Private Electronic Money, Fiat Money and the Payments System

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Abstract

This paper provides insight on how a modern system of private electronic money would work and how the necessary network shall function. We present a model with two types of private electronic currencies with one being local, and the other being global. Both of them display transactional advantages and dominate fiat money in rate of return. However, in spite of these different returns, the two electronic currencies and fiat money circulate in equilibrium. We further observe that the local electronic currency can be sold with a premium or with a discount, depending on several factors including the probability of relocation faced by the agents in this economy. The higher the probability of relocation, the higher is this discount, and the lower the share of the local electronic currency in the young creditors' portfolio.

JEL classification: E41, E52
Keywords: Private Money, Commodity Money, Network, Payments System, Competition.

1. Introduction

In this paper we introduce and model a new concept of private electronic money. One of the main defining characteristics of this new type of electronic currency is that it is not a surrogate of fiat money. We do so in the context of a closed production economy with a payments system where agents face the possibility of relocation. We build from Townsend (1980), the basic payments system delineated by Freeman (1996) and from the literature of bank-runs as modified by Smith and others [for a recent example, see Gomis-Porqueras and Smith (2003)] from the basic Diamond-Dybvig model (1983). We then introduce private electronic currencies and modify accordingly the fashion in which debt is settled and payments are made in the payments system in an attempt to answer the following questions: what are the characteristics that these private electronic currencies must have in order for them to circulate? Can these electronic currencies circulate together with fiat money? Under which circumstances is this possible? And what are the characteristics that the network needs to have for the introduction of these currencies to be successful?

There have been extremely valuable recent contributions to the theoretical literature of private electronic money. Among these attempts, the papers by Williamson (1999) and Temzelides and Williamson (2001) provide us with remarkable insight. Both papers use the approach of a random matching model with infinitely lived agents, where all currencies are *indivisible*. Private money takes the form of a medium of exchange issued by *financial intermediaries*. Also, transaction costs, informational frictions, and related factors are the main factors explaining the discounts observed when trading different private currencies. However, as argued convincingly by Schreft (2001), even though the models offered by Williamson (1999) and Temzelides and Williamson (2001) are a very good representation of how the financial system worked during the 19th century, they do not provide a clear insight on how a modern system of private electronic money would work and how the necessary network shall function. Interestingly, Williamson (1999) and Temzelides and Williamson (2001) provide a very good analysis and interpretation regarding, for example, the private banking notes circulating in the Suffolk Banking System, as presented by Rolnick and Weber (1988), Rolnick, Smith and Weber (1998), and Smith and Weber (1998). We understand this to be a very interesting departing point by its own right. Thus, in their environment, private banknotes are debt and, at the end of the day, they take the form of privately issued fiat money, or, in other words, a *surrogate of fiat money*.

The recent advancements in electronic commerce are enormous. As of today, several companies are exploring a next step that would increase the potential use of the Internet: electronic micropayments and a clearinghouse to account for these transactions. It is said that approximately forty percent of today's online companies would sell contents they are currently giving away if they had a viable micropayment system. As an example, people could sell many different new creations on the Internet if only mechanisms existed to facilitate small payments. However, the main problem faced today is that transaction costs, such as fees from banks and credit card companies erase any profit. Companies such as Peppercoin are working in the introduction of easier-to-use technology that allows web sites to accept tiny payments by effectively processing them in batches, thereby cutting down on bank fees[1].

As far as we know, almost all of today's existing electronic moneys are *surrogates of fiat money*: they are denominated in fiat currency, and their values fluctuate with the value of the denominating currency. More importantly, they are *fiat* in the sense there is no backing of these electronic moneys, and thus they are not redeemable to the issuers. Firms like BitPass[2] allow web surfers to set up accounts with US dollars charged to a credit card or internet money-exchange service such as PayPal, and make micropayments from such accounts. However, in our mind, these firms act more like a payment intermediary rather than an electronic money issuer. In our eyes, a true electronic money issuer would be some firm who issues electronic money in its own denomination.

Modeling an object and an institution whose time is coming close but do not exist yet in its full form presents a very difficult task. As we mentioned before, the work of Williamson (1998) and Temzelides and Williamson (2001) constitute a strong point from which future efforts have to build. Future successful attempts must be able to provide useful insight on the proper mechanisms to be designed within the payments system. It is

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[1] See http://www.ecommercetimes.com/perl/story/32440.html and Bergstein (2003) for a more comprehensive account.

[2] See https://www.bitpass.com for more information.

our point of view that success would need a more comprehensive effort than just that of theoretical monetary economics. It is for this reason that in this paper we present the results of the combined effort of two disciplines: Macro/Monetary Economics and Management and Information Systems (MIS).

We propose a form of private electronic currency *issued by firms* who have the capability of doing so profitably, as opposed to financial intermediaries. This electronic currency constitutes debt issued by the firm, but at the same time, it is an advanced sale of the commodity (or basket of commodities) produced by such a firm. Thus, circulating private electronic currency, as we understand it, is a *circulating real option* that consumers can exercise or otherwise use as a transactional medium. Because of the positive network externalities involved in this type of trade, this advanced sale of commodities is different from that of traditional commodities. First, for this externality to be present, there have to be some transactional advantages: a divisible currency that can be used in transactions of arbitrarily large or small size at a *very low cost*, and a currency that provides privacy and anonymity to consumers who require them to perform specific transactions. The latter can be considered as necessary although not sufficient conditions. Second, in the case of traditional commodities, consumers decide whether to purchase them or not simply based upon the usefulness of these commodities. However, in the case of the private electronic currency that we propose, a consumer's choice of whether to acquire it or not takes also in consideration whether other agents in the economy are accepting it or not. Thus, this private electronic currency takes the form of some sort of commodity money which uses the issuer's products as its backing. However, the people's beliefs on the issuer's willingness and ability to honor its promised advanced sale will also play a very important role on the consumer's decision to acquire this currency.

There are, at least, two additional issues that must be addressed. First, there is the issue of whether or not alternative private electronic currencies can circulate simultaneously, with some of them being local, as posed by Schreft (2001). What is the meaning of an electronic currency being local and what are the requirements of the implied network? Second, there is the issue of both electronic currencies and fiat money circulating in equilibrium. We take the view that different currencies circulate in equilibrium because they provide for different needs, in spite of having different rates of return.

We thus propose a simple environment to reproduce how payments are settled in the economy. We focus on the two issues mentioned above and thus, we abstract from the complicated dynamic nature of contracting that considering agents with infinite lives would imply. We consider then heterogeneous agents with finite lives that are spatially separated in two outer locations, in the spirit of Townsend (1980), Freeman (1996) and Gomis-Porqueras and Smith (2003). However, the interpretation that we give to locations does not relate necessarily to space but rather to different networks or Internet communities. There have to be different outer locations or Internet communities, at least two types of electronic currencies, and agents facing the possibility of relocation if we want to be able to address the issue of alternative electronic currencies circulating and local electronic currencies. There is also a centralized location or network in this economy where clearing is present and contracts can be enforced. This centralized network, as apposed to outer networks, relates to a very well established network or community, with high entry barriers and strong enforcement mechanisms[3]. In an environment like the one we consider, the possibility of clearing and supervised trade in secondary markets for currencies could prove to be welfare improving.

