# Habit Formation and Oligopolistic Competition

Alejandro Tatsuo Moreno[1]

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**Abstract**

In this paper I introduce competition in the habit formation literature by extending the idea of habit formation to the characteristics of the products. I model a two-period game in which two firms can enter a market and compete with each other, and individuals' favorite characteristics in the second period are the characteristics of the product they consumed in the first period. I find that if two firms enter the market, they do it sequentially. That is, one firm enters in the first period and attracts individuals' preferences to the characteristics of its product, while the other firm enters in the second period and competes for the individuals that have grown to prefer the characteristics of the original product. However, the second firm's product is similar to the original one, but not exactly the same. The model also applies to habit formation for different markets with characteristics in common. For example, sweetness is a common characteristic of sodas and ice-cream, and consumption of a product in one market affects the preferences for products in the other market. I find that new firms produce products with similar characteristics not only to a product that has entered the same market, but to products that have entered other markets with characteristics in common. I apply my model to durable goods and find a new explanation for fashion: firms can take advantage of habit formation by showing their products in the media to generate demand for new durable goods.

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# 1 Introduction

Psychologists have long observed that individuals increase their enjoyment of many products just by consuming them repeatedly.[2] This applies to many types of food, beverages, music, etc. Economists have formalized this phenomenon in models of habit formation. However, no model to date that I am aware of has analyzed competition in markets with habit formation. The goal of this paper is to introduce competition in the habit formation literature.

Psychologists have also observed that individuals not only increase their preference for a product, but for its characteristics as well. I extend habit formation for the characteristics of the products. This offers several advantages. First, my model can analyze how the consumption of a product affects the preferences for the characteristics of other products, whose design are defined over the same charateristic space, even in different markets. Second, it allows us to understand how habit formation affects the firms' design choices over the characteristics of the products.

In section 2 I introduce my model. I extend habit formation into the characteristics of the products by modifying Hotelling's linear city. Hotelling (1929) models competition in a market where consumers have different preferences for the characteristics of a product distributed uniformly on [0,1]. Every consumer has a favorite level of the characteristic (favorite location in the characteristic space) and their utility decreases as the characteristic of the product differs from this level. However, the market will provide only a limited variety of products, and therefore individuals with different preferences will consume the same products.

Because habit formation is a fundamentally dynamic phenomenon, it is necessary to extend the classically static Hotelling model to more periods. I work with the simplest possible extension, a two period model. In the first period I assume that individuals do not have a preference over the characteristics of the products and they receive a constant utility from consumption. The basic assumption of my model is that in the second period individuals' favorite location is the location of the products they consumed in the first period.

Once an individual has consumed a product and learned to enjoy its characteristics, she will prefer a product with the same characteristics, even if it is a product from a different firm. If different individuals grow to prefer the characteristics of the products

[2] For a review of the psychological literature see Bornstein, (1989).

they consume, and if they consume the same products, then they will grow to prefer the same characteristics as other individuals. Thus habit formation results in individuals' favorite locations being clustered in the location of the products that are available. As a result, the characteristics of the products will explain preferences as much as preferences explain the characteristics of the products.

In section 2.1 I analyze the case of two firms that have the option of entering a market where individuals have habit formation. I show that when both firms enter the market, they do it sequentially: one firm enters in the first period and the other firm enters in the second period. I assume that individuals differ in their willingness to pay for these characteristics (different transportation costs), and that the utility from consumption, gross of transportation cost, is not large. The first firm attracts individuals' preferences toward the characteristics of its product and the second firm steals the low-transportation cost individuals, who prefer the original product but are not willing to pay much more for it. The entrant produces a similar product to the original one, but not exactly the same (to relax competition) and sells it at a lower price. I find that the firm that enters in the first period has a first-mover advantage, as its profits are higher than those of the second firm.

In section 2.2 I apply the model to analyze how firms compete to increase the availability of their products. If individuals learn to enjoy the characteristics of a product by consuming it in one outlet, they will value the same product more, and products with similar characteristics in other outlets. For example, an individual that consumes Coca-Cola in vending machines will be willing to pay a higher price for Coca-Cola than a product with different characteristics, let's say Max-Cola, in supermarkets, where both products are sold. If Coca-Cola monopolizes the market for vending machines it will be able to charge a higher price than Max-Cola in supermarkets. Max-Cola will respond by producing a product that is similar, but not exactly the same as Coca-Cola, to attract some of the consumer that grew to prefer Coca-Cola.

In section 2.3 I use my model to give another explanation of fads and fashion. I model how firms can to use habit formation to create demand for new durable goods. If individuals grow to prefer the characteristics of a product they consume and if it is a durable product, then these individuals will not buy a new product, since they can consume the old product by free, which at the same time becomes their most favorite product. But if there is a second market that is especially important for the influence of individuals' preferences, for example the media, firms may be able to change individuals preferences toward products with different characteristics by making them available in this market. In this way a firm can "depreciate" an old product by introducing a new product with different characteristics in the media.

In section 3 I extend my model to multiple markets. Products in different markets often share several characteristics. For example, sweetness is a characteristic that is shared by products in many markets. If individuals learn to enjoy the characteristics of the products they consume, then the consumption of one product would affect the preferences for other products that have the same characteristics, even in different markets. For instance, the level of sweetness in sodas will affect the preference for the level of sweetness for ice-creams. I analyze the simplest case: two markets that share one characteristic.

I first analyze the case of a firm that enters in one of the markets in the first period and a second firm that enters in the other market in the second period. Individuals that consume the product in the first period are going to prefer its characteristics in both markets in the second period. The firm that enters in the second period will produce with the same level of the characteristic that both markets have in common. In other words, there is path dependency even in different markets and firms will take in consideration what other firms have produced in other markets that share the same characteristics.

I also analyze the case where two firms enter in the first period, one in each market. If both firms can change location in the second period, then they do not have to produce with the same characteristics in the first period, but if they do not produce with the same characteristics, they would influence each other, that is, they would change their characteristics toward the characteristics of each other in the second period. If firms cannot change location in the second period, they will produce with the same characteristics as the firm in the other market.

In the case that two firms can enter each market, I show that when both firms enter, they do it sequentially. However, the first firm to enter each market has to enter in the same location as each other. If they do not enter in the same location they will produce a product that is not the individuals' most preferred. In this case entering in the first period without the same characteristics of the other firm can be a disadvantage for the incumbent, that new firms can exploit in the second period.

In section 4 I extend my model to include an error in the production process. Production processes are normally done with variability and products have deviations from their intended specifications. I assume that in these markets firms can invest in reducing the variability of the products, however, it is costly to do it. In many markets, as in the case of wine, variability from one year to the other is considerable. I analyze the case of two firms that can enter the market. I show that both firms choose to enter sequentially and produce with the same product specifications, differentiating instead in the variability of their products.

