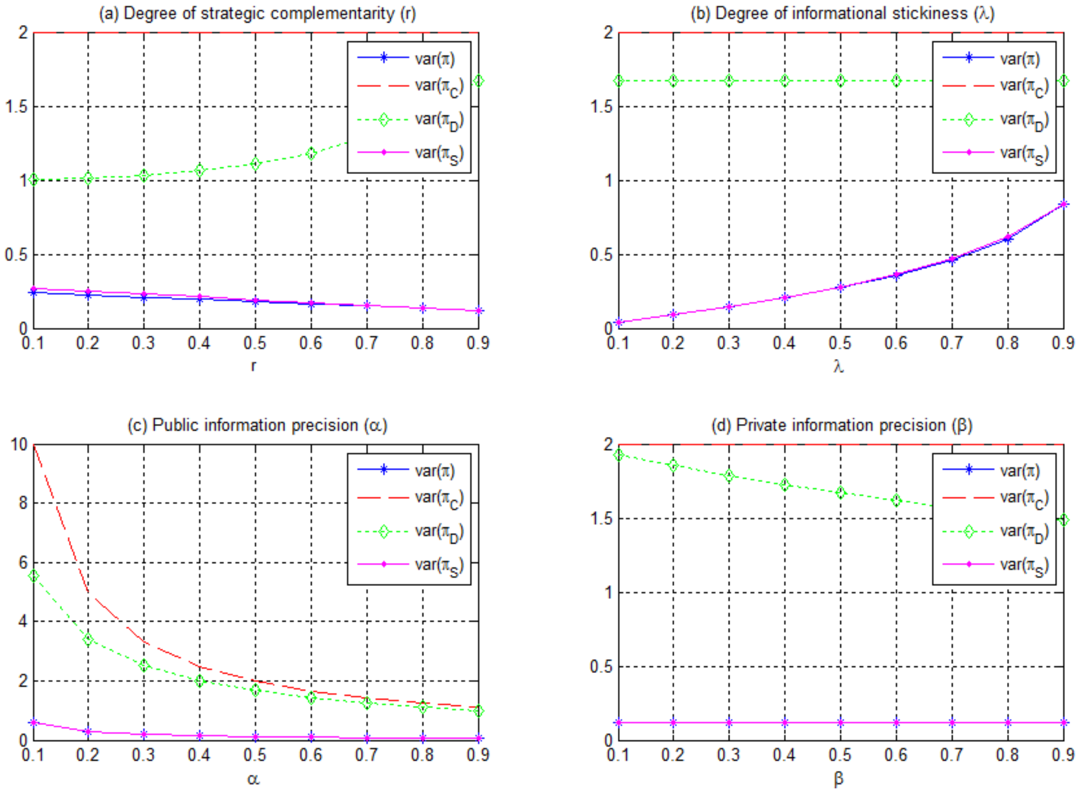

Figure 3: Variances of SDI inflation π_t, complete-information inflation $\pi_{C,t}$, dispersed-information inflation $\pi_{D,t}$, and sticky-information inflation $\pi_{S,t}$ as a function of $(r, \lambda, \alpha, \beta)$.

the complete information benchmark. The reason is as follows. As discussed throughout the paper, the combination of sticky and dispersed information with strategic interdependence in price setting leads to inflationary inertia, which, in turn, reduces, the variance of inflation under SDI.

Figure 3 plots $Var\left[\pi_t\right]$ as well as $Var\left[\pi_{C,t}\right]$, $Var\left[\pi_{D,t}\right]$, and $Var\left[\pi_{S,t}\right]$ as a function of $(r, \lambda, \alpha, \beta)$.[8]

As can be seen from Figure 3, the variances of complete-information inflation $Var\left[\pi_{C,t}\right]$ and dispersed-information inflation $Var\left[\pi_{D,t}\right]$ are always higher than SDI inflation's, $Var\left[\pi_t\right]$, and sticky-information inflation's, $Var\left[\pi_{S,t}\right]$. Notice, moreover, that $Var\left[\pi_t\right]$ and $Var\left[\pi_{S,t}\right]$ have a similar behavior and only seem to be affected by the degree of informational stickiness

[8] We plot $\Omega_{\bar{k}} \equiv \sum_{j=0}^{\bar{k}} c_j^2$ rather than Ω for computational reasons.

