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Revenue Sharing and Competitive Balance in a Dynamic Contest Model

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Abstract

This paper presents a dynamic model of talent investments in a team sports league with an infinite time horizon. We show that the clubs' investment decisions and the effects of revenue sharing on competitive balance depend on the following three factors: (i) the cost function of talent investments, (ii) the clubs' market sizes, and (iii) the initial endowments of talent stock. We analyze how these factors interact in the transition to the steady state as well as in the steady state itself.

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1 Introduction

The uncertainty of outcome hypothesis is probably the most unique characteristic of the professional team sports industry. According to this hypothesis, fans prefer to attend contests with an uncertain outcome and enjoy close championship races.[1] Unlike Wal-Mart, Sony, and BMW who benefit from weak competitors in their industries, FC Barcelona and the New York Yankees need strong competitors to fill their stadiums. Since weak teams produce negative externalities on strong teams, many professional sports leagues have introduced revenue sharing arrangements to, at least partly, internalize these externalities and increase competitive balance. However, the economic effect of such revenue-sharing arrangements is heavily disputed in the literature.

Talent investments in professional sports clubs are a dynamic phenomenon. Since the majority of players sign multiple year contracts, most of the talent acquired in this season will also be available in the next season. Thus, today's talent investments determine tomorrow's talent stock and expected future profits. From our point of view, a major shortcoming of the sports economic literature is the disregard of this inter-temporal investment effect.

Almost all contributions consider static models focusing on one period only (e.g., see Atkinson et al. (1988), Dietl and Lang (2008), Fort and Quirk (1995), Szymanski and Késenne (2004) and Vrooman (1995, 2008)). Static models, however, do not analyze the dynamics leading to convergence or divergence of clubs' playing strengths, and therefore they cannot differentiate between the short and long run effects of revenue sharing on competitive balance.

One exception is El-Hodiri and Quirk (1971) who develop a dynamic decision-making model of a professional sports league. They confirm the "invariance proposition" and show that revenue sharing does not influence competitive balance. Their model, however, is based on some critical assumptions. First, they assume a fixed supply of talent because the total amount of talent is exogenously given in their model.[2] Second, the specification of the club's

[1] See e.g., Lee and Fort (2008).

[2] As Szymanski (2004) has shown, the assumption of fixed talent supply is often used to justify Walrasian fixed-supply instead of Contest-Nash conjectures. Under Walrasian fixed-supply conjectures, the quantity of

cost function is restrictive since they assume constant marginal costs. Our analysis shows that the cost function has a significant effect on the transitional dynamics in the model.

Grossmann and Dietl (2009) analyze the effect of revenue sharing in the context of a two-period model. They focus on the effect of different equilibrium concepts (open-loop and closed-loop equilibria) on clubs' optimal investment decisions. This two-period model, however, does not allow any conclusions regarding the possible convergence of clubs' playing strengths. An infinite period model is required to analyze these dynamics aspects.

In this paper, we account for the dynamic perspective of clubs' talent investments by developing a dynamic model with an infinite time horizon. In each period, two profit-maximizing clubs invest in playing talent in order to accumulate talent stock, which depreciates over time. The available stock of playing talent determines the clubs' winning percentages in each period, which ultimately, determine clubs' revenues. We show that the clubs' investment decisions and the effect of revenue sharing on competitive balance depend on a combination of the following three factors: (i) the cost function of talent investments, (ii) the clubs' market sizes and (iii) the initial endowments of talent stock. We analyze how these factors interact in the transition to the steady state (short run) as well as in the steady state itself (long run).

The remainder of this paper is organized as follows. In Section 2, we explain the model. The results are presented in Section 3. In Subsection 3.1, we solve the dynamic problem and analyze the efficiency conditions. In Subsection 3.2, we compute the steady states and derive comparative statics. In Subsection 3.3, we analyze the transitional dynamics of the model for symmetric initial endowments, and in Subsection 3.4 for asymmetric initial endowments. Finally, Section 4 concludes.

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talent hired by at least one club owner is determined by the choices of all other club owners.

2 Model Specification

The following dynamic model describes the investment behavior of two profit-maximizing clubs which compete in a professional team sports league. The investment horizon comprises an infinite number of periods in discrete time. We interpret one period as one season, where expected profits in period $t \in \{0, ..., \infty\}$ are discounted by β^t with $\beta \in (0, 1)$.

In each period t, club $i \in \{1, 2\}$ invests a certain amount $\tau_{i,t} \geqslant 0$ into playing talent in order to accumulate a stock of playing talent, $T_{i,t} \geq 0$, which depreciates over time. We assume that playing talent is measured in perfectly divisible units that can be hired at a competitive market for talent, generating strictly convex costs $c(\tau_{i,t})$. Thus, $c'(\tau_{i,t}) > 0$ and $c''(\tau_{i,t}) > 0$ for $\tau_{i,t} > 0$, $t \in \{0, ..., \infty\}$.[3]

The stock of playing talent $T_{i,t}$ linearly increases (ceteris paribus) through talent investments $\tau_{i,t}$ in period t. Thus, $T_{i,t}$ is a state variable and is given by the talent accumulation equation

$$T_{i,t} = (1 - \delta)T_{i,t-1} + \tau_{i,t}, \; i \in \{1, 2\}, \; t \in \{0, ..., \infty\} \tag{1}$$

where $\delta \in (0, 1)$ represents the depreciation factor. Equation (1) shows that replacements are necessary in order to maintain the existing stock of playing talent. Before the competition starts, i.e., in period $t = -1$, each club i is assumed to have initial endowments of talent stock given by $T_{i,-1} \geq 0$.

In each period t, the talent stock determines the clubs' win percentages. The win percentage of club i is characterized by the contest-success function (CSF) which maps club i's and club j's talent stock $(T_{i,t}, T_{j,t})$ into probabilities for each club.[4] We apply the logit approach, which is the most widely used functional form of a CSF in sporting contests.[5] The

[3] Note that, in Section 3.4.1, we consider linear costs $c(\tau_{i,t}) = \theta\tau_{i,t}$ with a constant marginal cost parameter θ such that $c'(\tau_{i,t}) = \theta > 0$ and $c''(\tau_{i,t}) = 0$.

[4] In the subsequent analysis $i, j \in \{1, 2\}$, $j \neq i$ and $t \in \{0, ..., \infty\}$, if not otherwise stated.

[5] The logit CSF was generally introduced by Tullock (1980) and subsequently axiomatized by Skaperdas (1996) and Clark and Riis (1998). An alternative functional form would be the probit CSF (e.g., Lazear and Rosen, 1981; Dixit, 1987) and the difference-form CSF (e.g., Hirshleifer, 1989).

win percentage of club i in period t is then given by

$$w_i(T_{i,t}, T_{j,t}) = \frac{T_{i,t}^{\gamma}}{T_{i,t}^{\gamma} + T_{j,t}^{\gamma}}. \tag{2}$$

Note that club i's win percentage is an increasing function of its own talent stock. We define $w_i(T_{i,t}, T_{j,t}) := 1/2$, if $T_{i,t}^{\gamma} = T_{j,t}^{\gamma} = 0$. Given that the win percentages must sum up to unity, we obtain the adding-up constraint: $w_j = 1 - w_i$.