We propose a model with the characteristic mentioned above where there are two types of firms that can issue each, a specific type of electronic currency (this currency cannot be issued by anybody else in the economy) denominated in its own units. Each type of this privately issued electronic currency represents not only debt issued by the producer to finance investment but also an advanced sale of the commodities produced by the corresponding type of firm: it is redeemable in the future in the exclusive commodity (or bundle of commodities) produced by that type of firm. However, only one type of electronic currency is accepted in both outer networks. Thus, the second electronic currency will be the one that we call the “local” electronic currency. The cost for consumers of accepting and carrying holdings of a given type of electronic currency from one period to the next is arbitrarily low due to the presence of the network externalities mentioned before. In addition, a stock of perfectly divisible fiat money

[3] See Parameswaran, Susarla and Whinston (2001) for a deeper analysis of the mentioned networks.

issued by the monetary authority circulates in the economy. The cost for consumers of accepting and holding fiat money from one period to the next (inflation) is higher than that of an electronic currency. Thus, in this environment fiat money is dominated in rate of return by the electronic currencies. However, the motive for agents in the economy to hold fiat money is given by a legal restriction imposing that taxes must be paid using this currency.

We offer a model with what we call *intentional* matching. The concept of intentional matching that we use in this paper differs from the one mentioned by Schreft (2001), in the following sense: different types of agents meet and exchange because it is in their best interest, given the physical environment and the structure of trade and travel patterns. In equilibrium, consumers hold both types of electronic money together with fiat money. Fiat money is always needed to pay taxes. However, as mentioned before, some agents face the possibility of relocation, and this event will be known only after they have the chance of choosing their portfolio of currencies. The latter implies that agents hold both types of electronic currency and that they later will look for the possibility of trading them depending on whether they have to relocate or not. It will be observed in equilibrium that the "local" electronic currency will be traded for the other electronic currency at a discount. We consider and delineate an environment general enough to allow for the transactional advantage of privacy and anonymity provided by electronic currencies. However, at this initial stage of our research we abstract from such possibility. This will be developed in the next stage of this research project. At this stage, we also abstract from the issue of willingness and ability by the issuers to hold to their promise, and leave this also for a later stage of our research. Thus, firms are willing and able to produce enough goods to honor all of their outstanding electronic currency. Also, all consumers exercise their real option and redeem all its holding of private electronic currency.

One of the main contributions of this paper is the introduction of a new type of private electronic currency, nonexistent yet but to be expected in the very near future. This type of currency shows to have desirable properties and positive network externalities, and it would circulate both as a complement and substitute of fiat money. We are one step ahead, then, and we can make ourselves ready about what to expect. This implies the need for a change on the way in which debt and payments are settled in the economy, and presents interesting challenges to both law makers and policy makers on how to design new settlement rules, how to regulate and supervise the financial system and the elements needed for this regulation and supervision to be a successful one.

We also study how the properties displayed by the equilibrium in this economy change according to different parameters of the system, such as different redemption rates of electronic currencies offered by firms, the transactional cost of electronic money and the inflation rate. In this sense, this paper has another important contribution: given the stylized nature of the model presented here, an experiment can be later designed and implemented in a laboratory, in order to provide insight to policy makers and potential future issuers of electronic money on the requirements that the design and implementation of such a network has within the payments system.

Finally, we should mention that this model could be extended to an open economy version, by, for example, considering one of the outer locations as a foreign location with a different fiat currency and introducing a market for foreign exchange. However, additional issues related to network design and regulation would arise and we abstract from them at this initial stage of our research.

The remainder of this paper is organized as follows. In Section 2 we present the general environment and the structure of the payments system. Next, in Section 3 we describe the incentives and behavior of the different types of agents that populate this economy. In Section 4 we present the market clearing and aggregate consistency conditions and in Section 5 we describe the properties displayed by equilibria in this environment. Finally, in Section 6 we present the main conclusions of our research and delineate future related research.

2. Environment

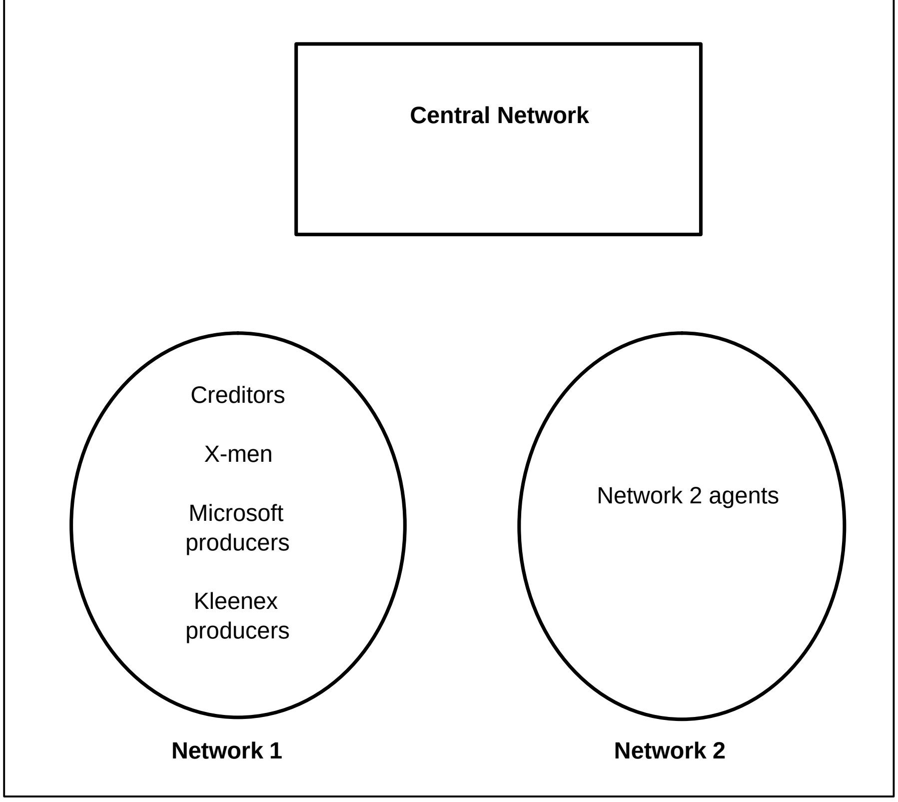
Consider a closed, production economy with overlapping generations of two-period lived agents. Time is discrete and indexed by t=0, 1, 2, … Each generation is composed of heterogeneous agents. One of the main features of this model is that a group of these agents are only dedicated to the production of producer-type-specific composite final goods. In particular, we consider two types of producers in this economy, and we will call them Microsoft producers and Kleenex producers, respectively, for reasons that will become apparent later. In addition, there is a group of agents who own a specific good whose consumption requires privacy and anonymity. We will label this good as good *X*, and the agents who own it will be called *X*-men, accordingly.

There are two outer networks or communities that are separated. These two networks surround a Central Network (or Central Island, as called in the payments system literature). Five types of agents are born in one of these two communities each period. In the first community there are four types of agents, which we will henceforth call creditors, *X*-men, Microsoft producers and Kleenex producers. In the second community there is a single type of agents, which we will henceforth call Network 2 agents. Population is constant and each generation has the same composition. A continuum with unit mass of agents of each type is born each period. There is a generation of initial old agents (creditors) endowed with the initial supply of fiat money M_0.

