

Mergers in asymmetric Stackelberg markets

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Abstract

It is well known that the profitability of horizontal mergers with quantity competition is scarce. However, in an asymmetric Stackelberg market we obtain that some mergers are profitable. Our main result is that mergers among followers become profitable when the followers are inefficient enough. In this case, leaders reduce their output when followers merge and this reduction renders the merger profitable. This merger increases price and welfare is reduced.

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1. Introduction

In a symmetric linear Cournot oligopoly setting with homogenous goods, Salant, Switzer and Reynolds (1983) (henceforth, SSR) showed that two-firm mergers are never profitable. Also, in an asymmetric Cournot model with linear demand and constant marginal costs the merger of symmetric firms is not profitable. Unprofitability comes from the fact that non-merging firms react to the merger by increasing their output.¹ In the Stackelberg model with linear demand and symmetric cost functions, Daughety (1990) showed that the merger of two followers is potentially profitable, and that this merger may be welfare-enhancing. However, he focused only on the merger between two followers resulting in a firm that becomes a leader, and why two followers should gain commitment power by merging is not discussed. On the other hand, in the same model, Huck, Konrad and Müller (2001) showed that only mergers between a leader and a follower are unambiguously profitable.

In the present paper we show that in the Stackelberg model the profitability of horizontal merger crucially depends on cost asymmetries. We extend the analysis by Huck, Konrad and Müller (2001) to the case where cost asymmetries and multilateral mergers are allowed for. We develop a model where a group of firms (leaders) choose output before another group of firms (followers). Followers may be less efficient than leaders. We show that leaders rarely have an incentive to merge. We also obtain that in the asymmetric case mergers can be profitable even if costs are linear. This is true in two cases; first, when a leader firm incorporates follower firms. In this case, the followers essentially disappear and the newly merged firm produces less quantity than the merged firms did prior to the merger. However, the price increases sufficiently to make this profitable. Second, in the main result of the paper, we show that mergers between followers become profitable when the marginal cost of the followers is high enough. We obtain that leaders reduce their output when followers merge and this reduction increases as followers become less

¹In fact, the profitability of horizontal merger depends on the degree of concavity of cost and demand functions (see for instance Perry and Porter (1985) or Faulí-Oller (1997)).

efficient. We also observe that in both cases, welfare is reduced. Our analysis proves useful because it allows to obtain that mergers of symmetric firms i.e. without efficiency gains maybe profitable in a setting where firms choose output. For instance, a merger of two followers may be profitable and they need not to be the only two firms in the industry. Furthermore, they need not to be even the only two firms of their type in the industry.

Although we have focused on the asymmetric Stackelberg model from a theoretical viewpoint, our paper is also motivated by the profitability of the real mergers. An example could be the semiconductor industry such as the DRAM (Dynamic Random Access Memory) industry, where the leading manufacturers announce their production plan in advance. The manufacturers that enter the market late correspond by adjusting their quantity of DRAM produced (see Cho, D.-S.; D.-J. Kim and D.K Rhee (1998)). For the last decades an overdue wave of mergers has been reshaping this industry. Another typical example could be the international market competition among domestic and foreign firms in a developing country. The domestic firms are often less efficient and decide their quantity after learning the output choice announced by the leading multinationals firms. In both cases, it seems plausible to analyze merger profitability in an asymmetric Stackelberg model.

The remainder of the paper is organized as follows. In Section 2 we briefly outline the model and we study the effects of merger. In a subsection we study the effect of merger on welfare. Another subsection provides a numerical example to illustrate the results. Section 3 tests the robustness of our results with a convex cost function —as the one proposed by Perry and Porter (1985)— and establishes that the main result continues to hold. Section 4 concludes. All proofs are grouped together in the appendix.

2. The model and merger profitability

We consider a market for a homogenous product with n firms. Inverse demand is given by $P(Q) = 1 - Q$ where Q is the industry output. Competition occurs in two stages. In the first stage, k firms (leaders) simultaneously choose the output they want to sell. In

the second stage, the remaining $n - k$ firms (followers), knowing the outputs chosen by the leaders in the first stage, choose also simultaneously their level of production. Apart from their strategic advantage, leaders are assumed to be more efficient than followers.² The (constant) marginal cost of production of leaders is normalized to 0, whereas the unit cost of followers is given by $c \geq 0$. When $c = 0$ we are back to the standard (symmetric) Stackelberg model. We assume

$$c < \frac{1}{k + 1 + k(n - k)} < 1, \quad (2.1)$$

so that followers are active and the equilibrium is interior. The output sold in equilibrium by a leader and a follower and the market price are given respectively by:

