Inflation and Breaks: the validity of the Dickey-Fuller test

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June 2007 (Draft Version)

Abstract

This article proves the asymptotic efficiency of the Dickey Fuller (DF) test when the Data Generating Process of the variable under consideration is in fact mean stationary with breaks. Monte Carlo simulations show that asymptotic properties remain valid for sample sizes of practical interest. We illustrate its performance by studying inflation rate series, a variable that should be stationary if the monetary authority follows an effective inflation targeting regime: shocks are short-lived, therefore, inflation fluctuates randomly around pre-specified targets.

Keywords: Dickey-Fuller test, Mean Stationary Process, Structural Breaks.
JEL classification: C12, C22, E31.

1 Introduction

Whether or not inflation follows a stationary process is an important and conflicting issue for a broad range of economic analyses and policymaking questions. For instance, evidence that inflation behaves as a stationary process may imply that it is being controlled by certain monetary policy—the monetary authority is able to offset shocks that might induce significant deviations from a pre-specified inflation target. Moreover, if inflation rates are $I(1)$, then price levels would be $I(2)$. This in turn implies—for the long-run PPP relationship to hold—that either price levels and nominal exchange rate are cointegrated of order 2,2, denote $CI(2,2)$ or price levels are $CI(2,1)$ and that this linear combination is cointegrated with the nominal exchange rate.

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The literature dealing with the statistical properties of inflation rate series is vast, and these have been analyzed from different perspectives. Gregoriou and Kontonikas (2006) actually assert that the inflation process in economies where the Central Bank adopted an explicit inflation targeting monetary policy should be stationary around the target, and find evidence to support their claim by running a Unit Root (UR) test that allows for a non-linear mean reverting process in the alternative. Lai (1997) uses a modified DF test based on weighted symmetric least square estimation to show that the use of different data frequencies and different prices indexes to measure inflation can lead to conflicting evidence; his empirical results show considerable evidence of stationarity for monthly inflation rates, and mixed evidence for quarterly inflation rates. Along the same lines, Culver and Papell (1997) and Basher and Westerlund (2006) accept the null hypothesis of stationarity by using a panel data UR test. In contrast, Bai and Ng (2004) focus on the common trend component of inflation, rather than on the series themselves and find mixed evidence with a new testing methodology, known as PANIC. A further possibility, as considered by Hassler and Wolters (1995) and Arize, Malindretos, and Nam (2005), is that inflation follows long memory processes; this would explain why standard tests fail to reject the UR hypothesis. Persistence shifts in inflation have also been studied and there appears to be a changes of regime in the series from an $I(1)$ process to a mean stationary one (Sollis 2001, Taylor 2005, Chiquiar, Noriega, and Ramos-Francia 2007). Nevertheless, these results should be regarded with caution; Cavaliere and Taylor (2006) identified severe size distortions in persistence tests in the presence of a volatility shift.

The considerable effort devoted to discriminate between UR from stationarity has led to the current plethora of UR tests, yet, as Phillips and Xiao (1998) have pointed out: "*The immense literature and diversity of UR tests can at times be confusing even to the specialist and presents a truly daunting prospect to the uninitiated. In consequence, much empirical work makes use of the simplest testing procedures because it is unclear from the literature [...] wich tests if any are superior...*"

Hence, the standard DF remains the most popular methodology when testing for stationarity. Nevertheless, DF test is subject to certain amount of criticism which, in turn, has led theorists and practitioners to be skeptical about the conclusions to be drawn from it. In particular, Perron (1989) showed that the effectiveness of the UR tests decreases significantly in the presence of structural breaks. This is, if the true Data Generating process (DGP) of economic time series is in fact broken-trend stationary, tests of UR would under-reject the null hypothesis.