In addition, there is a Central Network. The Central Community is a well established network and think of it as where the civil and economic authority is located. All contracts can be enforced in the Central Network, and entry barriers are very high.

Figure 1 illustrates the structure of this economy.

Figure 1: Central Network and Outer Networks



At this point, it will be useful to have an overview of this economy. There are five goods each period: the creditors' endowment good, good X, the Network-2-specific good, the Microsoft composite final good and the Kleenex composite final good. Each composite final good can be thought of as a bundle of commodities produced by each producer. In addition, there are two types of physical capital each period: the Microsoft-specific physical capital and the Kleenex-specific physical capital. There are two types of labor each period: Microsoft-specific labor and Kleenex-specific labor. There are three types of currency: fiat money, Microsoft private electronic money and Kleenex private electronic money. Fiat money is issued by the monetary authority of this economy and, by definition, is not redeemable. Taxes must be paid using fiat money. Private producers issue private electronic money. Thus, Microsoft electronic money is issued only by Microsoft producers and it is redeemable in the Microsoft composite final good. Kleenex electronic money is issued only by Kleenex producers and it is redeemable in the Kleenex composite final good. Notice that Kleenex electronic money is what we will call the local electronic currency.

2.1 Private Electronic Currencies and the Networks

As we mentioned before, almost all of the electronic currencies existing today are *surrogates of fiat money*: they are denominated in fiat currency, and their values fluctuate with the value of the denominating currency. The form of private electronic currency that we propose is *issued by firms* who have the capability of doing so profitably, as opposed to financial intermediaries. There are several advantages with this revolutionary electronic money: its value will not be fluctuating with fiat currencies any more, but instead it will depend upon the value of commodity bundle backing it. In some sense, this type of electronic currency belongs to commodity money: like gold and silver, it is now the product bundle that serves as the anchor. Freed from financial risk, this new form of electronic money should have the potential of being a currency with more stable value than fiat currency. As the money for the Internet, it has the potential to become a global currency by eliminating the need for foreign exchange. This is also why in our minds the issuer should be some firms which are world-wide recognized, like Microsoft. In this sense, the existing form of electronic money e-gold[4] has the flavor of what we are proposing: e-gold users' account balance are denominated in various precious metals, it enables people to make payment with specific weights of gold. However, notice that while the limited availability of gold serves as the mechanism to stabilize its value, it also disables gold to serve as the world's global currency. In fact, that is exactly the same cause for the breakdown of Bretton Woods System. In contrast, the form of electronic money we are proposing has the ability to become a global currency. Notice, however, that the mechanism that would stabilize its value is not the limited availability of the commodities backing these currencies. Instead, the value-stabilizing mechanism would be given by the issuer's profit-maximizing incentive to keep its product bundle's value and maintain its reputation as an issuer.

Technically speaking, for any electronic money to be accepted by the public, three problems must be solved: security, privacy and low cost. Above all, open networks like the Internet need to generate *trust* in an electronic environment, thus the kernel of electronic payment is its security. Cryptographic security mechanisms, including data encryption and digital signature schemes, are often used to provide security for the transaction. Also, privacy has become a much bigger concern in today's information world: despite its potential flaw[5], an ideal electronic money should have the same property as cash, which can provide anonymity for the consumer. This problem is usually solved by the use of zero-knowledge proofs. Last but not the least, the wide acceptance of electronic money also hinges critically on the issuer's ability to provide a low cost currency, especially for the micropayments. So far, most of the research on decreasing the cost has been focusing on reducing the computational intensiveness of public key operations. However, there are also economic and judicial ways to reduce the cost. It is based on these engineering characteristics of electronic money that we explicitly model as privacy and low cost in our environment. Although security is not explicitly mentioned in our model, it is also implied since security is often associated with privacy.

[4] See http://www.e-gold.com/ for more information.

[5] There is wide concern that this anonymous currency can be a "safe heaven" for criminal activities such as money laundering, and blackmailing.

2.2 Endowments and Preferences

Each young creditor is endowed with *y* units of a specific endowment good. Old creditors have no endowment. Young Microsoft producers have no endowments of goods but they have exclusive access to an investment technology that allows them to transform one unit of the creditors' endowment good at time *t* into one unit of Microsoft-specific capital at time *t+1*. Old Microsoft producers are endowed with one unit of Microsoft-specific labor. Also, old Microsoft producers have exclusive access to a constant returns to scale production technology that transforms the Microsoft-specific capital stock they got from last period $\left(K_t^M\right)$and Microsoft-specific labor $\left(L_{t+1}^M\right)$into the Microsoft-specific composite final good. K_t^M is essential in the production of the Microsoft composite final good. To fix ideas, we let the Microsoft-specific production function take the form $\left(K_t^M\right)^a \left(L_{t+1}^M\right)^{1-a}$, where $0 < a < 1$. In addition, for simplicity we assume that old Microsoft producers work only for themselves and, thus, $L_{t+1}^M = 1$.

Young Kleenex producers have no endowments of goods but they have exclusive access to an investment technology that allows them to transform one unit of the creditors' endowment good at time *t* into one unit of Kleenex-specific capital at time *t+1*. Old Kleenex producers are endowed with one unit of Kleenex-specific labor. Also, old Kleenex producers have exclusive access to a constant returns to scale production technology that transforms the Kleenex-specific capital stock they got from last period $\left(K_t^K\right)$and Microsoft-specific labor $\left(L_{t+1}^K\right)$into the Microsoft-specific composite final good. To fix ideas, we let the Kleenex-specific production function take the form $\left(K_t^K\right)^b \left(L_{t+1}^K\right)^{1-b}$, where $0 < b < 1$. Also for simplicity we assume that old Kleenex producers work only for themselves and, thus, $L_{t+1}^K = 1$.

X-men are endowed with *w* units of good *X* when young and with nothing when old. Finally, young Network 2 agents are endowed with *e* units of a Network-2-specific good that we will call nuts, for simplicity. Old Network 2 agents have no endowments.

Creditors do not derive utility of consuming when young. They face the possibility of relocation when old in the following way[6]: with probability p , and old creditor must relocate to Network 2. If an old creditor born at t does relocate, she derives utility only from consuming nuts $\left(c_{t+1}^{n}\right)$, which are the Network-2 specific endowment good. However, with probability $(1-p)$ an old creditor does not have to relocate, and she can stay in Network 1. If an old creditor born at t does not relocate, she derives utility from consuming the Microsoft composite final good $\left(c_{t+1}^{M}\right)$, the Kleenex composite final good $\left(c_{t+1}^{K}\right)$, the creditors' endowment good $\left(c_{t+1}^{c}\right)$ and good X $\left(c_{t+1}^{x}\right)$. Thus, the expected lifetime utility of a creditor born at t is of the form

$$pu\left(c_{t+1}^{n}\right)+(1-p)v\left(c_{t+1}^{M},c_{t+1}^{K},c_{t+1}^{c},c_{t+1}^{x}\right) \quad (1)$$

where $u''<0<u'$, $v_{11}<0<v_{1}$, $v_{22}<0<v_{2}$, $v_{33}<0<v_{3}$ and $v_{44}<0<v_{4}$. Both $u(\cdot)$ and $v(\cdot)$ satisfy the Inada conditions. We further assume, for simplicity, that $v(\cdot)$ is additively separable, but this assumption is not required.