My model is related to the switching cost literature. The switching cost literature analyzes competition when individuals face a cost if they consume a different product or brand than the one they consumed in previous periods. Some examples of switching costs are the costs of learning how to use a product and the need for compatibility with existing equipment, but it also can be interpreted as brand loyalty [Klemperer 1995].

In a sense, habit formation creates a switching cost for changing a location in a characteristic space, rather than for switching the chosen brand. Once a consumer has consumed a product, she will prefer a product with the same characteristics. With habit formation the size of the cost of switching to another product depends on the characteristics of that product and therefore the size of the switching cost is now endogenous to the firms. By choosing their characteristics, new firms are choosing the size of the switching cost to an original product.

The difference is that the switching cost literature does not analyze individuals' preferences for the characteristics of the products or the products' design while my model focus on these.

## 2 Model

Hotelling (1929) models a linear city of length one. A number of individuals live in this city and are uniformly distributed across it. There are two firms that sell a product that consumers see as identical other than the location of the firm where they purchase it. Consumers have a transportation cost to travel to each store and each consumer demands at most one unit of the product. Two firms choose their location and then compete on prices to attract customers. We can also interpret this model as describing the location of preferences in a characteristic space (for example, the level of sweetness) where the location of the consumers represents their favorite level of the characteristic and the location of the firms is the level of this characteristic in their products.

An assumption of Hotelling's model is that preferences are fixed and individuals' present consumption do not affect their future utility. However, psychologists have observed that when individuals consume many products, they not only increase their preference for the products, but also for its characteristics. Several studies have documented that an increase (decrease) in the level of salt, sweetness, fat, etc. in many of the products that we consume for a few weeks will increase (decrease) our preferred level of those characteristics, even when these characteristics are found in other products. For example, in one experiment, Bertino and Beauchmp (1986) gave students food with more salt than what they normally consumed. After only three weeks of the higher-salt diet, these students began to develop a preference for saltier foods. After four weeks the students were allowed to choose the amount of salt that they preferred and it was observed that they continued using a high concentration of salt.[3] It is important to note that these students not only increased their preference for the specific salty products they consumed, but they grew to prefer salty food in general.

I modify Hotelling's linear city model to incorporate habit formation by including a second period. I assume that in the first period individuals do not have a favorite location. In many products, individuals do not have a innate preference for any specific characteristics of the products and sometimes they cannot even recognize the difference between different products the first time they consume them. But if individuals have habit formation, they will increase their preference for the characteristics of the products they previously consumed. I assume that in the second period individuals' preferences are determined by the product they consumed in the first period. I assume that the transportation cost is quadratic in the distance to their most favorite location.

Individuals' utility function in the first period is given by:

$$U_1 = \upsilon - p_1$$

In the second period, if individuals consumed a product in the first period, their utility function is:

$$U_2 = S - \theta(l_1 - l_2)^2 - p_2 \tag{1}$$

zero otherwise, where $\upsilon$ is the surplus from consumption in the first period, $S$ is the surplus from consumption in the second period (gross of transportation cost), $l_2$ is the location of the product that individuals consume in the second period, $l_1$ is the location of the product that individuals consumed in the first period and $\theta$ is a

[3] Other studies that have shown that variations in dietary sodium alter salt preferences are Bertino et al. (1982) and Blais et al. (1986).

parameter that represents the disutility individuals face when consuming away from their favorite location (transportation cost). I assume that $\theta$ differs for each individual and is uniformly distributed in the interval $[a, b]$. For simplicity, I normalize the number of consumers to one by making $a = b - 1$.

This definition implies that in the second period, individuals' favorite location becomes the location of the product they consumed in the first period and their utility decreases as the product they consume in the second period differs from this location. Although the assumption that individuals like the characteristics of one product just by consuming it one time is extreme, it captures the idea that individuals grow to prefer the characteristics of the products they consume and allows us to model habit formation in a simple way. Since my main interest is studying competition between firms, and not how individuals manage their habit formation, I assume that individuals are not forward looking, that is, they do not take in consideration their utility in the second period when they choose which product to consume in the first period.

Hotelling's model assumes that individuals' utility gross of price is large enough that any individual is willing to consume a product, no matter its characteristics. However, this is highly unrealistic. I assume instead that no individual, independently of her transportation cost or favorite location, would receive a positive utility from consuming a product that is in the furthest extreme in the characteristic space from her favorite location.

I assume that two firms can choose to enter the market and if they enter, they have the option of entering in the first or second period. If they enter, firms have to pay a fixed cost ($f$) each period. I assume that both firms have no capacity constraint, their marginal cost is zero, and they cannot change location from the first to the second period. The motivation for this is that it is difficult for firms to design new products.

The timing of the game is the following:

*First period:*

1. Both firms decide whether to enter or not.
2. After a firm chose to enter, it chooses the characteristics of its product.
3. Firms choose prices.
4. Firms produce and sell.
5. Individuals decide to buy or not and if they do, which product to consume.

*Second period:*

1. If they did not enter in the first period, firms decide whether to enter or not in the second period.

2. Firms choose the characteristics of their products.

3. Firms choose prices.

4. Firms produce and sell.

5. Individuals decide to buy or not and if they do, which product to consume.

The equilibrium concept of the game is subgame perfect Nash equilibrium, and I restrict my analysis to pure strategies. The model is similar to the model of quality by Gabszewics and Thiesse (1979) [4], where firms can differentiate in quality without any cost. In the case that both firms charge the same price for their products in the first period, I assume that with 50% percent probability each firm sells to every consumer. This tie rule simplifies the analysis, as it rules out the possibility of a mixed strategy in the second period.

**Proposition 1** *There is a threshold $\bar{b}$, where if $b \leq \bar{b}$, both firms choose to enter the market sequentially, one firm (incumbent) enters in the first period and the other (entrant) enters in the second period. In the first period the incumbent can choose any location and charges a price of $\upsilon$ for its product. In the second period the entrant will locate at a distance from the incumbent:*

$$d_e = \sqrt{\frac{3S}{b+1}}$$

*and firms price at:*

$$p_{2i} = S$$
$$p_{2e} = \frac{2-b}{b+1}S$$

*If $b > \bar{b}$, only one firm enters the market. It will enter in the first period and price at $\upsilon$. In the second period it prices at $S$, where the threshold $\bar{b}$ is given by:*

$$\bar{b} = \frac{3f + 4S - 3\sqrt{f^2 + 4fS}}{2S}$$

**Proof.** I divide the proof of proposition 1 into two cases:

4 With the difference that $S$, individuals' utility, gross of transportation cost, is the same for every consumer.

1) If $b \leq \bar{b}$, it is profitable for both firms to enter the market. I show that one firm chooses to enter in the first period and the other firm chooses to enter in the second period while the original firm remains in the market.