Moreover, we assume that the supply of talent is elastic. As a consequence, we consider the so-called Nash equilibrium model rather than the Walrasian equilibrium model, and we thus adopt the "Contest-Nash conjectures" $\frac{\partial \tau_{i,t}}{\partial \tau_{j,t}} = 0$.[6]

The parameter $\gamma > 0$ is called the "discriminatory power" of the CSF and reflects the degree to which talent affects the win percentage.[7] As γ increases, the win percentage for the club with the higher talent stock increases, and differences in the talent stock affect the win percentage in a stronger way. In the limiting case where γ goes to infinity, we would have a so-called 'all-pay auction', i.e., a perfectly discriminating contest.

The revenue function of club i is given by $R_i(w_i, m_i)$ and is assumed to have the following properties:[8] either $\frac{\partial R_i}{\partial w_i} > 0$ and $\frac{\partial^2 R_i}{\partial w_i^2} \leq 0$ for all $w_i \in [0,1]$ or $\exists\ w_i^* \in [0,1]$ such that if $w_i \geq w_i^*$, then $\frac{\partial R_i}{\partial w_i} < 0$, otherwise $\frac{\partial R_i}{\partial w_i} > 0$, and $\frac{\partial^2 R_i}{\partial w_i^2} \leq 0$ everywhere. In order to guarantee an equilibrium, we assume that $w_i^* \geq 0.5$ for at least one club. The parameter $m_i > 0$ represents the market size of club i. To make further progress and to derive closed form solutions, we have to simplify the model. We assume that the revenue function of club i is

[6] According to Szymanski (2004), only the Contest-Nash conjectures are consistent with the concept of Nash equilibrium (see also Szymanski and Késenne, 2004 and Késenne, 2007). However, the disagreement regarding "Nash conjectures" vs. "Walrasian conjectures" remains an open area for research. For instance, Fort and Quirk (2007) describe a competitive talent market model, which is consistent with a unique rational expectation equilibrium (see also Fort, 2006).

[7] We are grateful to an anonymous referee who suggested that we integrate this parameter in our model. See also Dietl et al. (2008) and Fort and Winfree (2009) for an analysis of the parameter γ in a static model.

[8] See Szymanski and Késenne (2004) on p. 168.

linear in its own win percentage and is specified by

$$R_i(w_i(T_{i,t}, T_{j,t}), m_i) = m_i \cdot w_i(T_{i,t}, T_{j,t}).$$

This revenue function has the desired properties and is consistent with the revenue function used e.g., by Dietl et al. (2009), Hoehn and Szymanski (1999), Szymanski (2003) and Vrooman (2007, 2008).[9] Note that Szymanski and Késenne (2004) also use an identical revenue function in Section III on page 171.

Moreover, we introduce a gate revenue sharing arrangement. The after-sharing revenues of club i, denoted by $\widehat{R}_i$, can be written as

$$\widehat{R}_i = \alpha R_i + (1-\alpha) R_j = \alpha \frac{m_i T_{i,t}^{\gamma}}{T_{i,t}^{\gamma} + T_{j,t}^{\gamma}} + (1-\alpha) \frac{m_j T_{j,t}^{\gamma}}{T_{i,t}^{\gamma} + T_{j,t}^{\gamma}},$$

with $\alpha \in (\frac{1}{2}, 1]$. From the home match, club i obtains share α of its own revenues R_i, and from the away match, it obtains share $(1-\alpha)$ of club j's revenues R_j. Note that a higher parameter α represents a league with a lower degree of redistribution. Thus, the limiting case of $\alpha = 1$ describes a league without revenue sharing.

Club i's expected profits $E[\pi_{i,t}]$ in period t are given by after-sharing revenues minus costs, i.e.,

$$E[\pi_{i,t}] = \widehat{R}_i(T_{i,t}, T_{j,t}) - c(\tau_{i,t}).$$

Club i maximizes its expected discounted profits $\sum_{t=0}^{\infty} \beta^t E[\pi_{i,t}]$ with respect to the stream $\{\tau_{i,t}\}_{t=0}^{\infty}$ and subject to $T_{i,t} = (1-\delta)T_{i,t-1} + \tau_{i,t}$. We assume that both clubs have an outside option of zero profits before the competition starts.

In order to solve the model in an infinite horizon model, we use the open-loop equilibrium concept, which facilitates computations.[10]

[9] Even though the revenue function is quadratic in own win percentages in the mentioned articles, only the part where $\frac{\partial R_i}{\partial w_i} > 0$ is relevant for their analysis. It is obvious that equilibria in which $\frac{\partial R_i}{\partial w_i} < 0$ holds do not exist. Moreover, the following proofs hold for all $\gamma \in (0, \infty)$. However, if $\gamma > 1$, the revenue function has both convex and concave parts. Therefore, the existence of a maximum is only guaranteed, if $0 < \gamma \leq 1$.

[10] See, for instance, Fudenberg and Tirole (1991). Their paper discusses the differences between the two

3 Results

3.1 Dynamic Program

We solve the dynamic program for club i by Bellman:[11]

$$v(T_{i,t-1}) = \max_{\tau_{i,t}, T_{i,t}} \left\{ \widehat{R}_i(T_{i,t}, T_{j,t}) - c(\tau_{i,t}) + \beta v(T_{i,t}) \right\} \qquad \text{s.t.} \qquad T_{i,t} = (1-\delta)T_{i,t-1} + \tau_{i,t}$$

Note that $v(\cdot)$ represents the club's value function. Moreover, club i takes $T_{j,t}$ as given in period $t \in \{0, ..., \infty\}$ according to the open-loop concept. The associated Lagrangian $\mathcal{L}$ with multiplier λ_t has the following form:

$$\mathcal{L} = \alpha \frac{m_i T_{i,t}^{\gamma}}{T_{i,t}^{\gamma} + T_{j,t}^{\gamma}} + (1-\alpha) \frac{m_j T_{j,t}^{\gamma}}{T_{i,t}^{\gamma} + T_{j,t}^{\gamma}} - c(\tau_{i,t}) + \beta v(T_{i,t}) + \lambda_t[(1-\delta)T_{i,t-1} + \tau_{i,t} - T_{i,t}]$$

The corresponding first order conditions are given by

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \tau_{i,t}} &= -c'(\tau_{i,t}) + \lambda_t = 0, \\
\frac{\partial \mathcal{L}}{\partial T_{i,t}} &= \alpha \frac{\gamma m_i T_{i,t}^{\gamma-1} T_{j,t}^{\gamma}}{\left(T_{i,t}^{\gamma} + T_{j,t}^{\gamma}\right)^2} - (1-\alpha) \frac{\gamma m_j T_{i,t}^{\gamma-1} T_{j,t}^{\gamma}}{(T_{i,t}^{\gamma} + T_{j,t}^{\gamma})^2} + \beta \frac{\partial v(T_{i,t})}{\partial T_{i,t}} - \lambda_t = 0, \\
\frac{\partial \mathcal{L}}{\partial \lambda_t} &= (1-\delta)T_{i,t-1} + \tau_{i,t} - T_{i,t} = 0.
\end{aligned} \tag{3}$$

The envelope theorem gives us $\frac{\partial \mathcal{L}}{\partial T_{i,t-1}} = \frac{\partial v(T_{i,t-1})}{\partial T_{i,t-1}} = \lambda_t(1-\delta)$. Using the first order conditions and the updated envelope theorem, and assuming that clubs have identical market sizes, i.e.,

concepts. Generally, they argue that in case of many agents the differences between the closed-loop and open-loop equilibria are negligible. Moreover, Grossmann and Dietl (2009) show that the open-loop and closed-loop equilibrium coincide in a similar two-period model if costs are linear.

