The Relationship between Real Interest Rates and Inflation

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Abstract

In the recent decade, a huge amount of papers, describing monetary policy rules based on nominal interest rates, has been written. As it is, however, well known, it is in fact the real and not the nominal interest rate, that can influence spending decisions of enterprises and households and thus inflation. One way, to describe the relationship between real interest rates and inflation, is based on our experience with the monetary theory of the price level. The quantity theory of money can be used under certain assumptions as a good description of the long-run relationship between money and prices. In this respect the best known empirical application is probably the P-star model of Hallman, Porter and Small (1991).

In this paper we use two simple descriptions of the long run link between real interest rates and inflation, and subsequently test their empirical performance, using similar techniques as employed in P-star modeling. In an empirical study, based on cointegration analysis, we show that the gap between the real and natural rate of interest does not determine inflation, as it is often postulated, but its growth rate. We find that this relationship describes reasonably well the long run influence of the interest rate gap on inflation. Simultaneously we calculate the average natural rate of interest.

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1 Introduction

For many years money has been a central issue in monetary policy making. Central banks used to set monetary targets and academics used to teach monetary policy, as a story about how central bankers adjusts the money supply. Even the name of the main activity of central banks took its origins from the word "money". Thus, it is no wonder that many economic papers describing inflationary phenomena still assume that central banks control the money supply.

Thanks to its important role in monetary policy, a lot of research has been done on testing the long-run relationship between money and inflation. The probably best known study, is based on the quantity theory of money, and called P-star[1]. The model shows that the quantity equation, being a very simplified description of the relationship between money and prices, can be used for monetary targeting and inflation forecasting, provided that some additional assumptions are fulfilled. The most important one is related to the long-run stability (or at least predictability) of velocity. A positive verification of the quantity equation states that there is a long-run path for the general price level, determined by the quantity of money that the actual price level is cointegrated with.

However, the world is changing, and targeting monetary aggregates becomes less and less fashionable. The main reason is probably the growing instability of money demand functions. In reaction, monetary authorities move from targeting the money supply towards controlling nominal interest rates at the money market. As a result, in the recent decade, a huge amount of papers, describing monetary policy rules based on nominal interest rates, has been written. As it is, however, well known, assuming there is no money illusion, it is in fact the real and not the nominal interest rate, that can influence spending decisions of enterprises and households. Monetary authorities can alter real rates (at least in the short run) as long as prices and inflationary expectations are sticky[2]. Thus, it is crucial for a central banker not only to look at the level of nominal interest rates, but also to monitor the behaviour of real rates.

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[1] See J. Hallman, R.Porter, D.Small (1991), Deutsche Bundesbank (1991) or M.Brzoza-Brzezina, J.Kotłowski (2001).

[2] A simple, but comprehensive description of these mechanisms is given by A.Blinder (1998).

Despite the growing importance of interest rate oriented policies, our knowledge on this topic is still unsatisfactory. The first approach to describe the relationship between real interest rates and inflation is often ascribed to K.Wicksell (1898, 1907). However already 100 years earlier, two British economists, H.Thornton and T.Joplin, described economic processes resulting from the central bank's influence on the real rate of interest (T.M.Humphrey 1993). Nevertheless, not much has been done in this field since. Recent papers, among others by M.Woodford (1999, 2000), revived the (now called) Wicksellian idea of inflationary processes being determined by the gap between the real and natural[3] rates of interest. In a very recent study K.Neiss and E.Nelson (2001) use a stochastic general equilibrium model to examine the properties of the interest rate gap as an inflation indicator. The above mentioned studies are strongly in favour of using the gap as a measure of the stance of monetary policy that could be used by central bankers in their day-to-day (or rather month-to-month) policy setting.

This paper aims to test, whether a simple equation, of the form introduced to the economic literature by the quantity theory, can be found and empirically verified for the long-run relationship between the real interest rate gap and inflation. In other words, we will check whether there is a long-run path for inflation, determined by the interest rate gaps that actual inflation is cointegrated with. When describing the relationship, we will naturally ignore short-run dynamics, and the influence of external shocks, which should be the reasons for temporary divergences between actual and equilibrium inflation rates.

A brief look into the related literature, in search for an appropriate equation, reveals that at least 2 different specifications should be considered. In some descriptions a closed gap (real interest rate equal to the natural one) results in a stable price level, in others in stable inflation. A detailed description of the two models will be presented in section 2 of the paper.

The rest of the paper is structured as follows. In section 2, two models are described, one relating the interest rate gap to inflation, the other one - to its growth rate.

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[3] The natural rate of interest is sometimes called „the neutral rate", see A.Blinder (1998), Economist (1999), E.Retting (1999).

Empirical verification of the models finds place in section 3 of the paper. As we are looking for long-run relationships, cointegration analysis will be used for assessment, if any of the models fits the data. The estimation results will allow us calculate among others the average natural rate of interest for the US. Section 4 concludes, and an appendix presents some of the empirical results in detail.