The effects of structural breaks in UR tests have been widely studied in the corresponding literature. In Kim, Leybourne, and Newbold's (2004) study of the DF auxiliary regression with constant and trend using a broken trend stationary DGP; they observed that the nature and location of the break significantly affects the DF test. Montañés and Sansó (2001) studied the performance of the DF test using the same trend stationary process together with the inclusion of a seasonal pattern using dummy variables; they showed that, in spite of the

seasonal component, the DF tends to reject the UR null hypothesis. Montañés (1997) showed that Perron's UR test is asymptotically efficient in the case of breaking date misspecification when a mean stationary variable is analyzed. Montañés and Reyes (1998) analyzed the asymptotic properties of the DF test when the process under analysis has a break in the trend function; they found that the DF test is biased towards the non-rejection of the UR hypothesis for small sample sizes. To our knowledge, no research has yet been carried out to analyze the performance of the DF test in relation to mean stationary processes with level breaks, a plausible DGP for series such as controlled inflation.

The contribution of this article is that of demonstrating that the standard DF test is asymptotically effective when used to differentiate between an $I(1)$ and a mean stationary process with breaks; both are relevant DGPs when studying inflation[1]. In addition, we show that the asymptotic results also apply in finite samples—although the effects that autocorrelation, location and break size on the DF test are not negligible in relatively small samples—by means of a Monte Carlo study.

The article is organized as follows: in Section 2, we show the asymptotic behavior of the DF test as well as several particularly revealing properties of the asymptotic expressions. Section 3 presents Monte Carlo simulations. Section 4 presents an empirical application of the DF test using inflation rate series for the OECD countries, whilst conclusions are drawn in Section 5. Mathematical derivations are provided in the Appendix.

2 The validity of the DF test in the presence of structural breaks

We are interested in testing stationarity of a time series generated by:

$$x_t = \mu_x + \sum_{i=1}^{N_x} \theta_x DU_{i,t} + u_{xt} \tag{1}$$

where μ_x is a constant, $u_{xt} = \phi_z u_{xt-1} + \epsilon_u$, $|\phi_x| < 1$, ϵ_u is $iid(0, \sigma_\epsilon^2)$, and $DU_{i,t}$ are dummy variables allowing changes in the mean, that is, $DU_{i,t} = \mathbf{1}(t > T_{b_i})$, where $\mathbf{1}(\cdot)$ is the indicator function, and T_{b_i} is the unknown date of the ith break in x. We define the break fraction as $\lambda_i = (T_{b_i}/T) \in (0,1)$, where T is the sample size. We use an AR(1) structure for the innovations u_{xt} as in Kim, Lee, and Newbold (2004) and Noriega and Ventosa-Santaulària (2006), although it can also be assumed that innovations obey the (general-level) conditions stated in Phillips (1986). If we use the following four DF auxiliary regressions

$$\Delta x_t = \delta x_{t-1} + u_t \tag{2}$$

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[1]Under the null, inflation contains a UR; this implies an uncontrolled inflation process. Under the alternative, inflation follows a mean stationary process with level breaks; this implies that monetary policy actions ensure that inflation oscillates randomly around specific inflation targets—although the target may change throughout time.

$$\Delta x_t = \alpha + \delta x_{t-1} + u_t \tag{3}$$

$$\Delta x_t = \alpha + \delta x_{t-1} + \beta t + u_t \tag{4}$$

$$\Delta x_t = \alpha + \delta x_{t-1} + \beta t + \gamma \Delta x_{t-1} + u_t, \tag{5}$$

we can assert the following proposition:

Proposition 1 *Let x_t be generated by DGP 1 and be used to estimate regression (2), (3), (4) or (5). Hence, the t-statistic associated with $\hat{\delta}$ diverges:*

$$t_{\hat{\delta}} = O_p\left(T^{\frac{1}{2}}\right)$$

Proof: See appendix

Remark 1 *Where only one break exists, the asymptotic t-statistic associated with $\hat{\delta}$ in equation (2) is:*

$$T^{-\frac{1}{2}}t_{\hat{\delta}} \xrightarrow{p} -\sqrt{\frac{\sigma_u^2(1-\rho_{1x})}{2[\mu_x^2 + \theta_x(1-\lambda_x)(2\mu_x+\theta_x)] + \sigma_x^2(1+\rho_{1x})}},$$

where $\xrightarrow{p}$ indicates convergence in probability and ρ_{1x} is the first autocorrelation of x_t.