[11] In order to solve the model, we follow King et al. (1988): In a first step, we solve the dynamic problem and analyze the efficiency conditions (Euler equations). Then we compute the steady states (long run), and afterwards, we analyze the transitional dynamics (short run). Note that, henceforth the results are only presented for club i. The corresponding results for club j can be found by changing subscripts i and j.

$m_i = m_j = m$,[12] we get the following Euler equation for club i:

$$(2\alpha - 1)\frac{\gamma m T_{i,t}^{\gamma-1} T_{j,t}^{\gamma}}{(T_{i,t}^{\gamma} + T_{j,t}^{\gamma})^2} = c'(\tau_{i,t}) - \beta(1-\delta)c'(\tau_{i,t+1}) \tag{4}$$

Equation (4) reflects the well-known inter-temporal trade-off: marginal benefit of an investment into talent (left hand side) must equal marginal cost of talent (right hand side) in an optimum. Note that the marginal benefit of an investment is increasing in α and m. The first term on the r.h.s of the equation indicates the instantaneous marginal cost of an investment, whereas the second term on the r.h.s. represents the inter-temporal effect of today's investment. That is, an investment of one unit today reduces marginal costs tomorrow, which has to be discounted by $\beta(1-\delta)$.

Moreover, we can solve the Euler equation (4) recursively forward and get the following result for club i:

$$(2\alpha - 1)\gamma m \sum_{k=0}^{T} \left\{ [\beta(1-\delta)]^k \frac{T_{i,t+k}^{\gamma-1} T_{j,t+k}^{\gamma}}{\left(T_{i,t+k}^{\gamma} + T_{j,t+k}^{\gamma}\right)^2} \right\} = c'(\tau_{i,t}) - \underbrace{[\beta(1-\delta)]^{T+1} c'(\tau_{i,t+T+1})}_{T \overset{\to \infty}{=} 0}$$

Note that the second term on the right hand side vanishes as T converges to infinity since $\beta(1-\delta) \in (0,1)$ such that

$$(2\alpha - 1)\gamma m \sum_{k=0}^{\infty} \left\{ [\beta(1-\delta)]^k \frac{T_{i,t+k}^{\gamma-1} T_{j,t+k}^{\gamma}}{\left(T_{i,t+k}^{\gamma} + T_{j,t+k}^{\gamma}\right)^2} \right\} = c'(\tau_{i,t}).$$

Today's marginal cost of an investment (r.h.s.) equals the sum of today's and (all) discounted future expected marginal benefits (l.h.s.).

[12] In Section 3.4.1, we extend our model and allow for clubs that have different market sizes. For this purpose, we simultaneously have to simplify the cost function by assuming linear costs.

3.2 Steady States

Generally, in a steady state all variables grow with a constant rate. In this model, however, we have a stationary economy such that the growth rate is zero. Thus, $T_{i,t} = T_{i,t+1} \equiv T_i$ in a steady state. Equation (1) implies that $\tau_i = \delta T_i$ in a steady state, i.e., the amount of playing talent which is lost through depreciation is replaced by newly recruited players.

By neglecting the time subscript t, we rewrite the Euler equation (4) for club i as follows:[13]

$$(2\alpha - 1)\frac{\gamma m T_i^{\gamma-1} T_j^{\gamma}}{(T_i^{\gamma} + T_j^{\gamma})^2} = (1 - \beta(1-\delta))c'(\tau_i) \tag{5}$$

Dividing equation (5) by the corresponding Euler equation for the other club j, we derive $\frac{T_j}{T_i} = \frac{c'(\tau_i)}{c'(\tau_j)}$ and can establish the following proposition:

Proposition 1

If $m_i = m_j$, then $T_i = T_j \equiv T$ and $\tau_i = \tau_j \equiv \tau$ in the steady state (independent of the distribution of initial endowments). As a consequence, revenue sharing has no effect on competitive balance in the long run.

Proof. See Appendix 5.1 ■

Proposition 1 implies that talent investments and the talent stock are identical for both clubs in the steady state, i.e., there is not only relative convergence but also absolute convergence of talent stocks in the long run as long as clubs have identical market sizes. This result holds even if clubs started with different initial endowments $T_{i,-1}$ and $T_{j,-1}$. It follows that revenue sharing has no effect on competitive balance in the steady state, and therefore, the invariance proposition holds in the long run.

Nonetheless, the question remains whether and how quickly the steady state is achieved. The transitional dynamics are discussed in the next sections, where we show how revenue sharing influences competitive balance in the short run, i.e., during the talent accumulation

[13] Henceforth, variables without a time subscript indicate steady states.

process. Prior to this, we first derive the comparative statics of the steady state talent stock and investment.

According to equation (5) and the results of Proposition 1, we implicitly get the steady state values T and $\tau = \delta T$:

$$(2\alpha - 1)\frac{\gamma m}{4T} = (1 - \beta(1 - \delta))c'(\delta T) \tag{6}$$

Comparative statics lead to the following proposition:

Proposition 2

(i) Talent stock T in the steady state is increasing in m, γ, β and α, but decreasing in δ.

(ii) Talent investment τ in the steady state is increasing in m, γ, β, α and δ.

Proof. See Appendix 5.2 ■

Proposition 2 (i) shows that a larger market size, a higher discriminatory power, a higher discount rate and/or a lower degree of revenue sharing (i.e., a higher α) imply a higher talent stock in the steady state.[14] On the other hand, a higher depreciation rate reduces incentives to accumulate talent in the steady state.

Since higher parameters m, γ, β and α imply a higher talent stock T in the steady state, it is necessary to increase the steady state talent investment τ in order to sustain this higher talent stock T. Thus, τ is increasing in the aforementioned parameters as stated in Proposition 2 (ii). Furthermore, a higher depreciation factor also increases the steady state talent investment.[15]

[14] If the market size and/or the revenue sharing parameter are increasing, then it is quite intuitive that incentives to invest in talent are also increasing due to higher marginal benefits of talent investments. A higher discriminatory power implies a higher marginal revenue in the steady state, which also leads to a higher talent stock. Furthermore, we observe a higher talent stock in the long run for a higher discount rate β. Hence, as future expected profits get less discounted, clubs invest more in talent accumulation.

