Multiple Reserve Requirements, Exchange Rates, Sudden Stops and Equilibrium Dynamics in a Small Open Economy[♦]

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Abstract

We model a typical Asian-crisis-economy using dynamic general equilibrium techniques. Exchange rates obtain from nontrivial fiat-currencies demands. Sudden stops/bank-panics are possible, and key for evaluating the merits of alternative exchange rate regimes. Strategic complementarities contribute to the severe indeterminacy of the continuum of equilibria. The scope for existence and indeterminacy of equilibria and dynamic properties are associated with the underlying policy regime. Binding multiple reserve requirements promote stability under floating but increase the scope for panic equilibria under both regimes. Backing the money supply acts as a stabilizer only in fixed regimes, but reduces financial fragility under both regimes.

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1. Introduction

In this paper, we study the interaction between monetary policies and alternative exchange rate regimes to ascertain the probability of a crisis, building from the characteristics of the Asian-crisis countries in 1997. Our broader goal is to reinforce and fill in the link between the overexpansion of the financial system, banking crises, and exchange rate regimes/monetary policy that we find lacking in the literature. With this in mind, we build a Dynamic Stochastic General Equilibrium Model (DSGE) --from micro-foundations-- replicating a small, open economy (SOE) with a nontrivial banking system, such as one of the 1997 East Asian countries. Two words of caution to the reader: First, this paper does not aim, from a historical point of view, to show the success of a particular monetary policy in place either in defending the national currency or in managing contagion at the time of the crisis. Our goal, instead, takes the form of a theoretical treatise on a "what if:" what if a typical Asian-crisis-country were to implement a policy of multiple reserve requirements with backing of the domestic money supply, and how would it work under alternative exchange rate arrangements? Thus, we look forward and aim to suggest policy options that may help these countries maintain stability in case a similar crisis was to hit again. Second, at this time, we do not consider economic activity explicitly other than in the financial sector.

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Our model captures all five stylized facts of the East Asian countries at the time of the crisis. It is general knowledge that Indonesia, South Korea, and Thailand were the countries most affected by the East Asian 1997/98 crisis, followed by Malaysia, Laos and the Philippines. There are five stylized facts shared by these countries at the time of the crisis that we want to emphasize. 1) Increased risky-lending behavior by banks led to a boom in private borrowing financed by non-performing loans[1]. 2) The lack of sound financial structure worsened with the ill-oriented process of financial and capital liberalization[2]. 3) Banks' financial assets constituted the majority of their total assets –instead, for instance, of financing in capital markets. 4) Borrowing from foreign banks was a significant portion of domestic banks' loans. 5) The majority of these countries had intermediate pegs in place. According to the standard chronology of the crisis, the floating of the baht in July 1997 in Thailand triggered the crisis. A subsequent change in expectations led to the depreciation of most currencies in the region, bank runs, rapid withdrawals of foreign capital --a *sudden stop*—and a dramatic economic downturn followed. Unlike previous crises originated from fiscal imbalances and/or trade deficits, the Asian crises shed light on the increased risky behavior and the overexpansion of the banking system.

To build the framework that we just described, we used three building blocks that took us closer to our goal systematically. In the first block, we model explicitly the behavior of individuals and obtain the microfoundations for our general equilibrium model. In the second block, we introduce alternative exchange rate regimes with their associated monetary policy rules. It is a well-established fact that for economies open to international capital flows, the choice of exchange rate regime is central to explain the vulnerability and fragility of financial markets, as well as domestic price stability and long-run viability. Table 1.A summarizes the exchange rate arrangements in the Asian countries. During most of the 1980s and the first part of the 1990s, Indonesia, South Korea, Thailand and Malaysia had managed floating arrangements --an intermediate peg--, while Philippines had free floating. However, there were some important differences after the 1997 crises: Philippines continued with free floating, Indonesia, Korea and Thailand moved from intermediate pegs to free floating as well, but Malaysia turned to a very hard peg. These facts make our comparison of the relative merits of the two sets of policy rules relevant in the presence of binding multiple reserve requirements.

> The third building block may allow one to infer behavior from a particular set of circumstances: we may be able to separate and identify causes and consequences by studying separately and jointly the main stylized facts of sudden stops and bank-runs in similar economies[3].

We consider two potential causes of crises: a crisis comes to our model either in the form of a sudden stop of foreign credit (*intrinsic* uncertainty) or in the form of a panic among national depositors (*extrinsic* uncertainty.) We put most of our effort on the distinguishing characteristic of the former but do not neglect the fact that a self-fulfilling panic and run may implicitly aggravate a crisis.

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[1] One may also think of this fact in the context of the 2007 U.S. sub-prime mortgage crisis. See Hernandez-Verme (2009.)
[2] See Lindgren et. a.l. (1999) and Kishi and Okuda (2001.)
[3] See Kaminsky (2003) for details.

We believe that we improve on Chang and Velasco (2000a, 2000b and 2001) (C-V) in at least three dimensions. First, C-V attached intrinsic value to currencies they intended to be fiat[4.] We instead take multiple fiat currencies –domestic and foreign—a bit more seriously, and introduce non-trivial demands for them. In particular, banks must hold a fraction of their deposits as unremunerated currency reserves: a fraction to be held in the form of domestic currency and another fraction in the form of foreign currency. Then, fiat money instead enters our model by the regulation that governs the multiple reserve requirements in this economy[5], the implications being that: 1) there is a meaningful nominal exchange rate in our model, and 2) this nominal exchange rate will be determined according to the exchange rate regime and the monetary policy in place. In second place, we use a DSGE model in an economy with an infinite horizon, as is the Overlapping Generations (OG.) Thus, we are able to discuss the interesting equilibrium dynamics defining each exchange rate arrangement, as opposed to both Diamond and Dybvig (1983) (D-D) and C-V. In third place, we improve the way in which we introduce and treat potential crises, The potential for strategic complementarities and the realization of self-fulfilling prophesies is ever present in alternative versions of the OG model with outside assets in general, and models with one or more fiat currencies in particular[6], and, of course, in our model[7]. In such a context, the presence of informational and institutional frictions can exacerbate situations that are already problematic, such as credit rationing, financial repression and endogenously arising volatility, thus complicating the standard analysis of separating and pooling equilibria. Thus, the appropriate utilization of the information and action sets available to agents at all points in time is critical. In this respect, we reformulate the sequential checking algorithm used by depositors and devise a re-optimization problem by banks after a sudden stop. In particular, one argument of the C-V framework was that when the probability of a crisis is public information, each agent in this economy must use this information when contemplating optimal plans of action at the beginning of every period, and, as result, the optimal behavior of households is invariant with respect to whether the crisis was realized or not. Alternatively, we introduce the potential for uncertainty of the crisis by using a sunspot variable: a random variable unconnected to the fundamentals of the economy and that expresses the extrinsic uncertainty as a shock to the depositors' beliefs.

Our results show the existence of a continuum of equilibria that are indeterminate in two ways: 1) an allocation may be consistent with a continuum of relative price vectors, and 2) a vector of relative prices may be consistent with several different allocations. There is a strong association between the scope for existence and indeterminacy of equilibria, the properties along dynamic paths and the underlying policy regime. Binding multiple reserve requirements promote stability under floating but increase the scope for panic equilibria under both regimes. Alternatively, the backing of the domestic money supply acts as a stabilizer only in fixed regimes, but it

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[4] In their model, people held domestic currency because they derived utility from it, attaching intrinsic value to the currency.
[5] See Hernandez-Verme (2004) for the original discussion.
[6] The recent literature on open economy macroeconomics has used intensively self-fulfilling prophecies as a tool that may lead to very important underlying explanations for financial fragility, currency crises and/or speculative attacks. See Cole and Kehoe (1996) for an example.
[7] We also introduce a new dimension to Bencivenga and Smith (2002.)

reduces financial fragility under both regimes. We will show that a floating exchange rate regime is Pareto superior to a hard peg. Moreover, a different trade-off in policy implementation will be observed in each regime when the goals of high welfare, increases stability and reduced financial fragility are considered together.

The remainder of the paper proceeds as follows. In Sections 2 and 3, we analyze the properties of stationary and dynamic equilibria under the alternative exchange rate regimes where no crises are possible in equilibrium. In Section 4, we allow for the possibility of crises by introducing extrinsic and intrinsic uncertainties. Section 5 concludes.

2. Floating Exchange Rates: the Case of Indonesia, Korea and Thailand

In this section, we build the model of a SOE that captures the main stylized characteristics shared by the Indonesian, Korean and Thai economies at the time of the crisis. Here, we focus on the construction of the general equilibrium without extrinsic or intrinsic uncertainty and, thus, we do not allow for any event that could lead to a crisis of any type. The reader interested will find the analysis of crises and financial fragility in Section 4.

The (private) banking sector in this economy is a net debtor with respect to the rest of the world, and there is an exogenous and binding upper limit to foreign credit faced by domestic banks at each point in time, so that credit is always rationed. We will observe *ex-ante* identical domestic households who face uncertainty as to their preferences types. The distribution of this shock is public information, but its realization is known only by the private households. Our model has also the potential for strategic complementarities, taking the form of a standard problem in coordination that may lead to crises of a self-fulfilling type: the decisions made by individual households will be intertwined with the choices of other households, giving rise to strategic interdependence between a household's actions and the actions of others. We will see that two fiat national currencies can potentially circulate simultaneously: a domestic fiat currency and a foreign fiat currency. The legal regulations in financial intermediation and foreign exchange establish the following: 1) all intermediated domestic investment is subject to multiple, unremunerated and binding reserve requirements. 2) A flexible exchange rate regime is in place, and thus the nominal exchange rate will be market-determined; and 3) there are no legal domestic restrictions on either using foreign currency or on obtaining foreign credit.

2.1 The Environment

Consider a *pure exchange*, SOE consisting of an infinite sequence of two-period-lived, overlapping generations. Time is discrete, and indexed by $t = 1, 2, 3, \ldots$. Standard analysis of overlapping-generations economies typically groups households into two categories: all the future generations versus the generation of initial old. Moreover, we will observe four groups of players in this model economy: households (depositors,) domestic banks, foreign banks and the domestic monetary authority. Foreign banks will lend to domestic banks inelastically at the world interest rates and up to an exogenous, binding limit. The monetary authority in this model economy is in charge of choosing the combination of monetary policies consistent with floating exchange rates.

On the one hand, each of the future generations consists of a continuum of households with unit mass. A household born at period t is young during period t and old during period $t+1$. Since banks will prove to be coalitions of households, we will label them "young" banks or "old" banks in correspondence to the age of the households. Households within a generation are *ex ante* identical, but they can become of one of the following types before the end of their youth: impatient, with probability $l \in (0,1)$, or patient, with probability $(1-l)$. The parameter l is public information, but each particular household's realization is private, and banks will try to infer the household's type from her behavior. Impatient households will derive utility solely from consuming before the end of their youth ($c_{1,t}$,) while patient households will derive utility only from consuming in their old age ($c_{2,t+1}$.) On the other hand, at $t=0$, there is a generation of initial old. The initial old consist of a continuum of old households with unit mass. A fraction $(1-l)$ of these initial old are of the patient type.

Each period has two parts that we will call morning and afternoon, since different types of interaction will take place in each of them[8]. As is standard in the D-D-related literature, domestic banks will turn out to be coalitions of individual households, they will be competitive, and we can assume then that they are identical. The latter facilitates the analysis by allowing to focus on the examination of a representative bank.

Every period, there is a single endowment, tradable good. This good is homogeneous across households and countries, but it cannot be produced anywhere. When young, a household receives w units of the single good first thing in the morning, as if it were *manna* falling from the sky. Old households receive no endowments of any type. The following expression represents the expected lifetime utility of an individual born at period t, with the information available at the beginning of this period:

$$E_t\left[u\left(c_{1,t},c_{2,t+1}\right)\right] = l \cdot \ln\left(c_{1,t}\right) + \left(1-l\right) \cdot \ln\left(c_{2,t+1}\right). \qquad (1)$$

Households and domestic banks have access to the following storage/investment technology: for one unit of good invested at the beginning of period t, the household receives the return $R>1$ goods at the end of period $t+1$. However, she would receive only a return of $r<1$ if she were to liquidate the investment early, by the end of period t[9]. The reader may want to think of this as a "refrigerator" technology: the *manna* is put inside the refrigerator; if the refrigerator is opened to soon, part of the *manna* disappears. Then, the condition to promote truth-telling can be written as

$$c_{2,t+1} \geq r \cdot c_{1,t}\,. \qquad (2)$$

The inequality above must hold as the incentive-compatibility or self-selection condition that allows no motivation for young patient households to misrepresent their types. In the remainder of this section and in Section 3, we assume that the inequality in (2) holds while we build the general equilibrium.

[8] The sequence in which events take place and information becomes available determines the type of coordination problem present.

[9] The only available technology for the short-term investment is this same storage technology. For instance, if an agent liquidates early, she would get the return $r<1$ per good at the end of period t, and if she stores the proceeds again between the end of period t and the end of period $t+1$, then she will get the return $r \cdot r = r^2 < 1 < R$ per good at the end of period $t+1$. Thus, the long-term storage technology dominates early liquidation in rates of return.

Multiple Fiat Currencies. Two fiat and outside national currencies may circulate in this SOE at any point in time. The first one is the domestic fiat national currency, while the second is a foreign currency that serves also the purpose of international currency. To fix ideas, we will call the domestic currency the *won* and the foreign/international currency the US dollar.

On the first hand, it is apparent that the monetary authority in the domestic country has the monopoly in issuing *wons*, where M_t is the outstanding nominal stock of *wons* at the end of period t. The domestic price level p_t represents the number of *wons* to be exchanged for one unit of the single good (*manna*) at period t, and $(p_t / p_{t+1}) > 0$ is the gross real return on *wons*. As is standard in economies with floating exchange rates, the monetary authority has the control over the nominal aggregate supply of *wons*, as we will see in detail in section 2.3. On the other hand, the US dollar may circulate in the domestic economy together with the *won*, and Q_t represents the outstanding stock of foreign currency in the domestic country at the end of period t. The exogenous world price level p_t^* represents the number of US dollars that households need to exchange for one unit of the single good at period t, while $\left(p_t^* / p_{t+1}^*\right) = \left(1 + s^*\right)^{-1} > 0$ represents the constant, gross real return on US dollars, where $s^* > -1$ is the exogenous net inflation rate in the rest of the world. It is apparent that Q_t is endogenous, and it will depend on a group of variables such as foreign credit constraints binding, relative prices and policy rules, among others. Finally, we use e_t to denote the market-determined nominal exchange rate, measured as the number of *wons* exchanged for one US dollar.

We also assume that there is free international capital mobility, free international trade with homogeneous *manna* and no legal restrictions to the use of foreign currency in the domestic country. As a result, the Law of One Price will hold in equilibrium, so that $e_t \cdot p_t^* = p_t$ is always satisfied.

The domestic monetary authority accomplishes all injections and/or withdrawals of *wons* through lump-sum transfers. In particular, each young household will receive the equivalent in *wons* of t_t goods *ex ante*, at the beginning of period *t*, regardless of type. Two reasons justify our choice of this particular transfer scheme: first, it enters both sides of the self-selection constraint, and second, it does not require the monetary authority to have additional information that could be of a private nature.

At $t = 0$, the initial-old generation behaves as any old agent from the future generations would. In particular, the fraction $\left(1 - l\right)$ of these initial old individuals is of the patient type and wishes to consume $c_{2,0}$ goods now. In standard models with overlapping generations, the initial conditions of the economy describe the initial stock of the different assets that exist in the economy as well as the "endowments" to the members of the initial old generation .In the present case, there are two initial conditions: $M_0 > 0$ and $Q_0 > 0$ are given, and they are distributed equally among the patient initial old such that each consumes $c_{2,0} = \left(M_0 + e_0 \cdot Q_0\right) / \left[\left(1 - l\right) \cdot p_0\right]$.

2.2 Financial Intermediation

In our model, foreign banks play the somehow impersonal role of suppliers of international liquidity through foreign-credit instruments that may take different forms. Domestic banks follow the standard D-D set up: a representative bank arises endogenously and pools together the resources owned by young households with the purpose of providing them with partial insurance against the uncertainty of their potential types and with feasible allocations that are Pareto superior to autarky[10]. In the context of a SOE, the same principle holds whenever such economy does not display aggregate uncertainty, as is the case in our model economy.

Following the standard practice in the literature, a representative young bank that starts business at period t will maximize the expected utility of the individual households born at period t, given by equation (1.) Henceforth, when we use the term "banks" without further qualification, we refer to domestic banks starting business at t.

Access to Foreign Credit Markets. Only banks may access the world credit markets by trading with foreign banks in several primary debt markets, the idea being that the debt-instruments available may provide them with liquidity in a variety of terms and/or dates of maturity.

Banks have access to three different foreign-debt instruments, as shown in Table 2. The amounts traded of the different instruments form the debt-structure vector denoted by $\left(d_{0,t}, d_{1,t+1}, d_{2,t+1}\right)$, where all amounts are expressed in terms of the single good[11]. The vector of relevant prices associated with this debt-structure is time-invariant, and it is denoted by $\left(r_0^*, r_1^*, r_2^*\right) >> 1$. $d_{0,t}$ is a short-term *intra-period* loan issued early-morning in period t and maturing in late-afternoon of the same period, while $d_{1,t+1}$ stands for a short-term *inter-period* (bail out) loan issued in the late afternoon of period t and maturing late in the afternoon of period $t+1$. Finally, $d_{2,t+1}$ is a *long-term* loan issued early in the morning of period t and maturing late in the afternoon of period $t+1$. We assume that each of the elements in the price vector $\left(r_0^*, r_1^*, r_2^*\right) >> 1$ is a time-invariant and exogenous gross real interest rate determined in the appropriate world financial market such that $\left(d_{0,t}, d_{1,t+1}, d_{2,t+1}\right) > 0$ and the vector $(0,0,0)$ never obtains in equilibrium. It then follows that banks are net debtors of the rest of the world.

Banks also face exogenous borrowing constraints on the standing debt at the beginning and at the end of period t, respectively. These constraints are given by

$$d_{0,t} + d_{2,t+1} \leq f_0, \tag{3a}$$

$$d_{1,t+1} + d_{2,t+1} \leq f_1. \tag{3b}$$

$f_0 > 0$ and $f_1 > f_0$ are time-invariant and measured in *manna* (the single good.) They are chosen exogenously by foreign banks, and represent the maximum foreign credit available at the beginning and at the end of period t, respectively. We restrict our attention to allocations where (3a) and (3b) bind, so that foreign credit is rationed.

Fractional-Reserves Banking and Multiple Reserve Requirements. The multiple, unremunerated reserve requirements in our model follow Hernandez-Verme (2004.) All investment is done by banks and it is subject to the financial regulations of the domestic country. Out of the total deposits, a fraction must be held as currency reserves and only the remainder can be invested. In particular, the policy parameter $f_d \in (0,1)$ denotes the fraction

[10] See the section on deposit contracts for more details.
[11] This treatment is standard in the literature.

of total deposits that the banks must hold as currency reserves in the form of *wons*. When held between periods t and $t+1$, domestic currency reserves earn the same return as real *wons* balances, namely $\left(p_t / p_{t+1}\right)$. Similarly, $f_f \in (0,1)$ denotes the fraction of deposits that banks must hold in the form of foreign currency, with the real rate of return $\left(p_t^* / p_{t+1}^*\right)$. Obviously, $f_d + f_f < 1$ must hold, where $\left(1 - f_d - f_f\right) > 0$ stands for the fraction of total deposits that banks can invest long-term. Finally, we must mention that we will focus on allocations where both reserves requirements are binding. This will transpire when $\left(p_t / p_{t+1}\right) < R$ and $\left(p_t^* / p_{t+1}^*\right) < R$ hold.

Timing of Transactions. In our model, there are no transactions among individual households of any age or type, either domestically or with the rest of the world. All transactions take place through the banks. Thus, banks in this model are inherently financial intermediaries.

Households born at t live for four sub-periods: the morning and afternoon of period t, and the morning and afternoon of period $t+1$. We now proceed to describe the transactions that take place each sub-period. Notice that only in this section we setup the budget constraints with the amount of early liquidation l_t to facilitate a full understanding of the general bank's problem, but in what follows we will return to the case where $l_t = 0$.

The morning of period t: young households born early in the morning of period t have two sources of funds at this point: their endowment of w units of the single good, and the transfer of t_t goods from the monetary authority. Each of these young households deposits the total of $w + t_t$ goods in a young bank. On the other side of the financial market, there are young banks receiving these deposits. These young banks also have the domestic monopoly on borrowing $d_{0,t} + d_{2,t+1}$ units of *manna* from the rest of the world. They must also set aside the required currency reserves $f_d \cdot (w + t_t) + f_f \cdot (w + t_t)$ and deposit them into their reserves accounts held with the monetary authority, where they will remain until the end of period $t+1$. The young banks combine their resources in order to finance the long-term investment in the amount of k_{t+1} units of manna, which they will place in their "refrigerators," leading to the budget constraint

$$k_{t+1} \leq d_{0,t} + d_{2,t+1} + (1 - f_f - f_d) \cdot (w + t_t). \tag{4}$$

The afternoon of period t: individual households learn their types in the early afternoon of time *t*. Each impatient household will withdraw $c_{1,t}$ goods and consume them, while a patient agent would not withdraw, provided that (2) holds. The young banks must pay $l \cdot c_{1,t}$ to depositors[12] and must repay $r_0^* \cdot d_{0t}$ as well. They also have a new source of funds in the inter-period foreign loan of $d_{1,t+1}$ goods. In case more funds were required, banks could liquidate early the amount $l_t \leq k_{t+1}$ of their long-term investment, but they will try to avoid doing so, since early liquidation is costly[13]. Summarizing, the budget constraint faced by a young bank at the end of period *t* is

$$l \cdot c_{1,t} + r_0^* \cdot d_{0,t} \leq r \cdot l_t + d_{1,t+1}. \tag{5}$$

[12] The parameter l is public information, but all the banks know is that there are l households of the impatient type that will each withdraw $c_{1,t}$ goods, following a sequential service constraint of the form "first come, first served." Banks do not know the identities of the impatient households, and so they would pay claims by patient households pretending to be of the impatient type.

[13] In the sense that it obtains the effective return $r < 1 < R$ instead. Moreover, one could think of d_1 and l as substitute sources of liquidity for banks, but d_1 is cheaper, since $r_0^* < R$ holds. If the bank were to exhaust its resources before covering all liabilities, the bank would close, and any future payments contracted by the bank would be lost.

The morning of period $t+1$***:*** there is no action by either old households or "old" banks in this sub-period; they wait for their investment to multiply and their debt to mature. If all households behaved according to their true type at t, then all impatient households have already consumed.

The afternoon of period $t+1$***:*** households of the patient type who behave according to their true type wish to withdraw funds to consume in their old-age. Repayments of the long-term foreign debt $\left(r_2^* \cdot d_{2,t+1}\right)$ and the inter-period foreign debt $\left(r_1^* \cdot d_{1,t+1}\right)$ are due as well. An old bank still in operation will use the return of the remaining long-term investment -given by $R \cdot \left(k_{t+1} - l_t\right)$ goods- and the gross real return on its currency reserves to pay its obligations. We must emphasize at this point that one of the consequences of the regulations on reserve requirements is that old banks have an additional source of funds at the end of $t+1$ –even if they each yield a dominated real rate of return. Patient households know this and will think twice before running on banks at the end of period *t*. In brief, the budget constraint faced by a bank in late-afternoon of period $t+1$ is given by

$$\left(1-l\right) \cdot c_{2,t+1} + r_2^* \cdot d_{2,t+1} + r_1^* \cdot d_{1,t+1} \leq R \cdot (k_{t+1} - l_t) + f_d \cdot \left(\frac{p_t}{p_{t+1}}\right) \cdot \left(w + t_t\right) + \frac{f_f \cdot \left(w + t_t\right)}{\left(1+s^*\right)}. \quad (6)$$

Deposit Contracts. Domestic individual households face uncertainty about the type they will become at the end of their youth. Moreover, once realized, this information is private to each individual agent. Under these circumstances, the representative bank aims to protect itself by using some kind of self-selection mechanism. Such a mechanism is designed to give individual households the right incentives to behave according to their true type, and it takes the form of the truth-telling constraint given in (2). The severity of the private information problem in a particular state of the world will determine whether (2) will hold at the end of t or not. For our convenience, henceforth, we focus on to the general problem solved by a representative young bank born at t.