X-men derive utility only from consuming the Microsoft composite final good $\left(h_{t}^{M}\right)$ and the Kleenex composite final good $\left(h_{t}^{K}\right)$when young. Thus, the lifetime utility of an *X*-man born at t is given by $a\left(h_{t}^{M},h_{t}^{K}\right)$, where $a_{11}<0<a_{1}$ and $a_{22}<0<a_{2}$. Microsoft producers derive utility only from consuming the Microsoft composite final good when old $\left(g_{t+1}\right)$. Hence, the lifetime utility of a Microsoft producer born at t is given by $i\left(g_{t+1}\right)$, where $i''<0<i'$. Kleenex producers derive utility only from consuming the Kleenex composite final good when old $\left(b_{t+1}\right)$. The lifetime utility of a Kleenex producer born at t is given by $j\left(b_{t+1}\right)$, where $j''<0<j'$. Finally, Network 2 agents derive utility only from consuming the Microsoft composite final good when young $\left(l_{t}\right)$. Thus, the lifetime utility of a Network 2 agent born at t is given by $o\left(l_{t}\right)$, where $o''<0<o'$. Notice that $a(\cdot)$, $i(\cdot)$, $j(\cdot)$, and $o(\cdot)$ satisfy the Inada conditions

[6] We borrow from the relocation literature of bank runs, introduced by Bruce D. Smith.

2.3 Trade and Travel Patterns

Each period has two parts. In the first part of the period both intra- and inter-generational trade takes place: young agents trade with other young agents in the outer networks, old agents trade with other old agents in the Central Network, and young agents trade with old agents in the outer networks. During the second part of period intra- and intergenerational trade takes place: old agents trade with other old agents in the outer networks and young agents trade with old agents in the outer networks. We now explain in detail these transactions.

Young agents, first part of the period

During the first part of the period, young Microsoft producers issue Microsoft electronic money and exchange it for the creditors' endowment good, which they need for investment. One unit of Microsoft electronic money is a promise made by a young Microsoft producer to pay r^M units of the Microsoft composite final good next period. At the same time, the young Kleenex producers issue Kleenex electronic money and exchange it for the creditors' endowment good, which they need for investment. One unit of Kleenex electronic money is a promise made by a young Kleenex producer to pay r^K units of the Kleenex composite final good next period. Holding one unit of electronic money (either Microsoft money or Kleenex money) from one period to the other is costly: the electronic money holding lose value at the rate e . However, by assumption, the cost of holding electronic money is lower than the cost of holding fiat currency. The young creditors will accept the electronic money issued by the Microsoft young producers and by the Kleenex young producers in exchange for part of their endowment good. The remainder of the young creditors' endowment will be exchanged for fiat money. Creditors must use fiat money to pay a lump-sum tax of t_1 goods next period, measured in terms of the creditors' endowment good.

The Microsoft young producers take the creditors' endowment good they purchased and invest it into the Microsoft-specific physical capital. Similarly, the Kleenex young producers take the creditors' endowment good they purchased and invest it into the Kleenex-specific physical capital.

Old agents, first part of the period

During the first part of the period, all the old creditors must travel to the Central Network. Old creditors use their holdings of fiat money to pay the tax they owe to the economic authority. At this moment, while on the Central Network, the old creditors learn whether they must relocate or not. If an old creditor must relocate, her holdings of Kleenex electronic money are of no use to her, since only Microsoft electronic money will be accepted in Network 2 (Network 2 agents wish to consume only the Microsoft composite final good). However, an old creditor who does not relocate has use for both Microsoft and Kleenex electronic money. At this point, a secondary market for electronic currencies opens: old creditors who must relocate will sell their holdings of Kleenex electronic money to the old creditors who do not relocate, in exchange for Microsoft electronic money. Let q_t denote the price of one unit of Kleenex electronic money in terms of Microsoft electronic money at t. After these transactions take place, the old agents who do not relocate receive a transfer of $t_{2,t}$ units of fiat money from the monetary authority. Later, all the old creditors who do not relocate go back to Network 1 and all the old creditors who relocate travel to Network 2.

Young with old agents, first part of the period

The old creditors who do not relocate go back to Network 1. Here, they use the fiat money they bring from the Central Island to purchase the creditors' endowment good from the young creditors.

Second part of the period

The old Microsoft producers use the Microsoft-specific physical capital and their labor to produce the Microsoft composite final good. They have to produce enough of the good to stand ready to pay back to all the agents who show up with the Microsoft electronic money they issued last period. Similarly, the old Kleenex producers use the Kleenex-specific physical capital and their labor to produce the Kleenex composite final good. They also have to produce enough of the good to stand ready to pay back to all the agents who show up with the Kleenex electronic money they issued last period.

The old creditors who relocate, once in Network 2, use their holdings of Microsoft electronic money to purchase nuts from the young Network 2 agents. The young Network 2 agents then travel to Network 1 and use (redeem) their holdings of

Microsoft electronic money to obtain and consume the Microsoft composite final good from the old Microsoft producers.

The old creditors who do not relocate use part of their holdings of Microsoft electronic money and Kleenex electronic money to secretly and anonymously purchase good *X*. The young *X*-men exchange their good for electronic currency. Later, the young *X*-men use their holdings of Microsoft electronic money to purchase the Microsoft composite final good from the old Microsoft producers and consume it, while they use their holdings of Kleenex electronic money to purchase the Kleenex composite final good from the old Kleenex producers and consume it.

The old creditors use the remainder of their holdings of Microsoft electronic money to purchase the Microsoft composite final good from the old Microsoft producers. They also use the remainder of their holdings of Kleenex electronic money to purchase the Kleenex composite final good from the old Kleenex producers.

The old Microsoft producers consume whatever is left of the Microsoft composite final good that they produced, and the old Kleenex producers whatever is left of the Kleenex composite final good that they produced.

It is important to mention at this point that, in order to gain insight of the fundamental properties displayed by equilibria with three types of currency circulating in this environment, in the remainder of this preliminary version of the paper, we depart from the general setup of the model and solve the case where there is no good *X* or *X*-men. As a consequence, equation (1) is modified accordingly:

$$pu\left(c_{t+1}^{n}\right)+\left(1-p\right)v\left(c_{t+1}^{M},c_{t+1}^{K},c_{t+1}^{c}\right) \qquad (1')$$

3. Agents' Behavior

3.1 Creditors

Consider a creditor born at *t*. Let m_t denote the nominal holdings of fiat money by a young creditor, and $\tilde{E}_t^M$ and $\tilde{E}_t^K$ denote her nominal holdings of Microsoft electronic money and Kleenex electronic money, respectively. Let also P_t denote the price of the creditors' endowment good in terms of fiat money at *t*, and P_t^M and P_t^K denote the price of the creditor's endowment good in terms of Microsoft electronic money and Kleenex electronic money, respectively. Then, a young creditor born at t faces the following budget constraint:

$$y = \frac{m_t}{P_t} + \frac{\tilde{E}_t^M}{P_t^M} + \frac{\tilde{E}_t^K}{P_t^K} \quad (2)$$