2) If $b > \bar{b}$, it is profitable for only one firm to enter the market.

**Case 1:** $b \leq \bar{b}$

If both firms enter the market there are three alternatives: 1) that both firms enter in the second period, 2) that one firm enters in the first period and the other firm enters in the second period and 3) both firms enter in the second period. I show that firms enter sequentially by showing that profits are higher for both firms if they enter sequentially than if both firms enter in the same period. Then, I show that both firms choose to enter the market by showing that profits are positive (when $b \leq \bar{b}$) for both firms if they enter sequentially.

**Both firms enter in the second period**

If neither of both firms enter in the first period, individuals' utility from consuming any product is zero in the second period and neither firm will be able to sell its product at a positive price. Their profits are negative once we take in consideration the fixed cost.

**Sequential entry**

In this part I analyze the case where firms enter sequentially using backward induction.

**Second Period**

If the firms enter sequentially, the first firm attracts individuals' preferences toward its product and the second firm will compete for some of these individuals. The new firm decides how similar its product will be to the product of the original firm. The new firm can eliminate any advantage the incumbent has by duplicating the same characteristics. However, this would increase competition as individuals are indifferent to both products. Or the new firm can relax competition by selling products with different characteristics, but this will increase the transportation cost for consumers and individuals may not want to consume its product. I find that the new firm will produce a product that is similar to the original product to attract some of its consumers, but that it will not produce the exact same product in order to relax competition. This result seems to capture an important aspect of reality, that latecomers to a market produce similar products to original ones, but try to differentiate themselves in some way

First I look for the individual ($\theta^*$) that is indifferent to the original and the new product:

$$S - p_i = S - \theta^* d_e^2 - p_e$$

where $d_e$ is the distance from the location of the original product $(i)$ to the product of the entrant $(e)$, and $p_i$ and $p_e$ are the prices of the incumbent and entrant respectively. I solve for $\theta^*$:

$$\theta^* = \frac{p_i - p_e}{d_e^2}$$

Individuals with higher transportation costs than $\theta^*$ consume the product of the incumbent, and individuals with lower transportation costs consume the product from the entrant.

The profit function for the incumbent is:

$$\pi_i = (b - \theta^*)\, p_i - f$$

and the profit function for the entrant is:

$$\pi_e = (\theta^* - a)\, p_e - f$$

I first solve for the first order condition with respect to price and obtain that prices and profits are the following:

$$p_i = \frac{1}{3}(b+1)\left(d_e^2\right) \tag{2}$$

$$p_e = \frac{1}{3}(2-b)\left(d_e^2\right) \tag{3}$$

$$\pi_i = \frac{1}{9}(b+1)^2\left(d_e^2\right) - f \tag{4}$$

$$\pi_e = \frac{1}{9}(2-b)^2\left(d_e^2\right) - f \tag{5}$$

While the entrant has an incentive to differentiate itself as much as possible to relax competition and increase prices, there is a limit to this as prices cannot go higher than an individual's total valuation for a product. At the distance

$$d_e = \sqrt{\frac{3S}{b+1}}$$

the price of the incumbent reaches $S$. If the entrant differentiates beyond the point, the incumbent will not raise prices and the profits for the entrant are given by the function:

$$\pi_e = \frac{1}{4}\left(\frac{S^2}{d_e^2} - S(b-1)\right)$$

We can see that the profits for the entrant decreases if it differentiate beyond this point. Therefore, the entrant will differentiate until the price of the incumbent reaches $S$, but no more. At this distance the prices are:

$$p_{2i} = S$$
$$p_{2e} = \frac{2-b}{b+1}S$$

We know that $b \geq 1$ because $a = b - 1$ and $a$ cannot be less than zero. Given that $b \geq 1$, we can see from equations (2)-(5) that the price and profits for the incumbent are higher than those of the entrant. The incumbent has an advantage by entering in the first period in markets with habit formation. The entrant will differentiate from the incumbent by producing a different product than the one individuals have grown to prefer and sell it to a lower price than the original product.

**First Period**

In the first period the incumbent can choose any location as individuals will grow to prefer any location. Given that individuals are not forward looking, consumers are only willing to pay the utility they receive for the product in the first period and therefore the firm will price at $v$.

**Simultaneous entry in the first period**

In the first period individuals are indifferent to the characteristics of any product. If both firms enter in the first period, individuals will consume the product with the lowest price and the firm that prices lower will sell to every consumer. This intensifies competition as each firm has an incentive to charge a lower price than the other firm until neither of them earns anything by decreasing the price further. This happens until the price of the product (negative price) compensates for any difference in profit for selling to every consumer. The only equilibrium is that both firms price at $p_1 = \pi_{non} - \pi_{all}$, where $\pi_{all}$ are the profits in the second period if selling to every consumer in the first period and $\pi_{non}$ are the profits in the second period of not selling to any consumer in the first period. That is, that they are going to decrease their price until they pay individuals to consume their product and the negative price would compensate for any profit they get by getting every consumer. Therefore, the profits for both firms are $\pi_{non}$. If both firms enter in the first period they would choose the location that maximizes $\pi_{non}$. This is the same problem of the entrant in the sequential case and the solution is the same. $\pi_{non} = \pi_e$, that is they are equal to the profits of the entrant in the sequential game. Nevertheless, the profits for entering in the first period have to include the fixed cost of the first period. Therefore the profits of both firms, if they enter simultaneously in the first period, are lower by $f$ than the profits of the entrant in the second period if they enter sequentially.

**Profits are positive for both firms if they enter the market**

As I showed, if both firms enter the market they choose to enter sequentially. Now I have to check if the profits for the firm that enters in the second period are positive. By solving for the values of $b$ for which this firm has positive profits I get the threshold of $\overline{b} = \frac{3f+4S-3\sqrt{f^2+4fS}}{2S}$. For values lower than $\overline{b}$ it is profitable for both firms to enter the market.

**Case 2:** $b > \overline{b}$ :

It is not profitable for a second firm to enter the market when $b > \overline{b}$. For higher values of $b$ the incumbent is going to compete more aggressively for the consumers and the entrant will have to price at a lower level to attract some of these consumers. ■

## 2.1 Comparative Statics

In the second period the entrant chooses how similar to produce with respect to the product of the incumbent. These are the comparative statics of the differentiation chosen by the entrant.

$$\frac{\partial d_e^*}{\partial S} = \frac{1}{2}\sqrt{\frac{3}{S(b+1)}} > 0$$

The difference between the product of the entrant and the original product increases with $S$. As $S$ increases, individuals are willing to pay higher prices. Given that the entrant has an incentive to differentiate until the incumbent charges the highest possible price, a higher value of $S$ results in a higher differentiation.

$$\frac{\partial d_e^*}{\partial b} = -\sqrt{\frac{3S}{(b+1)^3}} < 0$$

The difference between the products decreases with $b$. An increase in $b$ means that it is costlier for consumers to consume away from their favorite location. The entrant has to produce a more similar product to the original product to attract some of the consumers of the incumbent.