Figure 1.*a* clearly illustrates the implication of the formula of the asymptotic t-statistic associated with $\hat{\delta}$ in equation (2): where the structural break is positive, the DF test decreases its power, the larger the size of the break and the smaller the break fraction. Furthermore, where the structural break is negative, the DF test decreases its power when there is an increase in the size of the break only when ($\mu_x < -\theta_x$); analogously, an increase in the break fraction biases the DF test toward the acceptance of the null hypothesis if ($\frac{-2\mu_x}{\theta_x} < 1$). Similarly, figure 1.*b* shows that positive autocorrelation decreases the power of the test, and negative autocorrelation biases the DF test toward the rejection of the null hypothesis. The Monte Carlo section will show that the problems such as an increase in autocorrelation, the size and location of the breaks can be overcome by increasing the sample size.

3 A Monte Carlo study

We present a Monte Carlo study in which the finite sample behavior of the DF test is analyzed in the case where the variable under examination is mean stationary with breaks—see equation (1). Table (1) shows the rejection rates of the null hypothesis of the DF test with neither constant nor time trend. The computed parameter values selected were similar to those obtained in empirical estimates. Sample sizes range from $T = 25$ to $T = 500$, whilst the number of replications is R=10,000. The results in this table are consistent with the
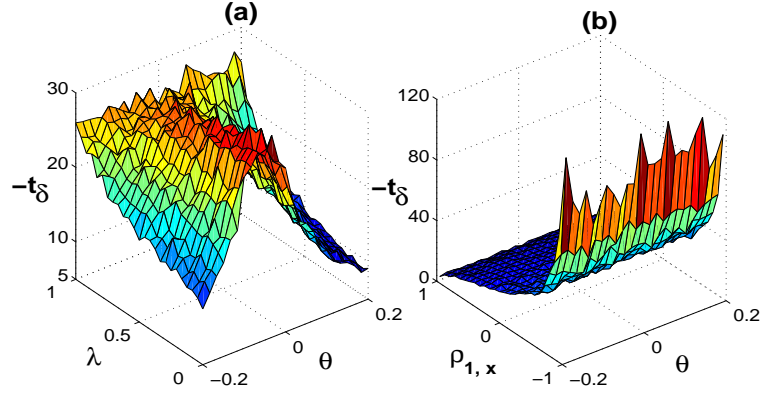
Figure 1: t–statistic behavior according to the size and location of the break, and the degree of autocorrelation in the noise

theoretical results obtained in the previous section: firstly, columns 2 and 3 exemplify the effect of autocorrelation. The only difference between the DGPs in these two columns is that the first uses $i.i.d$ innovations whereas the second uses an stationary $AR(1)$ process. It is clear that rejection rates decrease in the presence of positive autocorrelation. Secondly, columns 4 and 5 illustrate that with an smaller positive break, rejection rates are greater, in the case of small sample sizes at least; and thirdly, columns 6 and 7 show the location of the break effect. With a positive break, the smaller the break fraction, the smaller the rejection rates.

T	DGP_1	DGP_2	DGP_3	DGP_4	DGP_5	DGP_6
25	0.83	0.03	0.26	0.00	0.17	0.92
50	0.99	0.07	0.93	0.00	0.88	0.99
100	1.00	0.26	1.00	0.18	1.00	1.00
250	1.00	0.97	1.00	1.00	1.00	1.00
500	1.00	1.00	1.00	1.00	1.00	1.00

Table 1: Monte Carlo Experiment

The values of the parameters in the DGPs are as follows: All DGPs $\sigma_\epsilon = 0.02$, $\mu_x = 0.05$
$DGP_1 : \lambda_1 = 0.25 \ \lambda_2 = 0.75 \ \theta_1 = 0.09 \ \theta_2 = -0.09 \ \rho_x = 0$
$DGP_2 : \lambda_1 = 0.25 \ \lambda_2 = 0.75 \ \theta_1 = 0.09 \ \theta_2 = -0.09 \ \rho_x = 0.9$
$DGP_3 : \lambda_1 = 0.50 \ \theta_1 = 0.195 \ \rho_x = 0 \quad DGP_4 : \lambda_1 = 0.50 \ \theta_1 = 0.495 \ \rho_x = 0$
$DGP_5 : \lambda_1 = 0.10 \ \theta_1 = 0.15 \ \rho_x = 0 \quad DGP_6 : \lambda_1 = 0.90 \ \theta_1 = 0.15 \ \rho_x = 0$