λ. Both variances, $Var[\pi]$ and $Var[\pi_{S,t}]$, increase with the degree of information stickiness.

As the signals become more precise, more similar are the information sets of the firms. As a result, dispersed-information inflation $Var[\pi_{D,t}]$ decreases considerably as information precision α and β increase. $Var[\pi_{D,t}]$ is also affected by the degree os strategic complementarity r. As r increases, more weight is given by a firm to its forecast about the forecast of the others, increasing $Var[\pi_{D,t}]$.

6 Conclusion

Costs to acquire and process information make its diffusion through the economy slow: i.e., information is sticky. Likewise, heterogeneity in the sources and interpretation of new information is likely to make relevant information about the economy dispersed across agents. In this paper, we have considered the impact of sticky and dispersed information on individual price setting decisions, and the resulting effect on the aggregate price level and the inflation rate.

Compared to a setting in which information is solely sticky as in Mankiw and Reis (2002), sticky *and* dispersed information always leads to non-trivial effects on prices regardless of assumptions about the agents' capability to predict equilibrium behavior by their opponents. Moreover, the effects of information on aggregate prices and inflation rates will be more pronounced: aggregate prices and inflation rates will be more inertial than their sticky information counterparts.

There are several interesting dimensions in which our model of price setting under SDI can be extended. Perhaps the most important one is to explore the policy implications of dispersed information. In a world in which information is dispersed, a benevolent central banker's optimal communication policy is far from trivial. On the one hand, any information disclosed by the central banker about the state will have the benefit of allowing the agents to count on an additional piece of information about the state when deciding on their prices. On the other hand, from a social perspective, agents will place too much weight on any information disclosed by the central banker as this is a public signal (e.g., Morris and Shin (2002) and Angeletos and Pavan (2007)). One can remedy this latter effect by setting a tax that corrects the incentives the agents have to "coordinate" on such public signal. Our derivation of the equilibrium played by firms and the prevailing Phillips curve when information is sticky and dispersed is a necessary first step toward answering the policy questions suggested above.

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7 Appendix

7.1 Prior Distribution

At this appendix, we calculate the distribution of the fundamental θ_{t-j} given that the firm updated its information set at period $t-j$. We can compute $f\left(\theta_{t-j} \mid \Theta_{t-j-1}, x_{t-j}\right)$ as

$$
\begin{aligned}
f\left(\theta_{t-j} \mid \theta_{t-j-1}, x_{t-j}\right) &= \frac{f\left(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}\right)}{\int_{-\infty}^{\infty} f\left(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}\right) d\theta_{t-j}} \\
&= \frac{f\left(\theta_{t-j-1}, x_{t-j} \mid \theta_{t-j}\right) f\left(\theta_{t-j}\right)}{\int_{-\infty}^{\infty} f\left(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}\right) d\theta_{t-j}} \\
&= \frac{f\left(\theta_{t-j-1} \mid \theta_{t-j}\right) f\left(x_{t-j} \mid \theta_{t-j}\right) f\left(\theta_{t-j}\right)}{\int_{-\infty}^{\infty} f\left(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}\right) d\theta_{t-j}}
\end{aligned}
$$

where the last equality holds due to the independence of $\xi_t(z)$ and ϵ_{t-j}. As

$$
\begin{aligned}
x_{t-j}(z) &= \theta_{t-j} + \xi_{t-j}(z) \\
\theta_{t-j-1} &= \theta_{t-j} - \epsilon_{t-j}.
\end{aligned}
$$

where $\xi_t(z) \sim N\left(0, \beta^{-1}\right)$ and $\epsilon_{t-j} \sim N\left(0, \alpha^{-1}\right)$, we have that $f\left(x_{t-j} \mid \theta_{t-j}\right) = N\left(\theta_{t-j}, \beta^{-1}\right)$ and $f\left(\theta_{t-j-1} \mid \theta_{t-j}\right) = N\left(\theta_{t-j}, \alpha^{-1}\right)$. If the dynamics of θ_t was

$$
\theta_{t-j-1} = \rho\theta_{t-j} - \epsilon_{t-j}.
$$

we would have

$$
\begin{aligned}
E\left[\theta_{t-j}\right] &= E\left[\theta_t\right] = \frac{E\left[\epsilon_t\right]}{1-\rho} = 0 \\
Var\left[\theta_{t-j}\right] &= Var\left[\theta_t\right] = \frac{Var\left[\epsilon_t\right]}{1-\rho^2} = \frac{\alpha^{-1}}{1-\rho^2}.
\end{aligned}
$$