[15] A higher depreciation factor δ, however, has two effects on the steady state talent investments $\tau = \delta T(\delta)$. First, a higher δ reduces the talent stock $T(\delta)$ such that the steady state investment τ is lower in order to maintain the talent stock. Second, a higher δ also implies that clubs have to invest more in talent in order to maintain the steady state talent stock. Thus, a higher depreciation factor implies higher talent investments. The second effect dominates the first effect in the model such that τ is increasing in δ.

3.3 Transitional Dynamics with Symmetric Initial Endowments

In this section, we assume that both clubs have identical initial endowments. That is, the initial talent stock in period $t = -1$ is the same for both clubs with $T_{i,-1} = T_{j,-1} \equiv T_{-1}$.[16] This assumption has special implications for the clubs' optimal investment behavior. Equation (4) implies

$$\frac{T_{j,t}}{T_{i,t}} = \frac{c'(\tau_{i,t}) - \beta(1-\delta)c'(\tau_{i,t+1})}{c'(\tau_{j,t}) - \beta(1-\delta)c'(\tau_{j,t+1})} \tag{7}$$

We derive the following results:

Proposition 3

If $T_{i,-1} = T_{j,-1} \equiv T_{-1}$, then $\tau_{i,t} = \tau_{j,t} \equiv \tau_t$ for all $t \in \{0, ..., \infty\}$. Therefore, symmetric initial endowments imply that clubs' talent investment and talent stock are identical in each period.

Proof. See Appendix 5.3 ■

Proposition 3 shows that both clubs optimally invest an identical amount in talent in each period as long as initial endowments of talent stock ($T_{i,-1}$ and $T_{j,-1}$) are identical. Thus, we can neglect clubs' subscripts i and j in this subsection.

The optimal path of talent investments, however, cannot be explicitly determined in case of a general cost function. The dynamics are implicitly characterized by the Euler equation (4), the talent accumulation equation (1), the initial endowments, and the results of Proposition 3. Even though we are not able to explicitly solve the model, we can plot the dynamics in a phase diagram, where we have to consider the dynamics of T_t and τ_t separately.[17]

For all initial endowments T_{-1}, there is a unique value τ_0 such that the dynamic path leads into the steady state. The unique value τ_0 is determined by the saddle path in Figure 1.

[16] Note that, even in a perfectly symmetric contest, symmetric club investments are not compulsory ex ante. We can show in this section, however, that a symmetric investment behaviour is the unique solution in our model.

[17] Note that the dynamics are just approximately true in a phase diagram because the model is based on discrete time and the phase diagram rather qualifies for continuous time. Nevertheless, we use the phase diagram to strengthen our intuition. In Appendix 5.4, we derive the computations for this phase diagram.

This saddle path is consistent with the efficiency conditions and the accumulation equations. Note that if $T_{-1} < T$, then initial talent investments τ_0 are higher than the steady state talent investments τ. Otherwise, if $T_{-1} > T$, then initial investments τ_0 are lower than the steady state talent investments τ. In both cases, the dynamic path leads to the steady state.

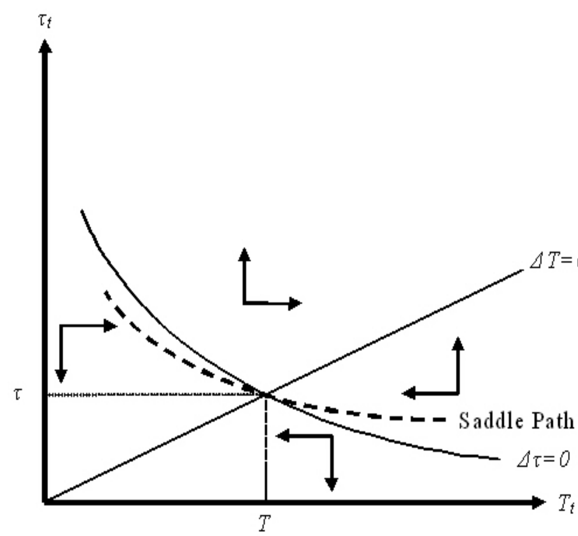

Figure 1: Saddle Path in the Phase Diagram

3.4 Transitional Dynamics with Asymmetric Initial Endowments

In this section, we assume that both clubs have different initial endowments in period $t = -1$, i.e., $T_{i,-1} \neq T_{j,-1}$. Again, the Euler equation, the talent accumulation equation and the initial endowments represent the dynamics of the model and characterize the clubs' optimal investment behavior.

It is not possible to solve this model explicitly to provide an explicit computation of the investment path in the transition to the steady state. As a consequence, we further specify the cost function and consider linear cost in the next subsection. In case of linear costs, we are able to explicitly compute the steady state variables and to determine the clubs' optimal investment in each period. In Subsection 3.4.2, we consider a quadratic cost function and derive the optimal investment path through a simulation.[18]

3.4.1 Linear Cost Function

In this subsection, we consider linear costs $c(\tau_{i,t}) = \theta\tau_{i,t}$ and, simultaneously, allow for different market sizes.[19] Without loss of generality, we assume that club i has a larger market size than club j such that $m_i > m_j > 0$. Due to the larger market size, club i generates higher revenues for a given win percentage than club j. We get the following Euler equation for club i:

$$(\alpha m_i - (1-\alpha)m_j)\frac{\gamma T_{i,t}^{\gamma-1}T_{j,t}^{\gamma}}{(T_{i,t}^{\gamma}+T_{j,t}^{\gamma})^2} = \theta[1-\beta(1-\delta)]$$

Hence, club i's talent stock in each period $t \in \{0, ..., \infty\}$ is given by

$$T_{i,t} = \frac{\gamma\left(\alpha m_i - (1-\alpha)m_j\right)^{\gamma+1}\left(\alpha m_j - (1-\alpha)m_i\right)^{\gamma}}{\theta[1-\beta(1-\delta)]\left[\left(\alpha m_j - (1-\alpha)m_i\right)^{\gamma} + \left(\alpha m_i - (1-\alpha)m_j\right)^{\gamma}\right]^2}.$$

Thus, the steady state is attained immediately in the first period, i.e. in period zero, regardless of initial endowments of talent stock. Moreover, we derive that club i's talent stock is higher than club j's talent stock in each period because[20]

$$\frac{T_{i,t}}{T_{j,t}} = \frac{\alpha m_i - (1-\alpha)m_j}{\alpha m_j - (1-\alpha)m_i} > 1,$$

for all $t \in \{0, ..., \infty\}$. It follows that club i is the dominant team that has a higher win percentage in each period $t \in \{0, ..., \infty\}$ compared to club j because $\left(\frac{w_{i,t}}{w_{j,t}}\right)^{\frac{1}{\gamma}} = \frac{T_{i,t}}{T_{j,t}} > 1$ independent of initial endowments. It follows that, even if club j had higher initial endowments in $t = -1$, there would be an immediately leapfrogging by club i such that club i would

[18] Furthermore, we briefly discuss the main results of an n-club league with $n > 2$ in Appendix 5.5.

[19] Note that we are able to relax the restrictive assumption of identical market sizes in this subsection since we have simplified the model by using linear costs.