$$\begin{aligned} q_l &= \frac{c(n - k) + 1}{k + 1} \\ q_f &= \frac{c(-k(n - k) - 1 - k) + 1}{(k + 1)(n - k + 1)} \end{aligned} \quad (2.2)$$

$$p(n, k, c) = \frac{1 + c(n - k)}{1 + k(n - k) + n}. \quad (2.3)$$

Contrary to the symmetric case, the output levels of the leaders depend on the number of followers. We note that q_l decreases with the number of leaders and increases with the number of followers. On the other hand, q_f is decreasing with the number of followers but the effect of a change in the number of leaders depends on whether leaders are a majority or a minority. In particular, q_f increases with k when $k > \frac{1 + cn - \sqrt{(1 - c)(1 + c(1 + n))}}{c} = f(c, n)$ and decreases with k when $k < f(c, n)$. It can also be verified that when $k = f(c, n)$, q_f does not vary with k , and welfare is maximized.³

The expressions from (2.2) lead to the following equilibrium profits obtained respec-

²This is assumed following the reasoning of the “folk theorem” that relatively large firms are committed leaders and small firms are followers. Sadanand and Sadanand (1996) obtain a formal result for sufficiently small amounts of uncertainty.

³This extends the result on welfare of Daughety (1990) to the asymmetric case. Observe that $f(c, n)$ is always increasing with c . In particular, for all possible values of c , the optimal number of leaders ranges from $\frac{n}{2}$ to $\frac{n+1}{2}$.

tively by leaders and followers:

$$\begin{aligned}\Pi_l(n, k, c) &= \frac{(c(n-k) + 1)^2}{(k+1)^2(n-k+1)} \\ \Pi_f(n, k, c) &= \frac{(c(k+1 + k(n-k)) - 1)^2}{(k+1)^2(n-k+1)^2}\end{aligned}$$

We consider three different types of mergers. (a) a merger of $m+1$ leaders, (b) the merger between a leader and m followers and (c) a merger between $m+1$ followers. In case (b) the merged entity chooses output (only) in the first stage. A merger is considered to be profitable if the profits of merging firms increase after merger. In case (a) it implies that:

$$\Pi_l(n-m, k-m, c) - (m+1)\Pi_l(n, k, c) > 0 \quad (2.4)$$

For a merger of case (b) it implies that:

$$\Pi_l(n-m, k, c) - m\Pi_f(n, k, c) - \Pi_l(n, k, c) > 0$$

and a merger of type (c) is profitable if the following condition holds:

$$\Pi_f(n-m, k, c) - (m+1)\Pi_f(n, k, c) > 0$$

Regarding cases (a) and (b), the results of the symmetric case analyzed by Huck, Konrad and Müller (2001) extend to the asymmetric case: leaders rarely have an incentive to merge, and the merger between a leader and a group of followers is always profitable. In particular, condition (2.4) can be rewritten as $\frac{\Pi_l(n-m, k-m)}{\Pi_l(n, k)} > (m+1)$, and it is the same condition as in the symmetric case because the left hand side does not depend on c . For case (b), merger profitability can only increase when followers become inefficient because the merger has the additional positive effect of allowing some cost savings by transferring output from a high cost firm to a low cost firm.

Proposition 1. *For all $m < k$, a merger between $m+1$ leaders is only profitable if*

$$m \geq k - \frac{1}{2}(\sqrt[2]{4k+5} - 1).$$

This result parallels the results by SSR because cost asymmetry does not play a role.⁴

Proposition 2. *For all $m \leq n - k$, a merger between a leader and m followers is always profitable.*

Thus, with quantity competition, a merger can be profitable even if costs are linear in the case of a leader firm incorporating follower firms. As can be seen from (2.2), the newly merged firm produces even less quantity than the leader prior to merger. Intuitively, market price increases and this overcompensates the decrease in the joint quantity sold. When $c > 0$, the leader-follower firm internalizes even more of the benefits from the price increases because it reduces output less drastically than symmetric firms would. Note also that a leader has incentives to incorporate as many followers as possible.