### 2.1.1 Social Welfare

Now I analyze how the entry of a second firm affects social welfare relative to a monopoly that serves the entire market.

**Proposition 2** *Social welfare diminishes with the entry of the second firm.*

**Proof.** In the case of a monopoly social welfare is given by the total utility of all the individuals that consume the product (gross of price) minus the cost of production (the fixed cost of producing in both periods).

$$W_{monopoly} = \upsilon + S - 2f$$

where $W_{monopoly}$ is the social welfare with a monopoly. The social welfare if a second firm enters in the second period is given by the utility of those individuals that consume from the incumbent plus the utility of those who consume from the entrant minus the fixed cost for both firms

$$W_{entry} = \upsilon + (b - \theta^*) S + \int_{b-1}^{\theta^*} \left(S - \theta l_e^2\right) d\theta - 2f - f$$

where $W_{entry}$ is the social welfare when two firms enter the market.By solving the integral we get:

$$W_{entry} = \upsilon + S - \frac{l_e^2}{2} \left(\theta^{*2} - (b-1)^2\right) - 3f$$

Because the right hand side of the last equation is negative, the social welfare is higher under the monopoly case than with competition:

$$W_{monopoly} > W_{entry}$$

■

Even without taking in consideration the additional fixed cost of the entrant there would be a loss in social welfare with the entry of a second firm. This is due to the fact that with the entry of another firm some consumers consume a product that does not have their favorite characteristics. In the case of monopoly there is no transportation cost as individuals consumer the same product that they have grown to prefer, although the monopoly is able to take all consumer surplus. Although a new firm charges a lower price and its consumers are better off, there is a loss in social welfare as the firm produces a different product and its consumers have a transportation cost.

My model results in excessive entry with just two firms as social welfare diminishes with the entry of a second firm. However, my model is not taking into consideration the positive effects of competition: lower prices would increase consumption and increase welfare. This is the result of the assumption that each individual consumes at most one unit. However my model isolates the effect that the variety of products has in social welfare. Variety reduces social welfare as consumers with habit formation do not like to consume products with characteristics that are different to those of the products that they consumed in previous periods.

## 2.2 Vending Machines Vs. Supermarkets

In this section I apply my model to analyze the effects of the availability of a product. In the previous section I showed that a firm that enters in the first period has an advantage with respect to a firm that enters in the second period. Now I analyze how a firm can enjoy the same advantage by selling its product in more outlets or more stores.

If individuals learn to enjoy the characteristics of a product by consuming it in one outlet, they will value the same product and products with similar characteristics more in other outlets. For example, an individual that consumes and "learns" to enjoy Coca-Cola in vending machines will be willing to pay a higher price for Coca-Cola than a product with different characteristics, let's say Max-Cola, in supermarkets, where both products are sold. If one of both firms, let's say Coca-Cola, monopolizes the outlet of vending machines, it will be able to charge a higher price than Max-Cola in supermarkets. Max-Cola will respond by producing a product that is similar, but not exactly the same as Coca-Cola, to attract some of the consumer that grow to prefer Coca-Cola.

For expositional purposes I will use as an example the market of cola sodas. I assume that individuals consume cola sodas in two different places: from vending machines and from supermarkets. There are two firms that can sell their products in both outlets: Coca-Cola and Max-Cola.

I simplify the problem by assuming that individuals only consume sodas from vending machines in the first period and in the second period individuals only consume sodas from supermarkets. Therefore the preferences are determined in the vending machines. Individuals want to pay a higher price in supermarkets for the product that is sold in vending machines. The solution is equivalent to the one of sequential entry in the previous section. If both firms enter the market of soda colas, one chooses to sell in both outlets, while the other only sell in the supermarkets. The firm whose product is available in vending machines and supermarkets will attract the high transportation cost individuals and will be able to charge a higher price. The firm whose product is available only in supermarkets will decrease competition by producing a product with different characteristics and will attract the low transportation individuals by charging a lower price.

**Proposition 3** *If $b \leq \bar{b}$, one firm (Coca-Cola) chooses to enter in the vending machines and supermarket markets and the other firm (Max-Cola) enters only in the supermarket markets. Coca-Cola will produce sodas with any characteristics and Max-Cola will locate at a distance from Coca-Cola:*

$$d_{MC2} = \sqrt{\frac{3S}{2b-a}}$$

*and the firms will price at:*

$$\begin{aligned} p_{CC1} &= \upsilon \\ p_{CC2} &= S \\ p_{MC2} &= \frac{2-b}{b+1}S \end{aligned}$$

*where $d_{MC2}$ is the distance of Max-Cola to Coca-Cola, $p_{CC1}$ and $p_{CC2}$ are the prices of Coca-Cola in the vending machines and supermarkets respectively and $p_{MC2}$ is the price of Max-Cola in the supermarkets.*

*In the case that $b > \bar{b}$ only one firm enters the market for soda colas and it sells in both outlets. In the vending machines it prices at $\upsilon$ and in supermarkets it prices at $S$.*

**Proof.** The proof is the same as Proposition 1. ■

As before, there are two cases depending of the value of $b$. When $b \leq \bar{b}$ it is profitable for both firms to enter the market. Coca-Cola will sell in both outlets while Max-Cola will sell in only supermarkets. When $b > \bar{b}$ it is profitable for only one firm to enter the market.

## 2.3 Durable Goods, Fads and Change in Preferences

In this section I apply my model to durable goods and find another explanation for fashion and fads: firms can use habit formation to generate demand for new durable goods by influencing individuals' preferences. I assume that, in order to influence preferences, firms can place their products in other markets where individuals also consume. We can think of the music industry, where record companies have been known for paying music stations to play new releases and influence consumers preferences toward their products, and then profit by selling them in record stores.

Pesendorfer (1995) models fashion as a signaling device for a dating game and Karni and Schmeidler (1990) as result of the products' social attributes like social distinction. Both models have explained fashion assuming constant preferences. My model gives another explanation: preferences indeed change as firms use the media to generate demand for new durable products.