Therefore, the distribution of θ_{t-j} would be given by $f\left(\theta_{t-j}\right) = N\left(0, \Psi^{-1}\right)$ where $\Psi =.\alpha\left(1-\rho^2\right)$. Thus, we would obtain

$$
\begin{aligned}
f\left(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}\right) &= c \exp\left\{-\frac{1}{2}\left[\frac{\left(x_{t-j}(z) - \theta_{t-j}\right)^2}{\beta^{-1}} + \frac{\left(\theta_{t-j-1} - \rho^{-1}\theta_{t-j}\right)^2}{\left(\rho^2\alpha\right)^{-1}} + \frac{\theta_{t-j}^2}{\Psi^{-1}}\right]\right\} \\
&= c \exp\left\{-\frac{1}{2}\left[\left(\beta + \alpha + \Psi\right)\theta_{t-j}^2 - 2\left(\beta x_{t-j}(z) + \alpha\rho\theta_{t-j-1}\right)\theta_{t-j}\right]\right\} \\
&\quad \times \exp\left\{-\frac{1}{2}\left[\beta x_{t-j}^2(z) + \alpha\rho^2\theta_{t-j-1}^2\right]\right\} \\
&= cd\frac{1}{\sqrt{2\pi}\sigma\Sigma} \exp\left\{-\frac{1}{2}\frac{\left(\theta_{t-j} - \mu\right)^2}{\Sigma^2}\right\}
\end{aligned}
$$

where

$$
\begin{array}{ll}
c = (2\pi)^{-3/2} (\beta\alpha\Psi)^{1/2} & d = \sqrt{2\pi}\sigma \exp\left\{-\frac{1}{2}\left[-\mu^2\Sigma^{-2} + \beta x_{t-j}^2(z) + \alpha\rho^2\theta_{t-j-1}^2\right]\right\} \\
\mu = \left[\Delta x_{t-j}(z) + (1-\Delta) z_{t-j-1}\right] & \Delta = \beta(\beta+\alpha+\Psi)^{-1} \\
z_{t-j-1} = \Lambda\rho\theta_{t-j-1} & \Lambda = \alpha(\beta+\alpha)^{-1} \\
\Sigma^2 = (\beta+\alpha+\Psi)^{-1} &
\end{array}
$$

As $\rho \to 1$, we have $\Psi \to 0$, $\Delta \to \delta$, and $\Sigma^2 \to (\beta+\alpha)^{-1}$. Thus $f(\theta_{t-j} \mid \theta_{t-j-1}, x_{t-j}) = N(\mu, \sigma^2)$ where $\mu = [\delta x_{t-j}(z) + (1-\delta)\theta_{t-j-1}]$, and $\sigma^2 = (\beta+\alpha)^{-1}$.

7.2 Higher Order Beliefs

In this appendix we derive the general formula of the k-th order average expectation

$$\bar{E}^k[\theta_t] = \lambda \sum_{m=0}^{\infty} (1-\lambda)^m \left[\kappa_{m,k}\theta_{t-m} + \delta_{m,k}\theta_{t-m-1}\right]$$

with the weights $(\kappa_{m,k}, \delta_{m,k})$ are recursive defined for $k \geq 1$

$$\begin{bmatrix} \kappa_{m,k+1} \\ \delta_{m,k+1} \end{bmatrix} = \begin{bmatrix} (1-\delta) \\ \delta \end{bmatrix} \left[1-(1-\lambda)^m\right]^k + A_m \begin{bmatrix} \kappa_{m,k} \\ \delta_{m,k} \end{bmatrix},$$

where the matrix A_m is given by

$$A_m \equiv \begin{bmatrix} \left[(1-\delta)\left[1-(1-\lambda)^{m+1}\right] + \delta\left[1-(1-\lambda)^m\right]\right] & 0 \\ \delta\left[\left[1-(1-\lambda)^{m+1}\right] - \left[1-(1-\lambda)^m\right]\right] & \left[1-(1-\lambda)^{m+1}\right] \end{bmatrix},$$

and the initial weights are $(\kappa_{1,k}, \delta_{1,k}) \equiv (1-\delta, \delta)$.