[20] We assume that $\alpha m_j - (1-\alpha)m_i > 0$ in order to guarantee positive equilibrium investments by club j (see also Szymanski and Késenne (2004)).

overtake club j with respect to the talent stock and win percentage in $t = 0$ (see Figure 2).

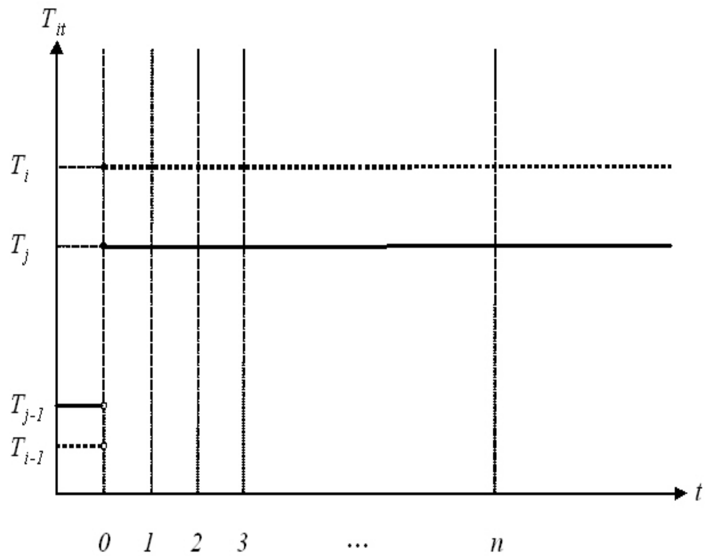

Figure 2: Leapfrogging of Talent Stocks

These results show that, if costs are linear and $m_i \neq m_j$, convergence to different steady states occurs in the first period, i.e., in $t = 0$, such that the league is characterized through a persistent inequality.[21]

What is the effect of revenue sharing in this case? We derive that the ratio $\frac{w_{i,t}}{w_{j,t}}$ is decreasing in the revenue sharing parameter α for all $t \in \{0, ..., \infty\}$. As a consequence, we get the following proposition:

Proposition 4

If costs are linear and $m_i \neq m_j$, a higher degree of revenue sharing (i.e., a lower α) decreases competitive balance (independent of the distribution of initial endowments).

Proof. Straightforward and therefore omitted. ■

[21] In case of identical market sizes, i.e., $m_i = m_j = m$, clubs' talent stocks also converge in $t = 0$ but the steady states are identical with $T_t = \frac{(2\alpha-1)\gamma m}{4\theta(1-\beta(1-\delta))} = T_{i,t} = T_{j,t}$ such that the league is perfectly balanced in the long run. Initial talent investments are then given by $\tau_{i,0} = \frac{(2\alpha-1)\gamma m}{4\theta(1-\beta(1-\delta))} - (1-\delta)T_{i,-1}$. Thus, club i invests more in the first period, the lower its initial endowments $T_{i,-1}$. For $t \geq 1$, both clubs exactly replace depreciated talent such that talent investments are given by $\tau_t = \delta T_t = \delta \frac{(2\alpha-1)\gamma m}{4\theta(1-\beta(1-\delta))}$.

This proposition shows that revenue sharing produces a more unbalanced league and thus the invariance principle does not hold. Note that our result in this dynamic setting generalizes the static finding of Szymanski and Késenne (2004).

3.4.2 Quadratic Cost Function

In this subsection, we consider a strictly convex cost function $c(\tau_i) = \frac{1}{2}\tau_i^2$. In order to focus on the effect of different initial endowments in the transition, we have to simplify matters by assuming that clubs have identical market sizes, i.e., $m_i = m_j = m$, such that the clubs' talent stocks are identical in the long run. According to equation (4), we derive the following Euler equation for club i:

$$(2\alpha - 1)\frac{\gamma m T_{i,t}^{\gamma-1} T_{j,t}^{\gamma}}{(T_{i,t}^{\gamma} + T_{j,t}^{\gamma})^2} = \tau_{i,t} - \beta(1-\delta)\tau_{i,t+1} \tag{8}$$

Together with the talent accumulation equation (1) and the initial endowments of talent stock $T_{i,-1}$, equation (8) determines club i's optimal behavior. In contrast to the previous subsection with linear costs, it is not possible to solve the model explicitly in the case of quadratic costs to derive equations for the talent stock and investment in each period. However, we are able to run three different simulations to get more insights into the transitional dynamics of the model.

For the three simulations, we fix the exogenous parameters as follows: $\delta = 0.05, \beta = 0.99, \gamma = 1$ and $m = 100$. For this parameterization, the steady state values, which are independent of initial endowments, are given by $T = 91.670$ and $\tau = 4.583$ for each club. Moreover, in the first two simulations, we consider a league without revenue sharing (i.e., $\alpha = 1$), whereas in the third simulation we vary α in order to analyze the effect of revenue sharing on competitive balance.[22]

[22] Note that the initial investments and optimal investment paths are computed by the "shooting method". We separately choose initial investments for each club in order to undershoot and overshoot the corresponding steady state talent stocks. In this way, we approximately determine the optimal investment paths.

Different Initial Endowments and the Speed of Convergence In a first simulation, we concentrate on the effect of different initial endowments of talent stocks. The results of the simulation are summarized in Table 1. Initial endowments of talent stock (initial investments) are illustrated in rows 1 and 2 (3 and 4). Note that we only vary initial endowments $T_{j,-1}$ for club j. For the benchmark case, represented in column 4, we consider clubs with identical initial endowments $T_{i,-1} = 50$ and $T_{j,-1} = 50$. The variables $half(T_i)$ and $half(T_j)$ in rows 5 and 6 measure the speed of convergence and indicate the period in which the talent stocks $T_{i,t}$ and $T_{j,t}$, respectively, have passed half of the way to the steady state talent stock.

$Simulation$	1	2	3	4	5	6	7
$T_{i,-1}$	50	50	50	50	50	50	50
$T_{j,-1}$	20	30	40	50	60	70	80
$\tau_{i,0}$	5.684	5.764	5.803	5.814	5.805	5.782	5.749
$\tau_{j,0}$	7.378	6.767	6.253	5.814	5.434	5.101	4.808
$half(T_i)$	8	8	8	8	8	8	8
$half(T_j)$	8	8	8	8	8	9	9
$Variation$	$T_{j,-1}$	$T_{j,-1}$	$T_{j,-1}$	$Benchmark$	$T_{j,-1}$	$T_{j,-1}$	$T_{j,-1}$

Table 1: Different Initial Endowments and the Speed of Convergence

Table 1 shows that the club with lower initial endowments invests more in talent compared to the other club in $t = 0$. It also follows that a higher difference in initial endowments implies an (inversely) higher difference in talent investments in the first period. The values $half(T_i)$ and $half(T_j)$ indicate that heterogeneity with respect to initial endowments does not have a large impact on the talent stocks' speed of convergence. Both clubs pass half of the way to the steady state talent stock after 8 or 9 periods.[23]

Moreover, in contrast to linear costs, convergence to the steady state does not occur in the first period if clubs have quadratic costs. Clubs' talent stocks smoothly converge over time.