In this environment, since taxes must be paid using fiat money, a young creditor holds fiat money only to be able to pay the tax of t_1, of her endowment goods when old. Thus,

$$\frac{m_t}{P_t} = \frac{P_{t+1}}{P_t} t_1 \quad (3)$$

Combining (2) and (3), we obtain the following budget constraint:

$$y = \frac{P_{t+1}}{P_t} t_1 + \frac{\tilde{E}_t^M}{P_t^M} + \frac{\tilde{E}_t^K}{P_t^K} \qquad (4)$$

An old creditor who does not relocate faces the following budget constraints:

$$c_{t+1}^M = r^M (1-e) \tilde{E}_{t+1}^{M,N} \quad (5)$$

$$c_{t+1}^K = r^K (1-e) \tilde{E}_{t+1}^{K,N} \quad (6)$$

$$\tilde{E}_{t+1}^{K,N} = \tilde{E}_t^K + \frac{1}{q_{t+1}} \left[\tilde{E}_t^M - \tilde{E}_{t+1}^{M,N} \right] \qquad (7)$$

and

$$c_{t+1}^c = \frac{t_{2,t+1}}{P_{t+1}}, \quad (8)$$

where e is the cost of holding electronic money between t and $t+1$, and $t_{2,t+1}$ is the nominal monetary transfer received from monetary authority by the old creditors who do not relocate. Also, $\tilde{E}_{t+1}^{M,N}$ denotes the nominal holdings of Microsoft electronic money by an old creditor who relocates, and $\tilde{E}_{t+1}^{K,N}$ denotes her nominal holdings of Kleenex electronic money.

The budget constraint (5) reflects the fact that an old creditor can purchase the Microsoft composite final good only with Microsoft electronic money. The budget constraint (6) reflects the fact that an old creditor can purchase the Kleenex composite final good only with Kleenex electronic money. The budget constraint (7) reflects this old creditor's transactions in the secondary market for electronic money with old creditors who must relocate. Finally, the budget constraint (8) indicates that an old creditor who does not relocate uses fiat money to purchase the young creditors' endowment good (since her holdings of electronic money cannot be redeemed into her endowment good afterwards).

An old creditor who relocates faces the following budget constraints:

$$n_{t+1}c^n_{t+1} = (1-e)\tilde{E}^{M,R}_{t+1} \quad (9)$$

$$\tilde{E}^{M,R}_{t+1} = \tilde{E}^M_t + q_{t+1}\tilde{E}^K_t \quad (10)$$

and

$$\tilde{E}^{K,R}_{t+1} = 0 \qquad (11)$$

where $\tilde{E}^{M,R}_{t+1}$ denotes the nominal holdings of an old creditor who does not relocate and $\tilde{E}^{K,R}_{t+1}$ denotes her holdings of Kleenex electronic money.

After some simple manipulations, we obtain:

$$c^n_{t+1} = \frac{(1-e)}{n_{t+1}}\left[\tilde{E}^M_t + q_{t+1}\tilde{E}^K_t\right] \quad (12)$$

$$c^K_{t+1} = r^K(1-e)\left\{\tilde{E}^K_t + \frac{1}{q_{t+1}}\left[\tilde{E}^M_t - \frac{c^M_{t+1}}{r^M(1-e)}\right]\right\} \qquad (13)$$

A young creditor born at t chooses $\tilde{E}^M_t$, $\tilde{E}^K_t$ and c^M_{t+1} in order to maximize her lifetime utility (1') subject to (4), (5), (8), (12) and (13). Let l be the Lagrange multiplier associated with (4). Thus, the first order conditions determining $\tilde{E}^M_t$, $\tilde{E}^K_t$ and c^M_{t+1} are given by:

$$\frac{p(1-e)}{n_{t+1}}u'(\cdot)+\frac{(1-p)r^K(1-e)}{q_{t+1}}v_2=\frac{l}{P_t^M} \qquad (14)$$

$$\frac{p(1-e)q_{t+1}}{n_{t+1}}u'(\cdot)+(1-p)r^K(1-e)v_2=\frac{l}{P_t^K} \qquad (15)$$

$$(1-p)v_1-(1-p)\frac{r^K}{r^M q_{t+1}}v_2=0 \qquad (16)$$

and by the budget constraint (4). By combining (14) and (15), we obtain:

$$q_{t+1}=\frac{P_t^M}{P_t^K} \qquad (17)$$

Also, combining (16) and (17) yields:

$$\frac{v_1\left(c_{t+1}^M\right)}{v_2\left(c_{t+1}^K\right)}=\frac{r^K}{r^M q_{t+1}} \qquad (18)$$

3.2 Microsoft Producers

Consider a Microsoft producer born at *t*. Recall that g_{t+1} denotes the consumption of Microsoft composite final good by an old Microsoft producer at $t+1$. A young Microsoft producer issues $\hat{E}_t^M$ units of Microsoft electronic money at *t* and then invests the $\left(\frac{\hat{E}_t^M}{P_t^M}\right)$ units of the creditors' endowment good she purchased into Microsoft-specific physical capital. When old, a Microsoft producer will produce $\left(\frac{\hat{E}_t^M}{P_t^M}\right)^a$ units of the Microsoft composite final good. Part of her production will be used to repay the holders of the electronic money she issued when young in the amount of $r^M(1-e)\hat{E}_t^M$. She will use the remainder of her production for her own consumption. Thus, an old Microsoft producer faces the following budget constraint:

$$\left(\frac{\hat{E}_t^M}{P_t^M}\right)^a=(1-e)r^M\hat{E}_t^M+g_{t+1} \qquad (19)$$

A young Microsoft producer born at t chooses $\hat{E}_t^M$ in order to maximize her life-time utility:

$$i\left(\left(\frac{\hat{E}_t^M}{P_t^M}\right)^a - (1-e)r^M \hat{E}_t^M\right) \quad (20)$$

taking P_t^M, e, and r^M as given. The choice by a Microsoft producer can be expressed as:

$$\hat{E}_t^M = P_t^M\left[\frac{a}{(1-e)r^M P_t^M}\right]^{\frac{1}{1-a}} \quad (21)$$

3.3 Kleenex Producers

Consider a Kleenex producer born at t. Recall that b_{t+1} denotes the consumption of the Kleenex composite final good by an old Kleenex producer at $t+1$. A young Kleenex producer issues $\tilde{E}_t^K$ units of Kleenex electronic money at t. Later, she invests $\left(\frac{\hat{E}_t^K}{P_t^K}\right)$ units of the creditors' endowment good she purchased into Kleenex-specific physical capital. When old, a Kleenex producer will produce $\left(\frac{\hat{E}_t^K}{P_t^K}\right)^b$ units of the Kleenex composite final good. She will use part of her production to repay the holders of the electronic money she issued when young in the amount of $r^K(1-e)\hat{E}_t^K$. She will use the remainder of her production for her own consumption. Thus, an old Kleenex producer faces the following budget constraint:

$$\left(\frac{\hat{E}_t^K}{P_t^K}\right)^b = (1-e)r^K \hat{E}_t^K + b_{t+1} \quad (22)$$