We start by computing $\bar{E}^1[\theta_t]$ as

$$
\begin{aligned}
\bar{E}^1[\theta_t] &= \sum_{j=0}^{\infty} \int_{\Lambda_j} E\left[\bar{E}^0[\theta_t] \mid I_{t-j}(z)\right] dz \\
&= \sum_{j=0}^{\infty} \int_{\Lambda_j} E\left[\theta_t \mid I_{t-j}(z)\right] dz \\
&= \sum_{j=0}^{\infty} \int_{\Lambda_j} \left[(1-\delta) x_{t-j}(z) + \delta\theta_{t-j-1}\right] dz \\
&= \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \left[(1-\delta)\theta_{t-j} + \delta\theta_{t-j-1}\right].
\end{aligned}
$$

We can use this result to obtain $\bar{E}^2[\theta_t]$ as

$$\begin{aligned}
\bar{E}^2[\theta_t] &= \textstyle\sum_{m=0}^{\infty} \int_{\Lambda_m} E\left[\bar{E}^1[\theta_t] \mid I_{t-m}(z)\right] dz \\
&= \lambda \textstyle\sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=0}^{\infty} (1-\lambda)^j E\left[(1-\delta)\theta_{t-j} + \delta\theta_{t-j-1} \mid I_{t-m}(z)\right] dz.
\end{aligned}$$

We know that

$$E[\theta_{t-j} \mid I_{t-m}(z)] = \begin{cases} (1-\delta) x_{t-m}(z) + \delta\theta_{t-m-1} & : m \geq j \\ \theta_{t-j} & : m < j \end{cases}.$$

Thereafter

$$\begin{aligned}
\bar{E}^2[\theta_t] &= \lambda \textstyle\sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=0}^{m-1} (1-\lambda)^j \left\{(1-\delta) E[\theta_{t-j} \mid I_{t-m}(z)] + \delta E[\theta_{t-j-1} \mid I_{t-m}(z)]\right\} dz \\
&\quad + \lambda \textstyle\sum_{m=0}^{\infty} \int_{\Lambda_m} (1-\lambda)^m \left\{(1-\delta) E[\theta_{t-m} \mid I_{t-m}(z)] + \delta\theta_{t-m-1}\right\} dz \\
&\quad + \lambda \textstyle\sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=m+1}^{\infty} (1-\lambda)^j \left[(1-\delta)\theta_{t-j} + \delta\theta_{t-j-1}\right] dz \\
&= \lambda \textstyle\sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=0}^{m-1} (1-\lambda)^j \left[(1-\delta) x_{t-m}(z) + \delta\theta_{t-m-1}\right] dz \\
&\quad + \lambda \textstyle\sum_{m=0}^{\infty} \int_{\Lambda_m} (1-\lambda)^m \left[(1-\delta)\left[(1-\delta) x_{t-m}(z) + \delta\theta_{t-m-1}\right] + \delta\theta_{t-m-1}\right] dz \\
&\quad + \lambda \textstyle\sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=m+1}^{\infty} (1-\lambda)^j \left[(1-\delta)\theta_{t-j} + \delta\theta_{t-j-1}\right] dz \\
&= \lambda^2 \textstyle\sum_{m=0}^{\infty} (1-\lambda)^m \left[(1-\delta)\theta_{t-m} + \delta\theta_{t-m-1}\right] \sum_{j=0}^{m-1} (1-\lambda)^j \\
&\quad + \lambda^2 \textstyle\sum_{m=0}^{\infty} (1-\lambda)^{2m} \left[(1-\delta)^2 \theta_{t-m} + \left[1-(1-\delta)^2\right] \theta_{t-m-1}\right] \\
&\quad + \lambda^2 \textstyle\sum_{j=1}^{\infty} (1-\lambda)^j \left[(1-\delta)\theta_{t-j} + \delta\theta_{t-j-1}\right] \sum_{m=0}^{j-1} (1-\lambda)^m \\
&= \lambda \textstyle\sum_{m=0}^{\infty} (1-\lambda)^m \left[(1-\delta)\theta_{t-m} + \delta\theta_{t-m-1}\right] \left[1-(1-\lambda)^m\right] \\
&\quad + \lambda^2 \textstyle\sum_{m=0}^{\infty} (1-\lambda)^{2m} \left[(1-\delta)^2 \theta_{t-m} + \left[1-(1-\delta)^2\right] \theta_{t-m-1}\right] \\
&\quad + \lambda \textstyle\sum_{j=1}^{\infty} (1-\lambda)^j \left[(1-\delta)\theta_{t-j} + \delta\theta_{t-j-1}\right] \left[1-(1-\lambda)^j\right] \\
&= \lambda \textstyle\sum_{m=0}^{\infty} (1-\lambda)^m \, 2\left[1-(1-\lambda)^m\right] \left[(1-\delta)\theta_{t-m} + \delta\theta_{t-m-1}\right] \\
&\quad + \lambda^2 \textstyle\sum_{m=0}^{\infty} (1-\lambda)^{2m} \left[(1-\delta)^2 \theta_{t-m} + \left[1-(1-\delta)^2\right] \theta_{t-m-1}\right].
\end{aligned}$$