Remark 1. *The incentive for a leader to merge with $m + 1$ followers increases with m*

We turn now our attention to a merger of type (c). The main contribution of this paper is that merger of followers are profitable if c is high enough. In a Cournot setting, mergers are unprofitable because nonparticipants expand their output after the merger. However, the profitability of mergers in the asymmetric Stackelberg model is explained by the fact that leaders reduce their output after the merger of two followers. In particular, the marginal reduction in the output of leaders given a marginal decrease in the number of followers is given by

$$\frac{\partial q_l}{\partial(n-k)} = \frac{c}{(1+k)}$$

It is zero for the symmetric case and negative for the asymmetric case if $c > 0$. The reduction becomes more important as followers become more inefficient. This explains why mergers are only profitable when c is high enough.⁵

⁴Note that the inequality in Proposition 1 is exactly the same in SSR for the Cournot case: for any k , it is sufficient for a merger between $m + 1$ leaders to be unprofitable that less than 80% of the leaders merge.

⁵Note that this reduction is also more important when the number of leaders is small.

Proposition 3. *For all $m < n - k$, a merger between $m + 1$ followers is profitable if their marginal cost is high enough.*

The intuition of Proposition 3 is that as c increases leaders take less into account the rivalry of the followers. When the number of inefficient followers is reduced, leaders use less their “strategic power” to anticipate a large output, and consequently leaders reduce production. Since followers act as Cournot quantity-setting firms facing a residual demand, it seems plausible that merger profitability for case (c) follows the results by SSR. In this case, SSR established that a merger between two (or more) like firms to produce a firm of the same type is nearly always unprofitable. However, this is not true when the merger involves “inefficient” followers forming a follower. Interestingly, for example if two followers merge, they do not need to be neither the only two firms in the industry nor the only two followers for the merger to be profitable.

2.1. Welfare

In absence of synergies, after the merger of $m + 1$ followers there will be k leaders and $n - k - m$ followers, which is exactly the same market configuration after a merger between a leader and m followers. Since market price is given by (2.3), it is easy to see that

$$\frac{\partial p(n, k, c)}{\partial n} = \frac{c - 1}{(1 + k)(1 + n - k)^2} < 0. \quad (2.5)$$

Therefore, as far as welfare is concerned both types of profitable mergers have the same effect: market price increases and welfare is reduced. Notice also that in the asymmetric case the effect of the profitable mergers on welfare is smaller than in the symmetric case. Intuitively, the more “inefficient” the followers, the smaller is the increase in market price due to the merger. This is true as (2.5) is larger in absolute value with $c = 0$ than with $c > 0$. Observe that our results contrast with those obtained by Daughety (1990): if followers do not gain commitment power by merging, welfare is reduced.

2.2. Example

A numerical example is provided to illustrate better the range of parameters over which the mergers analyzed in this paper can be profitable. We consider an industry where $n = 8$ and $k = 5$.

Type (a): a merger between $m + 1$ leaders is profitable if: $\frac{(1+3c)^2}{144} \left(\frac{36}{(m-6)^2} - (m+1) \right) > 0$.

This does not depend on c and is only positive if $m > 3$.

Type (b): a merger between a leader and m followers is profitable if

$$\frac{m(m+c(160-42m)+c^2(457m-1824))}{576(4-m)} > 0, \text{ which is always true for all } m \in (1, 3).$$

Type (c): for the equilibrium to be interior we assume that $c < \frac{1}{k+1+k(n-k)} = 0.047$.

A merger between two followers is profitable if $\frac{122c-1921c^2-1}{2592} > 0$. The last expression equal to zero has 2 roots: $c = 0.01$ and $c = 0.05$. Therefore, when $c \in (0.01, 0.047)$ the equilibrium is interior, and a merger between two followers is profitable. Observe also that price prior to merger was $\frac{3c+1}{24}$ and after the merger is $\frac{2c+1}{18}$. Thus, when the merger is profitable ($c \in (0.01, 0.047)$) market price increases and welfare is reduced.

3. Extensions

To test the robustness of our main result, it is natural to analyze mergers of type (c). following the formulation by Perry and Porter (1985). For simplicity, we consider an industry with one leader and n followers where the (constant) marginal cost of production of the leader is normalized to 0, whereas the cost of followers is given by the following function: $c(q) = cq + \frac{d}{2}q^2$. We assume $c < \frac{2}{4+n}$ so that followers are active in equilibrium. Note that in this case, since the merger may give rise to scale economies, it does not reduce the number of firms. When $d = 1$, it can be verified that profits of a follower before and after the merger of m followers (denoted by $\Pi_b(c, n)$ and $\Pi_a(c, n, m)$ respectively) are given by:

$$\begin{aligned} \Pi_b(c, n) &= \frac{3(-2+c(4+n))^2}{32(2+n)^2} \\ \Pi_a(c, n, m) &= \frac{(1+2m)(2(1+m)+c(-4+m^2-n-m(5+n)))^2}{8(1+m)^2(2+n+m(3-m+n))^2} \end{aligned}$$