I assume that individuals already have bought and consumed a product and learned to enjoy its characteristics in previous periods. Additionally, I assume that the products are durable, so if an individual once buy a product, she can consumed the same product every period. In this situation no firm can introduce a new product, since individuals prefer the old product and can consume it for free. However, if there is a second market where individuals consume and influence consumer preferences, like radio stations, a second firm could alter the preferences by letting individuals consume a product with different characteristics in this market. In this way a new firm can "depreciate" the first product by introducing the new product in both markets. For simplicity I assume that every individual consumes the product of the second market (radio stations).

I assume that the cost of placing its product in the second market is an increasing function of the transportation cost to the old product, specifically, I assume that this cost is $(d_{new})^4$, where $d_{new}$ is the distance from the new product to the old product. We can think of this cost as the bribe that the record companies have to pay to radio stations[5] to play their new releases and this bribe increases as the new product differs from the music that individuals want to hear. Additionally, I assume that once individuals consume the new product their preferences will change completely towards its characteristics, and the price in the second market is zero, as it is the case of radio stations.

**Proposition 4** *The firm will locate the new product at a distance:*

$$d_{new} = \frac{b}{\sqrt{8}}$$

*and will price at:*

$$p_{new} = \frac{1}{16} b^3$$

where $p_{new}$ is the price of the new product.

**Proof.** In the appendix.

[5] This practice is known as payola.

Individuals with high transportation cost prefer to buy the new product and individuals with low transportation consume the old product at zero cost.

# 3 Multiple Markets

Until now I have analyzed habit formation in the characteristic space in only one market. However, products in different markets often share several characteristics. For example, sweetness is a characteristic that is shared by many markets, like sodas, ice-cream, etc. If individuals learn to enjoy the characteristics of the products they consume, then the consumption of one product would affect the preferences for other products that have the same characteristics, even in different markets. In this section I analyze the case of two markets that share one characteristic. For expositional purposes I solve the case of two markets, but the analysis can easily be extended to many more markets.

In the case that multiple markets share one or more characteristics, the value of a product depends not only on where its characteristics are in relation to other products in the same market, but also in relation to products in other markets that share the same characteristics. Individuals want to consume products that minimize their transportation cost to the products they have consumed, even if these products are in different markets. I extend the definition of individuals' utility function to two markets that share one characteristic.

In the first period, individuals' utility function for a product in market $i$ is given by:

$$U_{1i} = \upsilon - p_{1i}$$

and in the second period, the utility function for a product in market $i$ is the following:

$$U_{2i} = S - \theta(l^*_{2i} - l_{2i})^2 - p_{2i} \tag{6}$$

and the most preferred location in market $i$ in the second period is given by:

$$l^*_{2i} = \phi\, l_{1i} + (1-\phi)l_{1j}$$

or

$$l^*_{2i} = l_{1j}$$

if individuals did not consume any product in the first period in market $i$,

or

$$l^*_{2i} = l_{1i}$$

if individuals did not consume any product in the first period in market $j$. If individuals did not consume any product in both markets in the first period individuals' utility in the second period is zero, where $\phi > 0$, $i$ and $j = 1, 2$ and $i \neq j$. $(1 - \phi)$ represents the influence that products in one market have in shaping the preferences in the other market. In the case that there is no consumption in one market in the first period, individuals preferences in that market will be shaped completely by the consumption in the other market.

The difference between this definition and the definition of utility function for a single market is that in the second period individuals' favorite locations in market $i$ are a function of previous consumption of products not only in market $i$, but also in market $j$.

## 3.1 One Firm enters Each Market

I analyze the case where only one firm enters in each market.

**Proposition 5** *If only one firm enters in one of the markets in the first period, a second firm that enters in the second period in the other market will produce in the same location as the product in the first period in the characteristic they share. The first firm prices at $v$ in the first period and both firms price at $S$ in the second period.*

Proof. See appendix.

Proposition (5) shows that there is path dependency in different markets in the characteristics they share. If only one firm enters in the first period, it will determine the preferences in both markets for that characteristic. After that, if another firm enters it will produce with the same characteristic to attract the consumers that have to learn the level of that characteristic.

For example, in a country where individuals eat spicy products, those individuals will grow to prefer spicy food in general. If a firm wants to introduce a new product, it would make sense to produce a product with the same level of spice of the other products people consume. We observe that in Mexico some brands of potato chips have chile, in United States have barbecue sauce, in Japan seaweed and in India curry. Those products have adapted to the local taste where individuals' preferences have been shaped by other markets.

Now I analyze the case where two firms enter in the first period, one in each market. If firms can change the characteristics of their products, each firm does not have to produce with the same characteristics in the first period, but as the following proposition shows, if they do not do it, they would change their characteristics toward the characteristics of each other in the second period.

**Proposition 6** *If one firm enters each market in the first period and both firms can change their location, if they produce with different characteristics as the product in other market, in the second period they will produce with characteristics* $\phi\, l_{1i} + (1 - \phi) l_{1j}$*, that is, they will move toward the characteristics of the product consumed in the other market in the first period. Both firms charge* $\upsilon$ *in the first period and* $S$ *in the second period.*

Proof. See appendix.

The reason for this is that in the second period individuals' favorite characteristics are an average of the characteristics of the products they consume in the first period. In the second period firms have an incentive to change their product to match individuals preferences.

If firms cannot change location in the second period they will produce in the same location in the characteristics they share.

**Proposition 7** *If one firm enters in each market in the first period and if both firms cannot change their location from the first period to the second, firms in both markets will produce with the same specifications in the characteristic they share.*

Proof. See appendix.

Proposition (7) shows that in the case where only one firm enters in the first period in each market and firms cannot change the product specifications, the products will be aligned in the characteristic they share. If a firm produces a product that is not aligned with the location of other products that share the same characteristics, individuals' preferences will not correspond with the specifications of the product and the firm would earn higher profits by moving its product to the same location of the product in the other market.

## 3.2 Two Firms Can Enter Each Market

Now I analyze the case where two firms can enter in each market. Individuals' favorite locations in the second period are the average of the characteristics of the products they consumed in both markets in the first period.

**Proposition 8** *If $b \leq \bar{b}$, both firms choose to enter the market $i$ sequentially, one firm (incumbent) enters in the first period and the other (entrant) enters in the second period. In the first period the incumbent chooses any location, as long as it is the same location of the incumbent in market $j$ and charges a price of $\upsilon$ for its product. In the second period the entrant locates at a distance from the incumbent:*

$$d_e = \sqrt{\frac{3S}{b+1}}$$

*and firms price at:*

$$p_{2i} = S$$
$$p_{2e} = \frac{2-b}{b+1}S$$

*If $b > \bar{b}$, only one firm enters each market. They will enter in the first period and price at $\upsilon$. In the second period it prices at $S$. They would choose to enter in the same location as each other.*

Proof. See appendix.