Autarkic Equilibrium. In the absence of financial intermediation, individuals cannot benefit from pooling their resources and there are no insurance schemes available to them. Thus, they could save only through their investment/storage technology. Before the information about types is realized, the individual households face a feasible set in the space of state-contingent commodities $\left(c_{1,t}, c_{2t+1}\right)$ that contains only the *point* $\left\{r \cdot \left(w + t_t\right), R \cdot \left(w + t_t\right)\right\}$.

Financial Intermediation. Recall that representative banks are coalitions of households in our model economy. These banks will offer to individuals a deposit contract consisting of the state-contingent pair $\left(c_{1,t}, c_{2,t+1}\right)$. Banks design this contract by choosing the pair $\left(c_{1,t}, c_{2,t+1}\right)$ that maximizes the individuals' lifetime utility described in (1), subject to the constraints (2)-(6). Interestingly, when financial intermediation is available to individuals, the feasible set consists of a continuum of state-contingent commodities. These contracts may provide households with allocations that are Pareto-superior to that of autarky, since banks offer some insurance against the uncertainty of types, and they are capable of doing so due to their ability to pool the households' resources. Individuals, in turn, are willing to sacrifice a little of potential future returns in exchange for this insurance, so that $r \cdot \left(w + t_t\right) < c_{1,t} < c_{2,t+1} < R \cdot \left(w + t_t\right)$ holds, ensuring that $c_{1,t}$ and $c_{2,t+1}$ lie closer to each other.

2.3 Conducting Monetary Policy under Floating Exchange Rates

In this model economy, the nominal exchange rate is left to float and the monetary authority prioritizes two important aspects when conducting monetary policy: 1) the setting of the rate of domestic money growth, and 2) the choice of the appropriate backing of the domestic money supply with foreign-reserve assets[14].

In the case of the first aspect, the growth rate of the domestic money supply becomes a tool of monetary policy of the utmost importance, as opposed to situations where the money supply is subordinated to other primarily chosen policy objectives. Many reasons may explain the special interest on this policy tool, other than the obvious --controlling the evolution of the domestic money supply--. First, it contributes to the control, up to some degree, over the determination of inflation rates, especially --but not only-- in the long-run. Second, it is a mean of giving or taking away incentives to the actors involved with real activity –in our case, financial activity. Third, it can influence the formation of public expectations by keeping reasonable stable rates of inflation as well as the value of the national currency. We adopt the simplest scenario for our model economy, which contemplates the choice and setting, once-and-for-all, of a constant rate of money growth $s > -1$ by the domestic monetary authority. Such a policy sets the evolution of the supply of *wons* by the rule

$$M_{t+1} = (1+s) \cdot M_t, \forall t > 0, M_0 > 0. \tag{7}$$

The monetary authority injects/withdraws money through lump-sum transfers to all young households in the amount of t_t goods each.

With respect to the second aspect of policy, backing the domestic money supply is by itself a precautionary mechanism aimed to protecting this economy against potential reversals in the World financial market. Typically, the monetary authority chooses and sets a fixed fraction of the dollar value of the domestic money supply to be backed. The backing takes the form of holdings of foreign, interest-bearing reserve assets, and it aims at stabilizing the perceived value of domestic money and the willingness of the public to hold it. Thus, the monetary authority holds B_t^* dollars in the form foreign-reserve assets that yield the world interest rate $\tilde{r} \in (1, R)$ every period. These reserve-holdings evolve over time according to the rule

$$B_t^* = q \cdot \left(\frac{M_t}{e_t} \right), \tag{8}$$

where $q \in [0,1]$ is the exogenous and constant fraction of the *dollar-value* of the supply of *wons* backed by the monetary authority. This policy is a variation of the one used in Hernandez-Verme (2004)[15].

We now define $z_t \equiv (M_t / p_t)$ as the real balances of *wons* per household and $b_t^* \equiv (B_t^* / p_t^*)$ as the holdings of foreign-reserve assets per household. When the two policy rules in (7) and (8) are adopted and combined by the monetary authority, its budget constraint at each period takes the following form:

[14] We do not concern ourselves with the choice of the optimal s or q, but we provide with some useful guidelines regarding existence and the properties of the system along dynamic equilibrium paths.
[15] We assume for simplicity that there is no backing of domestic deposits.

$$t_t = \frac{M_t - M_{t-1}}{p_t} - \frac{B_t^* - \tilde{r} \cdot \left(p_t^* / p_{t-1}^* \right) \cdot B_{t-1}^*}{p_t^*} = \left(\frac{s}{1+s} \right) \cdot z_t - \left(b_t^* - \tilde{r} \cdot b_{t-1}^* \right) = \left[\left(\frac{s}{1+s} \right) - q \right] \cdot z_t + \tilde{r} \cdot q \cdot z_{t-1} . \tag{9}$$

Notice that equation (9) is a linear first order difference equation that describes the evolution of the real monetary transfer (t_t,) and that its dynamic behavior is inherited from z_t.

2.4 General Equilibrium with Floating Exchange Rates

In the remainder of this section, we use the symbol "^" over the values of the different endogenous variables that arise from a floating exchange rate regime. Several sets of conditions must be satisfied simultaneously.

First, there are two conditions on international transactions that apply to our small open economy. One, since *manna* is homogeneous across countries and there are no restrictions on the international trade of goods, it follows that purchasing power parity must hold, and, moreover, that the market-determined exchange rate e_t adjusts to ensure this condition is satisfied:

$$e_t \cdot p_t^* = \hat{p}_t . \tag{10a}$$

Also, we confine our attention to vectors of prices of foreign debt $\left(r_0^*, r_1^*, r_2^* \right) >> 1$ that satisfy the following no arbitrage condition, which controls for the different maturities of the three foreign-debt instruments:

$$R = r_2^* = r_0^* \cdot r_1^* . \tag{10b}$$

Second, there are two constraints on the gross real returns on currencies that guarantee that both reserve requirements bind in equilibrium:

$$R > \left(\hat{p}_t / \hat{p}_{t+1} \right) \tag{11a}$$

$$R > \left(p_t^* / p_{t+1}^* \right) = \left(1 + s^* \right)^{-1} . \tag{11b}$$

Third, we have all the conditions associated with the market for *wons*. One, the domestic price level $\hat{p}_t$ clears the market for domestic real money balances:

$$\hat{M}_t / \hat{p}_t = \hat{z}_t = f_d \left(w + \hat{t}_t \right) . \tag{12a}$$

The latter, in turn, leads to the equilibrium return of domestic real money balances

$$\hat{p}_t / \hat{p}_{t+1} = \left(1 + s \right)^{-1} \left(\hat{z}_{t+1} / \hat{z}_t \right) \tag{12b}$$

and, –using also (9) in the process, to the equilibrium laws of motion of z_t and t_t, respectively

$$\hat{z}_t = a_1(s) + a_2(s) \cdot \hat{z}_{t-1} , \tag{12c}$$

$$\hat{t}_t = b_1(s) + b_2(s) \cdot \hat{z}_{t-1} , \tag{12d}$$

where the reduced-form coefficients are given by $a_1(s) \equiv [f_d \cdot w \cdot (1+s)] / \mathrm{M}(s)$, $a_2(s) \equiv [q \cdot f_d \cdot \tilde{r} \cdot (1+s)] / \mathrm{M}(s)$, $b_1(s) \equiv [a_1(s) - f_d \cdot w] / f_d$, $b_2(s) \equiv [a_2(s)] / f_d$, and $\mathrm{M}(s) \equiv \{ 1 + q \cdot f_d + s \cdot [1 - f_d \cdot (1-q)] \}$.

In fourth place, we have the market for foreign currency, which clears when

$$\hat{q}_t \equiv \left(e_t \cdot \hat{Q}_t / \hat{p}_t \right) = f_f \cdot \left(w + \hat{t}_t \right) = f_f \cdot \hat{z}_t / f_d . \tag{13a}$$

In equilibrium, q_t and b_t^* are governed by the following two reduced-form equations

$$\hat{q}_t = \left[f_f \cdot a_1(s) \right] / f_d + \left\{ \left[f_f \cdot a_2(s) \right] / f_d \right\} \cdot \hat{z}_{t-1} , \tag{13b}$$

$$\hat{b}_t^* = q \cdot a_1(s) + q \cdot a_2(s) \cdot \hat{z}_{t-1} . \tag{13c}$$

Moreover, the endogenous growth rate of the supply of dollars in the domestic economy is given by

$$\left(\hat{Q}_{t+1}/\hat{Q}_t\right)=\left[\left(1+s^*\right)\cdot\hat{z}_{t+1}\right]/\hat{z}_t\ , \tag{13d}$$

while the nominal exchange rate follows:

$$\left(e_{t+1}/e_t\right)=\left[\left(1+s\right)\cdot\hat{z}_t\right]/\left[\left(1+s^*\right)\cdot\hat{z}_{t+1}\right]. \tag{13e}$$

Finally, there are several conditions that characterize deposit contracts in equilibrium. One, the truth-telling condition in (2) holds. Two, the constraints on foreign credit must bind, and thus

$$\hat{d}_{0,t}+\hat{d}_{2,t+1}=f_0 \text{ and } \hat{d}_{1,t+1}+\hat{d}_{2,t+1}=f_1. \tag{14a}$$

Three, the representative bank's long-term investment in equilibrium follows

$$\hat{k}_{t+1}=x_1\left(s\right)+x_2\left(s\right)\cdot\hat{z}_{t-1}. \tag{14b}$$

(14b) is a first order linear difference equation, and its reduced-form coefficients are given by $x_1\left(s\right)\equiv f_0+\left[\left(1-f_d-f_f\right)\cdot w\cdot \tilde{N}\left(s\right)\right]/\mathrm{M}\left(s\right)$ and $x_2\left(s\right)\equiv\left[\left(1-f_d-f_f\right)\cdot q\cdot\tilde{r}\cdot\left(1+s\right)\right]/\mathrm{M}\left(s\right)$, where $\tilde{N}\left(s\right)\equiv\left\{2+q\cdot f_d+s\cdot\left[2-f_d\cdot\left(1-q\right)\right]\right\}$. Next, the withdrawals offered by banks to impatient and patient individuals in equilibrium are, respectively

$$\hat{c}_{1,t}=\left[\left(f_1-f_0\right)/l\right]-\left[\left(r_0^*-1\right)/l\right]\cdot\hat{d}_{0,t}, \tag{14c}$$

$$\hat{c}_{2,t+1}=\backsim_{\circ}\left(s\right)-\frac{\left(r_2^*-r_1^*\right)}{\left(1-l\right)}\cdot\hat{d}_{2,t+1}+\frac{\Theta_1\left(s\right)}{\left(1-l\right)}\cdot\hat{z}_{t-1}+\frac{\Theta_2\left(s\right)}{\left(1-l\right)}\cdot\left(\frac{\hat{z}_{t+1}}{\hat{z}_t}\right)+\frac{\Theta_3\left(s\right)}{\left(1-l\right)}\cdot\hat{z}_{t+1}+\frac{\Theta_4\left(s\right)}{\left(1-l\right)}\cdot\hat{z}_t \tag{14d}$$

where the intercept is $\backsim_{\circ}\left(s\right)\equiv\left[r_2^*\cdot x_1\left(s\right)\cdot\left(1-l\right)^{-1}+f_d\cdot w\cdot\left(1-l\right)^{-1}\cdot\left(1+s^*\right)^{-1}-r_1^*\cdot f_1\right]$. The reduced-form coefficients are given by $\Theta_1\left(s\right)\equiv r_2^*\cdot x_2\left(s\right)$, $\Theta_2\left(s\right)\equiv\left[f_d\cdot w/\left(1+s^*\right)\right]$, $\Theta_3\left(s\right)\equiv\left(1+s\right)^{-1}$ and $\Theta_4\left(s\right)\equiv f_f\cdot\left[f_d\cdot\left(1+s^*\right)\right]^{-1}$. Interestingly, equation (14d) is a second order, nonlinear difference equation in $\hat{z}_t$, and contemporaneous in $\hat{d}_{2,t+1}$, which anticipates some potentially complex dynamics. Last, using the guidelines by the IMF and the equilibrium conditions, we define the current account balance as:

$$C\hat{A}_t=\left(r_0^*-1\right)\cdot\hat{d}_{2.t+1}-\left(r_2^*-r_1^*\right)\cdot\hat{d}_{2,t}-\left(r_0^*-1\right)\cdot f_0-\left(r_1^*-1\right)\cdot f_1. \tag{14e}$$

Notice that (14e) is a first order difference equation in $\hat{d}_{2,t}$. As we will see in the next section, the reduced-form equations for the debt-structure vector $\left(\hat{d}_{0,t},\hat{d}_{1,t+1},\hat{d}_{2,t+1}\right)$ and, of course, the couple of state-contingent commodities $\left(\hat{c}_{1,t},\hat{c}_{2,t+1}\right)$ will depend on the particular set the equilibria belong to.

2.5 Multiplicity and Indeterminacy of Stationary Equilibria under Floating Exchange Rates

Here we discuss the set of ***separating*** stationary equilibria that arise with floating exchange rates. These equilibria are such that: 1) there are no misrepresentations of types; 2) there are no problems of liquidity or solvency, and 3) young banks do not liquidate early their long-term investment. Before we proceed we must point the reader to Figure 3, which illustrates the structure of causality of this economy. There is a *"core"* of variables which are independent of the foreign interest rates, and another set that contains the debt-structure vector and the state-contingent commodities that are determined as a result and *do* depend upon the world interest rates.

The Core in a Stationary Equilibrium. The core consists of a vector of five key variables: $\left(\hat{z}_t,\hat{t}_t,\hat{q}_t,\hat{b}_t^*,\hat{k}_t\right)$. These five variables are always determinate in equilibrium, since they do not depend on the interest rates

$\left(r_0^*, r_1^*, r_2^*\right)$. Interestingly, the core dynamic system is de-coupled, inheriting its dynamics from $\hat{z}_t$. The stationary values of core variables can be found in next five equations:

$$\hat{z} = a_1(s)/[1-a_2(s)] = [f_d \cdot w \cdot (1+s)/\gamma(s)], \qquad (15a)$$

$$\hat{t} = (\hat{z}/f_d) - w = \left\langle w \cdot \{q \cdot f_d \cdot (\tilde{r}-1) + s \cdot f_d \cdot [q \cdot (\tilde{r}-1)+1]\}/\gamma(s)\right\rangle, \qquad (15b)$$

$$\hat{q} = \left(f_f \cdot \hat{z}/f_d\right) = \left[f_f \cdot w \cdot (1+s)/\gamma(s)\right], \qquad (15c)$$

$$\hat{b}^* = q \cdot \hat{z} = [q \cdot f_d \cdot w \cdot (1+s)/\gamma(s)], \qquad (15d)$$

$$\hat{k} = x_1(s) + x_2(s) \cdot \hat{z}, \qquad (15e)$$

where $\gamma(s) \equiv s \cdot [1 - f_d \cdot (1 + q \cdot (\tilde{r}-1))] + 1 - f_d \cdot q \cdot (\tilde{r}-1)$. Notice that $\left(\hat{z}, \hat{q}, \hat{b}^*\right)$ are increasing in the policy parameters (s, f_d, q) and that, as expected, $\hat{q}$ is increasing in f_f. In addition, $\hat{t}$ is nonlinear in both s and f_d but monotonically increasing in q. Finally, $\hat{k}$ is increasing in s, but nonlinear in $\left(f_d, f_f\right)$. With respect to the steady-state gross returns on domestic and foreign real money balances, the growth of the nominal exchange rate and the growth rate of the real exchange rate, they are all constant and equal to $(1+s)^{-1}$, $\left(1+s^*\right)$, $(1+s) \cdot \left(1+s^*\right)^{-1}$ and 1, respectively.

Foreign Debt in a Stationary Equilibrium. The amount that banks borrow from abroad is constant and non-negative in a stationary equilibrium, for all types of foreign debt-instrument, provided, of course that (10b) holds. Thus, the structure of foreign debt of a bank in a steady-state equilibrium is given by the triplet $\left(\hat{d}_0, \hat{d}_1, \hat{d}_2\right) > 0$. This debt structure vector permits us to calculate the current account balance in a stationary equilibrium:

$$C\hat{A} = \left(1 - r_0^*\right) \cdot \hat{d}_0 + \left(1 - r_1^*\right) \cdot \hat{d}_1 + \left(1 - r_2^*\right) \cdot \hat{d}_2 < 0 \qquad (16a).$$

We must remark that the deficit of the current account in stationary equilibria poses significant doubt on the long-run viability of this economy, as one might expect.

A stationary equilibrium is defined as the set of vectors $\left(\hat{z}, \hat{t}, \hat{q}, \hat{b}^*, \hat{k}\right) \in \mathbf{R}^5$, $\left(\hat{d}_0, \hat{d}_1, \hat{d}_2\right) \in \mathbf{R}^3_+$ and $\left(\hat{c}_1, \hat{c}_2\right) \in \mathbf{R}^2_{++}$ such that $\hat{l} = 0$ and all the above conditions are satisfied. The particular type of equilibrium that obtains as well as its properties will depend on the composition of the vector $(\hat{d}_0, \hat{d}_1, \hat{d}_2)$. We will discuss this issue as we go along.

Existence and Local Uniqueness of Stationary Equilibria. Before discussing fully the issue of existence, we must discuss the different types of equilibria that may arise[16], based on the properties of the structure of foreign debt issued by domestic banks. We will observe multiple stationary equilibria in this model economy with floating exchange rates. There are three cases, which we discuss below. The second subscript on variables denotes Case *j*, where $j = 1, 2, 3$.

Case 1: Equilibria with no intra-period debt $\hat{d}_0$**.** These are stationary allocations characterized by the debt-structure vector $\left(\hat{d}_{0,1}, \hat{d}_{1,1}, \hat{d}_{2,1}\right) = (0, f_1 - f_0, f_0)$. This allocation can be thought of as banks willing to borrow arbitrarily large values of $\hat{d}_{2,1}$ but since foreign credit is rationed, banks must content themselves with $\hat{d}_{2,1} = f_0$. The pair of state-contingent commodities that obtain in this case is given by

[16] This general classification will also apply to dynamic equilibria, as we will see in the next section.

$$l \cdot \hat{c}_{1,1} = f_1 - f_0 , \tag{16b}$$

$$(1-l)\cdot \hat{c}_{2,1} = ?_{\ 0}(s) - \left(R - r_1^*\right)\cdot f_0 + \Theta_2(s) + \hat{z}\cdot\left[\Theta_1(s) + \Theta_3(s) + \Theta_4(s)\right], \tag{16c}$$

while the steady-state expected utility transpires from

$$\hat{U}_1 = l \cdot \ln\left(\hat{c}_{1,1}\right) + (1-l)\cdot \ln\left(\hat{c}_{2,1}\right) \tag{16d}$$

Case 2: Interior Solution of Debt-Structure. All these stationary allocations are characterized by a debt-structure vector of the form $(\hat{d}_{0,2}, \hat{d}_{1,2}, \hat{d}_{2,2}) = \left(f_0 - \hat{d}_{2,2}, f_1 - \hat{d}_{2,2}, \hat{d}_{2,2}\right) >> 0$, and there is a continuum of allocations that satisfy this criterion. The steady-state consumption vector and the steady-state expected utility follow

$$l \cdot \hat{c}_{1,2} = f_1 - r_0^* \cdot f_0 + \left(r_0^* - 1\right)\cdot\left[\Theta_0(s) + \Theta_2(s)\right] + \hat{z}\cdot\left[\Theta_1(s) + \Theta_3(s) + \Theta_4(s)\right] \tag{16e}$$

$$(1-l)\cdot \hat{c}_{2,2} = ?_{\ 0}(s) - \left(R - r_1^*\right)\cdot\Theta_0(s) + \Theta_2(s) + \hat{z}\cdot\left[\Theta_1(s) + \Theta_3(s) + \Theta_4(s)\right] \tag{16f}$$

$$\hat{U}_2 = l \cdot \ln\left(\hat{c}_{1,2}\right) + (1-l)\cdot \ln\left(\hat{c}_{2,2}\right) \tag{16g}$$

Case 3: Equilibria with no long-term debt $\hat{d}_2$**.** These equilibria are characterized by the debt-structure vector $(\hat{d}_{0,3}, \hat{d}_{1,3}, \hat{d}_{2,3}) = (f_0, f_1 - f_0, 0)$. One could think of an explanation along the lines we used for Case 1: domestic banks are willing to borrow arbitrarily large amounts of intra-period debt, but they must content themselves with $\hat{d}_{0,3} = f_0$. Regarding the steady-state consumption vector and expected utility, they obtain from

$$l \cdot \hat{c}_{1,3} = f_1 - r_0^* \cdot f_0 \tag{16h}$$

$$(1-l)\cdot \hat{c}_{2,3} = ?_{\ 0}(s) + \Theta_2(s) + \hat{z}\cdot\left[\Theta_1(s) + \Theta_3(s) + \Theta_4(s)\right] \tag{16i}$$

$$\hat{U}_3 = l \cdot \ln\left(\hat{c}_{1,3}\right) + (1-l)\cdot \ln\left(\hat{c}_{2,3}\right) \tag{16j}$$

The properties displayed by the stationary debt-structure in equilibrium depend, among other things, on the different values that the policy parameter s --the growth rate of the domestic money supply-- may take. The aforementioned properties have to do with existence, the number of equilibria, and the case to which the equilibria belong to. Thus, s is a bifurcation parameter of the steady-state allocation given by $\left\{\left(\hat{z}, \hat{t}, \hat{q}, \hat{b}^*, \hat{k}\right), \left(\hat{d}_{0,j}, \hat{d}_{1,j}, \hat{d}_{2,j}\right), \left(\hat{c}_{1,j}, \hat{c}_{2,j}\right) \middle| \ \hat{l} = 0\right\}$, and so is the structure of the interest rates $\left(r_0^*, r_1^*, r_2^*\right) >> 1$. Proposition 1 illustrates the general properties of the separating steady-state equilibria in our model economy.

Proposition 1. Define the set $\mathfrak{P} = \left\{s, r_0^*, r_1^*, r_2^*\right\} \in \mathbf{R}^4$ to be the space of bifurcation parameters under a floating exchange rate regime. Bifurcation values of these parameters partition $\mathfrak{P}$ into three subsets with defining characteristics that we describe below.

Subset 1= $\mathfrak{P}_1$*:* Existence of Case 1 Equilibria. Given $\hat{e}$ as defined in the Appendix, the two mutually exclusive conditions must hold for equilibria of Case 1 to exist:

- Condition 1: $\hat{e} > 0$ must hold.
- Condition 2: when $\hat{e} < 0$ obtains, $s < \hat{s} \equiv \left[\hat{e} + l \cdot f_d \cdot w / \left(-\hat{e}\right)\right]$ must hold

Subset 2= $\mathfrak{P}_2$*:* Existence of Case 2 Equilibria. This type of equilibrium always exists. Therefore, $\mathfrak{P}_2 = \mathfrak{P}$.