A young Kleenex producer born at t chooses $\hat{E}_t^K$ in order to maximize her life-time utility:

$$j\left(\left(\frac{\hat{E}_t^K}{P_t^K}\right)^b - (1-e)r^K \hat{E}_t^K\right) \quad (23)$$

taking P_t^K, e, and r^K as given. The choice by a Kleenex producer can be expressed as:

$$\hat{E}_t^K = P_t^K \left[\frac{b}{(1-e)r^K P_t^K}\right]^{\frac{1}{1-b}} \quad (24)$$

3.4 Network 2 Agents

Consider a Network 2 agent born at *t*. A young Network 2 agent sells her endowment of nuts in exchange for Microsoft electronic money. Later, the young Network 2 agent uses her holdings of Microsoft electronic money to purchase the Microsoft composite final good l_t. Let L_t^M denote the nominal holdings of Microsoft electronic money by a young Network 2 agent at t, and n_t denote the price of nuts in terms of Microsoft electronic money. Thus, a young Network 2 agent faces the following budget constraint:

$$L_t^M = n_t e \quad (25)$$

$$l_t = r^M L_t^M \quad (26)$$

A young Network 2 agent wants to maximize her lifetime utility:

$$o(r^M n_t e) \quad (27)$$

3.5 The Government

The monetary authority in this economy prints fiat money at the real net rate s. Thus, the nominal supply of fiat money evolves according to:

$$M_{t+1} = (1+s)M_t,\ M_0 > 0 \quad (28)$$

Also, the government collects taxes from old creditors and gives monetary transfers to old creditors who do not relocate.

4. Market Clearing Conditions

In our model, because of the relocation, a secondary market for electronic money will emerge between the old creditors who are and are not relocated. Since for those creditors who are relocated, Kleenex electronic money is useless, they exchange all their holdings of this currency for Microsoft electronic money, which is the only useful transactional medium in Network 2. The creditors who are not relocated will accept the Kleenex electronic money because they will be able to redeem it and consume the Kleenex composite good.

Thus, the secondary market for electronic money clears when demand equals supply:

$$(1-p)\left[\tilde{E}_t^{M} - \frac{1}{r^M(1-e)} c_{t+1}^M\right] = q_{t+1} p\, \tilde{E}_t^{K} \qquad (29)$$

where q_{t+1} is the exchange rate which clears the secondary market: it is the price of a unit of Kleenex electronic money in terms of Microsoft electronic money. If $q_{t+1} < 1$, it means that Kleenex electronic money is sold at a discount.

In the primary market for Microsoft electronic money, the demand should equal the supply in equilibrium:

$$\tilde{E}_t^{M} = \hat{E}_t^{M} = E_t^M \qquad (30)$$

Equivalently, in the primary market for Kleenex electronic money, the demand should equal the supply in equilibrium:

$$\tilde{E}_t^{K} = \hat{E}_t^{K} = E_t^K \qquad (31)$$

Obviously, in the market for fiat money, the following condition holds:

$$m_t = M_t = (1-p)t_{2,t} \quad (32)$$

That means that the money supply is equally distributed among the old creditors who do not relocate.

Also, Substituting (3) into (32) gives us:

$$M_t = m_t = P_{t+1} t_1, \; \forall t = 0,1,2... \qquad (33)$$

and thus, the market for fiat money clears when:

$$\frac{P_{t+1}}{P_t} t_1 = \frac{(1-p)t_{2,t}}{P_t} \qquad (34)$$

The real return on fiat money in equilibrium is given by:

$$\frac{P_t}{P_{t+1}} = \frac{1}{(1+s)} \quad (35).$$

Since $(1-e)(1+s) > 1$, fiat money is dominated in rate of return by both electronic currencies.

For every young creditor, her endowment good has three uses: it can be used to purchase Microsoft electronic money, to purchase Kleenex electronic money, or sold to old creditors who are not relocated, in exchange for fiat money that she will be used to pay her tax when she travels to the central island the next period.

Thus, the market for the young creditors' endowment good clears when

$$y = \frac{E_t^M}{P_t^M} + \frac{E_t^K}{P_t^K} + (1+s)t_1 \qquad (36)$$

Let G_t denote the total production of Microsoft composite good at *t*. The market for Microsoft composite final good clears when supply equals demand:

$$G_t = (1-p)c_t^M + l_t + g_t \qquad (37)$$

Let B_t denote the total production of Kleenex composite good at *t*. The market for Kleenex composite final good clears when supply equals demand:

$$B_t = (1-p)c_t^K + b_t \qquad (38)$$

In Network 2, the consumption of nuts by the relocated old creditors (demand) will equal the Network 2 agent's endowment of coconuts (supply):

$$e = p \cdot c_t^n \qquad (39)$$

Finally, the following aggregate consistency conditions must hold:

$$(1-p)c_{t+1}^M + l_{t+1} = r^M(1-e)E_t^M \qquad (40)$$

$$(1-p)c_t^K = r^K(1-e)E_t^K \qquad (41)$$

5. Equilibrium

Combining (4), (21), (24), (30) and (31), and solving for P_t^K as a function of P_t^M yields:

$$P_t^K = P_t^K\left(P_t^M\right) \equiv \frac{b}{r^K(1-e)\left\{y-(1+s)t_1-\left[\dfrac{a}{(1-e)r^M P_t^M}\right]^{\frac{1}{1-a}}\right\}^{1-b}} \quad (42)$$

Combining (4), (21), (29), (30) and (31), and solving for c_{t+1}^M as a function of P_t^M yields:

$$c_{t+1}^M = c_{t+1}^M\left(P_t^M\right) \equiv \frac{r^M(1-e)P_t^M}{1-p}\left\{\left[\frac{a}{(1-e)r^M P_t^M}\right]^{\frac{1}{1-a}} - p\left[y-(1+s)t_1\right]\right\} \quad (43)$$

Combining (13), (21), (24), (30), (31) and (42), and solving for c_{t+1}^K as a function of P_t^M yields:

$$c_{t+1}^K = c_{t+1}^K\left(P_t^M\right) \equiv \frac{b}{1-p}\left\{y-(1+s)t_1-\left[\frac{a}{(1-e)r^M P_t^M}\right]^{\frac{1}{1-a}}\right\}^{b} \quad (44)$$

Using (42), (43), (44) and (18), we obtain the expression:

$$r^M P_t^M v_1\left[c_{t+1}^M\left(P_t^M\right)\right] = r^K P_t^K\left(P_t^M\right) v_2\left[c_{t+1}^K\left(P_t^M\right)\right] \quad (45)$$

Then, the equilibrium P_t^M is such that equation (45) is satisfied.

In this preliminary version of the property, we illustrate the property displayed by equilibria by using a numerical example. In later versions of this paper, we will include analytical results.