The difference with the results for only one market is that each incumbent does not have first mover advantage unless it produces with the same characteristics of the incumbent in the other market. If the incumbents produce with different characteristics, moving in the first period can become a disadvantage as they produce products that individuals do not like in the second period and the entrants can produce the products that individuals prefer.

# 4 Production with an error in characteristics

In this section I analyze the case where firms produce with an error in the characteristics of their products. Production process is normally done with variability and products have deviations from their intended specifications, [Merton (2003), page 3.] In the case of many markets, the characteristics of the products vary significantly through time. If the characteristics of a product change, even if it is from the same firm, an individual with habit formation will suffer a loss in utility. Individuals prefer products with lower variability and are willing to pay more for them. I assume that firms cannot choose the exact characteristics of its product, but only the mean of the production process and the error is the same for all the products of one firm. I am thinking of a market like wine where the characteristics changes from year to year, but are stable for the bottles of the same year.

I define the specification of a product as the mean of the distribution of the production process.

$$y = E[l]$$

where $y$ is the product specifications and $l$ is a random variable that represents the production process. The variability allows firms another dimension to differentiate themselves. I show that in markets with considerable variability in the production process firms produce with the same specifications and differentiate in the variability of their production process. The model is an application of the model analyzed in Cremmer and Thiesse (1994) where it is costly to increase the quality of the products.

In some markets some specifications are associated with a lower variability. For example, in the case of wine, the reduction in the variability of the grapes is related to their content of sugar. In order to reduce the variability with which the grapes ripen they have to be produced in low yield crops which in turn increases the level of sugar in the grapes. If a firm wants to reduce the variability in the grapes, it has to produce a wine with a high content of sugar [Mark Greenspan (2005).] In section 4.2 I include this phenomenom in my model by assuming that the cost of reducing the variability is lower in some specifications than others. I show that if the cost of decreasing the variability is a function of the specifications of the products firms will choose to produce with the specifications that minimize this cost. If individuals have habit formation, firms want them to learn the characteristics that decrease their cost of reducing the variability.

I assume individuals observe the characteristics of the products after they consume them. The expected value of individuals utility in the second period is given by the following equation:

$$EU_2 = S - \theta \left( m_2^2 + \sigma_2^2 \right) - p_2$$

where $\sigma_2^2$ is the variance of the production process in the second period and $m_2$ as the difference from the characteristics of the product individuals consume in the first period to the product specifications of the second period.

I assume that firms can choose the product specifications and variance of the production process in both periods. However, it is costly to reduce their variability. I assume that the error of the production process is i.i.d. between both periods and the marginal cost is constant in the number of units produced, but it increases as firms make an effort to reduce the variability of the production process. Following Mussa and Rosen (1978) I assume that the cost per unit is given by: $C = t(k - \sigma^2)^2$, where $t$ and $k$ are two parameters of the cost of reducing the variability, $k$ representing the cost of reducing the variability to zero. My analysis does not need any specific distribution. I assume that the size of the line is one, but the realization of the error can be outside the line. We can think of a regulation that does not allow products in a market to have specifications outside certain range, but the realization of the products can be outside this range. I assume that the fixed cost is not large, and both firms wants to enter the market, that is $f < \frac{3}{16t}$.

I assume that firms can produce a product with so much variability that individuals expected utility would be negative. That is,

$$k \geq S$$

I have to solve the model using backward induction.

**Second Period** In the second period, individuals want to consume a product with characteristics that are similar to the ones of the product they consumed in the first period. The closer the specifications of the product in the second period to the characteristics of the product in the first period and the lower its variability the lower the individuals' expected transportation cost. Therefore, in the second period, individuals want to consume products that have the same specifications as the characteristics of product they consumed in the first period and that are produced with low variability.

I assume that in the second period both firms decide the specifications and variability of their products at the same time.

To solve the model, I first look for the individual that is indifferent between the low variability product and the high variability product. I write the transportation cost of this individual as $\theta^*$. Individuals that have a higher transportation than $\theta^*$ will consume product $h$, while individuals that have a lower transportation cost than $\theta^*$ will consume product $l$, where $h$ stands for high transportation cost individuals, while $l$ stands for low transportation cost individuals.

$$\upsilon - p_1 + S - \theta^* \left(\sigma_{2l}^2 + m_{2l}^2\right) - p_{2l} = \upsilon - p_1 + S - \theta^* \left(\sigma_{2h}^2 + m_{2h}^2\right) - p_{2h} \quad (7)$$

I solve for $\theta^*$ :

$$\theta^* = \frac{p_{2l} - p_{2h}}{\sigma_{2h}^2 + m_{2h}^2 - \sigma_{2l}^2 - m_{2l}^2}$$

Demand for product $h$ is given by $b-\theta^*$ and for product $l$ is $\theta^*-a$, where $b-a=1$, and the profit functions for the high and low variability firms are the following:

$$\pi_l = (\theta^* - a)\left(p_l - t\left(k - \sigma_l^2\right)^2\right) - f$$

$$\pi_h = (b - \theta^*)\left(p_h - t\left(k - \sigma_h^2\right)^2\right) - f$$

**Proposition 9** *Both firms choose to enter the market sequentially, one firm (incumbent), enters in the first period and the other (entrant) enters in the second period. In the first period the incumbent chooses to produce with any specifications, variability $k$ and prices at $v$. In the second period the entrant chooses the same specifications of the incumbent. In the second period one of the firms chooses to produce with a high variance and the other with low variance. The variances chosen by both firms are the following:*

$$\sigma_{2h}^2 = k - \frac{1}{8t}(1+4b)$$
$$\sigma_{2l}^2 = k + \frac{1}{8t}(5-4b)$$

*and the prices are the following:*

$$p_{2h} = \frac{1}{64}\left(25 + 8b + 16b^2\right)$$

$$p_{2l} = \frac{1}{64}\left(49 - 40b + 16b^2\right)$$

Proof. See appendix.

where $\sigma_h^2$ represents the variance of the product consumed by the high-transportation-cost individuals and $\sigma_l^2$ is the variance of the product consumed by the low-transportation-cost individuals.

**First period** In the first period the incumbent can choose any location as individuals will like any location. The firm will price $v$ for its product as consumers are only willing to pay for the product the utility they receive in the first period.

Proposition (9) shows that there is path dependency in the introduction of products. Once the first firm enters, it attracts individuals preferences toward the characteristics of its product. Subsequent firms would choose as specifications the characteristics of the original product. The preferences and the specifications of the product in the second period are based on the characteristics of the product in the first period, not the intended specifications, but the actual realization of the production, including the error.

Therefore, an unexpected error in the production will affect the preferences and design of products in the following periods.

Which firm produces with high variance and which firm produces with low variance in the second period is not determined by the order in which each firm enters the market.