Subset 3= $\mathfrak{P}_3$*:* Existence of Case 3 Equilibria. Given the expressions *A*, *B* and *C*, as defined in the Appendix, equilibria of Type 3 exist when

- Condition 3: $\max\left\{A, B\right\} < r_1^* \leq C$

Table 3 summarizes the results on existence of stationary equilibria, and presents as well how the scope for existence varies with s. It is apparent that the stationary allocations of state-contingent commodities $\left(\hat{c}_{1,j}, \hat{c}_{2,j}\right)$ follow directly from $\left(\hat{d}_{0,j}, \hat{d}_{1,j}, \hat{d}_{2,j}\right)$. The issue of multiple equilibria raises a new set of questions related to the properties of local uniqueness and determinacy, which focus on the mapping from allocations to prices in equilibrium. The practice in standard General Equilibrium theory is to attempt to construct a particular economic environment with the aim of ensuring that the equilibria are *"regular,"* which means that: 1) the number of equilibria is finite, and, 2) there is a one-to-one mapping between the vectors of relative prices and the excess demand functions in a neighborhood of the equilibrium allocation. However, our model economy violates the two conditions of regularity. First, there is typically a continuum of equilibria. Second, the mapping between the vectors of relative prices and the excess demand *correspondence* is not one-to-one. Thus, steady-state equilibria in our economy are *"irregular,"* and thus, they are not locally unique nor are determinate.

Let us illustrate the nature of the irregularity of equilibria in our model economy. Note that the core in the steady-state $\left(\hat{z}, \hat{t}, \hat{q}, \hat{b}^*, \hat{k}\right)$ is always unique and determinate, since it is not associated with the vector $\left(r_0^*, r_1^*, r_2^* = R\right)$. However, for a fixed point in the parameter-space, and for each stationary debt-structure vector $\left(\hat{d}_{0,j}, \hat{d}_{1,j}, \hat{d}_{2,j}\right)$ that belongs to case j there is typically a continuum of vectors of interest rates satisfying the equilibrium conditions. To fix ideas, let us discuss briefly the equilibrium in Case 1, in which this issue appears to be a little simpler. Here, the debt-structure $\left(0, f_1 - f_0, f_0\right)$ is unique. However, as the reader can observe in Figure 1, there is a continuum of vector prices $\left(r_0^*, r_1^*, r_2^* = R\right)$ that satisfy the equilibrium conditions, and is associated with the allocation $\left(0, f_1 - f_0, f_0\right)$. This is illustrated by the thick gray line on the plane $r_2^* = R$, which defines the set of possible equilibrium price vectors. The range of $\left(r_0^*, r_1^*, r_2^*\right)$ that obtains in an equilibrium that belongs to Case 1 depends on the following three boundary functions: $\tilde{A}(s) \equiv r \cdot \tilde{B}(s) \cdot \left[\, l \cdot r + (1 - l) \cdot r_0^* \right]^{-1}$, $\tilde{B}(s) \equiv \sigma \cdot \left(f_1 - f_0\right)^{-1}$ and $\tilde{C}(s) \equiv [\tilde{B}(s) - (1 - l) \cdot r] \cdot l^{-1}$. In this particular case, stationary equilibria exist when $\tilde{A}(s) < r_1^* < \tilde{B}(s) < \tilde{C}(s)$ holds.

Turning back to the general case, we must emphasize that the indeterminacy of equilibria in this case goes two ways: 1) for a given vector $\left(\hat{d}_{0,j}, \hat{d}_{1,j}, \hat{d}_{2,j}\right)$ there is a continuum of vectors $\left(r_0^*, r_1^*, r_2^*\right)$ consistent with equilibrium conditions, but also 2) for a given vector $\left(r_0^*, r_1^*, r_2^*\right)$, there may be more than one vector $\left(\hat{d}_{0,j}, \hat{d}_{1,j}, \hat{d}_{2,j}\right)$ consistent with equilibrium. Moreover, these vectors may belong to the three the different cases.

Steady-State Social Welfare. Interestingly, we found that, under a floating exchange rate regime, $\hat{c}_{1,2}, \hat{c}_{2,1}, \hat{c}_{2,2}$ and $\hat{c}_{2,3}$ are concave and increasing functions of the rate of domestic money growth, s, and so are $\hat{U}_1, \hat{U}_2$ and $\hat{U}_3$. Regarding the impatient households, the ranking $\hat{c}_{1,3} < \hat{c}_{1,1} < \hat{c}_{1,2}$ obtains, and impatient households are always better-off in equilibria of Case 2. However, quite the opposite is observed for patient households, since $\hat{c}_{2,2} < \hat{c}_{2,1} < \hat{c}_{2,3}$ obtains; thus, patient households prefer equilibria of Case 3, and Case 1 is always intermediate.

Nonetheless, the households learn of their type ex-post, after making their choices, and the concept of expected utility must be used to classify the equilibria of the different cases. We found that under floating exchange rates the ranking $\hat{U}_1 \geq \hat{U}_3 > \hat{U}_2$ transpires in most cases, as shown in Figure 2. The latter implies that Cases 1 and 3 Pareto dominate Case 2. Below we describe the effects of the different structural and policy parameters on expected utility. The effects of the world interest rates can be found in the Appendix.

The number of impatient households. Adding to l reduces expected utility in Cases 1 and 3. Case 2, however, is an exception: for $l < 0.5$, adding to l increases expected utility and $\hat{U}_2$ is an increasing function of the rate of domestic money growth; but further increases of l reduce welfare, and the expected utility in Case 2 becomes a decreasing function of s. Interestingly, as $l \to 0$, the ranking changes slightly to $\hat{U}_3 \geq \hat{U}_1 > \hat{U}_2$, but as $l \to 1$ the ranking is reversed to $\hat{U}_2 > \hat{U}_1 > \hat{U}_3$, and Case 2 Pareto dominates Cases 1 and 3.

The reserve requirements. Increasing both currencies reserves, provided that $f_d = f_f$, reduces expected utility, and the welfare ranking is preserved. Increasing only the foreign currency reserves $\left(f_f\right)$ produces the same results, and it would appear that higher foreign currency real balances hurt the economy. However, results are mixed in the face of augmenting the domestic currency reserves $\left(f_d\right)$: higher domestic currency real balances seem to add to welfare in Cases 1 and 3, but to lessen it in Case 2. In all cases, the welfare ranking is preserved.

The backing of the domestic money supply. Augmenting q boosts welfare in Cases 1 and 3, but it lessens $\hat{U}_2$. In all cases, the changes in welfare are very small, and a slight change in the welfare ranking is observed: as $q \to 0$, $\hat{U}_3 \geq \hat{U}_1 > \hat{U}_2$ obtains but, as $q \to 1$, it turns to $\hat{U}_1 \geq \hat{U}_3 > \hat{U}_2$.

2.6 Dynamic Equilibria under a Floating Exchange Rate Regime

The dynamic system for this economy has three parts: the core dynamic system, the dynamic system for the debt-structure vector $\left(\hat{d}_{0,t}, \hat{d}_{1,t+1}, \hat{d}_{2,t+1}\right)$, and the dynamic system for the space-contingent commodities $\left(\hat{c}_{1,t}, \hat{c}_{2,t+1}\right)$. Obviously, the latter will depend on the arrangement of foreign debt and the particular case the equilibrium belongs to. We point the reader again to Figure 3.

The Core Dynamic System with Floating. The core dynamic system in equilibrium involves the five core variables $\left(\hat{z}_t, \hat{t}_t, \hat{q}_t, \hat{b}_t^*, \hat{k}_{t+1}\right)$ and it consists of equations (12c), (12d), (13b), (13c) and (14b). The dynamic system is de-coupled and all dynamics originates from the real balances of *wons*, $\hat{z}_t$, in equation (12c). Proposition 2 discusses the main dynamic properties of the core system.

Proposition 2 Under floating exchange rates, the quintet $\left(\hat{z}_t, \hat{t}_t, \hat{q}_t, \hat{b}_t^*, \hat{k}_{t+1}\right)$ displays monotonic dynamics along the equilibrium paths around the stationary core. Let $i = 1, 2, 3, 4, 5$ index the variables in the core and their associated eigenvalues. All eigenvalues $?_i(s)$ are smooth functions of all the policy parameters, and four non-overlapping cases may arise: there is no dynamics, the monotonic dynamics is stable, the dynamics display unit roots or there is monotonic divergence. In particular, we find:

i) For a fixed combination of returns, the policy parameters $\left(f_d, f_f, q\right)$ and $\forall i$, $?_i(s)$ is a monotonically increasing and concave function of the rate of domestic money growth, s. Moreover, all eigenvalues have a bifurcation value at $\tilde{s} \equiv (2 - \tilde{r})/(\tilde{r} - 1) > 0$ such that $?_i(\tilde{s}) = 1$. For $s > (<) \tilde{s}$, dynamic equilibria are stable (unstable).
ii) The interaction between all the policy parameters and the nature of the dynamics is highly complex. Interestingly, there are many different sub-regions in the parameter-space that determine characteristic dynamic properties. As an example, we illustrate the case of the eigenvalue $?_1(s) = a_2(s)$ in Table 3. There are many different restrictions/combinations of parameters that yield monotonic dynamics, a unit root or monotonic divergence.

The Dynamic System of the Debt-Structure under Floating. We now turn to discuss the dynamics of the debt-structure vector that result from a floating exchange rate regime. Below, we discuss each case.

Foreign-Debt Dynamics in Cases 1 and 3: the equilibrium debt-structure in either of these extreme cases is always stationary. On the first hand, the composition of foreign credit in Case 1 is given by

$\left(\hat{d}_{0,t,1},\hat{d}_{1,t+1,1},\hat{d}_{2,t+1,1}\right)=\left(\hat{d}_{0,1},\hat{d}_{1,1},\hat{d}_{2,1}\right)=\left(0,f_1-f_0,f_0\right),\forall t\geq 0$, and the corresponding current account $\hat{CA}_1=\left(1-r_1^*\right)\cdot f_1+\left(r_1^*-r_2^*\right)\cdot f_0<0$ displays a nontrivial deficit. On the other hand, in Case 3, foreign credit is given by $\left(\hat{d}_{0,t,3},\hat{d}_{1,t+1,3},\hat{d}_{2,t+1,3}\right)=\left(\hat{d}_{0,3},\hat{d}_{1,3},\hat{d}_{3,1}\right)=\left(f_0,f_1,0\right),\forall t\geq 0$, while there is also a deficit in the associated current account balance given by $\hat{CA}_3=\left(1-r_1^*\right)\cdot f_1+\left(r_0^*-r_1^*\right)\cdot f_0<0$.

Foreign-Debt Dynamics in Case 2: the debt-structure vector displays non trivial dynamics. We present the results for the long-term, foreign-debt instrument (the results for $\hat{d}_{0,t,2}=f_0-\hat{d}_{2,t+1,2}$ and $\hat{d}_{1,t+1,2}=f_1-\hat{d}_{2,t+1,2}$ follow directly.) The dynamics of $\hat{d}_{2,t+1,2}$ is governed by

$$\hat{d}_{2,t+1,2}=\Theta_0\left(s\right)+\Theta_1\left(s\right)\cdot\hat{z}_{t-1}+\Theta_2\left(s\right)\cdot\left(\hat{z}_{t+1}/\hat{z}_t\right)+\Theta_3\left(s\right)\cdot\hat{z}_{t+1}+\Theta_4\left(s\right)\cdot\hat{z}_t\,. \qquad (17)$$

Equation (17) is a second order, nonlinear, difference equation in $\hat{z}_t$. To solve for the dynamic path, we first augment the state-space by using $\hat{y}_{t+1}=\hat{z}_t$, which reduces the order of the system to a first order dynamic system in $\left(\hat{z}_t,\hat{y}_t\right)$.

Local Stability Analysis. We took a First Order Taylor approximation around the steady-state. The trace and determinant of the Jacobian matrix associated with this system in the steady-state are continuous monotonic functions of the parameters, and given by the expressions:

$$Tr\left(J\right)=\frac{f_d\cdot w}{\left(f_d\cdot w+\hat{z}\right)}-\frac{\left(1+s\right)\cdot f_f\cdot\hat{z}}{f_d\cdot\left(1+s^*\right)\cdot\left(f_d\cdot w+\hat{z}\right)}, \qquad (18a)$$

$$Det\left(J\right)=\frac{\left(1+s\right)\cdot r_2^*\cdot x_2\left(s\right)\cdot\hat{z}}{\left(f_d\cdot w+\hat{z}\right)}>0\,. \qquad (18b)$$

It is apparent, from (18b), that the pair of eigenvalues has the same sign. Notice that $Tr\left(J\right)$ is a monotonically decreasing function of s and $Det\left(J\right)$ is a monotonically increasing function of s, while the discriminant $\Delta\equiv\left[Tr\left(J\right)\right]^2-4\cdot Det\left(J\right)$ and eigenvalues are highly nonlinear in the same parameter. Moreover, the world inflation rate and the policy parameters interact with the rate of domestic money growth, altering the dynamic properties of the system. We now describe how the dynamic properties of the long-term debt and bifurcations change as we contemplate scenarios with different combinations of parameters[17]. In particular, we present first the "*baseline sequence*" in Proposition 3, which describes how the dynamic properties of the long-term debt vary with the rate of domestic money growth in the *baseline scenario*. We describe how the *baseline sequence* changes with different values of $\left(s^*,f_d=f_f,q\right)$ and $\left(\tilde{r},r_0^*,r_1^*,r_2^*\right)>>1$ in the Appendix. For our convenience, we define the following notation: (+) sink is a sink with positive eigenvalues, (-) sink is a sink with negative eigenvalues, (-) saddle is a saddle with a negative stable eigenvalue, (-) source is a source with negative eigenvalues, (+) complex-stable indicates complex conjugates with a positive real part that is less than one, (-) complex-stable indicates complex

[17] Due to limited space, we only present a summary of the results of our simulations. The detailed results and the simulation files are available upon request. The parameter values used in the *baseline scenario* are: $w=2$, $f_d=f_f=0.1$, $q=0.2$, $l=0.2$, $s^*=0.05$, $\tilde{r}=1.1$, $R=r_2^*=1.2$, $r_0^*=1.08$ and $r_1^*=1.11$. The *baseline scenario* only represents a reasonable starting point, since this is not a calibration exercise.

conjugates with a negative real part that is less than one, and (-) complex-unstable indicates complex conjugates with a real part that is negative and outside of the unit circle.

Proposition 3 The *baseline sequence* as a function of s under floating exchange rates consists of: (+) sink, (+) complex-stable, (-) complex-stable, (-) complex-unstable, (-) source and (-) saddle. This means that for values of s that are low enough, the steady-state $\hat{d}_{2,2}$ is a sink with positive eigenvalues and indeterminacy is observed; as s increases, the eigenvalues become complex conjugates with a positive and stable real part that next turns into a negative stable real part, displaying cyclical and non-cyclical stable fluctuations; with higher values of s, the real part of complex conjugates become negative and unstable, displaying unstable cyclical oscillations and causing equilibrium sequences to vanish; as s continues to increase, equilibrium sequences do not exist since the steady-state becomes a source with large and negative eigenvalues, dynamic equilibria are unstable and display large and exploding non-cyclical fluctuations along dynamic paths; finally, for values of the rate of domestic money growth that are high enough, equilibrium sequences reappear, since the steady state becomes a saddle with non-cyclical stable fluctuations along the stable manifold. The scope for determinacy dominates, but so does the scope for endogenously arising volatility.

Proposition 3 illustrates the rich dynamics of the long-term debt asset in Case 2. Interestingly, market-generated volatility is always present and dominates along dynamic paths.

The Dynamic System of the Space-Contingent Commodities. We now turn to discuss the properties of the pair $\left(\hat{c}_{1,t,j}, \hat{c}_{2,t+1,j}\right) \gg 0$ in dynamic equilibria with floating exchange rates. This dynamic system consists of equations (14c) and (14d), which inherit their dynamics from the dynamic system of the debt-structure. We first discuss the dynamic properties of the consumption by impatient households in three different cases of equilibria, and we follow with the analysis of the consumption by patient households.

Dynamics of the Consumption by Impatient Households. In general, the consumption by impatient households is given by $l \cdot \hat{c}_{1,t,j} = f_1 - f_0 - \left(r_0^* - 1\right) \cdot \hat{d}_{0,t,j}$. In particular, for each case, we obtain

$$l \cdot \hat{c}_{1,t,1} = f_1 - f_0, \text{ for } j = 1, \tag{19a}$$

$$l \cdot \hat{c}_{1,t,2} = f_1 - r_0^* \cdot f_0 + \left(r_0^* - 1\right) \cdot \hat{d}_{2,t+1,2}, \text{ for } j = 2, \tag{19b}$$

$$l \cdot \hat{c}_{1,t,3} = f_1 - r_0^* \cdot f_0, \text{ for } j = 3. \tag{19c}$$

Notice that $\hat{c}_{1,t,1} = \hat{c}_{1,1}$ and $\hat{c}_{1,t,3} = \hat{c}_{1,3}$, $\forall t \geq 0$ and thus, the consumption by impatient households is always stationary in Cases 1 and 3. However, in Case 2 the evolution of $\hat{c}_{1,t,2}$ is governed by the term $\left(r_0^* - 1\right) \cdot \hat{d}_{2,t+1,2}$, which indicates non-trivial dynamics, according to our discussion in the previous section. Specifically, the dynamic properties of $\hat{c}_{1,t,2}$ just scale down from the dynamics of $\hat{d}_{2,t+1,2}$ by the factor $0 < r_0^* - 1 < 1$.

Dynamics of the Consumption by Patient Households. By combining (14d) with (17), it is apparent that the dynamic behavior of $\hat{c}_{2,t+1,j}, \forall j$ is inherited from $\hat{d}_{2,t+1,2}$, regardless of the Case the equilibrium belongs to. In particular, for each case we obtain:

$$(1 - l) \cdot \hat{c}_{2,t+1,1} \approx \Theta_0(s) - \left(R - r_1^*\right) \cdot f_0 + \Theta_1(s) \cdot \hat{z}_{t-1} + \Theta_2(s) \cdot \left(\hat{z}_{t+1} / \hat{z}_t\right) + \Theta_3(s) \cdot \hat{z}_{t+1} + \Theta_4(s) \cdot \hat{z}_t, \tag{19d}$$

$$(1 - l) \cdot \hat{c}_{2,t+1,2} \approx \Theta_0(s) - \left(R - r_1^*\right) \cdot \Theta_0(s) + \left(1 + r_1^* - R\right) \cdot \left[\Theta_1(s) \cdot \hat{z}_{t-1} + \Theta_2(s) \cdot \left(\hat{z}_{t+1} / \hat{z}_t\right) + \Theta_3(s) \cdot \hat{z}_{t+1} + \Theta_4(s) \cdot \hat{z}_t\right] \tag{19e}$$

$$(1 - l) \cdot \hat{c}_{2,t+1,3} \approx \Theta_0(s) + \Theta_1(s) \cdot \hat{z}_{t-1} + \Theta_2(s) \cdot \left(\hat{z}_{t+1} / \hat{z}_t\right) + \Theta_3(s) \cdot \hat{z}_{t+1} + \Theta_4(s) \cdot \hat{z}_t \tag{19f}$$

Obviously, (18a) and (18b) still apply to the consumption by patient households in all cases. Proposition 4 summarizes the results for the pair of state-contingent commodities $\left(\hat{c}_{1,t,j}, \hat{c}_{2,t+1,j}\right)$.

Proposition 4 The stationary pair $\hat{c}_{1,1}$ and $\hat{c}_{1,3}$ takes up from the also stationary values $\hat{d}_{2,1} = f_0$ and $\hat{d}_{2,3} = 0$, respectively, while $\hat{c}_{1,t,2}$ inherits its dynamics from $\hat{d}_{2,t+1,2}$. Regarding $(\hat{c}_{2,t+1,1}, \hat{c}_{2,t+1,2}, \hat{c}_{2,t+1,3})$, they come into their dynamics from $\hat{d}_{2,t+1,2}$, and share the same baseline sequence. Thus, endogenously-arising volatility is observed along equilibrium dynamic paths.

Summary of Properties along Equilibrium Dynamic Paths under Floating. There is a nontrivial scope for complex eigenvalues that contributes to both cyclical and non-cyclical fluctuations that are exacerbated by inflation. In some cases, the fluctuations can be significantly large and explosive, ruling out the existence of equilibrium sequences, since $\hat{d}_{2,t+1,j}$ is bounded above by f_0. The scope for instability --and indeterminacy-- is typically small, and the scope for determinacy typically dominates, but fluctuations are observed on the stable manifold. Moreover, unstable and oscillating divergence occurs for some values of s, causing the vanishing of equilibria.

The reserve requirements stabilize, at least partially, the dynamic equilibria in this model economy, while backing the domestic supply plays the opposite role. These results provide us with policy recommendations for this regime regarding reduced instability: to keep the rate of domestic money growth sufficiently low, to implement high and binding reserve requirements, but keeping the backing of the money supply to a minimum. In the extreme case of $q = 0$, the order of the dynamic system is reduced, which one can interpret as the ultimate stabilization of dynamic equilibria.

The Trade-off between Steady-State Welfare and Stability with Floating Exchange Rates. The policy recommendations seem to vary significantly when we look at steady-state welfare versus stability, and trade-offs are always present. First, regarding the rate of domestic money growth, we find that increasing steady-state inflation adds to steady-state welfare, but at the cost of introducing endogenously-arising volatility and instability when s is too high. Second, augmenting the reserve-requirement reduces steady-state welfare but it increases the scope for stability along dynamic paths. Finally, raising the backing of the domestic money supply increases the steady-state utility slightly in all cases, but it introduces instability along dynamic paths, and equilibrium dynamic sequences may vanish.

Our results illustrate that the difficulty of policy-making increases exponentially in an open economy. It is clear that the choice of the appropriate combination of policy parameters will require at least two things from policy makers. The first is the setting of explicit and non contradictory policy goals together with commitment - all tasks that policy makers in most emergent open economies almost always overlook. The second requirement is a very careful choice of the policy parameters consistent with furthering these goals.

3. Fixed Exchange Rates: the Case of Malaysia

In this section, we consider a SOE that aims at reproducing the case of Malaysia at the time of the crisis. This economy is identical to the one discussed in Section 2 in every respect but for the exchange rate regime: this economy operates under a fixed exchange rate regime instead. We continue to focus on equilibria with truth-telling and no crises. As in Hernandez-Verme (2004), we construct this fixed exchange rate regime such that a currency board emerges as a special case. We restrict our attention to very hard pegs, where the nominal exchange rate is set once and for all and it remains constant over time, as opposed to the case of intermediate pegs. We continue to use the *won* as the domestic currency and the *US dollar* as the foreign, international currency.

3.1 Conducting Monetary Policy under Fixed Exchange Rates

In this model economy, the monetary authority has different priorities as to the aspects of monetary policy rules to be implemented. In the first place, policymakers worry about the setting, once-and-for-all, of the nominal exchange rate at $t=0$. Secondarily, the monetary authority must choose the appropriate backing of the domestic money supply with regards to the maintenance and sustainability of the peg. The policy of binding multiple and unremunerated reserve requirements is still in place.

With respect to the setting of the nominal exchange rate, we let $e > 0$ denote, without loss of generality, the time-invariant and exogenous nominal exchange rate set by the monetary authority at *t*=0. It is essential, under this policy, that the central bank must stand ready to buy or sell foreign currency at the fixed exchange rate, causing the domestic money supply to change as needed to meet this target. The latter implies that the nominal supply of domestic currency is endogenous: by fixing the nominal exchange rate, the monetary authority gives up full control of M_t, and we say that the monetary policy is subordinated[18]. The withdrawals/injection of domestic currency (*wons*), as in Section 2, are accomplished through the monetary lump-sum transfers in the amount of t_t units of *manna* (goods) to the *ex ante* identical young households. There are many reasons that may explain the choice of a fixed nominal exchange rate regime in spite of the lack of an independent monetary policy. Among them, we must mention that, under a very hard peg, the domestic country typically inherits the world inflation rate, contributing to a relatively quick stabilization and the reduction of domestic rates of inflation. This also helps promote the credibility of this policy, in general, and the monetary authority, in particular, among the general public.