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4 Empirical evidence

Theoretical results discussed earlier are illustrated by way of an empirical application; we analyze the results of the DF test applied to inflation rate series corresponding to the 30 OECD countries. Quarterly inflation rate series[2] were constructed from monthly Consumer Price Index data retrieved from the International Monetary Fund's International Financial Statistics data; the data span for these countries varies, the maximum sample period being 1957:01 to 2005:12.

Results of the DF test—with a constant, time trend and one lag[3]—shown in Table (2) attest the asymptotic properties of the DF test. For all the countries presented in this table (except Italy), evidence in favor of broken mean stationarity is stronger the larger the sample size. It is important to notice that the DF tests were performed to allow for first order autocorrelation only—we decided to use the specification studied in our asymptotic results, in the kowledge that the tests could suffer a consequent from loss of power[4], nevertheless, in several cases, the null is rejected for sample sizes as small as 25. As shown in the Monte Carlo section, rejection rates found empirically are high for sample sizes of practical interest.

5 Conclusions

Structural breaks do affect the performance of UR tests. Nevertheless, we have shown the robustness of the DF test in the presence of structural breaks where the DGP of the series under analysis is mean stationary with breaks. In that case the t–statistic of the DF test diverges at rate $\sqrt{T}$ implying that the null will, at some point, be rejected. However, our asymptotic results prove that the DF test is sensitive not only to the size of breaks, but also to their sign and their location, as well as to autocorrelation.

Monte Carlo experiments and empirical evidence show that the DF test provides adequate results when the sample size is sufficiently large. In fact, the DF is more sensitive to the presence of autocorrelation, regardless of the type of the break, although this phenomenon can also be alleviated by increasing the size of the sample or simply by running an Augmented DF test. UR testing can be reliably performed using a simple DF test if the sample size is large enough. In the specific case of inflation rates, the alternative of broken mean stationarity

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[2]Quarterly inflation series were computed as the logged differences of CPI at three-monthly intervals. For Australia and New Zealand we have quarterly CPIs; the quarterly inflation series are the logged differences of successive quarters.

[3]The test was also performed under different specifications (DF with intercept, as well as intercept and time trend), providing similar results. Due to space constraints, we report this specification only and a subsample of countries. The other results are available upon request.

[4]As it is well known, UR tests generally suffer from two problems: low power and size distortions. In particular, loss of power occurs when the innovations have an autocorrelated structure, that is, they can be represented as an $AR(p)$ polynomial. If the DF test does not adequately correct the autocorrelation in the disturbances, we should expect such loss of power—we will under-reject the null when in fact it is false.