Numerical example with a loglinear utility function. To fix ideas, in this section we let equation (1') take the following functional form:

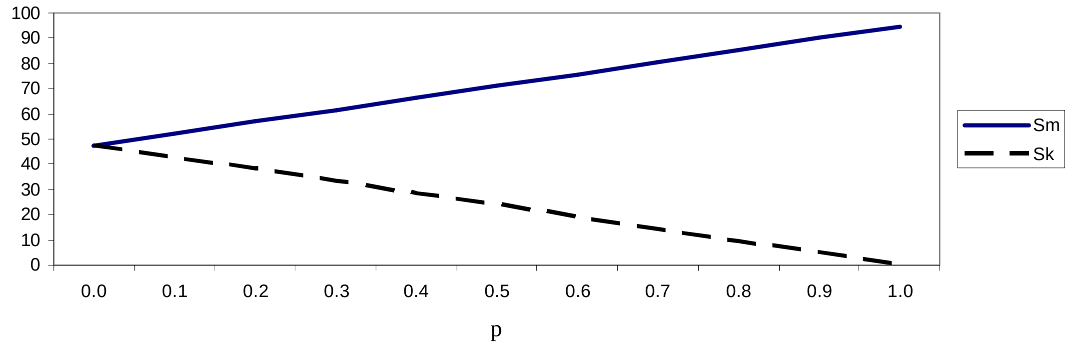
$$p\,\ln\left(c_{t+1}^n\right) + (1-p)\left[\ln\left(c_{t+1}^M\right) + \ln\left(c_{t+1}^K\right) + \ln\left(c_{t+1}^c\right)\right] \quad (46)$$

We set the following parameter values across all scenarios: $e = 0.01$, *y*=2, and *e*=1. We define three cases as corresponding to the following pairs of (a, b): (0.3,0.2), (0.3,0.3) and (0.2,0.3). For each pair (a, b), we define first eleven scenarios, each corresponding to the following values of *p*: 0, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90 and 0.99999.Then, for each value of *p*, we use the following values of s: 0.05, 0.075 and 0.1. Finally, for each value of p and each value of s, we use the following pairs of values for (r^M, r^K): (1.1, 1.05), (1.075, 1.075) and (1.05, 1.1). Thus, we evaluate a total of 297 scenarios that we use to analyze the properties of the equilibrium.

One of the advantages of using a log linear utility function is that it ensures the uniqueness of the stationary equilibrium. A second property of the log linear utility function is that the different pairs (r^M, r^K) considered have no effect on the real holdings of each type of electronic currency by a young creditor. But, as we will see later, different combinations of (r^M, r^K) do affect q, which is the relative price of Kleenex electronic money in terms of Microsoft electronic money in the secondary market.

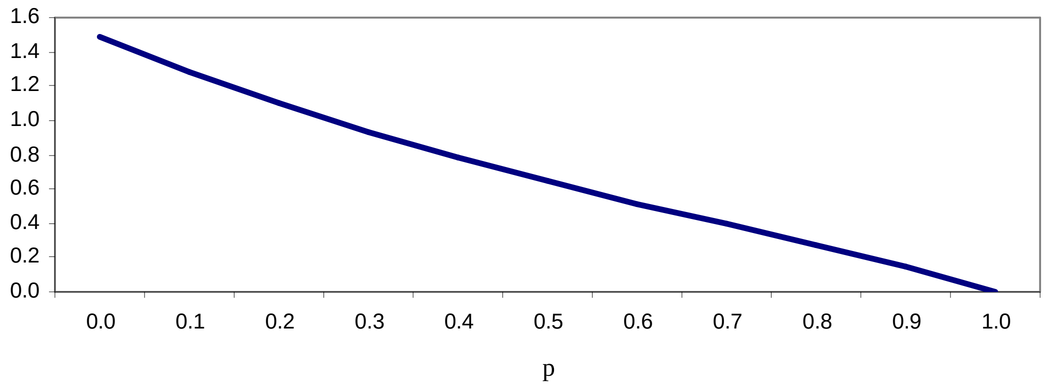
Let *Sm* denote the share of real holdings of Microsoft electronic currency in a young creditor's portfolio, and *Sk* denote the share of real holdings of Kleenex electronic currency in a young creditor's portfolio. We find that *Sm* is always an increasing function of the probability of relocation (p), while *Sk* always decreases as p increases. This property is illustrated in Figure 1. Notice that when the probability of relocation is zero, young creditors hold equal amounts, in real terms, of the Kleenex and Microsoft electronic currency. In the limit, as p grows close enough to 1, young creditors will hold almost nothing of Kleenex currency, but only fiat money and Microsoft electronic currency.

Figure 1: Shares of Microsoft and Kleenex electronic money on the young creditor's portfolio (%) as a function of p
Loglinear utility



A second important property of the stationary equilibrium is that, as it should be expected, the price of Kleenex electronic currency in terms of Microsoft electronic currency (q) decreases as the probability of relocation increases. This property is illustrated in Figure 2. In the limit, as p grows close enough to 1, there is no need to hold Kleenex electronic currency, and thus its price becomes zero.

Figure 2: Price of Kleenex electronic money in terms of Microsoft electronic money (q)
Loglinear utility



However, whether Kleenex electronic money will be sold at a discount (q<1) or at a premium (q>1) seems to be a more complicated matter. In order to answer this question we also need to take into account the relative capital-intensity of each sector and the combination of $\left(r^M, r^K\right)$. A summary of results can be found in Tables 1 and 2.

In Table 1, we consider the case where both electronic currencies have the same redemption rates, i.e.: we keep $r^M = r^K$. This exercise allows us to focus on the effects of relative capital intensity across sectors on q. When the production of the Microsoft composite final good is more capital intensive $(a > b)$, it is possible to observe that the Kleenex electronic currency is sold at a premium (q>1) when the probability of relocation is low enough. One possible explanation is that, in relative terms, the Kleenex industry requires less debt in order to produce. However, if the Kleenex industry is more capital intensive $(a < b)$, the Kleenex currency is always sold at a discount (q<1).

Table 1
q(p), Loglinear utility
$r^m=r^k$=1.075, s=0.075

p	a>b	a=b	a<b
0.00	1.492	1.000	0.670
0.10	1.283	0.869	0.577
0.20	1.098	0.753	0.496
0.30	0.933	0.648	0.423
0.40	0.783	0.553	0.358
0.50	0.645	0.463	0.298
0.60	0.516	0.379	0.242
0.70	0.393	0.297	0.189
0.80	0.273	0.215	0.136
0.90	0.151	0.127	0.080
1.00	0.000	0.000	0.000

In Table 2, we consider the case where both industries have the same capital intensity in production: we keep $a=b$. This exercise allows us to focus on the effects of different redemption rates of electronic currencies on q. Only when $r^M < r^K$ and the probability of relocation is low enough, do we observe that Kleenex electronic currency will be sold at a premium (q>1).

Table 2
q(p), Loglinear utility
a=b, s=**0.075**

p	$r^M>r^K$	$r^M=r^K$	$r^M<r^K$
0.00	0.955	1.000	1.048
0.10	0.829	0.869	0.910
0.20	0.719	0.753	0.789
0.30	0.619	0.648	0.679
0.40	0.527	0.553	0.579
0.50	0.442	0.463	0.486
0.60	0.362	0.379	0.397
0.70	0.283	0.297	0.311
0.80	0.205	0.215	0.225
0.90	0.122	0.127	0.133
1.00	0.000	0.000	0.000

Numerical example with a CRRA utility function. In this section, we use a more general utility function, namely a constant relative risk aversion (CRRA) utility function.