The monetary authority may also choose to hold reserves in the form of interest-bearing, foreign-reserve assets. This rule takes the same form than in Section 2. However, the general purpose of this policy rule is somehow different under fixed exchange rates. These reserve-holdings aim to back the dollar-value of the domestic money supply but with the purpose instead of meeting the needs of the public who wants to buy or sell foreign currency at the fixed exchange rate e, so that speculative attacks on the domestic currency can be avoided or at least minimized. Thus, this rule also contributes to the sustainability of the very hard peg in place. We continue to use the same notation in as much as possible. Notice that, at the same time at period 0, the monetary authority sets both e and q. Next, we modify equation (8) accordingly, and obtain

$$B_t^* = q \cdot \left(\frac{M_t}{e} \right). \tag{20}$$

[18] The reader may recall "Tinbergen's principle": for each policy goal, one policy tool is needed, since a trade-off is always present, while obtaining two or more goals by using only one tool is generally unfeasible. This relates to the impossibility of fixing exchange rates <u>and</u> keeping full control of the domestic money supply under a hard peg.

As we mentioned before, a currency board arrangement obtains as a particular case, when the monetary authority sets $q = 1$ once and for all at $t = 0$. In what follows, we study the general case where $q \in [0,1]$.

The new regime of monetary and foreign exchange policy thus defined imposes different restrictions on the resources effectively available to the monetary authority. For instance, the nominal supply of domestic currency is endogenous in that, for all $t \geq 0$, it adjusts to keep the nominal exchange rate at the level e set by the government. In addition, a change in the foreign-reserve position of the monetary authority affects the resources available to the government, as it was the case with floating exchange rates. Of course, the monetary authority must account for these two sources of resources, resulting in the following budget constraint:

$$t_t = \frac{M_t - M_{t-1}}{p_t} - \frac{B_t^* - \tilde{r} \cdot \left(p_t^* / p_{t-1}^* \right) \cdot B_{t-1}^*}{p_t^*} = f_d \cdot (w + t_t) - \left(p_{t-1} / p_t \right) \cdot f_d \cdot (w + t_{t-1}) - \left(b_t^* - \tilde{r} \cdot b_{t-1}^* \right) \cdot \tag{21}$$

The first two terms on the right hand side of equation (21) represent the real value of any changes in the nominal supply needed to sustain the fixed nominal exchange rate. The third term indicates the effects of any changes in the real foreign-reserve position of the government.

3.2 General Equilibrium in a Fixed Exchange Rate Regime

In the remainder of this section, we use $\overline{x}_t$ to denote the value that the endogenous variable x_t takes under a hard peg. The general equilibrium under fixed exchange rates is also very complex. We try to shorten our presentation, and, thus, we continue to proceed by groups, while we substitute equations when necessary.

First, the restrictions on international transactions, equations (10a) and (10b) still hold: purchasing power parity and no arbitrage, respectively. Second, domestic and foreign currencies must be dominated in rate of return for the reserve requirements to be binding, implying that (11a) and (11b) still hold.

Third, regarding the market for *wons*, the condition (12a) still holds. However, the rate of return on real won balances changes significantly under a hard peg, and we substitute equation (12b) by the equation

$$\left(\overline{p}_t / \overline{p}_{t+1} \right) = \left(p_t^* / p_{t+1}^* \right) = \left(1 + s^* \right)^{-1}. \tag{22b}$$

This last equation implies that the domestic country inherits the world inflation rate, as we mentioned in the previous section. However, notice that s^* is not a parameter under the control of the monetary authority, reflecting the lack of control of the domestic money supply. Accordingly, the equilibrium laws of motion in equations (12c) and (12d) must be modified as well, and the following two equations obtain:

$$\overline{t}_t = h_1\left(s^*\right) + h_2\left(s^*\right) \cdot \overline{t}_{t-1}, \tag{22c}$$

$$\overline{z}_t = r_1\left(s^*\right) + r_2\left(s^*\right) \cdot \overline{t}_{t-1}, \tag{22d}$$

where the coefficients are $h_1\left(s^*\right) \equiv \left\langle f_d \cdot w \cdot \left\{ \left(1+s^*\right) \cdot \left[q \cdot (\tilde{r}-1)+1\right] - 1 \right\} / ?\left(s^*\right) \right\rangle$, $h_2\left(s^*\right) \equiv \left\{ f_d \cdot \left[q \cdot \tilde{r} \cdot \left(1+s^*\right) - 1 \right] / ?\left(s^*\right) \right\}$, $r_2\left(s^*\right) \equiv f_d \cdot h_2\left(s^*\right)$, $r_1\left(s^*\right) \equiv f_d \cdot \left[w + h_1\left(s^*\right) \right]$, and $?\left(s^*\right) \equiv \left(1+s^*\right) \cdot \left[1 - f_d \cdot (1-q)\right]$. Notice that equations (22c) and (22d) are first order linear difference equations in t_t. We must remark again that, under this hard peg, the dynamics of the system originates in t_t instead of z_t, as it was the case under floating exchange rates.

We now turn to the market for foreign currency in our fourth group. Obviously, the market clearing condition (13a) still holds, but we must modify the equilibrium laws of motion in equations (13b) and (13c) to represent the hard peg instead. Thus, the following two equilibrium laws of motion obtain:

$$\overline{q}_t = c_1\left(s^*\right) + c_2\left(s^*\right)\cdot \overline{\tau}_{t-1}, \tag{23b}$$

$$\overline{b}_t^* = y_1\left(s^*\right) + y_2\left(s^*\right)\cdot \overline{\tau}_{t-1}, \tag{23c}$$

where $c_1\left(s^*\right) \equiv f_f \cdot r_1\left(s^*\right) / f_d$, $c_2\left(s^*\right) \equiv f_f \cdot r_2\left(s^*\right) / f_d$, $y_1\left(s^*\right) \equiv q \cdot r_1\left(s^*\right)$ and $y_2\left(s^*\right) \equiv q \cdot r_2\left(s^*\right)$. Next, the rate of growth of the supply of foreign currency in circulation in the domestic economy $\left(Q_t\right)$ remains unchanged and governed by (13d), while now equation (13e) becomes

$$\left(\overline{e}_{t+1}/\overline{e}_t\right) = \left(e/e\right) = 1. \tag{23d}$$

The last group of equilibrium conditions in this new policy regime has to do with the deposit contract offered by banks. One, the self-selection constraint in (2) holds. Two, the constraints on foreign credit in (14a) continue to bind. Three, the equilibrium law of motion for the long-term investment is now given by

$$\overline{k}_{t+1} = V_1\left(s^*\right) + V_2\left(s^*\right)\cdot \overline{\tau}_{t-1}, \tag{24a}$$

where $V_1\left(s^*\right) \equiv f_0 + \left(1 - f_d - f_f\right)\cdot r_1\left(s^*\right) / f_d$ and $V_2\left(s^*\right) \equiv \left(1 - f_d - f_f\right)\cdot r_2\left(s^*\right) / f_d$. Four, the total return on domestic and foreign currency reserves under this policy regime are given, respectively, by the following two equations:

$$f_d \cdot \left(\overline{p}_t / \overline{p}_{t+1}\right)\cdot\left(w + \overline{\tau}_t\right) = m_1\left(s^*\right) + m_2\left(s^*\right)\cdot \overline{\tau}_{t-1}, \tag{24b}$$

$$f_f \cdot \left(p_t^* / p_{t+1}^*\right)\cdot\left(w + \overline{\tau}_t\right) = u_1\left(s^*\right) + u_2\left(s^*\right)\cdot \overline{\tau}_{t-1}, \tag{24c}$$

where the coefficients are $m_1\left(s^*\right) \equiv r_1\left(s^*\right) / \left(1 + s^*\right)$, $m_2\left(s^*\right) \equiv r_2\left(s^*\right) / \left(1 + s^*\right)$, $u_1\left(s^*\right) \equiv c_1\left(s^*\right) / \left(1 + s^*\right)$ and $u_2\left(s^*\right) \equiv c_2\left(s^*\right) / \left(1 + s^*\right)$. Five, the space-contingent commodities are governed by

$$l \cdot \overline{c}_{1,t,j} = f_1 - r_0^* \cdot f_0 + \left(r_0^* - 1\right)\cdot \overline{d}_{2,t+1,j} \tag{24d}$$

$$\left(1 - l\right)\cdot \overline{c}_{2,t+1,j} = w_1\left(s^*\right) + w_2\left(s^*\right)\cdot \overline{\tau}_{t-1} - r_1^* \cdot f_1 - \left(r_2^* - r_1^*\right)\cdot \overline{d}_{2,t+1,j}, \tag{24e}$$

where the parameters are $w_1\left(s^*\right) \equiv r_2^* \cdot V_1\left(s^*\right) + m_1\left(s^*\right) + u_1\left(s^*\right)$ and $w_2\left(s^*\right) \equiv r_2^* \cdot V_2\left(s^*\right) + m_2\left(s^*\right) + u_2\left(s^*\right)$. Six and last, the definition of the current account balance given in equation (14e) still holds.

We want to highlight two issues at this time: 1) all the reduced-form coefficients are a function of s^*, but s^* is not a policy parameter since it is determined in the rest of the world; 2) the structure of the general equilibrium system under fixed exchange rates shares the general structure of causality. In particular, as shown in Figure 5, all dynamics originates from $\overline{\tau}_{t-1}$, and we observe the same set of core variables $\left(\overline{\tau}_t, \overline{z}_t, \overline{q}_t, \overline{b}_t^*, \overline{k}_{t+1}\right)$ that determine the vectors $\left(\overline{d}_{0,t,j}, \overline{d}_{1,t+1,j}, \overline{d}_{2,t+1,j}\right)$ and $\left(\overline{c}_{1,t,j}, \overline{c}_{2,t+1,j}\right)$.

3.3 Multiplicity and Indeterminacy of Stationary Equilibria under a Hard Peg

In this section, we discuss the set of ***separating*** stationary equilibria that obtain from a fixed exchange regime. In these equilibria, all households behave according to their true type and there are no problems of liquidity. We will proceed by first characterizing the core in steady-state equilibria. Next, we characterize the stationary debt-structure vectors and the steady-state vectors, which define the stationary space-contingent commodities.

The Core in the Steady-State Equilibrium. The five variables that belong to the core, $\left(\overline{\iota}_t, \overline{z}_t, \overline{q}_t, \overline{b}_t^*, \overline{k}_{t+1}\right)$, are also determinate under a fixed exchange rate regime whenever an equilibrium exists, since they do no depend on the foreign interest rates $\left(r_0^*, r_1^*, r_2^*\right)$. We impose stationarity on equations (22c), (22d), (23b), (23c) and (24a), and obtain the steady-state values for the variables in the core that we display in the following five expressions:

$$\overline{\iota} = h_1\left(s^*\right) \Big/ \left[1 - h_2\left(s^*\right)\right] = \left\langle f_d \cdot w \cdot \left\{\left(1+s^*\right) \cdot \left[1 + q \cdot \left(\tilde{r} - 1\right)\right] - 1\right\} \Big/ ?\left(s^*\right)\right\rangle, \tag{25a}$$

$$\overline{z} = \left[f_d \cdot w \cdot \left(1+s^*\right) \Big/ ?\left(s^*\right)\right], \tag{25b}$$

$$\overline{q} = \left[f_f \cdot w \cdot \left(1+s^*\right) \Big/ ?\left(s^*\right)\right], \tag{25c}$$

$$\overline{b^*} = \left[q \cdot f_d \cdot w \cdot \left(1+s^*\right) \Big/ ?\left(s^*\right)\right], \tag{25d}$$

$$\overline{k} = f_0 + \left(1 - f_d - f_f\right) \cdot w + \left\langle f_d \cdot \left(1 - f_d - f_f\right) \cdot \left\{\left(1+s^*\right) \cdot \left[q \cdot \left(\tilde{r} - 1\right) + 1\right] - 1\right\} \Big/ ?\left(s^*\right)\right\rangle. \tag{25e}$$

Here, $?\left(s^*\right) \equiv \left(1+s^*\right) - f_d \cdot \left\{\left(1+s^*\right) \cdot \left[1 + q \cdot \left(\tilde{r}-1\right)\right] - 1\right\}$, where $\left(1+s^*\right) \cdot \left[1 + q \cdot \left(\tilde{r}-1\right)\right] > 1$ holds, $\forall s^* > -1$. Given the latter, we find it reasonable to restrict our attention to allocations where $\left(1+s^*\right) > f_d \cdot \left\{\left(1+s^*\right) \cdot \left[1 + q \cdot \left(\tilde{r}-1\right)\right] - 1\right\} > 0$ holds, $\forall s^* > -1$. It follows, from (25a,) that $\overline{\iota} > 0$, and $\left(\partial \overline{\iota} \big/ \partial s^*\right) > 0$ obtains.

Foreign Debt in a Stationary Equilibrium. It is also the case in this policy regime that the amount borrowed by banks from the rest of the world is constant and non-negative in a stationary equilibrium, regardless of the type of foreign debt instrument and provided that there is no arbitrage of the interest rates. The same three cases of equilibria still apply, and thus, the structure of foreign debt of a bank in a steady-state equilibrium that belongs to Case j is given by the triplet $\left(\overline{d}_{0,j}, \overline{d}_{1,j}, \overline{d}_{2,j}\right) > 0$.

As before, the stationary debt-structure vector permits us to calculate the current account balance in a stationary equilibrium that belongs to Case j:

$$\overline{CA_j} = \left(1 - r_0^*\right) \cdot \overline{d}_{0,j} + \left(1 - r_1^*\right) \cdot \overline{d}_{1,j} + \left(1 - r_2^*\right) \cdot \overline{d}_{2,j} < 0. \tag{27a}$$

Notice that, under this policy as well, there is always a long-run deficit in the current account balance, which troubles the concept of long-run sustainability of this regime.

Stationary equilibria under fixed exchange rates are defined by allocations such that $\left\{\left(\overline{\iota}, \overline{z}, \overline{q}, \overline{b^*}, \overline{k}\right), \left(\overline{d}_{0,j}, \overline{d}_{1,j}, \overline{d}_{2,j}\right), \left(\overline{c}_{1,j}, \overline{c}_{2,j}\right) \middle| \ \overline{l} = 0\right\} \in \mathbf{R}_{++}^5 \times \mathbf{R}_{+}^3 \times \mathbf{R}_{++}^2$, which satisfy all the conditions given above. Of course, the particular case of equilibrium that arises and its properties will depend on the composition of the vector $\left(\overline{d}_{0,j}, \overline{d}_{1,j}, \overline{d}_{2,j}\right)$, as we will see when checking for existence, uniqueness and determinacy.

Existence and Local Uniqueness under a Fixed Exchange Rate Regime. In our model economy with fixed exchange rates, the domestic and foreign inflation are always equal, even in non-stationary allocations. In particular, in situations where the world inflation rate is high (low,) banks would have no (strong) incentives to borrow long-term funds from abroad because inflation would undermine (boost) the real return of the currency reserves that banks must use when such debt is due. Moreover, different combinations of foreign interest rates and world inflation may produce a variety of situations that translate into different equilibrium sets, since both foreign interest rates and world inflation represent costs that banks must face. Notice that high rates of world inflation may exacerbate the effect of the foreign interest rates, and thus, we focus on the role that s^* plays in determining the existence of equilibria. The reader may notice some similarities in the proposition presented below with respect to existence under floating exchange rates, except for the fact the relevant parameter is s^* instead of s.

Proposition 5. Define the set $\mathcal{Q} = \left\{ s^*, r_0^*, r_1^*, r_2^* \right\} \in \mathbf{R}^4$ as the set of bifurcation parameters under a fixed exchange rate regime. Bifurcation values of these parameters partition $\mathcal{Q}$ into three subsets with defining characteristics that we describe below.

Subset 1= $\mathcal{Q}_1$ *:* Existence of Case 1 Equilibria. Given $\overline{e}$ as defined in the Appendix, equilibria of Case 1 exist only when the following condition holds:

Condition 1: $\overline{e} > 0$ must hold, and $\mathcal{Q}_1$ is smaller than $\mathcal{P}_1$, thus reducing the likeliness of borrowing large amounts long-term in equilibrium with respect to floating exchange rates.

Subset 2= $\mathcal{Q}_2$ *:* Existence of Case 2 Equilibria. This type of equilibrium always exists. Therefore, $\mathcal{Q}_2 = \mathcal{Q}$, and equilibria of Case 2 may coexist with equilibria of Cases 1 and 3.

Subset 3= $\mathcal{Q}_3$ *:* Existence of Case 3 Equilibria. Given the expressions $\overline{A}, \overline{B}$ and $\overline{C}$ as defined in the Appendix, equilibria of Case 3 exist when:

Condition 2: $\max\left\{ \overline{A}, \overline{B} \right\} < r_1^* \le \overline{C}$ holds.

This second model economy violates as well the two standard conditions of regularity. Regarding the number of equilibria, there is typically a continuum of equilibria in this economy. Moreover, the mapping between the vectors of relative prices and the excess demand *correspondence* is not unique, causing the steady-state equilibria in our economy to be *"irregular,"* and they are not locally unique or they are determinate.

Allow us to elaborate more on the last statement. The core in the steady-state, given by the quintet $\left(\overline{t}, \overline{z}, \overline{q}, \overline{b}^*, \overline{k} \right)$, is always a unique and determinate vector, since it is not associated with the relative-price vector $\left(r_0^*, r_1^*, r_2^* \right)$. The problem, though, concerns the debt-structure vector, affecting through it the vector of state-contingent commodities as well. To begin with, given a fixed combination of parameters, each stationary debt-structure vector $\left(\overline{d}_{0,j}, \overline{d}_{1,j}, \overline{d}_{2,j} \right)$ is typically associated with a continuum of vectors of interest rates $\left(r_0^*, r_1^*, r_2^* = R \right)$, which are all consistent with the equilibrium conditions. Second, for a given price vector $\left(r_0^*, r_1^*, r_2^* = R \right)$, there may be more than one vector $\left(\overline{d}_{0,j}, \overline{d}_{1,j}, \overline{d}_{2,j} \right)$ that satisfies the equilibrium conditions. Thus, the nature of the absence of local uniqueness and determinacy is very similar to what we observe with floating exchange rates, saving of course different particulars and details such as the different role played by the world inflation rate in both regimes. The existence of the steady-state equilibria under both floating and fixed exchange rate regimes is strongly conditioned on the domestic policy parameter s, for floating exchange rates, and the world inflation rate s^* for a fixed exchange rate regime. In general, when one compares such two economies, $s \neq s^*$ would be typically observed. The latter implies that, a priori, the scopes for existence and indeterminacy are different, to some extent, under alternative exchange rate regimes. However, these properties cannot be compared strictly without additional information, making the task of ranking the alternative regimes in this respect somewhat difficult.

We now turn to describe the debt-structure and state-contingent consumption vectors that obtain in each case.

Case 1: $j = 1$ ***and no intra-period debt*** $\overline{d}_0$**.** The debt-structure vector $\left(\overline{d}_{0,1}, \overline{d}_{1,1}, \overline{d}_{2,1} \right) = \left(0, f_1 - f_0, f_0 \right)$ obtains, and the value of consumption for impatient and patient agents transpires from

$$l \cdot \overline{c}_{1,1} = f_1 - f_0, \tag{27b}$$

$$(1 - l) \cdot \overline{c}_{2,1} = \Sigma_0 \left(s^* \right) - \left(R - r_1^* \right) \cdot f_0 + \Sigma_1 \left(s^* \right) \cdot \overline{t}, \tag{27c}$$

where $\Sigma_0(s^*) \equiv w_1(s^*) - r_1^* \cdot f_1$ and $\Sigma_1(s^*) \equiv w_2(s^*)$. The steady-state expected utility in this case follows

$$\overline{U}_1 = l \cdot \ln(\overline{c}_{1,1}) + (1-l) \cdot \ln(\overline{c}_{2,1}) \quad (27d)$$

Case 2: $j=2$ ***and interior solution.*** In an interior solution, we observe the foreign-debt-structure vector $(\overline{d}_{0,2}, \overline{d}_{1,2}, \overline{d}_{2,2}) = (f_0 - \overline{d}_{2,2}, f_1 - \overline{d}_{2,2}, \overline{d}_{2,2}) >> 0$. Specifically, the foreign long-term debt in a stationary equilibrium of Case 2 with a hard peg is given by

$$\overline{d_{2,2}} = \Omega_0(s^*) + \Omega_1(s^*) \cdot \overline{t} \quad , \quad (27e)$$

where $\Omega_0(s^*) \equiv \{ l \cdot (1+s^*) r_2^* \cdot V_1(s^*) + l \cdot (f_d + f_f)[h_1(s^*) - w] - r_1^* \cdot (1+s^*) \cdot (r_0^* - r_1^*)\} / [r_1^* \cdot (1+s^*) \cdot (r_0^* - r_1^*)]$, and $\Omega_1(s^*) \equiv l \cdot [(1+s^*) \cdot r_2^* \cdot V_2(s^*) + (f_d + f_f) \cdot h_2(s^*)] / [r_1^* \cdot (1+s^*) \cdot (r_0^* - r_1^*)]$. The vector of state-contingent consumption and the expected utility obtain from

$$l \cdot \overline{c}_{1,2} = f_1 - r_0^* \cdot f_0 + (r_0^* - 1) \cdot \Omega_0(s^*) + (r_0^* - 1) \cdot \Omega_1(s^*) \cdot \overline{t} \,, \quad (27f)$$

$$(1-l) \cdot \overline{c}_{2,2} = \Sigma_0(s^*) - (R - r_1^*) \cdot \Omega_0(s^*) + [\Sigma_1(s^*) - (R - r_1^*) \cdot \Omega_1(s^*)] \cdot \overline{t} \,, \quad (27g)$$

$$\overline{U}_2 = l \cdot \ln(\overline{c}_{1,2}) + (1-l) \cdot \ln(\overline{c}_{2,2}) \,. \quad (27h)$$

Case 3: $j=3$ ***and no long-term debt*** $\overline{d}_2$***.*** This case gives rise to the vector $(\overline{d}_{0,3}, \overline{d}_{1,3}, \overline{d}_{2,3}) = (f_0, f_1 - f_0, 0)$ and the state contingent consumption

$$l \cdot \overline{c}_{1,3} = f_1 - r_0^* \cdot f_0 \,, \quad (27i)$$

$$(1-l) \cdot \overline{c}_{2,3} = \Sigma_0(s^*) + \Sigma_1(s^*) \cdot \overline{t} \,. \quad (27j)$$

Of course, the steady-state expected utility in this case is given by

$$\overline{U}_3 = l \cdot \ln(\overline{c}_{1,3}) + (1-l) \cdot \ln(\overline{c}_{2,3}) \,. \quad (27k)$$

Steady-State Social Welfare. Under a hard peg, we observe that the steady-state consumption of impatient and patient households follow the same rankings than under floating exchange rates: $\overline{c}_{1,3} < \overline{c}_{1,1} < \overline{c}_{1,2}$ and $\overline{c}_{2,2} < \overline{c}_{2,1} < \overline{c}_{2,3}$, respectively. Interestingly, the ranking of expected utility changes significantly relative to the one observed under floating: $\overline{U}_2 > \overline{U}_3 \geq \overline{U}_1$ arises instead of $\hat{U}_1 > \hat{U}_3 > \hat{U}_2$, as shown in Figure 4. Moreover, consumption and expected utility in all cases prove to be monotonically decreasing in the inflation rate s^*. Below we present the effects of the different policy and structural parameters on expected utility.

The number of impatient households. Enlarging the population of impatient households, for $l < 0.5$, augments welfare in all cases. However, further increases of l reduce expected utility in Cases 1 and 3, while equilibria of Case 2 cease to exist. In most cases, the welfare ordering is preserved. However, as $l \to 1$, $\overline{U}_1 > \overline{U}_3$ arises instead.

The reserve requirements. The effects of the reserve requirements on social welfare are fairly complex under a hard peg. Increasing the reserve requirements, provided that $f_d = f_f$, augment welfare for most negative values of s^* but shrink expected utility when s^* is sufficiently large. In all cases, the welfare ranking is preserved.