We can write this expression as

$$\bar{E}^2[\theta_t] = \lambda \textstyle\sum_{j=0}^{\infty} (1-\lambda)^j \left[\kappa_{j,2}\theta_{t-j} + \delta_{j,2}\theta_{t-j-1}\right]$$

where

$$\begin{aligned}
\kappa_{j,2} &= \left(1-\delta^2\right)\left[1-(1-\lambda)^j\right]+(1-\delta)^2\left[1-(1-\lambda)^{j+1}\right] \\
&= \left[1-(1-\lambda)^{j+1}\right]\kappa_{j,1}^2+\left[1-(1-\lambda)^j\right]\left(1-\delta_{j,1}^2\right), \\
\delta_{j,2} &= \delta^2\left[1-(1-\lambda)^j\right]+\left[1-(1-\delta)^2\right]\left[1-(1-\lambda)^{j+1}\right] \\
&= \left[1-(1-\lambda)^{j+1}\right]\left(1-\kappa_{j,1}^2\right)+\left[1-(1-\lambda)^j\right]\delta_{j,1}^2.
\end{aligned}$$

Note that

$$\kappa_{j,2}+\delta_{j,2}=\textstyle\sum_{n=0}^{1}\left[1-(1-\lambda)^j\right]^n\left[1-(1-\lambda)^{j+1}\right]^{1-n}.$$

We use induction to obtain the general case. Suppose that (17) holds for $k-1$. Then

$$\bar{E}^{k-1}\left[\theta_t\right]=\lambda\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m\left[\kappa_{m,k-1}\theta_{t-m}+\delta_{m,k-1}\theta_{t-m-1}\right],$$

where

$$\textstyle\sum_{j=0}^{m-1}(1-\lambda)^j\left(\kappa_{j,k-1}+\delta_{j,k-1}\right)=\dfrac{1}{\lambda}\left[1-(1-\lambda)^m\right]^{k-1}.$$