2 The models

As it has already been noted, various descriptions (definitions) of the natural rate of interest can be met across economic papers. This section describes two most frequently postulated models that relate the interest rate gap to inflation and to its growth rate respectively. It should be noted that these descriptions can be equally treated as definitions of the natural rate. **Thus, we are not going to check whether so defined natural rates exist (because they do by definition), but whether any of them is stable enough to become a useful benchmark for monetary policy.** In what follows we will verify the existence of the two natural rates under the identifying assumption of constancy[4].

2.1 Model 1: interest rate gap as a determinant of inflation

The basic property of the model described in this section is that the gap between the real and natural rates of interest determines, after all lags have worked themselves out, the rate of inflation[5]. This kind of long-run relationship can be described by means of a simplified equation:

$$(1) \qquad \pi_{t+1} \equiv p_{t+1} - p_t = \psi(r^* - r_t) \qquad \psi > 0,$$

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[4] Useful as it is for our purpose, this identifying assumption cannot be satisfying as a thorough research on the natural rate. However, I hope that a paper devoted exclusively to estimating the historical time series of the NRI using the Blanchard-Quah method will proceed soon.

[5] This kind of relationship is what K.Wicksell (1907) probably thought about the influence of the interest rate gap on inflation (see J.Amato (2001)). A similar equation (although in forward-looking form) is presented by M.Woodford (1999, pp. 40-41) as solution to a general equilibrium model. See also W.Kerr and R.King (1996) for a broad discussion of various systems of macroeconomic equations. Note that this relationship (as well as equation 6) can be considered as a definition of the natural rate.

where π is the inflation rate, p the log price level, r^* the natural rate of interest[6] and r the real rate of interest. It is worth noting that a popular description[7] of the relationship between the interest rate gap and inflation, of the form $\pi_t = \alpha\pi_{t-1} + \psi(r^* - r_t)$, $0 < \alpha < 1$ exhibits the same steady state properties as equation (1):

- when the interest rate gap is closed, inflation is zero and prices are stable. Loose monetary policy ($r^* > r$) will eventually cause inflation, restrictive monetary policy ($r^* < r$) will induce deflation[8].
- permanently higher rates of inflation are related to a permanently lower real rate of interest[9] (assuming that the natural rate is quite stable, as has been postulated by K.Neiss and E.Nelson 2001).

Table 1. Properties of model 1

	Model 1
	$\pi \equiv \Delta p = \psi(r^* - r)$
r=r*	p = const.
	$\pi = 0$
	$\Delta\pi = 0$
r>r*	p $\downarrow$
	$\pi < 0$
	$\Delta\pi = 0$
r<r*	p $\uparrow$
	$\pi > 0$
	$\Delta\pi = 0$

[6] Throughout the rest of the paper it will be assumed that the natural rate of interest is constant. Although this is certainly not true (it depends among others on the marginal product of capital and on the subjective discount rate of agents), our assumption will be based on the empirical result of K.Neiss and E.Nelson (2001), who found that the variance of r^* is much smaller than the variance of r. This means that the interest rate gap is to a great extent determined by changes in r and thus r^* can be assumed for simplicity constant.

[7] See for instance the model estimated by K.Neiss and E.Nelson (2001, p. 30-32).

[8] I am however, fully aware that measuring the stance of monetary policy relying only on interest rates is a simplified view of the central bank business.

[9] This seems to be in line with empirical studies done among others by R.King and M.W.Watson (1992, 1997).

Thus the model 1 economy works like a car driven by the central banker. When he presses the accelerator (i.e. opens the interest rate gap: $r^*>r$), the car goes faster (i.e. inflation picks up), when he puts the gear stick into neutral (i.e. closes the gap: $r^*=r$) the car will start slowing down until it stops (i.e. inflation falls to zero). Stable speed (stable inflation) necessitates a permanently pressed accelerator (permanently open interest rate gap). Basic properties of model 1 are presented in table 1.

To prepare the model for empirical analysis, some transformations will have to be performed. This is because, as it has already been noted, this paper is to describe the long run equilibrium and uses cointegration analysis. As the interest rate gap is expected to be stationary[10], equation (1) has to be transformed one level of integration "upwards", to allow for order 1 integration of the variables. This conclusion is a result of the model's theoretical specification. Empirical integration tests will be presented in section 3. The price level can be calculated from the definition of π:

$$(2) \qquad p_t \equiv p_{t-1} + \pi_t = p_0 + \sum_{i=1}^{t} \pi_i \ ,$$

and yields after substituting from equation (1):

$$(3) \qquad p_t = p_0 + \psi \sum_{i=0}^{t-1} (r^* - r_i) \ .$$

Before empirical analysis is conducted, one more fact has to be noted. Equation (3) has been postulated for a stationary economy, with a constant level of potential output. However, as it is widely accepted, the permanent growth of potential output will *ceteris paribus* lower the general price level. This fact is for example incorporated into the QTM equation through the presence of Y. Accordingly our model has to be enlarged by the potential output growth impact on prices[11]:

[10] The main reason is arbitrage between investment in financial instruments (return r) and physical capital (return $f'(k)=r^*$).

[11] For simplicity we ignore the relationship between potential output (especially between the productivity growth rate) and the natural rate of interest.