Augmenting f_d alone leads to higher utility in all cases when the world inflation rate is sufficiently low. There is a middle area where utility in all cases decreases, to later increase when s^* is sufficiently large, converging to the same value. Thus, the shape of the expected utility changes with f_d, becoming decreasing for low values of

s^* but increasing with higher values of s^*, converging asymptotically. As $f_d \rightarrow 0$, the welfare ranking changes dramatically to $\overline{U}_3 > \overline{U}_2 > \overline{U}_1$, but it remains unchanged for other values of this policy parameter.

Increasing the foreign currency reserves of banks adds to welfare for low values of s^* in all cases, but lessens it significantly as s^* augments further. The welfare ranking remains unchanged. The intuition behind these changes seems to be that, when the rate of return on currency is high, holding more of it is welfare improving.

The backing of the domestic money supply. Adding to the backing of the money supply (q) increases welfare in all cases, but the magnitude of these changes is very small and the welfare ranking remains unchanged.

3.4 Discussion: Social Welfare and Alternative Exchange Rate Regimes

It is evident that $\overline{c}_{1,1} = \hat{c}_{1,1}$ and $\overline{c}_{1,3} = \hat{c}_{1,3}$ obtains. However, the comparison of $\overline{c}_{1,2}$ versus $\hat{c}_{1,2}$ and that of $\overline{c}_{2,j}$ against $\hat{c}_{2,j}$ are exponentially complex, and we had to resort to the use of simulations that spanned across a vast number of scenarios to make our point. The same applies to expected utility. To make both regimes as comparable as possible, we set $s = s^*$ in all scenarios.

We found that $\hat{U}_1 > \overline{U}_1$, $\hat{U}_2 > \overline{U}_2$ and $\hat{U}_3 > \overline{U}_3$ obtains unambiguously. Thus, a floating exchange rate regime Pareto dominates a fixed exchange regime in all stationary allocations where $s = s^*$ holds, and a policy of floating is clearly Pareto superior to its counterpart with a hard peg in this respect.

3.5 Dynamic Equilibria under a Fixed Exchange Rate Regime

Under a fixed exchange rate regime, the dynamics originates from the monetary authority's budget constraint, and the fact that both the money supply and the holdings of foreign-reserve assets must adjust to keep the nominal exchange rate at its set level, $e > 0$. Next, we proceed by steps: we start with the core in dynamic equilibria, followed by foreign-debt dynamics, and we conclude with the dynamics of the state-contingent commodities. The structure of causality under a hard peg is shown in Figure 5.

The Core Dynamic System under a Hard Peg. The core dynamic system consists of the first order difference equations (22c), (22d), (23b), (23c) and (24a). Moreover, the dynamic system of the core is a de-coupled system where the dynamics is inherited from t_t (see Figure 4.) We proceed by describing the dynamic behavior of each of the core variables, both with respect to s^* and q. Let $i = 1,2,3,4,5$ index each of the core variables in $\left(\overline{t}_t, \overline{z}_t, \overline{q}_t, \overline{b}_t^*, \overline{k}_{t+1}\right)$, while $\left(?_1, ?_2, ?_3, ?_4, ?_5\right)$ denotes the vector of associated eigenvalues. Proposition 6 below describes the dynamic properties of the core system.

Proposition 6 The dynamic properties of the core system depend strongly upon the world inflation rate (s^*) and the fraction of the domestic supply (q) backed with foreign-reserve assets by the monetary authority.

i) <u>The World Inflation Rate</u>. For each core variable i, where $i = 1,2,3,4,5$, the eigenvalue $?_i(s^*, q)$ is a monotonically increasing function of s^*. In particular, $\forall i$, there exists a vector of the form $\left(\hat{s}_i^*, \breve{s}_i^*, \overline{s}_i^*\right)$, such that $-1 < \hat{s}_i^* < \breve{s}_i^* < \overline{s}_i^* < \infty$, $?_i\left(\hat{s}_i^*\right) = -1$, $?_i\left(\breve{s}_i^*, q\right) = 0$ and $?_i\left(\overline{s}_i^*, q\right) = 1$. Each entry in this vector is one of three bifurcation values that partition the support set of s^* into four regions with defining characteristics:

a) When $s^* \in \left(-1, \hat{s}_i^*\right)$, $?_i\left(s^*, q\right) < -1$ transpires, and unstable non-cyclical fluctuations are observed along dynamic paths around the stationary core.
b) For $s^* \in \left(\hat{s}_i^*, \breve{s}_i^*\right)$, $?_i\left(s^*, q\right) \in (-1, 0)$ obtains, resulting in damped oscillations around the stationary.
c) $?_i\left(s^*, q\right) \in (0,1)$ occurs when $s^* \in \left(\breve{s}_i^*, \overline{s}_i^*\right)$, causing stable monotonic dynamics around the steady-state core.

d) For values of the world inflation rate that are sufficiently high (i.e. when $s^* > \overline{s}_i^*$ holds), $\overline{?}_i\left(s^*\right)>1$ ensues and dynamic paths diverge monotonically from the core steady-state.

ii) Backing of the Money Supply. Interesting dynamics and bifurcations are also observed for the different values that $q \in [0,1]$ may take. Each eigenvalue $\overline{?}_i\left(s^*, q\right)$, for $i = 1,2,3,4,5$, is a monotonically increasing function of the policy parameter q. Specifically, for each variable indexed by i that belongs to the core system, there exists a vector $\left(\hat{q}_i, \breve{q}_i, \overline{q}_i\right)$ of bifurcation values of q such that $\hat{q}_i < \breve{q}_i < \overline{q}_i$, $\overline{?}_i\left(s^*, \hat{q}_i\right) = -1$, $\overline{?}_i\left(s^*, \breve{q}_i\right) = 0$ and $\overline{?}_i\left(s^*, \overline{q}_i\right) = 1$ hold. Four sets of situations may arise, and we describe them below.

a) When $\hat{q}_i < 0 < \overline{q}_i < 1$, $\overline{?}_i\left(s^*, q\right) < -1$ obtains, $\forall q \in [0,1]$, no unstable fluctuations are observed. Stable fluctuations arise, followed by monotonic stable dynamics and monotonic divergence as q increases.

b) In the case where $0 < \hat{q}_i < \overline{q}_i < 1$ arises, this rules out monotonic divergence. For $q \in \left[0, \hat{q}_i\right)$, unstable fluctuations govern the dynamics around the stationary core. As q increases, the oscillations gradually converge toward the steady-state core. Stable monotonic divergence ensues as q continues to increase.

c) When $\hat{q}_i < 0 < 1 < \overline{q}_i$ obtains, there is no diverging dynamics. Damped oscillations are observed along dynamic paths, giving place to stable monotonic dynamics as q increases.

d) For $0 < \hat{q}_i < \overline{q}_i < 1$, the full spectrum dynamics comes about. Exploding oscillations take place for $q \in \left[0, \hat{q}_i\right)$, giving place to damped oscillations when $q \in \left[\hat{q}_i, \breve{q}_i\right)$. As q continues to increase, stable monotonic dynamics ensue, followed by monotonic dynamics when $q > \overline{q}_i$.

The Dynamic System of the Debt-Structure under a Hard Peg. We now move to discuss the composition and dynamic properties of the foreign-debt vector in dynamic equilibria that result from a fixed exchange rate regime. Below, we start our discussion of each case.

Foreign-Debt Dynamics in Cases 1 and 3: the equilibrium debt-structure in either of these extreme cases is stationary, as it was also the case with floating. Interestingly, the debt-structure vectors under a hard peg are equal to their counterparts under floating. In particular, in Case 1, we observe that $\left(\overline{d}_{0,1}, \overline{d}_{1,1}, \overline{d}_{2,1}\right) = \left(\hat{d}_{0,1}, \hat{d}_{1,1}, \hat{d}_{2,1}\right) = \left(0, f_1 - f_0, f_0\right), \forall t \geq 1$, and there is a deficit in the current account. Concerning stationary equilibria in Case 3, instead, it transpires that $\left(\overline{d}_{0,3}, \overline{d}_{1,3}, \overline{d}_{2,3}\right) = \left(\hat{d}_{0,3}, \hat{d}_{1,3}, \hat{d}_{2,3}\right) = \left(f_0, f_1 - f_0, 0\right), \forall t \geq 1$, while there is also a deficit in the associated current account balance.

Foreign-Debt Dynamics in Case 2: the debt-structure vector displays non trivial dynamics. As in Section 2, we present only the results for the long-term foreign debt instrument. Thus, the dynamics of $\overline{d}_{2,t+1,2}$ is governed by

$$\overline{d}_{2,t+1,2} = \Omega_0\left(s^*\right) + \Omega_1\left(s^*\right) \cdot \overline{t}_{t-1}, \tag{28}$$

It is evident that (28) is a first order, linear difference equation in $\overline{t}_t$. The reader may notice that, so far, the dynamics under a fixed exchange rate is very different from what observed in the case of floating exchange rates.

Local Stability Analysis. The reduced-form coefficient $\Omega_1\left(s^*\right)$ is the eigenvalue associated with $\overline{d}_{2,t+1,2}$. It depends on the combination of parameters $\left(s^*, f_d, f_f, q, l\right)$ and $\left(\tilde{r}, r_0^*, r_1^*, r_2^*\right)$. We want to point out that the structural parameter l plays an important role in fixed-exchange-rate-dynamics, as opposed to the case with floating. Proposition 7 describes the *baseline sequence* as a function of the world inflation rate.

Proposition 7 Under a hard peg, the eigenvalue $\Omega_1\left(s^*\right)$ is a monotonically increasing function of the world inflation rate. The *baseline sequence* consists of a small range with unstable and very large non-cyclical fluctuations for values of s^* sufficiently close to -1, where equilibrium sequences vanish. As s^* increases, the eigenvalue comes to lie inside the unit circle, with damped oscillations around the steady-state that turn gradually into stable monotonic dynamics when s^* is sufficiently large.

We discuss the dynamic properties with respect to policy parameters and rates of return in the Appendix.

The Dynamic System of the State-Contingent Commodities under a Hard Peg. Finally, we turn to discuss the dynamic properties of the pair $\left(\overline{c}_{1,t,j}, \overline{c}_{2,t+1,j}\right) >> 0$ in equilibria with a fixed exchange rate regime in place. This dynamic system in equations (14c) and (24b) inherits its equilibrium laws of motion from the core system but mostly from the dynamic system of the foreign-debt structure. We start by discussing the dynamic properties of consumption by impatient households in each of the different cases of equilibria. Next, we do the same for the consumption by patient households.

Dynamics of the Consumption by Impatient Households. The consumption by impatient households is given by $l \cdot \overline{c}_{1,t,j} = f_1 - f_0 - \left(r_0^* - 1\right) \cdot f_0 + \left(r_0^* - 1\right) \cdot \overline{d}_{2,t+1,j}$. In particular, for each case, we obtain

$$l \cdot \overline{c}_{1,t,1} = f_1 - f_0, \text{ for } j = 1, \tag{29a}$$

$$l \cdot \overline{c}_{1,t,2} = f_1 - r_0^* \cdot f_0 + \left(r_0^* - 1\right) \cdot \overline{d}_{2,t+1,2}, \text{ for } j = 2, \tag{29b}$$

$$l \cdot \overline{c}_{1,t,3} = f_1 - r_0^* \cdot f_0, \text{ for } j = 3. \tag{29c}$$

The consumption by impatient individuals is always stationary in Cases 1 and 3 and identical to their counterparts under floating. However, in Case 2 the dynamic properties of $\overline{c}_{1,t,2}$ are inherited from $\overline{d}_{2,t+1,2}$, indicating non-trivial dynamics. Specifically, the reduced-form dynamic equation for $\overline{c}_{1,t,2}$ is given by

$$\overline{c}_{1,t,2} = \Sigma_0\left(s^*\right) - \left(r_2^* - r_1^*\right) \cdot f_0 + \Sigma_1 \cdot \overline{t}_{t-1}. \tag{30}$$

The *baseline sequence* for $\overline{c}_{1,t,2}$ consists of a very small range of unstable fluctuations that turn very fast into damped oscillations as s^* increases. For values of s^* that are sufficiently high, there is a very large range of stable monotonic dynamics, which dominates the general dynamics. The minimum eigenvalue is extremely large and negative, indicating the presence of fairly large and unstable fluctuations for values of s^* that are close enough to -1, and equilibrium sequences vanish.

Dynamics of the Consumption by Patient Households. The dynamic behavior of $\overline{c}_{2,t+1,j}$ is strongly influenced by $\overline{d}_{2,t+1,2}$, with some variations that depend upon the Case the particular equilibria belong to. This is apparent from the three first order linear difference equations below:

$$\left(1 - l\right) \cdot \overline{c}_{2,t+1,1} = \Sigma_0\left(s^*\right) - \left(r_2^* - r_1^*\right) \cdot f_0 + \Sigma_1\left(s^*\right) \cdot \overline{t}_{t-1}, \tag{31a}$$

$$\left(1 - l\right) \cdot \overline{c}_{2,t+1,2} = \Sigma_0\left(s^*\right) - \left(r_2^* - r_1^*\right) \cdot \Omega_0\left(s^*\right) + \left[\Sigma_1\left(s^*\right) - \left(r_2^* - r_1^*\right) \cdot \Omega_1\left(s^*\right)\right] \cdot \overline{t}_{t-1}, \tag{31b}$$

$$\left(1 - l\right) \cdot \overline{c}_{2,t+1,3} = \Sigma_0\left(s^*\right) + \Sigma_1\left(s^*\right) \cdot \overline{t}_{t-1}. \tag{31c}$$

From (31a) and (31c), it is straightforward that the eigenvalues of $\overline{c}_{2,t+1,1}$ and $\overline{c}_{2,t+1,3}$ are identical and, thus, these two variables share the same dynamic properties. All the eigenvalues in (31a)-(31c) share the same baseline sequence, with some minor variations. Proposition 8 summarizes our findings about the dynamic system of the state-contingent commodities.

Proposition 8 Let $m = 1, 2, 3, 4$ index the variables $\left(\overline{c}_{1,t,2}, \overline{c}_{2,t+1,1}, \overline{c}_{2,t+1,2}, \overline{c}_{2,t+1,3}\right) >> 0$ and the vector $\left(\overline{g}_1, \overline{g}_2, \overline{g}_3, \overline{g}_4\right)$ represent the associated vector of eigenvalues, where $\overline{g}_2 = \overline{g}_4$. All eigenvalues $\overline{g}_m\left(s^*\right)$ are monotonically increasing functions of the world inflation rate, and, thus, all these variables display similar baseline sequences. The variations lie on the bifurcation values and the size of the minimum eigenvalue. The latter indicates the maximum size of the unstable fluctuations. The baseline sequence starts with a very small scope for unstable fluctuation when s^* is very close to -1, where equilibrium sequences vanish. Next, a small range with stable fluctuations arises as s^* continues to increase, leading to a large and dominant scope for stable monotonic dynamics. The sizes of the unstable fluctuations are ranked as follows: $\min \overline{g}_1 < \min \overline{g}_2 = \min \overline{g}_4 < \min \overline{g}_3$. Thus, the consumption by patient households in Case 2 exhibits the largest unstable fluctuations, while the consumption by impatient households --also in Case 2, presents the smallest magnitude of diverging fluctuations. Finally, all the dynamic properties of $\overline{d}_{2,t+1,2}$ apply.

Summary of Properties along Equilibrium Dynamic Paths under a Hard Peg. Equilibrium dynamics under a hard peg is less complex than its counterpart with floating exchange rates. Moreover, the properties of the dynamic system are quite different. In the first place, higher steady-state inflation promotes stable monotonic dynamics, eliminating instability and fluctuations altogether. Second, high reserve requirements increase the scope for endogenously-arising volatility. Finally, adding to the backing of the domestic money supply reduces the scope for unstable fluctuations while increasing the scope for stable monotonic dynamics.

A combination of low reserve requirements with high backing of the domestic money supply seems to be a good policy recommendation for furthering economic stability, since unstable and stable endogenously arising volatility disappears, and monotonic convergence is observe along the dynamic paths.

The Trade-off between Steady-State Welfare and Stability under a Hard Peg. With a fixed exchange rate regime we observe a bit less of a trade-off than under floating, in the sense that augmenting the backing of the domestic money supply increases welfare and also promotes stable monotonic dynamics. However, adding to the world inflation rate promotes stable monotonic dynamics but at the cost of worsening expected utility. The same idea applies to the currency reserves held by banks: enlarging both increases endogenously-arising volatility, but it increases welfare when the world inflation rate is sufficiently high. One word of advice, though: the monetary authority has no control over s^*, but policy-making must clearly be subordinated to the particular value observed of this variable. The same discussion of policy-making applies to this policy regime as well.

3.6 Discussion: Equilibrium-Stability Properties under Alternative Exchange Rate Regimes.

We now summarize the results obtained along dynamic paths with a fixed exchange rate regime, and compare them against their counterpart under floating exchange rates. In the first place, all dynamic systems under a hard peg have first order difference equations, eliminating the possibility of cyclical fluctuations in the debt-structure vector that are typically associated with complex eigenvalues. Second, regarding the core, the full spectrum dynamics can be observed under fixed, while floating allowed only for monotonic dynamics; the latter implies that a fixed exchange rate regime promotes endogenously-arising volatility around the stationary core, while floating does not. There is a trade-off, however, vis-à-vis the foreign-debt structure and the state-contingent consumption: floating promotes higher order and very complicated dynamics that allow for nontrivial regions with complex eigenvalues in which cyclical and non-cyclical fluctuations are intertwined. In some cases, fluctuations can be significantly large and explosive, and the volatility may arise from a very large and unstable real part together with explosively-large and diverging amplitude of the cyclical component. A hard peg, instead, prevents the latter from occurring, and there is only a very small range for first order, simpler oscillatory dynamics. Finally, the policy recommendations vary drastically across regimes: i) high and binding reserve requirements promote and extend the stability of dynamic equilibria under floating, while they increase endogenously arising volatility under fixed; ii) the backing of the money supply is a de-stabilizing policy parameter under floating, but it promotes stability under fixed; iii) the policy recommendations are exact opposites; floating requires very high reserve requirements and a very low backing of the money supply but an economy with fixed exchange regime improves stability with a combination of very low reserve requirements and a very high backing of the money supply.

In the next section, we bring in an additional layer and trade-off to policy making by introducing the possibility of crises in our model economies.

4. Potential for Crises and the Vulnerability of Banks

In sections 2 and 3, we studied the case where there was only uncertainty on the households' type in our model economy, but one of two alternative exchange rate regimes was in place. We observed that domestic households would withdraw from banks according to their true types, and domestic banks would anticipate this perfectly (i.e., there were no bank runs in equilibrium.) Interestingly, not only banks would not liquidate early the long-term asset in equilibrium, but all the banks would be liquid and solvent. In particular, at the beginning of period *t*, the domestic banks would have chosen the state-contingent consumption vector $\left(c_{1,t}, c_{2,t+1}\right) >> 0$ and would have also formulated a plan that involved setting $l_t = 0$, the vector of core variables $\left(z_t, t_t, q_t, b_t^*, k_{t+1}\right)$ and the debt-structure vector $\left(d_{0,t}, d_{1,t+1}, d_{2,t+1}\right) \geq 0$. The reader may notice, however, that not all *ex ante* choices would have been made effective at this time: $c_{1,t}$ and $d_{1,t+1}$ would be made effective at the end of period t, according to the initial contingent plan, while $c_{2,t+1}$ would be effective only at the end of period $t+1$.

In this section, we focus instead on the case where an unanticipated shock will hit the economy late in the afternoon of period t, immediately after depositors (households) learn their type. This shock may take one of two forms: a shock to the depositors' beliefs (i.e. a bad dream) or a sudden stop of foreign capital. In some cases, banks will be allowed to re-optimize, deviating from their ex-ante contingent plan. In the remainder of this section, the notation $\tilde{x}$ will indicate the re-optimized value of the variable x.

4.1 Sources of Uncertainty

Extrinsic Uncertainty typically exacerbates the beliefs of the general public –the depositors in this case— without it being associated with any change in the fundamentals of the economy. This type of shock is frequently linked with outcomes of a self-fulfilling type: a prophecy which outcome becomes true through the beliefs and actions of the main economic actors. The literature frequently illustrates the properties of this type of shock by using the example of all of the young domestic depositors having a "*bad dream*" in which banks will close, but without any change to the fundamentals of the economy. Interestingly, there is a coordination problem in our model economy in which complementarity is present in the strategic interaction between the individual depositors: if individual households cannot observe the actions or types of others, they may panic if they believe that everyone in the economy has had the same bad dream. In other words, if an individual depositor believes that everybody else is going to run and banks have a sequential service constraint in place, she will run as well, and, in the aggregate, all depositors will run on banks[19]. In particular, whether depositors choose to run or not will be determined by the results they obtain from the sequential checking mechanism, which is shown in Figure 6. This mechanism is informationally efficient and it is defined as the algorithm followed by domestic depositors, in which they check several conditions related to the banks' positions. In the event of a shock, banks do not get a chance to re-optimize, since there is no change in the fundamentals of the economy, but all domestic depositors may run on the banks to withdraw their resources immediately --aiming at being the first in line-- and the financial system would have a fully blown self-fulfilling attack on banks in its hands. As we will see later in detail, there exists a particular set of circumstances under which this "*bad dream*" has the potential of greatly affecting the outcomes in the economy by leading to panic-equilibria with significantly lower welfare.

Intrinsic uncertainty is the second type of uncertainty that we explore in this paper. It has to do with an unanticipated change in one or more of the fundamentals of the economy after households have formulated their contingent plans and learned their true types. A sudden stop certainly belongs in this category, and we try to replicate it in this paper by an unanticipated and exogenous reduction of the amount of new credit available from the rest of the world ($f_1 > 0$.) Recall that the pair $\{f_0, f_1\}$ was set exogenously in the rest of the world at the beginning of period t, and that all contingent plans were made at that point. The two constraints on foreign credit were binding and banks had already acted on their choices of $d_{0,t}$ and $d_{2,t+1}$, but not on their choices of $d_{1,t+1}$ and l_t. Moreover, the deposit contract $\{c_{1,t}, c_{2,t+1}\}$ cannot be modified. When a sudden stop hits the economy, it abruptly reduces the resources available at the end of period t to f_1' goods, where $0 < f_0 < f_1' < f_1$ obtains. The relevant foreign-credit constraint now becomes

$$d_{2,t+1} + \tilde{d}_{1,t+1} = f_1' , \tag{32}$$

where $\tilde{d}_{1,t+1}$ denotes the re-optimized value of $d_{1,t+1}$, since the latter is not feasible after the shock. We will see later in this section that this intrinsic shock may reduce both liquidity and solvency in the financial sector[20], thus having the potential of triggering a domestic crisis that may take the form of generalized attacks on banks, bankruptcy and closure in the domestic financial system.

Once such a shock hits the economy, this event becomes public information and the availability of future resources changes irreversibly. Under this new set of circumstances, it may be in the depositors' best interest to withdraw from banks as much as possible immediately, and the presence of a sequential service constraint may only exacerbate this problem. In these circumstances, it may not be optimal for any agent –specially, households of the patient type-- to wait until the next period to withdraw from the banks, since they believe that banks could be facing bankruptcy and closure. Both banks and depositors will need to re-optimize to account for the change in circumstances, leading again to the Sequential Checking Mechanism. This algorithm consists of three steps. The

[19] The reader may also be interested in suspension of convertibility as a measure that could preclude a bank run from happening. However, we only focus in the simplest case here.

[20] One could also argue that unanticipated reductions in foreign credit may trigger a shock to the preferences of depositors. If such a shock induces a crisis of a self-fulfilling nature, this may only exacerbate the existing problems in this economy. In this paper, for simplicity, we abstract from this possibility.

first step is an evaluation of the liquidity position of banks. Next, evaluating the banks' solvency becomes the next priority, followed by checking whether the resulting allocations are incentive-compatible or not.