## 4.1 Variability as a Function of the Location

In this section I analyze the case where the cost of reducing the variability of the production process depends of the selection of the specifications. In some markets some specifications are associated with a lower variability than others. Greenspan (2005) explains how the reduction in wine yielding can improve the quality of the wine by reducing the variability of the grapes. However, the reduction in the variability of the grapes is related to their content of sugar. In order to reduce the variability with which the grapes ripe they have to be produced in low-yield crops which in turn increases the level of sugar in the grapes.

I will incorporate this evidence by assuming that the cost of decreasing the variability is a function of the specifications by making the parameter $t$ a function of $y$. I will assume that there is one product specifications $y^*$ where the cost of reducing the variability is minimized.

**Proposition 10** *If there are specifications $y^*$ where the cost $t(y)$ is minimized, then both firms choose to enter the market sequentially, one firm (incumbent) enters in the first period and the other (entrant) enters in the second period. In the first period the incumbent chooses to produce with specifications $y^*$, variability $k$ and charge a price of $\upsilon$. In the second period the incumbent and entrant will produce with the same specifications as the characteristics of the product of the first period. The variances chosen by both firms in the second period are the following:*

$$\sigma_{2h}^2 = k - \frac{1}{8t}(1+4b)$$

$$\sigma_{2l}^2 = k + \frac{1}{8t}(5-4b)$$

*and the prices are:*

$$p_{2h} = \frac{1}{64}\left(25+8b+16b^2\right)$$
$$p_{2l} = \frac{1}{64}\left(49-40b+16b^2\right)$$

Proof. See appendix.

The incumbent chooses the specifications where it can reduce the variability at a lower price. This is consistent with the literature in quality in management that stresses the development of products' designs that minimizes their variability in their production.

## 5 Conclusions

I have introduced competition in the habit formation literature by extending the idea of habit formation to the characteristics of the products. I analyzed two firms that compete when individuals grow to prefer the characteristics of the products they consume. This approach offers several advantages:

First, it allows us to analyze how firms choose the design of their products to respond to habit formation. Second, we can model how the availability of a product affects individuals' preference for it. Third, we can model how firms can use habit formation to create demand for new durable products by showing its product in the media and therefore, we can give another explanation for fads and fashion using habit formation. Fourth, it allows us to analyze how products affect the preferences in other markets that share the same characteristics. Finally, we can analyze competition when there is an error in the production process.

There are numerous ways this research can be expanded. The first natural extension is to study markets where consumers are forward looking. In some products like wine and cigars, individuals try to "refine" their preference by learning the characteristics of certain products like "fine" wines or "fine" cigars. An extension of my model may be able to explain what are the characteristics that define a "fine" product. For example, if individuals want to learn the characteristics of a wine from certain regions, then the fine wines would be considered those that better represent the characteristics of such regions.

Extending the model to analyze social interaction is essential to understand the impact of the media. For example, if our concept for beauty is influenced through habit, as some psychological evidence suggest, there would be a huge social impact of the media in the markets where beauty is important, like the marriage and the labor market.

My model assumes that individuals start without any initial preference for the characteristics of the products. However, if individuals with habit formation have initial preferences that are uniformly distributed in the characteristic space, it is possible that two or more firms enter in the first period and split the consumers between them. The results of my model would apply to new firms that enter after them and compete for the consumers that have grown to prefer the characteristics of each product.

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Appendix

Proof of Proposition 4

Individuals' utility from consuming the old product is:

$$S - \theta(d_{new})^2$$

Individuals' utility from consuming the new product is:

$$S - p_{new}$$

I solve for the transportation cost of the individual ($\theta^*$) that is indifferent between both products,

$$\theta^* = \frac{p_{new}}{(d_{new})^2}$$

Individuals with higher transportation cost than $\theta^*$ consume the new product. The profits for the firm is given by the demand $(b - \theta^*)$ times the price of the new product minus the cost of showing the new product in the media:

$$\pi_{new} = (b - \theta^*)p_{new} - (d_{new})^4$$

$$\pi_{new} = (b - \frac{p_{new}}{(d_{new})^2})p_{new} - (d_{new})^4$$

By solving for the first order condition with respect to price I get:

$$p_{new} = \frac{b(d_{new})^2}{2}$$

By substituting this price in the profit function I get:

$$\pi_{new} = (\frac{bd_{new}}{2})^2 - d_{new}^4$$

I solve for the first order conditions with respect to the distance and I get the optimal values of the distance and price:

$$d_{new} = \frac{b}{\sqrt{8}}$$

$$p_{new} = \frac{b^3}{16}$$

Proof of Proposition 5

If individuals only consume in market $j$ in the first period, then by the definition of utility function in multiple markets, individuals' favorite location in market $i$ in the second period is given by the product they consume in market $j$ in the first period.

$$l_{2i}^{*} = l_{1j}$$

That is, individuals utility from consuming from a new firm in market $i$ in the second period is:

$$U_{2i} = S - \theta(l_{2i} - l_{1j})^2 - p_{2i}$$

The location that maximizes the profits for the new firm in market $i$ is individuals' favorite location, that is the same location of the product individuals consume in the first period in market $j$.

$$l_{2i} = l_{2i}^{*}$$

Firms charge the highest possible price, that is $v$ in the first period and $S$ in the second period.

Proof of Proposition 6

Both firms can produce in any location in period one and still charge price $v$ as individuals do not care about any characteristics. In the second period individuals favorite location is an average of the location of the products in both markets:

$$l_{2i}^{*} = \phi l_{1i} + (1 - \phi) l_{1j}$$

If firms can change location from the first to the second period, both firms would move to this location as it is the only location where individuals are willing to pay $S$. Therefore, the characteristics of both products in the second period will be an average of the characteristics of the products in the first period.

Proof of Proposition 7

If firms cannot change location from the first to the second period and if both firms do not produce in the same location that each other in the first period, in the second period individuals' favorite characteristics will not be the same of the products and individuals would not be willing to pay as much for the products. It is easy to see that both firms can increase the price they charge for their product to $S$ by changing their location to the same location of the firm that produces in the other market.