(4) $$p_t = p_0 + \psi \sum_{i=0}^{t-1} (r^* - r_i) - y_t^* .$$

Another important assumption has to be made in order to empirically estimate the equation. As the natural rate of interest is not observable, we will assume that it is a stationary variable and that its variance is small as related to the variance of the real interest rate[12]. This allows us keeping r^* constant and taking it out from below the sum:

(5) $$p_t = p_0 + \psi \cdot t \cdot r^* - \psi \sum_{i=0}^{t-1} r_i - y_t^* ,$$

where t denotes the time trend. An implicit message of equation (5) is that the general price level will depend on the whole history of interest rate gaps. This specification can be subject to cointegration analysis that will be presented in section 3.

2.2 Model 2: interest rate gap as a determinant of inflation growth

The basic property of the model described in this section, will be that the gap between the real and natural rates of interest determines, after all lags have worked themselves out, the change of the inflation rate[13]. This kind of long-run relationship can be described by means of a simplified equation:

(6) $$\Delta\pi_t = \psi (r^* - r_{t-1}) .$$

This model exhibits the following properties:

- when the interest rate gap is closed, inflation growth is zero and inflation is stable. Loose monetary policy ($r^*>r$) will start the process of inflation acceleration, restrictive monetary policy ($r^*<r$) will reduce inflation. The bigger the gap the faster will the inflation rate change.

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[12] This result has been described by E.Nelson and K.Neiss (2001).

[13] This kind of relationship has been advocated among others by J.C.Fuhrer and G.R.Moore (1995), T.Henckel, A.Ize and A.Kovanen (1999) as well as J.Andres, R.Mestre and J.Valles (1997).

- if the central bank wants to lower inflation, it has to raise interest rates to open the gap on the restrictive side, and wait for inflation to fall to the desired level. Once this has happened, the gap should be closed again. An undeniable advantage of model 2, is its accordance with the principle of only short run influence of the central bank on the real rate.

The model 2 economy works like a spacecraft driven by the central banker. When he presses the accelerator (i.e. opens the interest rate gap: $r^* > r$), the spacecraft goes faster (i.e. inflation rises), but once the engines are turned off (i.e. the gap is closed: $r^* = r$) the shuttle will fly at a constant speed (i.e. inflation will stay stable).

In this specification expectations are the (implicit) driving force behind inflation persistence. If inflation (for whatever reason) stabilized at a certain level, rational agents observe the behavior of the central bank. If they do not see any sign of policy tightening (i.e. opening the gap) they expect inflation not to change in the next period and thus increase wages and prices by the inflation rate. Basic properties of model 2 are presented in table 2.

Table 2. Properties of model 2.

	Model 2
	$\Delta\pi = \psi(r^* - r)$
r=r*	p = ?
	π = const.
	$\Delta\pi = 0$
r>r*	p = ?
	π ↓
	$\Delta\pi < 0$
r<r*	p = ?
	π ↑
	$\Delta\pi > 0$

As before, the model has to be transformed to the cointegrating representation. As with respect to model 1, it is assumed that the interest rate gap is a stationary variable. This implies that the model has to be transformed one level of integration „upwards":

$$(7)\quad \pi_t = \pi_0 + \psi \sum_{i=0}^{t-1} (r^* - r_i)\,.$$

Proceeding further as in case of equations (3) and (4), the model is enlarged by the influence potential output exerts on inflation. Additionally, we keep r^* constant as in equation (5):

$$(8)\quad \pi_t = \pi_0 + \psi \cdot t \cdot r^* - \psi \sum_{i=0}^{t-1} r_i - \Delta y_t^*\,.$$

This equation can be subject to cointegration analysis.

3 Empirical results

In what follows we will use cointegration analysis[14], to verify whether any of the described models is a reasonable description of the long run relationship between the real interest rate gap and inflation (or its growth rate).

3.1 The data

As long run relationships will be tested, it is desirable to work with respectively long data series[15]. The exchange rate regime being another obstacle, only a few countries in the world have such long and reliable time series. In this situation the US seem to be a good candidate. In this big, open economy, the influence of exchange rate fluctuations has only a small impact on the domestic price level. As an additional advantage, contrary to the European countries, the US has had a floating exchange rate regime since it abandoned the Bretton Woods system in the early 70's. For empirical studies, half-yearly data for the period 1954-1999 was used. Following raw data series were utilised:

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[14] All the econometric tools used in this paper have been described in detail in M.Brzoza-Brzezina, J.Kotłowski (2001).

[15] In a recent study D.Hendry (1999) used time series starting in 1874 to asses the impact money has on prices. Nevertheless he complains about not being able to reach deeper.

Table 3. Raw data

Variable	Description
CPI	Consumer price index (fixed basis)
DEF	GDP deflator (fixed basis)
FED	Federal funds rate
TBOND5	Yield on 5 year T-bonds
CPIEX	Expected inflation (Livingstone)
GDP	Real GDP (fixed prices)

In the next step auxiliary time series were constructed, containing the data for testing equations (5) and (8). Nominal interest rates were deflated[16] with inflationary expectations from Livingstone polls[17]. Potential output was estimated by means of the Hoddrick-Prescott filter[18].