[23] Even if $T_{j,-1} = 200$ and $T_{i,-1} = 50$, club i (club j) would pass half of the way in period 10 (period 9).

Redistribution and the Speed of Convergence In a second simulation, we concentrate on the effect of redistribution in clubs' initial endowments on the speed of convergence. That is, in contrast to the first simulation, we vary not only the initial endowments of club j but also the initial endowments of club i such that the sum of initial endowments remains constant. Table 2 summarizes the main results.

$Simulation$	1	2	3	4	5
$T_{i,-1}$	30	35	40	45	50
$T_{j,-1}$	70	65	60	55	50
$\tau_{i,0}$	6.628	6.425	6.221	6.017	5.814
$\tau_{j,0}$	4.999	5.203	5.407	5.610	5.814
$half(T_i)$	8	8	8	8	8
$half(T_j)$	9	9	8	8	8
$half(w_i)$	4	4	4	4	0
$half(w_j)$	4	4	4	4	0
$Variation$	$T_{i,-1}, T_{j,-1}$	$T_{i,-1}, T_{j,-1}$	$T_{i,-1}, T_{j,-1}$	$T_{i,-1}, T_{j,-1}$	$Benchmark$

Table 2: Redistribution and the Speed of Convergence

The simulation shows that redistribution of initial endowments also does not change the speed of convergence of the state variables because $half(T_i)$ and $half(T_j)$ do not vary significantly.

A league's policy maker, however, might also be interested in the speed of convergence of the win percentages. Therefore, we additionally consider the variables $half(w_i)$ and $half(w_j)$, representing the period in which the win percentages of club i and club j, respectively, have passed half of the way to the steady state win percentage given by 0.5. In this case also, we derive that redistribution of initial endowments has no effect on the speed of convergence of the win percentages.[24]

Revenue Sharing and the Speed of Convergence In a third simulation, we analyze how revenue sharing affects the speed of convergence of the win percentages. We consider

[24]It is clear that in the benchmark case with identical initial endowments (column 5), $half(w_i)$ and $half(w_j)$ equal zero because the stock of talent for both clubs will be identical in all periods (see also Section 3.3).

the same distribution of initial endowments as in the second simulation, however, now the revenue-sharing parameter α varies. Table 3 summarizes the main results.

Simulation	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
$half(w_i)$	11	8	6	5	4
$half(w_j)$	11	8	6	5	4

Table 3: Revenue Sharing and the Speed of Convergence

According to the simulation, we derive the following proposition:

Proposition 5

If costs are quadratic and $m_i = m_j$*, a higher degree of revenue sharing (i.e., a lower* α*) implies a lower speed of convergence of the win percentages in the transition (independent of redistribution of initial endowments).*

Proof. Follows from the simulation. ■

According to this proposition, a league's policy maker should implement a lower degree of revenue sharing in order to increase the speed of convergence of the win percentages in the transition.

Example 1 *For initial endowments* $T_{i,-1} = 30$ *and* $T_{j,-1} = 70$*, consider Figure* 3*. This figure shows that a lower* α *implies a lower speed of convergence of the win percentages. Note that the steady state win percentages are given by* $w_i = w_j = 0.5$ *and the variables representing half of the way to the steady states are* $half(w_i) = 0.4$ *and* $half(w_j) = 0.6$*. The respective win percentages pass* $half(w_i)$ *and* $half(w_j)$ *according to Table 3.*[25]

[25] Note that we obtain qualitatively similar figures after a redistribution of initial endowments, e.g., $T_{i,-1} = 40$ and $T_{j,-1} = 60$.

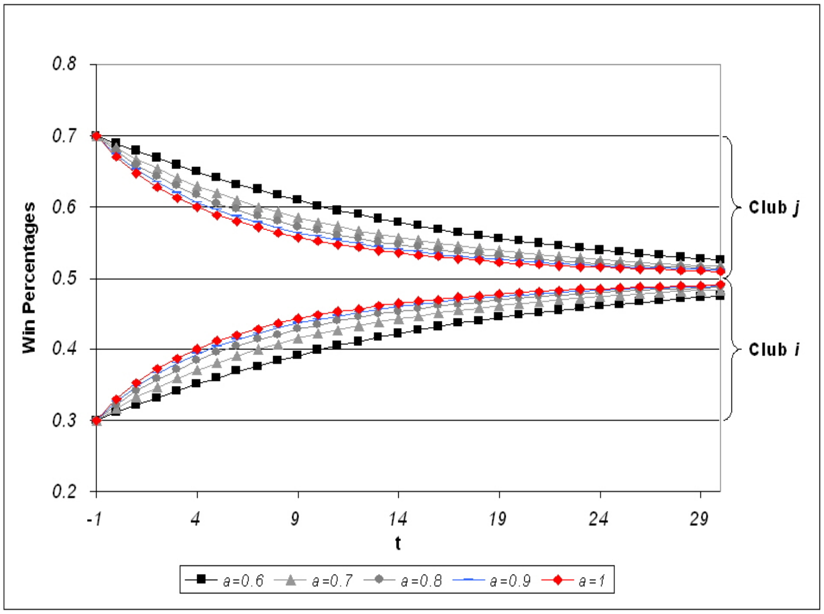

Figure 3: The Effect of Revenue Sharing on the Speed of Convergence

4 Conclusion

Investment decisions in professional team sports leagues are a dynamic economic phenomenon. Today's talent investments determine tomorrow's talent stock and expected future profits. We develop an infinite period model of a professional team sports league to show that, even if clubs have different initial talent endowments, the transitional dynamics will lead to a fully balanced league in the long run as long as clubs have the same market size. In this case, revenue sharing has no effect on competitive balance and thus the famous invariance principle holds.

Moreover, we show that the dynamics are influenced mainly by the cost function. In case of linear costs, convergence occurs immediately: the steady state is attained in the first period. Furthermore, if clubs differ in market size, then the steady state variables also differ, and the league is characterized by a persistent inequality regardless of the initial endowments. In this case, revenue sharing decreases competitive balance.

In case of a quadratic cost function, convergence to the steady state does not occur in the first period. Our simulation further shows that initial endowments affect initial investments. The club with lower initial endowments invests more in the first period than the club with higher initial endowments. Moreover, we derive that redistribution of initial endowments affects neither the speed of convergence of the state variables nor the speed of convergence of the win percentages. In this case, revenue sharing decreases the speed of convergence of the win percentages in the transition.

The current revenue-sharing schemes vary widely among professional sports leagues all over the world. The most prominent is possibly that operated by the National Football League (NFL), where the visiting club secures 40% of the locally earned television and gate receipt revenue. In 1876, Major League Baseball (MLB) introduced a 50-50 split of gate receipts that was reduced over time. Since 2003, all the clubs in the American League have put 34% of their locally generated revenue (gate, concession, television, etc.) into a central pool, which is divided then equally among all the clubs. In the Australian Football League (AFL), gate receipts were at one time split evenly between the home and the visiting team. This 50-50 split was finally abolished in 2000.