Proposition 4. *When $d = 1$, a merger between two followers is only profitable if $c > \frac{6(4(56+\sqrt{15}\sqrt{(2+n)^2(4+3n)^2})+n(32-n(100-21n)))}{1088-n(16+3n)(64+7n(16+3n))}$.*

The intuition is the same as in Proposition 3: as c increases the leader reduces production, and this reduction renders the merger profitable. In particular, it can be verified that, for any d , the variation in the leader's production due to the merger of m followers is given by the following expression: $-\frac{c(-1+m)m}{2(1+d)(d+m)} < 0$. Observe that the reduction becomes more important as c increases but is smaller as the followers have more convex costs (namely, d increases).

On the other hand, it can easily be verified that price variation due to the merger of two followers is given by the following expression: $\frac{12(n+1)-44c+cn(-84+n(-28+9n(2+n)))}{6(1+n)(2+n)}$, which is positive for all $c < \frac{2}{4+n}$ and $n \geq 2$. Thus, the merger increases price, and welfare is reduced.

4. Concluding remarks

In this paper, we showed that in a simple generalized Stackelberg market with k leaders and $n - k$ followers although firms choose output some mergers are profitable. In a merger between a leader and a group of followers the joint payoff of the merging firms is increased independently of the “inefficiency” of the followers. On the other hand, as followers become relatively less efficient, leaders take less into account the rivalry of the followers and, therefore, some of the nonmerging firms reduce their output after the merger. In this case, when the number of inefficient followers is reduced, leaders use less their “strategic power” to anticipate a large output. The consequence is that leaders cut production and this reduction renders the merger between followers profitable. Therefore, although in a linear (symmetric or asymmetric) Cournot model the merger of two symmetric firms is not profitable, we found that merger profitability with quantity competition depends crucially on the involved firms’ “strategic power” and on cost asymmetries.

Regarding the welfare analysis, the literature on the subject states that, in the absence

of any cost saving,⁶ many welfare-lowering mergers are unprofitable for the involved firms (and thus unlikely to happen), and that some profitable mergers are welfare raising (see for instance Daughety (1990)). However, we showed that in a two-stage oligopoly model where firms compete in quantities, welfare-lowering mergers may also be profitable.

Appendix

Proof of Proposition 1. The merger is profitable if:

$\Pi_l(n - m, k - m, c) - (m + 1)\Pi_l(n, k, c) = \frac{(\frac{1}{(1-m+k)^2} - \frac{1+m}{(1+k)^2})(1+c(n-k))^2}{(1-k+n)} > 0$. We can see that it does not depend on c if $c > 0$. It is only positive if $m \geq k - \frac{1}{2}(\sqrt[2]{4k+5} - 1)$. ■

Proof of Proposition 2. For the symmetric case ($c = 0$) the merger is profitable if: $\Pi_l(n - m, k, 0) - m\Pi_f(n, k, 0) - \Pi_l(n, k, 0) = \frac{m^2}{(1+k)^2(1-m+n-k)(1+n-k)^2} > 0$. Then, we only have to check that $\frac{\partial(\Pi_l(n-m,k,c)-m\Pi_f(n,k,c)-\Pi_l(n,k,c))}{\partial c} \Big|_{c=0} = 2(\frac{\frac{1}{m+k-n-1} + \frac{1+k}{(1-k+n)^2} + \frac{1+k+km}{1-k+n}}{(1+m)^2}) > 0$.

■

Proof of Remark 1. The merger of a leader and $m + 1$ followers is profitable if: $\Pi_l(n - m, k, c) - m\Pi_f(n, k, c) - \Pi_l(n, k, c) = \Pi^m(n, k, m, c) > 0$. Therefore, we have to prove that $\frac{\partial \Pi^m(n, k, m, c)}{\partial m} = \frac{-c^2(1+k^2) + \frac{2(c-1)ck}{-1+k-n} + \frac{(c-1)^2}{(-1+k+m-n)^2} - \frac{(c-1)^2}{(1-k+n)^2}}{(1+k)^2} > 0$. It is easy to see that $\frac{(c-1)^2}{(-1+k+m-n)^2} - \frac{(c-1)^2}{(1-k+n)^2} > 0$. Then, $-c^2(1+k^2) + \frac{2(c-1)ck}{-1+k-n}$ can be written as $\frac{-c(1+k^2-k^3+n+k^2n)+2k}{1+n-k}$ which is a decreasing function of c . We only have to check that this expression is positive when evaluated at the highest possible c given by (2.1). In this case, the expression equals to $\frac{(-1+k^2)(-1+k-n)}{-1+k^2-k(1+n)} > 0$. ■