Table 4. Data used in the models

Variable	Transformation
LCPI	ln (CPI)
LDEF	ln (DEF)
RFEDFUND	(1+FED)/(1+CPIEX)
RTBOND5	(1+TBOND5)/(1+CPIEX)
RFEDFUNDSUM	Σ ln (RFEDFUND)
RTBOND5SUM	Σ ln (RTBOND5)
LGDP	ln (GDP)
LGDPTREND	HP filter (100) (LGDP)

[16] Calculation of real interest rates is not an easy or straightforward task. The method employed is only one of the possibilities. For more information on calculating real rates see ECB 1999, pp. 16-18, and N.Anderson, J.Sleath 2001.

[17] Data from Federal Reserve Bank of Minneapolis.

[18] This is certainly a shortcut and it could be interesting to see how the models behave under alternative specifications of potential output. However, in our study the multiplicity of models to estimate would have been overwhelming.

3.2 Integration tests

For integration analysis two tests have been chosen: Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP). The lag order for ADF has been chosen to account for autocorrelation of residuals, and for PP according to the Newey-West criterion, which pointed at 3 lags in our case. The results[19] are presented in table 5.

Table 5. Unit root tests

Variable	Lag order in ADF test	ADF statistics without constant	ADF statistics with constant	PP statistics without constant	PP statistics with constant
LCPI	4		0,09		-0,17
d(LCPI)	3	-1,56	-2,96**	-1,42	-2,97**
dd(LCPI)	3	-6,18***	-6,15***	-9,20***	-9,15***
d(LDEF)	0	-1,02	-1,94	-1,06	-2,06
dd(LDEF)	0	-9,43***	-9,38***	-9,43***	-9,38***
RFEDFUNDSUM	1	2,41	-0,03	7,21	0,58
d(RFEDFUNDSUM)	1	-1,08*	-3,41**	-1,85*	-3,65***
dd(RFEDFUNDSUM)	1	-6,04***	-6,01***	-9,50***	-9,45***
RTBOND5SUM	2		0,53		1,15
d(RTBOND5SUM)	2	-1,06	-3,27**	-1,02	-2,88*
dd(RTBOND5SUM)	1	-6,54***	-6,51***	-7,75***	-7,70***
LGDPTREND	4		-0,91		
d(LGDPTREND)	4	-0,15	-2,59*	-0,23	-1,87
dd(LGDPTREND)	3	-4,33***	-4,31***	-2,88***	-2,88*

* denotes rejection of H_0 at 10%.

** denotes rejection of H_0 at 5%.

*** denotes rejection of H_0 at 1%.

The analysis of the results is not an easy task. According to the unit root tests, it cannot be unambiguously decided, what the level of integration of most variables is. The inflation rate may be stationary, if measured with CPI, or I(1), if measured with the GDP deflator. However

[19] ADF and PP critical values come from R.Davidson, J.MacKinnon (1993).

it has to be born in mind that the sample incorporates the 1973-79 period, when oil price shocks induced a huge rise in inflation rates. Their influence can result in imprecise test results that may overstate the number of unit roots. Taking this additional handicap into account, it cannot be said precisely, whether the general price level in the US is I(1) or I(2).

Similar conclusions can be drawn for real interest rates and for potential output. According to our estimates real interest rates are probably stationary, although there is a possibility that they are I(1). Potential output is probably an I(2) variable, but even this cannot be stated definitely.

The unit root tests disappoint not only because they impede the choice of appropriate econometric tools for data analysis. It is worth noting that knowing with certainty the integration level of prices could help eliminating the wrong model without further estimation. As it has been noted before, economic theory predicts that the interest rate gap should be a stationary variable. It follows from equation (1) that for model 1 to comply with the data, the LHS should also be I(0), which means that inflation should be stationary. In contrary, for model 2 to be consistent with the data set, inflation should be I(1), and its growth rate $\Delta\pi$ stationary. However, as the real level of integration of the price level cannot be stated unambiguously from table 5, none of the models can be rejected on the basis of unit root tests. The integration level of the GDP deflator points at model 2, but the results obtained for the CPI can be compliant with both models. It does not however seem to be reasonable to distinguish the models assigning each of them „its" respective price index. The price indices are only imperfect approximations of what economists call „the general price level" and which should be explained by the theory, and so the ambiguity should be rather explained as a result of imperfection of the indices or low power of integration tests.

3.3 Cointegration tests - model 1

As it has been previously noted, this paper aims at finding the long-run relationship between the interest rate gap and inflation (or its growth rate). Thus appropriate econometric tools have to be chosen, that can test for cointegration in economic systems. One of the possible techniques, and probably the most popular one at present, has been proposed by Johansen (1991). In our case, it will be based on the basis of a Vector Error Correction Model (VECM) build for three variables, the price level (p), the sum of all previous real interest rates ($\sum_{i=0}^{t-1} r_i$), and potential output (y^*). In what follows, a short description, of what will be done further in this section is presented:

1. The lag order for the VECM will be specified on the basis of information criteria and sequential test.
2. The cointegrating relationship for equation (5) will be found. This means finding a vector [1, ψ, 1] with time trend and constant such, that the residuals ε from equation (9) are stationary.

$$p_t = p_0 + \psi \cdot t \cdot r^* - \psi \sum_{i=0}^{t-1} r_i - y_t^* + \varepsilon_t , \qquad (9)$$

3. The signs of vector components will be verified to comply with the theoretical model.
4. The parameter on y^* will be restricted to 1 (as in the theoretical model), the validity of restriction will be tested.
5. The average natural rate of interest will be calculated and compared to estimates from other papers.
6. The error correction adjustment coefficients will be tested to show whether the causality is compliant with theory (the only significant error correction mechanism should obtain in the inflation equation)

Only models passing all stages of verification will be considered a proof for the existence of a long run relationship, connecting the price level to the sum of interest rate gaps. Of course if the analysis breaks down at some point (for instance no cointegrating relationships will be found), subsequent steps will be cancelled.