We try to answer the question that how the young creditors' portfolio composition and how q change when the creditors become more risk averse. We let equation (1') take the following functional form:

$$p\,\frac{\left(c_{t+1}^{n}\right)^{1-x}}{(1-x)}+(1-p)\left[\frac{\left(c_{t+1}^{M}\right)^{1-x}}{(1-x)}+\frac{\left(c_{t+1}^{K}\right)^{1-x}}{(1-x)}+\frac{\left(c_{t+1}^{c}\right)^{1-x}}{(1-x)}\right] \qquad (47)$$

To focus on the effects purely due to electronic monies' global/local characteristics, we keep other properties of electronic currency the same, i.e. $a = b = 0.3$, $\left(r^{M}, r^{K}\right)$= (1.075, 1.075), and use the same parameter values as in the loglinear utility case: $e = 0.01$, y=2, $s = 0.075$, and e=1. Again, we define first eleven scenarios, each corresponding to the following values of p: 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1. Finally, for each value of p, we change the value of ? from 0.5, 1.5, to 3. Thus, we have a total of 33 scenarios that we use to analyze the properties of the equilibrium. Since the loglinear utility function is simply one special case of CRRA utility function with $x = 1$, it would be helpful to compare the results from CRRA utility function with those from loglinear utility function under the same set of parameter values.

In each scenario, given $a = b$, there is a unique stationary equilibrium. Like in the loglinear utility function, the different pairs $\left(r^{M}, r^{K}\right)$ considered still have no effect on the real holdings of each type of electronic currency by a young creditor, but they do affect q, which is the relative price of Kleenex electronic money in terms of Microsoft electronic money in the secondary market.

In the figures below, we present the results obtained from our numerical example.

Figure 3. Shares of Microsoft and Kleenex electronic money in the young creditor's portfolio (%)

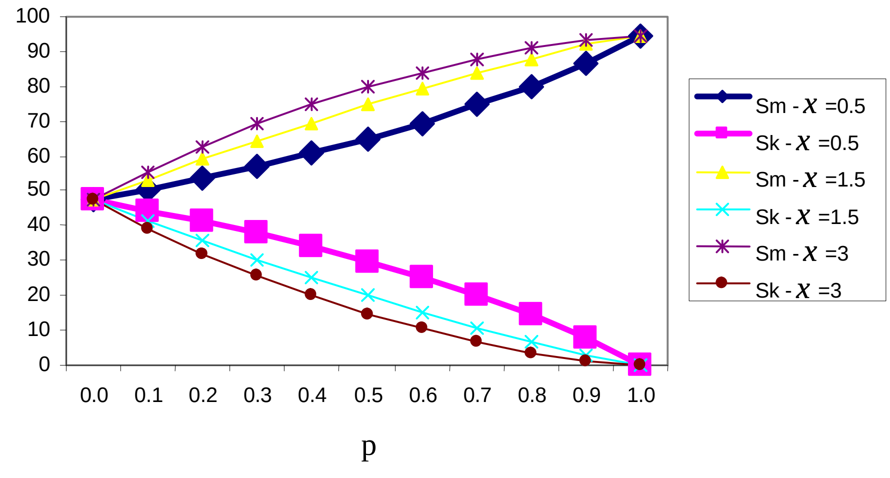
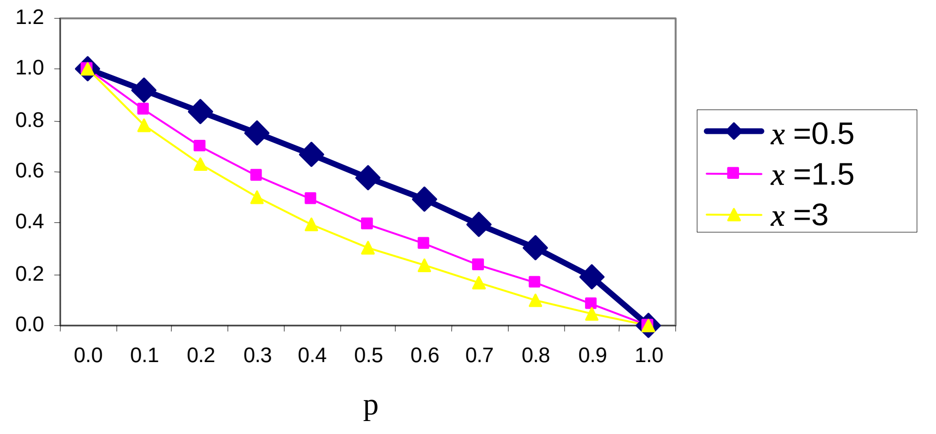


Figure 4. Price of Kleenex electronic money in terms of Microsoft electronic money (q)



As can be observed, for a given relocation probability p, a larger x is always associated with a larger *Sm*, a smaller *Sk*, and a smaller *q*. This result makes perfect sense intuitively since for any given relocation probability, a more risk averse creditor will want to put a larger fraction of her portfolio into the safer "global" Microsoft electronic money, and reduce her real holding of the "local" Kleenex electronic money; therefore, the price of Kleenex electronic money in terms of Microsoft money in the secondary market will be lower when the creditors become more risk averse.

Another thing to notice is that in all these scenarios, q is always smaller than 1, which means Kleenex electronic money is always sold at a discount in the secondary market. This is due to the reason that both electronic monies in these scenarios are identical in every aspect except the fact that one is global and the other is local. In other words, no matter how small the relocation probability is, Kleenex electronic money will always be sold at a discount; the discount is going to be heavier (q smaller) when the relocation possibility becomes larger. All other qualitative behavior of *Sm*, *Sk* and q with respect to p is similar to the loglinear utility case.

6. Conclusions and Extensions

We present a model with two types of private electronic currencies that display transactional advantages and dominate fiat money in rate of return. One of these electronic currencies is a local electronic currency in the following sense: it is accepted only in one network. However, in spite of these different returns, the two electronic currencies and fiat money circulate in equilibrium.

The type of electronic currencies introduced in this paper has the important property that it is not a surrogate of fiat money. It departs from the banknotes present in the financial system during the 19th century and looks forward to a next stage to be introduced soon in electronic commerce: electronic micropayments. We also offer insight on how payments are settled in such networks and how a clearinghouse shall be designed and regulated within the financial and payments system. This poses new and interesting challenges for both lawmakers and policy makers.

In this model of the payments system, debt takes the form of electronic currency issued by firms, and it is settled with a bundle of commodities produced by the issuer in a decentralized network, instead of being settled with fiat money. In this early version of the model, issuers of debt are committed to repay, and there is no uncertainty in their future stream of profits. However, the introduction of this type of uncertainty in the economy would imply a role to be played by the monetary authority. In addition, even in this simple version of the model, clearing and supervision of a secondary market for electronic currencies proves to be welfare improving and implies that the monetary authority could play a central role.

We do observe that the local electronic currency can be sold with a premium or with a discount, depending on E-money producers' relative capital intensity, E-monies' redemption rates, and the probability of relocation faced by the agents in this economy. Compared to that of Microsoft producer, the more capital intensive of Kleenex producer's production, the higher this discount; the smaller Kleenex money's redemption rate, the higher this discount; The higher the probability of relocation, the higher this discount and the lower the share of the local electronic currency in the young creditors' portfolio.

A natural extension, already mentioned, would be the introduction of uncertainty in the future stream of profits of the firms issuing electronic currency. In addition, a deeper study of the technical characteristics of the network will soon follow. Another extension would be the introduction of a new transactional advantage of electronic currency, by allowing consumers to have privacy and anonymity in specific types of transactions.

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