4.2 The Sequential Checking Mechanism

Checking Liquidity. Chang and Velasco (2000a, 2000b and 2001) were among the first in the literature to mention the need to evaluate the liquidity position of banks in the context of financial crises in the emerging markets, and we follow them in this paper. The illiquidity condition summarizes the current situation of banks after a shock hits the domestic economy at the end of period *t*, assuming that a bank cannot commit *not* to liquidate fully the long-term investment if needed. We now describe this condition.

On the one hand, if all the depositors decided to withdraw their deposits, the real value of the bank's short-term obligations due at the end of period t would be $c_{1,t}+r_0^*\cdot d_{0,t}$ goods. On the other hand, if the bank were to liquidate early the full amount of its long-term investment at the end of period *t*, $\tilde{l}_t=k_{t+1}$ would obtain, with a return of $r\cdot\tilde{l}_t=r\cdot k_{t+1}$ goods from this transaction. Moreover, the illiquidity condition assumes that no additional funds are available at this point in time (i.e. no bailing out and $\tilde{d}_{1,t+1}=0$.) In summary, one would say that the representative bank has an illiquid position when the real value of its short-term obligations at the end of period *t* exceeds the liquidation value of the long-term investment, or equivalently, when the following inequality holds:

$$c_{1,t}+r_0^*\cdot d_{0,t}>r\cdot k_{t+1}. \tag{33}$$

Of course, when the inequality in (33) does not obtain, we say that the bank has a liquid position.

Checking Solvency. One of our main points is that an illiquid position is a necessary but not sufficient condition for a bank-run equilibrium to obtain in our model. The banks' illiquidity may only be a temporary matter caused by the shock that could be solved if foreign lenders would provide them with a provisional bail-out in the amount of $0<\tilde{d}_{1,t+1}\leq f_1'$ goods. The following inequality describes the condition for the insolvency of a representative bank at the end of period *t*:

$$c_{1,t}+r_0^*\cdot d_{0,t}>r\cdot k_{t+1}+\tilde{d}_{1,t+1} \tag{34}$$

The inequality in (34) means that if the real value of the new short-term foreign debt $\tilde{d}_{1,t+1}$ (i.e., the bail out) is enough to alleviate the temporary problem of liquidity, then it would be in the best interest of the foreign creditors to help this bank out instead of forcing it to close, so that they can still recover at least the amount $d_{2,t+1}$ that they lent long-term to domestic banks at the beginning of period t.The rule followed by foreign creditors would be to bail out solvent banks, but let the insolvent ones close. We can summarize this idea with the following saying: *"why throw away good money after bad money?"*

Checking Incentive Compatibility. In a situation where banks are illiquid but solvent, it is still possible that a fraction of the patient households may still have incentives to misrepresent their type and withdraw early, leading to panics and closure. Thus, we incorporate a last step in this algorithm, by which individual depositors check whether the Incentive Compatibility constraint in (2) is satisfied or not.

We now summarize the actions that individual depositors and foreign creditors may take, depending on the results they obtain in the steps of this algorithm. First, in a case where (33) does not obtain, banks have liquid positions and no further checking is needed: domestic depositors will choose not run on banks. Second, in cases where (33) and (34) hold, banks are illiquid and insolvent: foreign creditors will not bail banks out[21] and domestic depositors will run on banks. Third, when (33) and (2) hold, but (34) is not satisfied, it is in the foreign creditors' best interest to bail banks out, and depositors choose not to run on banks. Finally, when (33) holds but (34) and (2) do not, foreign creditors will not bail banks out, and domestic depositors will find it in their best interest to run on banks.

4.3 Type of Equilibria and Re-optimization

After a shock hits our model economy, banks may need to formulate a new plan. In the case of extrinsic uncertainty, then $\left(\tilde{d}_{1,t+1},\tilde{l}_t\right)=\left(d_{1,t+1},l_t\right)$, since no fundamentals have changed. However, in the case of intrinsic uncertainty, $\left(\tilde{d}_{1,t+1},\tilde{l}_t\right)\neq\left(d_{1,t+1},l_t\right)$, and one would typically expect that $\tilde{d}_{1,t+1}<d_{1,t+1}$ and $\tilde{l}_t>l_t=0$. Following the simple logic of our sequential checking mechanism, we were able to differentiate four different sets of possible equilibrium outcomes:

a) Equilibria of Type 1: This equilibrium transpires when (33) does not hold. Liquidity implies solvency, and (2) must hold as well. Thus, banks have a liquid and solvent position, the allocation is incentive compatible, and there are no panics in equilibrium. In addition, there is no need for bail out. We will call this outcome a *separating non-panic equilibrium with liquid banks.*
b) Equilibria of Type 2: This equilibrium comes to pass when (33) and (2) hold but (34) does not. Banks have an illiquid position, but they are solvent and incentive-compatible. Foreign creditors choose to bail out domestic banks and, subsequently, depositors choose not to run on banks. Thus, no panics occur. This outcome is a *separating non-panic equilibrium with illiquid banks*.
c) Equilibria of Type 3: This equilibrium emerges when (33) is satisfied but (34) and (2) are not. Banks have an illiquid and solvent position, but the latter is not incentive-compatible. Foreign creditors choose not to bail out banks and depositors choose to run on banks, and banks must close. Thus, this equilibrium will display panics, and it will be called a *pooling equilibrium with panics but solvent banks.*
d) Equilibria of Type 4: This outcome occurs when (33) and (34) both hold and (2) does not. Banks have an illiquid and insolvent position. Foreign creditors choose not to bail out banks, and domestic depositors, finding their initial beliefs verified, choose to run on banks. Thus, this equilibrium will display panics, and we call it a *pooling equilibrium with panics, illiquid and insolvent banks.*

In summary, equilibria of Types 1 and 2 are good separating equilibria where depositors behave according to their true type. Panics do not occur in good separating equilibria, since these allocations are always incentive compatible. However, equilibria of Types 3 and 4 are pooling equilibria in which foreign creditors do not bail out banks and domestic depositors choose to misrepresent their types and run on banks. Obviously, different levels of social welfare will be attached to each type of equilibria, and social welfare will be positively related with the amount of resources that are available to banks at the time when shocks are realized. Proposition 9 below discusses this issue.

[21] This may sound familiar to the reader: as of September of 2008, the Federal authority has decided to evaluate whether a similar criterion must be used on investment banks, given the recent events subsequent to the subprime mortgage crisis. However, the origins of the aforementioned crisis are different from a sudden stop.

Proposition 9 The following ranking of social welfare obtains:

i. Equilibria of Type 1 Pareto-dominate equilibria of Type 2, Type 3 and Type 4.
ii. Equilibria of Type 2 are Pareto superior to equilibria of Type 3.
iii. Equilibria of Type 3 Pareto-dominate equilibria of Type 4.

A Sudden Stop and the Re-Optimization Problem At the beginning of period t, banks choose their plan of action based upon, among other things, the pair $\{f_0, f_1\}$. Banks formulate their plans by maximizing the *ex-ante* expected utility of a representative individual, subject to the relevant constraints. Before the end of period t, but after all individual depositors learn their true type (impatient or patient,) the banks learn the realization of $f_1' \in (f_0, f_1)$. This realization constitutes an irreversible reduction in the future resources available to banks that merits a revision of plans. Under the new credit constraint (32,) it may be possible that some decision rules that were optimal *ex ante* are not optimal anymore. However, at this point in time, only two choice variables remain elastic to the banks: $d_{1,t+1}$ and l_t, since $d_{0,t}, d_{2,t+1}, k_{t+1}$ and the reserve-holdings were chosen and made effective at the beginning of t. Interestingly, the banks choose to deviate from their original plans, and they do so by finding a new combination $\left(\tilde{d}_{1,t+1}, \tilde{l}_t\right)$ that is optimal under the new circumstances.

After the sudden stop hits the domestic financial sector, this news is public, and everybody realizes that the amount of new borrowing from abroad is less than it was when they formulated their original plans. Fortunately, households can still try to behave optimally by sequentially checking the situation of the banks. Thus, the need to re-optimize and re-formulate their plans triggers a new sequential checking to try to determine the best course of action under the new circumstances, and equilibria obtain accordingly. The new borrowing constraint faced by banks at the end of period *t* implies that

$$\tilde{d}_{1,t+1} \leq f_1' - d_{2,t+1}. \tag{35}$$

It follows directly that $\tilde{d}_{1,1} < \tilde{d}_{1,2} < \tilde{d}_{1,3}$. The sequential checking mechanism now requires re-evaluating (33), (34) and (2) by taking into account (35) as well. Banks now try to maximize the expected utility in (1) by choosing only $\left(\tilde{d}_{1t+1}, \tilde{l}_t\right)$, subject to the new budget constraint in (35), the other relevant budget constraints and the exchange rate regime in place.

4.4 Early Liquidation and Existence of Equilibria after a Sudden Stop

Henceforth, we focus on stationary allocations where $f_1' > f_0$ obtains. We now describe the new equilibria that result after re-optimization and the sequential checking process. Before proceeding, we must point out that, at the time of the shock, the variables d_0, d_2 and *k* are already effective. Similarly, the pair (c_1, c_2) is already determined by the deposit contract. Thus, d_0, d_2, c_1, c_2 cannot be changed in the re-optimization process. The only actions left to banks are, thus, to change d_1 to $\tilde{d}_1$ and l from zero to $\tilde{l} > 0$. To proceed, we first set $\tilde{d}_1 = f_1' - d_2 < d_1$, and $\tilde{l} \neq 0$, and use it in equations (5) and (6) to solve for c_1 and c_2, respectively, as functions of $\tilde{l}$. Next, we impose equality in (2) and solve for $\tilde{l}$. Below we present the results for early liquidation after a sudden stop, under floating and fixed exchange rates, respectively.

$$\hat{\tilde{l}}_j = \frac{l\cdot(1-l)\left[r\cdot\left(r_0^*-1\right)\cdot f_0 - r_1^*\cdot f_1'\right]}{(1-l)\cdot r^2 + lR} + \left[\frac{l}{(1-l)\cdot r^2 + lR}\right]\cdot\left\{R\cdot\hat{k} + \left(w+\hat{t}\right)\left[\frac{f_d}{(1+s)} + \frac{f_f}{\left(1+s^*\right)}\right]\right\} - \left[\frac{l\cdot\left(R-r_0^*\right)+(1-l)\cdot\left(r_0^*-1\right)}{(1-l)\cdot r^2 + lR}\right]\cdot\hat{d}_{2,j}, \qquad (36a)$$

$$\bar{\tilde{l}}_j = \frac{l\cdot(1-l)\left[r\cdot\left(r_0^*-1\right)\cdot f_0 - r_1^*\cdot f_1'\right]}{(1-l)\cdot r^2 + lR} + \left[\frac{l}{(1-l)\cdot r^2 + lR}\right]\cdot\left\{R\cdot\hat{k} + \frac{\left(w+\hat{t}\right)\cdot\left(f_d+f_f\right)}{\left(1+s^*\right)}\right\} - \left[\frac{l\cdot\left(R-r_0^*\right)+(1-l)\cdot\left(r_0^*-1\right)}{(1-l)\cdot r^2 + lR}\right]\cdot\bar{d}_{2,j}. \qquad (36b)$$

We must point out a couple of interesting things from (36a-b). First, $\tilde{l}$ is monotonically decreasing in f_1', which ensures that a positive amount of early liquidation in equilibrium, after the economy is hit by a sudden stop. Second, $\tilde{l}$ is also a monotonically decreasing function of d_2, indicating that economies that borrow larger amounts long-term will experience smaller amounts of early liquidation of the long-term investment. It follows directly that $0<\tilde{l}_1<\tilde{l}_2<\tilde{l}_3$ holds.

When the sudden stop of foreign credit hits the economy, the anxious domestic depositors and foreign creditors start checking the bank's capacity of operation. Under illiquidity, the credit crunch among foreign creditors will directly impact the bank's solvency. We summarize our results in Table 5. In particular, in order to determine the type of equilibrium, we must find the appropriate regions in the space of foreign interest rates and monetary policy parameters where liquidity, solvency and self-selection intersect. As we pointed out before, equilibria of Type 4 yield the lowest social welfare possible –even worse than autarky, and they can be considered an outcome that the society as a whole will be willing to prevent. The latter obtains only when the banks are illiquid and lack solvency at the same time, and the bank will close and claim a bankruptcy. Otherwise, panic equilibria exist only when the outcomes are not incentive-compatible. In what follows, we proceed by first classifying the equilibria into the three possible cases that we have examined before, and then examine the scope for existence of non-panic and panic equilibria in each subset of the parameter space. Whether a particular type of equilibria obtains or not will depend on the amount of the sudden stop, i.e. f_1', and this results hold under floating and under fixed exchange rates (see Technical Appendix for details.)

Equilibria in Case 1. In Case 1, the debt-structure vector $\left(d_{0,1},\tilde{d}_{1,1},d_{2,1}\right)=\left(0,\, f_1'-f_0,\, f_0\right)$ obtains, and banks must liquidate early $\tilde{l}_1>0$ units of their long-term investment. Interestingly, as shown in Table 4, equilibria of Type 4 do not obtain in Case 1. For values of f_1 that are large enough, the best non-panic equilibria (Type 1) may still obtain. However, the economy is not free of panic equilibria: equilibria of Type 3 will be observed for values of f_1 that sufficiently low.

Equilibria in Case 2. In this case, there is an interior solution for long-term debt, and the vector $\left(d_{0,2},\tilde{d}_{1,2},d_{2,2}\right)=\left(f_0-d_{2,2},\, f_1'-d_{2,2},\, d_{2,2}\right)$ transpires. In addition, banks must liquidate early the amount of

$\tilde{l}_2 > \tilde{l}_1 > 0$. In this particular subset of the parameter space, equilibria of Type 1 and of Type 4 do not exist and so the economy will not experience the best non-panic equilibria but neither the worst panic equilibria.

Equilibria in Case 3. In this case, there is no long-term debt. We find that $(d_{0,3}, \tilde{d}_{1,3}, d_{2,3}) = (f_0, f_1^{'}, 0)$ and banks must liquidate early amount of $\tilde{l}_3$ units of their long-term investment. Only equilibria of Type 1 and Type 3 exist in this case. Thus, the best non-panic equilibria can be achieved for values of f_1 that are sufficiently large.

Discussion. In Case 1, banks borrow a relatively large amount of foreign long-term debt $(d_2 = f_0)$ at the beginning of period t, restricting the bank's fund-raising ability in the future: after the sudden stop of foreign credit, the access to the new short-term funds becomes even scarcer. Nevertheless, at period *t*, $d_0 = 0$ obtains and there are no foreign obligations to repay by the end of this period; the only liability the bank has to deal with is the withdrawals from the impatient depositors $(l \cdot c_1)$ if there is no panic in the economy. Given $d_{2,1} = f_0$, banks find it difficult to finance their obligations through short-term borrowing $(\tilde{d}_{1,1} = f_1' - f_0)$, and illiquidity will force the bank to liquidate part of its high-earning investment to fulfill the need of depositor's withdrawals. However, because $d_{0,1} = 0$, the early liquidation of the long-term investment need not be large and banks will wait instead for a higher return at the end of *t*+1. In Case 1, banks are never insolvent and equilibria of Type 4 do not obtain, but panics can still be observed if the realization of f_1 is low enough to violate incentive compatibility.

Regarding equilibria of Case 2, there is a scope for allocations that are not incentive compatible but banks are always illiquid and solvent, implying that equilibria of Type 1 and Type 4 do not obtain. Even though foreign lending could serve as a last resource for the illiquid bank, the depositors' beliefs may deteriorate and take the economy into a panic equilibrium. Thus, equilibria of Type 3 may exist since incentive compatibility is violated for particular values of f_1, the monetary policy parameter s and the world interest rates. Because $d_{0,2} > d_{0,1} = 0$, early liquidation in this case is larger than in Case 1.

Unlike Case 1, in Case 3 banks take out no long-term loans from abroad at period t. Banks are always solvent, but their positions may be liquid or illiquid. The latter entails a greater flexibility for banks regarding short-term borrowing if needed, and ensures that equilibria of Type 4 do not obtain. Banks with liquid positions produce allocations that are incentive compatible, giving rise to equilibria of Type 1. However, allocations generated by illiquid banks are never incentive comparable, leading to panics and equilibria of Type 3. It also implies that the total amount to be repaid by banks at period *t*+1 is relatively manageable. Because $d_{2,3} = 0$, if there is a shock that permanently reduces the sources of funds to the bank, this bank has the ability to gather sufficient amount of bail-out funds $(\tilde{d}_{1,2} = f_1')$ from foreign creditors, eliminating the likelihood of observing equilibria of Type 4.

To sum up, the economy may pivot from Type 1 (with the highest social welfare) to Type 3 (with the second to lowest social welfare) due to infinitesimal changes in f_1 and/or in policy parameters, illustrating the very fragile and highly volatile environment faced by the financial system, which could lead to panics and generalized bankruptcy. The good news is that the worst type of panic equilibria are ruled out under either exchange rate regimes. The bad news is that the best of non-panic equilibria cannot be obtained when $0 < d_2 < f_0$ holds. The recommendation to the banks in the small open economy seems to be to borrow either a lot or nothing long-term. Anywhere in between (Case 2) will eliminate the best type of equilibria.

4.5 Potential for Crises and the Role of Monetary Policy

Through out the paper, it has become evident that our model provides a suitable framework for analyzing the interaction among various types of monetary policies: fixed versus floating exchange rate regimes, controlling the domestic rate of money growth, regulations of multiple reserve requirements and the backing of the domestic money supply. We have also examined the policy trade-offs between steady-state welfare and stability. In this section, we add another layer to the analysis of monetary policy: how changes in policy parameters alter the ranges of existence for the different non panic and panic equilibria that we have discussed so far. Tables 5, 6 and 7 present our results for all the different cases and exchange rate regimes. We now proceed to the analysis for each policy regime.

Floating Exchange Rates. One important factor to recall is that, under floating, the rate of domestic money growth –which is also the steady-state inflation—is under the full control of the monetary authority, providing with an additional policy tool.

The Rate of Domestic Money Growth. We find that the rate of domestic money growth can have significant positive effects regarding the potential for crises present in this model economy. In particular, adding to s --within reason—increases the likelihood for equilibria of Type 1 in Cases 1 and 3, while reducing the scope for panic equilibria in all cases. On a different note, exogenous increases in the world inflation rate reduce the scope for non panic equilibria and increase the likelihood of observing panic equilibria.

The Reserve Requirements. Increasing either of the reserve requirements leads to some worrisome effects. First, it always reduces the scope for the equilibria with the highest welfare (Type 1.) Second, it increases the scope for equilibria of Type 2, which is not in itself a bad outcome. But, third, it augments the likelihood of observing the panic equilibria (Type3.)

The Backing of the Domestic Money Supply. Augmenting q always increases the scope for non-panic equilibria and reduces the likelihood of observing panics in equilibrium.

To sum up, the combination of policy parameters that maximizes the likelihood of equilibria of Type 1 and minimizes the odds of panic equilibria under floating seems to be one with a low to medium rate of domestic money growth, very low but positive reserve requirements and a high backing of the domestic money supply.

Fixed Exchange Rates. One of the advantages of pegging the domestic currency to the international currency is that the economy inherits the world inflation rate, which is usually smaller than the domestic inflation rates for economies that implement stabilization policies. While this reduces domestic inflation drastically, the monetary authority is giving up one tool of monetary policy: the economy is subordinated to the world inflation rate but it has no control over it.

The World Inflation Rate. A rise in the world inflation rate $\left(s^*\right)$ increases the scope for Equilibria of Type 1 in Cases 1 and 3, while it makes panic equilibria more likely in all cases. Regarding non panic equilibria with illiquid banks, the results are mixed: their probability is lower in Case 1, but higher in Case 2. The reader may notice that the effects on the economy are different from those obtained under floating, in particular, the widening of the scope for equilibria of Type 1.

The Reserve Requirements. The effects regarding equilibria of Types 1 and 3 are identical to the ones observed under floating. However, regarding equilibria of Type 2, their scope increases in Case 1, but it lessens in Case 3.

The Backing of the Domestic Money Supply. Same as with floating.

In a nutshell, the combination of policy parameters that maximizes the likelihood of equilibria of Type 1 and minimizes the odds of panic equilibria under a hard peg seems to be one with very low but positive reserve requirements and a high backing of the domestic money supply. A low to medium world inflation rate would also be desirable, but this is outside the control of the domestic monetary authority. See Table A.1 in the Appendix for the effects of the World Interest Rates.

4.6 Trade-offs between Welfare, Stability and Financial Fragility in the Presence of Alternative Exchange Rate Regimes

We present the summary of our results in Table 8. The reader may notice that each policy seems to produce mixed results in terms of welfare, unstable fluctuations and the scope for non panic and panic equilibria.

A propos the rate of domestic money growth under floating, the general recommendation seems to be for the monetary authority to choose low to medium-range values of s for which the benefits in terms of welfare and the scope for panic and non panic equilibria are obtained but unstable fluctuations are minimized. If, instead, there is a hard peg in place, the monetary authority must be aware that high world inflation rates reduce volatility, increases stability and augments the scope for equilibria of Type 1, but at the cost of reduced welfare, increased scope for panic equilibria and lower likelihood of observing equilibria of Type 2.

As regards the effects of the reserve requirements, trade-offs are also observed under both exchange rate regimes. If the goal is to maximize the scope for Type 1 equilibria and minimize panic equilibria, then the monetary authority must chose relatively low values for both reserve requirements under any of the exchange rate regimes studied, though such a policy reduces welfare under a hard peg. Finally, when it comes to the backing of the domestic money supply, higher values of q promote higher welfare and a very small potential for crisis-equilibria. However, under floating this also increases unstable fluctuations and perhaps a medium value of q will work better instead.

In brief, on the one hand, a combination of low but non zero reserve requirements and a medium-range backing of the money supply are welfare improving/preserving, maximize the likelihood of non panic equilibria and minimize the scope for panic equilibria under a floating exchange rate regime. On the other hand, a combination of low reserve requirements and a very high backing of the money supply will work best under a hard peg.

Then again, we must mention that the monetary authority that chooses a floating exchange rate regime has one additional tool to her disposal: choosing low to medium values of the rate of domestic money growth to further her goals. Of course, we are aware of the complexity of this issue, and further studies under this framework will be needed to provide with more exact predictions.

5. Conclusions

In this paper, we investigate whether particular combinations of monetary policy can help prevent financial crises that originate in illiquid positions by banks as well as promote high welfare and reduced unstable endogenously-arising volatility. We pay particular mind to a policy of multiple reserve requirements, and compare the advantages and disadvantages of alternative *de facto* exchange rate regimes in achieving economy stability.

In order to call attention to the questions that we are interested in, we built a dynamic, stochastic, general equilibrium model of a small, open economy that displays nontrivial demands for fiat currencies, unexpected sunspots and financial/banking crises originated by sudden stops of foreign capital inflows. We motivated the effective demands for domestic and foreign fiat money with a policy of banks holding reserves of these different currencies. These reserves partly prevent banks from financing domestic investment, but they may also provide banks with access to liquid resources in their time of need. Under some particular circumstances, reserve requirements may reduce the likelihood of observing equilibria with illiquid and insolvent banks.

In situations where no crises are present, we observe that the monetary rule in place determines to an important extent the existence and properties of equilibria. Typically, there is a continuum of stationary equilibria, and local uniqueness and determinacy are lacking. Three cases may arise that correspond to different debt-structure vectors. With respect to dynamic equilibria under floating exchange rates, there is a nontrivial scope for complex eigenvalues that contributes to both cyclical and non-cyclical fluctuations; in some cases, the fluctuations can be significantly large and explosive. The scope for stability --and indeterminacy-- is typically small, and the scope for determinacy typically dominates, but fluctuations are observed on the stable manifold. Moreover, unstable and oscillating divergence is observed in general. The reserve requirements play the role of stabilizing, at least partially, the dynamic equilibria in this model economy, while backing the domestic supply plays the opposite role. Thus, these results provide us with policy recommendations: to implement high and binding reserve requirements, but keeping the backing of the money supply to a minimum. In the extreme case of $q = 0$, the order of the dynamic system is reduced, which one can interpret as the ultimate stabilization of dynamic equilibria.