According to the unit root tests, from the formal point of view, CPI is the only price index that can be used for empirical tests of this model. This is because the GDP deflator is an I(2) variable and, as such, cannot be consistently included into the system. Moreover, it has been decided, that two different data sets will be used, a long sample (1954-1999) and a short one (1972-1999). This is because of the exchange rate regime change in the early 70's[20], which could have caused important changes in the way, in which interest rates transmit to prices.

[20] More on the breakdown of the Bretton Woods regime see in H.R.Wüffli (1979).

Both interest rates, the federal funds rate and the 5 year T-bond rate were included. In addition, as it was difficult to decide how long one lag is (interest rates enter equation (9) with a lag), two different approaches were used, where the lag was interpreted as half a year and one year, respectively. Therefore, 8 different models were tested, based on:

- 1 price index,
- 2 data samples,
- 2 interest rates,
- 2 different lags.

In the first step, the lag order of the VAR model was determined, based on the sequential test (LR), Akaike information criterion (AIC), Schwarz criterion (SC) and Hannan-Quinn criterion (HQ)[21]. It was arbitrarily assumed that the maximal lag order should not exceed 6 and than the lag indicated by most criteria was chosen. In case of an ambiguous result, the smaller lag was chosen, provided that the VECM residuals did not show autocorrelation. The results are presented in Appendix 1.

As the next step, cointegrating relationships were searched. Table 6 contains the maximum eigenvalue and trace test statistics for the 8 models[22]. According to equation (9), a constant and time trend have been included.

As it can be seen, for the long sample, both tests rejected the hypothesis of 2 in favour of 3 cointegrating vectors, which would normally indicate that the variables are stationary. As they certainly are not, this case will not be examined any further.

Table 6. Cointegration tests - model 1

Variables	Hypothesis		Trace test statistics		Maximum eigenvalue statistics	
	H_0	H_1	1954-1999	1972-1999	1954-1999	1972-1999
LCPI;	r = 0	r = 1 (r ≥ 1)	52,76**	57,07**	23,71	30,86**
RFEDFUNDSUM(-1);	r = 1	r = 2 (r ≥ 2)	29,04*	26,21*	16,15	14,12
LGDPTREND	r = 2	r = 3	12,89*	12,08	12,89*	12,08

[21] The criteria are described in detail in H.Lütkepohl (1995).

[22] Critical values come from M.Osterwald-Lenum (1992).

LCPI; RFEDFUNDSUM(-2); LGDPTREND	r = 0 r = 1 r = 2	r = 1 (r ≥ 1) r = 2 (r ≥ 2) r = 3	63,24** 30,28* 13,74*	64,69** 23,99 9,76	32,95** 16,54 13,74*	40,80** 14,44 9,76
LCPI; RTBOND5SUM(-1); LGDPTREND	r = 0 r = 1 r = 2	r = 1 (r ≥ 1) r = 2 (r ≥ 2) r = 3	63,74** 32,66** 13,60*	55,74** 26,75* 9,30	31,08** 19,05* 13,60*	28,98* 17,44 9,30
LCPI; RTBOND5SUM(-2); LGDPTREND	r = 0 r = 1 r = 2	r = 1 (r ≥ 1) r = 2 (r ≥ 2) r = 3	72,29** 36,10** 13,26*	57,94** 26,09* 10,82	36,18** 22,80* 13,26*	31,85** 15,27 10,82

r denotes the number of cointegrating vectors

* denotes rejection of H_0 at 5%.

** denotes rejection of H_0 at 1%.

As regards the short sub-sample, it seems possible to find a long run relationship in the data. In all four cases, the maximum eigenvalue test indicated one cointegrating vector, whereby the trace statistic indicated one or two vectors. In what follows, the estimated cointegrating relationships will be presented. In all cases the existence of one vector has been assumed. Table 7 presents the results:

Table 7. Cointegrating vectors[23]

	RFEDFUNDSUM(-1)	RFEDFUNDSUM(-2)	RTBOND5SUM(-1)	RTBONDSUM(-2)
LCPI	1,00	1,00	1,00	1,00
LGDPTREND	10,77	12,44	8,63	7,60
Interest rate	**-0,71**	**-0,46**	**-0,67**	**-0,05**
TREND	-0,17	-0,26	-0,13	-0,14
CONSTANT	-83,57	-96,47	-67,44	-60,06

As it can be easily noticed, all four relationships fail to fulfil the criterion on coefficient signs. In all cases the elements of the cointegrating vector standing with the sum of interest rates are negative, which implies a positive relationship between real rates and inflation and thus contradicts the theoretical model.