As a result

$$\begin{aligned}
\bar{E}^k[\theta_t] &= \textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m} E\left[\bar{E}^{k-1}[\theta_t] \mid I_{t-m}(z)\right] dz \\
&= \textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m} E\left[\lambda\sum_{j=0}^{\infty}(1-\lambda)^j\left[\kappa_{j,k-1}\theta_{t-j}+\delta_{j,k-1}\theta_{t-j-1}\right] \mid I_{t-m}(z)\right] dz \\
&= \lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=0}^{m-1}(1-\lambda)^j\left\{\kappa_{j,k-1}E\left[\theta_{t-j}\mid I_{t-m}(z)\right]+\delta_{j,k-1}E\left[\theta_{t-j-1}\mid I_{t-m}(z)\right]\right\} dz \\
&\quad +\lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}(1-\lambda)^m\left\{\kappa_{m,k-1}E\left[\theta_{t-m}\mid I_{t-m}(z)\right]+\delta_{m,k-1}\theta_{t-m-1}\right\} dz \\
&\quad +\lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=m+1}^{\infty}(1-\lambda)^j\left[\kappa_{j,k-1}\theta_{t-j}+\delta_{j,k-1}\theta_{t-j-1}\right] dz \\
&= \lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=0}^{m-1}(1-\lambda)^j\left(\kappa_{j,k-1}+\delta_{j,k-1}\right)\left[(1-\delta)x_{t-m}(z)+\delta\theta_{t-m-1}\right] dz \\
&\quad +\lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}(1-\lambda)^m\left[\kappa_{m,k-1}\left[(1-\delta)x_{t-m}(z)+\delta\theta_{t-m-1}\right]+\delta_{m,k-1}\theta_{t-m-1}\right] dz \\
&\quad +\lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=m+1}^{\infty}(1-\lambda)^j\left[\kappa_{j,k-1}\theta_{t-j}+\delta_{j,k-1}\theta_{t-j-1}\right] dz \\
&= \lambda^2\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m\left[(1-\delta)\theta_{t-m}+\delta\theta_{t-m-1}\right]\sum_{j=0}^{m-1}(1-\lambda)^j\left(\kappa_{j,k-1}+\delta_{j,k-1}\right) \\
&\quad +\lambda^2\textstyle\sum_{m=0}^{\infty}(1-\lambda)^{2m}\left[\kappa_{m,k-1}(1-\delta)\theta_{t-m}+\left[\kappa_{m,k-1}\delta+\delta_{m,k-1}\right]\theta_{t-m-1}\right] \\
&\quad +\lambda^2\textstyle\sum_{j=1}^{\infty}(1-\lambda)^j\left[\kappa_{j,k-1}\theta_{t-j}+\delta_{j,k-1}\theta_{t-j-1}\right]\sum_{m=0}^{j-1}(1-\lambda)^m \\
&= \lambda\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m\left[1-(1-\lambda)^m\right]^{k-1}\left[(1-\delta)\theta_{t-m}+\delta\theta_{t-m-1}\right] \\
&\quad +\lambda^2\textstyle\sum_{m=0}^{\infty}(1-\lambda)^{2m}\left[\kappa_{m,k-1}(1-\delta)\theta_{t-m}+\left[\kappa_{m,k-1}\delta+\delta_{m,k-1}\right]\theta_{t-m-1}\right] \\
&\quad +\lambda\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m\left[1-(1-\lambda)^m\right]\left[\kappa_{m,k-1}\theta_{t-m}+\delta_{m,k-1}\theta_{t-m-1}\right].
\end{aligned}$$

We can rewrite the last three as

$$\bar{E}^k[\theta_t] = \lambda\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m\left[\kappa_{m,k}\theta_{t-m}+\delta_{m,k}\theta_{t-m-1}\right],$$

where

$$\begin{aligned}
\kappa_{m,k} &\equiv (1-\delta)\left[1-(1-\lambda)^m\right]^{k-1}+\left[(1-\delta)\lambda(1-\lambda)^m+\left[1-(1-\lambda)^m\right]\right]\kappa_{m,k-1} \\
&= (1-\delta)\left[1-(1-\lambda)^m\right]^{k-1} \\
&\quad +\left[(1-\delta)\left[1-(1-\lambda)^{m+1}\right]+\delta\left[1-(1-\lambda)^m\right]\right]\kappa_{m,k-1} \\
\delta_{m,k} &\equiv \delta\left[1-(1-\lambda)^m\right]^{k-1}+\delta\lambda(1-\lambda)^m\kappa_{m,k-1}+\left[\lambda(1-\lambda)^m+\left[1-(1-\lambda)^m\right]\right]\delta_{m,k-1} \\
&= \delta\left[1-(1-\lambda)^m\right]^{k-1} \\
&\quad +\delta\left[\left[1-(1-\lambda)^{m+1}\right]-\left[1-(1-\lambda)^m\right]\right]\kappa_{m,k-1}+\left[1-(1-\lambda)^{m+1}\right]\delta_{m,k-1}
\end{aligned}$$

since

$$\lambda(1-\lambda)^m = \left[1-(1-\lambda)^{m+1}\right]-\left[1-(1-\lambda)^m\right].$$

Rewriting these weights in matrix format, we obtain

$$\begin{bmatrix} \kappa_{m,k+1} \\ \delta_{m,k+1} \end{bmatrix} = \begin{bmatrix} (1-\delta) \\ \delta \end{bmatrix} [1-(1-\lambda)^m]^k + A_m \begin{bmatrix} \kappa_{m,k} \\ \delta_{m,k} \end{bmatrix},$$

where the matrix A_m is given by

$$A_m \equiv \begin{bmatrix} \left[(1-\delta)\left[1-(1-\lambda)^{m+1}\right] + \delta\left[1-(1-\lambda)^m\right]\right] & 0 \\ \delta\left[\left[1-(1-\lambda)^{m+1}\right] - \left[1-(1-\lambda)^m\right]\right] & \left[1-(1-\lambda)^{m+1}\right] \end{bmatrix},$$

which is exactly our result.

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