Our analysis suggests that a league policy maker should implement a lower degree of revenue sharing in order to increase the competitive balance (in case of linear costs) or the speed of convergence of clubs' win percentages (in case of quadratic costs). Whether clubs have linear or quadratic costs remains an empirical question and is left for further research.

5 Appendix

5.1 Proof of Proposition 1

First, we prove that $T_i = T_j$ and $\tau_i = \tau_j$ in a steady state. We provide a proof by contradiction: Suppose that $T_j > T_i$: using equation (5) for club i and club j, we get $\frac{T_j}{T_i} = \frac{c'(\tau_i)}{c'(\tau_j)}$. This implies that $c'(\tau_i) > c'(\tau_j)$. Strict convexity of the cost function yields $\tau_i > \tau_j$. Using $\tau_i = \delta T_i$ and $\tau_j = \delta T_j$ we get $T_i > T_j$, which is a contradiction to $T_j > T_i$. By symmetry, there is a contradiction, if we suppose that $T_i > T_j$. Therefore, we conclude that $T_i = T_j$. Furthermore, we get $\tau_i = \tau_j$ because $\tau_i = \delta T_i$ and $\tau_j = \delta T_j$.

Note that $T_i = T_j$ holds independent of α. Thus, $\frac{T_i^\gamma}{T_i^\gamma + T_j^\gamma} = w_i(T_i, T_j) = \frac{1}{2} = w_j(T_i, T_j) = \frac{T_j^\gamma}{T_i^\gamma + T_j^\gamma}$ is constant, which implies that revenue sharing has no effect on competitive balance in the long run. ■

5.2 Proof of Proposition 2

Here, we prove the comparative statics results. First, we define the function $F(T, \delta, \beta, m, \gamma, \alpha) \equiv (2\alpha - 1)\frac{\gamma m}{4T} - (1 - \beta(1-\delta))c'(\delta T)$. In a steady state $F(T, \delta, \beta, m, \gamma, \alpha) = 0$. Using the implicit function theorem we get:

$$\begin{aligned}
\frac{\partial T}{\partial m} &= -\frac{\frac{\partial F(T,\delta,\beta,m,\gamma,\alpha)}{\partial m}}{\frac{\partial F(T,\delta,\beta,m,\gamma,\alpha)}{\partial T}} = -\frac{(2\alpha-1)\frac{\gamma}{4T}}{-(2\alpha-1)\frac{\gamma m}{4T^2} - (1-\beta(1-\delta))c''(\delta T)\delta} > 0 \\
\frac{\partial T}{\partial \gamma} &= -\frac{\frac{\partial F(T,\delta,\beta,m,\gamma,\alpha)}{\partial \gamma}}{\frac{\partial F(T,\delta,\beta,m,\gamma,\alpha)}{\partial T}} = -\frac{(2\alpha-1)\frac{m}{4T}}{-(2\alpha-1)\frac{\gamma m}{4T^2} - (1-\beta(1-\delta))c''(\delta T)\delta} > 0 \\
\frac{\partial T}{\partial \beta} &= -\frac{\frac{\partial F(T,\delta,\beta,m,\gamma,\alpha)}{\partial \beta}}{\frac{\partial F(T,\delta,\beta,m,\gamma,\alpha)}{\partial T}} = -\frac{(1-\delta)c'(\delta T)}{-(2\alpha-1)\frac{\gamma m}{4T^2} - (1-\beta(1-\delta))c''(\delta T)\delta} > 0 \\
\frac{\partial T}{\partial \alpha} &= -\frac{\frac{\partial F(T,\delta,\beta,m,\gamma,\alpha)}{\partial \alpha}}{\frac{\partial F(T,\delta,\beta,m,\gamma,\alpha)}{\partial T}} = -\frac{\frac{\gamma m}{2T}}{-(2\alpha-1)\frac{\gamma m}{4T^2} - (1-\beta(1-\delta))c''(\delta T)\delta} > 0 \\
\frac{\partial T}{\partial \delta} &= -\frac{\frac{\partial F(T,\delta,\beta,m,\gamma,\alpha)}{\partial \delta}}{\frac{\partial F(T,\delta,\beta,m,\gamma,\alpha)}{\partial T}} = -\frac{-\beta c'(\delta T) - (1-\beta(1-\delta))c''(\delta T)T}{-(2\alpha-1)\frac{\gamma m}{4T^2} - (1-\beta(1-\delta))c''(\delta T)\delta} < 0
\end{aligned}$$

Thus, we conclude that T is increasing in m, γ, β, and α, but it is decreasing in δ, as stated in Proposition 2(i).

Second, we define the function $G(\tau, \delta, \beta, m, \gamma, \alpha) \equiv (2\alpha - 1)\frac{\delta\gamma m}{4\tau} - (1 - \beta(1-\delta))c'(\tau)$. In a steady state, $G(\tau,\delta,\beta,m,\gamma,\alpha)=0$. Using the implicit function theorem we get:

$$\begin{aligned}
\frac{\partial\tau}{\partial m} &= -\frac{\frac{\partial G(\tau,\delta,\beta,m,\gamma,\alpha)}{\partial m}}{\frac{\partial G(\tau,\delta,\beta,m,\gamma,\alpha)}{\partial\tau}} = -\frac{(2\alpha-1)\frac{\delta\gamma}{4\tau}}{-(2\alpha-1)\frac{\delta\gamma m}{4\tau^2}-(1-\beta(1-\delta))c''(\tau)} > 0\\
\frac{\partial\tau}{\partial\gamma} &= -\frac{\frac{\partial G(\tau,\delta,\beta,m,\gamma,\alpha)}{\partial\gamma}}{\frac{\partial G(\tau,\delta,\beta,m,\gamma,\alpha)}{\partial\tau}} = -\frac{(2\alpha-1)\frac{\delta m}{4\tau}}{-(2\alpha-1)\frac{\delta\gamma m}{4\tau^2}-(1-\beta(1-\delta))c''(\tau)} > 0\\
\frac{\partial\tau}{\partial\beta} &= -\frac{\frac{\partial G(\tau,\delta,\beta,m,\gamma,\alpha)}{\partial\beta}}{\frac{\partial G(\tau,\delta,\beta,m,\gamma,\alpha)}{\partial\tau}} = -\frac{(1-\delta)c'(\tau)}{-(2\alpha-1)\frac{\delta\gamma m}{4\tau^2}-(1-\beta(1-\delta))c''(\tau)} > 0\\
\frac{\partial\tau}{\partial\alpha} &= -\frac{\frac{\partial G(\tau,\delta,\beta,m,\gamma,\alpha)}{\partial\alpha}}{\frac{\partial G(\tau,\delta,\beta,m,\gamma,\alpha)}{\partial\tau}} = -\frac{\frac{\delta\gamma m}{2\tau}}{-(2\alpha-1)\frac{\delta\gamma m}{4\tau^2}-(1-\beta(1-\delta))c''(\tau)} > 0\\
\frac{\partial\tau}{\partial\delta} &= -\frac{\frac{\partial G(\tau,\delta,\beta,m,\gamma,\alpha)}{\partial\delta}}{\frac{\partial G(\tau,\delta,\beta,m,\gamma,\alpha)}{\partial\tau}} = -\frac{(2\alpha-1)\frac{\gamma m}{4\tau}-\beta c'(\tau)}{-(2\alpha-1)\frac{\delta\gamma m}{4\tau^2}-(1-\beta(1-\delta))c''(\tau)} > 0
\end{aligned}$$