Proof of Proposition 3. This merger is profitable if $\Pi_f(n - m, k, c) - (m + 1)\Pi_f(n, k, c) = \Pi^m(n, k, m, c) > 0$. If we denote the number of followers by s , ($s = n - k$), the incentives to merge can be also written as a function of s :

$\Pi^m(n, k, m, c) \Big|_{n=k+s} = \Pi^m(s, k, m, c) = \frac{(-1+c(1+k(1-m+s)))^2}{(1+k)^2(1-m+s)^2} - \frac{(1+m)(-1+c(1+k(1+s)))^2}{(1+s)^2(1+k)^2}$. It is tedious but straightforward to show that $\frac{\partial^2 \Pi^m(s, k, c, m)}{\partial^2 c} < 0$ and thus, $\Pi^m(s, k, c, m)$ is a concave function. Also, $\exists c_1, c_2$ such

⁶See Farrell and Shapiro (1990) for a formal definition of mergers creating “synergies” so that the marginal costs of the firms do not remain unchanged after the merger.

that $\Pi^m(s, k, m, c_1) = \Pi^m(s, k, m, c_2) = 0$ where

$$c_1 = \frac{1+m+\sqrt{k^2(1+m)(1+s)^2(1-m+s)^2-(m-s)(m-s+k(-1+m-s)(1+s))}}{1+m-m(m-s+k(-1+m-s)(1+s))} \text{ and}$$

$$c_2 = \frac{1+m-\sqrt{k^2(1+m)(1+s)^2(1-m+s)^2-(m-s)(m-s+k(-1+m-s)(1+s))}}{1+m-m(m-s+k(-1+m-s)(1+s))}, \text{ being } c_1 < c_2. \text{ Condition}$$

(2.1) is equivalent to $c < \frac{1}{1+k(1+s)} < 1$. Then, we have that $c_2 > \frac{1}{1+k(1+s)} \forall k, s, m > 0$. We have 2 different cases:

1) When $m > s - \frac{1}{2}(\sqrt{4s+5} - 1)$. Note that this condition is exactly the same in SSR for the Cournot case (here means that at least 80% of the followers merge). In this case, $\Pi^m(s, k, m, 0) > 0$, $c_1 < 0$ and $\forall c < \frac{1}{1+k(1+s)}$ a merger between $m+1$ followers is profitable.

2) When $m \leq s - \frac{1}{2}(\sqrt{4s+5} - 1)$, we have that $\Pi^m(s, k, m, 0) \leq 0$, $c_1 \geq 0$ and $\Pi^m(s, k, m, c) \geq 0 \forall c \geq c_1$. Less than 80% of the followers merge and a merger between $m+1$ followers is only profitable if $c > c_1$. ■

Proof of Proposition 4. The merger of two followers is profitable if $\Pi_a(c, n, 2) - \Pi_b(c, n) = \frac{-3(-2+c(4+n))^2}{32(2+n)^2} + \frac{5(-6+c(10+3n))^2}{72(4+3n)^2} > 0$. We have two different cases:

If $n = 2$, then $\Pi_a(c, 2, 2) - \Pi_b(c, 2) = \frac{9+(42-191c)}{5760}$ which is positive for all $c < \frac{2}{4+n} = \frac{1}{3}$.

If $n \geq 3$, then $\Pi_a(0, n, 2) - \Pi_b(0, n) = \frac{5}{2(4+3n)^2} - \frac{3}{8(2+n)^2}$ which is negative for all $n \geq 3$.

In this case, $\Pi_a(c, n, 2) - \Pi_b(c, n) = 0$ has two roots in c . We denote them by:

$$c_1 = \frac{6(4(56+\sqrt{15}\sqrt{(2+n)^2(4+3n)^2})+n(32-n(100-21n)))}{1088-n(16+3n)(64+7n(16+3n))} \text{ and}$$

$$c_2 = \frac{6(4(-56+\sqrt{15}\sqrt{(2+n)^2(4+3n)^2})+n(-32+n(100-21n)))}{-1088+n(16+3n)(64+7n(16+3n))}. \text{ It is straightforward to check that}$$

$c_1 < \frac{2}{4+n} < c_2$. We note also that $c_1 > 0$ if $n \geq 3$. Then, the merger is only profitable when $c > c_1$. ■

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