Proof of Proposition 8

The proof is the same as proposition 1, with the difference that if firms do not enter in the same location, they can increase their profits by producing with in the same location as the firm in the other market. There are two cases:

a) If the firm enters close from the location of the other incumbent, the new firm will attract some of the low transportation cost consumers by producing further product from the one individuals prefer. I solve for the profits of each firm:

$$\pi_i = \frac{1}{9}(b+1)^2\left(d_e^2 - d_i^2\right) - f \tag{8}$$

$$\pi_e = \frac{1}{9}(2-b)^2\left(d_e^2 - d_i^2\right) - f$$

where $d_i$ and $d_e$ are the distances from the incumbent and entrant to the individuals favorite location, that is, to $l_{2i}^* = \phi l_{1i} + (1-\phi)l_{1j}$. We can see from above that the incumbent wants to produce a product in the first period that makes $d_i^2 = 0$ and the only location is where $l_{1i} = l_{1j}$.

b) If the firm enters far from the location of the other incumbent. In this case the new firm chooses to produce in the individuals' favorite location, that is location:

$$l_{2i}^* = \phi l_{1i} + (1-\phi)l_{1j}$$

The highest profits of the incumbent is given by equation () that is lower than the profits it gets by location in the same location as the first firm in the market that shares the same characteristic.

Proof of Proposition 9

The profit function for the firm that attracts the individuals with low transportation costs is:

$$\pi_l = \left(\frac{p_l - p_h}{\sigma_h^2 + m_h^2 - \sigma_l^2 - m_l^2} - a\right)\left(p_l - (k - \sigma_l^2)^2\right)$$

The profit function for the firm that attracts the individuals with high transportation costs is:

$$\pi_h = \left(b - \frac{p_l - p_h}{\sigma_h^2 + m_h^2 - \sigma_l^2 - m_l^2}\right)\left(p_h - (k - \sigma_h^2)^2\right)$$

I solve for the first order conditions with respect to the price for each firm:

$$p_l = \frac{1}{2}\left(a\left(\sigma_h^2 + m_h^2 - \sigma_l^2 - m_l^2\right) + (k - \sigma_l^2)^2 + p_h\right)$$

$$p_h = \frac{1}{2}\left(-b\left(\sigma_h^2 + m_h^2 - \sigma_l^2 - m_l^2\right) + (k-\sigma_h^2)^2 + p_l\right)$$

Solving for $p_l$ and $p_h$ I get the price for each firm:

$$p_l = \frac{1}{3}\left((2a-b)\left(\sigma_h^2 + m_h^2 - \sigma_l^2 - m_l^2\right) + 2(k-\sigma_l^2)^2 + (k-\sigma_h^2)^2\right)$$

$$p_h = \frac{1}{3}\left((a-2b)\left(\sigma_h^2 + m_h^2 - \sigma_l^2 - m_l^2\right) + (k-\sigma_l^2)^2 + 2(k-\sigma_h^2)^2\right)$$

The Second Derivatives with respect to price are negative, so these price maximize firms' profits:

$$\frac{\partial^2 \pi_l}{\partial p_l^2} = \frac{2}{m_h^2 + m_h^2 - \sigma_l^2 - m_l^2} < 0$$
$$\frac{\partial^2 \pi_h}{\partial p_h^2} = \frac{2}{m_h^2 + m_h^2 - \sigma_l^2 - m_l^2} < 0$$

Substituting $p_l$ and $p_h$ in the profit functions I get that the profit functions are:

$$\pi_l = \frac{1}{9}\left((b-2a) + \frac{(k-\sigma_l^2)^2 - (k-\sigma_h^2)^2}{\sigma_h^2 + m_h^2 - \sigma_l^2 - m_l^2}\right)\left((2a-b)\left(\sigma_h^2 + m_h^2 - \sigma_l^2 - m_l^2\right) + (k-\sigma_h^2)^2 - (k-\sigma_l^2)^2\right) \tag{9}$$

$$\pi_h = \frac{1}{9}\left((2b-a) - \frac{(k-\sigma_l^2)^2 - (k-\sigma_h^2)^2}{\sigma_h^2 + m_h^2 - \sigma_l^2 - m_l^2}\right)\left((a-2b)\left(\sigma_h^2 + m_h^2 - \sigma_l^2 - m_l^2\right) - (k-\sigma_h^2)^2 + (k-\sigma_l^2)^2\right) \tag{10}$$

The derivative of the profits with respect to $ml$ is:

$$\frac{\partial \pi_l}{\partial m_l} = \frac{1}{9}\left((b-2a)^2(2m_l) - 2m_l\frac{\left(\left(k-\sigma_l^2\right)^2 - \left(k-\sigma_h^2\right)^2\right)^2}{(\sigma_h^2 + m_h^2 - \sigma_l^2 - m_l^2)^2}\right)$$

The derivative of the profits with respect to $mh$ is:

$$\frac{\partial \pi_h}{\partial m_h} = \frac{1}{9}\left(-(2b-a)^2(2m_h) + 2m_h\frac{\left(\left(k-\sigma_l^2\right)^2 - \left(k-\sigma_h^2\right)^2\right)^2}{(\sigma_h^2 + m_h^2 - \sigma_l^2 - m_l^2)^2}\right)$$

$$\frac{\partial \pi_h}{\partial \sigma_h^2} = \frac{1}{9}\left( -4(k-\sigma_h^2)\frac{\left(k-\sigma_l^2\right)^2 - \left(k-\sigma_h^2\right)^2}{\sigma_h^2 + m_h^2 - \sigma_l^2 - m_l^2} + 2\left(a-2b\right)\left[-2\left(k-\sigma_h^2\right) + \frac{2b-a}{2}\right] + \frac{\left(\left(k-\sigma_l^2\right)^2 - \left(k-\sigma_h^2\right)^2\right)}{(\sigma_h^2 + m_h^2 - \sigma_l^2 - m_l^2)^2} \right.$$

$$\frac{\partial \pi_l}{\partial \sigma_l^2} = \frac{1}{9}\left( 4(k-\sigma_l^2)\frac{\left(k-\sigma_l^2\right)^2 - \left(k-\sigma_h^2\right)^2}{\sigma_h^2 + m_h^2 - \sigma_l^2 - m_l^2} + 2\left(b-2a\right)\left[2\left(k-\sigma_l^2\right) + \frac{b-2a}{2}\right] - \frac{\left(\left(k-\sigma_l^2\right)^2 - \left(k-\sigma_h^2\right)^2\right)^2}{(\sigma_h^2 + m_h^2 - \sigma_l^2 - m_l^2)^2} \right)$$

By solving for the values of $m_l, m_h, \sigma_l^2$ and $\sigma_h^2$ I get the solution of the proposition.

$$\begin{aligned} m_l &= 0 \\ m_h &= 0 \\ \sigma_{2h}^2 &= k - \frac{1}{8t}(1+4b) \\ \sigma_{2l}^2 &= k + \frac{1}{8t}(5-4b) \end{aligned}$$

Proof of Proposition 10

Firms' profits in the second period are:

$$\pi_l = \pi_h = \frac{3}{16t(l)}$$

Given that the profits for the firm that enters in the first period decrease with $t(l)$, the first firm enters in the location $l^*$ where $t(l)$ is minimized.