Thus, we conclude that the steady state value τ is increasing in m, γ β and α, as stated in Proposition 2(ii). Moreover, τ is increasing in δ iff $(2\alpha-1)\frac{\gamma m}{4\tau} > \beta c'(\tau)$. Note that $(2\alpha-1)\frac{\gamma m}{4\tau} > \beta c'(\tau)$ is always satisfied. Using the steady state condition $(2\alpha-1)\frac{\delta\gamma m}{4\tau} = (1-\beta(1-\delta))c'(\tau)$, we get:

$$(2\alpha-1)\frac{\gamma m}{4\tau} = \frac{(1-\beta(1-\delta))c'(\tau)}{\delta} > \beta c'(\tau) \Leftrightarrow 1 > \beta$$

Thus, we get $\frac{\partial\tau}{\partial\delta} > 0$ iff $1 > \beta$, which is true by assumption. ■

5.3 Proof of Proposition 3

In a first step, we prove that if $T_{i,t-1} = T_{j,t-1} \equiv T_{t-1}$, then $\tau_{i,t} = \tau_{j,t} \equiv \tau_t$ for all $t \in \{0, ..., \infty\}$. Suppose that $T_{i,t-1} = T_{j,t-1}$ and $\tau_{i,t} > \tau_{j,t}$. Equation (4) implies

$$\kappa \equiv \frac{T_{j,t}}{T_{i,t}} = \frac{c'(\tau_{i,t}) - \beta c'(\tau_{i,t+1})(1-\delta)}{c'(\tau_{j,t}) - \beta c'(\tau_{j,t+1})(1-\delta)}.$$

Note that $\kappa < 1$ because $T_{i,t} > T_{j,t}$. Rewriting the last equation, we yield

$$\kappa \left[c'(\tau_{j,t}) - \beta c'(\tau_{j,t+1})(1-\delta)\right] = c'(\tau_{i,t}) - \beta c'(\tau_{i,t+1})(1-\delta). \tag{9}$$

Combining equation (9) with $\tau_{i,t} > \tau_{j,t}$ we conclude that $\tau_{i,t+1} > \tau_{j,t+1}$. This result implies that $T_{i,t+1} > T_{j,t+1}$, which itself implicates divergence of the state variables $T_{i,t}$ and $T_{j,t}$. It follows from this divergence of the state variables that the winning probabilities also diverge across clubs such that the transversality condition (positive expected discounted profits for both clubs) is violated. However, $\tau_{i,t} = \tau_{j,t}$ $(t = 0, ..., \infty)$ is consistent both with the combined Euler equation (7) and with the transversality condition. This result proves that if $T_{i,t-1} = T_{j,t-1} \equiv T_{t-1}$, then $\tau_{i,t} = \tau_{j,t} \equiv \tau_t$ for all $t \in \{0, ..., \infty\}$.

Using this result, we recursively conclude that both clubs invest an identical amount in each period. It directly follows that the state variables are also identical. Thus, under the restriction $T_{i,-1} = T_{j,-1}$, clubs' decisions are symmetric. This result is summarized in Proposition 3. ■

5.4 Derivation of the Phase Diagram

First, we investigate the dynamics of τ_t. Combining equation (4) with the results of Proposition 3, yields

$$(2\alpha - 1)\frac{\gamma m}{4T_t} = c'(\tau_t) - \beta(1-\delta)c'(\tau_{t+1}).$$

We note that $\Delta\tau_{t+1} = \tau_{t+1} - \tau_t = 0$ if $(2\alpha - 1)\frac{\gamma m}{4T_t} = (1 - \beta(1-\delta))c'(\tau_t)$. Therefore, we get a decreasing function in the $(\tau, T)-$space if $\Delta\tau = 0$. This curve is represented in Figure 1. In the northeast of this curve, it holds that $\Delta\tau_{t+1} > 0$ such that τ increases. In the southwest of this curve, it holds that $\Delta\tau_{t+1} < 0$ and τ decreases. The directions of motion are summarized by the vertical arrows in Figure 1.

Second, we investigate the dynamics of T_t. The talent accumulation equation combined with Proposition 3 implies that $T_t = (1-\delta)T_{t-1} + \tau_t \Leftrightarrow \Delta T_t = -\delta T_{t-1} + \tau_t$. Hence, $\Delta T_t = 0$

if $\tau_t = \delta T_t$. Note that $\tau_t = \delta T_t$ is also represented in Figure 1. In the southeast of this curve, it holds that $\Delta T_t < 0$, whereas in the northwest it holds that $\Delta T_t > 0$. Once again, the horizontal arrows indicate the directions of motion in the $(\tau, T)-$space.

5.5 Extension: n-Club League

In this extension, we consider a league with $n > 2$ clubs and show that many results from the two-club league still hold. We allow for heterogeneous clubs with respect to initial endowments but assume that clubs have identical market sizes, i.e., $m_i = m$ for all $i \in \{1, ..., n\}$. Moreover, we consider a league without revenue sharing, i.e., $\alpha = 1$, because we focus on the effect of more clubs.

The win percentage of club $i \in \{1, ..., n\}$ is now defined as

$$w_i = \frac{n}{2} \frac{T_{i,t}^{\gamma}}{\sum_{j=1}^{n} T_{j,t}^{\gamma}}$$

We derive the following Euler equation for club $i \in \{1, ..., n\}$

$$\frac{n}{2} \frac{\gamma m T_{i,t}^{\gamma-1} \sum_{j \neq i} T_{j,t}^{\gamma}}{\left(\sum_{j=1}^{n} T_{j,t}^{\gamma}\right)^2} = c'(\tau_{i,t}) - \beta(1-\delta)c'(\tau_{i,t+1}),$$

with $t \in \{0, ..., \infty\}$. As in the two-club league, in the steady state it holds that $T_i = T$ and $\tau_i = \tau = \delta T$ for all $i \in \{1, ..., n\}$. Moreover, we obtain the following implicit function for the talent stock in the steady state

$$\frac{n-1}{n} \frac{\gamma m}{2T} = [1 - \beta(1-\delta)]c'(\delta T).$$

It is easy to see that the talent stock in the steady state is increasing with the number of clubs in the league.

In the following, we specify the cost function by assuming linear costs $c(\tau_{i,t}) = \theta \tau_{i,t}$. In this case, we are able to determine the transitional path of the talent stocks. As in the two-club league, the steady state is immediately attained in the first period, regardless of initial endowments of talent stock. The steady state is given by

$$T_t = \frac{n-1}{n} \frac{\gamma m}{2[1-\beta(1-\delta)]\theta} = T$$

with $t \in \{0, ..., \infty\}$. According to the last equation, we derive that a higher number of clubs in the league also increases the talent stock in each period.

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