In this respect, further analysis of model 1 seems purposeless, and will be given up, making room for the empirical verification of model 2.

[23] In the following tables numbers in bold denote results not compliant with the model.

3.4 Cointegration tests - model 2

As before, Johansen tests will be used for the analysis of cointegrating relationships and the 6-step procedure will be adopted:

1. The lag order for the VECM will be specified on the basis of the sequential test and information criteria.
2. The cointegrating relationship for equation (8) will be found. This means finding a vector [1, ψ, 1] with time trend and constant such, that the residuals ε from equation (10) are stationary.

$$\pi_t = \pi_0 + \psi \cdot t \cdot r^* - \psi \sum_{i=0}^{t-1} r_i - \Delta y_t^* + \varepsilon_t \ . \tag{10}$$

3. The signs of vector components will be verified to comply with the theoretical model.
4. The average natural rate of interest will be calculated, and compared to estimates from other papers.
5. The error correction adjustment coefficients will be tested to show whether the causality is compliant with theory (the only significant error correction mechanism should obtain in the inflation equation).
6. The parameter on y^* will be restricted to 1 (as in the theoretical model), the validity of restriction will be tested.

As before, 2 different measures of interest rates were taken, and the tests were conducted separately for 2 data samples. As earlier, two different lag structures have been adopted, but in contrary to model 1, two different measures of the price level could be introduced. This is because for cointegration analysis of equation (10) inflation has to be an I(1) variable, and as it can be seen from table 5, according to unit root tests, both the CPI level and the GDP deflator can be integrated of order 2, which means that the inflation measures can be I(1). Thus model 2 will be tested in 16 cases, consisting of:

- 2 price indices,
- 2 data samples,
- 2 interest rates,
- 2 different lags.

Table 8. Cointegration tests - model 2

Variables	Hypothesis H_0	H_1	Trace test statistics 1954-1999	Trace test statistics 1972-1999	Maximum eigenvalue statistics 1954-1999	Maximum eigenvalue statistics 1972-1999
D(LCPI);	r = 0	r = 1 (r ≥ 1)	56,16**	43,94*	39,79**	24,30
RFEDFUNDSUM(-1);	r = 1	r = 2 (r ≥ 2)	16,36	19,63	13,78	10,13
LGDPTREND	r = 2	r = 3	2,58	9,50	2,58	9,50
D(LCPI);	r = 0	r = 1 (r ≥ 1)	41,74	38,60	25,22	19,58
RFEDFUNDSUM(-2);	r = 1	r = 2 (r ≥ 2)	16,52	19,02	13,65	11,69
LGDPTREND	r = 2	r = 3	2,877	7,32	2,87	7,32
D(LCPI);	r = 0	r = 1 (r ≥ 1)	61,36**	39,87	43,03**	20,72
RTBOND5SUM(-1);	r = 1	r = 2 (r ≥ 2)	18,32	19,15	14,80	12,42
LGDPTREND	r = 2	r = 3	3,52	6,72	3,52	6,72
D(LCPI);	r = 0	r = 1 (r ≥ 1)	50,17**	39,94	36,31**	28,39*
RTBOND5SUM(-2);	r = 1	r = 2 (r ≥ 2)	13,85	11,54	9,62	7,42
LGDPTREND	r = 2	r = 3	4,23	4,12	4,23	4,12
D(LDEF);	r = 0	r = 1 (r ≥ 1)	44,97*	41,00	26,32*	20,02
RFEDFUNDSUM(-1);						
LGDPTREND	r = 1	r = 2 (r ≥ 2)	18,64	20,97	15,96	12,60
	r = 2	r = 3	2,68	8,37	2,68	8,37
D(LDEF);	r = 0	r = 1 (r ≥ 1)	41,07	35,59	25,29	20,90
RFEDFUNDSUM(-2);	r = 1	r = 2 (r ≥ 2)	15,782	14,68	12,22	9,22
LGDPTREND	r = 2	r = 3	3,55	5,46	3,55	5,46
D(LDEF);	r = 0	r = 1 (r ≥ 1)	52,08**	46,33*	31,40**	21,86
RTBOND5SUM(-1);	r = 1	r = 2 (r ≥ 2)	20,67	24,46	17,91	16,82
LGDPTREND	r = 2	r = 3	2,75	7,64	2,75	7,64
D(LDEF);	r = 0	r = 1 (r ≥ 1)	45,10*	38,66	29,94*	23,71
RTBOND5SUM(-2);	r = 1	r = 2 (r ≥ 2)	15,15	14,95	11,90	9,91
LGDPTREND	r = 2	r = 3	3,25	5,04	3,25	5,04

r denotes the number of cointegrating vectors.

* denotes rejection of H_0 at 5%.

** denotes rejection of H_0 at 1%.

As before, the lag order of the VECM augmentations, has been derived from four information criteria, which are presented in Appendix 1. Table 8 contains the outcome of cointegration tests.

Let us start describing the results with the cases based on the federal funds rate. As it can be seen only in 3 out of 8 cases, a cointegrating vector has been reported. This result seems poor, but it still might be worth taking a look at the estimated vectors (tab. 9).