Country	Sample period	Observations used in the test (N)				
		25	50	100	150	195
Austria	1957:01-2005:12	H_a^{**}	H_a^{***}	H_a^{***}	H_a^{***}	H_a^{***}
Czech Republic	1993:01-2005:12	H_0	H_a^{**}			
Finland	1957:01-2005:12	H_0	H_a^{**}	H_a^{***}	H_a^{***}	H_a^{***}
France	1957:01-2005:12	H_a^{*}	H_a^{***}	H_a^{***}	H_a^{***}	H_a^{***}
Grecce	1957:01-2005:12	H_a^{**}	H_a^{***}	H_a^{***}	H_a^{***}	H_a^{***}
Hungary	1976:01-2005:12	H_a^{**}	H_a^{***}	H_a^{***}		
Iceland	1983:01-2005:12	H_a^{**}	H_a^{***}	H_a^{***}		
Ireland	1997:01-2005:12	H_0	H_a^{*}			
Italy	1957:01-2005:12	H_a^{*}	H_a^{**}	H_a^{***}	H_a^{*}	H_a^{**}
Korea	1970:01-2005:12	H_0	H_a^{*}	H_a^{***}	H_a^{***}	
Norway	1957:01-2005:12	H_0	H_a^{***}	H_a^{***}	H_a^{***}	H_a^{***}
Poland	1988:01-2005:12	H_a^{*}	H_a^{***}	H_a^{***}		
Slovakia	1993:01-2005:12	H_a^{**}	H_a^{***}			
Sweden	1957:01-2005:12	H_a^{**}	H_a^{***}	H_a^{***}	H_a^{***}	H_a^{***}
Switzerland	1957:01-2005:12	H_0	H_a^{*}	H_a^{***}	H_a^{***}	H_a^{***}
Turkey	1969:01-2005:12	H_a^{*}	H_a^{*}	H_a^{***}	H_a^{***}	
United Kingdom	1957:01-2005:12	H_a^{**}	H_a^{***}	H_a^{***}	H_a^{***}	H_a^{***}
United States	1957:01-2005:12	H_a^{*}	H_a^{**}	H_a^{**}	H_a^{*}	H_a^{***}

Table 2: Results of the Dickey-Fuller test with intercept, time trend and one lag. Note: the *, **, and *** denote statistical significance at the 10%, 5% and 1% level respectively.

may be understood as the existence of an inflation targeting regime with time-varying inflation targets.

A   Appendix

Proof of Proposition 1. We present a guide on how to obtain the order in probability of one of the four t-statistics appearing in proposition (1), by using the DF regression (2) for which $\Delta x_t = \sum_{i=1}^{N_x} \theta_i DP_{it} + u_{xt} - u_{xt-1}$ where $DP_{i,t} = DU_{i,t} - DU_{i,t-1}$. The other three cases follow the same steps. Proof as such was provided with the aid of *Mathematica 4.1* software. The corresponding codes are available at **http://www.ventosa-santaularia.com/VSG_06b.zip**. We shall now describe the process involved in establishing the aforementioned proof. The DF regression equation shall be $\Delta x_t = \alpha + \delta x_{t-1} + u_t$ in matrix form: $\Delta X = X_1 \beta + u$. The vector of *OLS* estimators is $\widehat{\beta} = (\widehat{\alpha}\ \widehat{\delta})' = (X_1' X_1)^{-1} X_1' \Delta X$, and the *t*-statistic of interest $t_{\widehat{\delta}} = \widehat{\delta} \left[\widehat{\sigma}_u^2 (X_1' X_1)_{22}^{-1} \right]^{-1/2}$, where $(X_1' X_1)_{22}^{-1}$ is the 2nd diagonal element of $(X_1' X_1)^{-1}$ and $\widehat{\sigma}_u^2 = T^{-1} \sum_{t=1}^{T} \widehat{u}_t^2 = T^{-1} \sum_{t=1}^{T} \left(\Delta x_t - \widehat{\alpha} - \widehat{\delta} x_{t-1} \right)^2$. $t_{\widehat{\delta}}$ is a function of the following expressions:

$\sum_{t=1}^{T} \Delta x_t = O_p(1)$	$\sum_{t=1}^{T} (\Delta x_t)^2 = O_p(T)$
$\sum_{t=1}^{T} x_{t-1} = O_p(T)$	$\sum_{t=1}^{T} x_t^2 = O_p(T)$
$\sum_{t=1}^{T} \Delta x_t x_{t-1} = O_p(T)$	$\sum_{t=1}^{T} x_{t-1} \cdot t = O_p(T^2)$
$\sum_{t=1}^{T} \Delta x_t \cdot t = O_p(T)$	$\sum_{t=1}^{T} \Delta x_t \Delta x_{t-1} = O_p(T)$

We can fill the matrix $(X_1'X_1)$ as well as the vector $(X_1'Y)$ and then compute the OLS parameter estimates $\beta = (X_1'X_1)^{-1} X_1'\Delta X$ and the t-statistic associated with δ. The program computes the asymptotics[5].

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