With respect to the properties of dynamic paths under a hard peg, we must point out following. In the first place, all dynamic systems under a hard peg have first order difference equations, eliminating the possibility of cyclical fluctuations in the debt-structure vector that are typically associated with complex eigenvalues. Second, regarding the core, the full spectrum dynamics can be observed under fixed, while floating allowed only for monotonic dynamics; the latter implies that a fixed exchange rate regime promotes endogenously-arising volatility around the stationary core, while floating does not. There is a trade-off, however, vis-à-vis the foreign-debt structure and state-contingent consumption: floating promotes higher order and very complicated dynamics that allow for nontrivial regions with complex eigenvalues in which cyclical and non-cyclical fluctuations are intertwined. In some cases, fluctuations can be significantly large and explosive, and the volatility may arise from a very large and unstable real part together with explosively-large and diverging amplitude of the cyclical component. A hard peg, instead, prevents the latter from occurring, and there is only a very small range for first order, simpler oscillating dynamics. Finally, the policy recommendations regarding dynamic properties vary drastically across regimes: i) high and binding reserve requirements promote and extend the stability of dynamic equilibria under floating, while the only preserve stability and prevent monotonic divergence under fixed; ii) the backing of the money supply is a de-stabilizing policy parameter under floating, but it promotes stability under fixed; iii) the policy recommendations are exact opposites; floating requires very high reserve requirements and a very low backing of the money supply but an economy with fixed exchange regime is better-off with a combination of very low reserve requirements and a very high backing of the money supply.

Next, we evaluated welfare under each exchange rate regime. We found that a policy of floating exchange rate regimes clearly and unambiguously dominates a hard peg in terms of steady-state expected utility in all cases and under all combinations of monetary policy, provided that $s = s^*$ for comparability. This is then a clear and very important advantage of floating versus fixed.

Last, we examined the potential for crises in the case of a sudden stop in a small, open economy that is a net borrower from the rest of the world. We show the existence of multiple equilibria of four types that we may rank based on the presence of binding information constraints and on social welfare. In particular, Type 1 equilibria are allocations where banks have liquid and solvent positions that are also incentive compatible, there are no panics and they yield the highest social welfare. Type 2 equilibria display no panics and banks have illiquid but solvent positions that are incentive compatible. Type 3 equilibria display panics since the banks positions are illiquid and solvent but not incentive compatible. The good news is that panic equilibria with illiquid and insolvent banks –Type 4 equilibria, which yield the lowest welfare—are not present in either of our model economies. Not surprisingly, we also find that the magnitude of bail out foreign credit (f_1) determines the type of equilibria present. Economies with a high bail out credit are more prone to display panic equilibria, since they will be more sensitive to a sudden stop. In this respect, the goal of the monetary authority would be to maximize the likelihood of Type 1 equilibria and to minimize the odds of Type 3 equilibria. Under floating, the policy combinations consistent with this goal display a high rate of domestic money growth, low but nonzero reserve requirements and a high backing of the domestic money supply. However, under a hard peg this goal is accomplished by low reserve requirements and low backing of the domestic money supply.

There is a clear trade-off in policy implementation when one regards the goals of high welfare, decreased scope for unstable fluctuations and the potential for crises together under both exchange rate regimes. Under floating, a high rate of domestic money growth and a high backing of the domestic money supply would increase welfare and would reduce the scope of panic equilibria. But there is a limit to this effect, since further increases of these two instruments may exacerbate unstable endogenously arising volatility. Under a hard peg, however, a high backing of the domestic money supply promotes high welfare, reduced unstable fluctuations, large scope for non panic equilibria and a small scope for panic equilibria. The only common point under both policies is the need for low but nonzero reserve requirements.

Overall, a policy of floating exchange rates generates higher steady-state expected utility but there is a high order dynamic system that promotes increased endogenously arising volatility that is not present under fixed exchange rates. Thus, floating promotes better steady-state properties while fixed displays better properties along equilibrium dynamic paths where equilibrium sequences do not vanish.

Table 1: Exchange Rate Regimes in the East-Asian Countries
Before the Crisis and After the Crisis

Country	Before/During the crisis	After the crisis
Japan	Free floating	Free floating
Philippines	Free floating	Free floating
China	Managed floating	Managed floating
Indonesia	Managed floating	Floating
Korea	Managed floating	Floating
Singapore	Managed floating	Managed floating
Thailand	Managed floating	Managed floating $\rightarrow$ floating
Malaysia	Managed floating	Fixed
Hong Kong	Fixed	Fixed

Source: Frankel et al (2002)

Table 2: The Different Foreign-Debt Instruments Available to Banks

Instrument	Term	Notation	Issued	Matures	World Interest Rate
Intra-period debt	Short	$d_{0,t}$	Early at *t*	End of *t*	r_0^*
Inter-period debt	Short	$d_{1,t+1}$	End of *t*	End of *t+1*	r_1^*
Long-term debt	Long	$d_{2,t+1}$	Early at *t*	End of t+1	r_2^*

Table 3: Existence of Steady-State Equilibria under Floating Exchange Rates, $\hat{l} = 0$

Case	Debt-structure	Existence	Scope for existence and s
Case 1	$d_2 = f_0$, $d_0 = 0$, $d_1 = f_1 - f_0$	1) $\hat{e} > 0$, or 2) $s < -(\hat{e} + l \cdot f_d \cdot w)/\hat{e}$	1) Increases with s when e>0 and $l \cdot w > r_1^* \cdot (f_1 - f_0)$ holds.
Case 2	$0 < d_2 < f_0$, $d_0 = f_0 - d_2$, $d_1 = f_1 - d_2$	Equilibria always exist	Unchanged with changes in s
Case 3	$d_2 = 0$, $d_0 = f_0$, $d_1 = f_1$	$\max\{A, B\} < r_1^* \le C$ must hold	1) When $B < A < r_1^* \le C$ holds, it increases with s. 2) When $A < B < r_1^* \le C$ holds, it increases with s only when $2 \cdot l \cdot f_1 < (f_1 - r_0^* \cdot f_0) \cdot \hat{\Omega}$ holds.

$$\hat{\Omega} \equiv \left| 1 - [(1-l)\cdot r \cdot f_1 - l \cdot R \cdot f_0 - \varpi\ ] \cdot \left\{ \sqrt{[(1-l)\cdot r \cdot f_1 - l \cdot R \cdot f_0 - \quad]^2 + 4 \cdot l \cdot (1-l) \cdot r \cdot R \cdot f\ \cdot_1\ f_0} \right\}^{-1} \right| .$$

Table 4: Dynamic Properties of the Real Balances of *wons* z_t - Eigenvalue -Floating Exchange Rates

q	f_d	s	$a_2(s)$
0	(0,1)	>-1	0
(0,1)	0	>-1	0
(0,1)	(0,1)	$< \tilde{s}$	<1
$< \hat{q}$	$< \hat{f}_d$	$\ge \tilde{s}$	<1
$\ge \hat{q}$	$\ge \hat{f}_d$	$\ge \tilde{s}$	≥ 1
(0,1)	1	$< \tilde{s}$	<1
$< \hat{q}$	1	$\ge \tilde{s}$	<1
$\ge \hat{q}$	1	$\ge \tilde{s}$	≥ 1
(0,1)	$< k$	∞	<1
$< \hat{q}$	$\ge k$	∞	<1
$\ge \hat{q}$	$\ge k$	∞	≥ 1
(0,1)	1	∞	≥ 1
1	1	∞	∞

Note: $\hat{q} \equiv [(\tilde{r}-1)\cdot(1+s)]^{-1}$, $\hat{f}_d \equiv (1+s)\cdot\{q\cdot(\tilde{r}-1) + s \cdot [q \cdot (\tilde{r}-1)+1]\}^{-1}$, $\tilde{s} \equiv (2-\tilde{r})/(\tilde{r}-1)$ *and* $k = (1/\tilde{r})$.

Table 5: Existence of Steady-State Equilibria after a Sudden Stop
Floating and Fixed Exchange Rates

Case	Condition	Type 1	Type 2	Type 3	Type 4
Case 1	$B_1<A_1<C_1$	May exist	May exist	May exist	Does not exist
Case 2	$A_2<B_2<C_2$	Does not exist	May exist	May exist	Does not exist
Case 3	$B_3<C_3<A_3$	May exist	Does not exist	May exist	Does not exist

Table 6: Summary of Policy Effects on the Scope for Existence of
Equilibria after a Sudden Stop in Case 1

Policy	Type 1	Type 2	Type 3	Type 4
Floating:				
?s	Widens	Narrows	Narrows	n.a.
?s*	--	Narrows	Widens	n.a.
?ϕ_d	Narrows	Widens	Widens	n.a.
?ϕ_f	Narrows	Widens	Widens	n.a.
?θ	Widens	Widens	Narrows	
Fixed:				
?s*	Widens	Narrows	Widens	n.a.
?ϕ_d	Narrows	Widens	Widens	n.a.
?ϕ_f	Narrows	Widens	Widens	n.a.
?θ	Widens	Widens	Narrows	n.a.

Table 7: Summary of Policy Effects on the Scope for Existence of Equilibria after a Sudden Stop in Case 2

Policy	Type 1	Type 2	Type 3	Type 4
Floating:				
?s	n.a.	Narrows	Narrows	n.a.
?s*	n.a.	Narrows	Widens	n.a.
?ϕ_d	n.a.	Widens	Widens	n.a.
?ϕ_f	n.a.	Widens	Widens	n.a.
?θ	n.a.	Widens	Narrows	n.a.
Fixed:				
?s*	n.a.	Widens	Widens	n.a.
?ϕ_d	n.a.	Narrows	Widens	n.a.
?ϕ_f	n.a.	Narrows	Widens	n.a.
?θ	n.a.	Widens	Narrows	n.a.

Table 8: Summary of Policy Effects on the Scope for Existence of Equilibria after a Sudden Stop in Case 3

Policy	Type 1	Type 2	Type 3	Type 4
Floating:				
?s	Widens	n.a.	Narrows	n.a.
?s*	Narrows	n.a.	--	n.a.
?ϕ_d	Narrows	n.a.	Widens	n.a.
?ϕ_f	Narrows	n.a.	Widens	n.a.
?θ	Widens	n.a.	Narrows	n.a.
Fixed:				
?s*	Widens	n.a.	Widens	n.a.
?ϕ_d	Narrows	n.a.	Widens	n.a.
?ϕ_f	Narrows	n.a.	Widens	n.a.
?θ	Widens	n.a.	Narrows	n.a.

Table 9: Summary of Policy Trade-offs

				Scope for Existence		
Policy	**Welfare**	**Volatility**	**Instability**	**Type 1**	**Type 2**	**Type 3**
Floating:						
?s	Increases	Increases	Increases	Widens	Narrows	Narrows
?ϕ_d,?ϕ_f	Decreases	--	Decreases	Narrows	Widens	Widens
?θ	Increases	Increases	Increases	Widens	Widens	Narrows
Fixed:						
?s*	Decreases	Decreases	Decreases	Widens	Narrows	Widens
?ϕ_d,?ϕ_f	Increases	Increases	--	Narrows	Widens	Widens
?θ	Increases	Decreases	Decreases	Widens	Widens	Narrows

Figure 1: Case 1 - Continuum of Equilibria

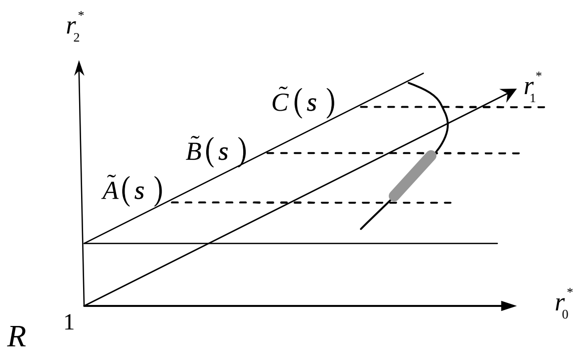


Figure 2: Steady-State Expected Utility, Floating Exchange Rates, Baseline Scenario

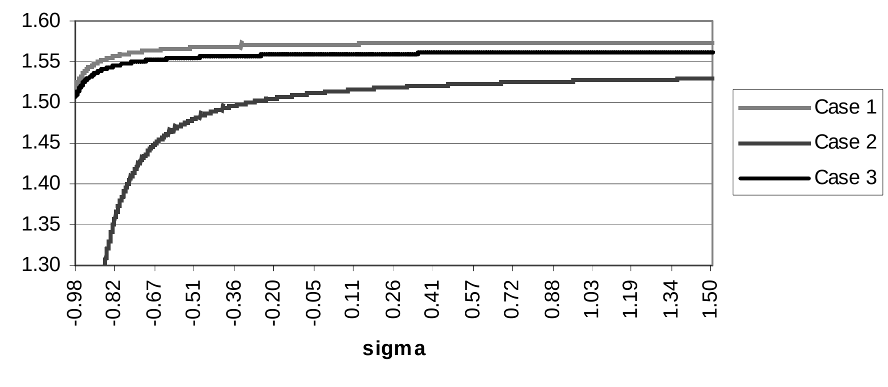


Figure 3: The Structure of the Dynamic System
Floating Exchange Rates

$$\hat{z}_{t-1} \to \underbrace{\hat{z}_t \to \left(\hat{t}_t, \hat{q}_t, \hat{b}_t^*, \hat{k}_{t+1}\right)}_{\text{The Core}} \to \underbrace{\left(\hat{d}_{0,t,j}, \hat{d}_{1,t+1,j}, \hat{d}_{2,t+1,j}\right) \to \left(\hat{c}_{1,t,j}, \hat{c}_{2,t+1,j}\right)}_{\text{Case } j\text{, for } j=1,2,3}$$

t-1

Figure 4: Steady-State Expected Utility, Fixed Exchange Rates, Baseline Scenario

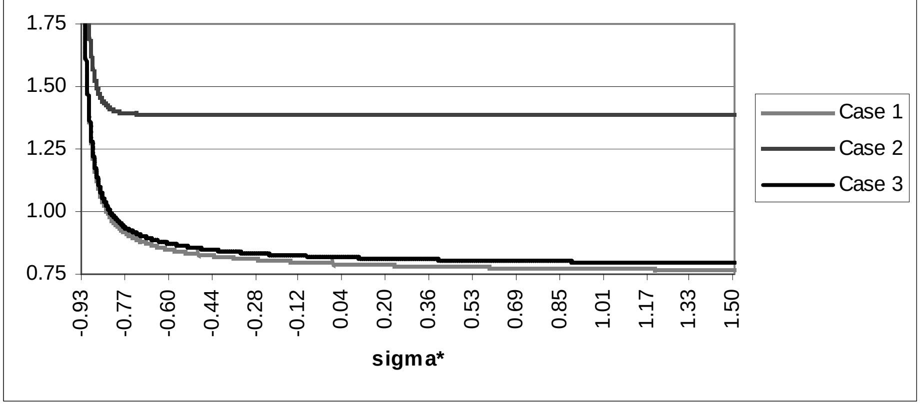
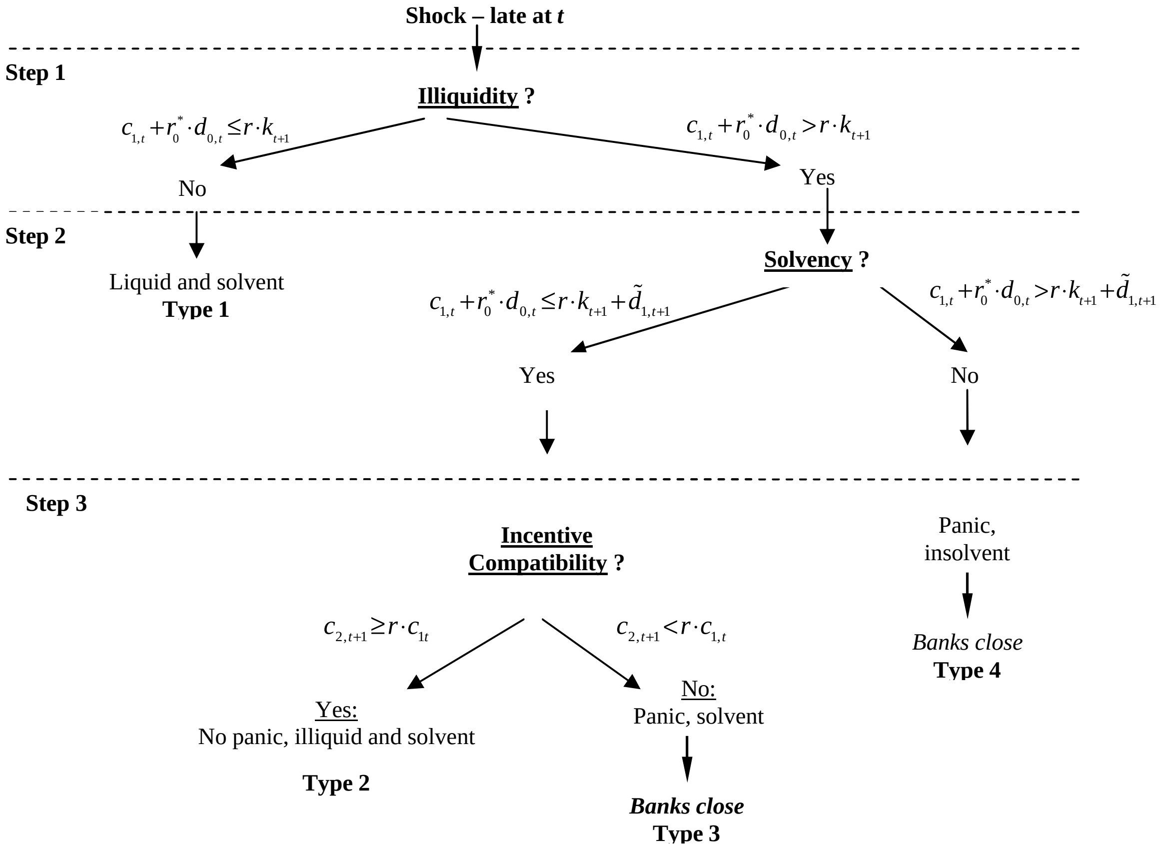


Figure 5: The Structure of the Dynamic System
Fixed Exchange Rates

$$\bar{t}_{t-1} \to \underbrace{\bar{t}_t \to \left(\bar{z}_t, \bar{q}_t, \bar{b}_t^*, \bar{k}_{t+1}\right)}_{\text{The Core}} \to \underbrace{\left(\bar{d}_{0,t,j}, \bar{d}_{1,t+1,j}, \bar{d}_{2,t+1,j}\right) \to \left(\bar{c}_{1,t,j}, \bar{c}_{2,t+1,j}\right)}_{\text{Case } j\text{, for } j=1,2,3}$$

t-1

Figure 6. The Sequential Checking Mechanism



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Technical Appendix

1. Existence of Equilibria under Floating Exchange Rate Regimes

1.1 Conditions for Existence in Case 1. We must define first the following expression: $\hat{e} \equiv l \cdot w \cdot \left\{ r_2^* \cdot \left(1 - f_d - f_f\right) + \left[f_f \big/ \left(1+s^*\right) \right] \right\} - r_1^* \cdot \left(f_1 - f_0 \right) \cdot \left\{ 1 - f_d \cdot \left[1 + q \cdot \left(\tilde{r} - 1 \right) \right] \right\}$. Next, we enumerate the two mutually exclusive conditions needed for this type of stationary equilibrium to exist under floating:

<u>Condition 1:</u> $\hat{e} > 0$ must hold.

<u>Condition 2:</u> when $\hat{e} < 0$, $s < \hat{s} \equiv \left[\hat{e} + l \cdot f_d \cdot w \big/ \left(-\hat{e} \right) \right]$ must hold.

Ceteris paribus, values of r_1^* that are high enough tend to increase the scope for which $d_0 = 0$ and are associated with situations where the relative cost of the debt-instrument d_0 is perceived as high. Thus, it is in the banks' best interest to avoid this expensive instrument. Condition 2 instead is a statement about the rate of domestic money growth: low enough values of s imply high return on the domestic currency reserves held by banks, and thus, banks are willing to borrow arbitrarily large amounts of the long-term instrument, which matures in the same period.

3.2 Conditions for Existence in Case 2. There is only one condition for the existence of the interior solution equilibria:

<u>Condition 3:</u> $r_0^* > r$ and $r_1^* > r$.

Notice that Condition 3 always holds, since $r < 1$. The amount borrowed of long-term debt in equilibrium is

$$\hat{d}_{2,2} = l \cdot \left(w + t \right) \cdot \left(r_2^* - r_1^* \right)^{-1} \left[r_2^* \cdot \left(1 - f_d - f_f \right) + f_d \cdot \left(1 + s \right)^{-1} + f_f \cdot \left(1 + s^* \right)^{-1} \right] - r_1^* \cdot \left(f_1 - r_0^* \cdot f_0 \right) \qquad \text{(A.3.1)}$$

while it is evident that $0 < \hat{d}_{2,2} < f_0$, $\hat{d}_{0,2} = f_0 - \hat{d}_{2,2} > 0$ and $\hat{d}_{1,2} = f_1 - \hat{d}_{2,2} > 0$ obtain.

3.3 Conditions for Existence in Case 3. The conditions for existence in this case are fairly complex and algebra-intensive. Define: $\hat{A} \equiv \left(2 \cdot l \cdot f_1 \right)^{-1} \cdot \left[\varpi + l \cdot r_2^* \cdot f_0 - \left(1 - l \right) \cdot \left(f_1 \cdot r_2^* / r \right) + \sqrt{\;} \right]$, $\hat{B} \equiv \varpi \cdot \left(f_1 - r_0^* \cdot f_0 \right)^{-1}$, $\hat{C} \equiv \left(2 \cdot l \cdot f_1 \right)^{-1} \cdot \left[\; + l \cdot r_2^* \cdot f_0 - \left(1 - l \right) \cdot r \cdot f_1 + \sqrt{\;} \right]$, $\cap \equiv \left[\left(1 - l \right) \cdot r \cdot f_1 - l \cdot r_2^* \cdot f_0 \varpi \right]^2 + 4 l \cdot \left(1 - l \right) \cdot r \cdot r_2^* \cdot f_1 \cdot f_0$, $\varpi \equiv \left[\left(1 - l \right) \cdot \left(f_1 \cdot r_2^* / r \right) - l \cdot r_2^* \cdot f_0 \varpi \right]^2 + 4 \cdot l \cdot \left(1 - l \right) \cdot \left[\left(r_2^* \right)^2 / r \right] \cdot f_1 \cdot f_0$, and $\varpi \equiv \left[r_2^* \cdot \left(1 - f_d - f_f \right) + f_d \cdot \left(1 + s \right)^{-1} + f_f \cdot \left(1 + s^* \right)^{-1} \right] \cdot l \cdot \left(w + t \right)$ Equilibria exist when the following condition holds:

<u>Condition 4:</u> $\max\left\{ \hat{A}, \hat{B} \right\} < r_1^* \le \hat{C}$.