Table 9. Cointegrating vectors for RFEDFUNDSUM

	D(LCPI) RFEDFUNDSUM(-1) 1954-1999	D(LDEF) RFEDFUNDSUM(-1) 1954-1999	D(LCPI) RFEDFUNDSUM(-1) 1972-1999
Inflation	1,0000	1,000	1,000
D(LGDPTREND)	**3,467910**	**3,855123**	2,774886
Interest rate	0,082327	0,059435	0,084818
TREND	-0,001953	-0,001304	-0,001931
CONSTANT	-0,134388	-0,129226	-0,131249
Error correction coefficient in the inflation equation (standard deviation)	-0,559009 (0,09282)	-0,232074 (0,08812)	-0,830450 (0,17431)
Implied average natural rate of interest	2,4%	2,2%	2,3%

Following the above-outlined steps of analysis, we can state that:

1. All parameter signs are as expected.

2. The natural rate of interest amounts to respectively 2,2%, 2,3% and 2,4% and thus is similar to other reported estimates (A.Blinder (1998)).

3. The error correction mechanism is quite strong, and significantly differs from zero.

4. The parameters standing with potential output amount to 3,46, 3,85 and 2,77, respectively.

Whether these numbers are significantly different from one, will be shown in table 10. The test has been proposed by Johansen and is based on the likelihood ratio statistics. It can be clearly seen from table 10 that only in one case the hypothesis of significant difference of the parameter from one could not be rejected at the 5% level.

Table 10. Testing the validity of restrictions imposed on the *y** parameter of the cointegrating vector.

Model	D(LCPI) RFEDFUNDSUM(-1) 1954-1999	D(LDEF) RFEDFUNDSUM (-1) 1954-1999	D(LCPI) RFEDFUNDSUM (-1) 1972-1999
Statistics	18,15566	10,33139	2,336606
p-value	**0,000020**	**0,001308**	0,126365

Let us now move to the cointegrating equations estimated with the long interest rate variable RTBOND5SUM. The 6 estimated vectors are presented in tables 11 and 12.

Table 11. Cointegrating vectors (1954-1999)

	D(LCPI) RTBOND5SUM(-1)	D(LCPI) RTBONDSUM(-2)	D(LDEF) RTBOND5SUM(-1)	D(LDEF) RTBONDSUM(-2)
Inflation	1,0000	1,0000	1,0000	1,0000
D(LGDPTREND)	1,373842	1,434585	**2,835506**	**2,544106**
Interest rate	0,083813	0,082602	0,053273	0,058477
TREND	-0,002541	-0,002505	-0,001500	-0,001664
CONSTANT	-0,091262	-0,089200	-0,102605	-0,098730
Error correction coefficient in the inflation equation (standard deviation)	-0,751238 (0,12032)	-0,609649 (0,14298)	-0,252958 (0,09064)	-0,277906 (0,10797)
Implied average natural rate of interest	3,0%	3,0%	2,8%	2,9%

Table 12. Cointegrating vectors (1972-1999)

	D(LCPI) RTBOND5SUM(-2)	D(LDEF) RTBOND5SUM(-1)
Inflation	1,0000	1,000
D(LGDPTREND)	1,506398	1,891945
Interest rate	0,049769	0,019799
TREND	-0,001236	-0,000112
CONSTANT	-0,092560	-0,096108
Error correction coefficient in the inflation equation (standard deviation)	-0,969133 (0,27029)	-0,688883 (0,15521)
Implied average natural rate of interest	2,5%	**0,6%**

Proceeding as before, it can be stated that:

1. In all six cases the signs of parameters are as expected,

2. With one exception, the natural rates of interest are of reasonable size (2.8-3.0%),

3. In all six cases there is significant error correction in the inflation equation,

4. As it can be seen from tables 13 and 14, in four cases the *y parameter does not significantly differ from one (at the 5% level).**

Table 13. Testing the validity of restrictions imposed on the *y** parameter of the cointegrating vector (1954-1999)

	D(LCPI) RTBOND5SUM(-1) 1954-1999	D(LCPI) RTBOND5SUM(-2) 1954-1999	D(LDEF) RTBOND5SUM(-1) 1954-1999	D(LDEF) RTBOND5SUM(-2) 1954-1999
Statistics	0,033257	0,697099	5,857772	6,678978
p-value	0,855297	0,403760	**0,015508**	**0,009756**

Table 14. Testing the validity of restrictions imposed on the *y** parameter of the cointegrating vector (1972-1999)

	D(LCPI) RTBOND5SUM(-2) 1972-1999	D(LDEF) RTBOND5SUM(-1) 1972-1999
Statistics	0,234689	0,539415
p-value	0,628068	0,462675

Summing up the results[24], it can be noted that from among sixteen cases under consideration four of the stated cointegrating relationships fulfilled all the criteria imposed on the model. Three successful models are based on the long term interest rate, whereby one includes the federal funds rate. It also seems important that in all four models with imposed restrictions the adjustment coefficients show a significant error correction mechanism in the inflation equation and no error correction in the two remaining equations. This implies a causal relationship of the type we would have expected, going from real interest rates to inflation. It thus can be said that the relationship described in model 2, linking inflation to the history of

[24] Though our unit root test results strongly support the I(2) result for trend GDP, it is often argued that this variable is I(1). If so, D(LGDPTREND) has to be excluded from the CV in Model 2. However, the results of such an exercise give comparable support to the concept described by Model 2 and place the natural rate of interest in the range 2.2-3.0% for the long rate and 1.5-1.9% for the federal funds rate.

real interest rate gaps and implicitly making the growth rate of inflation determined by the gap, describes the macroeconomic relationship between central bank instruments and inflation.

4 Conclusions

This paper aimed at testing, whether a simple equation, of the form introduced to the economic literature by the quantity theory, can be found and empirically verified for the long-run relationship between the real interest rate gap and inflation. Our results, based on cointegration analysis, show that such a stable long-run equation links the growth rate of inflation to the interest rate gap:

$$\Delta\pi_t = \psi(r^* - r_{t-1}) .$$

Thus, at least with respect to inflationary processes, the economy seems to work like a space shuttle, that once accelerated, will cruise at a stable speed without the use of engines. The central bank can open the interest rate gap to accelerate inflation, and once this has happened the gap can be closed and inflation will remain at the higher level. The natural rate of interest, although certainly not constant, is stable enough, to allow us determine the interest rate gap by means of changes in the real interest rate. Using the calculated average value of the natural rate we can easier guess what the current stance of monetary policy is.

This does not mean that estimating more precisely the natural rate of interest would not be helpful for monetary policy. We still do not know much about the behavior of this variable. Its determinants, among others the marginal product of capital, the productivity growth rate and the subjective discount rate of private agents are only hardly observable. With better estimates of the natural rate of interest, central banks could influence economic behavior with more precision. Disinflating countries would know, at what level to set real interest rates, after disinflation has been finished, without taking the risk of reflating the economy again.

As already mentioned, the adopted research technique only allowed us to calculate the average level of the natural rate; the actual time series is still unknown. There are various ways to proceed further. One possibility is to build a general equilibrium model of the economy, calibrate it and calculate the flexible price equilibrium level of real interest rates.

Another possibility is the use of advanced time-series techniques like the Kalman filter, to distinguish between permanent and temporary changes of real rates. The third solution could be based on introduction of the technique described by O.J.Blanchard and D.Quah (1989). This task however, will be left for another paper.

Appendix 1

The lag order chosen from the sequential test and information criteria:

Variables in the model	Sample	LR	AIC	SC	HQ	Choice
MODEL 1						
LCPI; RFEDFUNDSUM(-1); LGDPTREND	1954-1999	5	5	4	5	**5**
LCPI; RFEDFUNDSUM(-2); LGDPTREND	1954-1999	5	6	4	5	**5**
LCPI; RTBOND5SUM(-1); LGDPTREND	1954-1999	5	5	5	5	**5**
LCPI; RTBOND5SUM(-2); LGDPTREND	1954-1999	5	6	5	5	**5**
LCPI; RFEDFUNDSUM(-1); LGDPTREND	1972-1999	5	5	5	5	**5**
LCPI; RFEDFUNDSUM(-2); LGDPTREND	1972-1999	5	5	5	5	**5**
LCPI; RTBOND5SUM(-1); LGDPTREND	1972-1999	5	5	5	5	**5**
LCPI; RTBOND5SUM(-2); LGDPTREND	1972-1999	5	5	5	5	**5**
MODEL 2						
D(LCPI); RFEDFUNDSUM(-1); LGDPTREND	1954-1999	4	4	4	4	**4**
D(LCPI); RFEDFUNDSUM(-2); LGDPTREND	1954-1999	5	5	4	5	**5**
D(LCPI); RTBOND5SUM(-1); LGDPTREND	1954-1999	4	4	4	4	**4**
D(LCPI); RTBOND5SUM(-2); LGDPTREND	1954-1999	5	5	4	4	**4**
D(LDEF); RFEDFUNDSUM(-1); LGDPTREND	1954-1999	4	4	4	4	**4**
D(LDEF); RFEDFUNDSUM(-2); LGDPTREND	1954-1999	5	5	4	4	**4**
D(LDEF); RTBOND5SUM(-1); LGDPTREND	1954-1999	4	4	4	4	**4**
D(LDEF); RTBOND5SUM(-2); LGDPTREND	1954-1999	5	5	4	4	**4**
D(LCPI); RFEDFUNDSUM(-1); LGDPTREND	1972-1999	4	4	4	4	**4**
D(LCPI); RFEDFUNDSUM(-2); LGDPTREND	1972-1999	5	6	4	5	**5**
D(LCPI); RTBOND5SUM(-1); LGDPTREND	1972-1999	4	6	4	4	**4**
D(LCPI); RTBOND5SUM(-2); LGDPTREND	1972-1999	5	6	3	5	**5**
D(LDEF); RFEDFUNDSUM(-1); LGDPTREND	1972-1999	4	4	4	4	**4**
D(LDEF); RFEDFUNDSUM(-2); LGDPTREND	1972-1999	4	4	3	4	**4**
D(LDEF); RTBOND5SUM(-1); LGDPTREND	1972-1999	4	4	4	4	**4**
D(LDEF); RTBOND5SUM(-2); LGDPTREND	1972-1999	4	4	3	4	**4**

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