2. Steady-State Welfare and the World Interest Rates – Floating Exchange Rates

The world interest rate on intra-period debt. r_0^* is the real interest rate associated with the foreign debt instrument $\hat{d}_0$. Interestingly, changes in r_0^* do not affect the expected utility in Case 1, but the welfare ranking depends upon the value of $r_0^* \in \left(1, r_2^* \right)$. Specifically, for values of r_0^* sufficiently close to 1, $\hat{U}_3 > \hat{U}_1 > \hat{U}_2$ obtains, and thus, as one might expect, borrowing intra-period debt to the maximum Pareto dominates all other Cases, since it is relatively cheaper. However, as $r_0^* \to r_2^*$, this debt instrument becomes too expensive and $\hat{U}_1 > \hat{U}_3 > \hat{U}_2$ obtains instead. Moreover, the reader may notice that equilibria with $\hat{d}_{0,1} = 0$ and $\hat{d}_{0,3} = f_0$ always dominate equilibria with an interior solution. The latter may seem counterintuitive but for the fact that $r_2^* > r_0^*$ always holds.

The world interest rate on inter-period debt. The real interest rate r_1^* is the cost associated with $\hat{d}_1$, for which there is a nonzero solution in all Cases. In particular, $\hat{d}_{1,2} = f_1 - f_0 < \hat{d}_{1,2} < \hat{d}_{1,3} = f_1$ always holds. Interestingly, the expected utility in all cases depends on this interest rate: increases in r_1^* reduce $\hat{U}_1$ and $\hat{U}_3$ since in Cases 1 and 3 no substitution between $\hat{d}_1$ and the other assets is possible. However, the circumstances are different in Case 2, where banks can substitute the more expensive $\hat{d}_{1,2}$ for the relatively cheaper $\hat{d}_{0,1}$ or $\hat{d}_{2,2}$, and thus $\hat{U}_2$ increases with r_1^*. Moreover, as in the case of r_0^*, for values of r_1^* sufficiently close to 1, $\hat{U}_3 > \hat{U}_1 > \hat{U}_2$ obtains, and thus, as one might expect, borrowing inter-period debt to the maximum Pareto dominates all other Cases, since it is relatively cheaper. However, as $r_1^* \to r_2^*$, this debt instrument becomes too expensive and borrowing the least amount possible of inter-period debt is in the best interest of households and banks, and $\hat{U}_1 > \hat{U}_3 > \hat{U}_2$ obtains instead.

The world interest rate on long-term debt. The interest rate r_2^* corresponds to long-term borrowing, $\hat{d}_2$, and in equilibrium $r_2^* = R$. The expected utility in all cases and the welfare ranking depend strongly upon the interest rate on long-term debt, since $\hat{d}_{2,3} = 0 < \hat{d}_{2,2} < \hat{d}_{2,1} = f_0$ transpires in equilibrium. It appears that the role of r_2^* as the return on long-term investment dominates that as the cost on long-term debt in Cases 1 and 3, since higher values of r_2^* cause $\hat{c}_{2,1}$ and $\hat{c}_{2,3}$ to increase, thus augmenting the expected utility in Cases 1 and 3. However, the evidence is mixed in Case 2, given that substitution toward cheaper debt instruments can occur: welfare increases only for rates of domestic money growth that are high enough. Regarding the welfare ranking, we find that, for values of $r_2^* \to r_1^*$, $\hat{U}_1 > \hat{U}_3 > \hat{U}_2$ obtains, while $\hat{U}_3 > \hat{U}_1 > \hat{U}_2$ transpires for high values of r_2^*. The latter coincides with intuition, since $\hat{d}_{2,3} = 0$ and its price is high.

The world interest rate on foreign reserve assets. As one would expect, adding to $\tilde{r}$ is welfare-improving in Cases 1 and 3, since it increases domestic and foreign real money balances, as well as the real foreign reserves holdings by the monetary authority. However, it has almost not effect in Case 2. The welfare ordering remains unchanged.

3. Local Stability Analysis of Foreign Debt in Equilibria of Case 2 – Floating Exchange Rates

- <u>Dynamics and the world inflation rate.</u> The interaction with changing values of s^* introduces interesting variations to the sequence of dynamic properties that we describe below.
  a) For low enough values of the world inflation rate such that $s^* \in \left(-1, 0 \right)$ the sequence becomes: (+) sink, (+) complex-stable, (-) sink and (-) saddle, thus increasing the scope for determinacy and stable fluctuations with respect to the *baseline scenario*.

b) As s^* increases gradually, the economy goes back to the *baseline sequence*, but the scope for determinacy decrease gradually as well.

c) When s^* increases from inflation crises values to hyperinflation, the sequence becomes: (+) sink, (+) complex-stable, (-) complex-stable and (-) complex unstable, eliminating the scope for determinacy.

d) Next, for even higher values of s^*, the scope for complex eigenvalues with negative real parts decreases until it eventually disappears, and the sequence becomes: (+) sink, (+) complex-stable, but with almost explosive values of the discriminant which translate into very large cyclical fluctuations.

In conclusion, low enough rates of foreign inflation contribute to the goal of increased stability and determinacy of dynamic equilibria, as one might expect, but coupled with endogenously-arising volatility.

- Dynamics and world interest rates. In this case, we proceed by changing gradually the pair $\left(\tilde{r}, R = r_2^*\right)$ to study how the dynamic sequence changes. We describe our findings below.

  a) For $\tilde{r} = R = 1$ (i.e. zero net returns,) the associated dynamic sequence is: (+)sink, (+)complex-stable, (-) complex-stable, (-)sink and (-)saddle, where the scope for determinacy dominates. Thus, this combination of returns eliminates the scope for diverging non-cyclical fluctuations.

  b) As $\left(\tilde{r}, R\right)$ increase gradually, the dynamic sequence converges to the *baseline sequence*.

  c) As the pair $\left(\tilde{r}, R\right)$ continues to increase, the scope for complex eigenvalues increases significantly together with the source, while the scope for determinacy eventually disappears completely. Thus, high foreign interest rates promote instability and fairly large cyclical and non-cyclical fluctuations.

- Dynamics and reserve requirements. We proceed by increasing gradually both reserve requirements, with the condition that $f_d = f_f < 0.5$. The reader will notice that the reserve requirements play the role of stabilizing dynamic equilibria. We find:

  a) For $f_d = f_f$ close enough to zero the dynamic sequence consists of: (+) sink followed by (+) complex-stable, but with very high values of the discriminant, and thus very large cyclical fluctuations.

  b) As $f_d = f_f$ increase gradually, the scope for (+) complex-stable decreases, giving rise instead to (-) complex, (-) source and a (-) saddle until converging to the *baseline sequence*.

  c) High enough values of $f_d = f_f$ reduce the scope for complex eigenvalues and (-) source, which gives rise to a (-) sink and a significantly larger range of (-) saddle. Thus, high and binding reserve requirements promote stability and determinacy along the dynamic paths.

- Dynamics and backing of the money supply. Different fractions of the money supply to be backed by the monetary authority introduce interesting changes in the dynamic sequence, which we describe below. The reader will notice that the policy parameter q plays the role of de-stabilizing of dynamic equilibria.

  a) When $q = 0$, the order of the dynamic system decreases to a first order nonlinear difference equation. As a result, for low enough values of s, the eigenvalue is positive and stable, promoting monotonic dynamics. As s increases, the eigenvalue becomes negative but stable and damped oscillations are observed along dynamic paths. Finally, for values of s that are high enough, the negative eigenvalue turns unstable, and explosive oscillations take place.

  b) For very low, non-zero values of q, the system turns into a second order nonlinear difference equation where the following dynamic sequence arises: (+) sink, (+) complex-stable, (-) complex-stable, (-) sink and (-) saddle. This sequence displays both stability and determinacy along dynamic paths.

  c) As the backing of the domestic money supply increases, the dynamic sequence converges to the *baseline sequence*, where instability is observed.

  d) For values of q close enough to 1, the scope for complex eigenvalues increases significantly, eliminating gradually the (-) source as well as the (-) saddle. Dynamic equilibria are indeterminate and large cyclical and non-cyclical fluctuations dominate along dynamic paths.

- Dynamics and combinations of reserve requirements and backing of the domestic money supply. We now evaluate alternative combinations of reserve requirements and backing of the domestic money supply, in order to find the combination that promotes stability and determinacy. We now describe our findings.

  a) Low $\left(f_d = f_f, q\right)$. Under this combination, the dynamic sequence becomes: (+) sink, (+) complex-stable, (-) complex-stable, (-) complex-unstable, and (-) source. Thus, the scope for instability and fluctuations dominates.

  b) High $\left(f_d = f_f, q\right)$. Under this combination, the dynamic sequence turns into: (+) sink, (+) complex-stable, (-) complex-stable, (-) complex-unstable, (-) source and (-) saddle. Thus, the scope for determinacy increases.

  c) Low $f_d = f_f$ and high q. The dynamic sequence turns out to be: (+) sink, (+) complex-stable, (-) complex-stable, and (-) complex-unstable. Equilibria are indeterminate and display cyclical fluctuations that become unstable.

  d) High $f_d = f_f$ and low q. The dynamic sequence in this case is (+) sink, (+) complex-stable, (-) complex-stable, (-) sink and (-) saddle. Thus, this combination of policy parameters promotes both the stability and determinacy of dynamic equilibria.

4. Existence of Equilibria under a Hard Peg

Conditions for Existence in Case 1. In this case, we must define first the expression $\overline{e}$ as $\overline{e} \equiv l \cdot w \cdot r_2^* \cdot \left(1 - f_d - f_f\right) - r_1^* \cdot \left(f_1 - f_0\right) \cdot \left\{1 - f_d \cdot \left[1 + q \cdot \left(\tilde{r} - 1\right)\right]\right\}$. Next, we establish the condition for existence under a hard peg.

Condition 5: $\overline{e} > 0$ must hold.

Condition 5 is more likely to hold for values of r_1^* and/or q that are sufficiently low. The intuition behind this finding is very simple: when $\overline{d}_{1,1}$ is relative cheap, banks will borrow less of its short-term substitute $\overline{d}_{0,1}$ The latter may condition banks to borrow more of the long-term debt, $\overline{d}_{2,1}$.

Conditions for Existence in Case 2. The following condition must hold for an interior solution to exist equilibria:

Condition 6: $r_0^* > r$ and $r_1^* > r$ must hold.

Condition 6 is identical to Condition 3 with floating exchange rates. It always holds $\forall\left(r_0^*, r_1^*, r_2^*\right) >> 1$, since $r < 1$.

Conditions for Existence in Case 3. Under a fixed exchange rate regime, we must substitute the expression a by $\bar{\bar{a}} \equiv \left\{r_2^* \cdot (1-f_d - f_f) \pm \left[\left(f_d \pm f_f\right) / \left(1+s^*\right)\right]\right\} \cdot l \cdot \left(w+\overline{\tau}\right)$. Of course, the expressions $\left(A, B, C, a, c\right)$ must be modified accordingly, giving rise instead to $\left(\overline{A}, \overline{B}, \overline{C}, \overline{a}, \overline{c}\right)$. Thus, the condition for the existence of equilibria in Case 3 becomes:

Condition 7: $\max\left(\overline{A}, \overline{B}\right) < r_1^* \leq \overline{C}$ must hold.

Thus, Condition 7 requires the debt instrument $\overline{d}_{1,3}$ to be sufficiently expensive, so that banks are conditioned to acquire more of the instrument $\overline{d}_{0,3}$ instead. It follows, in the limit, that $\overline{d}_{0,3} = f_0$ and $\overline{d}_{2,3} = 0$ in equilibrium. Table 6 summarizes the conditions for existence as well as how the scope for existence depends on the world inflation rate s^*.

5. Steady-State Welfare and the World Interest Rates – Fixed Exchange Rates

The world interest rate on intra-period debt. As in the case of floating exchange rates, changes in r_0^* do not affect the expected utility in Case 1 but is lessens $\hat{U}_3$. The effect on $\hat{U}_2$ is mixed: for low values of r_0^*, $\hat{U}_2$ increases, while further increases in r_0^* reduce it. Thus, the welfare ranking depends upon the value of $r_0^* \in \left(1, r_2^*\right)$. Specifically, for values of r_0^* sufficiently close to 1, $\hat{U}_2 > \hat{U}_3 > \hat{U}_1$ obtains, and borrowing intra-period debt to the maximum Pareto dominates Case 1. However, as $r_0^* \rightarrow r_2^*$, this debt instrument becomes too expensive and $\hat{U}_2 > \hat{U}_1 \geq \hat{U}_3$ obtains instead. Moreover, the reader may notice that the equilibria with $\hat{d}_{0,2}$ always Pareto-dominate equilibria with corner solutions, and the ability to substitute for cheaper debt assets leads welfare. Recall that the opposite transpired with floating.

The world interest rate on inter-period debt. Interestingly, the expected utility in all cases depends on this interest rate: increases in r_1^* reduce $\hat{U}_1$ and $\hat{U}_3$ since in Cases 1 and 3 no substitution between $\hat{d}_1$ and the other assets is possible. However, the circumstances are different in Case 2, where banks can substitute the more expensive $\hat{d}_{1,2}$ for the relatively cheaper $\hat{d}_{0,1}$ or $\hat{d}_{2,2}$: for low values of r_0^*, $\hat{U}_2$ falls, while further increases in r_0^* add to it. Furthermore, as $r_1^* \rightarrow 1^+$ the ordering of expected utility changes dramatically to $\hat{U}_3 > \hat{U}_1 > \hat{U}_2$ and borrowing inter-period debt to the maximum Pareto dominates all other Cases, since it is the cheapest. Nonetheless, as $r_1^* \rightarrow r_2^*$, this debt instrument becomes too expensive and borrowing the least amount possible of inter-period debt improves on Case 3. However, the possibility of substitution for other debt assets always dominates the other cases, and $\hat{U}_2 > \hat{U}_1 \geq \hat{U}_3$ obtains instead.

The world interest rate on long-term debt. The expected utility in all cases depend strongly upon the interest rate on long-term debt, and adding to it is welfare improving in all cases. It appears then, that, under a hard peg, the role of r_2^* as the return on long-term investment dominates that as the cost on long-term debt. Regarding the welfare ranking, we find that, as $r_2^* \rightarrow r_1^*$, $\hat{U}_2 > \hat{U}_1 \geq \hat{U}_3$ obtains, while $\hat{U}_2 > \hat{U}_3 > \hat{U}_1$ transpires for high values of r_2^*. Moreover, as r_2^* augments, $\hat{U}_2$ becomes an increasing function of s^*.

The world interest rate on foreign reserve assets. Raising $\tilde{r}$ is welfare-improving in all cases, and especially in Case2. The welfare ranking remains unchanged.

6. Stability properties of long-term debt in Case 2 regarding other parameters under a Hard Peg

- Dynamics and Rates of return $\left(\tilde{r}, R\right)$: When $\tilde{r} = R = r_2^* = r_0^* = r_1^* = 1$ (zero net returns,) the scope for stability almost disappears, while the scopes for very large unstable fluctuations and very large monotonic divergence dominate. A very small increase in the returns eliminates the scope for monotonic divergence while the scope for stable monotonic dynamics dominates. As the returns increase, the dynamics converges toward the *baseline sequence*. In the limit, the maximum eigenvalue approaches to zero from the right. Thus, higher returns seem to stabilize dynamic equilibria.
- Dynamics and the Number of impatient households l : $\forall\, l \in \left(0,1\right)$, no monotonic divergence is observed, and only small variations around the *baseline sequence* take place. The scope for large unstable fluctuations increases with l, at the expense of damped oscillations. The scope for stable monotonic dynamics dominates and remains unchanged. The maximum eigenvalue increases with l, but it is always less than 1.
- Dynamics and Reserve Requirements $f_d = f_f$: no monotonic divergence is observed when $f_d = f_f \rightarrow 0$, and the dynamic sequence deviates only slightly from the baseline sequence. As $f_d = f_f$ increase, the scope and size of unstable fluctuations augment. Thus, fairly large oscillations may be observed around the steady-state while the scope for stable oscillations decreases. The scope for monotonic convergence dominates and it remains unaffected. Thus, binding reserve requirements preserve monotonic stability but at the cost of larger unstable fluctuations.
- Dynamics and q : the policy parameter q is a stabilizer for long-term debt dynamics. The value $q = 0$ eliminates monotonic dynamics, and fairly large unstable fluctuations may arise. As q increases and crosses the threshold value $q = 0.037$, stable monotonic dynamics arises, and the scope for large unstable fluctuations decreases. $q = 1$ alone is not enough to produce monotonic divergence, nor is it $l \rightarrow 1^-$. However, coupling $q = 1$ with a high enough value of l $\left(l > 0.766\right)$ increases the maximum eigenvalue beyond 1 for high values of s^* $\left(s^* > 23\right)$.
- Dynamics and combinations of $f_d = f_f$ and q : We now compare four alternative combinations of reserve requirements and backing of the domestic money supply against the baseline sequence, in order to find the combination that promotes monotonic stability.

a) Low $\left(f_d = f_f, q.\right)$ The range and size of unstable fluctuations decreases. At the same time, the scope of stable fluctuations increases at the expense of the scope of stable monotonic dynamics, but the latter still dominates.
b) High $\left(f_d = f_f, q.\right)$ This combination enlarges the range and size of unstable fluctuations. At the same time, the scope of stable fluctuations shrinks while the scope of stable monotonic dynamics raises and it still dominates.
c) Low $f_d = f_f$ and high q. This combination reduces significantly the range and size of unstable fluctuations together with a fall in the scope for stable fluctuations. Moreover, the range of stable monotonic dynamics increases dramatically and it dominates the general scope of dynamics. Thus, this combination of policy parameters promotes stable monotonic dynamics.
d) High $f_d = f_f$ and low q. This combination adds to the range and size of unstable fluctuations as well as to the scope for stable fluctuations. Consequently, the range of stable monotonic dynamics suffers, falling dramatically. Moreover, stable fluctuations dominate the general scope of dynamics. Notice that this policy had stabilizing effects under floating exchange rates but now it is the opposite.

7. The Functions in Table 4 - Steady State

$$A_1 \equiv (1+l\cdot r)\cdot f_0 + l\cdot r\cdot\left(1-f_d-f_f\right)\cdot(w+t) \quad \text{(A7.1)}$$

$$B_1 \equiv \left(f_1'/l\right) - \left[(1/l)+r-1\right]\cdot f_0 - r\cdot\left(1-f_d-f_f\right)\cdot(w+t) \quad \text{(A7.2)}$$

$$C_1 \equiv f_0 + l\cdot\left[l\cdot r_1^* + (1-l)\cdot r\right]^{-1}\cdot\left[R\cdot\left(1-f_d-f_f\right)\cdot(w+t)+RDC+RFC\right] \quad \text{(A7.3)}$$

$$RDC+RFC \equiv (w+t)\cdot f_d\cdot(1+s)^{-1} + (w+t)\cdot f_f\cdot(1+s^*)^{-1} \quad \text{(A7.4)}$$

$$A_2 \equiv r_0^*\cdot f_0 - y^{-1}\cdot\left(r_0^*-r\right)\cdot f_0 + y^{-1}\cdot(w+t)\cdot\left\{\left[(1-l)\cdot r_0^*-1\right]\cdot\left(r_2^*-r_1^*\right)^{-1}\cdot\left[\left(1-f_d-f_f\right)+f_d\cdot(1+s)^{-1}+f_f\cdot(1+s^*)^{-1}\right]-r\cdot\left(1-f_d-f_f\right)\right\} \quad \text{(A7.5)}$$

$$y \equiv l\cdot\left[1+\left(R/r_0^*\right)-R\cdot(1-l)\right] \quad \text{(A7.6)}$$

$$B_2 \equiv l^{-1}\left[1-(1-l)\cdot\left(r_2^*-r_1^*\right)\right]\cdot\left(f_1'-r_0^*\cdot f_0\right)+\left(r_0^*-r\right)\cdot f_0+\left[(1-l)r_0^*-r\right]\cdot\left(1-f_d-f_f\right)\cdot(w+t)+\left(r_1^*\right)^{-1}\cdot(1-l)\cdot(RDC+RFC) \quad \text{(A7.7)}$$

$$C_2 \equiv r_0^*\cdot f_0 + \left(y_1\right)^{-1}\cdot r\cdot\left[R\cdot\left(1-f_d-f_f\right)\cdot(w+t)+RDC+RFC\right] \quad \text{(A7.8)}$$

$$y_1 \equiv \left[r_1^*\cdot(1-l)^{-1}+r\cdot l^{-1}\right]\cdot\left\{1-l\cdot r_1^*\cdot\left(r_0^*-1\right)\right\} \quad \text{(A7.9)}$$

$$A_3 \equiv \left[r_1^*\cdot(1-l)^{-1}+r\cdot l^{-1}\right]\cdot f_0 + r\cdot l^{-1}\cdot\left(1-f_d-f_f\right)\cdot(w+t) \quad \text{(A7.10)}$$

$$B_3 \equiv l^{-1}\cdot f_1' + l^{-1}\left[(1-l)\cdot r_0^* - r\cdot l\right]\cdot f_0 - r\cdot\left(1-f_d-f_f\right)\cdot(w+t) \quad \text{(A7.11)}$$

$$C_3 \equiv r_0^*\cdot f_0 + l\cdot\left[l\, r_1^*+(1-l)\, r\right]^{-1}\cdot\left[R\cdot\left(1-f_d-f_f\right)\cdot(w+t)+RDC+RFC\right] \quad \text{(A7.12)}$$

Notice that for different exchange rate regimes, the corresponding government transfers are different, and so are the rates of return of domestic and foreign currency reserves.

8. Scope for Existence for the Different Types of Equilibria

If $f_1 < A_1$ $\left(f_1 > A_1\right)$, the allocation is liquid (illiquid.) When $f_1 < B_1$ $\left(f_1 > B_1\right)$ holds the allocation is insolvent (insolvent.) If $f_1 < C_1$ $\left(f_1 > C_1\right)$ obtains, the allocation is incentive compatible (not incentive compatible.)

8.1 Case 1. In Case 1, $B_1 < A_1 < C_1$ obtains.
Equilibria of Type 1 exist when $B_1 < f_1 < A_1 < C_1$ holds.
Equilibria of Type 2 exist when $B_1 < A_1 < f_1 < C_2$ obtains.
Equilibria of Type 3 exist when $B_1 < A_1 < C_1 < f_1$ transpires.
Equilibria of Type 4 do not exist in Case 1.

8.2 Case 2. In Case 2, $A_2 < B_2 < C_2$ obtains.
Equilibria of Type 1 do not exist in Case 2.
Equilibria of Type 2 exist when $A_2 < B_2 < f_1 < C_2$ holds.
Equilibria of Type 3 exist when $A_2 < B_2 < C_2 < f_1$ obtains.
Equilibria of Type 4 do not exist in Case 2.

8.3 Case 3. In Case 3, $B_3 < C_3 < A_3$ obtains.
Equilibria of Type 1 exist when $B_3 < f_1 < C_3 < A_3$ transpires.
Equilibria of Type 2 do not exist in Case 3.
Equilibria of Type 3 exist when $B_3 < C_3 < A_3 < f_1$ holds.
Equilibria of Type 4 do not exist in Case 3.

Table A.1: Summary of Effects of World Interest Rates on the Scope for Existence of Equilibria after a Sudden Stop

Case and Interest Rate	Type 1	Type 2	Type 3	Type 4
<u>Case 1:</u>				
$\uparrow r_0^*$	--	--	--	n.a.
$\uparrow r_1^*$	--	Narrows	Widens	n.a.
$\uparrow r_2^* = R$	--	Widens	Narrows	n.a.
$\uparrow \tilde{r}$	Widens	Widens	Narrows	n.a.
<u>Case 2:</u>				
$\uparrow r_0^*$	n.a.	Widens	Narrows	n.a.
$\uparrow r_1^*$	n.a.	Widens	Narrows	n.a.
$\uparrow r_2^* = R$	n.a.	Widens	Narrows	n.a.
$\uparrow \tilde{r}$	n.a.	Widens	Narrows	n.a.
<u>Case 3:</u>				
$\uparrow r_0^*$	Narrows	n.a.	--	n.a.
$\uparrow r_1^*$	Narrows	n.a.	Narrows	n.a.
$\uparrow r_2^* = R$	Widens	n.a.	--	n.a.
$\uparrow \tilde{r}$	Widens	n.a.	Narrows	n.a.